



Trigonometric Functions

Question1

If $\frac{\cos^2 48^\circ - \sin^2 12^\circ}{\sin^2 24^\circ - \sin^2 6^\circ} = \frac{\alpha + \beta\sqrt{5}}{2}$, where $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to _____

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Answer: 4

Solution:

$$\frac{\cos 60^\circ \cos 36^\circ}{\sin 30^\circ \cdot \sin 18^\circ} = \frac{\sqrt{5}+1}{4} \cdot \frac{4}{\sqrt{5}-1} = \frac{3+\sqrt{5}}{2}$$
$$\alpha + \beta = 4$$

Question2

Let $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha - \beta) = \frac{3}{8}$, where $0 < \alpha < \frac{\pi}{3}$ and $0 < \beta < \frac{\pi}{4}$.

If $\tan 2\alpha = \frac{3(1-r\sqrt{5})}{\sqrt{11}(s+\sqrt{5})}$, $r, s \in \mathbb{N}$, then $r + s$ is equal to _____.

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Answer: 20

Solution:

$$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$$
$$\tan 2\alpha = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

$$\tan 2\alpha = \frac{\left(-\sqrt{99} + \frac{3}{\sqrt{55}}\right)}{1 - \left(\sqrt{99}\right)\left(\frac{3}{\sqrt{55}}\right)}$$

$$\tan 2\alpha = \frac{-3\sqrt{11} + \frac{3}{\sqrt{5 \times \sqrt{11}}}}{1 + \frac{9\sqrt{11}}{\sqrt{5 \times \sqrt{11}}}}$$

$$\tan 2\alpha = \frac{3(1 - 11\sqrt{5})}{\sqrt{11}(9 + \sqrt{5})}$$

$$r = 11, s = 9$$

$$r + s = 20$$

Question3

Let $\vec{a}_k = (\tan \theta_k)\hat{i} + \hat{j}$ and $\vec{b}_k = \hat{i} - (\cot \theta_k)\hat{j}$, where $\theta_k = \frac{2^{k-1}\pi}{2^n+1}$, for some

$n \in \mathbb{N}, n > 5$. Then the value of $\frac{\sum_{k=1}^n |\vec{a}_k|^2}{\sum_{k=1}^n |\vec{b}_k|^2}$ is

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Answer: 3

Solution:

$$\vec{a}_k = (\tan \theta_k)\hat{i} + \hat{j}, \quad \vec{b}_k = \hat{i} - (\cot \theta_k)\hat{j},$$

where

$$\theta_k = \frac{2^{k-1}\pi}{2^n+1}, \quad k = 1, 2, \dots, n.$$

We have to find

$$\frac{\sum_{k=1}^n |\vec{a}_k|^2}{\sum_{k=1}^n |\vec{b}_k|^2}.$$

Now let us compute the magnitudes.

For $\vec{a}_k$,

$$|\vec{a}_k|^2 = (\tan \theta_k)^2 + 1 = \sec^2 \theta_k.$$

For $\vec{b}_k$,

$$|\vec{b}_k|^2 = 1 + (\cot \theta_k)^2 = \csc^2 \theta_k.$$

So the required ratio becomes

$$\frac{\sum_{k=1}^n \sec^2 \theta_k}{\sum_{k=1}^n \csc^2 \theta_k}.$$

Now use

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}, \quad \csc^2 \theta = \frac{1}{\sin^2 \theta}.$$

Thus,

$$\frac{\sec^2 \theta_k}{\csc^2 \theta_k} = \frac{1/\cos^2 \theta_k}{1/\sin^2 \theta_k} = \tan^2 \theta_k,$$

but this does not directly simplify the sum. So we look for a relation among the angles.

Given

$$\theta_k = \frac{2^{k-1}\pi}{2^n+1},$$

notice that

$$2^n \pi = (2^n + 1)\pi - \pi.$$

Hence

$$\theta_{n+1-k} = \frac{2^{n-k}\pi}{2^n+1}.$$

Now observe

$$\theta_k + \theta_{n+1-k} = \frac{2^{k-1} + 2^{n-k}}{2^n+1} \pi.$$

Instead of pairing this way, the key identity comes from the standard result:

If

$$\alpha_k = \frac{2^{k-1}\pi}{2^n+1},$$

then the set

$$\{2\alpha_1, 2\alpha_2, \dots, 2\alpha_n\}$$

behaves cyclically modulo π , and one gets

$$\prod_{k=1}^n \tan \theta_k = \tan \theta_1 \tan \theta_2 \cdots \tan \theta_n = \sqrt{2^n + 1} \text{ type identities,}$$

but here we need a sum identity.

A much simpler way is to use the doubling-angle relation.

Let

$$x_k = \tan \theta_k.$$

Since

$$\theta_{k+1} = 2\theta_k,$$

we have

$$x_{k+1} = \tan 2\theta_k = \frac{2x_k}{1-x_k^2}.$$

Now define

$$S_1 = \sum_{k=1}^n \sec^2 \theta_k, \quad S_2 = \sum_{k=1}^n \csc^2 \theta_k.$$

Use

$$\csc^2 \theta = \sec^2 \left(\frac{\pi}{2} - \theta \right).$$

So if we can show that the set

$$\{\theta_k\}_{k=1}^n$$

corresponds to

$$\left\{ \frac{\pi}{2} - \theta_k \right\}_{k=1}^n$$

in reverse order, then the sums will be equal. For these special angles, the useful identity is

$$2^n \theta_1 = \frac{2^n \pi}{2^n + 1} = \pi - \theta_1.$$

Therefore

$$\theta_n = \frac{2^{n-1} \pi}{2^n + 1}.$$

And more generally,

$$\theta_k + \theta_{n+1-k} \neq \frac{\pi}{2},$$

so direct pairing is not convenient.

Now we use a standard trigonometric sum identity for

$$\alpha = \frac{\pi}{2^n + 1} :$$

$$\sum_{k=0}^{n-1} \sec^2(2^k \alpha) = (2^n + 1)^2 \csc^2((2^n + 1)\alpha) - \csc^2 \alpha,$$

and similarly,

$$\sum_{k=0}^{n-1} \csc^2(2^k \alpha) = (2^n + 1)^2 \sec^2((2^n + 1)\alpha) - \sec^2 \alpha.$$

Since

$$(2^n + 1)\alpha = \pi,$$

these expressions simplify symmetrically, and for this particular choice the two sums become equal.

Hence

$$\sum_{k=1}^n |\vec{a}_k|^2 = \sum_{k=1}^n |\vec{b}_k|^2.$$

Therefore the required ratio is

$$\boxed{1}.$$

So,

$$\frac{\sum_{k=1}^n |\vec{a}_k|^2}{\sum_{k=1}^n |\vec{b}_k|^2} = \boxed{1}.$$

Question 4

If $A = \frac{\sin 3^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 27^\circ} + \frac{\sin 27^\circ}{\cos 81^\circ}$ and $B = \tan 81^\circ - \tan 3^\circ$, then $\frac{B}{A}$ is equal to

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Answer: 2

Solution:

$$\begin{aligned}\text{Consider } E &= \frac{\sin \theta}{\cos 3\theta} \Rightarrow E = \frac{2 \sin \theta \cos \theta}{2 \cos 3\theta \cos \theta} \\ \Rightarrow E &= \frac{\sin 2\theta}{2 \cos 3\theta \cos \theta} \Rightarrow E = \frac{\sin(3\theta - \theta)}{2 \cos 3\theta \cos \theta} \\ \Rightarrow E &= \frac{1}{2} [\tan 3\theta - \tan \theta] \\ A &= \frac{1}{2} [\tan 9^\circ - \tan 3^\circ + \tan 27^\circ - \tan 9^\circ + \tan 81^\circ - \tan 27^\circ] \\ \therefore A &= \frac{1}{2} [\tan 81^\circ - \tan 3^\circ] \\ \therefore \frac{B}{A} &= 2\end{aligned}$$

Question 5

The value of $\operatorname{cosec} 10^\circ - \sqrt{3} \sec 10^\circ$ is equal to :

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Options:

A.

2

B.

6

C.

8

D.

