



Vector Algebra

Question1

Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}$, $\lambda > 0$, make an obtuse angle with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle θ , $\frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z -axis. If the set of all possible values of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to _____.

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Answer: 5

Solution:

$$\vec{a} \wedge \hat{k} \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$$

$$\frac{\lambda}{\sqrt{(2+1+\lambda^2)}} \in \left(0, \frac{\sqrt{3}}{2}\right) \Rightarrow 0 < \frac{\lambda}{\sqrt{3+\lambda^2}} < \frac{\sqrt{3}}{2}$$

$$\lambda > 0 \& \lambda \in (-3, 3) \Rightarrow \lambda \in (0, 3) \dots \dots \dots (1)$$

$\vec{a} \wedge \vec{b}$ is obtuse

$$\vec{a} \cdot \vec{b} < 0 \Rightarrow -\sqrt{2}\lambda^2 - 4\sqrt{2} + 4\sqrt{2}\lambda < 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 4 > 0 \Rightarrow (\lambda - 2)^2 > 0$$

$$\Rightarrow \lambda \in \mathbb{R} - \{2\} \dots \dots \dots (2)$$

$$(1) \& (2) \Rightarrow \lambda \in (0, 3) - \{2\} \Rightarrow \alpha + \beta + \gamma = 5$$

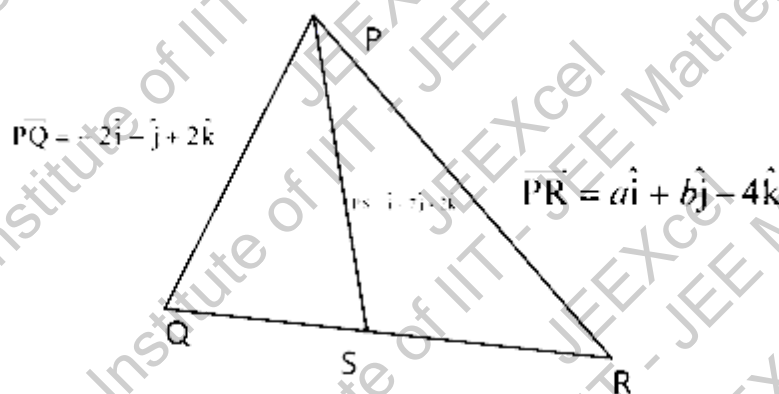
Question2

Let PQR be a triangle such that $\vec{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$, $a, b \in \mathbb{Z}$. Let S be the point on QR , which is equidistant from the lines PQ and PR . If $|\vec{PR}| = 9$ and $\vec{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is _____

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Answer: 37

Solution:



We are given that $|\vec{PR}| = 9$ Using the distance formula for the vector $PR = a\hat{i} + b\hat{j} - 4\hat{k}$:

$$\sqrt{a^2 + b^2 + (-4)^2} = 9$$

$$\Rightarrow a^2 + b^2 + 16 = 81$$

$$\Rightarrow a^2 + b^2 = 65$$

The problem states that point S lies on QR and is equidistant from lines PQ and PR . In a triangle, the locus of points equidistant from two sides is the angle bisector. Therefore, $\vec{PS}$ is the internal angle bisector of $\angle QPR$.

For an angle bisector $\vec{PS}$, it must be collinear with the sum of the unit vectors along the adjacent sides:

$$\vec{PS} = \lambda \left(\frac{\vec{PQ}}{|\vec{PQ}|} + \frac{\vec{PR}}{|\vec{PR}|} \right)$$

where λ is a scalar constant.

First, calculate $|\overrightarrow{PQ}|$:

$$|\overrightarrow{PQ}| = \sqrt{(-2)^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = 3$$

Now, set up the bisector equation :

$$\overrightarrow{PS} = \lambda \left(\frac{-2\hat{i} - \hat{j} + 2\hat{k}}{3} + \frac{a\hat{i} + b\hat{j} - 4\hat{k}}{9} \right)$$

Multiply the first term by $\frac{3}{3}$ to get a common denominator:

$$\overrightarrow{PS} = \frac{\lambda}{9} ((-6 + a)\hat{i} + (-3 + b)\hat{j} + (6 - 4)\hat{k})$$

$$\Rightarrow \overrightarrow{PS} = \frac{\lambda}{9} ((a - 6)\hat{i} + (b - 3)\hat{j} + 2\hat{k})$$

We are given

$$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$$

Comparing the $\hat{k}$ -components:

$$\Rightarrow \frac{\lambda}{9}(2) = 2 \Rightarrow \lambda = 9$$

Substitute $\lambda = 9$ into other components:

For $\hat{i}$:

$$a - 6 = 1 \Rightarrow a = 7$$

For $\hat{j}$:

$$b - 3 = -7 \Rightarrow b = -4$$

Check against the constraint :

$$a^2 + b^2 = 7^2 + (-4)^2 = 49 + 16 = 65$$

The values are consistent. value of $3a - 4b$:

$$3a - 4b = 3(7) - 4(-4) = 21 + 16 = \mathbf{37}$$

Question3

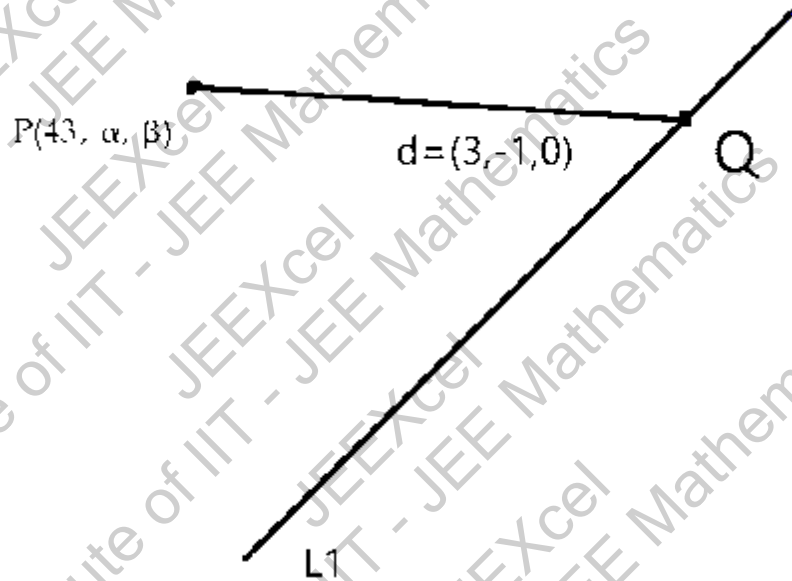
If the distance of the point $P(43, \alpha, \beta)$, $\beta < 0$, from the line

$\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, $\mu \in \mathbb{R}$ along a line with direction ratios $3, -1, 0$ is $13\sqrt{10}$, then $\alpha^2 + \beta^2$ is equal to _____

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Answer: 170

Solution:



- **Point P** : $(43, \alpha, \beta)$ with the constraint $\beta < 0$.
- **Target Line (L_1)** : Given by $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$.

A general point Q on this line is $(4 + 2\mu, 0, -1 + 3\mu)$.

- **Direction of measurement** : The distance is measured along a line with direction ratios $(3, -1, 0)$.

Find Point Q on the Target Line

Since the distance is measured along the direction $d = (3, -1, 0)$, the vector PQ must be parallel to d .

$$\begin{aligned} PQ &= Q - P = (4 + 2\mu - 43, 0 - \alpha, -1 + 3\mu - \beta) \\ PQ &= (2\mu - 39, -\alpha, 3\mu - \beta - 1) \\ PQ &= (2\mu - 39, -\alpha, 3\mu - \beta - 1) \end{aligned}$$

Because PQ is parallel to $(3, -1, 0)$, their components must be proportional :

$$1. \frac{2\mu - 39}{3} = \frac{-\alpha}{-1} \implies 2\mu - 39 = 3\alpha \implies 2\mu = 3\alpha + 39$$

$$\begin{aligned} 2. \frac{3\mu - \beta - 1}{0} &\text{ suggests the numerator must be zero for a finite ratio to exist:} \\ 3\mu - \beta - 1 &= 0 \implies \beta = 3\mu - 1 \end{aligned}$$

Calculate the Distance PQ

The distance PQ is given as $13\sqrt{10}$. Using the distance formula on the proportional components where the ratio is k :

$$PQ = k(3, -1, 0) = (3k, -k, 0)$$

$$|PQ| = \sqrt{(3k)^2 + (-k)^2 + 0^2} = \sqrt{10k^2} = |k|\sqrt{10}$$

Given $|PQ| = 13\sqrt{10}$, we find $|k| = 13$, so $k = \pm 13$.

Solve for α , μ , and β

From our proportionality equations :

- $-\alpha = -k \implies \alpha = k$

- $2\mu - 39 = 3k$

Case 1 : $k = 13$

- $\alpha = 13$

- $2\mu - 39 = 3(13) = 39 \implies 2\mu = 78 \implies \mu = 39$

- $\beta = 3\mu - 1 = 3(39) - 1 = 116$

- Constraint Check : $\beta < 0$ is False. (Reject this case)

Case 2 : $k = -13$

- $\alpha = -13$

- $2\mu - 39 = 3(-13) = -39 \implies 2\mu = 0 \implies \mu = 0$

- $\beta = 3(0) - 1 = -1$

- Constraint Check : $\beta < 0$ is True. (Accept this case)

$$\alpha^2 + \beta^2 = (-13)^2 + (-1)^2 = 169 + 1 = 170$$

The value of $\alpha^2 + \beta^2$ is 170.

Question4

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{c} \cdot (\vec{a} - 2\vec{b})$ is equal to _____.

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Answer: 3

Solution:

Let

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{j} - \hat{k}$$

We have to find

$$\vec{c} \cdot (\vec{a} - 2\vec{b})$$

given that

$$\vec{a} \times \vec{c} = \vec{b} \quad \text{and} \quad \vec{a} \cdot \vec{c} = 3$$

Let

$$\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$$

Then

$$\vec{a} = (1, 1, 1), \quad \vec{b} = (0, 1, -1)$$

Now compute $\vec{a} \times \vec{c}$:

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

Expanding,

$$\vec{a} \times \vec{c} = \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x)$$

Since this is equal to $\vec{b} = \hat{j} - \hat{k}$, we compare components:

$$z - y = 0$$

$$-(z - x) = 1 \Rightarrow x - z = 1$$

$$y - x = -1$$

Also, from $\vec{a} \cdot \vec{c} = 3$,

$$x + y + z = 3$$

Now solve the equations.

From $z - y = 0$,

$$z = y$$

From $x - z = 1$,

$$x = y + 1$$

Substitute into $x + y + z = 3$:

$$(y + 1) + y + y = 3$$

$$3y + 1 = 3$$

$$3y = 2 \Rightarrow y = \frac{2}{3}$$

So,

$$z = \frac{2}{3}, \quad x = \frac{5}{3}$$

Hence,

$$\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Now find $\vec{a} - 2\vec{b}$:

$$\begin{aligned}\vec{a} - 2\vec{b} &= (\hat{i} + \hat{j} + \hat{k}) - 2(\hat{j} - \hat{k}) \\ &= \hat{i} - \hat{j} + 3\hat{k}\end{aligned}$$

Therefore,

$$\begin{aligned}\vec{c} \cdot (\vec{a} - 2\vec{b}) &= \left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right) \cdot (1, -1, 3) \\ &= \frac{5}{3} - \frac{2}{3} + 2 \\ &= \frac{3}{3} + 2 = 1 + 2 = 3\end{aligned}$$

So, the required value is

3

Question 5

Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c} + \vec{d}| = \sqrt{29}$ and $\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$. If λ_1, λ_2 ($\lambda_1 > \lambda_2$) are the possible values of $(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$, then the equation $K^2x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to :

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Options:

A.

4

B.

-1

C.

2

D.

1

Answer: D

Solution:

$$|\vec{c} + \vec{d}| = \sqrt{29}$$

$$\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d} = 0$$

$$(\vec{c} + \vec{d}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow \vec{c} + \vec{d} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\Rightarrow \sqrt{29} = |\lambda|\sqrt{29}$$

$$\Rightarrow \lambda = \pm 1 \Rightarrow \vec{c} + \vec{d} = \pm(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\Rightarrow (\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k}) = \pm(-14 + 6 + 12) = \pm 4$$

$$\Rightarrow \lambda_1 = 4, \lambda_2 = -4$$

Given equation represents a circle

$$\Rightarrow K^2 = 3K + \frac{\lambda_2}{2} \text{ and } K^2 - 5K + \lambda_1 = 0$$

$$\Rightarrow K^2 = 3K - 2 \text{ and } K^2 - 5K + 4 = 0$$

$$\Rightarrow K = 1, 2 \text{ and } K = 4, 1$$

$$\Rightarrow K = 1$$

$\Rightarrow$ option (4) is correct.

Question 6

Let (α, β, γ) be the co-ordinates of the foot of the perpendicular drawn from the point $(5, 4, 2)$ on the line

$$\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k}).$$

Then the length of the projection of the vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ on the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is :

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Options:

A.

$$\frac{18}{7}$$

B.

$$\frac{15}{7}$$

C.

$$4$$

D.

$$3$$

Answer: A

Solution:

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$$

$$(2\lambda - 1, 3\lambda + 3, 1 - \lambda)$$

$$\vec{PF} = (2\lambda - 6)\hat{i} + (3\lambda - 1)\hat{j} + (-\lambda - 1)\hat{k}$$

$$\vec{PF} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$4\lambda - 12 + 9\lambda - 3 + \lambda + 1 = 0 \Rightarrow \lambda = 1$$

$$\Rightarrow F \equiv (1, 6, 0)$$

$$\Rightarrow \text{length of projection of } \hat{i} + 6\hat{j} \text{ on } 6\hat{i} + 2\hat{j} + 3\hat{k}$$

$$= \left| \frac{(\hat{i} + 6\hat{j}) \cdot (6\hat{i} + 2\hat{j} + 3\hat{k})}{|6\hat{i} + 2\hat{j} + 3\hat{k}|} \right|$$

$$= \frac{1}{7}(6 + 12) = \frac{18}{7}$$

Option (3) is correct.

Question7

Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{b}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, then $|\vec{a} + \vec{c}|^2$ is equal to :

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Options:

A.

30

B.

33

C.

27

D.

35

Answer: C

Solution:

$$\text{Let } \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ x & y & z \end{vmatrix}$$

$$= (2z - 2y)\hat{i} + (z + 2x)\hat{j} + (-y - 2x)\hat{k}$$

$$\equiv 8\hat{i} + 7\hat{j} - 3\hat{k}$$

$$\Rightarrow 2z - 2y = 8$$

$$z + 2x = 7$$

$$-y - 2x = 3 \quad \left. \vphantom{\begin{matrix} 2z - 2y = 8 \\ z + 2x = 7 \end{matrix}} \right\} y = 3 - 2x, z = 7 - 2x$$

$$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$$

$$x + y + z = 4$$

$$\text{Now } x + 3 - 2x + 7 - 2x = 4$$

$$\Rightarrow x = 2$$

$$y = -1$$

$$z = 3$$

$$\Rightarrow \vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$|\vec{a} + \vec{c}|^2 = |\hat{i} + \hat{j} + 5\hat{k}|^2 \\ = 27$$

Question8

For a triangle ABC , let $\vec{p} = \overrightarrow{BC}$, $\vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos \theta = \frac{1}{\sqrt{3}}$, where θ is the angle between $\vec{p}$ and $\vec{q}$, then $|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$ is equal to :

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Options:

A.

200

B.

220

C.

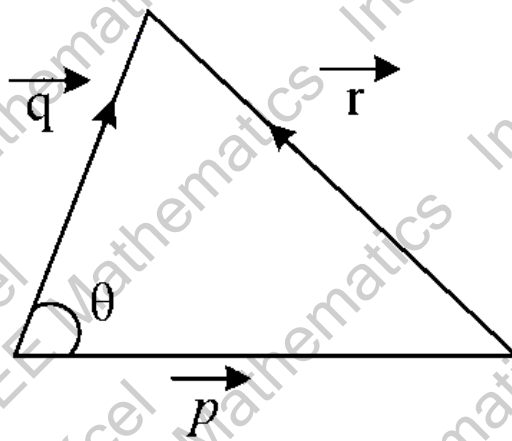
410

D.

340

Answer: A

Solution:



$$\Rightarrow \vec{r} = \vec{q} + \vec{p}$$

$$|\vec{r}| = \sqrt{q^2 + p^2 + 2pq \cos \theta}$$

$$|r|^2 = 4 + 12 + 2 \times \sqrt{3} \times 2 \times \frac{1}{\sqrt{3}}$$

$$|r|^2 = 24$$

$$|\vec{p} \times (\vec{q} - 3(\vec{q} - \vec{p}))|^2 + 3|\vec{r}|^2 = |-2\vec{p} \times \vec{q}|^2 + 3|\vec{r}|^2$$

$$= 4 \times |\vec{p}| |\vec{q}|^2 \sin^2 \theta + 3 \times 24 = 4 \times 12 \times 4 \times \left(\frac{2}{3}\right) + 72 = 128 + 72 = 200$$

Question9

Let $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal AC of the parallelogram ABCD be of length one unit. If α, β , where $\alpha > \beta$, be the roots of the equation $\lambda^2 x^2 - 6\lambda x + 5 = 0$, then $2\alpha - \beta$ is equal to

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Options:

A.

B.

6

C.

4

D.

1

Answer: A

Solution:

$$\overline{AB} = 2\vec{i} + 4\vec{j} - 5\vec{k}$$

$$\overline{AD} = \vec{i} + 2\vec{j} + \lambda\vec{k}$$

$$\vec{V} = \vec{i} + \vec{j} + \vec{k}$$

$$\vec{V} \cdot \overline{AC}$$

$$\frac{|\overline{AC}|}{|AC|}$$

$$\overline{AC} = \overline{AB} + \overline{BC}$$

$$18\lambda = 54$$

$$18\lambda = 54$$

Question 10

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}$, $\lambda \in \mathbf{Z}$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz -plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is equal to :

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Options:

A.

52

B.

26

C.

104

D.

208

Answer: D

Solution:

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 0 & \lambda & 2 \end{vmatrix} = (2 - \lambda)\vec{i} - \vec{j}(4) + (2\lambda)\vec{k}$$

$$|\vec{c}|^2 = 53 \Rightarrow (2 - \lambda)^2 + 16 + (2\lambda)^2 = 53 = 5\lambda^2 - 4\lambda - 33 = 0$$

$$\Rightarrow 5\lambda^2 - 15\lambda + 11\lambda - 33 = 0$$

$$\Rightarrow 5\lambda(\lambda - 3) + 11(\lambda - 3) = 0$$

$$\Rightarrow (\lambda - 3)(5\lambda + 11) = 0$$

$$\Rightarrow \lambda = 3, \lambda \neq -\frac{11}{5} \quad (\because \lambda \in \mathbb{Z})$$

$$\therefore \vec{c} = (-1, -4, 6)$$

$$\text{Let } \therefore \vec{d} = (0, y, z)$$

$$\text{Given } |\vec{d}| = 2 \Rightarrow y^2 + z^2 = 4$$

$$\vec{c} \cdot \vec{d} = 0 - 4y + 6z$$

$$(\vec{c} \cdot \vec{d})^2 = (-4y + 6z)^2$$

$$= 16y^2 + 36z^2 - 48yz$$

$$= 16y^2 + 36(4 - y^2) - 48y \cdot \sqrt{4 - y^2}$$

$$\text{Let } f(y) = -20y^2 + 144 - 48y\sqrt{4 - y^2}$$

$$f^1(y) = -40y - 48 \left(\sqrt{4 - y^2} \right) - 48y \left(\frac{1}{2\sqrt{4 - y^2}} \right) (-2y)$$

$$= -40y - 48 \left(\frac{4 - y^2 - y^2}{\sqrt{4 - y^2}} \right)$$

$$f^1(y) = 0 \Leftrightarrow 40y = 96 \left(\frac{y^2 - 2}{\sqrt{4 - y^2}} \right)$$

$$\Rightarrow 5y\sqrt{4 - y^2} = 12(y^2 - 2)$$

$$\Rightarrow 25y^2(4 - y^2) = 144(y^4 + 4 - 4y^2)$$

$$= 169y^4 - 676y^2 + 576 = 0$$

$$= 169y^4 - 208y^2 - 468y^2 + 576 = 0$$

$$\Rightarrow 13y^2(13y^2 - 16) - 36(13y^2 - 16) = 0$$

$$\Rightarrow y^2 = \frac{16}{13} \text{ (or) } \frac{36}{13}$$

$$\Rightarrow y = \pm \frac{4}{\sqrt{13}} \text{ (or) } \pm \frac{6}{\sqrt{13}}$$

$$\therefore \text{Maximum of } (\vec{c} \cdot \vec{d})^2 = 208$$

Question11

Let $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} - \vec{b}) \cdot \vec{d}$ is equal to :

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Options:

A.

2

B.

-2

C.

4

D.

-4

Answer: B

Solution:

$$(\vec{a} - \vec{b}) \cdot \vec{d} = \vec{a} \cdot \vec{d} - \vec{b} \cdot \vec{d}$$

$$\because \vec{d} = \vec{c} \times \vec{a} \Rightarrow \vec{a} \cdot \vec{d} = [\vec{a} \vec{c} \vec{a}] = 0$$

$$\text{And } \vec{b} \cdot \vec{d} = \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= \vec{b} \cdot ((\vec{a} \times \vec{b}) \times \vec{a})$$

$$\Rightarrow (\vec{a} - \vec{b}) \cdot \vec{d} = \vec{b} \cdot (\vec{a} \times (\vec{a} \times \vec{b}))$$

$$= \vec{b} \cdot [\vec{a} \cdot \vec{b} \vec{a} - \vec{a} \cdot \vec{a} \vec{b}]$$

$$= (\vec{a} \cdot \vec{b})^2 - (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b})$$

$$= (-8)^2 - 6 \times 11 = 64 - 66 = -2$$

Option (2) is correct.

Question 12

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p , then $9p^2$ is equal to

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Options:

A.

9

B.

6

C.

4

D.

12

Answer: D

Solution:

Given

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$$

$$\vec{v} = \vec{a} \times \vec{b}$$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$\vec{v} = \hat{i}(2 - 3) - \hat{j}(-1 - 6) + \hat{k}(1 + 4)$$

$$\vec{v} = -\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\vec{v} \cdot \vec{c} = 11$$

$$(-\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (\lambda\hat{i} + \hat{j} + \hat{k}) = 11$$

$$-\lambda + 7 + 5 = 11$$

$$-\lambda + 12 = 11$$

$$\lambda = 1$$

$$\vec{c} = \hat{i} + \hat{j} + \hat{k}$$

$$p = \frac{|\vec{b} \cdot \vec{c}|}{|\vec{c}|}$$

$$\vec{b} \cdot \vec{c} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= 2 + 1 - 1 = 2$$

$$|\vec{c}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$p = \frac{2}{\sqrt{3}}$$

$$p^2 = \frac{4}{3}$$

$$9p^2 = 12$$

Question 13

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. If

$|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60° , then $|\vec{a} \cdot \vec{c}|$ is equal to

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Options:

A.

2

B.

0

C.

1

D.

4

Answer: C

Solution:

Given that

$$\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$$

$$\vec{a} \times \vec{b} - 2(\vec{a} \times \vec{c}) = 0$$

$$\vec{a} \times (\vec{b} - 2\vec{c}) = 0$$

This means $\vec{a}$ and $\vec{b} - 2\vec{c}$ are parallel vectors

$$\vec{b} - 2\vec{c} = \lambda \vec{a}$$

$$\Rightarrow \vec{b} = \lambda \vec{a} + 2\vec{c} \quad (1)$$

It is given that $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$, angle between $\vec{b}$ and $\vec{c}$, $\theta = 60^\circ$.

$$|\vec{b}|^2 = |\lambda \vec{a} + 2\vec{c}|^2$$

$$\Rightarrow |\vec{b}|^2 = \lambda^2 |\vec{a}|^2 + 4|\vec{c}|^2 + 4\lambda(\vec{a} \cdot \vec{c})$$

$$\Rightarrow 16 = \lambda^2(1)^2 + 4(2)^2 + 4\lambda(\vec{a} \cdot \vec{c})$$

$$\Rightarrow \lambda^2 + 4\lambda(\vec{a} \cdot \vec{c}) = 0 \dots\dots(2)$$

Taking dot product with $\vec{c}$ in equation (1)

$$\vec{b} \cdot \vec{c} = (\lambda \vec{a} + 2\vec{c}) \cdot \vec{c}$$

$$\Rightarrow |\vec{b}||\vec{c}| \cos 60^\circ = \lambda(\vec{a} \cdot \vec{c}) + 2|\vec{c}|^2$$

$$\Rightarrow 4 \times 2 \times 1/2 = \lambda(\vec{a} \cdot \vec{c}) + 2(2)^2$$

$$\Rightarrow 4 = \lambda(\vec{a} \cdot \vec{c}) + 8$$

$$\Rightarrow \lambda(\vec{a} \cdot \vec{c}) = -4$$

From (2) and (3)

$$\lambda^2 + 4(-4) = 0$$

$$\Rightarrow \lambda^2 - 16 = 0$$

$$\Rightarrow \lambda = \pm 4$$

from (3)

$$\vec{a} \cdot \vec{c} = -4/\lambda$$

$$\text{if } \lambda = 4, \text{ then } \vec{a} \cdot \vec{c} = -1$$

$$\text{if } \lambda = -4, \text{ then } \vec{a} \cdot \vec{c} = 1$$

$$|\vec{a} \cdot \vec{c}| = 1$$

Question 14

Let the lines $L_1 : \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}), \lambda \in \mathbb{R}$ and

$L_2 : \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$, intersect at the point **R**.

Let **P** and **Q** be the points lying on lines L_1 and L_2 , respectively, such

that $|\overrightarrow{PR}| = \sqrt{29}$ and $|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}}$. If the point **P** lies in the first octant, then $27(\text{QR})^2$ is equal to

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Options:

A.

360

B.

348

C.

320

D.

Answer: A**Solution:**

$$L_1 : \vec{r} = \hat{i} + \hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$L_2 : \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

at the intersection point R, position vectors are equal.

$$(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

equating components of $\hat{i}$, $\hat{j}$ and $\hat{k}$.

$$1 + 2\lambda = 4 + 5\mu \implies 2\lambda - 5\mu = 3 \dots (1)$$

$$2 + 3\lambda = 1 + 2\mu \implies 3\lambda - 2\mu = -1 \dots (2)$$

$$3 + 4\lambda = \mu \implies \mu - 4\lambda = 3 \dots (3)$$

$$\text{eqn (1)} \times 3 - \text{eqn (2)} \times 2$$

$$-15\mu + 4\mu = 9 + 2 \implies -11\mu = 11 \implies \mu = -1$$

substitute in equation (1)

$$2\lambda - 5(-1) = 3 \implies \lambda = \frac{3-5}{2} = -1$$

Now, checking equation (3) $(-1) - 4(-1) = 3 = \text{R.H.S}$ consistent.

Now finding point R using $\lambda = -1$ in L_1

$$\vec{OR} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{OR} = -\hat{i} - \hat{j} - \hat{k}$$

Point P lies on L_1 so its position vector is $\vec{OP} = (1 + 2\lambda)\hat{i} + (2 + 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}$ It is given that

$$|\vec{PR}| = \sqrt{29} \implies |\vec{PR}|^2 = 29$$

$$\vec{PR} = \vec{OR} - \vec{OP}$$

$$\vec{PR} = (-2 - 2\lambda)\hat{i} + (-3 - 3\lambda)\hat{j} + (-4 - 4\lambda)\hat{k}$$

$$|\vec{PR}|^2 = 4(1 + \lambda)^2 + 9(1 + \lambda)^2 + 16(1 + \lambda)^2 = 29$$

$$29(1 + \lambda)^2 = 29 \implies (1 + \lambda)^2 = 1 \implies \lambda = 0, -2$$

point P lies in first octant, means position vector of P must be positive, so $\lambda = 0$

$$\vec{OP} = \hat{i} + 2\hat{j} + 3\hat{k}$$

similarly Q lies on L_2 , so position vector of Q is

$$\overrightarrow{OQ} = (4 + 5\mu)\hat{i} + (1 + 2\mu)\hat{j} + \mu\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{\frac{47}{3}} \Rightarrow |\overrightarrow{PQ}|^2 = \frac{47}{3} \quad (\text{given})$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\overrightarrow{PQ} = (3 + 5\mu)\hat{i} + (-1 + 2\mu)\hat{j} + (\mu - 3)\hat{k}$$

$$(3 + 5\mu)^2 + (-1 + 2\mu)^2 + (\mu - 3)^2 = \frac{47}{3}$$

$$30\mu^2 + 20\mu + 19 = \frac{47}{3}$$

$$90\mu^2 + 60\mu + 57 = 47$$

$$90\mu^2 + 60\mu + 10 = 0$$

$$9\mu^2 + 6\mu + 1 = 0$$

$$(3\mu + 1)^2 = 0 \Rightarrow \mu = -\frac{1}{3}$$

finding position vector of Q using $\mu = -1/3$

$$\overrightarrow{OQ} = \left(4 - \frac{5}{3}\right)\hat{i} + \left(1 - \frac{2}{3}\right)\hat{j} - \frac{1}{3}\hat{k}$$

$$\overrightarrow{OQ} = \frac{7}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k}$$

calculate $27|QR|^2$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$$

$$\overrightarrow{QR} = -\frac{10}{3}\hat{i} - \frac{4}{3}\hat{j} - \frac{2}{3}\hat{k}$$

$$|\overrightarrow{QR}|^2 = \frac{9}{100} + \frac{9}{16} + \frac{9}{4} = \frac{9}{120}$$

$$27|\overrightarrow{QR}|^2 = \frac{9}{120} \times 27 = 360$$

Question 15

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}$, $|\vec{c} \times \vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$.

Then $\vec{a} \cdot \vec{d}$ is equal to

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Options:

A.

0

B.

1

C.

3

D.

11

Answer: A

Solution:

$$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} - \hat{j}$$

$$|\vec{a}| = \sqrt{4 + 1 + 4} = 3, |\vec{b}| = \sqrt{1 + 1} = \sqrt{2}$$

$$\text{It is given that } \vec{c} = \vec{a} \times \vec{b}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$$

$$\vec{c} = \hat{i}(0 + 2) - \hat{j}(0 + 2) + \hat{k}(-2 - 1)$$

$$\vec{c} = 2\hat{i} - 2\hat{j} - 3\hat{k}$$

$$|\vec{c}| = \sqrt{4 + 4 + 9} = \sqrt{17} = 4.123$$

$$\text{It is given that } |\vec{c} \times \vec{d}| = 3 \text{ \& angle between } \vec{c} \text{ \& } \vec{d} \text{ is } \pi/4$$

$$|\vec{c}||\vec{d}| \sin \frac{\pi}{4} = 3$$

$$3|\vec{d}| \times \frac{1}{\sqrt{2}} = 3 \Rightarrow |\vec{d}| = \sqrt{2}$$

$$|\vec{d} - \vec{a}| = \sqrt{11} \text{ (given)}$$

$$|\vec{d} - \vec{a}|^2 = 11$$

$$|\vec{d}|^2 + |\vec{a}|^2 - 2\vec{d} \cdot \vec{a} = 11$$

$$2 + 9 - 2\vec{d} \cdot \vec{a} = 11 \text{ use } |\vec{a}| = 3$$

$$11 - 2\vec{d} \cdot \vec{a} = 11$$

$$-2\vec{d} \cdot \vec{a} = 0$$

$$\vec{d} \cdot \vec{a} = 0$$

Question 16

Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$. If $\vec{c}$ is a vector such that $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0}$ and $(\vec{a} - \vec{b}) \cdot \vec{c} = -97$, then $|\vec{c} \times \hat{k}|^2$ is equal to

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Options:

A.

193

B.

205

C.

218

D.