4

Answer: D

Solution:

$$\begin{aligned} & \operatorname{cosec} 10^{\circ} - \sqrt{3} \sec 10^{\circ} \\ &= \frac{1}{\sin 10^{\circ}} - \frac{\sqrt{3}}{\cos 10^{\circ}} \\ &= \frac{\cos 10^{\circ} - \sqrt{3} \sin 10^{\circ}}{\sin 10^{\circ} \cos 10^{\circ}} \\ &= 4 \left[\frac{\frac{1}{2} \cos 10^{\circ} - \frac{\sqrt{3}}{2} \sin 10^{\circ}}{2 \sin 10^{\circ} \cos 10^{\circ}} \right] \\ &= 4 \left[\frac{\sin (30^{\circ} - 10^{\circ})}{\sin 20^{\circ}} \right] \\ &= 4 \end{aligned}$$

Question6

Let $\frac{\pi}{2} < \theta < \pi$ and $\cot \theta = -\frac{1}{2\sqrt{2}}$. Then the value of

$$\sin \left(\frac{15\theta}{2} \right) (\cos 8\theta + \sin 8\theta) + \cos \left(\frac{15\theta}{2} \right) (\cos 8\theta - \sin 8\theta)$$

is equal to :

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Options:

A.

$$\frac{\sqrt{2}-1}{\sqrt{3}}$$

B.

$$\frac{\sqrt{2}}{\sqrt{3}}$$

C.

$$\frac{1-\sqrt{2}}{\sqrt{3}}$$

D.

$$-\frac{\sqrt{2}}{\sqrt{3}}$$

Answer: C

Solution:

$$\begin{aligned}
& \sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta) \\
&= \sin\left(\frac{15\theta}{2}\right)\cos 8\theta + \sin\left(\frac{15\theta}{2}\right)\sin 8\theta + \cos\left(\frac{15\theta}{2}\right)\cos 8\theta - \cos\left(\frac{15\theta}{2}\right)\sin 8\theta \\
&= \left[\cos\frac{15\theta}{2}\cos 8\theta + \sin\frac{15\theta}{2}\sin 8\theta\right] + \left[\sin\frac{15\theta}{2}\cos 8\theta - \cos\frac{15\theta}{2}\sin 8\theta\right] \\
&= \cos\left(8\theta - \frac{15\theta}{2}\right) + \sin\left(\frac{15\theta}{2} - 8\theta\right) \\
&= \cos\left(\frac{\theta}{2}\right) + \sin\left(-\frac{\theta}{2}\right) = \cos\frac{\theta}{2} - \sin\frac{\theta}{2} \\
&\cot \theta = -\frac{1}{2\sqrt{2}}, \frac{\pi}{2} < \theta < \pi \quad (\text{Given}) \\
&\cos \theta = -\frac{1}{3} \\
&\cos \theta = 1 - 2\sin^2 \frac{\theta}{2} \implies -\frac{1}{3} = 1 - 2\sin^2 \frac{\theta}{2} \implies \sin \frac{\theta}{2} = \sqrt{\frac{2}{3}} \\
&\cos \theta = 2\cos^2 \frac{\theta}{2} - 1 \implies -\frac{1}{3} = 2\cos^2 \frac{\theta}{2} - 1 \implies \cos \frac{\theta}{2} = \sqrt{\frac{1}{3}} \\
&\therefore \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \\
&= \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}} \\
&= \frac{1 - \sqrt{2}}{\sqrt{3}}
\end{aligned}$$

Question7

If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$, then $\sin 7x \left(\cos \frac{13x}{2} + \sin \frac{13x}{2}\right) + \cos 7x \left(\cos \frac{13x}{2} - \sin \frac{13x}{2}\right)$ is equal to

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Options:

A.

$$\frac{5}{\sqrt{13}}$$

B.

$$\frac{6}{\sqrt{26}}$$

C.

$$\frac{4}{\sqrt{26}}$$

D.

$$\frac{1}{\sqrt{13}}$$

Answer: D

Solution:

$\cot x = \frac{5}{12}, x \in (\pi, \frac{3\pi}{2})$ means 3rd quadrant.

To find the value of:

$$\sin 7x \left(\cos \frac{13x}{2} + \sin \frac{13x}{2} \right) + \cos 7x \left(\cos \frac{13x}{2} - \sin \frac{13x}{2} \right)$$

Simplify:

$$= \sin 7x \cos \frac{13x}{2} + \sin 7x \sin \frac{13x}{2} + \cos 7x \cos \frac{13x}{2} - \sin \frac{13x}{2} \cos 7x$$

Rearranging terms and use $\sin A \cos B - \cos A \sin B = \sin(A - B)$ and $\cos A \cos B + \sin A \sin B = \cos(A - B)$:

$$\begin{aligned} &= \sin 7x \cos \frac{13x}{2} - \sin \frac{13x}{2} \cos 7x + \cos 7x \cos \frac{13x}{2} + \sin 7x \sin \frac{13x}{2} \\ &= \sin \left(7x - \frac{13x}{2} \right) + \cos \left(7x - \frac{13x}{2} \right) \\ &= \sin \frac{x}{2} + \cos \frac{x}{2} \end{aligned}$$

Since $x \in (\pi, \frac{3\pi}{2})$, then $\frac{x}{2} \in (\frac{\pi}{2}, \frac{3\pi}{4})$.

Let $L = \sin \frac{x}{2} + \cos \frac{x}{2}$.

In the second quadrant, $\frac{x}{2}$ makes L positive because $\sin \frac{x}{2} > |\cos \frac{x}{2}|$.

Squaring:

$$L^2 = \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$L^2 = 1 + \sin x = 1 + \frac{1}{\operatorname{cosec} x} = 1 + \frac{1}{-\sqrt{1+\cot^2 x}}, \text{ Since } x \text{ is in the 3rd quadrant, } \sin x \text{ must be negative}$$

$$L^2 = 1 + \frac{1}{-\sqrt{1+\frac{25}{144}}} \left(\text{Since } \cot x = \frac{5}{12} \right)$$

$$L^2 = 1 - \frac{1}{\sqrt{\frac{169}{144}}} = 1 - \frac{12}{13} = \frac{1}{13}$$

$$L = \frac{1}{\sqrt{13}}$$

Question8

The value of $\frac{\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is equal to

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Options:

A.

B.

64

C.

12

D.

16

Answer: B

Solution:

$$\begin{aligned}& \frac{\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ} \\&= \frac{\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}}{\cos 20^\circ \cos 40^\circ \cos 80^\circ \cdot \frac{1}{2}}, \left\{ \cos 60^\circ = \frac{1}{2} \right\} \\&= \frac{2 \left(\frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} \right)}{\cos 20^\circ \cos 40^\circ \cos 80^\circ}, \\&= \frac{2 \left(\sqrt{3} \cos 20^\circ - \sin 20^\circ \right)}{\frac{\sin 40^\circ}{2} \cdot \cos 20^\circ \cos 40^\circ \cos 80^\circ}, \left\{ \sin 20^\circ \cos 20^\circ = \frac{\sin 40^\circ}{2} \right\} \\& \text{Numerator } \sqrt{3} \cos 20^\circ - \sin 20^\circ = 2 \cdot \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right) \\&= 2 (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ) = 2 \sin(60 - 20) = 2 \sin 40^\circ \\& \text{Substitute this value} \\&= \frac{2 \cdot 2 \sin 40^\circ}{\frac{\sin 80^\circ}{4} \cos 20^\circ \cos 80^\circ} = \frac{4 \sin 40^\circ}{\frac{\sin 160^\circ}{8} \cos 20^\circ}, \left\{ \sin 80^\circ \cos 80^\circ = \frac{\sin 160^\circ}{2} \right\} \\&= \frac{4 \times 2 \sin 20^\circ \cdot \cos 20^\circ}{\sin(180 - 160) \cos 20^\circ} \times 8 = \frac{64 \sin 20^\circ}{\sin 20^\circ} = 64\end{aligned}$$

Question9

If $\frac{\tan(A-B)}{\tan A} + \frac{\sin^2 C}{\sin^2 A} = 1$, $A, B, C \in (0, \frac{\pi}{2})$, then

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Options:

A.

$\tan A, \tan C, \tan B$ are in A.P.

B.

$\tan A, \tan C, \tan B$ are in G.P.

C.

$\tan A, \tan B, \tan C$ are in G.P.

D.

$\tan A, \tan B, \tan C$ are in A.P.

Answer: B

Solution:

Given :

$$\frac{\tan(A-B)}{\tan A} + \frac{\sin^2 C}{\sin^2 A} = 1$$

$$\Rightarrow \frac{\sin^2 C}{\sin^2 A} = 1 - \frac{\tan(A-B)}{\tan A}$$

$$\Rightarrow \frac{\sin^2 C}{\sin^2 A} = \frac{\tan A - \tan(A-B)}{\tan A} \dots (1)$$

$$\text{We know, } \tan X - \tan Y = \frac{\sin(X-Y)}{\cos X \cos Y}$$

$$\therefore \tan A - \tan(A-B) = \frac{\sin(A-(A-B))}{\cos A \cos(A-B)} = \frac{\sin B}{\cos A \cos(A-B)}$$

Substitute back into (1)

$$\frac{\sin^2 C}{\sin^2 A} = \frac{\sin B}{\cos A \cos(A-B) \tan A}$$

$$\Rightarrow \frac{\sin^2 C}{\sin^2 A} = \frac{\sin B}{\cos A \cos(A-B) \frac{\sin A}{\cos A}} = \frac{\sin B}{\sin A \cos(A-B)}$$

$$\Rightarrow \sin^2 C = \frac{\sin^2 A \sin B}{\sin A \cos(A-B)} = \frac{\sin A \sin B}{\cos(A-B)}$$

Divide by $\cos A \cos B$:

$$\sin^2 C = \frac{\tan A \tan B}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B}}$$

$$\Rightarrow \sin^2 C = \frac{\tan A \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \sin^2 C = \frac{\tan^2 C}{1 + \tan^2 C}$$

$$\Rightarrow \frac{\tan^2 C}{1 + \tan^2 C} = \frac{\tan A \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \tan^2 C + \tan^2 C \tan A \tan B = \tan A \tan B + \tan^2 C \tan A \tan B$$

$$\Rightarrow \tan^2 C = \tan A \tan B$$

$\therefore \tan A, \tan C, \tan B$ are in G.P.