233

Answer: C

Solution:

Given

$$\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}, \quad \vec{b} = \hat{i} - \hat{j} + 3\hat{k}$$

$$2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0}$$

$$2\vec{a} \times \vec{c} + 3\vec{b} \times \vec{c} = \vec{0}$$

$$(2\vec{a} + 3\vec{b}) \times \vec{c} = \vec{0}$$

This means $2\vec{a} + 3\vec{b}$ and $\vec{c}$ are parallel.

$$\Rightarrow \vec{c} = \lambda(2\vec{a} + 3\vec{b})$$

$$\vec{c} = \lambda[2(2\hat{i} - 5\hat{j} + 5\hat{k}) + 3(\hat{i} - \hat{j} + 3\hat{k})]$$

$$\vec{c} = \lambda(7\hat{i} - 13\hat{j} + 19\hat{k})$$

Given

$$(\vec{a} - \vec{b}) \cdot \vec{c} = -97$$

$$\begin{aligned}\vec{a} - \vec{b} &= (2\hat{i} - 5\hat{j} + 5\hat{k}) - (\hat{i} - \hat{j} + 3\hat{k}) \\ &= \hat{i} - 4\hat{j} + 2\hat{k}\end{aligned}$$

Put in equation:

$$(\hat{i} - 4\hat{j} + 2\hat{k}) \cdot \lambda(7\hat{i} - 13\hat{j} + 19\hat{k}) = -97$$

$$\lambda(7 + 52 + 38) = -97$$

$$\lambda(97) = -97 \Rightarrow \lambda = -1$$

$$\vec{c} = -1(7\hat{i} - 13\hat{j} + 19\hat{k})$$

$$\vec{c} = -7\hat{i} + 13\hat{j} - 19\hat{k}$$

$$\vec{c} \times \hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & 13 & -19 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}(13) - \hat{j}(-7) + \hat{k}(0)$$

$$= 13\hat{i} + 7\hat{j}$$

$$|\vec{c} \times \hat{k}|^2 = 13^2 + 7^2 = 169 + 49 = 218$$

Question 17

Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

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Options:

A.

$$\frac{\sqrt{35}}{2}$$

B.

13

C.

$$\frac{\sqrt{21}}{2}$$

D.

7

Answer: A

Solution:

Given

$$a = 2\hat{i} - \hat{j} - \hat{k}, b = \hat{i} + 3\hat{j} - \hat{k}, \text{ and } c = 2\hat{i} + \hat{j} + 3\hat{k}.$$

v is in the plane of vectors a and b ,

$$\text{so } v = a + \lambda b = (2 + \lambda)\hat{i} + (-1 + 3\lambda)\hat{j} + (-1 - \lambda)\hat{k}.$$

It is given that length of projection of v on c is $\frac{1}{\sqrt{14}}$.

$$\left| \frac{v \cdot c}{|c|} \right| = \frac{1}{\sqrt{14}}$$

$$|v \cdot c| = \frac{1}{\sqrt{14}} \times |c|$$

$$= \frac{1}{\sqrt{14}} \times \sqrt{(2)^2 + (1)^2 + (3)^2}$$

$$= \frac{1}{\sqrt{14}} \times \sqrt{14} = 1.$$

$$|(2 + \lambda)\hat{i} + (-1 + 3\lambda)\hat{j} + (-1 - \lambda)\hat{k} \cdot (2\hat{i} + \hat{j} + 3\hat{k})| = 1,$$

$$|4 + 2\lambda - 1 + 3\lambda - 3 - 3\lambda| = 1,$$

$$|2\lambda| = 1 \implies \lambda = \pm \frac{1}{2} \implies \lambda^2 = \frac{1}{4}.$$

$$|v|^2 = |a + \lambda b|^2,$$

$$|v|^2 = |a|^2 + \lambda^2 |b|^2 + 2\lambda(a \cdot b). \{a \cdot b = 0\}$$

$$= 6 + 11\lambda^2 + 0.$$

$$|v| = \sqrt{6 + 11\lambda^2}.$$

$$\text{Put value } \lambda^2 = \frac{1}{4} :$$

$$|v| = \sqrt{6 + \frac{11}{4}},$$

$$|v| = \sqrt{\frac{24 + 11}{4}},$$

$$|v| = \sqrt{\frac{35}{4}}$$

$$|v| = \frac{\sqrt{35}}{2}$$

Question 18

For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9 \text{ and } |2\vec{a} + k\vec{b} + k\vec{c}| = 3,$$

the positive value of k is :

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Options:

A.

4

B.

5

C.

6

D.

3

Answer: B

Solution:

$$\text{Given, } |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b} \cdot \vec{c} + |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} = 9$$

$$\Rightarrow 6 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 9 \quad \{|\vec{a}| = |\vec{b}| = |\vec{c}| = 1\}$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

also,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 3 + 2\left(-\frac{3}{2}\right) = 3 - 3 = 0$$

So

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2 = 1$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow \vec{a} \cdot \vec{b} = -\frac{1}{2}$$

Similarly,

$$\vec{b} \cdot \vec{c} = -\frac{1}{2} \quad \text{and} \quad \vec{a} \cdot \vec{c} = -\frac{1}{2}$$

so,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = -\frac{1}{2}$$

It is given that

$$|2\vec{a} + k\vec{b} + k\vec{c}| = 3$$

$$\Rightarrow |2\vec{a} + k\vec{b} + k\vec{c}|^2 = 9$$

$$\Rightarrow 4|\vec{a}|^2 + k^2|\vec{b}|^2 + k^2|\vec{c}|^2 + 4k\vec{a} \cdot \vec{b} + 2k^2\vec{b} \cdot \vec{c} + 4k\vec{a} \cdot \vec{c} = 9$$

$$\Rightarrow 4(1)^2 + k^2(1)^2 + k^2(1)^2 + 4k\left(-\frac{1}{2}\right) + 2k^2\left(-\frac{1}{2}\right) + 4k\left(-\frac{1}{2}\right) = 9$$

$$\Rightarrow 4 + 2k^2 - 2k - k^2 - 2k = 9$$

$$\Rightarrow k^2 - 4k - 5 = 0$$

$$\Rightarrow (k - 5)(k + 1) = 0$$

$$\Rightarrow k = 5, -1$$

Question 19

Let P be a point in the plane of the vectors $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and

$\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and

AC. If $|\overrightarrow{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is :

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Options:

A.

$$\frac{3}{2}$$

B.

$$\frac{\sqrt{30}}{4}$$

C.

$$\frac{\sqrt{26}}{4}$$

D.

$$2$$

Answer: B

Solution:

Point P is equidistant from the lines AB and AC . Geometrically, this means point P must lie on the angle bisector of the angle formed by vectors AB and AC .

First, we find the unit vectors along AB and AC :

- $|AB| = \sqrt{3^2 + 1^2 + (-1)^2} = \sqrt{11}$
- $|AC| = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{11}$

Since the magnitudes are equal, the angle bisector v is simply the sum of the two vectors:

$$v = AB + AC = (3 + 1)\hat{i} + (1 - 1)\hat{j} + (-1 + 3)\hat{k} = 4\hat{i} + 2\hat{k}$$

The vector AP will be in the direction of the unit vector $\hat{v}$:

$$\hat{v} = \frac{4\hat{i} + 2\hat{k}}{\sqrt{4^2 + 2^2}} = \frac{4\hat{i} + 2\hat{k}}{\sqrt{20}} = \frac{4\hat{i} + 2\hat{k}}{2\sqrt{5}} = \frac{2\hat{i} + \hat{k}}{\sqrt{5}}$$

Determine Vector AP

Given that $|AP| = \frac{\sqrt{5}}{2}$, we can write:

$$AP = |AP|\hat{v} = \frac{\sqrt{5}}{2} \left(\frac{2\hat{i} + \hat{k}}{\sqrt{5}} \right) = \frac{1}{2}(2\hat{i} + \hat{k}) = \hat{i} + \frac{1}{2}\hat{k}$$

3. Calculate the Area of Triangle ABP

The area of a triangle formed by two vectors $\vec{AB}$ and $\vec{AP}$ is given by the formula:

$$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AP}|$$

First, compute the cross product $\vec{AB} \times \vec{AP}$:

$$\begin{aligned}\vec{AB} \times \vec{AP} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & 0 & \frac{1}{2} \end{vmatrix} \\ &= \hat{i} \left(\frac{1}{2} - 0 \right) - \hat{j} \left(\frac{3}{2} - (-1) \right) + \hat{k}(0 - 1) \\ &= \frac{1}{2}\hat{i} - \frac{5}{2}\hat{j} - \hat{k}\end{aligned}$$

$$|\vec{AB} \times \vec{AP}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{5}{2}\right)^2 + (-1)^2} = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{26}{4} + \frac{4}{4}} = \sqrt{\frac{30}{4}} = \frac{\sqrt{30}}{2}$$

Finally, the area of the triangle is:

$$\text{Area} = \frac{1}{2} \left(\frac{\sqrt{30}}{2} \right) = \frac{\sqrt{30}}{4}$$

Question 20

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then the maximum value of $3|(3\vec{a} + 2\vec{b})| + 4|(3\vec{a} - 2\vec{b})|$ is:

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Options:

A.

30

B.

36

C.

60

D.

Answer: C

Solution:

$$\begin{aligned}
 E &= 3\sqrt{9a^2 + 4b^2 + 12\vec{a} \cdot \vec{b}} + 4\sqrt{9a^2 + 4b^2 - 12\vec{a} \cdot \vec{b}} \\
 &= 3\sqrt{36 + 36 + 12 \times 6 \cos \theta} + 4\sqrt{36 + 36 - 72 \cos \theta} \\
 &= 3\sqrt{72 + 72 \cos \theta} + 4\sqrt{72 - 72 \cos \theta} \\
 &= 18\sqrt{2}\sqrt{1 + \cos \theta} + 24\sqrt{2}\sqrt{1 - \cos \theta} \\
 &= 18\sqrt{2}\sqrt{2} \cos \frac{\theta}{2} + 24\sqrt{2} \left(\sqrt{2} \sin \frac{\theta}{2} \right) \\
 &= 36 \left(\cos \frac{\theta}{2} \right) + 48 \sin \left(\frac{\theta}{2} \right) \\
 E_{\max} &= \sqrt{(36)^2 + (48)^2} = \sqrt{6^4 + 6^2 \times 8^2} = 60
 \end{aligned}$$

Question 21

Two adjacent sides of a parallelogram PQRS are given by

$\overrightarrow{PQ} = \hat{j} + \hat{k}$ and $\overrightarrow{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2 \left(\frac{5\alpha}{2} \right) - \sin^2 \left(\frac{\alpha}{2} \right)$ is equal to:

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Options:

A.

$$\frac{1}{2}$$

B.

$$\frac{\sqrt{3}}{2}$$

C.

$$\frac{\sqrt{3}}{4}$$

D.

$$\frac{2\sqrt{3}}{5}$$

Answer: B

Solution:

Let angle between $\vec{PQ}$ & $\vec{PS}$ is θ

$$\cos \theta = \frac{\vec{PQ} \cdot \vec{PS}}{|\vec{PQ}| |\vec{PS}|}$$

$$\cos \theta = \frac{0-1+0}{\sqrt{2}\sqrt{2}}$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\alpha = \frac{2\pi}{3} - \frac{\pi}{2} \text{ So } \alpha = \frac{\pi}{6}$$

$$\sin^2 \frac{5\alpha}{2} - \sin^2 \frac{\alpha}{2} = \sin \left(\frac{5\alpha}{2} + \frac{\alpha}{2} \right) \sin \left(\frac{5\alpha}{2} - \frac{\alpha}{2} \right)$$

$$= \sin \frac{6\alpha}{2} \times \sin \frac{4\alpha}{2}$$

$$= \sin 3\alpha \times \sin 2\alpha$$

$$= \sin \frac{\pi}{2} \times \sin \frac{\pi}{3}$$

$$= 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Question 22

Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbb{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$.

If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to :

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Options:

A.

8

B.

12

C.

14

D.

15

Answer: B

Solution:

$$\vec{c} = \lambda \vec{a} + \mu \vec{b}$$

$$\vec{c} = (\mu - \lambda)\hat{i} + (\lambda + 3\mu)\hat{j} + (3\lambda + \mu)\hat{k}$$

$$\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$$

$$3(\mu - \lambda) - 6(\lambda + 3\mu) + 2(3\lambda + \mu) = 10$$

$$-3\lambda - 13\mu = 10 \quad \dots (1)$$

$$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$$

$$5\mu + 3\lambda = -2 \quad \dots (2)$$

Solving (1) & (2)

$$\lambda = 1, \mu = -1$$

$$\vec{c} = -2\hat{i} - 2\hat{j} + 2\hat{k}$$

$$|\vec{c}|^2 = 12$$

Question 23

Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that

$|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$. If $\vec{A} = \lambda \hat{u} + \hat{v} + (\hat{u} \times \hat{v})$, then λ is equal to:

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Options:

A.

$$\frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$$

B.

$$\frac{2}{3}(\vec{A} \cdot \hat{u}) - \frac{1}{3}(\vec{A} \cdot \hat{v})$$

C.

$$\frac{4}{3}(\vec{A} \cdot \hat{u}) + \frac{2}{3}(\vec{A} \cdot \hat{v})$$

D.

$$(\vec{A} \cdot \hat{u}) - \frac{1}{2}(\vec{A} \cdot \hat{v})$$

Answer: A

Solution:

Since $\hat{u}$ and $\hat{v}$ are unit vectors,

$$|\hat{u} \times \hat{v}| = |\hat{u}||\hat{v}|\sin \theta = \sin \theta$$

Given

$$|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$$

So,

$$\sin \theta = \frac{\sqrt{3}}{2}$$

The angle is acute, hence

$$\theta = 60^\circ \Rightarrow \hat{u} \cdot \hat{v} = \cos 60^\circ = \frac{1}{2}$$

Now,

$$\vec{A} = \lambda \hat{u} + \hat{v} + (\hat{u} \times \hat{v})$$

We take dot product with $\hat{u}$:

$$\vec{A} \cdot \hat{u} = \lambda(\hat{u} \cdot \hat{u}) + (\hat{v} \cdot \hat{u}) + (\hat{u} \times \hat{v}) \cdot \hat{u}$$

Now,

$$\hat{u} \cdot \hat{u} = 1, \quad \hat{v} \cdot \hat{u} = \frac{1}{2}, \quad (\hat{u} \times \hat{v}) \cdot \hat{u} = 0$$

Therefore,

$$\vec{A} \cdot \hat{u} = \lambda + \frac{1}{2}$$

So,

$$\lambda = \vec{A} \cdot \hat{u} - \frac{1}{2} \quad (1)$$

Now take dot product with $\hat{v}$:

$$\vec{A} \cdot \hat{v} = \lambda(\hat{u} \cdot \hat{v}) + (\hat{v} \cdot \hat{v}) + (\hat{u} \times \hat{v}) \cdot \hat{v}$$

Using

$$\hat{u} \cdot \hat{v} = \frac{1}{2}, \quad \hat{v} \cdot \hat{v} = 1, \quad (\hat{u} \times \hat{v}) \cdot \hat{v} = 0$$

we get

$$\vec{A} \cdot \hat{v} = \frac{\lambda}{2} + 1$$

Hence,

$$\frac{\lambda}{2} = \vec{A} \cdot \hat{v} - 1 \Rightarrow \lambda = 2(\vec{A} \cdot \hat{v}) - 2 \quad (2)$$

Now let

$$x = \vec{A} \cdot \hat{u}, \quad y = \vec{A} \cdot \hat{v}$$

Then from above,

$$x = \lambda + \frac{1}{2} \quad \text{and} \quad y = \frac{\lambda}{2} + 1$$

From the first,

$$\lambda = x - \frac{1}{2}$$

Also,

$$x - 2y = \left(\lambda + \frac{1}{2}\right) - 2\left(\frac{\lambda}{2} + 1\right) = \lambda + \frac{1}{2} - \lambda - 2 = -\frac{3}{2}$$

So we form λ as a linear combination of x and y .

Check Option A:

$$\frac{4}{3}x - \frac{2}{3}y = \frac{4}{3}\left(\lambda + \frac{1}{2}\right) - \frac{2}{3}\left(\frac{\lambda}{2} + 1\right) \\ = \frac{4\lambda}{3} + \frac{2}{3} - \frac{\lambda}{3} - \frac{2}{3} = \lambda$$

Hence,

$$\lambda = \frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$$

So the correct option is:

Option A

Question 24

Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + 2\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$, then $|3\vec{r}|^2$ is equal to:

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Options:

A.

44

B.

54

C.

86

D.

132

Answer: A

Solution:

Given

$$\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}, \quad \vec{b} = \hat{j} + 2\hat{k}$$

and

$$\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}, \quad \vec{r} \cdot \vec{a} = 0$$

We have to find $|3\vec{r}|^2$.

From

$$\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}$$

note that

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

So,

$$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$$

Hence,

$$(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}$$

This means $\vec{r} - \vec{b}$ is parallel to $\vec{a}$. Therefore,

$$\vec{r} = \vec{b} + \lambda \vec{a}$$

for some scalar λ .

Now use the condition

$$\vec{r} \cdot \vec{a} = 0$$

Substitute $\vec{r} = \vec{b} + \lambda \vec{a}$:

$$(\vec{b} + \lambda \vec{a}) \cdot \vec{a} = 0$$

So,

$$\vec{b} \cdot \vec{a} + \lambda(\vec{a} \cdot \vec{a}) = 0$$

Now calculate these dot products.

1. Find $\vec{b} \cdot \vec{a}$

$$\vec{b} = (0, 1, 2), \quad \vec{a} = (\sqrt{7}, 1, -1)$$

Thus,

$$\vec{b} \cdot \vec{a} = 0 \cdot \sqrt{7} + 1 \cdot 1 + 2 \cdot (-1) = 1 - 2 = -1$$

2. Find $\vec{a} \cdot \vec{a}$

$$\vec{a} \cdot \vec{a} = (\sqrt{7})^2 + 1^2 + (-1)^2 = 7 + 1 + 1 = 9$$

Therefore,

$$-1 + 9\lambda = 0$$

So,

$$\lambda = \frac{1}{9}$$

Hence,

$$\vec{r} = \vec{b} + \frac{1}{9}\vec{a}$$

Now find $|\vec{r}|^2$.

Using

$$\vec{r} = \vec{b} + \frac{1}{9}\vec{a}$$

we get

$$|\vec{r}|^2 = \left| \vec{b} + \frac{1}{9}\vec{a} \right|^2$$

Expand:

$$|\vec{r}|^2 = |\vec{b}|^2 + \frac{1}{81}|\vec{a}|^2 + \frac{2}{9}(\vec{a} \cdot \vec{b})$$

Now,

$$|\vec{b}|^2 = 0^2 + 1^2 + 2^2 = 5$$

$$|\vec{a}|^2 = 9$$

$$\vec{a} \cdot \vec{b} = -1$$

So,

$$|\vec{r}|^2 = 5 + \frac{9}{81} + \frac{2}{9}(-1)$$

$$= 5 + \frac{1}{9} - \frac{2}{9} = 5 - \frac{1}{9} = \frac{44}{9}$$

Therefore,

$$|3\vec{r}|^2 = 9|\vec{r}|^2 = 9 \cdot \frac{44}{9} = 44$$

So the correct answer is:

44

Hence, Option A.

Question 25

Let O be the origin, $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OQ} = \vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OP} = 5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to :

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Options:

A.

$$\frac{1}{5}(\vec{a} - 4\vec{b})$$

B.

$$\frac{1}{5}(\vec{b} - 4\vec{a})$$

C.

$$\frac{1}{5}(-\vec{a} + 4\vec{b})$$

D.

$$\frac{1}{5}(-\vec{b} + 4\vec{a})$$

Answer: B

Solution:

$$\vec{OR} = \frac{\vec{OP}}{5} = \frac{\vec{a}}{5}$$

$$\vec{RM} = \frac{\vec{OQ}}{5} = \frac{\vec{b}}{5}$$

$$\vec{OM} - \vec{OR} = \frac{\vec{b}}{5}$$

$$\vec{OM} = \frac{\vec{a} + \vec{b}}{5}$$

$$\vec{PM} = \vec{OM} - \vec{OP}$$

$$\begin{aligned} & \frac{\vec{a} + \vec{b}}{5} - \vec{a} \\ &= \frac{\vec{b} - 4\vec{a}}{5} \end{aligned}$$

Question 26

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is :

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Options:

A.

B.

900

C.

1800

D.

2400

Answer: C

Solution:

Given

$$\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}, \quad \vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$$

We need the square of the area of the triangle whose adjacent sides are

$$2\vec{a} + 3\vec{b} \quad \text{and} \quad \vec{a} - \vec{b}$$

For two adjacent sides $\vec{u}$ and $\vec{v}$, area of triangle is

$$\frac{1}{2} |\vec{u} \times \vec{v}|$$

So, square of the area is

$$\left(\frac{1}{2} |\vec{u} \times \vec{v}|\right)^2 = \frac{1}{4} |\vec{u} \times \vec{v}|^2$$

Let

$$\vec{u} = 2\vec{a} + 3\vec{b}, \quad \vec{v} = \vec{a} - \vec{b}$$

First find $\vec{u}$:

$$2\vec{a} = 2(2\hat{i} + 3\hat{j} + 3\hat{k}) = 4\hat{i} + 6\hat{j} + 6\hat{k}$$

$$3\vec{b} = 3(6\hat{i} + 3\hat{j} + 3\hat{k}) = 18\hat{i} + 9\hat{j} + 9\hat{k}$$

Therefore,

$$\vec{u} = 2\vec{a} + 3\vec{b} = 22\hat{i} + 15\hat{j} + 15\hat{k}$$

Now find $\vec{v}$:

$$\vec{v} = \vec{a} - \vec{b} = (2 - 6)\hat{i} + (3 - 3)\hat{j} + (3 - 3)\hat{k} = -4\hat{i}$$

Now compute $\vec{u} \times \vec{v}$:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 22 & 15 & 15 \\ -4 & 0 & 0 \end{vmatrix}$$

Expanding,

$$\begin{aligned} \vec{u} \times \vec{v} &= \hat{i}(15 \cdot 0 - 15 \cdot 0) - \hat{j}(22 \cdot 0 - 15 \cdot (-4)) + \hat{k}(22 \cdot 0 - 15 \cdot (-4)) \\ &= 0\hat{i} - 60\hat{j} + 60\hat{k} \end{aligned}$$

So,

$$|\vec{u} \times \vec{v}|^2 = 0^2 + (-60)^2 + 60^2 = 3600 + 3600 = 7200$$

Hence square of the area of the triangle is

$$\frac{1}{4} \times 7200 = 1800$$

Therefore, the correct answer is

$$\boxed{1800}$$

So, Option C is correct.

Question 27

Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ be such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$.

If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to :

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Options:

A.

-6

B.

-5

C.

Answer: B

Solution:

The unknown vector $\vec{c}$ is represented in 3D: $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Cross product of 2 vectors $\vec{a}$ and $\vec{b}$:

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 10 & 2 & -1 \end{vmatrix} = -5\hat{i} + 34\hat{j} + 18\hat{k} \dots (i)$$

Cross product of 2 vectors $\vec{b}$ and $\vec{c}$:

$$(\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 2 & -1 \\ x & y & z \end{vmatrix} = (2z + y)\hat{i} - (10z + x)\hat{j} + (10y - 2x)\hat{k} \dots (ii)$$

We know, $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = 0$

$$\Rightarrow 2(-5\hat{i} + 34\hat{j} + 18\hat{k}) + 3\{(2z + y)\hat{i} - (10z + x)\hat{j} + (10y - 2x)\hat{k}\} = 0$$

$$\Rightarrow (6z + 3y - 10)\hat{i} + (68 - 30z - 3x)\hat{j} + (36 + 30y - 6x)\hat{k} = 0$$

As individual vector components of a null vector are zero.

$$x - 5y = 6 \dots P_1$$

$$y + 2z = \frac{10}{3} \dots P_2$$

$$10z + x = \frac{68}{3} \dots P_3$$

$$(\vec{a}) \cdot (\vec{c}) = 15 \Rightarrow (4\hat{i} - \hat{j} + 3\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 4x - y + 3z = 15 \dots (iv)$$

Equation set (iii) is mathematically constructed to form a linear combination of 2 planes P_1 and P_2 generating the 3rd plane P_3 :

$$x - 5y + (\lambda) \cdot (y + 2z) = \frac{68}{3} \Rightarrow \lambda = 5 \Rightarrow P_1 + 5P_2 = P_3 \dots (v)$$

Equation (v) is conclusive in generating a family of planes $\{P_1, P_2, P_3\}$ indicating infinite solutions for the system equation (iii)

We need an auxiliary equation provided through (iv) to render an unique solution for the vector $\vec{c}$:

$$\begin{bmatrix} x - 5y = 6 & \dots & P_1 \\ y + 2z = \frac{10}{3} & \dots & P_2 \\ 4x - y + 3z = 15 & \dots & P_4 \end{bmatrix} \dots (vi)$$

Equation (vi) can be solved using Determinants and Cramer's rule :

$$\Delta = \begin{vmatrix} 1 & -5 & 0 \\ 0 & 1 & 2 \\ 4 & -1 & 3 \end{vmatrix} \leftarrow \mathbf{R}_3 = \mathbf{R}_3 - 4\mathbf{R}_1 = \begin{vmatrix} 1 & -5 & 0 \\ 0 & 1 & 2 \\ 0 & 19 & 3 \end{vmatrix} = -35 \dots (vii)$$

$$\Delta_x = \begin{vmatrix} 6 & -5 & 0 \\ \frac{10}{3} & 1 & 2 \\ 15 & -1 & 3 \end{vmatrix}$$

$$\mathbf{R}_1 = \mathbf{R}_1 + 5\mathbf{R}_2$$

$$\mathbf{R}_3 = \mathbf{R}_3 + \mathbf{R}_2$$

$$= \begin{vmatrix} \frac{68}{3} & 0 & 10 \\ \frac{10}{3} & 1 & 2 \\ \frac{55}{3} & 0 & 5 \end{vmatrix}$$

$$C_1 \leftrightarrow C_2$$

$$= - \begin{vmatrix} 0 & \frac{68}{3} \\ 1 & \frac{10}{3} \\ 0 & \frac{10}{3} \\ \frac{55}{3} & 0 \end{vmatrix} = \left(\frac{68}{3}\right) \cdot (5) - \left(\frac{55}{3}\right) \cdot (10) = -70 \dots (viii)$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 0 \\ 0 & \frac{10}{3} & 2 \\ 4 & 15 & 3 \end{vmatrix} \leftarrow \mathbf{R}_3 = \mathbf{R}_3 - 4\mathbf{R}_1 = \begin{vmatrix} 1 & 6 & 0 \\ 0 & \frac{10}{3} & 2 \\ 0 & -9 & 3 \end{vmatrix} = 28 \dots (ix)$$

$$\Delta_z = \begin{vmatrix} 1 & -5 & 6 \\ 0 & 1 & \frac{10}{3} \\ 4 & -1 & 15 \end{vmatrix} \leftarrow \mathbf{R}_3 = \mathbf{R}_3 - 4\mathbf{R}_1 = \begin{vmatrix} 1 & -5 & 6 \\ 0 & 1 & \frac{10}{3} \\ 0 & 19 & -9 \end{vmatrix} = -\frac{217}{3} \dots (x)$$

Equations (vii), (viii), (ix) and (x) provides the solution to system equation (vi) from Cramer's rule :

$$\begin{bmatrix} x = \frac{\Delta_x}{\Delta} = \frac{70}{35} = \\ y = \frac{\Delta_y}{\Delta} = -\frac{28}{35} = -\frac{4}{5} \\ z = \frac{\Delta_z}{\Delta} = \frac{217}{(3) \cdot (35)} = \frac{31}{15} \end{bmatrix}$$

$$\Rightarrow \vec{c} = (2)\hat{i} - \left(\frac{4}{5}\right)\hat{j} + \left(\frac{31}{15}\right)\hat{k}$$

$$\therefore (\vec{c}) \cdot (\hat{i} + \hat{j} - 3\hat{k})$$

$$\equiv \left\{ (2)\hat{i} - \left(\frac{4}{5}\right)\hat{j} + \left(\frac{31}{15}\right)\hat{k} \right\} \cdot (\hat{i} + \hat{j} - 3\hat{k})$$

$$= 2 - \frac{4}{5} - \frac{31}{5}$$

$$= -\frac{4+31-10}{5} = -5$$

Question 28

Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}$, $\lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____.

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Answer: 16

Solution:

To find the projection vector $\vec{c}$ of $\vec{b} = \lambda\hat{i} + 4\hat{k}$ (where $\lambda > 0$) onto vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, we use the formula for the projection of a vector:

$$\vec{c} = \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a}$$

Calculate the dot product $\vec{b} \cdot \vec{a}$:

$$\vec{b} \cdot \vec{a} = (\lambda\hat{i} + 4\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = \lambda \cdot 1 + 4 \cdot 2 = \lambda + 8$$

Calculate the magnitude of $\vec{a}$:

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

Now substitute to find $\vec{c}$:

$$\vec{c} = \left(\frac{\lambda+8}{9} \right) (\hat{i} + 2\hat{j} + 2\hat{k})$$

We know that the magnitude $|\vec{a} + \vec{c}| = 7$. Thus, substituting $\vec{c}$ in this equation, we resolve it to find λ :

$$|\vec{a} + \vec{c}| = 7 \Rightarrow (\lambda = 4)$$

This indicates that λ has a value of 4.

Next, calculating the area of the parallelogram formed by vectors $\vec{b}$ and $\vec{c}$ involves finding the cross product $|\vec{b} \times \vec{c}|$:

$$\vec{b} = 4\hat{i} + 4\hat{k}, \quad \vec{c} = \left(\frac{12}{9} \right) (\hat{i} + 2\hat{j} + 2\hat{k}) = \left(\frac{4}{3} \right) \hat{i} + \left(\frac{8}{3} \right) \hat{j} + \left(\frac{8}{3} \right) \hat{k}$$

The cross product is:

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4 \end{vmatrix}$$

Solving the determinant:

$$\begin{aligned} \vec{b} \times \vec{c} &= \left(\left(\frac{8}{3} \times 4 - 0 \times \frac{8}{3} \right) \hat{i} - \left(\frac{4}{3} \times 4 - 4 \times \frac{8}{3} \right) \hat{j} + \left(\frac{4}{3} \times 0 - \frac{8}{3} \times 4 \right) \hat{k} \right) \\ &= \left(\frac{32}{3} \hat{i} + 0 \hat{j} + \left(-\frac{32}{3} \right) \hat{k} \right) \end{aligned}$$

Then, the magnitude is calculated as:

$$\begin{aligned} |\vec{b} \times \vec{c}| &= \sqrt{\left(\frac{32}{3} \right)^2 + 0 + \left(-\frac{32}{3} \right)^2} = \sqrt{\left(\frac{32}{3} \right)^2 + \left(\frac{32}{3} \right)^2} = \sqrt{\frac{2048}{9}} \\ &= \frac{\sqrt{2048}}{3} = \frac{16}{3} \times 3 \approx 16 \end{aligned}$$

Therefore, the area of the parallelogram formed by $\vec{b}$ and $\vec{c}$ is 16.

Question 29

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{d} = \vec{a} \times \vec{b}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - 2\vec{a}|^2 = 8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$ is equal to _____.

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Answer: 6

Solution:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{d} = \vec{a} \times \vec{b}$$

$$= -\hat{i} + \hat{j}$$

$$|\vec{c} - 2\vec{a}|^2 = 8$$

$$|\vec{c}|^2 + 4|\vec{a}|^2 - 4(\vec{a} \cdot \vec{c}) = 8$$

$$c^2 + 12 - 4c = 8$$

$$c^2 - 4c + 4 = 0$$

$$|c| = 2$$

$$\vec{d} = \vec{a} \times \vec{b}$$

$$\vec{d} \times \vec{c} = (\vec{a} \times \vec{b})^2 \times \vec{c}$$

$$\left(|d||c| \sin \frac{\pi}{4}\right)^2 = ((\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{b} \cdot \vec{c}) \cdot \vec{a})^2$$

$$4 = 4b^2 + (\vec{b} \cdot \vec{c})^2 2(a)^2 - 2(\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{b})$$

$$\text{Let } \vec{b} \cdot \vec{c} = x$$

$$4 = 36 + 3x^2 - 20x$$

$$3x^2 - 20x + 32 = 0$$

$$3x^2 - 12x - 8x + 32 = 0$$

$$x = \frac{8}{3}, 4$$

$$\vec{b} \cdot \vec{c} = \frac{8}{3}, 4$$

$$\vec{b} \cdot \vec{c} = \frac{8}{3}$$

$$\text{Now } |10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$$

$$|10 - 8| + (2)^2$$

$$\Rightarrow 6 \text{ Ans.}$$

Question30

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to _____

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Answer: 5

Solution:

Given the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, along with $\vec{c} = \lambda\hat{j} + \mu\hat{k}$, and $\hat{d}$ being a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$, we proceed as follows:

Determine $\hat{d}$:

Since $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$, we have:

$$(\vec{a} - \vec{b}) \times \hat{d} = 0$$

Thus, $\hat{d}$ is parallel to $\vec{a} - \vec{b}$. Calculate:

$$\vec{a} - \vec{b} = (-2\hat{i} + \hat{j} + 2\hat{k})$$

Therefore, we can express $\hat{d}$ as:

$$\hat{d} = \frac{-2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Solve for λ and μ :

Using the condition $\vec{c} \cdot \hat{d} = 1$:

$$\frac{-\lambda}{3} + \frac{2\mu}{3} = 1$$

Simplify to:

$$-\lambda + 2\mu = 3 \quad \dots(i)$$

Since $\vec{c}$ is perpendicular to $\vec{a}$:

$$\vec{c} \cdot \vec{a} = 0$$

Which gives:

$$\lambda + \mu = 0 \Rightarrow \lambda = -\mu$$

Substitute $\lambda = -\mu$ into equation (i):

$$\mu + 2\mu = 3 \Rightarrow 3\mu = 3 \Rightarrow \mu = 1$$

Therefore, $\lambda = -1$.

Find $|3\lambda\hat{d} + \mu\vec{c}|^2$:

Calculate:

$$|3\lambda\hat{d} + \mu\vec{c}|^2 = 9\lambda^2|\hat{d}|^2 + \mu^2|\vec{c}|^2 + 2 \cdot 3 \cdot \lambda \cdot \mu \cdot \vec{c} \cdot \hat{d}$$

Substituting the determined values:

$$|\hat{d}|^2 = 1 \text{ (since } \hat{d} \text{ is a unit vector)}$$

$$\vec{c} \cdot \hat{d} = 1$$

Thus:

$$\begin{aligned}
 &= 9(-1)^2 \cdot 1 + 1^2 \cdot (1^2 + 1^2) + 2 \cdot 3 \cdot (-1) \cdot 1 \cdot 1 \\
 &= 9 + 2 - 6 \\
 &= 5
 \end{aligned}$$

Hence, $|3\lambda\hat{d} + \mu\vec{c}|^2 = 5$.

Question 31

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 4$. Then $|(\vec{a} \times \vec{d})|^2$ is equal to _____.

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Answer: 128

Solution:

Given Conditions :

The condition $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ implies that $(\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$.

This indicates that $\vec{d}$ is parallel to $\vec{b} - \vec{c}$.

Expressing $\vec{b} - \vec{c}$:

Calculate $\vec{b} - \vec{c} = (3\hat{i} - 3\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} - 2\hat{j} + \hat{k}$.

Express $\vec{d}$:

Since $\vec{d}$ is parallel to $\vec{b} - \vec{c}$, let $\vec{d} = \lambda(\hat{i} - 2\hat{j} + \hat{k})$.

Using the condition $\vec{a} \cdot \vec{d} = 4$:

Compute $\vec{a} \cdot \vec{d} = (\hat{i} + 2\hat{j} + \hat{k}) \cdot (\lambda(\hat{i} - 2\hat{j} + \hat{k}))$.

This simplifies to $\lambda(1 \cdot 1 + 2 \cdot (-2) + 1 \cdot 1) = \lambda(1 - 4 + 1) = -2\lambda$.

Set $-2\lambda = 4$, which gives $\lambda = -2$.

Determine $\vec{d}$ using λ :

Therefore, $\vec{d} = -2(\hat{i} - 2\hat{j} + \hat{k}) = -2\hat{i} + 4\hat{j} - 2\hat{k}$.

Calculate $\vec{a} \times \vec{d}$:

Use the determinant form for the cross product:

$$\vec{a} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 4 & -2 \end{vmatrix}$$

Calculate the determinant:

$$\text{For } \hat{i}: (2 \times -2) - (1 \times 4) = -4 - 4 = -8$$

$$\text{For } \hat{j}: -(1 \times -2) - (1 \times -2) = 2 - 2 = 0$$

$$\text{For } \hat{k}: (1 \times 4) - (2 \times -2) = 4 + 4 = 8$$

$$\text{Thus, } \vec{a} \times \vec{d} = -8\hat{i} + 0\hat{j} + 8\hat{k}.$$

Calculate the magnitude squared:

$$\text{Find } |\vec{a} \times \vec{d}|^2 = (-8)^2 + 0^2 + 8^2 = 64 + 0 + 64 = 128.$$

$$\text{Therefore, } |(\vec{a} \times \vec{d})|^2 = 128.$$

Question 32

Let the three sides of a triangle ABC be given by the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6 \left(|\vec{AG}|^2 + |\vec{BG}|^2 + |\vec{CG}|^2 \right)$ is equal to _____.

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Answer: 164

Solution:

Assuming Vertex A to be origin

$$\vec{A} = \vec{a}_1 = \vec{0}$$

$$\vec{B} = \vec{a}_1 + \vec{u} = \vec{u} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{C} = \vec{a}_1 + \vec{v} = \vec{v} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

One solving

$\vec{A} = \vec{0}$, $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, are the position vector of vertices A , B and C respectively.

$$\vec{G} = \frac{1}{3}(\vec{A} + \vec{B} + \vec{C}) = \frac{1}{3}(\vec{0} + \vec{B} + \vec{C}) = \frac{1}{3}(\vec{B} + \vec{C})$$

$$\Rightarrow \vec{G} = \frac{5}{3}\hat{i} - \frac{5}{3}\hat{j} - \hat{k}$$

$$\vec{AG} = \vec{G} - \vec{A} = \vec{G}$$

$$|\vec{AG}|^2 = \left(\frac{5}{3}\right)^2 + \left(-\frac{5}{3}\right)^2 + (-1)^2 = \frac{25}{9} + \frac{25}{9} + 1 = \frac{50}{9} + 1 = \frac{59}{9}$$

$$\vec{BC} = \vec{C} - \vec{B}$$

$$\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{BG}|^2 = \left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + 4 = \frac{1}{9} + \frac{4}{9} + 4 = \frac{5}{9} + 4 = \frac{41}{9}$$

$$\vec{CG} = \vec{G} - \vec{C}$$

$$\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$|\vec{CG}|^2 = \left(-\frac{4}{3}\right)^2 + \left(-\frac{7}{3}\right)^2 + 9 = \frac{16}{9} + \frac{49}{9} + 9 = \frac{65}{9} + 9 = \frac{65}{9} + \frac{81}{9} = \frac{146}{9}$$

$$6 \left(|\vec{AG}|^2 + |\vec{BG}|^2 + |\vec{CG}|^2 \right) = 6 \cdot \left(\frac{59}{9} + \frac{41}{9} + \frac{146}{9} \right) = 6 \cdot \frac{246}{9} = 164$$

Question 33

Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1, 3]$ is :

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Options:

A. 1

B. 3

C. 2

D. 0

Answer: D

Solution:

We are given two unit vectors $\vec{a}$ and $\vec{b}$ with an angle of $\frac{\pi}{3}$ between them. This means:

$$\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = 1 \text{ (since they are unit vectors)}$$

$$\vec{a} \cdot \vec{b} = \cos \frac{\pi}{3} = \frac{1}{2}$$

We need to find the number of values of λ in the interval $[-1, 3]$ for which the vectors $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular, meaning their dot product is zero.

Here's how to solve the problem step by step:

Set up the perpendicular condition by equating the dot product to zero:

$$(\lambda\vec{a} + 2\vec{b}) \cdot (3\vec{a} - \lambda\vec{b}) = 0.$$

Expand the dot product using the distributive property:

$$\begin{aligned}(\lambda\vec{a} + 2\vec{b}) \cdot (3\vec{a} - \lambda\vec{b}) &= \lambda \cdot 3(\vec{a} \cdot \vec{a}) - \lambda^2(\vec{a} \cdot \vec{b}) + 2 \cdot 3(\vec{b} \cdot \vec{a}) - 2\lambda(\vec{b} \cdot \vec{b}) \\&= 3\lambda - \lambda^2 \left(\frac{1}{2} \right) + 6 \left(\frac{1}{2} \right) - 2\lambda \\&= 3\lambda - \frac{\lambda^2}{2} + 3 - 2\lambda.\end{aligned}$$

Simplify the expression:

$$3\lambda - 2\lambda = \lambda,$$

so the equation becomes:

$$\lambda - \frac{\lambda^2}{2} + 3 = 0.$$

Multiply the entire equation by 2 to eliminate the fraction:

$$2\lambda - \lambda^2 + 6 = 0,$$

which can be rearranged as:

$$-\lambda^2 + 2\lambda + 6 = 0.$$

Multiply both sides by -1 to get a standard quadratic form:

$$\lambda^2 - 2\lambda - 6 = 0.$$

Solve the quadratic equation using the quadratic formula:

$$\lambda = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-6)}}{2} = \frac{2 \pm \sqrt{4 + 24}}{2} = \frac{2 \pm \sqrt{28}}{2}.$$

Notice that:

$$\sqrt{28} = 2\sqrt{7},$$

thus:

$$\lambda = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}.$$

Evaluate the solutions numerically:

$$\lambda = 1 + \sqrt{7} \approx 1 + 2.6458 \approx 3.6458,$$

$$\lambda = 1 - \sqrt{7} \approx 1 - 2.6458 \approx -1.6458.$$

Check if the solutions are in the interval $[-1, 3]$:

$$1 + \sqrt{7} \approx 3.6458 \text{ is greater than } 3.$$

$$1 - \sqrt{7} \approx -1.6458 \text{ is less than } -1.$$

Since neither value lies within the interval $[-1, 3]$, there are no valid values of λ in that interval.

Therefore, the number of values of λ in the interval $[-1, 3]$ is:

0

Question 34

Let the arc AC of a circle subtend a right angle at the centre O . If the point B on the arc AC , divides the arc AC such that

$$\frac{\text{length of arc } AB}{\text{length of arc } BC} = \frac{1}{5}, \text{ and } \vec{OC} = \alpha \vec{OA} + \beta \vec{OB}, \text{ then}$$

$\alpha + \sqrt{2}(\sqrt{3} - 1)\beta$ is equal to

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Options:

A. $2\sqrt{3}$

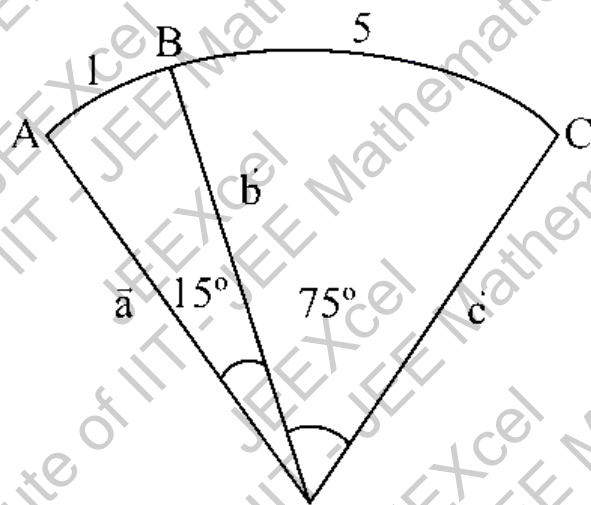
B. $5\sqrt{3}$

C. $2 + \sqrt{3}$

D. $2 - \sqrt{3}$

Answer: D

Solution:



$$\vec{c} = \alpha \vec{a} + \beta \vec{b} \dots (1)$$

$$\vec{a} \cdot \vec{c} = \alpha \vec{a} \cdot \vec{a} + \beta \vec{a} \cdot \vec{b}$$

$$0 = \alpha + \beta \cos 15^\circ \dots (2)$$

$$(1) \Rightarrow \vec{b} \cdot \vec{c} = \alpha \vec{a} \cdot \vec{b} + \beta \vec{b} \cdot \vec{b}$$

$$\Rightarrow \cos 75^\circ = \alpha \cos 15^\circ + \beta \dots (3)$$

$$(2) \& (3) \Rightarrow \cos 75^\circ = -\beta \cos^2 15^\circ + \beta$$

$$\beta = \frac{\cos 75^\circ}{\sin^2 15^\circ} = \frac{1}{\sin 15^\circ} = \frac{2\sqrt{2}}{\sqrt{3} - 1}$$

$$(2) \Rightarrow \alpha = \frac{-\cos 15^\circ}{\sin 15^\circ} = \frac{-(\sqrt{3} + 1)}{(\sqrt{3} - 1)}$$

$$\therefore \vec{c} = \frac{-(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \vec{a} + \left(\frac{2\sqrt{2}}{\sqrt{3} - 1} \right) \vec{b}$$

Now

$$\alpha + \sqrt{2}(\sqrt{3} - 1)\beta = \frac{-(\sqrt{3} + 1)}{(\sqrt{3} - 1)} + \frac{\sqrt{2}(\sqrt{3} - 1) \cdot 2\sqrt{2}}{\sqrt{3} - 1}$$

$$\begin{aligned}
 &= \frac{-(\sqrt{3} + 1)^2}{2} + 4 \\
 &= \frac{-3 - 1 - 2\sqrt{3} + 8}{2} \\
 &= 2 - \sqrt{3}
 \end{aligned}$$

Question 35

Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E.

If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

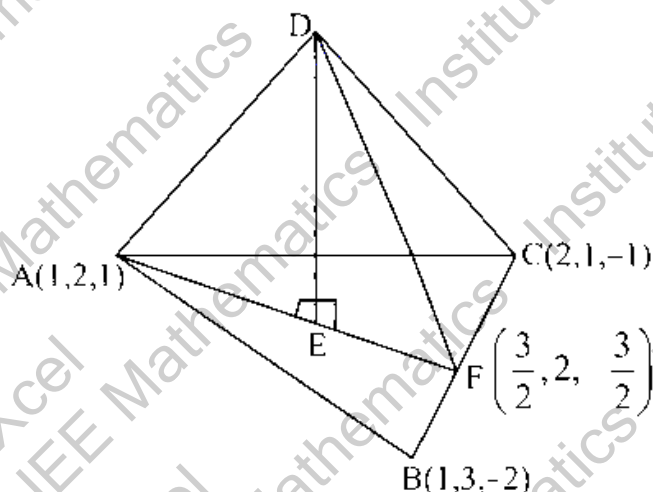
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Options:

- A. $\frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$
- B. $\frac{1}{12}(7\hat{i} + 4\hat{j} + 3\hat{k})$
- C. $\frac{1}{6}(12\hat{i} + 12\hat{j} + \hat{k})$
- D. $\frac{1}{2}(\hat{i} + 4\hat{j} + 7\hat{k})$

Answer: A

Solution:



$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2} |5\hat{i} + 3\hat{j} + \hat{k}| = \frac{1}{2} \sqrt{35}$$

volume of tetrahedron

$$= \frac{1}{3} \times \text{Base area} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$$

$$\frac{1}{3} \times \frac{1}{2} \sqrt{35} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$$

$$h = \sqrt{\frac{23}{2}}$$

$$AE^2 = AD^2 - DE^2 = \frac{13}{18} \therefore AE = \sqrt{\frac{13}{18}}$$

$$\vec{AE} = |AE| \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left(\frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right) = \frac{\hat{i} - 5\hat{k}}{6}$$

$$\text{P.V. of E} = \frac{\hat{i} - 5\hat{k}}{6} + \hat{i} + 2\hat{j} + \hat{k} = \frac{1}{6} (7\hat{i} + 12\hat{j} + \hat{k})$$

Question 36

Let the point A divide the line segment joining the points P(-1, -1, 2) and Q(5, 5, 10) internally in the ratio $r : 1$ ($r > 0$). If O

is the origin and $(\vec{OQ} \cdot \vec{OA}) - \frac{1}{5} |\vec{OP} \times \vec{OA}|^2 = 10$, then the value of r is :

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Options:

A. $\sqrt{7}$

B. 14

C. 7

D. 3

Answer: C

Solution:

$$A \equiv \left(\frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1} \right)$$

$$(\vec{OQ} \cdot \vec{OA}) - \frac{|\vec{OP} \times \vec{OA}|^2}{5} = 10 \quad \dots (1)$$

$$\vec{OQ} \cdot \vec{OA} = \frac{10}{r+1} (15r+1)$$

$$|\vec{OP} \times \vec{OA}|^2 = \frac{r^2}{(r+1)^2} (800)$$

so by equation (1)

$$\frac{10}{r+1} (15r+1) - \frac{1}{5} \frac{r^2(800)}{(r+1)^2} = 10$$

$$2r^2 - 14r = 0$$

$$r = 7, r \neq 0$$

Question 37

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{C}$ is perpendicular to $\vec{b}$ and $\vec{a} \cdot \vec{c} = 5$, then $|\vec{c}|$ is equal to

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Options:

A. $\sqrt{\frac{11}{6}}$

B. $\frac{1}{3\sqrt{2}}$

C. 18

D. 16

Answer: A

Solution:

$$\begin{aligned}\vec{c} &= \lambda(\vec{b} \times (\vec{a} \times \vec{b})) \\ &= \lambda((\vec{b} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b}) \\ &= \lambda(11\vec{a} - 2\vec{b}) = \lambda(11\vec{i} + 22\vec{j} + 33\vec{k} - 6\vec{i} - 2\vec{j} + 2\vec{k}) \\ &= \lambda(5\vec{i} + 20\vec{j} + 35\vec{k}) \\ &= 5\lambda(5\vec{i} + 4\vec{j} + 7\vec{k}) \\ &= \text{Given } \vec{c} \cdot \vec{a} = 5 \\ &= 5\lambda(1 + 8 + 21) = 5 = \lambda = \frac{1}{30} \\ \Rightarrow \vec{c} &= \frac{1}{6}(\vec{i} + 4\vec{j} + 7\vec{k}) \\ |\vec{c}| &= \frac{\sqrt{1 + 16 + 49}}{6} = \sqrt{\frac{11}{6}}\end{aligned}$$

Question 38

Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is :

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Options:

A. $2\sqrt{7}$

B. $3\sqrt{7}$

C. $\sqrt{14}$

D. $2\sqrt{14}$

Answer: D

Solution:

$$\begin{aligned}\vec{b} &= \vec{a} \times (\hat{i} - 3\hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}\end{aligned}$$

$$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j}$$

$$\vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$$

Projection of $(\hat{i} - 2\hat{j})$ on $\vec{a}$

$$(\vec{c} - 2\hat{j}) \cdot \hat{a} = \frac{\langle 8, -4, 0 \rangle \cdot \langle 3, -1, 2 \rangle}{\sqrt{14}}$$

$$= \frac{28}{\sqrt{14}} = 2\sqrt{14}$$

Question 39

Let the position vectors of three vertices of a triangle be $4\vec{p} + \vec{q} - 3\vec{r}$, $-5\vec{p} + \vec{q} + 2\vec{r}$ and $2\vec{p} - \vec{q} + 2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$ and $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$ respectively, then $\alpha + 2\beta + 5\gamma$ is equal to :

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Options:

A. 4

B. 3

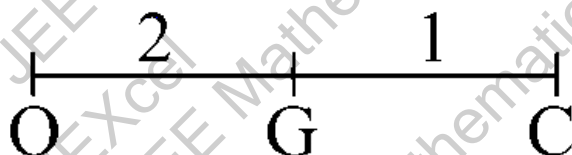
C. 1

D. 6

Answer: B

Solution:

We know that,



$$O \text{ (orthocentre)} \quad \frac{\vec{p} + \vec{q} + \vec{r}}{4}$$

$$C \text{ (circum centre)} \quad \alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$$

$$C \text{ (centroid)} = \frac{\vec{p} + \vec{q} + \vec{r}}{3}$$

by relation

$$\Rightarrow 2(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}) + \frac{\vec{p} + \vec{q} + \vec{r}}{4} = 3 \left(\frac{\vec{p} + \vec{q} + \vec{r}}{3} \right)$$

$$\Rightarrow 8(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}) = 3(\vec{p} + \vec{q} + \vec{r})$$

$$\Rightarrow 8\alpha = 3, 8\beta = 3, 8\gamma = 3$$

$$\alpha = \frac{3}{8}, \beta = \frac{3}{8}, \gamma = \frac{3}{8}$$

$$\therefore \alpha + 2\beta + 3\gamma$$

$$\frac{3}{8} + \frac{6}{8} + \frac{15}{8} = \frac{24}{8} = 3$$

Question 40

Let A, B, C be three points in xy -plane, whose position vector are given by $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $a\hat{i} + (1-a)\hat{j}$ respectively with respect to the origin O . If the distance of the point C from the line

bisecting the angle between the vectors $\vec{OA}$ and $\vec{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :

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Options:

A.

2

B.

0

C.

$\frac{9}{2}$

D.

1

Answer: D

Solution:

Equation of angle bisector : $x - y = 0$

$$\left| \frac{a(1-a)}{\sqrt{2}} \right| = \frac{9}{\sqrt{2}} \Rightarrow a = 5 \text{ or } -4$$

$$\text{Sum} = 5 + (-4) = 1$$

Question 41

If the components of $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively, are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

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Options:

A.

16

B.

23

C.

26

D.

18

Answer: C**Solution:**

let

 $\vec{a}_{11}$ = component of $\vec{a}$ along $\vec{b}$ $\vec{a}_1$ = component of $\vec{a}$ perpendicular to $\vec{b}$

$$\vec{a}_{11} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{a}_1 = \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$$

$$\therefore \vec{a} = \vec{a}_{11} + \vec{a}_1$$

$$\therefore \vec{a} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) + \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$$

$$= \frac{44}{11}\hat{i} + \frac{11}{11}\hat{j} - \frac{33}{11}\hat{k}$$

$$\vec{a} = 4\hat{i} + \hat{j} - 3\hat{k}$$

$$\alpha = 4 \quad \beta = 1 \quad \gamma = -3$$

$$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$$

Question 42

Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$. Then the maximum value of $|\vec{c}|^2$ is :

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Options:

A.

77

B.

154

C.

308

D.

462

Answer: C

Solution:

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$$

$$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$$

$$(\vec{a} + \vec{b}) \times \vec{c} = 0$$

$$\Rightarrow \vec{c} = \lambda(\vec{a} + \vec{b})$$

$$\vec{c} = \lambda(5\hat{i} - 6\hat{j} + 4\hat{k}) \dots (1)$$

$$|\vec{c}|^2 = \lambda^2(25 + 36 + 16)$$

$$|\vec{c}|^2 = 77\lambda^2$$

$$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$$

$$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 168$$

$$14 + \vec{c} \cdot (\vec{a} + \vec{b}) + 77\lambda^2 = 168$$

using equation (1)

$$\lambda|5\hat{i} - 6\hat{j} + 4\hat{k}|^2 + 77\lambda^2 = 154$$

$$77\lambda + 77\lambda^2 - 154 = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda = -2, 1$$

$\therefore$ Maximum value of $|\vec{c}|^2$ occurs when $\lambda = -2$

$$\begin{aligned} |\vec{c}|^2 &= 77\lambda^2 \\ &= 77 \times 4 \\ &= 308 \end{aligned}$$

Question43

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let

$L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda\vec{a}, \lambda \in \mathbf{R}$ and $L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu\vec{b}, \mu \in \mathbf{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2 and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point:

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Options:

A.

$$(-1, -1, 1)$$

B.

$$(2, 8, 5)$$

C.

$$(8, 26, 12)$$

D.

$$(5, 17, 4)$$

Answer: C

Solution:

$$L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{r} = (\lambda - 1)\hat{i} + 2(\lambda + 1)\hat{j} + (\lambda + 1)\hat{k}$$

$$L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu(2\hat{i} + 7\hat{j} + 3\hat{k})$$

$$\Rightarrow \vec{r} = 2\mu\hat{i} + (1 + 7\mu)\hat{j} + (1 + 3\mu)\hat{k}$$

For point of intersection equating respective components

$$\Rightarrow \lambda - 1 = 2\mu \quad \dots\dots (1)$$

$$2(\lambda + 1) = 1 + 7\mu \quad \dots\dots (2)$$

$$\lambda + 1 = 1 + 3\mu \quad \dots\dots (3)$$

We get

$$\Rightarrow \lambda = 3 \text{ and } \mu = 1$$

$$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}$$

$$L_3 : \vec{r} = 2\hat{i} + 8\hat{j} + 4\hat{k} + \alpha(3\hat{i} + 9\hat{j} + 4\hat{k})$$

$$\text{For } \alpha = 2, \vec{r} = 8\hat{i} + 26\hat{j} + 12\hat{k}$$

Question44

Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - \hat{k}$, and $\hat{a}$ makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i} + \hat{j} + \hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i} + \alpha\hat{j} + \hat{k}$, then the value of a is:

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

$$\sqrt{3}$$

B.

$$\sqrt{6}$$

C.

$$-\sqrt{6}$$

D.

$$-\sqrt{3}$$

Answer: C

Solution:

$$\text{Let } \vec{v} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= -7\hat{i} + 7\hat{j} + 7\hat{k}$$

$$= -7(\hat{i} - \hat{j} - \hat{k})$$

$$\text{Now } \hat{a} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}} \text{ or } \frac{-\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\downarrow$$

$$\downarrow$$

$$\cos \theta = \frac{\hat{a} \cdot \vec{v}}{|\hat{a}| |\vec{v}|} = \frac{1 - 1 - 1}{\sqrt{3}\sqrt{3}} = \frac{-1}{3} \quad \cos \theta = \frac{\hat{a} \cdot \vec{v}}{|\hat{a}| |\vec{v}|} = \frac{-1 + 1 + 1}{3} = \frac{1}{3} \quad (\text{rejected})$$

$$\Rightarrow \hat{a} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

$$\text{Now } \cos \frac{\pi}{3} = \frac{\hat{a} \cdot (\hat{i} + \alpha \hat{j} + \hat{k})}{\sqrt{1 + \alpha^2 + 1}}$$

$$\Rightarrow \frac{1}{2} = \frac{1 - \alpha - 1}{\sqrt{3}\sqrt{\alpha^2 + 2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \sqrt{\alpha^2 + 2} = -\alpha \quad (\because \alpha < 0)$$

$$3\alpha^2 + 6 = 4\alpha^2$$

$$\Rightarrow \alpha = -\sqrt{6}$$

Question 45

If $\vec{a}$ is a nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is :

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Options:

A. $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} + 5\hat{k})$

B. $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} - 5\hat{k})$

C. $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} - 5\hat{k})$

D. $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$

Answer: D

Solution:

Projection of $\vec{a}$ on $\vec{v}$

$$\begin{aligned} &= \frac{\vec{a} \cdot \vec{v}}{|\vec{v}|} \\ &\Rightarrow \frac{\vec{a} \cdot (2\hat{i} - \hat{j} + 2\hat{k})}{3} = \frac{\vec{a} \cdot \hat{k}}{1} = \frac{\vec{a} \cdot (\hat{i} + 2\hat{j} - 2\hat{k})}{3} \\ &\Rightarrow \vec{a} \cdot (2\hat{i} - \hat{j} - \hat{k}) = 0 \text{ and } \vec{a} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) = 0 \\ &\Rightarrow \vec{a} \perp (2\hat{i} - \hat{j} - \hat{k}) \text{ and } (\hat{i} + 2\hat{j} - 5\hat{k}) \\ &\Rightarrow \vec{a} \parallel (2\hat{i} - \hat{j} - \hat{k}) \times (\hat{i} + 2\hat{j} - 5\hat{k}) \\ &\Rightarrow \vec{a} = \pm k \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & -1 & -1 \\ 1 & 2 & -5 \end{vmatrix} = \pm k(7\hat{i} + 9\hat{j} - 5\hat{k}) \\ &\Rightarrow \text{Unit vector will be } \frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k}) \end{aligned}$$

Question 46

Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\vec{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a} \cdot \vec{d}|$ is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 15

B. 18

C. 12

D. 9

Answer: A

Solution:

To solve the problem, we start with the given equation:

$$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}.$$

Expanding this, we have:

$$\vec{a} \times \vec{b} - \vec{c} \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}.$$

Let $\vec{d} = \vec{c} \times \vec{b}$. Substituting this into the equation, we get:

$$\vec{a} \times \vec{b} + \vec{d} = -18\hat{i} - 3\hat{j} + 12\hat{k}.$$

Now, we take the dot product of both sides with $\vec{a}$:

$$\vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{a} \cdot \vec{d} = \vec{a} \cdot (-18\hat{i} - 3\hat{j} + 12\hat{k}).$$

Since the dot product $\vec{a} \cdot (\vec{a} \times \vec{b})$ is zero (the dot product of any vector with the cross product of itself with another vector is zero), we have:

$$\vec{a} \cdot \vec{d} = -36 + 9 + 12 = -15.$$

Thus, the magnitude of $\vec{a} \cdot \vec{d}$ is:

$$|\vec{a} \cdot \vec{d}| = 15.$$

Question 47

Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where

$\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $\left|\vec{v}_1\right|^2 + \left|\vec{v}_2\right|^2$ is equal to

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. $\frac{23}{2}$

B. $\frac{25}{2}$

C. 10

D. 14

Answer: D

Solution:

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos \theta$$

$$\Rightarrow 6 - 1 = \sqrt{10} \cdot \sqrt{5 + \lambda^2} \cdot \frac{\sqrt{5}}{2\sqrt{7}}$$

$$\Rightarrow 1 = \sqrt{2} \cdot \sqrt{5 + \lambda^2} \cdot \frac{1}{2\sqrt{7}}$$

$$\Rightarrow 14 = 5 + \lambda^2$$

$$\Rightarrow \lambda^2 = 9$$

$$\Rightarrow \lambda = 3$$

$$\vec{v}_1 = k\vec{u}$$

$$\vec{v} = \vec{v}_1 + \vec{v}_2$$

$$\Rightarrow \vec{v} = k\vec{u} + \vec{v}_2$$

$$\vec{v} \cdot \vec{u} = k \cdot |\vec{u}|^2$$

$$\Rightarrow 5 = k \cdot 10 \Rightarrow k = \frac{1}{2}$$

$$\therefore \vec{v}_1 = \frac{\vec{u}}{2} = \frac{3\hat{i}}{2} - \frac{\hat{j}}{2}$$

$$|\vec{v}_1|^2 = \frac{10}{4}$$

$$\vec{v}_2 = \vec{v} - \vec{v}_1$$

$$= \frac{1}{2}\hat{i} + \frac{3\hat{j}}{2} - 3\hat{k}$$

$$|\vec{v}_2|^2 = \frac{10}{4} + 9$$

$$|\vec{v}_1|^2 + |\vec{v}_2|^2 = \frac{10}{4} + \frac{10}{4} + 9 = 14$$

Question 48

Let the angle θ , $0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$, then the value of $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$ is

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 31

B. 29

C. 24

D. 27

Answer: B

Solution:

To solve the problem, we begin by examining the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$.

Let's calculate:

$$\vec{c} \cdot \hat{a} = 3 + 6(\hat{a} \cdot \hat{b}).$$

$$\vec{c} \cdot \hat{b} = 3(\hat{a} \cdot \hat{b}) + 6.$$

We need to find the value of $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$.

Substituting the expressions, we get:

$$9(\vec{c} \cdot \hat{a}) = 9(3 + 6\hat{a} \cdot \hat{b}) = 27 + 54(\hat{a} \cdot \hat{b})$$

$$3(\vec{c} \cdot \hat{b}) = 3(3\hat{a} \cdot \hat{b} + 6) = 9\hat{a} \cdot \hat{b} + 18$$

Therefore,

$$9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b}) = (27 + 54(\hat{a} \cdot \hat{b})) - (9\hat{a} \cdot \hat{b} + 18)$$

Simplifying,

$$= 27 - 18 + 54(\hat{a} \cdot \hat{b}) - 9(\hat{a} \cdot \hat{b})$$

$$= 9 + 45(\hat{a} \cdot \hat{b})$$

Given $\sin \theta = \frac{\sqrt{65}}{9}$, and knowing that for unit vectors $\cos \theta = \hat{a} \cdot \hat{b}$, we use the identity $(\cos \theta)^2 = 1 - (\sin \theta)^2$:

$$(\hat{a} \cdot \hat{b})^2 = 1 - \left(\frac{\sqrt{65}}{9}\right)^2$$

$$= 1 - \frac{65}{81}$$

$$= \frac{16}{81}$$

Thus, $\hat{a} \cdot \hat{b} = \frac{4}{9}$.