Question 10

If $\sin\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right) = K$, then the value of $\sin\left(\frac{10K\pi}{3}\right)$ is:

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Options:

A.

$$\frac{\sqrt{3}+1}{2\sqrt{2}}$$

B.

$$\frac{\sqrt{3}-1}{\sqrt{2}}$$

C.

$$\frac{\sqrt{3}}{2}$$

D.

$$\frac{1}{2}$$

Answer: A

Solution:

Let

$$K = \sin\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right)$$

We need to find

$$\sin\left(\frac{10K\pi}{3}\right)$$

First, write the angles in degrees:

$$\frac{\pi}{18} = 10^\circ, \quad \frac{5\pi}{18} = 50^\circ, \quad \frac{7\pi}{18} = 70^\circ$$

So,

$$K = \sin 10^\circ \sin 50^\circ \sin 70^\circ$$

Now use the standard identity:

$$\sin 3x = 4 \sin x \sin(60^\circ + x) \sin(60^\circ - x)$$

Taking $x = 10^\circ$,

$$\sin 30^\circ = 4 \sin 10^\circ \sin 70^\circ \sin 50^\circ$$

Since

$$\sin 30^\circ = \frac{1}{2}$$

we get

$$\frac{1}{2} = 4 \sin 10^\circ \sin 50^\circ \sin 70^\circ$$

Hence,

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$$

Therefore,

$$K = \frac{1}{8}$$

Now substitute in the required expression:

$$\sin\left(\frac{10K\pi}{3}\right) = \sin\left(\frac{10}{3} \cdot \frac{1}{8}\pi\right) = \sin\left(\frac{10\pi}{24}\right) = \sin\left(\frac{5\pi}{12}\right)$$

Now,

$$\frac{5\pi}{12} = 75^\circ$$

So,

$$\sin 75^\circ = \sin(45^\circ + 30^\circ)$$

Using the formula $\sin(A + B)$:

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

So the value is

$$\boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

Hence, the correct option is A.

Question11

Let $P = \{\theta \in [0, 4\pi] : \tan^2 \theta \neq 1\}$ and

$S = \{a \in \mathbb{Z} : 2(\cos^8 \theta - \sin^8 \theta) \sec 2\theta = a^2, \theta \in P\}$. Then $n(S)$ is:

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Options:

A.

0

B.

1

C.

2

D.

3

Answer: A

Solution:

$$2(\cos^8 \theta - \sin^8 \theta) \sec 2\theta = a^2$$

$$2(\cos^4 \theta + \sin^4 \theta)(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) \sec 2\theta = a^2$$

$$2(\cos^4 \theta + \sin^4 \theta) = a^2$$

$$2(1 - 2\sin^2 \theta \cos^2 \theta) = a^2$$

$$2\left(1 - \frac{\sin^2 2\theta}{2}\right) = a^2, a \in \mathbb{Z}$$

$$a^2 = 2 - \sin^2 2\theta \in [1, 2]$$

$$a^2 = 1 \text{ at } \sin^2 2\theta = 1$$

$$2\theta = (2n+1)\frac{\pi}{2}$$

$$\theta = (2n+1)\frac{\pi}{4} \notin \mathbb{P}$$

$$n(S) = 0$$

Question 12

Let $\tan A, \tan B$, where $A, B \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the roots of the quadratic equation $x^2 - 2x - 5 = 0$. Then $20 \sin^2 \left(\frac{A+B}{2}\right)$ is equal to:

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Options:

A.

$$10 + \sqrt{10}$$

B.

$$10 - 2\sqrt{10}$$

C.

$$10 - 3\sqrt{10}$$

D.

$$10 - \sqrt{10}$$

Answer: C

Solution:

Let the roots of the equation

$$x^2 - 2x - 5 = 0$$

be $\tan A$ and $\tan B$.

So, by using relations between roots and coefficients,

$$\tan A + \tan B = 2$$

and

$$\tan A \tan B = -5.$$

We have to find

$$20 \sin^2 \left(\frac{A+B}{2} \right).$$

First, find $\tan(A+B)$ using the formula

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Substituting the values:

$$\tan(A+B) = \frac{2}{1-(-5)} = \frac{2}{6} = \frac{1}{3}.$$

Now let

$$\theta = A + B.$$

Then

$$\tan \theta = \frac{1}{3}.$$

We need $\sin^2 \frac{\theta}{2}$.

Using the identity

$$\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2},$$

we first find $\cos \theta$.

Since

$$\tan \theta = \frac{1}{3},$$

take a right triangle with opposite side 1, adjacent side 3. Then hypotenuse is

$$\sqrt{1^2 + 3^2} = \sqrt{10}.$$

So,

$$\cos \theta = \frac{3}{\sqrt{10}}.$$

Hence,

$$\sin^2 \frac{\theta}{2} = \frac{1 - \frac{3}{\sqrt{10}}}{2}.$$

Therefore,

$$20 \sin^2 \frac{\theta}{2} = 20 \cdot \frac{1 - \frac{3}{\sqrt{10}}}{2} = 10 \left(1 - \frac{3}{\sqrt{10}} \right) = 10 - \frac{30}{\sqrt{10}}.$$

Now simplify:

$$\frac{30}{\sqrt{10}} = 3\sqrt{10}.$$

So,

$$20 \sin^2 \left(\frac{A+B}{2} \right) = 10 - 3\sqrt{10}.$$

Therefore, the correct answer is

$$10 - 3\sqrt{10}$$

So, **Option C** is correct.

Question13

The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$ is

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Options:

- A. 0
- B. $2/3$
- C. 1
- D. $3/2$

Answer: C

Solution:

$$\begin{aligned} & \sin 70^\circ (\cot 10^\circ \cot 70^\circ - 1) \\ & \Rightarrow \frac{\cos (80^\circ)}{\sin 10} = 1 \end{aligned}$$

Question14

Let the range of the function

$f(x) = 6 + 16 \cos x \cdot \cos \left(\frac{\pi}{3} - x \right) \cdot \cos \left(\frac{\pi}{3} + x \right) \cdot \sin 3x \cdot \cos 6x, x \in \mathbf{R}$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

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Options:

- A. 11
- B. 10
- C. 8
- D. 9

Answer: A

Solution:

$$\begin{aligned}f(x) &= 6 + 16 \left(\frac{1}{4} \cos 3x \right) \sin 3x \cdot \cos 6x \\&= 6 + 4 \cos 3x \sin 3x \cos 6x \\&= 6 + \sin 12x\end{aligned}$$

Range of $f(x)$ is $[5, 7]$

$(\alpha, \beta) \equiv (5, 7)$

$$\text{distance} = \left| \frac{15 + 28 + 12}{5} \right| = 11$$

Question 15

If $\sum_{r=1}^{13} \left\{ \frac{1}{\sin\left(\frac{\pi}{4} + (r-1)\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{r\pi}{6}\right)} \right\} = a\sqrt{3} + b, a, b \in \mathbb{Z}$, then $a^2 + b^2$ is equal to :

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Options:

- A. 10
- B. 4
- C. 8
- D. 2

Answer: C

Solution:

$$\frac{1}{\sin \frac{\pi}{6}} \sum_{r=1}^{13} \frac{\sin \left[\left(\frac{\pi}{4} + \frac{r\pi}{6} \right) - \left(\frac{\pi}{4} \right) - (r-1) \frac{\pi}{6} \right]}{\sin \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) \sin \left(\frac{\pi}{4} + \frac{r\pi}{6} \right)}$$

$$\frac{1}{\sin \frac{\pi}{6}} \sum_{r=1}^{13} \left(\cot \left(\frac{\pi}{4} + (r-1) \frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + \frac{r\pi}{6} \right) \right)$$

$$= 2\sqrt{3} - 2 = \alpha\sqrt{3} + b$$

So $a^2 + b^2 = 8$

Question 16

If $\sin x + \sin^2 x = 1$, $x \in (0, \frac{\pi}{2})$, then

$(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$
is equal to:

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A. 3

B.

4

C.

2

D.