Substitute back into the equation:

$$9 + 45 \times \frac{4}{9} = 9 + 20 = 29$$

Therefore, the value is 29.

Question 49

Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that

$$\frac{|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}| - |\vec{a} - \vec{b}|} = \sqrt{2} + 1. \text{ Then } \frac{|\vec{a} + \vec{b}|^2}{|\vec{a}|^2} \text{ is :}$$

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

$$2 + \sqrt{2}$$

B.

$$2 + 4\sqrt{2}$$

C.

$$4 + 2\sqrt{2}$$

D.

$$1 + \sqrt{2}$$

Answer: A

Solution:

$$\frac{|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}| - |\vec{a} - \vec{b}|} = \sqrt{2} + 1$$

Apply componendo and dividendo

$$\begin{aligned}
 &\Rightarrow \frac{2|\vec{a} + \vec{b}|}{2|\vec{a} - \vec{b}|} = \frac{\sqrt{2} + 2}{\sqrt{2}} \\
 &\Rightarrow |\vec{a} + \vec{b}| = (1 + \sqrt{2})|\vec{a} - \vec{b}| \\
 &\Rightarrow |\vec{a} + \vec{b}|^2 = (3 + 2\sqrt{2})|\vec{a} - \vec{b}|^2 \\
 &\Rightarrow 2|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} = (3 + 2\sqrt{2})(2|\vec{a}|^2 - 2\vec{a} \cdot \vec{b}) \\
 &\Rightarrow 2|\vec{a}|^2(2 + 2\sqrt{2}) = 2\vec{a} \cdot \vec{b}(4 + 2\sqrt{2}) \\
 &\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} = \frac{2 + 2\sqrt{2}}{4 + 2\sqrt{2}} = \frac{1}{\sqrt{2}}
 \end{aligned}$$

Now

$$\begin{aligned}
 \frac{|\vec{a} + \vec{b}|^2}{|\vec{a}|^2} &= 1 + \frac{|\vec{b}|^2}{|\vec{a}|^2} + \frac{2\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \\
 &= 1 + 1 + 2\left(\frac{1}{\sqrt{2}}\right) = 2 + \sqrt{2}
 \end{aligned}$$

Question 50

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is:

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

$$\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$$

B.

$$\frac{1}{\sqrt{5}}(\hat{j} - 2\hat{k})$$

C.

$$\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$$

D.

$$\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$$

Answer: A

Solution:

Let vector $\vec{p}$ in plane of $\vec{a}$ & $\vec{b} = K(\vec{a} + \lambda\vec{b})$

$$\vec{p} \perp \vec{a} \Rightarrow \vec{p} \cdot \vec{a} = 0$$

$$\Rightarrow K(\vec{a} + \lambda\vec{b}) \cdot \vec{a} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a} = 0$$

$$\Rightarrow 6 + \lambda(3) = 0$$

$$\Rightarrow \lambda = -2$$

$$\Rightarrow \vec{p} = (-3\hat{i} + 3\hat{k})$$

$$\text{Unit vector} \rightarrow \pm \frac{(-\hat{i} + \hat{k})}{\sqrt{2}}$$

Question 51

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that

$\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to :

[27-Jan-2024 Shift 1]

Options:

A. 32

B. 24

C. 20

D. 36

Answer: B

Solution:

$$\vec{a} \cdot [(\vec{c} \times \vec{b}) - \vec{b} - \vec{c}]$$

$$\vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} \dots\dots\dots(i)$$

$$\text{given } \vec{a} \times \vec{c} = \vec{b}$$

$$\Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} = \vec{b} \cdot \vec{b} = |\vec{b}|^2 = 27$$

$$\Rightarrow \vec{a} \cdot (\vec{c} \times \vec{b}) = \left[\begin{array}{ccc} \vec{a} & \vec{c} & \vec{b} \end{array} \right] = (\vec{a} \times \vec{c}) \cdot \vec{b} = 27 \dots\dots\dots(ii)$$

$$\text{Now } \vec{a} \cdot \vec{b} = 3 - 6 + 3 = 0 \dots\dots\dots(iii)$$

$$\vec{a} \cdot \vec{c} = 3 \dots(iv) \text{ (given)}$$

By (i), (ii), (iii) & (iv)

$$27 - 0 - 3 = 24$$

Question52

The least positive integral value of α , for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is ____
[27-Jan-2024 Shift 1]

Answer: 5

Solution:

$$\cos \theta = \frac{(\hat{a}_i - 2\hat{j} + 2\hat{k}) \cdot (\hat{a}_i + 2\hat{j} - 2\hat{k})}{\sqrt{a^2 + 4 + 4} \sqrt{a^2 + 4a^2 + 4}}$$

$$\cos \theta = \frac{a^2 - 4a - 4}{\sqrt{a^2 + 8} \sqrt{5a^2 + 4}}$$

$$\Rightarrow a^2 - 4a - 4 > 0$$

$$\Rightarrow a^2 - 4a + 4 > 8$$

$$\Rightarrow (a-2)^2 > 8$$

$$\Rightarrow a-2 > 2\sqrt{2} \text{ or } a-2 < -2\sqrt{2}$$

$$a > 2+2\sqrt{2} \text{ or } a < 2-2\sqrt{2}$$

$$a \in (-\infty, -0.82) \cup (4.82, \infty)$$

Least positive integral value of $a \Rightarrow 5$

Question53

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. if $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to
[29-Jan-2024 Shift 1]

Options:

A. 35

B. 30

C. -30

D. -25

Answer: A

Solution:

$$\vec{a} + 5\vec{b} = \lambda\vec{c}$$

$$\vec{b} + 6\vec{c} = \mu\vec{a}$$

Eliminating $\vec{a}$

$$\lambda\vec{c} - 5\vec{b} = \frac{6}{\mu}\vec{c} + \frac{1}{\mu}\vec{b}$$

$$\therefore \mu = \frac{-1}{5}, \lambda = -30$$

$$\alpha = 5, \beta = 30$$

Question 54

Let $\vec{OA} = \vec{a}$, $\vec{OB} = 12\vec{a} + 4\vec{b}$ and $\vec{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC , then

$\frac{\text{area of the quadrilateral } OABC}{\text{area of } S}$ is equal to

[29-Jan-2024 Shift 2]

Options:

A. 6

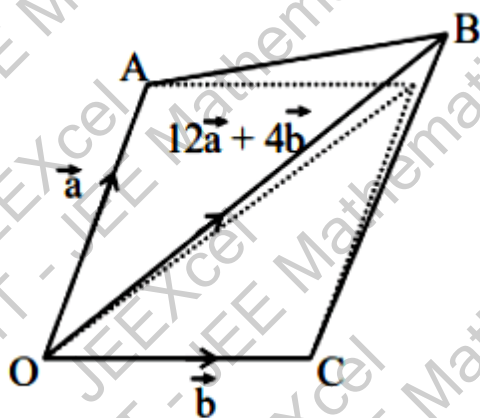
B. 10

C. 7

D. 8

Answer: D

Solution:



Area of parallelogram, $S = |\vec{a} \times \vec{b}|$

Area of quadrilateral = Area($\triangle OAB$) + Area($\triangle OBC$)

$$= \frac{1}{2} \{ |\vec{a} \times (12\vec{a} + 4\vec{b})| + |\vec{b} \times (12\vec{a} + 4\vec{b})| \}$$

$$= 8 |\vec{a} \times \vec{b}|$$

$$\text{Ratio} = \frac{8 |\vec{a} \times \vec{b}|}{|\vec{a} \times \vec{b}|} = 8$$

Question 55

Let $\vec{OA} = \vec{a}$, $\vec{OB} = 12\vec{a} + 4\vec{b}$ and $\vec{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC , then

$\frac{\text{area of the quadrilateral } OACB}{\text{area of } S}$ is equal to

[29-Jan-2024 Shift 2]

Options:

A. 6

B. 10

C. 7

D. 8

Answer: D

Question 56

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$; $\vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A. $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

B. $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

C. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

D. $\cos^{-1}\left(\frac{2}{3}\right)$

Answer: C

Solution:

Given $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2$

$$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$$

Dot product with $\vec{a}$ on both sides

$$\vec{c} \cdot \vec{a} = -6 \dots\dots(1)$$

Dot product with $\vec{b}$ on both sides

$$\vec{b} \cdot \vec{c} = -48 \dots\dots(2)$$

$$\vec{c} \cdot \vec{c} = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4[|a|^2 |b|^2 - (a \cdot b)^2] + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4[(1)(4)^2 - (4)] + 9(16)$$

$$|\vec{c}|^2 = 4[12] + 144$$

$$|\vec{c}|^2 = 48 + 144$$

$$|\vec{c}|^2 = 192$$

$$\therefore \cos \theta = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| |\vec{c}|}$$

$$\therefore \cos \theta = \frac{-48}{\sqrt{192} \cdot 4}$$

$$\therefore \cos \theta = \frac{-48}{8\sqrt{3} \cdot 4}$$

$$\therefore \cos \theta = \frac{-3}{2\sqrt{3}}$$

$$\therefore \cos \theta = \frac{-\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

Question 57

Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}$, $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If

$\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u} - \vec{v}|^2$ is equal to

[29-Jan-2024 Shift 2]

Options:

A. $\frac{11}{2}$

B. $\frac{5}{2}$

C. 9

D. 7

Answer: B

Solution:

$$\text{Unit vector } \hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{p}_1 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \vec{p}_2 = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{p}_3 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$\text{Now angle between } \hat{u} \text{ and } \vec{p}_1 = \frac{\pi}{2}$$

$$\hat{u} \cdot \vec{p}_1 = 0 \Rightarrow \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$$

$$\Rightarrow x + z = 0 \dots\dots\dots(i)$$

$$\text{Angle between } \hat{u} \text{ and } \vec{p}_2 = \frac{\pi}{3}$$

$$\hat{u} \cdot \vec{p}_2 = |\hat{u}| \cdot |\vec{p}_2| \cos \frac{\pi}{3}$$

$$\Rightarrow \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \Rightarrow y + z = \frac{1}{\sqrt{2}} \dots\dots\dots(ii)$$

$$\text{Angle between } \hat{u} \text{ and } \vec{p}_3 = \frac{2\pi}{3}$$

$$\hat{u} \cdot \vec{p}_3 = |\hat{u}| \cdot |\vec{p}_3| \cos \frac{2\pi}{3}$$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{-1}{2} \Rightarrow x + y = \frac{-1}{\sqrt{2}} \dots\dots\dots(iii)$$

from equation (i), (ii) and (iii) we get

$$x = \frac{-1}{\sqrt{2}} \quad y = 0 \quad z = \frac{1}{\sqrt{2}}$$

$$\text{Thus } \hat{u} \cdot \vec{v} = \frac{-1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{k} - \frac{1}{\sqrt{2}} \hat{i} - \frac{1}{\sqrt{2}} \hat{j} - \frac{1}{\sqrt{2}} \hat{k}$$

$$\hat{u} \cdot \vec{v} = \frac{-2}{\sqrt{2}} \hat{i} - \frac{1}{\sqrt{2}} \hat{j}$$

$$|\hat{u} \cdot \vec{v}|^2 = \left(\sqrt{\frac{4}{2} + \frac{1}{2}} \right)^2 = \frac{5}{2}$$

Question 58

Let $A(2, 3, 5)$ and $C(-3, 4, -2)$ be opposite vertices of a parallelogram ABCD if the diagonal $\vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to
[30-Jan-2024 Shift 1]

Options:

A. $\frac{1}{2}\sqrt{410}$

B. $\frac{1}{2}\sqrt{474}$

C. $\frac{1}{2}\sqrt{586}$

D. $\frac{1}{2}\sqrt{306}$

Answer: B

Solution:

$$\text{Area} = |\vec{AC} \times \vec{BD}|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2} |-17\hat{i} - 8\hat{j} + 11\hat{k}| = \frac{1}{2}\sqrt{474}$$

Question 59

Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$, $\alpha, \beta \in \mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$, If $\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2)|\vec{a} \times \vec{b}|^2$ is equal to
[30-Jan-2024 Shift 2]

Options:

A. 90

B. 75

C. 95

D. 85

Answer: A

Solution:

$$|\vec{b}|^2 = 6; |\vec{a}| |\vec{b}| \cos \theta = 3\sqrt{2}$$

$$|\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta = 18$$

$$|\vec{a}|^2 = 6$$

$$\text{Also } 1 + \alpha^2 + \beta^2 = 6$$

$$\alpha^2 + \beta^2 = 5$$

to find

$$(\alpha^2 + \beta^2) |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta$$

$$= (5)(6)(6) \left(\frac{1}{2} \right)$$

$$= 90$$

Question 60

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then

$\left| \left(\vec{b} \times \vec{a} \right) - \vec{b} \right|^2$ is equal to

[30-Jan-2024 Shift 2]

Options:

A. 3

B. 5

C. 1

D. 4

Answer: B

Solution:

$$|\vec{b}| = 1 \text{ \& } |\vec{b} \times \vec{a}| = 2$$

$$(\vec{b} \times \vec{a}) \cdot \vec{b} = \vec{b} \cdot (\vec{b} \times \vec{a}) = 0$$

$$\begin{aligned} |(\vec{b} \times \vec{a}) - \vec{b}|^2 &= |\vec{b} \times \vec{a}|^2 + |\vec{b}|^2 \\ &= 4 + 1 = 5 \end{aligned}$$

Question61

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then

$\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to
[31-Jan-2024 Shift 1]

Options:

A. 24

B. 36

C. 28

D. 32

Answer: D

Solution:

$$\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$$

$$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$$

$$\vec{p} - \vec{c} = \lambda \vec{b} \Rightarrow \vec{p} = \vec{c} + \lambda \vec{b}$$

$$\text{Now, } \vec{p} \cdot \vec{a} = 0 \text{ (given)}$$

$$\text{So, } \vec{c} \cdot \vec{a} + \lambda \vec{a} \cdot \vec{b} = 0$$

$$(3 - 3 - 8) + \lambda(12 + 1 - 14) = 0$$

$$\lambda = -8$$

$$\vec{p} = \vec{c} - 8\vec{b}$$

$$\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$$

$$\text{So, } \vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$$

$$= -31 + 11 + 52$$

$$= 32$$

Question 62

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α , then $192 \sin^2 \alpha$ is equal to _____
[31-Jan-2024 Shift 1]

Answer: 48

Solution:

$$\vec{b} \cdot \vec{c} = (2\vec{a} \times \vec{b}) \cdot \vec{b} - 3|\vec{b}|^2$$

$$|\vec{b}||\vec{c}|\cos\alpha = -3|\vec{b}|^2$$

$$|\vec{c}|\cos\alpha = -12, \text{ as } |\vec{b}| = 4$$

$$\vec{a} \cdot \vec{b} = 2$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$|\vec{c}|^2 = |(2\vec{a} \times \vec{b}) - 3\vec{b}|^2$$

$$= 64 \times \frac{3}{4} + 144 = 192$$

$$|\vec{c}|^2 \cos^2\alpha = 144$$

$$192 \cos^2\alpha = 144$$

$$192 \sin^2\alpha = 48$$

Question63

Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that

$(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$. Then $|\vec{c}|^2$

is equal to

[31-Jan-2024 Shift 2]

Answer: 38

Solution:

$$(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$$

$$(5\hat{i} + \hat{j} + 4\hat{k}) \times \vec{c} = 2(7\hat{i} - 7\hat{j} - 7\hat{k}) + 24\hat{j} - 6\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ x & y & z \end{vmatrix} = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$$\Rightarrow \hat{i}(z - 4y) - \hat{j}(5z - 4x) + \hat{k}(5y - x) = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$$z - 4y = 14, 4x - 5z = 10, 5y - x = -20$$

$$(\vec{a} - \vec{b} + \vec{i}) \cdot \vec{c} = -3$$

$$(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot \vec{c} = -3$$

$$2x + 3y - 2z = -3$$

$$\therefore x = 5, y = -3, z = 2$$

$$|\vec{c}|^2 = 25 + 9 + 4 = 38$$

Question 64

Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = \left(\left(\left(\vec{a} \times \vec{b} \right) \times \hat{i} \right) \times \hat{i} \right) \times \hat{i}$.

Then $\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k})$ is equal to
[1-Feb-2024 Shift 1]

Options:

A. -12

B. -10

C. -13

D. -15

Answer: A

Solution:

$$\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$(\vec{a} \times \vec{b}) \times \hat{i} = (\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}$$

$$= -5\vec{b} - \vec{a}$$

$$= ((-5\vec{b} - \vec{a}) \times \hat{i}) \times \hat{i}$$

$$= ((-11\hat{j} + 23\hat{k}) \times \hat{i}) \times \hat{i}$$

$$\Rightarrow (11\hat{k} + 23\hat{j}) \times \hat{i}$$

$$\Rightarrow (11\hat{j} - 23\hat{k})$$

$$\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 11 - 23 = -12$$

Question 65

Consider a $\triangle ABC$ where $A(1, 2, 3)$, $B(-2, 8, 0)$ and $C(3, 6, 7)$. If the angle bisector of $\angle BAC$ meets the line BC at D , then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:
[1-Feb-2024 Shift 2]

Options:

A. $\frac{37}{2\sqrt{38}}$

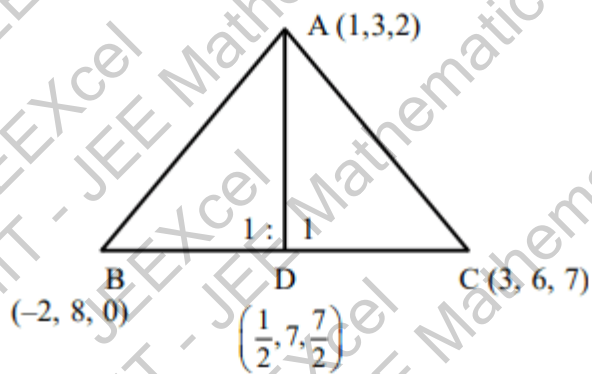
B. $\frac{\sqrt{38}}{2}$

C. $\frac{39}{2\sqrt{38}}$

D. $\sqrt{19}$

Answer: A

Solution:



$$A(1, 3, 2); B(-2, 8, 0); C(3, 6, 7);$$

$$\vec{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$AB = \sqrt{9 + 25 + 4} = \sqrt{38}$$

$$AC = \sqrt{4 + 9 + 25} = \sqrt{38}$$

$$\vec{AD} = \frac{1}{2}\hat{i} - 4\hat{j} - \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} - 8\hat{j} + 3\hat{k})$$

Length of projection of $\vec{AD}$ on $\vec{AC}$

$$= \left| \frac{\vec{AD} \cdot \vec{AC}}{|\vec{AC}|} \right| = \frac{37}{2\sqrt{38}}$$

Question 66

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ , then the greatest integer less than or equal to $\tan^2 \theta$ is :

[1-Feb-2024 Shift 2]

Answer: 38

Solution:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$$

$$(\vec{b} - \vec{c}) \times \vec{a} = 0$$

$$\vec{b} - \vec{c} = \lambda \vec{a}$$

$$\vec{b} = \vec{c} + \lambda \vec{a}$$

$$-\hat{i} - 8\hat{j} + 2\hat{k} = (4\hat{i} + c_2\hat{j} + c_3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$$

$$\lambda + 4 = -1 \Rightarrow \lambda = -5$$

$$\lambda + c_2 = -8 \Rightarrow c_2 = -3$$

$$\lambda + c_3 = 2 \Rightarrow c_3 = 7$$

$$\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\cos \theta = \frac{12 - 12 + 7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{2\sqrt{481}}$$

$$\tan^2 \theta = \frac{625 \times 3}{49}$$

$$[\tan^2 \theta] = 38$$

Question 67

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be

$$\vec{a} - \vec{b} + \vec{c}, \quad \lambda \vec{a} - 3\vec{b} + 4\vec{c}, \quad -\vec{a} + 2\vec{b} - 3\vec{c} \quad \text{and} \quad 2\vec{a} - 4\vec{b} + 6\vec{c}$$

respectively. If $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are coplanar, then λ is :

Official Ans. by NTA

[29-Jan-2023 Shift 1]

Answer: 2

Solution:

$$\vec{AB} = (\lambda - 1)\vec{a} - 2\vec{b} + 3\vec{c}$$

$$\vec{AC} = 2\vec{a} + 3\vec{b} - 4\vec{c}$$

$$\vec{AD} = \vec{a} - 3\vec{b} + 5\vec{c}$$

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$$

$$\Rightarrow (\lambda - 1) = 1 \Rightarrow \lambda = 2$$

Question 68

Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to

[24-Jan-2023 Shift 1]

Options:

A. 1

B. $\frac{3}{2}$

C. 2

D. $-\frac{2}{3}$

Answer: A

Solution:

$$\vec{u} = (1, -1, -2), \vec{v} = (2, 1, -1), \vec{v} \cdot \vec{w} = 2 \\ \vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v} \dots\dots\dots (1)$$

Taking dot with $\vec{w}$ in (1)

$$\vec{w} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{w} + \lambda \vec{v} \cdot \vec{w} \\ \Rightarrow 0 = \vec{u} \cdot \vec{w} + 2\lambda$$

Taking dot with $\vec{v}$ in (1)

$$\vec{v} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{v} + \lambda \vec{v} \cdot \vec{v} \\ \Rightarrow 0 = (2 - 1 + 2) + \lambda(6)$$

$$\lambda = -\frac{1}{2}$$

$$\Rightarrow \vec{u} \cdot \vec{w} = -2\lambda = 1$$

Question 69

Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of

$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

[24-Jan-2023 Shift 2]

Options:

A. 6

B. 11

C. 7

D. 9

Answer: C

Solution:

$$\text{Let } \vec{\beta}_1 = \lambda \vec{\alpha}$$

$$\text{Now } \vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$

$$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$$

$$\vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$$

$$\Rightarrow 4 - 16\lambda + 6 - 9\lambda - 25\lambda - 20 = 0$$

$$\Rightarrow 50\lambda = -10$$

$$\Rightarrow \lambda = -\frac{1}{5}$$

$$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$$

$$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$$

$$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$$

$$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$$

Question 70

$$\text{Let } \vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \quad \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \quad \vec{a} \cdot \vec{c} = 7,$$

$$2\vec{b} \cdot \vec{c} + 43 = 0, \quad \vec{a} \times \vec{c} = \vec{b} \times \vec{c}. \text{ Then } |\vec{a} \cdot \vec{b}| \text{ is equal to}$$

[24-Jan-2023 Shift 2]

Answer: 8

Solution:

$$\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a} \cdot \vec{c} = 7$$

$$\vec{a} \times \vec{c} - \vec{b} \times \vec{c} = \vec{0}$$

$$(\vec{a} - \vec{b}) \times \vec{c} = \vec{0} \Rightarrow (\vec{a} - \vec{b}) \text{ is paralleled to } \vec{c}$$

$$\vec{a} - \vec{b} = \mu \vec{c}, \text{ where } \mu \text{ is a scalar}$$

$$-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu \cdot \vec{c}$$

$$\text{Now } \vec{a} \cdot \vec{c} = 7 \text{ gives } 2\lambda^2 + 12 = 7\mu$$

$$\text{And } \vec{b} \cdot \vec{c} = -\frac{43}{2} \text{ gives } 4\lambda^2 + 82 = 43\mu$$

$$\mu = 2 \text{ and } \lambda^2 = 1$$

$$|\vec{a} \cdot \vec{b}| = 8$$

Question 71

The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is [25-Jan-2023 Shift 1]

Options:

A. $3\sqrt{2}$

B. 1

C. $\sqrt{6}$

D. $2\sqrt{3}$

Answer: A

Solution:

$$\vec{b} = \lambda \vec{a} \times (\vec{a} \times \hat{j})$$

$$\Rightarrow \vec{b} = \lambda(-2\hat{i} - 2\hat{j} + 2\hat{k})$$

$$|\vec{b}| = |\vec{a}| \therefore \sqrt{6} = \sqrt{12}|\lambda| \Rightarrow \lambda = \pm \frac{1}{\sqrt{2}}$$

$$\left(\lambda = \frac{1}{\sqrt{2}} \text{ rejected } \because \vec{b} \text{ makes acute angle with y axis } \right)$$

$$\vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$$

$$\left(3\vec{a} + \sqrt{2}\vec{b} \right) \cdot \frac{\vec{c}}{|\vec{c}|} = 3\sqrt{2}$$

Question 72

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} - \frac{\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then

$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to
[25-Jan-2023 Shift 1]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $-\frac{1}{4}$

D. $\frac{1}{4}$

Answer: D

Solution:

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} - \frac{\vec{c}}{2}$$

$$\vec{a} \cdot \vec{c} = \frac{1}{2}, \vec{a} \cdot \vec{b} = \frac{1}{2}$$

$$\therefore \vec{b} \cdot \vec{d} = \frac{1}{2}$$

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \vec{a} \cdot (\vec{b} \times (\vec{c} \times \vec{d}))$$

$$= \vec{a} \cdot ((\vec{b} \cdot \vec{d}) \vec{c} - (\vec{b} \cdot \vec{c}) \vec{d})$$

$$= (\vec{a} \cdot \vec{c}) (\vec{b} \cdot \vec{d}) = \frac{1}{4}$$

Question 73

If the four points, whose position vectors are

$3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

[25-Jan-2023 Shift 2]

Options:

A. $\frac{73}{17}$

B. $-\frac{107}{17}$

C. $-\frac{73}{17}$

D. $\frac{107}{17}$

Answer: A

Solution:

Let A : (3, -4, 2)

C : (-2, -1, 3)

B : (1, 2, -1) D : (5, -2 α , 4)

A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$$\Rightarrow \alpha = \frac{73}{17}$$

Question 74

Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to
[25-Jan-2023 Shift 2]

Options:

A. $3(\hat{i} - \hat{j} - \hat{k})$

B. $3(\hat{i} + \hat{j} + \hat{k})$

C. $3(\hat{i} - \hat{j} + \hat{k})$

D. $3(\hat{i} + \hat{j} - \hat{k})$

Answer: B

Solution:

$$\vec{a} \times \vec{b} = (\hat{i} - \hat{j})$$

Taking cross product with $\vec{a}$

$$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$$

$$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\Rightarrow 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\Rightarrow \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Question 75

If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to
[29-Jan-2023 Shift 1]

Options:

A. 0

B. 6

C. 24

D. 18

Answer: C

Solution:

$$\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$\lambda(10) - \mu(2) + 4(-14) = 0$$

$$10\lambda - 2\mu = 56$$

$$5\lambda - \mu = 28 \dots (1)$$

$$\frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|} = \sqrt{54}$$

$$-2\lambda + 4\mu - 8 / \sqrt{24} = \sqrt{54}$$

$$-2\lambda + 4\mu - 8 = \sqrt{54 \times 24} \dots (2)$$

By solving equation (1) & (2)

$$\Rightarrow \lambda + \mu = 24$$

Question 76

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be

$$\vec{a} - \vec{b} + \vec{c}, \quad \lambda \vec{a} - 3\vec{b} + 4\vec{c}, \quad -\vec{a} + 2\vec{b} - 3\vec{c} \quad \text{and} \quad 2\vec{a} - 4\vec{b} + 6\vec{c}$$

respectively. If $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are coplanar, then λ is :

Official Ans. by NTA

[29-Jan-2023 Shift 1]

Answer: 2

Solution:

$$\vec{AB} = (\lambda - 1)\vec{a} - 2\vec{b} + 3\vec{c}$$

$$\vec{AC} = 2\vec{a} + 3\vec{b} - 4\vec{c}$$

$$\overline{AD} = \overline{a} - 3\overline{b} + 5\overline{c}$$

$$\begin{vmatrix} \lambda-1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda-1)(15-12) + 2(-10+4) + 3(6-3) = 0$$

$$\Rightarrow (\lambda-1) = 1 \Rightarrow \lambda = 2$$

Question 77

Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c}$ is a vector such that

$\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ and projection of $\vec{c}$ on $\vec{a}$ is 1,

then the projection of $\vec{c}$ on $\vec{b}$ equals:

[29-Jan-2023 Shift 2]

Options:

A. $\frac{5}{\sqrt{2}}$

B. $\frac{1}{5}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{3}{\sqrt{2}}$

Answer: A

Solution:

$$\vec{a} \times \vec{b} = 15\hat{i} - 20\hat{j} - 25\hat{k}$$

$$\text{Let } \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow 15x - 20y - 25z + 25 = 0$$

$$\Rightarrow 3x - 4y - 5z = -5$$

$$\text{Also } x + y + z = 4$$

$$\text{and } \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} = 1 \Rightarrow 4x + 3y = 5$$

$$\Rightarrow \vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\text{Projection of } \vec{c} \text{ on } \vec{b} = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$$

Question 78

If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{k} + 4\hat{k}$ $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$ then $\vec{r} \cdot \vec{c}$ is equal to :

[29-Jan-2023 Shift 2]

Options:

A. 34

B. 12

C. 36

D. 30

Answer: A

Solution:

$$\begin{aligned}\vec{r} \times \vec{b} - \vec{c} \times \vec{b} &= \vec{0} \\ \Rightarrow (\vec{r} - \vec{c}) \times \vec{b} &= \vec{0} \\ \Rightarrow \vec{r} - \vec{c} &= \lambda \vec{b} \\ \Rightarrow \vec{r} &= \vec{c} + \lambda \vec{b}\end{aligned}$$

And given that $\vec{r} \cdot \vec{a} = 0$

$$\begin{aligned}\Rightarrow (\vec{c} + \lambda \vec{b}) \cdot \vec{a} &= 0 \\ \Rightarrow \vec{c} \cdot \vec{a} + \lambda \vec{b} \cdot \vec{a} &= 0 \\ \Rightarrow \lambda &= -\frac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}}\end{aligned}$$

$$\text{Now } \vec{r} \cdot \vec{c} = (\vec{c} + \lambda \vec{b}) \cdot \vec{c}$$

$$= \left(\vec{c} - \frac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}} \vec{b} \right) \cdot \vec{c}$$

$$= |\vec{c}|^2 - \left(\frac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}} \right) (\vec{b} \cdot \vec{c})$$

$$= 74 - \left[\frac{15}{3} \right] 8$$

$$= 74 - 40 = 34$$

Question 79

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha \vec{b} - \hat{n}$, ($\alpha \neq 0$) and $\vec{b} \cdot \vec{c} = 12$, then

$|\vec{c} \times (\vec{a} \times \vec{b})|$ is equal to :

[30-Jan-2023 Shift 1]

Options:

A. 15

B. 9

C. 12

D. 6

Answer: C

Solution:

$$\hat{n} \perp \vec{c} \quad \vec{a} = \alpha \vec{b} - \hat{n}$$

$$\vec{b} \cdot \vec{c} = 12$$

$$\vec{a} \cdot \vec{c} = \alpha (\vec{b} \cdot \vec{c}) - \hat{n} \cdot \vec{c}$$

$$\vec{a} \cdot \vec{c} = \alpha (\vec{b} \cdot \vec{c})$$

$$|\vec{c} \times (\vec{a} \times \vec{b})| = |(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}|$$

$$= |(\vec{c} \cdot \vec{b})\vec{a} - \alpha (\vec{b} \cdot \vec{c})\vec{b}|$$

$$= |(\vec{c} \cdot \vec{b})||\vec{a} - \alpha \vec{b}|$$

$$= 12 \times (|\hat{n}|)$$

$$= 12 \times 1$$

$$= 12$$

Question 80

Let $\lambda \in \mathbb{R}$, $\vec{a} = \lambda \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda \hat{j} + 2\hat{k}$.

If $\left((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) \right) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then

$\left| \lambda (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) \right|^2$ is equal to
[30-Jan-2023 Shift 2]

Options:

A. 140

B. 132

C. 144

D. 136

Answer: A

Solution:

$$\begin{aligned}\vec{a} &= \lambda \hat{i} + 2\hat{j} - 3\hat{k} \\ \vec{b} &= \hat{i} - \lambda\hat{j} + 2\hat{k} \\ \Rightarrow (\vec{b} - \vec{a}) \times ((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) &= 8\hat{i} - 40\hat{j} - 24\hat{k} \\ \Rightarrow ((\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}))(\vec{a} \times \vec{b}) &= 8\hat{i} - 40\hat{j} - 24\hat{k} \\ \Rightarrow 8(\vec{a} \times \vec{b}) &= 8\hat{i} - 40\hat{j} - 24\hat{k}\end{aligned}$$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$$

$$\begin{aligned}&= (4 - 3\lambda)\hat{i} - (2\lambda + 3)\hat{j} + (-\lambda^2 - 2)\hat{k} \\ \Rightarrow \lambda &= 1\end{aligned}$$

$$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{a} - \vec{b} = 3\hat{j} - 5\hat{k}$$

$$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2\hat{i} + 10\hat{j} + 6\hat{k}$$

$$\therefore \text{required answer} = 4 + 100 + 36 = 140$$

Question 81

Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is
[30-Jan-2023 Shift 2]

Options:

- A. -24
- B. -48
- C. -84
- D. -60

Answer: B

Solution:

$$\begin{aligned}\vec{c} &= (2\vec{a} \times \vec{b}) - 3\vec{b} \\ \vec{b} \cdot \vec{c} &= \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b} \\ &= -3|\vec{b}|^2 \\ &= -48\end{aligned}$$

Question82

Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statement:

- (A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.
- (B) $\vec{a}$ and $\vec{c}$ are always parallel

[31-Jan-2023 Shift 1]

Options:

- A. only (B) is correct
- B. neither (A) nor (B) is correct
- C. only (A) is correct

D. both (A) and (B) are correct.

Answer: C

Solution:

$$\begin{aligned} |\vec{a} + \vec{b} + \vec{c}|^2 &= |\vec{a} + \vec{b} - \vec{c}|^2 \\ 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} &= 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a} \\ 4\vec{a} \cdot \vec{c} &= 0 \end{aligned}$$

B is incorrect

$$\begin{aligned} |\vec{a} + \lambda \vec{c}|^2 &\geq |\vec{a}|^2 \\ \lambda^2 c^2 &\geq 0 \end{aligned}$$

True $\forall \lambda \in \mathbb{R}$ (A) is correct.

Question83

Let $\vec{a}$ and $\vec{b}$ be two vector such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 36

Solution:

$$\begin{aligned} |\vec{a}| &= \sqrt{14}, |\vec{b}| = \sqrt{6} \quad |\vec{a} \times \vec{b}| = \sqrt{48} \\ |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 &= |\vec{a}|^2 \times |\vec{b}|^2 \\ \Rightarrow (\vec{a} \cdot \vec{b})^2 &= 84 - 48 = 36 \end{aligned}$$

Question84

Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$. Then

25 $|\vec{r}|^2$ is equal to
[31-Jan-2023 Shift 2]

Options:

A. 449

B. 336

C. 339

D. 560

Answer: C

Solution:

$$\text{Sol. } \vec{a} = i + 2j + 3k$$

$$\vec{b} = i - j + 2k$$

$$\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$$

$$(\vec{r} - \vec{c}) \times \vec{b} = 0, \vec{r} \cdot \vec{a} = 0$$

$$\Rightarrow \vec{r} - \vec{c} = \lambda \vec{b}$$

$$\text{Also, } (\vec{c} + \lambda \vec{b}) \cdot \vec{a} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{c} + \lambda (\vec{a} \cdot \vec{b}) = 0$$

$$\therefore \lambda = \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = \frac{-8}{5}$$

$$\vec{r} = \frac{5(5\hat{i} - 3\hat{j} + 3\hat{k}) - 8(\hat{i} - \hat{j} + 2\hat{k})}{5}$$

$$\vec{r} = \frac{17\hat{i} - 7\hat{j} + \hat{k}}{5}$$

$$|\vec{r}|^2 = \frac{1}{25}(289 + 50)$$

$$25|\vec{r}|^2 = 339$$

Question 85

The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is (2, a, 4), $a \in \mathbb{N}$. If the volume of the tetrahedron OABC is 144unit^3 , then which of the

following points is NOT on P?
[31-Jan-2023 Shift 2]

Options:

A. (2, 2, 4)

B. (0, 4, 4)

C. (3, 0, 4)

D. (0, 6, 3)

Answer: C

Solution:

Equation of Plane:

$$(2\hat{i} + a\hat{j} + 4\hat{k}) \cdot [(x-2)\hat{i} + (y-a)\hat{j} + (z-4)\hat{k}] = 0$$

$$\Rightarrow 2x + ay + 4z = 20 + a^2$$

$$\Rightarrow A \equiv \left(\frac{20+a^2}{2}, 0, 0 \right)$$

$$B \equiv \left(0, \frac{20+a^2}{a}, 0 \right)$$

$$C \equiv \left(0, 0, \frac{20+a^2}{4} \right)$$

$\Rightarrow$ Volume of tetrahedron

$$= \frac{1}{6} [\vec{a} \cdot \vec{b} \times \vec{c}]$$

$$= \frac{1}{6} \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$\Rightarrow \frac{1}{6} \left(\frac{20+a^2}{2} \right) \cdot \left(\frac{20+a^2}{a} \right) \cdot \left(\frac{20+a^2}{4} \right) = 144$$

$$\Rightarrow (20+a^2)^3 = 144 \times 48 \times a$$

$$\Rightarrow a = 2$$

$\Rightarrow$ Equation of plane is $2x + 2y + 4z = 24$

Or $x + y + 2z = 12$

$\Rightarrow (3, 0, 4)$ Not lies on the Plane

$$x + y + 2z = 12$$

Question 86

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$.

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}} \right)^2$ is equal to _____.

[31-Jan-2023 Shift 2]

Answer: 3

Solution:

$$\begin{aligned} \text{Sol. } 2(\vec{a} \times \vec{b}) &= 3(\vec{c} \times \vec{a}) \\ \vec{a} \times (2\vec{b} + 3\vec{c}) &= 0 \\ \vec{a} &= \lambda(2\vec{b} + 3\vec{c}) \\ |\vec{a}|^2 &= \lambda^2 |2\vec{b} + 3\vec{c}|^2 \\ |\vec{a}|^2 &= \lambda^2 (4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b} \cdot \vec{c}) \\ 31 &= 31\lambda^2 \Rightarrow \lambda = \pm 1 \\ \vec{a} &= \pm(2\vec{b} + 3\vec{c}) \\ |\vec{b} \times \vec{c}|^2 &= |\vec{b}|^2 |\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2 = \frac{3}{4} \\ \left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}} \right)^2 &= 3 \end{aligned}$$

Question 87

Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$, and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $|\vec{u} \cdot \vec{v} \times \vec{w}|$ is $-\alpha\sqrt{3401}$, and $\left| \frac{\vec{u} \cdot \hat{i}}{\alpha} \right|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _____.

[1-Feb-2023 Shift 1]

Answer: 3501

Solution:

$$[\vec{u} \vec{v} \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$\min. (|\vec{u}| |\vec{v} \times \vec{w}| \cos \theta) = -\alpha \sqrt{3401}$$

$$\Rightarrow \cos \theta = -1$$

$$|\vec{u}| = \alpha \text{ (Given)}$$

$$|\vec{v} \times \vec{w}| = \sqrt{3401}$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$$\vec{v} \times \vec{w} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$$

$$|\vec{v} \times \vec{w}| = \sqrt{1 + 25\alpha^2 + 9\alpha^2} = \sqrt{3401}$$

$$34\alpha^2 = 3400$$

$$\alpha^2 = 100$$

$$\alpha = 10$$

$$(\text{as } \alpha > 0)$$

So

$$\vec{u} = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$$

$$|\vec{u}| = \sqrt{\lambda^2 + 25\alpha^2\lambda^2 + 9\alpha^2\lambda^2}$$

$$\alpha^2 = \lambda^2(1 + 25\alpha^2 + 9\alpha^2)$$

$$100 = \lambda^2(1 + 34 \times 100)$$

$$\lambda^2 = \frac{100}{3401} = \frac{m}{n}$$

Question 88

A(2, 6, 2), B(-4, 0, λ), C(2, 3, -1) and D(4, 5, 0), $|\lambda| \leq 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to _____.

[1-Feb-2023 Shift 1]

Answer: 11

Solution:

$$A(2, 6, 2), B(-4, 0, \lambda), C(2, 3, -1), D(4, 5, 0)$$

$$\text{Area} = \frac{1}{2} |\vec{BD} \times \vec{AC}| = 18$$

$$\vec{AC} \times \vec{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

$$= (3\lambda + 15)\hat{i} - \hat{j}(-24) + \hat{k}(-24)$$

$$\vec{AC} \times \vec{BD} = (3\lambda + 15)\hat{i} + 24\hat{j} - 24\hat{k}$$

$$= \sqrt{(3\lambda + 15)^2 + (24)^2 + (24)^2} = 36$$

$$= \lambda^2 + 10\lambda + 9 = 0$$

$$= \lambda = -1, -9$$

$$|\lambda| \leq 5 \Rightarrow \lambda = -1$$

$$5 - 6\lambda = 5 - 6(-1) = 11$$

Question 89

Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?
[1-Feb-2023 Shift 2]

Options:

A. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the p?

B. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the p?

C. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

D. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

Answer: D

Solution:

$$\begin{aligned}\vec{a} &= 5\hat{i} - \hat{j} - 3\hat{k} \\ \vec{b} &= \hat{i} - 3\hat{j} + 5\hat{k} \\ \vec{a} \cdot \vec{b} &= \frac{5-3-15}{\sqrt{35}} = -\frac{13}{\sqrt{35}}\end{aligned}$$

Question90

Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to :
[1-Feb-2023 Shift 2]

Options:

A. $\frac{11}{7}\sqrt{2}$

B. $\frac{11}{7}$

C. $\frac{11}{5}\sqrt{2}$

D. $\frac{\sqrt{914}}{7}$

Answer: A

Solution:

$$\begin{aligned}\vec{a} &= 2\hat{i} - 7\hat{j} + 5\hat{k} \\ \vec{b} &= \hat{i} + \hat{k} \\ \vec{c} &= \hat{i} + 2\hat{j} - 3\hat{k} \\ \vec{r} \times \vec{a} &= \vec{c} \times \vec{a} \Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0 \\ \therefore \vec{r} &= \vec{c} + \lambda \vec{a} \\ \vec{r} \cdot \vec{b} &= 0 \Rightarrow \vec{c} \cdot \vec{b} + \lambda \vec{b} \cdot \vec{a} = 0 \\ -2 + \lambda(7) &= 0 \Rightarrow \lambda = \frac{2}{7}\end{aligned}$$

$$\therefore \vec{r} = \vec{c} + 2\frac{\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$$

$$|\vec{r}| = \frac{11\sqrt{2}}{7}$$

Question91

Let the position vectors of the points A, B, C and D be

$5\hat{i} + 5\hat{j} + 2\lambda\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $-2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$. Let the set

$S = \{\lambda \in \mathbb{R} : \text{the points A, B, C and D are coplanar}\}$. Then $\sum_{\lambda \in S} (\lambda + 2)^2$ is equal to :

[6-Apr-2023 shift 1]

Options:

A. $\frac{37}{2}$

B. 13

C. 25

D. 41

Answer: D

Solution:

A, B, C, D are coplanar

$$\Rightarrow \begin{bmatrix} \vec{AB} & \vec{AC} & \vec{AD} \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{bmatrix} = 0$$

$$\Rightarrow -6[6\lambda - 12 - (\lambda - 5)(3 - 2\lambda)] + 0[] + (6 - 2\lambda)[20 - 4\lambda - 21]$$

$$\Rightarrow -6[6\lambda - 12 + 2\lambda^2 + 15 - 13\lambda] + (6 - 2\lambda)[-4\lambda - 1] = 0$$

$$\Rightarrow -12\lambda^2 + 42\lambda - 18 + 8\lambda^2 - 22\lambda - 6 = 0$$

$$\Rightarrow -4\lambda^2 + 20\lambda - 24 = 0 \Rightarrow \lambda^2 - 5\lambda + 6 = 0$$

$$\text{Now } \sum_{\lambda \in S} (\lambda + 2)^2 = 16 + 25 = 41$$

Question92

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a} \cdot \vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is equal to :

[6-Apr-2023 shift 1]

Options:

A. 760

B. 640

C. 720

D. 680

Answer: C

Solution:

$$\vec{d} = \lambda (\vec{b} \times \vec{c})$$

$$\text{For } \lambda : \vec{a} \cdot \vec{d} = 18 \Rightarrow \lambda [\vec{a} \cdot \vec{b} \times \vec{c}] = 18$$

$$\Rightarrow \lambda \begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 18$$

$$\Rightarrow \lambda(4 - 3 + 8) = 18 \Rightarrow \lambda = 2$$

$$\Rightarrow \vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$$

$$\begin{aligned} \text{Hence } |\vec{a} \times \vec{d}|^2 &= a^2 d^2 - (\vec{a} \cdot \vec{d})^2 \\ &= 29 \cdot 36 - (18)^2 = 18(58 - 18) \\ &= 18 \cdot 40 = 720 \end{aligned}$$

Question93

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a}$, $\vec{b} + \vec{c}$ and $\vec{a} + 2\vec{b} + 3\vec{c}$ is equal to :

[6-Apr-2023 shift 2]

Options:

A. 2V

B. 6V

C. 3V

D. V

Answer: D

Solution:

$$\vec{v}_1 = \left[\vec{a} \quad \vec{b} + \vec{c} \quad \vec{a} + 2\vec{b} + 3\vec{c} \right]$$

$$v_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} \left[\vec{a} \quad \vec{b} \quad \vec{c} \right]$$

$$v_1 = (3 - 2)v$$

$$= v$$

Ans. Option 4

Question94

The sum of all values of α , for which the points whose position

vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha + 1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$

are coplanar, is equal to :

[6-Apr-2023 shift 2]

Options:

A. -2

B. 2

C. 6

D. 4

Answer: B

Solution:

$$A = (1, -2, 3)$$

$$B = (2, -3, 4)$$

$$C = (\alpha + 1, 0, 2)$$

$$D = (9, \alpha - 8, 6)$$

$$[\vec{AB} \vec{AC} \vec{AD}] = 0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha - 6 & 3 \end{vmatrix} = 0$$

$$\Rightarrow (6 + \alpha - 6) + 1(3\alpha + 8) + (\alpha^2 - 6\alpha - 16) = 0$$

$$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$$

$$\Rightarrow \alpha = 4, -2$$

$$\Rightarrow \text{sum of all values of } \alpha = 2$$

Ans. option 2

Question 95

Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that

$\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a} \cdot \vec{c} = -12$, $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to _____.

[8-Apr-2023 shift 1]

Answer: 11

Solution:

$$\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{a} \times (\vec{c} - \vec{b}) = 0$$

$$\vec{a} \parallel (\vec{c} - \vec{b})$$

$$\therefore \vec{a} = \lambda (\vec{c} - \vec{b})$$

$$(6, 9, 12) = \lambda [x - \alpha, y - 11, z + 2]$$

$$\frac{x - \alpha}{2} = \frac{y - 11}{3} = \frac{z + 2}{4}$$

$$4y - 44 = 3z + 6$$

$$4y - 3z = 50$$

$$6x + 9y + 12z = -12$$

$$2x + 3y + 4z = -4$$

$$(\because x - 2y + z = 5)$$

$$2x - 4y + 2z = 10$$

$$+ \quad -$$

$$\hline 7y + 2z = -14 \quad \dots (2)$$

$$8y - 6z = 100$$

$$21y + 6z = -42$$

$$29y = 58$$

$$y = 2, z = -14$$

$$\therefore x - 4 - 14 = 5$$

$$x = 23$$

$$\vec{c} = (23, 2, -14)$$

$$c \cdot (1, 1, 1) = 23 + 2 - 14 = 11$$

Question 96

Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a+b+c)$ is equal to

[8-Apr-2023 shift 2]

Options:

A. 4

B. 12

C. 6

D. 0

Answer: B

Solution:

$$\begin{vmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \end{vmatrix} = 0 \quad \therefore \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow b - 1 + c - 1 + a(1 - bc) = 0$$

$$\therefore abc = a + b + c - 2$$

$$\left[\begin{array}{ccc} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{array} \right] = 0 \therefore \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$$\Rightarrow -2a(ac - bc - c^2) + 2c(a^2 + ab - ac) = 0$$

$$\Rightarrow -2a^2c + 2abc + 2ac^2 + 2a^2c + 2abc - 2ac^2 = 0$$

$$\Rightarrow 4abc = 0 \therefore abc = 0$$

$$\therefore a + b + c = 2 \therefore 6(a + b + c) = 12 \text{ Ans.}$$

Question 97

An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\vec{OP} = \vec{u}$, $\vec{OR} = \vec{v}$ and $\vec{OQ} = \alpha \vec{u} + \beta \vec{v}$, then α, β^2 are the roots of the equation :
[10-Apr-2023 shift 1]

Options:

A. $3x^2 - 2x - 1 = 0$

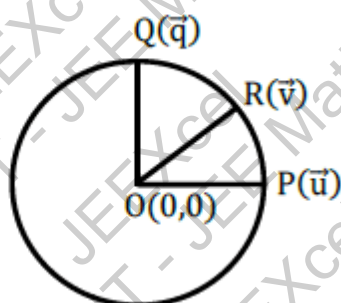
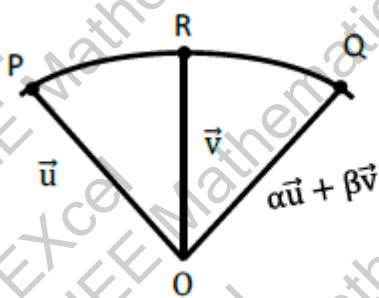
B. $3x^2 + 2x - 1 = 0$

C. $x^2 - x - 2 = 0$

D. $x^2 + x - 2 = 0$

Answer: C

Solution:



$$\text{Let } \vec{OP} = \vec{u} = \hat{i}$$

$$\vec{OQ} = \vec{q} = \hat{j}$$

$\therefore R$ is the mid point of $\vec{PQ}$

$$\text{Then } \vec{OR} = \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

Now

$$\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$$

$$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\beta = \sqrt{2}, \alpha + \frac{\beta}{\sqrt{2}} = 0 \Rightarrow \alpha = -1$$

Now equation

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-1 + 2)x + (-1)(2) = 0$$

$$x^2 - x - 2 = 0$$

Question98

Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 2\hat{k}$, Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$, and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. The

$(-\hat{i} + \hat{j} - \hat{k}) \cdot (\vec{c} \times \vec{d})$ -is equal to
[10-Apr-2023 shift 2]

Options:

A. 24

B. 42

C. 48

D. 44

Answer: D

Solution:

$$\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$$

$$\vec{b} = 3\hat{i} + 5\hat{k}$$

$$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$$

$$\vec{d} = \lambda(35\hat{i} - 13\hat{j} - 21\hat{k})$$

$$\lambda(35 + 13 - 42) = 12$$

$$\lambda = 2$$

$$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$$

$$(\hat{i} + \hat{j} - \hat{k})(\vec{c} \times \vec{d})$$

$$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$$

Question99

For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1$, $i = 1, 2, 3$, consider the following statements :

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3 \max\{|a_1|, |a_2|, |a_3|\}$

[11-Apr-2023 shift 1]

Options:

A. Only (B) is true

B. Both (A) and (B) are true

C. Neither (A) nor (B) is true

D. Only (A) is true

Answer: B

Solution:

Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$$|\vec{a}|^2 = |\vec{a}_1|^2 + |\vec{a}_2|^2 + |\vec{a}_3|^2 \geq (a_3)^2$$

$$\Rightarrow |\vec{a}| \geq |a_3| = \max\{|a_1|, |a_2|, |a_3|\}$$

A is true

$$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$$

$$\Rightarrow |\vec{a}|^2 \leq 3|a_3|^2$$

$$\Rightarrow |\vec{a}| \leq \sqrt{3} |\vec{a}_3| = \sqrt{3} \max\{|a_1|, |a_2|, |a_3|\}$$

$$\leq 3 \max\{|a_1|, |a_2|, |a_3|\}$$

(2) is true

Question100

Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}$, $\hat{i} + \hat{k}$ and $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then ordered pair $(\theta, |\vec{a} \times \vec{b}|)$ is equal to :
[11-Apr-2023 shift 1]

Options:

A. $\left(\frac{\pi}{3}, 6\right)$

B. $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$

C. $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$

D. $\left(\frac{\pi}{4}, 6\right)$

Answer: D

Solution:

$\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i} + \hat{j}$, $\hat{i} + \hat{k}$ and $\hat{i} - \hat{j}$, $\hat{i} - \hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \lambda |\vec{n}_2 \times \vec{n}_1|$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \lambda (-2\hat{j} + 2\hat{k})$$

$$\vec{a} \cdot \vec{b} = \lambda |0 + 4 + 2| = 6$$

$$\Rightarrow \lambda = 1$$

$$\vec{a} = -2\hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos \theta = \frac{6}{2\sqrt{2} \times 3} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

$$\text{Now } |\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$36 + |\vec{a} \times \vec{b}|^2 = 8 \times 9 = 72$$

$$|\vec{a} \times \vec{b}|^2 = 36$$

$$|\vec{a} \times \vec{b}| = 6$$

Question 101

If four distinct points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix}$ is equal to

[11-Apr-2023 shift 2]

Options:

A. $\left[\vec{d} \vec{c} \vec{a} \right] + \left[\vec{b} \vec{d} \vec{a} \right] + \left[\vec{c} \vec{d} \vec{b} \right]$

B. $\left[\vec{d} \vec{b} \vec{a} \right] + \left[\vec{a} \vec{c} \vec{d} \right] + \left[\vec{d} \vec{b} \vec{c} \right]$

C. $\left[\vec{a} \vec{d} \vec{b} \right] + \left[\vec{d} \vec{c} \vec{a} \right] + \left[\vec{d} \vec{b} \vec{c} \right]$

D. $\left[\vec{b} \vec{c} \vec{d} \right] + \left[\vec{d} \vec{a} \vec{c} \right] + \left[\vec{d} \vec{b} \vec{a} \right]$

Answer: A

Solution:

$$\vec{a}, \vec{b}, \vec{c}, \vec{d} \rightarrow \text{coplanar}$$

$$\left[\vec{a} \vec{b} \vec{c} \right] = ?$$

$$\vec{b} - \vec{a}, \vec{c} - \vec{b}, \vec{d} - \vec{c} \rightarrow \text{coplanar}$$

$$\left[\vec{b} - \vec{a} \vec{c} - \vec{b}, \vec{d} - \vec{c} \right] = 0$$

$$\Rightarrow (\vec{b} - \vec{a}) \cdot ((\vec{c} - \vec{b}) \times (\vec{d} - \vec{c})) = 0$$

$$(\vec{b} - \vec{a}) \cdot (\vec{c} \times \vec{b} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d}) = 0$$

$$[bcd] - [bca] - [bad] - [acd] = 0$$

$$\left[\vec{a} \vec{b} \vec{c} \right] = \left[\vec{d} \vec{c} \vec{a} \right] + \left[\vec{b} \vec{d} \vec{a} \right] + \left[\vec{c} \vec{d} \vec{b} \right]$$

Question102

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that

$\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to _____.

[11-Apr-2023 shift 2]

Answer: 285

Solution:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{b} \cdot (\vec{a} \times \vec{c}) = 27, \vec{a} \cdot \vec{b} = 0$$

$$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}$$

Let θ be angle between $\vec{b}$, $\vec{a} \times \vec{c}$

$$\text{Then } |\vec{b}| \cdot |\vec{a} \times \vec{c}| \sin \theta = 3\sqrt{14}$$

$$|\vec{b}| \cdot |\vec{a} \times \vec{c}| \cos \theta = 27$$

$$\Rightarrow \sin \theta = \frac{\sqrt{14}}{\sqrt{95}}$$

$$\therefore |\vec{b}| \times |\vec{a} \times \vec{c}| = 3\sqrt{95}$$

$$\Rightarrow |\vec{a} \times \vec{c}| = \sqrt{3} \times \sqrt{95}$$

Question 103

Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 1

B. 2

C. -2

D. -1

Answer: A

Solution:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

$$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(1-b) = 0$$

$$a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$$

$$\frac{a}{1-a} + \frac{a}{1-b} + \frac{a}{1-c} = 0$$

$$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Question 104

Let $\lambda \in \mathbb{Z}$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}$, $\vec{a} \cdot \vec{c} = -17$ and $\vec{b} \cdot \vec{c} = -20$. Then $\left| \vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k}) \right|^2$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 62

B. 53

C. 49

D. 46

Answer: D

Solution:

$$(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}$$

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$$

$$\vec{c} = \alpha(\vec{a} + \vec{b}) = \alpha(\lambda + 3)\hat{i} + \alpha\hat{k}$$

$$\vec{b} \cdot \vec{c} = -20 \Rightarrow 3\alpha(\lambda + 3) + 2\alpha = -20$$

$$\vec{a} \cdot \vec{c} = -17 \Rightarrow \alpha\lambda(\lambda + 3) - \alpha = -17$$

$$\Rightarrow \alpha(3\lambda + 9 + 2) = -20$$

$$\alpha(\lambda^2 + 3\lambda - 1) = -17$$

$$17(3\lambda + 11) = 20(\lambda^2 + 3\lambda - 1)$$

$$20\lambda^2 + 9\lambda - 207 = 0$$

$$\lambda = 3 \quad (\lambda \in \mathbb{Z})$$

$$\Rightarrow \alpha = -1 \Rightarrow \vec{c} = -(6\hat{i} + \hat{k})$$

$$\vec{v} = \vec{c} \times (3\hat{i} + \hat{j} + \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 1 \end{vmatrix} = \hat{i} + 3\hat{j} - 6\hat{k}$$

$$|\vec{v}|^2 = (-1)^2 + 3^2 + 6^2 = 46$$

Question 105

Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that

$\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to _____.

[13-Apr-2023 shift 1]

Answer: 66

Solution:

$$|\vec{a}| = \sqrt{11}, |\vec{c}| = \sqrt{22}$$

$$|\vec{a}| = |\vec{b} \times \vec{c}| = |\vec{b}| |\vec{c}| \sin \theta$$

$$\sqrt{11} = \sqrt{50} \sqrt{22} \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1}{10}$$

$$|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c}$$

$$= |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}| |\vec{c}| \cos \theta$$

$$= 50 + 22 + 2 \times \sqrt{50} \times \sqrt{22} \times \frac{\sqrt{99}}{10}$$

$$= 72 + 66$$

$$|72 - |\vec{b} + \vec{c}|^2| = 66$$

Question 106

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to -
[13-Apr-2023 shift 1]

Options:

- A. 323
- B. 423
- C. 413
- D. 313

Answer: C

Solution:

$$\begin{aligned}\vec{d} \times \vec{b} &= \vec{c} \times \vec{b} \\ \Rightarrow (\vec{d} - \vec{c}) \times \vec{b} &= 0 \\ \Rightarrow \vec{d} &= \vec{c} + \lambda \vec{b} \\ \text{Also } \vec{d} \cdot \vec{a} &= 24 \\ \Rightarrow (\vec{c} + \lambda \vec{b}) \cdot \vec{a} &= 24 \\ \lambda &= \frac{24 - \vec{a} \cdot \vec{c}}{\vec{b} \cdot \vec{a}} = \frac{24 - 6}{9} = 2 \\ \Rightarrow \vec{d} &= \vec{c} + 2(\vec{b}) \\ &= 8\hat{i} - 5\hat{j} + 18\hat{k} \\ \Rightarrow |\vec{d}|^2 &= 64 + 25 + 324 = 413\end{aligned}$$

Question107

Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$.

Then $\left| (\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b}) \right|^2$ is equal to
[13-Apr-2023 shift 2]

Options:

- A. 482

B. 841

C. 882

D. 441

Answer: C

Solution:

$$\cos\left(\frac{\pi}{4}\right) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\vec{a} \cdot \vec{b}}{(2)(3)} \Rightarrow \vec{a} \cdot \vec{b} = 3\sqrt{2}$$

$$\text{Let } \vec{p} = \vec{a} + 2\vec{b}$$

$$\vec{q} = 2\vec{a} - 3\vec{b}$$

$$|\vec{p}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4(\vec{a} \cdot \vec{b})$$
$$= 4 + 36 + 12\sqrt{2}$$
$$= 40 + 12\sqrt{2}$$

$$|\vec{q}|^2 = 4|\vec{a}|^2 + 9|\vec{b}|^2 - 12(\vec{a} \cdot \vec{b})$$
$$= 16 + 81 - 36\sqrt{2}$$
$$= 97 - 36\sqrt{2}$$

$$\vec{p} \cdot \vec{q} = 2|\vec{a}|^2 - 6|\vec{b}|^2 + \vec{a} \cdot \vec{b}$$
$$= 8 - 54 + 3\sqrt{2}$$
$$= -46 + 3\sqrt{2}$$

$$|\vec{p} \times \vec{q}| = (|\vec{p}| |\vec{q}|)^2 - (\vec{p} \cdot \vec{q})^2$$
$$= (40 + 12\sqrt{2})(97 - 36\sqrt{2}) - (3\sqrt{2} - 46)^2$$
$$= (3016 - 276\sqrt{2}) - (2134 - 276\sqrt{2})$$
$$= 882$$

Question 108

Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \mu\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda, \mu) \in S} 80(\lambda^2 + \mu^2)$

is equal to
[15-Apr-2023 shift 1]

Options:

A. 2130

B. 2210

C. 2290

D. 2370

Answer: C

Solution:

$$\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0 \quad \&\lambda - \mu = 5$$

$$\lambda(10 + 4\mu) + (5 - 3\mu) + (-10) = 0$$

$$(\mu + 5)(4\mu + 10) + 5 - 3\mu - 10 = 0$$

$$\mu = -15; \lambda = 5/4$$

$$\mu = -3; \lambda = 2$$

$$\text{Hence } \sum_{(\lambda, \mu) \in S} 80(\lambda^2 + \mu^2)$$

$$= 80 \left(\frac{250}{16} + 13 \right)$$

$$= 1250 + 1040$$

$$= 2290$$

Question 109

Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $(\vec{AB} - \vec{BC}) + (\vec{AD} - \vec{DC}) = k\vec{FE}$,

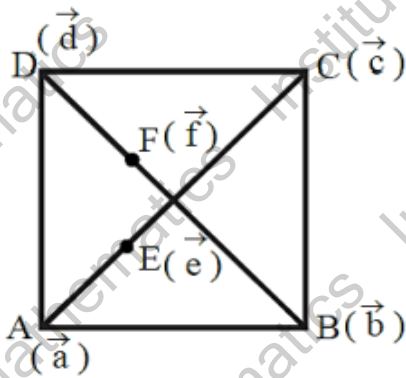
then k is equal to
[15-Apr-2023 shift 1]

Options:

- A. 4
- B. 2
- C. -2
- D. -4

Answer: D

Solution:



$$\begin{aligned}
 \vec{AB} - \vec{BC} + \vec{AB} - \vec{DC} &= k\vec{FE} \\
 (\vec{b} - \vec{a}) - (\vec{c} - \vec{b}) + (\vec{d} - \vec{a}) - (\vec{c} - \vec{d}) &= k\vec{FE} \\
 2(\vec{b} + \vec{d}) - 2(\vec{a} - \vec{c}) &= k\vec{FE} \\
 2(2\vec{f}) - 2(2\vec{e}) &= k\vec{FE} \\
 4(\vec{f} - \vec{e}) &= k\vec{FE} \\
 -4\vec{FE} &= k\vec{FE} \\
 k &= -4
 \end{aligned}$$

Question 110

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left| \left(\hat{a} + \hat{b} \right) + 2 \left(\hat{a} \times \hat{b} \right) \right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

(S1) : $2 \left| \hat{a} \times \hat{b} \right| = \left| \hat{a} - \hat{b} \right|$

(S2) : The projection of $\hat{a}$ on $\left(\hat{a} + \hat{b} \right)$ is $\frac{1}{2}$

[24-Jun-2022-Shift-2]

Options:

- A. Only (S1) is true.
- B. Only (S2) is true.
- C. Both (S1) and (S2) are true.
- D. Both (S1) and (S2) are false.