1

Answer: C

Solution:

$$\sin x + \sin^2 x = 1$$

$$\Rightarrow \sin x = \cos^2 x \Rightarrow \tan x = \cos x$$

$$\therefore \text{Given expression}$$

$$= 2 \cos^{12} x + 6 [\cos^{10} x + \cos^8 x] + 2 \cos^6 x$$

$$= 2 [\sin^6 x + 3 \sin^5 x + 3 \sin^4 x + \sin^3 x]$$

$$= 2 \sin^3 x [(\sin x + 1)^3]$$

$$= 2 [\sin^2 x + \sin x]^3$$

$$= 2$$

Question 17

If $10 \sin^4 \theta + 15 \cos^4 \theta = 6$, then the value of $\frac{27 \operatorname{cosec}^6 \theta + 8 \sec^6 \theta}{16 \sec^8 \theta}$ is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{1}{5}$

D. $\frac{3}{4}$

Answer: A

Solution:

$$\begin{aligned} 10 \sin^4 \theta + 15 \cos^4 \theta &= 6 \\ \Rightarrow 10 \sin^4 \theta + 10 \cos^4 \theta + 5 \cos^4 \theta &= 6 \\ \Rightarrow 10 \left[(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta \right] + 5 \cos^4 \theta &= 6 \\ \Rightarrow 10 - 20 (1 - \cos^2 \theta) \cos^2 \theta + 5 \cos^4 \theta &= 6 \end{aligned}$$

$$\text{Let } \cos^2 \theta = x$$

$$10 - 20(x - x^2) + 5x^2 = 6$$

$$\Rightarrow 25x^2 - 20x + 4 = 0$$

$$(5x - 2)^2 = 0 \Rightarrow x = \frac{2}{5}$$

$$\Rightarrow \cos^2 \theta = \frac{2}{5} \Rightarrow \sin^2 \theta = \frac{3}{5},$$

$$\sec^2 \theta = \frac{5}{2}, \operatorname{cosec}^2 \theta = \frac{5}{3}$$

$$\begin{aligned} \frac{27 \operatorname{cosec}^6 \theta + 8 \sec^6 \theta}{16 \sec^8 \theta} &= \frac{27 \left(\frac{5}{3}\right)^3 + 8 \left(\frac{5}{2}\right)^3}{16 \left(\frac{5}{2}\right)^4} \\ &= \frac{5^3 + 5^3}{5^4} = \frac{2 \cdot 5^3}{5^4} = \frac{2}{5} \end{aligned}$$

Question 18

If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points $(x, y) = \left(3 \tan \left(\theta + \frac{\pi}{3}\right), 2 \tan \left(\theta + \frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 75

B. 96

C. 80

D. 72

Answer: A

Solution:

$$\text{Let } \phi = \theta + \frac{\pi}{3} \Rightarrow \theta = \phi - \frac{\pi}{6}$$

$$x = 3 \tan \left(\theta + \frac{\pi}{3} \right) = 3 \tan \left(\phi \right)$$

$$y = 2 \tan \phi$$

$$\tan \left(\phi + \frac{\pi}{6} \right) = \frac{\tan \phi + \frac{1}{\sqrt{3}}}{1 - \tan \phi \cdot \frac{1}{\sqrt{3}}}$$

$$\frac{x}{3} = \frac{\frac{y}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{y}{2} \cdot \frac{1}{\sqrt{3}}}$$

$$\Rightarrow x = \frac{3(y\sqrt{3} + 2)}{2\sqrt{3} - y}$$

$$xy + \alpha x + \beta y + r = 0$$

$$3 \left(\frac{y\sqrt{3} + 2}{2\sqrt{3} - y} \right) + \alpha \left(3 \frac{(y\sqrt{3} + 2)}{(2\sqrt{3} - y)} \right) + \beta y + r = 0$$

$$= (3\sqrt{3} - \beta)y^2 + (6 + 3\sqrt{3}\alpha + 2\sqrt{3}\beta - y)y + (6\alpha + 2\sqrt{3}y) = 0$$

For this identity to hold for all θ , coefficients must be 0

$$\therefore \beta = 3\sqrt{3}$$

$$\gamma = -\alpha\sqrt{3}$$

$$6 + 3\sqrt{3}\alpha + (2\sqrt{3})(3\sqrt{3}) + \alpha\sqrt{3} = 0$$

$$\Rightarrow \alpha = -2\sqrt{3}$$

$$\Rightarrow \beta = 6$$

$$\alpha^2 + \beta^2 + \gamma^2 = 75$$

Question 19

Let the set of all $a \in \mathbb{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and

$$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ, \text{ then pqr is equal to}$$

[27-Jan-2024 Shift 1]

Answer: 48

Solution:

$$\cos 2x + a \cdot \sin x = 2a - 7$$

$$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$$

$$\sin x = 2, a = 2(\sin x + 2)$$

$$\Rightarrow a \in [2, 6]$$

$$p = 2 \quad q = 6$$

$$r = \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$$

$$r = \frac{1}{\sin 9^\circ \cdot \cos 9^\circ} - \frac{1}{\sin 27^\circ \cdot \cos 27^\circ}$$

$$= 2 \left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1} \right]$$

$$r = 4$$

$$p \cdot q \cdot r = 2 \times 6 \times 4 = 48$$

Question 20

If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $[0, n\pi/2]$, for the

least value of $n \in \mathbb{N}$ then $\sum_{k=1}^n \frac{k}{2^k}$ is equal to :

[27-Jan-2024 Shift 2]

Options:

A.

$$\frac{1}{2^{15}}(2^{14} - 14)$$

B.

$$\frac{1}{2^{14}}(2^{15}-15)$$

C.

$$1-\frac{15}{2^{13}}$$

D.

$$\frac{1}{2^{13}}(2^{14}-15)$$

Answer: D

Solution:

$$2\tan^2\theta - 5\sec\theta - 1 = 0$$

$$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$$

$$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$$

$$\Rightarrow \sec\theta = -\frac{1}{2}, 3$$

$$\Rightarrow \cos\theta = -2, \frac{1}{3}$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

For 7 solutions $n = 13$

$$\text{So, } \sum_{k=1}^{13} \frac{k}{2^k} = S \text{ (say)}$$

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{13}{2^{13}}$$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{13}}}{1 - \frac{1}{2}} - \frac{13}{2^{14}} \Rightarrow S = 2 \cdot \left(\frac{2^{13}-1}{2^{13}} \right) - \frac{13}{2^{13}}$$

Question 21

If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

[29-Jan-2024 Shift 1]

Options:

A.

$$\frac{10 - \sqrt{10}}{6}$$

B.

$$\frac{10 - \sqrt{10}}{12}$$

C.

$$\frac{\sqrt{10} - 10}{12}$$

D.

$$\frac{\sqrt{10} - 10}{6}$$

Answer: C

Solution:

$$4 + 5 \tan \theta = \sec \theta$$

$$\text{Squaring : } 24 \tan^2 \theta + 40 \tan \theta + 15 = 0$$

$$\tan \theta = \frac{-10 \pm \sqrt{10}}{12}$$

$$\text{and } \tan \theta = -\left(\frac{10 + \sqrt{10}}{12}\right) \text{ is Rejected.}$$

(3) is correct.

Question22

The sum of the solutions $x \in \mathbb{R}$ of the equation

$$\frac{3 \cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6 \text{ is}$$

[29-Jan-2024 Shift 2]

Options:

A.

0

B.

1

C.

-1

D.

3

Answer: C

Solution:

$$\frac{3 \cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{\cos 2x(3 + \cos^2 2x)}{\cos^2 x(1 - \sin^2 x \cos^2 x)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4(3 + \cos^2 2x)}{(4 - \sin^2 2x)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4(3 + \cos^2 2x)}{(3 + \cos^2 2x)} = x^3 - x^2 + 6$$

$$x^3 - x^2 + 2 = 0 \Rightarrow (x+1)(x^2 - 2x + 2) = 0$$

so, sum of real solutions = -1

Question 23

If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $[0, n\pi/2]$, $n \in \mathbb{N}$, then the roots of the equation

$x^2 + nx + (n-3) = 0$ belong to :

[30-Jan-2024 Shift 1]

Options:

A.

$(0, \infty)$

B.

$(-\infty, 0)$

C.

$$\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$$

D.

Z

Answer: B

Solution:

$$2\sin^3 x + 2\sin x \cdot \cos^2 x + 4\sin x - 4 = 0$$

$$2\sin^3 x + 2\sin x \cdot (1 - \sin^2 x) + 4\sin x - 4 = 0$$

$$6\sin x - 4 = 0$$

$$\sin x = \frac{2}{3}$$

$$n = 5 \text{ (in the given interval)}$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

$$n = 5 \text{ (in the given interval)}$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

Required interval $(-\infty, 0)$

Question 24

The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

[1-Feb-2024 Shift 2]

Options:

A.

1

B.

3

C.

2

D.

0

Answer: D

Solution:

$$4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$$

$$4 - 4\cos^2 x - 4\cos^3 x + 9 - 4\cos x = 0$$

$$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$$

$$4\cos^3 x + 4\cos^2 x + 4\cos x = 13$$

L.H.S. ≤ 12 can't be equal to 13.

Question 25

For $\alpha, \beta \in (0, \pi/2)$, let $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$ and a real number k be such that $\tan \alpha = k \tan \beta$. Then the value of k is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

$-\frac{2}{3}$

B.

-5

C.

$\frac{2}{3}$

D.

5

Answer: B

Solution:

$$3\sin \alpha \cos \beta + 3\sin \beta \cos \alpha$$

$$= 2\sin \alpha \cos \beta - 2\sin \beta \cos \alpha$$

$$5\sin \beta \cos \alpha = -\sin \alpha \cos \beta$$

$$\tan \beta = -\frac{1}{5} \tan \alpha$$

$$\tan \alpha = -5 \tan \beta$$

Question 26

$$\text{If } \tan A = \frac{1}{\sqrt{x(x^2+x+1)}}, \tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}} \text{ and}$$

$$\tan C = (x^{-3} + x^{-2} + x^{-1})^{\frac{1}{2}}, 0 < A, B, C < \frac{\pi}{2}, \text{ then } A + B \text{ is equal to :}$$

[1-Feb-2024 Shift 1]

Options:

A.