Answer: C

Solution:

$$\left| \hat{a} + \hat{b} + 2 \left(\hat{a} \times \hat{b} \right) \right| = 2, \theta \in (0, \pi)$$

$$\Rightarrow \left| \hat{a} + \hat{b} + 2 \left(\hat{a} \times \hat{b} \right) \right|^2 = 4$$

$$\Rightarrow \left| \hat{a} \right|^2 + \left| \hat{b} \right|^2 + 4 \left| \hat{a} \times \hat{b} \right|^2 + 2 \hat{a} \cdot \hat{b} = 4$$

$$\therefore \cos \theta = \cos 2\theta$$

$$\therefore \theta = \frac{2\pi}{3}$$

where θ is angle between $\hat{a}$ and $\hat{b}$.

$$\therefore 2|\hat{a} \times \hat{b}| = \sqrt{3} = |\hat{a} - \hat{b}|$$

(S1) is correct.

$$\text{And projection of } \hat{a} \text{ on } (\hat{a} + \hat{b}) = \left| \frac{\hat{a} \cdot (\hat{a} + \hat{b})}{|\hat{a} + \hat{b}|} \right| = \frac{1}{2} \text{ (S2) is correct.}$$

Question111

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ where $a_i > 0, i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90° . If $\vec{a}, \vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\sqrt{7}$

B. $\sqrt{2}$

C. 2

D. 7

Answer: B

Solution:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \Rightarrow \cos^2 \alpha = \frac{1}{3} \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\vec{a} = \frac{\lambda}{3}(\hat{i} + \hat{j} + \hat{k}), \lambda > 0$$

$$\frac{\lambda}{\sqrt{3}} \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (3\hat{i} + 4\hat{j})}{\sqrt{3^2 + 4^2}} = 7$$

$$\Rightarrow \frac{\lambda}{\sqrt{3}}(3+4) = 7 \times 5$$

$$\therefore \lambda = 5\sqrt{3}$$

$$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$$

$$\text{Let } \vec{b} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 \text{ and } [\vec{a} \vec{b} \hat{i}] = 0$$

$$\Rightarrow p + q + r = 0$$

$$\& \begin{vmatrix} p & q & r \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0 \Rightarrow q = r$$

$$\vec{b} = -2r\hat{i} + r\hat{j} + r\hat{k}$$

$$\vec{b} = r(-2\hat{i} + \hat{j} + \hat{k})$$

$$\text{Now } |\vec{a}| = |\vec{b}|$$

$$5\sqrt{3} = |r| \sqrt{b} \Rightarrow |r| = \frac{5}{\sqrt{2}}$$

$$\Rightarrow \text{Projection of } \vec{b} \text{ on } 3\hat{i} + 4\hat{j} = \left| \frac{\vec{b} \cdot (3\hat{i} + 4\hat{j})}{\sqrt{3^2 + 4^2}} \right|$$

$$= |r| \left| \frac{(-6+4)}{5} \right| = \left| \frac{-2r}{5} \right|$$

$$\text{Projection} = \frac{2}{5} \times \frac{5}{\sqrt{2}} = \sqrt{2}$$

$\therefore B$ is correct.

Question112

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4, |\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to ____
[25-Jun-2022-Shift-1]

Answer: 576

Solution:

$$\begin{aligned} & \left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2 \\ & \Rightarrow \left|\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}\right|^2 + 4(\vec{a} \cdot \vec{b})^2 \\ & \Rightarrow \left|2(\vec{a} \times \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2 \\ & \Rightarrow 4(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 \\ & \Rightarrow 4|\vec{a}|^2 |\vec{b}|^2 = 4 \cdot 16 \cdot 9 = 576 \end{aligned}$$

Question113

Let $\vec{b} = \hat{i} + \hat{j} + \lambda \hat{k}, \lambda \in \mathbb{R}$. If $\vec{a}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{a} \cdot \vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k})$ is equal to ____
[25-Jun-2022-Shift-2]

Answer: 14

Solution:

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

So, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix} = \hat{i}(\lambda y - z) + \hat{j}(z - \lambda x) + \hat{k}(x - y)$

$\Rightarrow \lambda y - z = 13, z - \lambda x = -1, x - y = -4$

and $x + y + \lambda z = -21$

$\Rightarrow$ Clearly, $\lambda = 3, x = -2, y = 2$ and $z = -7$

So, $\vec{b} - \vec{a} = 3\hat{i} - \hat{j} + 10\hat{k}$

and $\vec{b} + \vec{a} = -\hat{i} + 3\hat{j} - 4\hat{k}$

$\Rightarrow (\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) = 11 + 3 = 14$

Question 114

If $\vec{a} \cdot \vec{b} = 1$, $\vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of

$\left[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a}) \right]$ is :

[26-Jun-2022-Shift-1]

Options:

A. 0

B. $-6\vec{a} \cdot (\vec{b} \times \vec{c})$

C. $-12\vec{c} \cdot (\vec{a} \times \vec{b})$

D. $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

Answer: A

Solution:

$$\therefore \vec{a} \times (\vec{b} \times \vec{c}) = 3\vec{b} - \vec{c} = \vec{u}$$

$$\vec{b} \times (\vec{c} \times \vec{a}) = \vec{c} - 2\vec{a} = \vec{v}$$

$$\vec{c} \times (\vec{b} \times \vec{a}) = 3\vec{b} - 2\vec{a} = \vec{w}$$

$$\therefore \vec{u} + \vec{v} = \vec{w}$$

So, vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ are coplanar, hence their Scalar triple product will be zero.

Question 115

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors.

Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is

$\frac{2}{\sqrt{3}}$. If $\vec{v} \cdot \hat{j} = 7$, then $\vec{v} \cdot (\hat{i} + \hat{k})$ is equal to:

[26-Jun-2022-Shift-2]

Options:

A. 6

B. 7

C. 8

D. 9

Answer: D

Solution:

Let $\vec{v} = \lambda_1 \vec{a} + \lambda_2 \vec{b}$, where $\lambda_1, \lambda_2 \in \mathbb{R} = (\lambda_1 + 2\lambda_2)\hat{i} + (\lambda_1 - 3\lambda_2)\hat{j} + (2\lambda_1 + \lambda_2)\hat{k}$

$\therefore$ Projection of $\vec{v}$ on $\vec{c}$ is $\frac{2}{\sqrt{3}}$

$$\therefore \frac{\lambda_1 + 2\lambda_2 - \lambda_1 + 3\lambda_2 + 2\lambda_1 + \lambda_2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \lambda_1 + 3\lambda_2 = 1 \dots (i)$$

$$\text{and } \vec{v} \cdot \hat{j} = 7 \Rightarrow \lambda_1 - 3\lambda_2 = 7 \dots (ii)$$

from equation (i) and (ii)

$$\lambda_1 = 4, \lambda_2 = -1$$

$$\therefore \vec{v} = 2\hat{i} + 7\hat{j} + 7\hat{k}$$

$$\therefore \vec{v} \cdot (\hat{j} + \hat{k}) = 2 + 7$$

$$= 9$$

Question 116

Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is :
[27-Jun-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: A

Solution:

$$\vec{a} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\text{Now, } \vec{b} \times \vec{c} = \vec{a}$$

$$\vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot \vec{a}$$

$$\vec{c} \cdot \vec{a} = 0$$

$$\Rightarrow (\hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$$

$$= 2 - 3 - 2 = 0$$

$$\Rightarrow -3 = 0 \text{ (Not possible)}$$

$$\Rightarrow \text{No possible value of } \vec{b} \text{ is possible.}$$

Question 117

Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is

[27-Jun-2022-Shift-2]

Options:

A. $\frac{\pi}{4}$

B. $-\frac{\pi}{4}$

C. $\frac{5\pi}{6}$

D. $\frac{3\pi}{4}$

Answer: D

Solution:

$\because \vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$.

$$\therefore \frac{1}{2} |\vec{a} \times \vec{b}| = 2\sqrt{2}$$

$$|\vec{a}| |\vec{b}| \sin \theta = 4\sqrt{2}$$

$$\Rightarrow |\vec{b}| \sin \theta = 4\sqrt{2} \dots (i)$$

$$\text{and } \vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$$

$$|\vec{a}| |\vec{b}| \cos \theta = |\vec{a}| |\vec{b}| \sin \theta$$

$$\Rightarrow \tan \theta = 1$$

$$\therefore \theta = \frac{\pi}{4}$$

$$\text{By (i) } |\vec{b}| = 8$$

$$\text{Now } \vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$$

$$\Rightarrow \vec{c} \cdot \vec{b} = -2|\vec{b}|^2 = -128$$

$$\text{and } \vec{c} \cdot \vec{c} = 8|\vec{a} \times \vec{b}|^2 + 4|\vec{b}|^2$$

$$\Rightarrow |\vec{c}|^2 = 8.32 + 4.64$$

$$\Rightarrow |\vec{c}| = 16\sqrt{2}$$

From (ii) and (iii)

$$|\vec{c}| |\vec{b}| \cos \alpha = -128$$

$$\Rightarrow \cos \alpha = \frac{-1}{\sqrt{2}}$$

$$\alpha = \frac{3\pi}{4}$$

Question 118

If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5$, $\vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to **[28-Jun-2022-Shift-1]**

Answer: 150

Solution:

$$2C_1 + C_2 + 3C_3 = 5 \dots (i)$$

$$3C_1 + 3C_2 + C_3 = 0$$

$$\begin{aligned} \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} &= \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix} \\ &= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1) \end{aligned}$$

$$= 3C_3 + 7C_2 - 8C_1$$

$$\Rightarrow 8C_1 - 7C_2 - 3C_3 = 0 \dots\dots \text{(iii)}$$

$$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$$

$$\text{So } 122(C_1 + C_2 + C_3) = 150$$

Question119

Let $\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$, where $\alpha \in \mathbb{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to :
[28-Jun-2022-Shift-2]

Options:

A. 10

B. 7

C. 9

D. 14

Answer: D

Solution:

$$\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} = (2 + \alpha)\hat{i} - (\alpha - 2)\hat{j} + (\alpha^2 + 4)\hat{k}$$

$$\text{Now } |\vec{a} \times \vec{b}| = \sqrt{15(\alpha^2 + 4)}$$

$$\Rightarrow (2 + \alpha)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$$

$$\Rightarrow \alpha^4 - 5\alpha^2 - 36 = 0$$

$$\therefore \alpha = \pm 3$$

$$\text{Now, } 2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2 = 2 \cdot 14 - 14 = 14$$

Question 120

Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If

$\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is :

[28-Jun-2022-Shift-2]

Options:

A. $\frac{1}{3}$

B. 1

C. $\frac{5}{3}$

D. $\frac{7}{3}$

Answer: C

Solution:

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

and $\vec{a} \cdot (3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}) = 0 \Rightarrow 3a_1 + \frac{a_2}{2} + 2a_3 = 0 \dots (i)$

and $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$\Rightarrow a_2\hat{i} + (2a_3 - a_1)\hat{j} - 2a_2\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$\therefore a_2 = 2 \dots (ii)$

and $a_1 - 2a_3 = 13 \dots (iii)$

From eq. (i) and (iii) : $a_1 = 3$ and $a_3 = -5$

$\therefore \vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

$\therefore$ projection of $\vec{a}$ on $2\hat{i} + 2\hat{j} + \hat{k} = \frac{6+4-5}{3} = \frac{5}{3}$

Question 121

Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbb{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and

$\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to:

[29-Jun-2022-Shift-1]

Options:

A. 3

B. 4

C. 5

D. 6

Answer: A

Solution:

$$\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$$

$$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Projection of $\vec{a}$ on $\vec{c}$ is

$$\frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} = \frac{10}{3}$$

$$\frac{\alpha + 6 + 2}{\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{\alpha + 8}{3} = \frac{10}{3}$$

$$\therefore \alpha = 2$$

$$\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = (2\beta - 8)\hat{i} + 10\hat{j} + (6 + \beta)\hat{k} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$2\beta - 8 = -6 \text{ and } 6 + \beta = 7$$

$$\therefore \beta = 1$$

$$\alpha + \beta = 2 + 1 = 3$$

Question 122

Let A, B, C be three points whose position vectors respectively are

$$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are noncollinear, then the length of the median, in $\triangle ABC$, through A is :
[29-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{82}}{2}$

B. $\frac{\sqrt{62}}{2}$

C. $\frac{\sqrt{69}}{2}$

D. $\frac{\sqrt{66}}{2}$

Answer: A

Solution:

$$\vec{AB} \parallel \vec{AC} \text{ if } \frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

$$\text{Mid-point of BC} = M \left(\frac{5}{2}, 0, \frac{9}{2} \right)$$

$$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$$

Question 123

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that

$\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3(\vec{c} \cdot \vec{a})$ is equal

to _____
[29-Jun-2022-Shift-2]

Answer: 0

Solution:

$$\vec{a} + \vec{b} \times \vec{c} = 0$$

$$\vec{a} \times \vec{b} + |\vec{b}|^2 \vec{c} - 5\vec{b} = 0$$

$$\text{It gives } \vec{c} = \frac{1}{3}(10\hat{i} + 3\hat{j} + 2\hat{k})$$

$$\text{so } 3\vec{a} \cdot \vec{c} = 10$$

But it does not satisfy $\vec{a} + \vec{b} \times \vec{c} = 0$.

This question has data error.

Alternate (Explanation) :

According to given $\vec{a}$ & $\vec{b}$ $\vec{a} \cdot \vec{b} = 1 - 2 + 3 = 2 \dots$ (i)

but given equation

$$\vec{a} = -(\vec{b} \times \vec{c})$$

$$\Rightarrow \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

which contradicts.

Question 124

Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to :

[24-Jun-2022-Shift-1]

Options:

A. $6(3 - \sqrt{3})$

B. $3 + \sqrt{3}$

C. $6(3 + \sqrt{3})$

D. $6(\sqrt{3} + 1)$

Answer: C

Solution:

$$\therefore \hat{b} = \frac{\vec{c}}{c} + 2\left(\frac{\vec{c}}{c} \times \hat{a}\right)$$

$$\Rightarrow \hat{b} \cdot \frac{\vec{c}}{c} = \left|\frac{\vec{c}}{c}\right|^2$$

$$\therefore \hat{b} \cdot \vec{c} = 2\left(\frac{\vec{c}}{c} \times \vec{a}\right)$$

$$\Rightarrow |\hat{b}|^2 + \left|\frac{\vec{c}}{c}\right|^2 - 2\hat{b} \cdot \frac{\vec{c}}{c} = 4\left|\frac{\vec{c}}{c}\right|^2 \left|\vec{a}\right|^2 \sin^2 \frac{\pi}{12}$$

$$\Rightarrow 1 + \left|\frac{\vec{c}}{c}\right|^2 - 2\left|\frac{\vec{c}}{c}\right|^2 = 4\left|\frac{\vec{c}}{c}\right|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

$$\Rightarrow 1 = \left|\frac{\vec{c}}{c}\right|^2 (3 - \sqrt{3})$$

$$\Rightarrow 36\left|\frac{\vec{c}}{c}\right|^2 = \frac{36}{3 - \sqrt{3}} = 6(3 + \sqrt{3})$$

Question 125

If the shortest distance between the lines

$$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - \hat{a}\hat{j})$$

and $\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is

equal to

[24-Jun-2022-Shift-1]

Answer: 2

Solution:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix} = -a\hat{i} - \hat{j} + (a-1)\hat{k}$$

$$\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$$

$$\text{Shortest distance} = \left| \frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\Rightarrow \sqrt{\frac{2}{3}} = \frac{2(a-1)}{\sqrt{a^2+1+(a-1)^2}}$$

$$\Rightarrow 6(a^2-2a+1) = 2a^2-2a+2$$

$$\Rightarrow (a-2)(2a-1) = 0 \Rightarrow a = 2 \text{ because } a \in \mathbb{Z}.$$

Question 126

Let ABC be a triangle such that

$\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$, $\vec{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and

$\vec{b} \cdot \vec{c} = 12$. Consider the statements :

$$(S1) : |(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$(S2) : \angle ACB = \cos^{-1} \left(\sqrt{\frac{2}{3}} \right)$$

Then

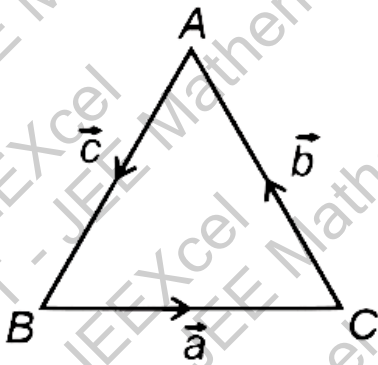
[25-Jul-2022-Shift-1]

Options:

- A. both (S1) and (S2) are true
- B. only (S1) is true
- C. only (S2) is true
- D. both (S1) and (S2) are false

Answer: D

Solution:



$$\because \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\text{then } \vec{a} + \vec{c} = -\vec{b}$$

$$\text{then } (\vec{a} + \vec{c}) \times \vec{b} = -\vec{b} \times \vec{b}$$

$$\therefore \vec{a} \times \vec{b} + \vec{c} \times \vec{b} = \vec{0} \dots (i)$$

$$\text{For (S1): } |\vec{a} \times \vec{b} + \vec{c} \times \vec{b}| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$|(\vec{a} + \vec{c}) \times \vec{b}| - |\vec{c}| = 6(2\sqrt{2} - 1)$$

$$|\vec{c}| = 6 - 12\sqrt{2} \text{ (not possible)}$$

$$\text{For (S2): from (i) } \vec{b} + \vec{c} = -\vec{a}$$

$$\Rightarrow \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b} = -\vec{a} \cdot \vec{b}$$

$$\Rightarrow 12 + 12 = -6\sqrt{2} \cdot 2\sqrt{3} \cos(\pi - \angle ACB)$$

$$\therefore \cos(\angle ACB) = \sqrt{\frac{2}{3}}$$

$$\therefore \angle ACB = \cos^{-1} \sqrt{\frac{2}{3}}$$

$$\therefore \text{S(2) is correct.}$$

Question 127

Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is:

[25-Jul-2022-Shift-2]

Options:

A. $\frac{2}{\sqrt{21}}$

B. $2\sqrt{\frac{3}{7}}$

C. $\frac{2}{3} \sqrt{\frac{7}{3}}$

D. $\frac{2}{3}$

Answer: A

Solution:

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$$

$$\vec{a} \cdot \vec{b} = 3$$

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2$$

$$\Rightarrow 5 + 9 = 6|\vec{b}|^2$$

$$\Rightarrow |\vec{b}|^2 = \frac{7}{3}$$

$$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}} = \sqrt{\frac{7}{3}}$$

$$\text{projection of } \vec{b} \text{ on } \vec{a} - \vec{b} = \frac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$$

$$= \frac{\vec{b} \cdot \vec{a} - |\vec{b}|^2}{|\vec{a} - \vec{b}|} = \frac{3 - \frac{7}{3}}{\sqrt{\frac{7}{3}}}$$

$$= \frac{2}{\sqrt{21}}$$

Question 128

Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to :
[26-Jul-2022-Shift-1]

Options:

A. $\frac{15}{2}$

B. 8

C. $\frac{13}{2}$

D. 7

Answer: D

Solution:

Given : $\vec{a} = (\alpha, 1, -1)$ and $\vec{b} = (2, 1, -\alpha)$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix}$$

$$= (-\alpha + 1)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$$

Projection of $\vec{c}$ on $\vec{d} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$= \left| \vec{c} \cdot \frac{\vec{d}}{|\vec{d}|} \right| = 30 \quad \text{Given}$$

$$\Rightarrow \left| \frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1 + 4 + 4}} \right| = 30$$

On solving $\alpha = \frac{-13}{2}$ (Rejected as $\alpha > 0$) and $\alpha = 7$

Question 129

Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

[27-Jul-2022-Shift-1]

Options:

A. 2

B. $\frac{39}{5}$

C. 9

D. $\frac{46}{5}$

Answer: D

Solution:

$$\vec{a} = \alpha \hat{i} + \hat{j} + \beta \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$4 + 5\beta = -1 \Rightarrow \beta = -1$$

$$-5\alpha - 3 = 12 \Rightarrow \alpha = -3$$

$$\vec{b} - 2\vec{a} = 3\hat{i} - 5\hat{j} + 4\hat{k} - 2(-3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{b} - 2\vec{a} = 9\hat{i} - 7\hat{j} + 6\hat{k}$$

$$\vec{b} + \vec{a} = (3\hat{i} - 5\hat{j} + 4\hat{k}) + (-3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{b} + \vec{a} = -4\hat{j} + 3\hat{k}$$

$$\text{Projection of } \vec{b} - 2\vec{a} \text{ on } \vec{b} + \vec{a} \text{ is } = \frac{(\vec{b} - 2\vec{a}) \cdot (\vec{b} + \vec{a})}{|\vec{b} + \vec{a}|}$$

$$= \frac{28 + 18}{5} = \frac{46}{5}$$

Question 130

Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $\left((\vec{a} \times \vec{b}) \times \hat{i} \right) \cdot \hat{k} = \frac{23}{2}$, then

$|\vec{b} \times 2\hat{j}|$ is equal to

[27-Jul-2022-Shift-1]

Options:

A. 4

B. 5

C. $\sqrt{21}$

D. $\sqrt{17}$

Answer: B

Solution:

Given, $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

$$\text{Also, } ((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow ((\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow (2 \cdot \vec{b} - \alpha \cdot \vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow 2 \cdot 2 - 5\alpha = \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$$

$$\begin{aligned} \text{Now, } |\vec{b} \times 2\hat{j}| &= |(\alpha\hat{i} + \beta\hat{j} + 2\hat{k}) \times 2\hat{j}| \\ &= |2\alpha\hat{k} + 0 - 4\hat{i}| \\ &= \sqrt{4\alpha^2 + 16} \\ &= \sqrt{4\left(\frac{-3}{2}\right)^2 + 16} = 5 \end{aligned}$$

Question 131

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors such that

$$\vec{a} \times \vec{b} = 4\vec{c}, \vec{b} \times \vec{c} = 9\vec{a} \text{ and } \vec{c} \times \vec{a} = \alpha\vec{b}, \alpha > 0$$

If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then α is equal to _____.

[27-Jul-2022-Shift-2]

Answer: 36

Solution:

Given,

$$\vec{a} \times \vec{b} = 4 \cdot \vec{c} \dots (i)$$

$$\vec{b} \times \vec{c} = 9 \cdot \vec{a} \dots (ii)$$

$$\vec{c} \times \vec{a} = \alpha \cdot \vec{b} \dots (iii)$$

Taking dot products with $\vec{c}$, $\vec{a}$, $\vec{b}$ we get

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

Hence,

$$(i) \Rightarrow |\vec{a}| \cdot |\vec{b}| = 4 \cdot |\vec{c}| \dots (iv)$$

$$(ii) \Rightarrow |\vec{b}| \cdot |\vec{c}| = 9 \cdot |\vec{a}| \dots (v)$$

$$(iii) \Rightarrow |\vec{c}| \cdot |\vec{a}| = \alpha \cdot |\vec{b}| \dots (vi)$$

Multiplying (iv), (v) and (vi)

$$\Rightarrow |\vec{a}| \cdot |\vec{b}| \cdot |\vec{c}| = 36\alpha \dots (vii)$$

$$\text{Dividing (vii) by (iv)} \Rightarrow |\vec{c}|^2 = 9\alpha \Rightarrow |\vec{c}| = 3\sqrt{\alpha} \dots (viii)$$

$$\text{Dividing (vii) by (v)} \Rightarrow |\vec{a}|^2 = 4\alpha \Rightarrow |\vec{a}| = 2\sqrt{\alpha}$$

$$\text{Dividing (vii) by (vi)} \Rightarrow |\vec{b}|^2 = 36 \Rightarrow |\vec{b}| = 6$$

$$\text{Now, as given, } 3\sqrt{\alpha} + 2\sqrt{\alpha} + 6 = \frac{1}{36} \Rightarrow \sqrt{\alpha} = \frac{-43}{36}$$

Question 132

Let the vectors $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}$, $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$ and

$\vec{c} = t\hat{i} - t\hat{j} + \hat{k}$, $t \in \mathbb{R}$ be such that for

$\alpha, \beta, \gamma \in \mathbb{R}$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is :

[28-Jul-2022-Shift-1]

Options:

A. a non-empty finite set

B. equal to $\mathbb{N}$

C. equal to $\mathbb{R} - \{0\}$

D. equal to $\mathbb{R}$.

Answer: C

Solution:

Clearly $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow (1+t)(1+t+2t) - (1-t)(1-t-2t) + 1(t^2-t-t-t^2) \neq 0$$

$$\begin{aligned} &\Rightarrow (3t^2 + 4t + 1) - (1 - t)(1 - 3t) - 2t \neq 0 \\ &\Rightarrow (3t^2 + 4t + 1) - (3t^2 - 4t + 1) - 2t \neq 0 \\ &\Rightarrow t \neq 0 \end{aligned}$$

Question 133

Let a vector $\vec{a}$, has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in \mathbb{R} \times \mathbb{R} - \{(0, 0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to:
[28-Jul-2022-Shift-1]

Options:

- A. $9\sqrt{3}$
- B. $27\sqrt{3}$
- C. 9
- D. 81

Answer: B

Solution:

$$\begin{aligned} &(\vec{a}x + \vec{b}y) \cdot (6y\vec{a} - 18x\vec{b}) = 0 \\ &\Rightarrow (6xy|\vec{a}|^2 - 18xy|\vec{b}|^2) + (6y^2 - 18x^2)\vec{a} \cdot \vec{b} = 0 \\ &\text{As given equation is identity} \\ &\text{Coefficient of } x^2 = \text{coefficient of } y^2 = \text{coefficient of } xy = 0 \\ &\Rightarrow |\vec{a}|^2 = 3|\vec{b}|^2 \Rightarrow |\vec{b}| = 3\sqrt{3} \\ &\text{and } \vec{a} \cdot \vec{b} = 0 \\ &|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta \\ &= 9 \cdot 3\sqrt{3} \cdot 1 = 27\sqrt{3} \end{aligned}$$

Question 134

Let S be the set of all $a \in \mathbb{R}$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$, ($b > 1$) is acute. Then S is equal to:
[28-Jul-2022-Shift-2]

Options:

A. $\left(-\infty, -\frac{4}{3}\right)$

B. $\emptyset$

C. $\left(-\frac{4}{3}, 0\right)$

D. $\left(\frac{12}{7}, \infty\right)$

Answer: B

Solution:

$$\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$$

$$\text{For acute angle } \vec{u} \cdot \vec{v} > 0$$

$$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$$

$$\because b > 1$$

$$\text{Let } \log_e b = t \Rightarrow t > 0 \text{ as } b > 1$$

$$at^2 + 6at - 12 > 0 \quad \forall t > 0$$

$$\Rightarrow a \in \emptyset$$

Question 135

Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is:

[29-Jul-2022-Shift-1]

Options:

A. -5

B. 5

C. 1

D. -1

Answer: A

Solution:

$$\vec{a} = 3\hat{i} + \hat{j} \text{ \& } \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} + \lambda\vec{c}$$

If $\vec{b}$ & $\vec{c}$ are non-parallel

$$\text{then } \vec{a} \cdot \vec{c} = 1 \text{ \& } \vec{a} \cdot \vec{b} = -\lambda$$

$$\text{but } \vec{a} \cdot \vec{b} = 5 \Rightarrow \lambda = -5$$

Question 136

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$.

If θ is the angle between the vectors $(\hat{a} + \hat{b})$ and $(\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))$, then the value of $164\cos^2\theta$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $90 + 27\sqrt{2}$

B. $45 + 18\sqrt{2}$

C. $90 + 3\sqrt{2}$

D. $54 + 90\sqrt{2}$

Answer: A

Solution:

$$\hat{a} \cdot \hat{b} = \frac{1}{\sqrt{2}} \text{ and } |\hat{a} \times \hat{b}| = \frac{1}{\sqrt{2}}$$

$$\frac{(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))}{|\hat{a} + \hat{b}| |\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|} = \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1 + 3\hat{a} \cdot \hat{b} + 2}{|\hat{a} + \hat{b}| |\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|}$$

$$|\hat{a} + \hat{b}|^2 = 2 + \sqrt{2}$$

$$|\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b})|^2 = 1 + 4 + 4|\hat{a} \times \hat{b}|^2 + 4\hat{a} \cdot \hat{b}$$
$$= 5 + 4 \cdot \frac{1}{2} + \frac{4}{\sqrt{2}} = 7 + 2\sqrt{2}$$

$$\text{So, } \cos^2 \theta = \frac{\left(3 + \frac{3}{\sqrt{2}}\right)^2}{(2 + \sqrt{2})(7 + 2\sqrt{2})} = \frac{9\sqrt{2}(5\sqrt{2} + 3)}{164}$$

$$\Rightarrow 164 \cos^2 \theta = 90 + 27\sqrt{2}$$

Question 137

If $(2, 3, 9)$, $(5, 2, 1)$, $(1, \lambda, 8)$ and $(\lambda, 2, 3)$ are coplanar, then the product of all possible values of λ is:
[29-Jul-2022-Shift-2]

Options:

A. $\frac{21}{2}$

B. $\frac{59}{8}$

C. $\frac{57}{8}$

D. $\frac{95}{8}$

Answer: D

Solution:

$\therefore A(2, 3, 9), B(5, 2, 1), C(1, \lambda, 8)$ and $D(\lambda, 2, 3)$ are coplanar.

$$\therefore \left[\overrightarrow{ABACAD} \right] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

$$\Rightarrow [-6(\lambda-3) - 1] - 8(1 - (\lambda-3)(\lambda-2)) + (6 + (\lambda-2)) = 0$$

$$\Rightarrow 3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$$

$$\Rightarrow 8\lambda^2 - 57\lambda + 95 = 0$$

$$\therefore \lambda_1 \lambda_2 = \frac{95}{8}$$

Question 138

Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and

$$\left(\vec{a} \times \vec{b} \right) \cdot \left(\vec{b} \times \vec{c} \right) + \left(\vec{b} \times \vec{c} \right) \cdot \left(\vec{c} \times \vec{a} \right) + \left(\vec{c} \times \vec{a} \right) \cdot \left(\vec{a} \times \vec{b} \right) = 168, \text{ then}$$

$|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:
[29-Jul-2022-Shift-2]

Options:

A. 10

B. 14

C. 16

D. 18

Answer: C

Solution:

$$|\vec{a}| |\vec{b}| |\vec{c}| = 14$$

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$$

$$\text{So, } \vec{a} \cdot \vec{b} = -\frac{1}{2}ab, \vec{b} \cdot \vec{c} = -\frac{1}{2}bc, \vec{a} \cdot \vec{c} = -\frac{1}{2}ac$$

(let)

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{b})$$

$$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$$

Similarly

$$(\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a})$$

$$= \frac{3}{4}abc^2$$

$$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b})$$

$$= \frac{3}{4}a^2bc$$

$$= \frac{3}{4}abc(a+b+c)$$

$$\text{So, } (a+b+c) = 16$$

Question 139

Let $\vec{a}$ and $\vec{b}$ be two vectors such that

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to _____.

[29-Jul-2022-Shift-2]

Answer: 14

Solution:

$$\therefore |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$$

$$\text{or } |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$$

$$\therefore |\vec{b}|^2 = 6$$

$$\text{Now } |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$75 = |\vec{a}|^2 \cdot 6 - 9$$

$$\therefore |\vec{a}|^2 = 14$$

Question 140

If $(1, 5, 35)$, $(7, 5, 5)$, $(1, \lambda, 7)$ and $(2\lambda, 1, 2)$ are coplanar, then the sum of all possible values of λ is

[26 Feb 2021 Shift 1]

Options:

A. $\frac{39}{5}$

B. $-\frac{39}{5}$

C. $\frac{44}{5}$

D. $-\frac{44}{5}$

Answer: C

Solution:

Let P(1, 5, 35), Q(7, 5, 5), R(1, λ , 7), S(2 λ , 1, 2)

Given P, Q, R, S are coplanar. Then, PQ, PR, PS lie on the same plane.

$$\vec{PQ} = (7-1)\hat{i} + (5-5)\hat{j} + (5-35)\hat{k} = 6\hat{i} - 30\hat{k}$$

$$\vec{PR} = (1-1)\hat{i} + (\lambda-5)\hat{j} + (7-35)\hat{k} = (\lambda-5)\hat{j} - 28\hat{k}$$

$$\vec{PS} = (2\lambda-1)\hat{i} + (1-5)\hat{j} + (2-35)\hat{k} = (2\lambda-1)\hat{i} - 4\hat{j} - 33\hat{k}$$

$\therefore$ PQ, PR and PS lie on same plane, then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

Expand along first row,

$$6[-33(\lambda-5) - 112] + 30[(2\lambda-1)(\lambda-5)] = 0$$

$$\Rightarrow 6(-33\lambda + 53) + 30(2\lambda^2 - 11\lambda + 5) = 0$$

$$\Rightarrow 60\lambda^2 - 528\lambda + 468 = 0$$

$$\Rightarrow 10\lambda^2 - 88\lambda + 78 = 0$$

$$\Rightarrow 5\lambda^2 - 44\lambda + 39 = 0 \dots (i)$$

Possible value of λ are roots of Eq. (i).