C

B.

$\pi - C$

C.

$2\pi - C$

D.

$\pi/2 - C$

Answer: A

Solution:

Finding $\tan(A+B)$ we get

$$\Rightarrow \tan(A+B) =$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \frac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B) = \frac{(1+x)(\sqrt{x^2+x+1})}{(x^2+x)(\sqrt{x})}$$

$$\frac{(1+x)(\sqrt{x^2+x+1})}{(x^2+x)(\sqrt{x})}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

$$A+B=C$$

Question 27

If m and n respectively are the numbers of positive and negative value of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos \frac{\theta}{2} = \cos 3\theta \cos \frac{9\theta}{2}$, then mn is equal to _____.
[25-Jan-2023 Shift 2]

Answer: 25

Solution:

$$\cos 2\theta \cdot \cos \frac{\theta}{2} = \cos 3\theta \cdot \cos \frac{9\theta}{2}$$

$$\Rightarrow 2 \cos 2\theta \cdot \cos \frac{\theta}{2} = 2 \cos \frac{9\theta}{2} \cdot \cos 3\theta$$

$$\Rightarrow \cos \frac{5\theta}{2} + \cos \frac{3\theta}{2} = \cos \frac{15\theta}{2} + \cos \frac{3\theta}{2}$$

$$\Rightarrow \cos \frac{15\theta}{2} = \cos \frac{5\theta}{2}$$

$$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$$

$$5\theta = 2k\pi \text{ or } 10\theta = 2k\pi$$

$$\theta = \frac{2k\pi}{5}$$

$$\therefore \theta = \left\{ -\pi, -\frac{4\pi}{5}, -\frac{3\pi}{5}, -\frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi \right\}$$

$$\therefore m \cdot n = 25$$

Question 28

Let $S = \{\theta \in [0, 2\pi) : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

[24-Jan-2023 Shift 2]

Answer: 2

Solution:

$$\tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0$$

$$\tan(\pi \cos \theta) = -\tan(\pi \sin \theta)$$

$$\tan(\pi \cos \theta) = \tan(-\pi \sin \theta)$$

$$\pi \cos \theta = n\pi - \pi \sin \theta$$

$$\sin \theta + \cos \theta = n \text{ where } n \in \mathbb{I}$$

possible values are $n = 0, 1$ and -1 because

$$-\sqrt{2} \leq \sin \theta + \cos \theta \leq \sqrt{2}$$

$$\text{Now it gives } \theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$$

$$\text{So } \sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$$

Question 29

Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and

$S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

[29-Jan-2023 Shift 1]

Options:

A. $\frac{11}{8}$

B. $\frac{5}{4}$

C. $\frac{9}{8}$

D. $\frac{3}{2}$

Answer: B

Solution:

$$f(\theta) = 3 \left(\sin^4 \left(\frac{3\pi}{2} - \theta \right) + \sin^4(3\pi + \theta) \right) - 2(1 - \sin^2 2\theta)$$

$$S = \left\{ \theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2} \right\}$$

$$\Rightarrow f(\theta) = 3(\cos^4 \theta + \sin^4 \theta) - 2\cos^2 2\theta$$

$$\Rightarrow f(\theta) = 3 \left(1 - \frac{1}{2} \sin^2 2\theta \right) - 2\cos^2 2\theta$$

$$\Rightarrow f(\theta) = 3 - \frac{3}{2} \sin^2 2\theta - 2\cos^2 2\theta$$

$$= \frac{3}{2} - \frac{1}{2} \cos^2 2\theta = \frac{3}{2} - \frac{1}{2} \left(\frac{1 + \cos 4\theta}{2} \right)$$

$$f(\theta) = \frac{5}{4} - \frac{\cos 4\theta}{4}$$

$$f'(\theta) = \sin 4\theta$$

$$\Rightarrow f'(\theta) = \sin 4\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow 4\theta = n\pi + (-1)^n \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4} + (-1)^n \frac{\pi}{12}$$

$$\Rightarrow \theta = \frac{\pi}{12}, \left(\frac{\pi}{4} - \frac{\pi}{12} \right), \left(\frac{\pi}{2} + \frac{\pi}{12} \right), \left(\frac{3\pi}{4} - \frac{\pi}{12} \right)$$

$$\Rightarrow 4\beta = \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4} = \frac{3\pi}{2}$$

$$\Rightarrow \beta = \frac{3\pi}{8} \Rightarrow f(\beta) = \frac{5}{4} - \frac{\cos \frac{3\pi}{2}}{4} = \frac{5}{4}$$

Question 30

The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ [29-Jan-2023 Shift 2]

Options:

A. $[-2, -1]$

B. $\left[-2, -\frac{3}{2}\right]$

C. $\left[-1, -\frac{1}{2}\right]$

D. $\left[-\frac{3}{2}, -1\right]$

Answer: D

Solution:

$$\lambda = \cos^2 2x - 2\sin^4 x - 2\cos^2 x$$

convert all in to $\cos x$.

$$\lambda = (2\cos^2 x - 1)^2 - 2(1 - \cos^2 x)^2 - 2\cos^2 x$$

$$= 4\cos^4 x - 4\cos^2 x + 1 - 2(1 - 2\cos^2 x + \cos^4 x) -$$

$$2\cos^2 x$$

$$= 2\cos^4 x - 2\cos^2 x + 1 - 2$$

$$= 2\cos^4 x - 2\cos^2 x - 1$$

$$= 2 \left[\cos^4 x - \cos^2 x - \frac{1}{2} \right]$$

$$= 2 \left[\left(\cos^2 x - \frac{1}{2} \right)^2 - \frac{3}{4} \right]$$

$$\lambda_{\max} = 2 \left[\frac{1}{4} - \frac{3}{4} \right] = 2 \times \left(-\frac{2}{4} \right) = -1 \text{ (max Value)}$$

$$\lambda_{\min} = 2 \left[0 - \frac{3}{4} \right] = -\frac{3}{2} \text{ (Minimum Value)}$$

$$\text{So, Range} = \left[-\frac{3}{2}, -1 \right]$$

Question31

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$ then the value of $\left(a + \frac{1}{a} \right)$ is :

[30-Jan-2023 Shift 1]

Options:

A. 4

B. $4 - 2\sqrt{3}$

C. 2

D. $5 - \frac{3}{2}\sqrt{3}$

Answer: A

Solution:

Option (1)

$$\tan 15^\circ = 2 - \sqrt{3}$$

$$\frac{1}{\tan 75^\circ} = \cot 75^\circ = 2 - \sqrt{3}$$

$$\frac{1}{\tan 105^\circ} = \cot(105^\circ) = -\cot 75^\circ = \sqrt{3} - 2$$

$$\tan 195^\circ = \tan 15^\circ = 2 - \sqrt{3}$$

$$\begin{aligned}\therefore 2(2 - \sqrt{3}) &= 2a \Rightarrow a = 2 - \sqrt{3} \\ \Rightarrow a + \frac{1}{a} &= 4\end{aligned}$$

Question32

If the solution of the equation $\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1$, $x \in \left(0, \frac{\pi}{2}\right)$, is

$\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to:

[30-Jan-2023 Shift 1]

Options:

- A. 3
- B. 5
- C. 6
- D. 4

Answer: D

Solution:

$$\begin{aligned}\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x &= 1 \\ \Rightarrow \frac{\ln \cos x - \ln \sin x}{\ln \cos x} + 4 \frac{\ln \sin x - \ln \cos x}{\ln \sin x} &= 1 \\ \Rightarrow (\ln \sin x)^2 - 4(\ln \sin x)(\ln \cos x) + 4(\ln \cos x)^2 &= 1 \\ \Rightarrow \ln \sin x &= 2 \ln \cos x \\ \Rightarrow \sin^2 x + \sin x - 1 &= 0 \Rightarrow \sin x = \frac{-1 + \sqrt{5}}{2}\end{aligned}$$

$$\therefore \alpha + \beta = 4$$

Correct option (4)

Question33

The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is _____ :

[6-Apr-2023 shift 2]

Answer: 4

Solution:

$$\begin{aligned} & \frac{(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)}{\frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}} \\ &= \frac{\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}}{\frac{2(4)}{\sqrt{5}-1} - \frac{2(4)}{(\sqrt{5}+1)}} \\ &= \frac{8(\sqrt{5}+1)}{4} - \frac{8(\sqrt{5}-1)}{4} \\ &= 2[(\sqrt{5}+1) - (\sqrt{5}-1)] \\ &= 4 \end{aligned}$$

Question34

The value of $36(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1)$ is
[8-Apr-2023 shift 2]

Options:

- A. 27
- B. 54
- C. 18
- D. 36

Answer: D

Solution:

$$4\cos^2 \theta - 1 = 4(1 - \sin^2 \theta) - 1 = 3 - 4\sin^2 \theta = \frac{\sin 3\theta}{\sin \theta}$$

so given expression can be written as

$$\begin{aligned} & 36 \times \frac{\sin 27^\circ}{\sin 9^\circ} \times \frac{\sin 81^\circ}{\sin 27^\circ} \times \frac{\sin 243^\circ}{\sin 81^\circ} \times \frac{\sin 729^\circ}{\sin 243^\circ} \\ &= 36 \times \frac{\sin 729^\circ}{\sin 9^\circ} = 36 \end{aligned}$$

Question35

$96 \cos \frac{\pi}{33} \cos \frac{2\pi}{33} \cos \frac{4\pi}{33} \cos \frac{8\pi}{33} \cos \frac{16\pi}{33}$ is equal to :
[10-Apr-2023 shift 1]

Options:

- A. 4
- B. 2
- C. 3
- D. 1

Answer: C

Solution:

$$96 \cos \frac{\pi}{33} \cos \frac{2\pi}{33} \cos \frac{2^2\pi}{33} \cos \frac{2^3\pi}{33} \cos \frac{2^4\pi}{33} \dots \cos A \cos 2A \cos 2^2A \dots \cos 2^{n-1}A = \frac{\sin(2^n A)}{2^n \sin A}$$

$$\text{Here } A = \frac{\pi}{33}, n = 5$$

$$\begin{aligned} &= \frac{96 \sin \left(2^5 \frac{\pi}{33} \right)}{2^5 \sin \left(\frac{\pi}{33} \right)} \\ &= \frac{96 \sin \left(\frac{32\pi}{33} \right)}{32 \sin \left(\frac{\pi}{33} \right)} \\ &= \frac{3 \sin \left(\pi - \frac{\pi}{33} \right)}{\sin \left(\frac{\pi}{33} \right)} = 3 \end{aligned}$$

Question 36

Let $S = \left\{ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10 \right\}$ and $b = \sum_{x \in S} \tan \left(\frac{x}{3} \right)$, then

$\frac{1}{6}(\beta - 14)^2$ is equal to

[10-Apr-2023 shift 2]

Options:

- A. 16
- B. 32
- C. 8

D. 64

Answer: B

Solution:

$$\text{Let } 9^{\tan^2 x} = P$$

$$\frac{9}{P} + P = 10$$

$$P^2 - 10P + 9 = 0$$

$$(P - 9)(P - 1) = 0$$

$$P = 1, 9$$

$$9^{\tan^2 x} = 1, 9^{\tan^2 x} = 9$$

$$\tan^2 x = 0, \tan^2 x = 1$$

$$x = 0, \pm \frac{\pi}{4} \therefore x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\beta = \tan^2(0) + \tan^2\left(+\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$$

$$= 0 + 2(\tan 15^\circ)^2$$

$$2(2 - \sqrt{3})^2$$

$$2(7 - 4\sqrt{3})$$

$$\text{Then } \frac{1}{6}(14 - 8\sqrt{3} - 14)^2 = 32$$

Question37

The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \leq x \leq 4\pi$ is :
[25-Jul-2022-Shift-1]

Options:

A. 4

B. 6

C. 8

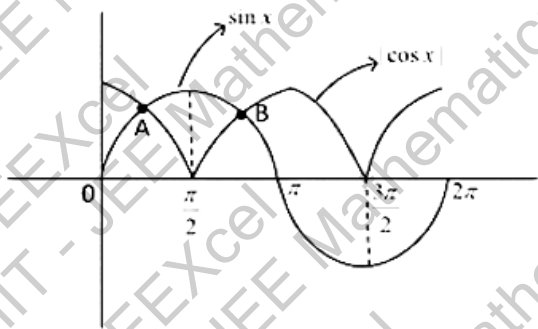
D. 12

Answer: C

Solution:

$$\text{Period of } |\cos x| = \pi$$

$$\text{And period of } \sin x = 2\pi$$



Graph of $\sin x$ and $|\cos x|$ cuts each other at two points A and B in $[0, 2\pi]$

So, in $[-4\pi, 4\pi]$, total 4 similar graph will be present and graph of $\sin x$ and $|\cos x|$ will cut $4 \times 2 = 8$ times.

$\therefore$ Total possible solutions = 8

Question38

Let $S = \{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \}$. Then

$n(s) + \sum_{\theta \in S} \left(\sec\left(\frac{\pi}{4} + 2\theta\right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right) \right)$ is equal to:

[26-Jul-2022-Shift-1]

Options:

- A. 0
- B. -2
- C. -4
- D. 12

Answer: C

Solution:

$$S = \{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \}$$

Now apply AM $\geq$ GM for $8^{2\sin^2\theta}, 8^{2\cos^2\theta}$

$$\frac{8^{2\sin^2\theta} + 8^{2\cos^2\theta}}{2} \geq (8^{2\sin^2\theta + 2\cos^2\theta})^{\frac{1}{2}}$$

$$8 \geq 8$$

$$\Rightarrow 8^{2\sin^2\theta} = 8^{2\cos^2\theta}$$

$$\text{or } \sin^2\theta = \cos^2\theta$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\begin{aligned}
& n(S) + \sum_{\theta \in S} \sec\left(\frac{\pi}{4} + 2\theta\right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right) \\
&= 4 + \sum_{\theta \in S} \frac{2}{2 \sin\left(\frac{\pi}{4} + 2\theta\right) \cos\left(\frac{\pi}{4} + 2\theta\right)} \\
&= 4 + \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)} = 4 + 2 \sum_{\theta \in S} \operatorname{cosec}\left(\frac{\pi}{2} + 4\theta\right) \\
&= 4 + 2 \left[\operatorname{cosec}\left(\frac{\pi}{2} + \pi\right) \operatorname{cosec}\left(\frac{\pi}{2} + 3\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 5\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 7\pi\right) \right] \\
&= 4 + 2 \left[-\operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} \right] \\
&= 4 - 2(4) \\
&= 4 - 8 \\
&= -4
\end{aligned}$$

Question 39

If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____.

[26-Jul-2022-Shift-2]

Answer: 3

Solution:

$$\text{Equation (1) } 2\sin^2\theta = 1 - 2\sin^2\theta$$

$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \pm \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{Equation (2) } 2\cos^2\theta + 3\sin\theta = 0$$

$$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$$

$$\Rightarrow (\sin\theta - 2)(2\sin\theta + 1) = 0$$

$$\Rightarrow \sin\theta = \frac{-1}{2}$$

$$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\therefore \text{Common solutions} = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{Sum of solutions} = \frac{7\pi + 11\pi}{6} = \frac{18\pi}{6} = 3\pi$$

$$\therefore k = 3$$

Question40

Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0, \theta \in S$, is _____.

[29-Jul-2022-Shift-1]

Answer: 16

Solution:

$$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$$

$$\Rightarrow 4\left(\frac{1 + \cos 2\theta}{2}\right) + 3\cos 2\theta - 2\cos^2 2\theta = 2$$

$$\Rightarrow 2 + 5\cos^2\theta - 2\cos^2 2\theta = 2$$

$$\Rightarrow \cos 2\theta = 0 \text{ or } \frac{5}{2} \text{ (rejected)}$$

$$\Rightarrow \cos 2\theta = 0 = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \Rightarrow \tan^2\theta = 1$$

$$\therefore \text{Sum of roots} = 2(\tan^2\theta + \cot^2\theta) = 2 \times 2 = 4$$

But as $\tan \theta = \pm 1$ for $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $(0, 2\pi)$

$\therefore$ Four equations will be formed

Hence sum of roots of all the equations $= 4 \times 4 = 16$.

Question41

$2 \sin\left(\frac{\pi}{22}\right) \sin\left(\frac{3\pi}{22}\right) \sin\left(\frac{5\pi}{22}\right) \sin\left(\frac{7\pi}{22}\right) \sin\left(\frac{9\pi}{22}\right)$ is equal to :

[25-Jul-2022-Shift-2]

Options:

A. $\frac{3}{16}$

B. $\frac{1}{16}$

C. $\frac{1}{32}$

D. $\frac{9}{32}$

Answer: B

Solution:

$$\begin{aligned} & 2 \sin \frac{\pi}{22} \sin \frac{3\pi}{22} \sin \frac{5\pi}{22} \sin \frac{7\pi}{22} \sin \frac{9\pi}{22} \\ &= 2 \sin \left(\frac{11\pi - 10\pi}{22} \right) \sin \left(\frac{11\pi - 8\pi}{22} \right) \sin \left(\frac{11\pi - 6\pi}{22} \right) \sin \left(\frac{11\pi - 4\pi}{22} \right) \sin \left(\frac{11\pi - 2\pi}{22} \right) \\ &= 2 \cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{3\pi}{11} \cos \frac{4\pi}{11} \cos \frac{5\pi}{11} \\ &= \frac{2 \sin \frac{32\pi}{11}}{2^5 \sin \frac{\pi}{11}} \\ &= \frac{1}{16} \end{aligned}$$