Then, sum of all possible values of λ = Sum of roots of Eq. (i)

$$= \frac{-(-44)}{5} = \frac{44}{5}$$

$$\{ \because ax^2 + bx + c = 0, \text{ sum of roots} = -b/a \}$$

Question 141

If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, then $\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times \mathbf{b})))$ is equal to
[26 Feb 2021 Shift 1]

Options:

A. 0

B. $\frac{1}{2} |\mathbf{a}|^4 \mathbf{b}$

C. $\mathbf{a} \times \mathbf{b}$

D. $|\mathbf{a}|^4 \mathbf{b}$

Answer: D

Solution:

$$\begin{aligned} & \mathbf{a} \times [\mathbf{a} \times \{\mathbf{a} \times (\mathbf{a} \times \mathbf{b})\}] \\ &= \mathbf{a} \times (\mathbf{a} \times [(\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b}]) \\ & [\text{Using } \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}] \\ &= \mathbf{a} \times [\mathbf{a} \times ((\mathbf{a} \cdot \mathbf{b})\mathbf{a} - |\mathbf{a}|^2 \mathbf{b})] \\ &= \mathbf{a} \times [(\mathbf{a} \times (\mathbf{a} \cdot \mathbf{b})\mathbf{a}) - |\mathbf{a}|^2 (\mathbf{a} \times \mathbf{b})] \\ &= \mathbf{a} \times [0 - |\mathbf{a}|^2 (\mathbf{a} \times \mathbf{b})] \\ &= -|\mathbf{a}|^2 [\mathbf{a} \times (\mathbf{a} \times \mathbf{b})] \\ &= -|\mathbf{a}|^2 [(\mathbf{a} \cdot \mathbf{b})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{b}] \\ &= -(\mathbf{a} \cdot \mathbf{b})\mathbf{a} |\mathbf{a}|^2 + |\mathbf{a}|^4 \mathbf{b} \\ &= 0 + |\mathbf{a}|^4 \mathbf{b} \\ &= |\mathbf{a}|^4 \mathbf{b} \end{aligned}$$

Question 142

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is
24 Feb 2021 Shift 1

Answer: 75

Solution:

$$\begin{aligned}\text{Let } \vec{c} &= \lambda (\vec{b} \times (\vec{a} \times \vec{b})) \\ &= \lambda ((\vec{b} \cdot \vec{b})\vec{a} - (\vec{b} \cdot \vec{a})\vec{b}) \\ &= \lambda (5(-\hat{i} + \hat{j} + \hat{k}) + 2\hat{i} + \hat{k}) \\ &= \lambda (-3\hat{i} + 5\hat{j} + 6\hat{k}) \\ \vec{c} \cdot \vec{a} &= 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7 \\ \lambda &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\therefore 2 \left| \left(\frac{-3}{2} - 1 + 2 \right) \hat{i} + \left(\frac{5}{2} + 1 \right) \hat{j} + (3 + 1 + 1) \hat{k} \right|^2 \\ = 2 \left(\frac{1}{4} + \frac{49}{4} + 25 \right) = 25 + 50 = 75\end{aligned}$$

Question 143

If vectors $\mathbf{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\mathbf{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + \hat{j} + z\hat{k}$ is
[26 Feb 2021 Shift 2]

Options:

A. $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

B. $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

C. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

D. $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

Answer: D

Solution:

Given, $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then $\frac{x}{1} = \frac{-1}{y} = \frac{1}{z} = \lambda$ (Say)

This gives $x = \lambda$, $y = -\frac{1}{\lambda}$, $z = \frac{1}{\lambda}$

Then, unit vector parallel to vector $\vec{a}_1 + \vec{a}_2$ will be

$$= \pm \left((\lambda)\hat{i} - \left(\frac{1}{\lambda}\right)\hat{j} + \left(\frac{1}{\lambda}\right)\hat{k} \right) \frac{1}{\sqrt{(\lambda)^2 + \left(\frac{-1}{\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2}}$$

$$= \pm \left(\lambda^2\hat{i} - \hat{j} + \frac{\hat{k}}{\lambda} \right) \frac{\lambda}{\lambda\sqrt{\lambda^4 + 2}} = \pm \left(\lambda^2\hat{i} - \hat{j} + \frac{\hat{k}}{\sqrt{\lambda^4 + 2}} \right)$$

Take, $\lambda = 1 = \pm \left(\hat{i} - \hat{j} + \frac{\hat{k}}{\sqrt{3}} \right)$

Question 144

Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to
[25 Feb 2021 Shift 2]

Answer: 2

Solution:

$$\begin{aligned} \text{Area of parallelogram} &= |\vec{a} \times \vec{b}| \\ &= \left| \left(\hat{i} + \alpha\hat{j} + 3\hat{k} \right) \times \left(3\hat{i} - \alpha\hat{j} + \hat{k} \right) \right| \\ (64)(3) &= 16\alpha^2 + 64 + 16\alpha^2 \quad (\text{given, area} = 8\sqrt{3}) \\ &= \left| \left(\hat{i} + \alpha\hat{j} + 3\hat{k} \right) \times \left(3\hat{i} - \alpha\hat{j} + \hat{k} \right) \right| \\ (64)(3) &= 16\alpha^2 + 64 + 16\alpha^2 \quad (\text{given, area} = 8\sqrt{3}) \\ &\quad (\text{squaring on both sides}) \\ \Rightarrow \alpha^2 &= 4 \\ \text{Now, } \vec{a} \cdot \vec{b} &= 3 - \alpha^2 + 3 \\ &= 6 - \alpha^2 = 6 - 4 = 2 \end{aligned}$$

Question 145

Let $\mathbf{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\mathbf{b} = \hat{i} - \hat{j}$ and $\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$ and $\mathbf{r} \cdot \mathbf{b} = 0$, then $\mathbf{r} \cdot \mathbf{a}$ is equal to

.....

[25 Feb 2021 Shift 1]

Answer: 12

Solution:

$$\mathbf{b} = \hat{i} - \hat{j}$$

$$\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$$

$$\mathbf{r} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$$

$$\Rightarrow \mathbf{r} \times \mathbf{a} - \mathbf{c} \times \mathbf{a} = 0$$

$$\Rightarrow (\mathbf{r} - \mathbf{c}) \times \mathbf{a} = 0$$

$$\therefore \mathbf{r} - \mathbf{c} = \lambda \mathbf{a}$$

$$\Rightarrow \mathbf{r} = \lambda \mathbf{a} + \mathbf{c}$$

$$\Rightarrow \mathbf{r} \cdot \mathbf{b} = \lambda \mathbf{a} \cdot \mathbf{b} + \mathbf{c} \cdot \mathbf{b} \quad (\text{taking dot w})$$

$$\Rightarrow 0 = \lambda \mathbf{a} \cdot \mathbf{b} + \mathbf{c} \cdot \mathbf{b}$$

$$\Rightarrow \lambda (\hat{i} + 2\hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j}) + (\hat{i} - \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j}) = 0$$

$$\Rightarrow \lambda(1 - 2) + 2 = 0$$

$$\Rightarrow \lambda = 2$$

$$\therefore \mathbf{r} = 2\mathbf{a} + \mathbf{c}$$

$$\Rightarrow \mathbf{r} \cdot \mathbf{a} = 2\mathbf{a} \cdot \mathbf{a} + \mathbf{c} \cdot \mathbf{a} \quad [\text{taking dot with a}]$$

$$= 2|\mathbf{a}|^2 + \mathbf{a} \cdot \mathbf{c}$$

$$= 2(1 + 4 + 1) + (1 - 2 + 1)$$

$$\mathbf{r} \cdot \mathbf{a} = 12$$

Question 146

A vector $\mathbf{a}$ has components $3p$ and 1 with respect to rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\mathbf{a}$ has components $p + 1$ and $\sqrt{10}$, then a value of p is equal to

[18 Mar 2021 Shift 1]

Options:

A. 1

B. $-\frac{5}{4}$

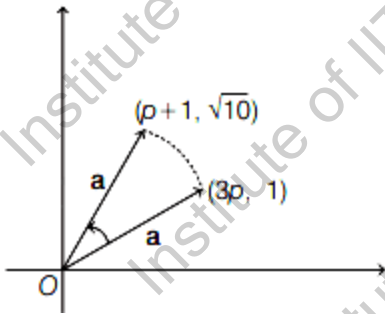
C. $\frac{4}{5}$

D. -1

Answer: D

Solution:

After counter clockwise (or anti-clockwise) rotation, the length of the vector a remains constant.



i.e. $|a|$ at old position = $|a|$ at new position

$$\Rightarrow (3p)^2 + (1)^2 = (p+1)^2 + (\sqrt{10})^2$$

$$\Rightarrow 9p^2 + 1 = p^2 + 1 + 2p + 10$$

$$\Rightarrow 8p^2 - 2p - 10 = 0 \Rightarrow 4p^2 - p - 5 = 0$$

$$\Rightarrow (p+1)(4p-5) = 0$$

$$\Rightarrow p = \frac{5}{4}, -1$$

Question 147

Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to
[16 Mar 2021 Shift 1]

Options:

A. $\frac{1}{2}$

B. 1

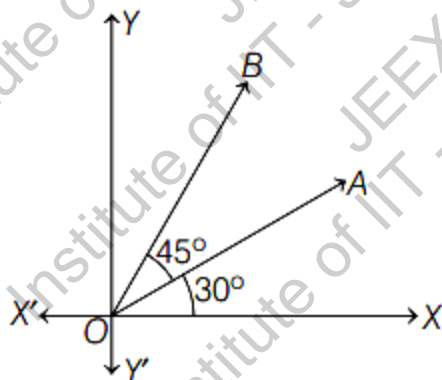
C. $\frac{1}{\sqrt{2}}$

D. $2\sqrt{2}$

Answer: A

Solution:

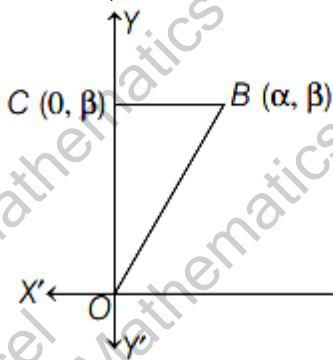
Let OA be $\sqrt{3}\hat{i} + \hat{j}$ and OB be $\alpha\hat{i} + \beta\hat{j}$.



As, we can notice in OA, $\frac{1}{\sqrt{3}} = \tan 30^\circ$. So, it makes an angle of 30° with the X-axis.

Now, when OA is rotated further by 45° anticlockwise, the resultant vector OB makes an angle of 75° with the X-axis.

So, $OB = |OA| (\cos 75^\circ \hat{i} + \sin 75^\circ \hat{j})$



Let $\triangle OBC$ be the required triangle whose area we have to determine.

Area of $\triangle OBC = (1/2) \times (\text{Base}) \times (\text{Height})$

$$= 1/2 \times \beta \times \alpha$$

$$= \frac{1}{2} (2 \sin 75^\circ) (2 \cos 75^\circ) = 2 \sin 75^\circ \cos 75^\circ$$

$$= \sin 150^\circ = \sin 30^\circ$$

$$= 1/2$$

Hence, the area is $1/2$ sq. unit.

Question 148

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to
[17 Mar 2021 Shift 2]

Answer: 486

Solution:

Let $x = \lambda a + \mu b$, where λ and μ are scalars.

$$\Rightarrow x = \lambda(2\hat{i} - \hat{j} + \hat{k}) + \mu(\hat{i} + 2\hat{j} - \hat{k})$$

$$x = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$$

Since, x is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$.

$$\text{Then, } x \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$$

$$\Rightarrow 3\lambda + 8\mu = 0$$

Also, given projection of x on a is $\frac{17\sqrt{6}}{2}$(i)

$$\therefore \frac{x \cdot a}{|a|} = \frac{17\sqrt{6}}{2}$$

$$\Rightarrow 2(2\lambda + \mu) + (\lambda - 2\mu) + (\lambda - \mu) = 51$$

$$6\lambda - \mu = 51 \dots (ii)$$

From Eqs. (i) and (ii),

$$\lambda = 8, \mu = -3$$

$$\therefore x = 13\hat{i} - 14\hat{j} + 11\hat{k}$$

$$\Rightarrow |x| = \sqrt{(13)^2 + (-14)^2 + (11)^2}$$

$$\therefore |x| = \sqrt{(13)^2 + (-14)^2 + (11)^2} = 486$$

Question 149

Let a and b be two non-zero vectors perpendicular to each other and $|a| = |b|$. If $|a \times b| = |a|$, then the angle between the vectors $[a + b + (a \times b)]$ and a is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

D. $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Answer: B

Solution:

Given, $a \perp b \dots (i)$

$|a| = |b| \dots (ii)$

and $|a \times b| = |a|$

$$\Rightarrow |a| |b| \sin 90^\circ = |a| \quad [\text{from Eq. (i)}]$$

$$\Rightarrow |b| = 1 = |a| \dots (iii) \quad [\text{from Eq. (ii)}]$$

From Eq. (iii), we can say that

$a \times b$ are mutually perpendicular unit vectors.

Let $\hat{a} = \hat{i}$ and $\hat{b} = \hat{j}$

$$\therefore \hat{a} \times \hat{b} = \hat{k}$$

$$\text{Now, } [\hat{a} + \hat{b} + (\hat{a} \times \hat{b})] = (\hat{i} + \hat{j} + \hat{k})$$

$$\therefore \cos \theta = \left(\hat{i} + \hat{j} + \hat{k} \right) \cdot \frac{\hat{i}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Question 150

Let $\hat{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\hat{b} = 7\hat{i} + \hat{j} - 6\hat{k}$. If

$\hat{r} \times \hat{a} = \hat{r} \times \hat{b}$, $\hat{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\hat{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to
[17 Mar 2021 Shift 1]

Options:

A. 12

B. 8

C. 13

D. 10

Answer: A

Solution:

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\mathbf{b} = 7\hat{i} + \hat{j} - 6\hat{k}$$

$$\text{If } \mathbf{r} \times \mathbf{a} = \mathbf{r} \times \mathbf{b}$$

$$\Rightarrow \mathbf{r} = \lambda(\mathbf{a} - \mathbf{b}) = \lambda(5\hat{i} + 4\hat{j} - 10\hat{k})$$

$$\text{Now, } \mathbf{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$$

$$\Rightarrow \lambda(5\hat{i} + 4\hat{j} + 10\hat{k}) \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$$

$$\Rightarrow \lambda(5 + 8 + 10) = -3$$

$$\Rightarrow \lambda = +1$$

$$\therefore \mathbf{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$$

$$\text{So, } \mathbf{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$$

$$= (-5\hat{i} - 4\hat{j} + 10\hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$$

$$= -10 + 12 + 10$$

$$= 12$$

Question 151

Let $\mathbf{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\mathbf{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If

$\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{r}$, $\mathbf{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$ and $\mathbf{r} \cdot (2\hat{i} + 5\hat{j} - \mathbf{a}) = -1$, $\alpha \in \mathbb{R}$, then the value of $\alpha + |\mathbf{r}|^2$ is equal to
[16 Mar 2021 Shift 2]

Options:

A. 9

B. 15

C. 13

D. 11

Answer: B

Solution:

$$\text{Given, } \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\text{and } \vec{b} = (2\hat{i} - 3\hat{j} + 5\hat{k})$$

$$\text{If } \vec{r} \times \vec{a} = \vec{b} \times \vec{r}, \vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$$

$$\Rightarrow \vec{r} \times \vec{a} = \vec{b} \times \vec{r}$$

$$(\vec{r} \times \vec{a}) = -(\vec{r} \times \vec{b}) \Rightarrow (\vec{r} \times \vec{a}) + (\vec{r} \times \vec{b}) = 0$$

$$\Rightarrow \vec{r} \times (\vec{a} + \vec{b}) = 0 \Rightarrow \vec{r} = \lambda(\vec{a} + \vec{b})$$

$$\Rightarrow \vec{r} = \lambda[(1+2)\hat{i} + (2-3)\hat{j} + (-3+5)\hat{k}]$$

$$\Rightarrow \vec{r} = \lambda(3\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\Rightarrow \lambda(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\Rightarrow \lambda(3\alpha - 2 + 2) = 3$$

$$\Rightarrow \alpha\lambda = 1$$

$$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$$

$$\Rightarrow \lambda(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$$

$$\Rightarrow \lambda(6 - 5 - 2\alpha) = -1 \Rightarrow \lambda(1 - 2\alpha) = -1$$

$$\Rightarrow \lambda - 2\alpha\lambda = -1 \Rightarrow \lambda - 2 = -1$$

$$\Rightarrow \lambda = 1$$

$$\text{So, } \alpha = 1$$

$$\vec{r} = (3\hat{i} - \hat{j} + 2\hat{k})$$

$$\Rightarrow |\vec{r}|^2 = 9 + 1 + 4 = 14$$

$$\therefore \alpha + |\vec{r}|^2 = 1 + 14 = 15$$

Question 152

Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and

$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$ then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to

[16 Mar 2021 Shift 2]

Answer: 28

Solution:

Since, c is perpendicular to a and b .

So, $c = \lambda(a \times b)$

$$a = \hat{i} + \hat{j} - \hat{k}$$

$$b = \hat{i} + 2\hat{j} + \hat{k}$$

$$\text{Now, } a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= (1+2)\hat{i} - (1+1)\hat{j} + (2-1)\hat{k}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow c = \lambda(3\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{Now, } c \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$$

$$\lambda(3\hat{i} - 2\hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$$

$$\Rightarrow 4\lambda = 8$$

$$\therefore \lambda = 2$$

$$\text{So, } c = 2(3\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{So, } c \cdot (a \times b) = 2(3\hat{i} - 2\hat{j} + \hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= 2(9 + 4 + 1)$$

$$= 28$$

Question 153

If $a = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$, $b = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and $c = \hat{i} - 2\hat{j} - \hat{k}$ such that $a \cdot b = 1$

and $b \cdot c = -3$, then $\frac{1}{3}[(a \times b) \cdot c]$ is equal to

[17 Mar 2021 Shift 1]

Answer: 2

Solution:

$$a = \langle \alpha, \beta, 3 \rangle$$

$$b = \langle -\beta, -\alpha, -1 \rangle$$

$$c = \langle 1, -2, -1 \rangle$$

$$a \cdot b = 1$$

$$-\alpha\beta - \alpha\beta - 3 = 1$$

$$\alpha\beta = -2$$

$$b \cdot c = -3$$

$$-\beta + 2\alpha + 1 = -3$$

$$\beta - 2\alpha = 4$$

$$\Rightarrow \beta - 2\left(-\frac{2}{\beta}\right) = 4$$

$$\Rightarrow \beta^2 + 4 = 4\beta \Rightarrow \beta^2 - 4\beta + 4 = 0$$

$$\Rightarrow (\beta - 2)^2 = 0 \Rightarrow \beta = 2$$

$$\alpha\beta = -2 \Rightarrow \alpha \cdot 2 = -2$$

$$\Rightarrow \alpha = -1$$

$$\text{Hence, } \frac{1}{3}[(a \times b) \cdot c]$$

$$a = \langle -1, 2, 3 \rangle$$

$$b = \langle -2, 1, -1 \rangle$$

$$c = \langle 1, -2, -1 \rangle$$

$$(a \times b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & 1 & -1 \end{vmatrix} = -5\hat{i} - 7\hat{j} + 3\hat{k}$$

$$(a \times b) \cdot c = (-5\hat{i} - 7\hat{j} + 3\hat{k}) \cdot (\hat{i} - 2\hat{j} - \hat{k})$$

$$= -5 + 14 - 3 = 6$$

$$\therefore \frac{1}{3}[(a \times b) \cdot c] = \frac{1}{3} \times 6 = 2$$

Question 154

Let O be the origin. Let $\vec{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and

$\vec{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in \mathbb{R}$, $x > 0$, be such that $|\vec{PQ}| = \sqrt{20}$ and the vector $\vec{OP}$ is perpendicular to $\vec{OQ}$. If $\vec{OR} = 3\hat{i} + 2\hat{j} - 7\hat{k}$, $z \in \mathbb{R}$, is coplanar with $\vec{OP}$ and $\vec{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to [17 Mar 2021 Shift 2]

Options:

A. 7

B. 9

C. 2

D. 1

Answer: B

Solution:

$$\text{Given, } \vec{OP} = x\hat{i} + y\hat{j} - \hat{k}$$

$$\vec{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$$

$$\vec{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$$

$$\text{and } |\vec{PQ}| = \sqrt{20}$$

$$\text{Now, } |\vec{PQ}| = |\vec{OQ} - \vec{OP}| = |\vec{OP} - \vec{OQ}|$$

$$= (x+1)\hat{i} + (y-2)\hat{j} - (1+3x)\hat{k}$$

$$\Rightarrow |\vec{PQ}|^2 = (\sqrt{20})^2 = 20$$

$$\Rightarrow (x+1)^2 + (y-2)^2 + (1+3x)^2 = 20$$

$$\Rightarrow (x+1)^2 + (2x-2)^2 + (1+3x)^2 = 20 \quad \left[\begin{array}{l} \because \vec{OP} \perp \vec{OQ} \\ \therefore \vec{OP} \cdot \vec{OQ} = 0 \\ \Rightarrow -x + 2y - 3x = 0 \\ \Rightarrow y = 2x \end{array} \right]$$

$$\Rightarrow x^2 + 1 + 2x + 4x^2 + 4 - 8x + 1 + 9x^2 + 6x = 20$$

$$\Rightarrow 14x^2 + 6 = 20 \Rightarrow 14x^2 = 14$$

$$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \text{ but } x \text{ must be positive as in question conditions i.e. } x > 0.$$

$$\therefore x = -1 \text{ (Rejected)}$$

$$\text{Hence, } x = 1$$

$$\therefore y = 2x = 2 \times 1 = 2$$

Now, $\vec{OP}$, $\vec{OQ}$ and $\vec{OR}$ are coplanar.

$$\therefore [\vec{OP} \vec{OQ} \vec{OR}] = 0$$

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1(-z - 6) = 0 \Rightarrow z = -2$$

$$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$$

Question 155

If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in

degrees) is _____.
[25 Jul 2021 Shift 2]

Answer: 60

Solution:

$$(\vec{a} + 3\vec{b}) \perp (7\vec{a} - 5\vec{b})$$

$$(\vec{a} + 3\vec{b}) \cdot (7\vec{a} - 5\vec{b}) = 0$$

$$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a} \cdot \vec{b} = 0 \dots\dots(1)$$

$$(\vec{a} - 4\vec{b}) \cdot (7\vec{a} - 2\vec{b}) = 0$$

$$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a} \cdot \vec{b} = 0 \dots\dots(2)$$

from (1) & (2)

$$|\vec{a}| = |\vec{b}|$$

$$\cos \theta = \frac{|\vec{b}|}{2|\vec{a}|} \therefore \theta = 60^\circ$$

Question 156

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ , with the vector

$\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to _____.

[20 Jul 2021 Shift 1]

Answer: 4

Solution:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}) = 3$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$

$$\begin{aligned}\vec{a}(\vec{a} + \vec{b} + \vec{c}) &= |\vec{a}| |\vec{a} + \vec{b} + \vec{c}| \cos \theta \\ \Rightarrow 1 &= \sqrt{3} \cos \theta \\ \Rightarrow \cos 2\theta &= -\frac{1}{3} \\ \Rightarrow 36 \cos^2 2\theta &= 4\end{aligned}$$

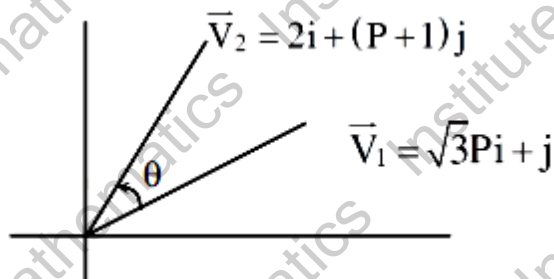
Question 157

For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan \theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to _____.

[20 Jul 2021 Shift 2]

Answer: 6

Solution:



$$\begin{aligned}|\vec{v}_1| &= |\vec{v}_2| \\ 3p^2 + 1 &= 4 + (p+1)^2 \\ 2p^2 - 2p - 4 &= 0 \Rightarrow p^2 - p - 2 = 0 \\ p &= 2, -1 \text{ (rejected)} \\ \cos \theta &= \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|} = \frac{2\sqrt{3}p + (p+1)}{\sqrt{(p+1)^2 + 4} \sqrt{3p^2 + 1}} \\ \cos \theta &= \frac{4\sqrt{3} + 3}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3} + 3}{13} \\ \tan \theta &= \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3} + 3} = \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3} = \frac{\alpha\sqrt{3} - 2}{4\sqrt{3} + 3} \\ \Rightarrow \alpha &= 6\end{aligned}$$

Question158

In a triangle ABC, if $|\vec{BC}| = 3$, $|\vec{CA}| = 5$ and $|\vec{BA}| = 7$, then the projection of the vector $\vec{BA}$ on $\vec{BC}$ is equal to
[20 Jul 2021 Shift 2]

Options:

A. $\frac{19}{2}$

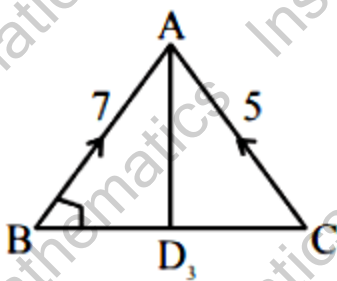
B. $\frac{13}{2}$

C. $\frac{11}{2}$

D. $\frac{15}{2}$

Answer: C

Solution:



Projection of $\vec{BA}$ on $\vec{BC}$ is equal to

$$\begin{aligned} &= |\vec{BA}| \cos \angle ABC \\ &= 7 \left| \frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3} \right| = \frac{11}{2} \end{aligned}$$

Question159

Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors.

If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors

$(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to

[25 Jul 2021 Shift 1]

Answer: 3

Solution:

$$\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k} \text{ (Given)}$$

$$\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\text{Now } (\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -2\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{r} = \pm \sqrt{3} \frac{((\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}))}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|} = \pm \frac{\sqrt{3}(-2\hat{i} - 2\hat{j} - 2\hat{k})}{\sqrt{2^2 + 2^2 + 2^2}}$$

$$\vec{r} = \pm(-\hat{i} - \hat{j} - \hat{k})$$

According to question

$$\vec{r} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\text{So } |\alpha| = 1, |\beta| = 1, |\gamma| = 1$$

$$\Rightarrow |\alpha| + |\beta| + |\gamma| = 3$$

Question 160

If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to :

[25 Jul 2021 Shift 2]

Options:

A. 6

B. 4

C. 3

D. 5

Answer: A

Solution:

$$\begin{aligned} |\vec{a}| &= 2, |\vec{b}| = 5 \\ |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| \sin \theta = \pm 8 \\ \sin \theta &= \pm \frac{4}{5} \therefore \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \\ &= 10 \cdot \left(\pm \frac{3}{5} \right) = \pm 6 \\ |\vec{a} \cdot \vec{b}| &= 6 \end{aligned}$$

Question 161

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product

$(\vec{a} + \vec{b}) \times \left(\left(\vec{a} \times \left(\left(\vec{a} - \vec{b} \right) \times \vec{b} \right) \right) \times \vec{b} \right)$ is equal to:

[27 Jul 2021 Shift 1]

Options:

A. $5(34\hat{i} - 5\hat{j} + 3\hat{k})$

B. $7(34\hat{i} - 5\hat{j} + 3\hat{k})$

C. $7(30\hat{i} - 5\hat{j} + 7\hat{k})$

D. $5(30\hat{i} - 5\hat{j} + 7\hat{k})$

Answer: B

Solution:

$$\begin{aligned} \vec{a} &= \hat{i} + \hat{j} + 2\hat{k} \\ \vec{b} &= -\hat{i} + 2\hat{j} + 3\hat{k} \\ \vec{a} + \vec{b} &= 3\hat{j} + 5\hat{k}; \vec{a} \cdot \vec{b} = -1 + 2 + 6 = 7 \end{aligned}$$

$$\begin{aligned}
 & ((\vec{a} \times ((\vec{a} - \vec{b}) \times \vec{b})) \times \vec{b}) \\
 & ((\vec{a} \times (\vec{a} \times \vec{b} - \vec{b} \times \vec{b})) \times \vec{b}) \\
 & (\vec{a} \times (\vec{a} \times \vec{b} - 0)) \times \vec{b} \\
 & (\vec{a} \times (\vec{a} \times \vec{b})) \times \vec{b} \\
 & ((\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}) \times \vec{b} \\
 & (\vec{a} \cdot \vec{b})\vec{a} \times \vec{b} - (\vec{a} \cdot \vec{a})(\vec{b} \times \vec{b}) \\
 & (\vec{a} \cdot \vec{b})(\vec{a} \times \vec{b})
 \end{aligned}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = -\hat{i} - 5\hat{j} + 3\hat{k}$$

$$\begin{aligned}
 & \therefore 7(-\hat{i} - 5\hat{j} + 3\hat{k}) \\
 & (\vec{a} + \vec{b}) \times (7(-\hat{i} - 5\hat{j} + 3\hat{k})) \\
 & 7(0\hat{i} + 3\hat{j} + 5\hat{k}) \times (-\hat{i} - 5\hat{j} + 3\hat{k})
 \end{aligned}$$

$$\begin{aligned}
 & \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix} \\
 & \Rightarrow 34\hat{i} - (5)\hat{j} + (3\hat{k}) \\
 & \Rightarrow 34\hat{i} - 5\hat{j} + 3\hat{k} \\
 & \therefore 7(34\hat{i} - 5\hat{j} + 3\hat{k})
 \end{aligned}$$

Question 162

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is 1, then the value of $3l^2$ is equal to _____.

[27 Jul 2021 Shift 1]

Answer: 2

Solution:

$$\vec{a} \times \vec{b} = \vec{c}$$

Take Dot with $\vec{c}$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2$$

Projection of $\vec{b}$ or $\vec{a} \times \vec{c} = 1$

$$\frac{|\vec{b} \cdot (\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|} = 1$$

$$\therefore 1 = \frac{2}{\sqrt{6}} \Rightarrow l^2 = \frac{4}{6}$$

$$3l^2 = 2$$

Question 163

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is θ ($0 < \theta < \frac{\pi}{2}$), then the value of $1 + \tan \theta$ is equal to:
[27 Jul 2021 Shift 2]

Options:

A. $\sqrt{3} + 1$

B. 2

C. 1

D. $\frac{\sqrt{3} + 1}{\sqrt{3}}$

Answer: B

Solution:

$$\vec{a} = (\vec{b} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{b}) \vec{c}$$

$$= 1 \cdot 2 \cos \theta \vec{b} - \vec{c}$$

$$\Rightarrow \vec{a} = 2 \cos \theta \vec{b} - \vec{c}$$

$$|\vec{a}|^2 = (2 \cos \theta)^2 + 2^2 - 2 \cdot 2 \cos \theta \vec{b} \cdot \vec{c}$$

$$\Rightarrow 2 = 4 \cos^2 \theta + 4 - 4 \cos \theta \cdot 2 \cos \theta$$

$$\Rightarrow -2 = -4 \cos^2 \theta$$

$$\begin{aligned}\Rightarrow \cos^2 \theta &= \frac{1}{2} \\ \Rightarrow \sec^2 \theta &= 2 \\ \Rightarrow \tan^2 \theta &= 1 \\ \Rightarrow \theta &= \frac{\pi}{4} \\ 1 + \tan \theta &= 2.\end{aligned}$$

Question 164

Let the vectors $(2 + a + b)\hat{i} + (a + 2b + c)\hat{j} - (b + c)\hat{k}$, $(1 + b)\hat{i} + 2b - b\hat{k}$ and $(2 + b)\hat{i} + 2b\hat{j} + (1 - b)\hat{k}$ $a, b, c, \in \mathbb{R}$ be co-planar. Then which of the following is true?
[25 Jul 2021 Shift 1]

Options:

- A. $2b = a + c$
- B. $3c = a + b$
- C. $a = b + 2c$
- D. $2a = b + c$

Answer: A

Solution:

If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix} = 0$$

Now $R_3 \rightarrow R_3 - R_2$, $R_1 \rightarrow R_1 - R_2$

$$\begin{aligned}\text{So } \begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} &= 0 \\ &= (a+1)2b - (a+c)(2b+1) - c(-2b) \\ &= 2ab + 2b - 2ab - a - 2bc - c + 2bc \\ &= 2b - a - c = 0\end{aligned}$$

Question 165

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?
[22 Jul 2021 Shift 2]

Options:

A. $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

B. Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2

C. $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} + \begin{bmatrix} \vec{c} & \vec{a} & \vec{b} \end{bmatrix} = 8$

D. $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

Answer: D

Solution:

$$\begin{aligned} (1) \vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) &= \vec{0} \\ &= \vec{a} \times (-\vec{b} \times \vec{c} + \vec{c} \times \vec{b}) = -2(\vec{a} \times (\vec{b} \times \vec{c})) \\ &= -2(\vec{a} \times \vec{a}) = \vec{0} \end{aligned}$$

$$\begin{aligned} (2) \text{Projection of } \vec{a} \text{ on } (\vec{b} \times \vec{c}) \\ &= \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{\vec{a} \cdot \vec{a}}{|\vec{a}|} = |\vec{a}| = 2 \end{aligned}$$

$$\begin{aligned} (3) \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} + \begin{bmatrix} \vec{c} & \vec{a} & \vec{b} \end{bmatrix} &= 2 \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} = 2\vec{a} \cdot (\vec{b} \times \vec{c}) \\ &= 2\vec{a} \cdot \vec{a} = 2|\vec{a}|^2 = 8 \end{aligned}$$

$$\begin{aligned} (4) \vec{a} \times \vec{b} &= \vec{c} \text{ and } \vec{b} \times \vec{c} = \vec{a} \\ \Rightarrow \vec{a}, \vec{b}, \vec{c} &\text{ are mutually perp vectors.} \end{aligned}$$

$$\therefore |\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}| |\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = \frac{|\vec{c}|}{2}$$

$$\text{Also, } |\vec{b} \times \vec{c}| = |\vec{a}| \Rightarrow |\vec{b}| |\vec{c}| = 2 \Rightarrow |\vec{c}| = 2 \text{ \& } |\vec{b}| = 1$$

$$\begin{aligned} |\vec{a} + \vec{b} - 2\vec{c}|^2 &= (3\vec{a} + \vec{b} - 2\vec{c}) \cdot (3\vec{a} + \vec{b} - 2\vec{c}) \\ &= 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 \\ &= (9 \times 4) + 1 + (4 \times 4) \\ &= 36 + 1 + 16 = 53 \end{aligned}$$

Question 166

Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$.

If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a

possible value of $[\vec{a} \vec{b} \vec{c}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{c} \vec{d}]$ equal to:

[22 Jul 2021 Shift 2]

Options:

A. -42

B. -40

C. -29

D. -38

Answer: A

Solution:

$$\vec{a} = \lambda \vec{b} + \mu \vec{c} = \hat{i}(2\lambda + \mu) + \hat{j}(\lambda - \mu) + \hat{k}(\lambda + \mu)$$

$$\vec{a} \cdot \vec{d} = 0 = 3(2\lambda + \mu) + 2(\lambda - \mu) + 6(\lambda + \mu)$$

$$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$$

$$\Rightarrow \vec{a} = (0)\hat{i} - 3\lambda\hat{j} + (-\lambda)\hat{k}$$

$$\Rightarrow |\vec{a}| = \sqrt{10}|\lambda| = \sqrt{10} \Rightarrow |\lambda| = 1$$

$$\Rightarrow \lambda = 1 \text{ or } -1$$

$$[\vec{a} \vec{b} \vec{c}] = 0$$

$$[\vec{a} \vec{b} \vec{c}] + [\vec{a} \vec{b} \vec{d}] + [\vec{a} \vec{c} \vec{d}] = [\vec{a} \vec{b} + \vec{c} \vec{d}]$$

$$= \begin{vmatrix} 0 & -3\lambda & \lambda \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= 3\lambda(12) + \lambda(6) = 42\lambda = -42$$

Question 167

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that

$\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$,

then the value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is

[20 Jul 2021 Shift 1]

Options:

A. $\frac{2}{3}$

B. 4

C. 3

D. $\frac{3}{2}$

Answer: D

Solution:

$$|\vec{a}| = 3 = a; \vec{a} \cdot \vec{c} = c$$

$$\text{Now } |\vec{c} - \vec{a}| = 2\sqrt{2}$$

$$\Rightarrow c^2 + a^2 - 2\vec{c} \cdot \vec{a} = 8$$

$$\Rightarrow c^2 + 9 - 2(c) = 8$$

$$\Rightarrow c^2 - 2c + 1 = 0 \Rightarrow c = 1 = |\vec{c}|$$

$$\text{Also, } \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\text{Given } (\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}| |\vec{c}| \sin \frac{\pi}{6}$$

$$= (3)(1)(1/2)$$

$$= 3/2$$

Question 168

Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $|2\mathbf{a} + 3\mathbf{b}| = |3\mathbf{a} + \mathbf{b}|$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 60° . If $\frac{1}{8}\mathbf{a}$ is a unit vector, then $|\mathbf{b}|$ is equal to

[31 Aug 2021 Shift 1]

Options:

A. 4

B. 6

C. 5

D. 8

Answer: C

Solution:

$$|2a + 3b| = |3a + b|$$

$$\Rightarrow |2a + 3b|^2 = |3a + b|^2$$

$$\Rightarrow 4|a|^2 + 9|b|^2 + 12a \cdot b = 9|a|^2 + |b|^2 + 6a \cdot b$$

$$\Rightarrow 5|a|^2 - 6a \cdot b - 8|b|^2 = 0$$

$\frac{a}{8}$ is a unit vector.

$$\text{So, } |a| = 8$$

And,

$$5 \cdot 64 - 6 \cdot 8|b| \left(\frac{1}{2}\right) - 8|b|^2 = 0$$

$$\Rightarrow |b|^2 + 3|b| - 40 = 0$$

$$(|b| + 8)(|b| - 5) = 0$$

$$|b| = 5$$

As, $|b| = -8$ Not possible.

Question 169

Let a , b and c be three vectors mutually perpendicular to each other and have same magnitude. If a vector r satisfies.

$a \times \{(r - b) \times a\} + b \times \{(r - c) \times b\} + c \times \{(r - a) \times c\} = 0$, then r is equal to

[31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{3}(a + b + c)$

B. $\frac{1}{3}(2a + b - c)$

C. $\frac{1}{2}(a + b + c)$

D. $\frac{1}{2}(a + b + 2c)$

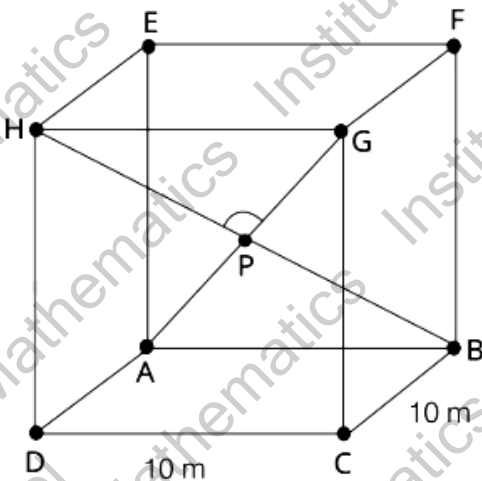
Answer: C

Solution:

$$\begin{aligned}a \times [(r-b) \times a] + b \times [(r-c) \times b] + c \times [(r-a) \times c] &= 0 \\ \Rightarrow a \cdot a(r-b) - (a \cdot (r-b))a + b \cdot b(r-c) - (b \cdot (r-c))b + c \cdot c(r-a) - (c \cdot (r-a))c &= 0 \\ \Rightarrow |a|^2(r-b) - (r \cdot a)a + |b|^2(r-c) - (r \cdot b)b + |c|^2(r-a) - (r \cdot c)c &= 0 \quad [\because a, b, c \text{ are mutually perpendicular; } \therefore a \cdot b = b \cdot c = c \cdot a = 0] \\ \Rightarrow |a|^2[3r - (a+b+c)] - [(r \cdot a)a + (r \cdot b)b + (r \cdot c)c] &= 0 \quad [\because |a| = |b| = |c|] \\ \Rightarrow |a|^2[3r - (a+b+c) - r] &= 0 \\ \therefore 3r - (a+b+c) - r &= 0 \\ \Rightarrow r &= \frac{a+b+c}{2}\end{aligned}$$

Question 170

A hall has a square floor of dimension $10\text{m} \times 10\text{m}$ (see the figure) and vertical walls. If the $\angle GPH$ between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in m) is



[26 Aug 2021 Shift 2]

Options:

A. 5

B. $2\sqrt{10}$

C. $5\sqrt{3}$

D. $5\sqrt{2}$

Answer: D

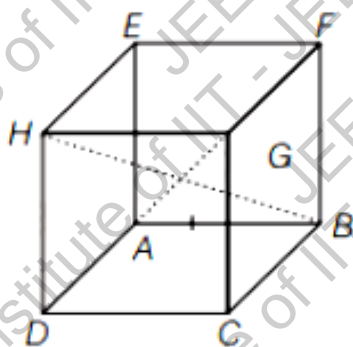
Solution:

Let $A = (0, 0, 0)$

$B = (10, 0, 0)$

$G = (10, 10, h)$

$H = (0, 10, h)$



$$\vec{AG} = 10\hat{i} + 10\hat{j} + h\hat{k}$$

$$\vec{BH} = -10\hat{i} + 10\hat{j} + h\hat{k}$$

$$\text{Since, } \vec{AG} \cdot \vec{BH} = |\vec{AG}| |\vec{BH}| \cos \theta$$

$$(-100 + 100 + h^2) = \sqrt{h^2 + 200} \cdot \sqrt{h^2 + 200} \cdot \left(\frac{1}{5}\right)$$

$$\Rightarrow 5h^2 = h^2 + 200$$

$$\Rightarrow h^2 = 50$$

$$\Rightarrow h = 5\sqrt{2}$$

Question 171

If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to
[26 Aug 2021 Shift 2]

Answer: 5

Solution:

$$\text{Let } \mathbf{a} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\mathbf{c} = -\lambda\hat{i} + 2\hat{j} + 3\hat{k}$$

Now, according to the questions,

$$\frac{\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})}{|\mathbf{b} + \mathbf{c}|} = 1$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} = |\mathbf{b} + \mathbf{c}|$$

$$\Rightarrow 5 + (-\lambda + 4 + 3) = |(2 - \lambda)\hat{i} + 6\hat{j} - 2\hat{k}|$$

$$\Rightarrow (12 - \lambda)^2 = (2 - \lambda)^2 + 36 + 4$$

$$\Rightarrow \lambda^2 + 144 - 24\lambda = \lambda^2 - 4\lambda + 4 + 40$$

$$\Rightarrow \lambda = 5$$

Question 172

Let $\mathbf{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\mathbf{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\mathbf{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\mathbf{b} \times \mathbf{c}| = 5\sqrt{3}$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$. Then, the greatest amongst the values of $|\mathbf{a}|^2$ is
[27 Aug 2021 Shift 1]

Answer: 90

Solution:

$$\text{Given, } \mathbf{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$$

$$\mathbf{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$$

$$\text{and } \mathbf{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\because \mathbf{a} \perp \mathbf{b}$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$$

$$\Rightarrow (\hat{i} + 5\hat{j} + \alpha\hat{k}) \cdot (\hat{i} + 3\hat{j} + \beta\hat{k}) = 0$$

$$\Rightarrow 1 + 15 + \alpha\beta = 0$$

$$\text{or } \alpha\beta = -16 \dots (i)$$

$$\text{Now, } \mathbf{b} \times \mathbf{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & \beta \\ -1 & 2 & -3 \end{vmatrix}$$

$$= \hat{i}(-9 - 2\beta) - \hat{j}(-3 + \beta) + \hat{k}(2 + 3)$$

$$\mathbf{b} \times \mathbf{c} = \hat{i}(-9 - 2\beta) + \hat{j}(3 - \beta) + 5\hat{k}$$

Given, $|b \times c| = 5\sqrt{3}$

$\Rightarrow |b \times c|^2 = 75$

$\Rightarrow (-9 - 2\beta)^2 + (3 - \beta)^2 + 25 = 75$

$\Rightarrow \beta^2 + 6\beta + 8 = 0$

$\Rightarrow \beta = -2, -4$

From Eq. (i), we get

For $\beta = -2, \alpha = 8$

For $\beta = -4, \alpha = 4$

For maximum value of $|a|^2, \alpha = 8$

$\therefore |\alpha|^2 = 1 + 25 + 64 = 90$

Question 173

If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$ then the value of $\tan p$ is

[26 Aug 2021 Shift 2]

Options:

A. $\frac{101}{102}$

B. $\frac{50}{51}$

C. 100

D. $\frac{51}{50}$

Answer: B

Solution:

Given, $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$

Now, $\sum \tan^{-1} \left(\frac{2}{1 + 4r^2 - 1} \right)$

$= \sum \tan^{-1} \left[\frac{(2r+1) - (2r-1)}{1 + (2r+1)(2r-1)} \right]$

$= \sum [\tan^{-1}(2r+1) - \tan^{-1}(2r-1)]$

$= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3) + \dots + \tan^{-1}101 - \tan^{-1}99$

$= \tan^{-1}(101) - \tan^{-1}1$

$= \tan^{-1} \left(\frac{101-1}{1+101} \right) = \tan^{-1} \left(\frac{50}{51} \right)$

$$\therefore \tan^{-1} \frac{50}{51} = p$$

$$\Rightarrow \tan p = \frac{50}{51}$$

Question 174

Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = \hat{j} - \hat{k}$. If $\mathbf{c}$ is a vector such that $\mathbf{a} \times \mathbf{c} = \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c} = 3$, then $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is equal to
[26 Aug 2021 Shift 1]

Options:

A. -2

B. -6

C. 6

D. 2

Answer: A

Solution:

Given, $\mathbf{a} \times \mathbf{c} = \mathbf{b}$

$\mathbf{a} \times (\mathbf{a} \times \mathbf{c}) = \mathbf{a} \times \mathbf{b}$

$(\mathbf{a} \cdot \mathbf{c})\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\mathbf{c} = \mathbf{a} \times \mathbf{b}$

We have, $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (0, 1, -1)$, $\mathbf{a} \cdot \mathbf{c} = 3$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{i} + \hat{j} + \hat{k}$$

So, $3\mathbf{a} - 3\mathbf{c} = (-2\hat{i} + \hat{j} + \hat{k})$

$$\Rightarrow (3\hat{i} + 3\hat{j} + 3\hat{k}) - 3\mathbf{c} = (-2\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow 3\mathbf{c} = (5\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\text{Now, } \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \left(\frac{1}{3}\right) \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 5 & 2 & 2 \end{vmatrix} = \frac{1}{3}(4 - 5 - 5) = -2$$

Question 175

Let $P_1, P_2, \dots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points $P_{\hat{i}}, P_{\hat{j}}$ and $P_{\hat{k}}$ such that $\hat{i} + \hat{j} + \hat{k} \neq 15$ is

[1 Sep 2021 Shift 2]

Options:

- A. 12
- B. 419
- C. 443
- D. 455

Answer: C

Solution:

$$\hat{i} + \hat{j} + \hat{k} = 15$$

$$\text{where, } \hat{i} = 1, \hat{j} + \hat{k} = 14$$

$$\Rightarrow (\hat{j} = 2, \hat{k} = 12), (\hat{j} = 3, \hat{k} = 11), (\hat{j} = 4, \hat{k} = 10),$$

$$(\hat{j} = 5, \hat{k} = 9), (\hat{j} = 6, \hat{k} = 8) \dots 5 \text{ ways}$$

$$\hat{i} = 2, \hat{j} + \hat{k} = 13$$

$$\Rightarrow (\hat{j} = 3, \hat{k} = 10), \dots, (\hat{j} = 6, \hat{k} = 7) \dots 4 \text{ ways}$$

$$\hat{i} = 3, \hat{j} + \hat{k} = 12$$

$$\Rightarrow (\hat{j} = 4, \hat{k} = 8), (\hat{j} = 5, \hat{k} = 7) \dots 2 \text{ ways}$$

$$\hat{i} = 4, \hat{j} + \hat{k} = 11$$

$$\Rightarrow (\hat{j} = 5, \hat{k} = 6) \dots 1 \text{ way}$$

$$\therefore \text{Total} = 12 \text{ ways}$$

Then, number of possible triangles using vertices

$$P_{\hat{i}}, P_{\hat{j}}, P_{\hat{k}} \text{ such that } \hat{i} + \hat{j} + \hat{k} \neq 15 \text{ is}$$

$${}^{15}C_3 - 12 = 455 - 12 = 443$$

Question 176

A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ ($\alpha, \beta \in \mathbb{R}$) lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

[Jan. 7, 2020 (I)]

Options:

A. $\vec{a} \cdot \hat{i} + 3 = 0$

B. $\vec{a} \cdot \hat{i} + 1 = 0$

C. $\vec{a} \cdot \hat{k} + 2 = 0$

D. $\vec{a} \cdot \hat{k} + 4 = 0$

Answer: C

Solution:

Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$$\vec{a} = \lambda(\hat{b} + \hat{c}) \text{ or } \vec{a} = \mu(\hat{b} - \hat{c})$$

$$\text{If } \vec{a} = \lambda \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}} \right)$$

$$= \frac{\lambda}{3\sqrt{2}} [3\hat{i} + 3\hat{j} + \hat{i} - \hat{j} + 4\hat{k}]$$

$$= \frac{\lambda}{3\sqrt{2}} [4\hat{i} + 2\hat{j} + 4\hat{k}]$$

$$\text{Compare with } \vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$$

$$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$$

$$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

Not satisfy any option

$$\text{Now consider } \vec{a} = \mu \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} - \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}} \right)$$

$$\vec{a} = \frac{\mu}{3\sqrt{2}} (3\hat{i} + 3\hat{j} - \hat{i} + \hat{j} - 4\hat{k})$$

$$= \frac{\mu}{3\sqrt{2}} (2\hat{i} + 4\hat{j} - 4\hat{k})$$

$$\text{Compare with } \vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$$

$$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2}$$

$$\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{a} \cdot \vec{k} + 2 = (\hat{i} + 2\hat{j} - 2\hat{k}) \cdot \hat{k} + 2 \\ = -2 + 2 = 0$$

Question 177

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ if

$$\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} \text{ and}$$

$$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}, \text{ then}$$

the ordered pair, $(\lambda, \vec{d})$ is equal to:

[Jan. 7, 2020 (II)]

Options:

A. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

B. $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$

C. $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

D. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

Answer: D

Solution:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$$

$$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a} \quad [\because \vec{c} = -\vec{a} - \vec{b}]$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$\vec{d} = 3(\vec{a} \times \vec{b})$$

Question 178

Let the volume of a parallelepiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos \theta$ can be:

[Jan. 8, 2020 (I)]

Options:

A. $\frac{7}{6\sqrt{6}}$

B. $\frac{7}{6\sqrt{3}}$

C. $\frac{5}{7}$

D. $\frac{5}{3\sqrt{3}}$

Answer: B

Solution:

It is given that $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$

Volume of d = $[\vec{u} \cdot \vec{v} \cdot \vec{w}]$

$$\Rightarrow \pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \Rightarrow -\lambda + 3 = \pm 1 \Rightarrow \lambda = 2 \text{ or } \lambda = 4$$

For $\lambda = 2$

$$\cos \theta = \frac{2+1+2}{\sqrt{6}\sqrt{6}} = \frac{5}{6}$$

For $\lambda = 4$

$$\cos \theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

Question 179

Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to:

[Jan. 8, 2020 (II)]

Options:

A. $-\frac{3}{2}$

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

Solution:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$$

$$-(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\Rightarrow -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} - \hat{k})$$

$$\Rightarrow -4\vec{c} = 2\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{c} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow \vec{b} \cdot \vec{c} = -\frac{1}{2}$$

Question 180

The projection of the line segment joining the points (1,-1,3) and (2,-4,11) on the line joining the points (-1,2,3) and (3,-2,10) is

 .
[NA Jan. 9, 2020 (I)]

Answer: 8

Solution:

Let P(1, -1, 3), Q(2, -4, 11), R(-1, 2, 3) and S(3, -2, 10)

Then, $\overrightarrow{PQ} = \hat{i} - 3\hat{j} + 8\hat{k}$

$$\text{Projection of } \overline{PQ} \text{ on } \overline{RS} \\ = \frac{\overline{PQ} \cdot \overline{RS}}{|\overline{RS}|} = \frac{4 + 12 + 56}{\sqrt{(4)^2 + (4)^2 + (7)^2}} = 8$$

Question181

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b} \cdot \vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{b}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to _____.

[NA Jan. 9, 2020 (II)]

Answer: 30

Solution:

$$\vec{b} \cdot \vec{c} = 10 \Rightarrow |\vec{b}| |\vec{c}| \cos\left(\frac{\pi}{3}\right) = 10 \\ \Rightarrow 5 \cdot |\vec{c}| \cdot \frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$$

Since, $\vec{b}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

$$\text{Now, } |\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}| |\vec{b} \times \vec{c}| \sin\left(\frac{\pi}{2}\right)$$

$$= \sqrt{3} \times |\vec{b}| |\vec{c}| \sin\frac{\pi}{3} \times 1$$

$$\text{Hence, } |\vec{a} \times (\vec{b} \times \vec{c})| = 30$$

Question182

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be such that $|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is

perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is _____.

[NA Sep. 05, 2020 (II)]

Answer: 6

Solution:

$\therefore$ Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$$\therefore \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$$

$$\text{Given, } \vec{b} \cdot \vec{c} = 0$$

$$\therefore |\vec{a} + \vec{b} - \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{a} \cdot \vec{c}$$

$$= 4 + 16 + 16 = 36$$

$$\Rightarrow |\vec{a} + \vec{b} - \vec{c}|^2 = 36$$

Question 183

If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and

$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ ($a \in \mathbb{R}$) are coplanar

and $3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is _____.

[NA Jan. 9, 2020 (I)]

Answer: 1

Solution:

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$\Rightarrow 3a+1=0 \Rightarrow a = -\frac{1}{3}$$

The given vectors

$$\vec{p} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k} = \frac{1}{3}(2\hat{i} - \hat{j} - \hat{k})$$

$$\vec{q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k})$$

$$\vec{r} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$$

$$\text{Now, } \vec{p} \cdot \vec{q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$$

$$\vec{r} \times \vec{q} = \frac{1}{9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$$

$$= \frac{1}{9}(i(4 - 1) - j(-2 - 1) + k(1 + 2))$$

$$= \frac{1}{9}(3i + 3j + 3k) = \frac{i + j + k}{3}$$

$$|\vec{r} \times \vec{q}| = \frac{1}{3}\sqrt{3} \Rightarrow |\vec{r} \times \vec{q}|^2 = \frac{1}{3}$$

$$3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \times \vec{q}|^2 = 0$$

$$\Rightarrow 3 \cdot \frac{1}{9} - \lambda \cdot \frac{1}{3} = 0 \Rightarrow \lambda = 1$$

Question 184

Let $a, b, c \in \mathbb{R}$ be such that $a^2 + b^2 + c^2 = 1$.

If $a \cos \theta - b \cos \left(\theta + \frac{2\pi}{3} \right) = c \cos \left(\theta + \frac{4\pi}{3} \right)$, where $\theta = \frac{\pi}{9}$ then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :
[Sep. 03, 2020 (II)]

Options:

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{9}$

D. 0

Answer: A

Solution:

$$a \cos \theta = b \cos \left(\theta + \frac{2\pi}{3} \right) = c \cos \left(\theta + \frac{4\pi}{3} \right) = k$$

$$a = \frac{k}{\cos \theta}, b = \frac{k}{\cos \left(\theta + \frac{2\pi}{3} \right)}, c = \frac{k}{\cos \left(\theta + \frac{4\pi}{3} \right)}$$

$$ab + bc + ca = k^2 \frac{\left[\cos \left(\theta + \frac{4\pi}{3} \right) + \cos \theta + \cos \left(\theta + \frac{2\pi}{3} \right) \right]}{\cos \left(\theta + \frac{4\pi}{3} \right) \cdot \cos \theta \cdot \cos \left(\theta + \frac{2\pi}{3} \right)}$$

$$= k^2 \left[\frac{\cos \theta + 2 \cos \left(\theta + \pi \right) \cdot \cos \left(\frac{\pi}{3} \right)}{\cos \theta \cdot \cos \left(\theta + \frac{2\pi}{3} \right) \cdot \cos \left(\theta + \frac{4\pi}{3} \right)} \right]$$

$$= k^2 \left[\frac{\cos \theta + 2 \cos \theta \cdot \frac{1}{2}}{\cos \theta \cdot \cos \left(\theta + \frac{2\pi}{3} \right) \cdot \cos \left(\theta + \frac{4\pi}{3} \right)} \right] = 0$$

$$\cos \phi = \frac{(\hat{a}\hat{i} + \hat{b}\hat{j} + \hat{c}\hat{k}) \cdot (\hat{b}\hat{i} + \hat{c}\hat{j} + \hat{a}\hat{k})}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{b^2 + c^2 + a^2}}$$

$$= ab + bc + ca = 0$$

$$\phi = \frac{\pi}{2}$$

Question 185

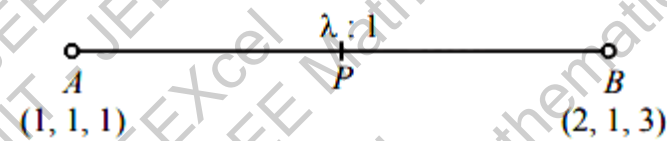
Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1 (\lambda > 0)$. If O is the origin and $\vec{OB} \cdot \vec{OP} - 3|\vec{OA} \times \vec{OP}|^2 = 6$, then λ is equal to _____.
[NA Sep. 02, 2020 (II)]

Answer: 0.8

Solution:

Let position vector of A and B be $\vec{a}$ and $\vec{b}$ respectively.

$$\therefore \text{Position vector of P is } \vec{OP} = \frac{\lambda \vec{b} + \vec{a}}{\lambda + 1}$$



$$\text{Given } \vec{OB} \cdot \vec{OP} - 3 |\vec{OA} \times \vec{OP}|^2 = 6$$

$$\Rightarrow \vec{b} \cdot \left(\frac{\lambda \vec{b} + \vec{a}}{\lambda + 1} \right) - 3 \left| \vec{a} \times \frac{\lambda \vec{b} + \vec{a}}{\lambda + 1} \right|^2 = 6$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b} + \lambda |\vec{b}|^2}{\lambda + 1} - \frac{3\lambda^2}{(\lambda + 1)^2} |\vec{a} \times \vec{b}|^2 = 6 \quad (\because \vec{a} \times \vec{b} = 2\vec{i} - \vec{j} - \vec{k} \text{ and } \vec{a} \cdot \vec{b} = 6)$$

$$\Rightarrow \frac{6 + 14\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$$

$$\Rightarrow 6 + \frac{8\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$$

$$\text{Let } \frac{\lambda}{\lambda + 1} = t$$

$$\Rightarrow 18t^2 - 8t = 0 \Rightarrow 2t(9t - 4) = 0$$

$$\Rightarrow t = 0, \frac{4}{9}$$

$$\therefore \frac{\lambda}{\lambda + 1} = \frac{4}{9} \Rightarrow \lambda = \frac{4}{5} = 0.8$$

Question 186

If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value

of $\sqrt{3} |\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is _____.

[NA Sep. 06, 2020 (I)]

Answer: 4

Solution:

Let angle between $\vec{a}$ and $\vec{b}$ be θ .

$$|\vec{a} + \vec{b}| = \sqrt{1 + 1 + 2 \cos \theta} = 2 \left| \cos \frac{\theta}{2} \right| \quad \left[\because |\vec{a}| = |\vec{b}| = 1 \right]$$

$$\text{Similarly, } |\vec{a} - \vec{b}| = 2 \left| \sin \frac{\theta}{2} \right|$$

$$\text{So, } \sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}| = 2 \left[\sqrt{3} \left| \cos \frac{\theta}{2} \right| + \left| \sin \frac{\theta}{2} \right| \right]$$

$$\therefore \text{Maximum value of } (a \cos \theta + b \sin \theta) = \sqrt{a^2 + b^2}$$

$$\therefore \text{Maximum value} = 2 \sqrt{(\sqrt{3})^2 + (1)^2} = 4$$

Question 187

If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is _____.

[NA Sep. 06, 2020 (II)]

Answer: 1

Solution:

$$\therefore |\vec{x} + \vec{y}| = |\vec{x}|$$

Squaring both sides we get

$$|\vec{x}|^2 + 2\vec{x} \cdot \vec{y} + |\vec{y}|^2 = |\vec{x}|^2$$

$$\Rightarrow 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y} = 0 \dots\dots\dots (i)$$

Also $2\vec{x} + \lambda\vec{y}$ and $\vec{y}$ are perpendicular

$$\therefore 2\vec{x} \cdot \vec{y} + \lambda\vec{y} \cdot \vec{y} = 0 \dots\dots\dots (ii)$$

Comparing (i) and (ii), $\lambda = 1$

Question 188

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$.

Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to _____.

[NA Sep. 02, 2020 (I)]

Answer: 2

Solution:

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -2$$

$$\text{Now, } |\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$$

$$= 2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 + 4(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}) = 2$$

Question 189

If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then:
[Sep. 05, 2020 (I)]

Options:

A. $\vec{a} \cdot \vec{c} = 17$

B. $\vec{b} \cdot \vec{c} = 10$

C. $n = 7$

D. $n = 9$

Answer: B

Solution:

We know that the volume of parallelopiped

$$= \left[\begin{array}{ccc} \vec{a} & \vec{b} & \vec{c} \end{array} \right]$$

$$\left| \begin{array}{ccc} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{array} \right| = 158$$

$$\Rightarrow (12 + n^2) - 1(6 + n) + n(2n - 4) = 158$$

$$\Rightarrow 3n^2 - 5n - 152 = 0$$

$$\Rightarrow 3n^2 - 24n + 19n - 152 = 0$$

$$\Rightarrow 3n(n - 8) + 19(n - 8) = 0$$

$$\Rightarrow n = 8 \text{ or } n = \frac{-19}{3}$$

$$\Rightarrow n = 8 (\because n \geq 0)$$

$$\therefore \vec{a} = \hat{i} + \hat{j} + 8\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 8\hat{k} \text{ and } \vec{c} = \hat{i} + 8\hat{j} + 3\hat{k}$$

$$\vec{a} \cdot \vec{c} = 1 + 8 + 24 = 33$$

$$\vec{b} \cdot \vec{c} = 2 + 32 - 24 = 10$$

Question 190

Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where

$\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of

$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is :

[Sep. 04, 2020 (I)]

Options:

A. -4

B. -30

C. 14

D. -22

Answer: D

Solution:

It is given that

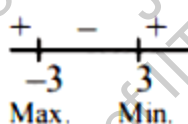
$$f(x) = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$$

$$\Rightarrow f(x) = x^3 - 27x + 26$$

$$\Rightarrow f'(x) = 3x^2 - 27$$

For critical point $f'(x) = 0$

$$\Rightarrow 3x^2 - 27 = 0 \Rightarrow x = -3, 3$$



The local maxima of $f(x)$ is, $x_0 = -3$.

Then $\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$

$$= -2x - 2x - 3 - 14 - 2x - x + 7x + 4 + 3x = 3x - 13$$

So, value at $x = x_0$, $= \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = 3x - 13$

$$= 3 \times (-3) - 13 = -22$$

Question 191

If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to _____.

[NA Sep. 04, 2020 (II)]

Answer: 18

Solution:

$$\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i} = \hat{j} + 2\hat{k}$$

$$\text{Similarly, } \hat{j} \times (\vec{a} \times \hat{j}) = 2\hat{i} + 2\hat{k}$$

$$\hat{k} \times (\vec{a} \times \hat{k}) = 2\hat{i} + \hat{j}$$

$$\therefore |\hat{j} + 2\hat{k}|^2 + |2\hat{i} + 2\hat{k}|^2 + |2\hat{i} + \hat{j}|^2 = 5 + 8 + 5 = 18$$