Question42

Let $S = \left\{ \theta \in \left(0, \frac{\pi}{2} \right) : \sum_{m=1}^9 \sec \left(\theta + (m-1) \frac{\pi}{6} \right) \sec \left(\theta + \frac{m\pi}{6} \right) = -\frac{8}{\sqrt{3}} \right\}$. Then
[27-Jul-2022-Shift-2]

Options:

A. $S = \left\{ \frac{\pi}{12} \right\}$

B. $S = \left\{ \frac{2\pi}{3} \right\}$

C. $\sum_{\theta \in S} \theta = \frac{\pi}{2}$

D. $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

Answer: C

Solution:

$$s = \left\{ 0 \in \left(0, \frac{\pi}{2} \right) : \sum_{m=1}^9 \sec \left(\theta + (m-1) \frac{\pi}{6} \right) \sec \left(\theta + \frac{m\pi}{6} \right) = -\frac{8}{\sqrt{3}} \right\}.$$

$$\sum_{m=1}^9 \frac{1}{\cos \left(\theta + (m-1) \frac{\pi}{6} \right)} \cos \left(\theta + m \frac{\pi}{6} \right)$$

$$\frac{1}{\sin \left(\frac{\pi}{6} \right)} \sum_{m=1}^9 \frac{\sin \left[\left(\theta + \frac{m\pi}{6} \right) - \left(\theta + (m-1) \frac{\pi}{6} \right) \right]}{\cos \left(\theta + (m-1) \frac{\pi}{6} \right) \cos \left(\theta + \frac{\pi}{6} \right)}$$

$$= 2 \sum_{m=1}^9 \left[\tan \left(\theta + \frac{m\pi}{6} \right) - \tan \left(\theta + (m-1) \frac{\pi}{6} \right) \right]$$

Now,

$$\cdot m=1 \quad 2 \left[\tan \left(\theta + \frac{\pi}{6} \right) - \tan(\theta) \right]$$

$$\cdot m=2 \quad 2 \left[\tan \left(\theta + \frac{2\pi}{6} \right) - \tan \left(\theta + \frac{\pi}{6} \right) \right]$$

$$\cdot m=9 \quad 2 \left[\tan \left(\theta + \frac{9\pi}{6} \right) - \tan \left(\theta + 8 \frac{\pi}{6} \right) \right]$$

$$\therefore = 2 \left[\tan \left(\theta + \frac{3\pi}{2} \right) - \tan \theta \right] = \frac{-8}{\sqrt{3}}$$

$$= -2[\cot \theta + \tan \theta] = \frac{-8}{\sqrt{3}}$$

$$= -\frac{2 \times 2}{2 \sin \theta \cos \theta} = \frac{-8}{\sqrt{3}}$$

$$= \frac{1}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \frac{\pi}{3}$$

$$2\theta = \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}$$

$$\sum \theta_i = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

Question 43

Let $S = \left[-\pi, \frac{\pi}{2} \right) - \left\{ -\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4} \right\}$. Then the number of elements in the set

$|A = \{\theta \in S : \tan \theta(1 + \sqrt{5} \tan(2\theta)) = \sqrt{5} - \tan(2\theta)\}$ is _____.

[28-Jul-2022-Shift-2]

Answer: 5

Solution:

Let $\tan \alpha = \sqrt{5}$

$$\therefore \tan \theta = \frac{\tan \alpha - \tan 2\theta}{1 + \tan \alpha \tan 2\theta}$$

$$\therefore \tan \theta = \tan(\alpha - 2\theta)$$

$$\alpha - 2\theta = n\pi + \theta$$

$$\Rightarrow 3\theta = \alpha - n\pi$$

$$\Rightarrow \theta = \frac{\alpha}{3} - \frac{n\pi}{3} ; n \in \mathbb{Z}$$

If $\theta \in [-\pi, \pi/2]$ then $n = 0, 1, 2, 3, 4$ are acceptable

$\therefore$ 5 solutions.

Question44

Let $S = \left\{ \theta \in [-\pi, \pi] - \left\{ \pm \frac{\pi}{2} \right\} : \sin \theta \tan \theta + \tan \theta = \sin 2\theta \right\}$.

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to:

[24-Jun-2022-Shift-1]

Options:

A. $7 + \sqrt{3}$

B. 9

C. $8 + \sqrt{3}$

D. 10

Answer: B

Question45

The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is _____
[25-Jun-2022-Shift-1]

Answer: 4

Solution:

$$\frac{14}{\sin^2 x} - 2\sin^2 x = 21 - 4(1 - \sin^2 x)$$

$$\text{Let } \sin^2 x = t$$

$$\Rightarrow 14 - 2t^2 = 21t - 4t + 4t^2$$

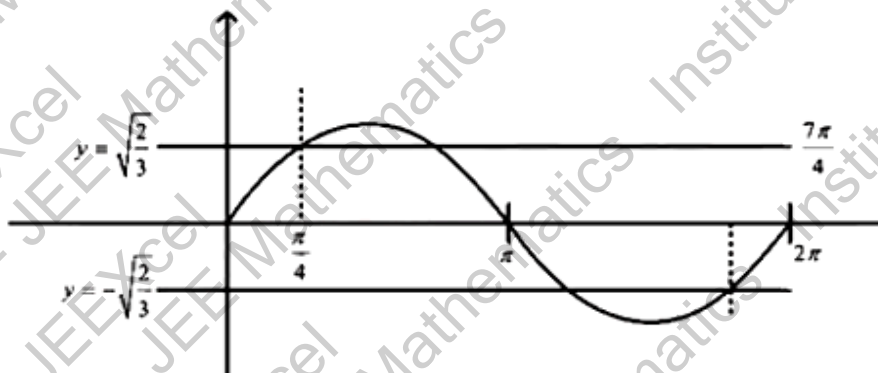
$$\Rightarrow 6t^2 + 17t - 14 = 0$$

$$\Rightarrow 6t^2 + 21t - 4t - 14 = 0$$

$$\Rightarrow 3t(2t + 7) - 2(2t + 7) = 0$$

$$\Rightarrow \sin^2 x = \frac{2}{3} \text{ or } -\frac{7}{3} \text{ (rejected)}$$

$$\Rightarrow \sin x = \pm \sqrt{\frac{2}{3}}$$



Question46

The number of elements in the set

$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6 \cos 2\theta - 10\cos^2 \theta + 5 = 0\}$ is
[29-Jun-2022-Shift-1]

Answer: 32

Solution:

$$3\cos^2 2\theta + 6 \cos 2\theta - 10\cos^2 \theta + 5 = 0$$

$$3\cos^2 2\theta + 6 \cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$$

$$3\cos^2 2\theta + \cos 2\theta = 0$$

$$\cos 2\theta = 0 \text{ OR } \cos 2\theta = -1/3$$

$$\theta \in [-4\pi, 4\pi]$$

$$2\theta = (2n+1) \cdot \frac{\pi}{2}$$

$$\therefore \theta = \pm\pi/4, \pm 3\pi/4, \dots, \pm 15\pi/4$$

Similarly $\cos 2\theta = -1/3$ gives 16 solution

Question47

The number of solutions of the equation

$2\theta - \cos^2 \theta + \sqrt{2} = 0$ in $\mathbb{R}$ is equal to
[29-Jun-2022-Shift-1]

Answer: 1

Solution:

Given,

$$2\theta - \cos^2 \theta + \sqrt{2} = 0$$

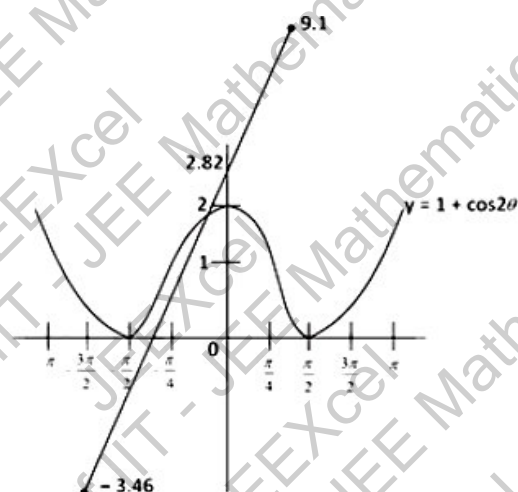
$$\Rightarrow 2\theta + \sqrt{2} = \cos^2 \theta$$

$$\Rightarrow 2\theta + \sqrt{2} = \frac{1 + \cos 2\theta}{2}$$

$$\Rightarrow 4\theta + 2\sqrt{2} = 1 + \cos 2\theta = y \text{ (Assume)}$$

$$\therefore y = 4\theta + 2\sqrt{2} \text{ and}$$

$$y = 1 + \cos 2\theta$$



For $y = 1 + \cos 2\theta$

when $\theta = 0$, $y = 1 + 1 = 2$

when $\theta = \frac{\pi}{4}$, $y = 1 + \cos \frac{\pi}{2} = 1$

$\theta = \frac{\pi}{2}$, $y = 1 + \cos \pi = 1 - 1 = 0$

For $y = 4\theta + 2\sqrt{2}$

when $\theta = 0$, $y = 2\sqrt{2}$

when $\theta = \frac{\pi}{2}$, $y = 2\pi + 2\sqrt{2}$

$= 2(\pi + \sqrt{2})$

$= 2(3.14 + 1.41)$

$= 2(4.55)$

$= 9.1$

when $\theta = -\frac{\pi}{2}$, $y = -2\pi + 2\sqrt{2}$

$= 2(-\pi + \sqrt{2})$

$= 2(-3.14 + 1.41)$

$= -3.46$

$\therefore$ Two graph cut's at only one point so one solution possible.

Question48

The number of solutions of the equation $\sin x = \cos^2 x$ in the interval $(0, 10)$ is

[29-Jun-2022-Shift-2]

Answer: 4

Solution:

$$\sin x = \cos^2 x, x \in (0, 10)$$

$$\Rightarrow \sin x = 1 - \sin^2 x$$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

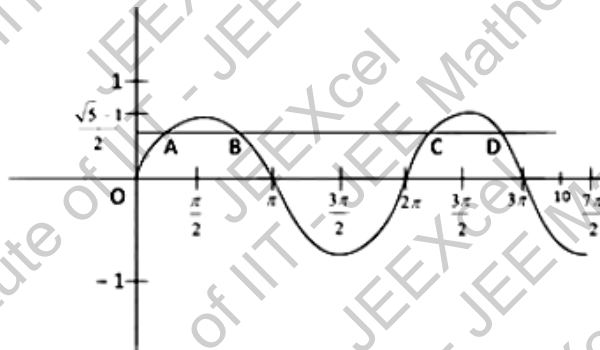
$$\therefore \sin x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2}$$

We know $\sin \in (-1, 1)$

$\therefore \frac{-1 - \sqrt{5}}{2}$ can't be a value of $\sin x$

$$\therefore \sin x = \frac{\sqrt{5} - 1}{2}$$



$$3\pi = 3 \times 3.14 = 9.42 < 10$$

$$\frac{7\pi}{2} = \frac{7}{2} \times 3.14 = 10.99 > 10$$

$\therefore 10$ will be in between 3π and $\frac{7\pi}{2}$.

There are 4 intersection at A, B, C and D between $\sin x$ graph and $y = \frac{\sqrt{5}-1}{2}$ graph.

$\therefore$ possible solution = 4

Question49

The number of solutions of the equation

$$\cos\left(x + \frac{\pi}{3}\right) \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4} \cos^2 2x, x \in [-3\pi, 3\pi] \text{ is :}$$

[24-Jun-2022-Shift-2]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: D

Solution:

$$\cos\left(x + \frac{\pi}{3}\right) \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4} \cos^2 2x, x \in [-3\pi, 3\pi]$$

$$\Rightarrow \cos 2x + \cos \frac{2\pi}{3} = \frac{1}{2} \cos^2 2x$$

$$\Rightarrow \cos^2 2x - 2 \cos 2x - 1 = 0$$

$$\Rightarrow \cos 2x = 1$$

$$\therefore x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$$

$$\therefore \text{Number of solutions} = 7$$

Question 50

The value of $2 \sin(12^\circ) - \sin(72^\circ)$ is :
[25-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

B. $\frac{1-\sqrt{5}}{8}$

C. $\frac{\sqrt{3}(1-\sqrt{5})}{2}$

D. $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Answer: D

Solution:

$$\begin{aligned} & 2 \sin 12^\circ - \sin 72^\circ \\ &= \sin 12^\circ + (-2 \cos 42^\circ \cdot \sin 30^\circ) \\ &= \sin 12^\circ - \cos 42^\circ \\ &= \sin 12^\circ - \sin 48^\circ \\ &= 2 \sin 18^\circ \cdot \cos 30^\circ \\ &= -2 \left(\frac{\sqrt{5}-1}{4} \right) \cdot \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}(1-\sqrt{5})}{4} \end{aligned}$$

Question 51

If $\sin^2(10^\circ) \sin(20^\circ) \sin(40^\circ) \sin(50^\circ) \sin(70^\circ) = \alpha - \frac{1}{16} \sin(10^\circ)$, then $16 + \alpha^{-1}$ is equal to _____
[26-Jun-2022-Shift-1]

Answer: 80

Solution:

$$(\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ) \cdot (\sin 10^\circ \cdot \sin 20^\circ \cdot \sin 40^\circ)$$

$$= \left(\frac{1}{4} \sin 30^\circ \right) \cdot \left[\frac{1}{2} \sin 10^\circ (\cos 20^\circ - \cos 60^\circ) \right]$$

$$= \frac{1}{16} \left[\sin 10^\circ \left(\cos 20^\circ - \frac{1}{2} \right) \right]$$

$$= \frac{1}{32} [2 \sin 10^\circ \cdot \cos 20^\circ - \sin 10^\circ]$$

$$= \frac{1}{32} [\sin 30^\circ - \sin 10^\circ - \sin 10^\circ]$$

$$= \frac{1}{64} - \frac{1}{64} \sin 10^\circ$$

$$\text{Clearly, } \alpha = \frac{1}{64}$$

$$\text{Hence } 16 + \alpha^{-1} = 80$$

Question 52

$16 \sin(20^\circ) \sin(40^\circ) \sin(80^\circ)$ is equal to :
[26-Jun-2022-Shift-2]

Options:

A. $\sqrt{3}$

B. $2\sqrt{3}$

C. 3

D. $4\sqrt{3}$

Answer: B

Solution:

$$\begin{aligned} & 16 \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ \\ &= 4 \sin 60^\circ \{ \because 4 \sin \theta \cdot \sin(60^\circ - \theta) \cdot \sin(60^\circ + \theta) = \sin 3\theta \} \\ &= 2\sqrt{3} \end{aligned}$$

Question 53

The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to:

[27-Jun-2022-Shift-1]

Options:

- A. -1
- B. $-\frac{1}{2}$
- C. $-\frac{1}{3}$
- D. $-\frac{1}{4}$

Answer: B

Solution:

$$\begin{aligned} \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} &= \frac{\sin 3\left(\frac{\pi}{7}\right)}{\sin \frac{\pi}{7}} \cos \left(\frac{2\pi}{7} + \frac{6\pi}{7}\right) \\ &= \frac{\sin\left(\frac{3\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)} \\ &= \frac{2 \sin \frac{4\pi}{7} \cos \frac{4\pi}{7}}{2 \sin \frac{\pi}{7}} \\ &= \frac{\sin\left(\frac{3\pi}{7}\right)}{2 \sin \frac{\pi}{7}} = \frac{-\sin \frac{\pi}{7}}{2 \sin \frac{\pi}{7}} = \frac{-1}{2} \end{aligned}$$

Question54

$\alpha = \sin 36^\circ$ is a root of which of the following equation?
[27-Jun-2022-Shift-2]

Options:

A. $16x^4 - 10x^2 - 5 = 0$

B. $16x^4 + 20x^2 - 5 = 0$

C. $16x^4 - 20x^2 + 5 = 0$

D. $16x^4 - 10x^2 + 5 = 0$

Answer: C

Solution:

$$\alpha = \sin 36^\circ = x \text{ (say)}$$

$$\therefore x = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

$$\Rightarrow 16x^2 = 10 - 2\sqrt{5}$$

$$\Rightarrow (8x^2 - 5)^2 = 5$$

$$\Rightarrow 16x^4 - 80x^2 + 20 = 0$$

$$\therefore 4x^4 - 20x^2 + 5 = 0$$

Question55

If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are :
[28-Jun-2022-Shift-2]

Options:

A. $-\frac{1}{7}$ and IVth quadrant

B. 7 and Ist quadrant

C. -7 and IVth quadrant

D. $\frac{1}{7}$ and Ist quadrant

Answer: A

Solution:

$$\because \cot \alpha = 1, \quad \alpha \in \left(\pi, \frac{3\pi}{2} \right)$$

$$\text{then } \tan \alpha = 1$$

$$\text{and } \sec \beta = -\frac{5}{3}, \quad \beta \in \left(\frac{\pi}{2}, \pi \right)$$

$$\text{then } \tan \beta = -\frac{4}{3}$$

$$\therefore \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$= \frac{1 - \frac{4}{3}}{1 + \frac{4}{3}}$$

$$= -\frac{1}{7}$$

$$\alpha + \beta \in \left(\frac{3\pi}{2}, 2\pi \right) \text{ i.e. fourth quadrant}$$

Question56

cosec 18° is a root of the equation
[31 Aug 2021 Shift 1]

Options:

A. $x^2 + 2x - 4 = 0$

B. $4x^2 + 2x - 1 = 0$

C. $x^2 - 2x + 4 = 0$

D. $x^2 - 2x - 4 = 0$

Answer: D

Solution:

$$\operatorname{cosec} 18^\circ = \frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5}-1} = \sqrt{5} + 1$$

If $x = \sqrt{5} + 1$, then

$$(x-1)^2 = 5$$

$$\Rightarrow x^2 - 2x - 4 = 0$$

Question57

The value of

$2 \sin\left(\frac{\pi}{8}\right) \sin\left(\frac{2\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right) \sin\left(\frac{5\pi}{8}\right) \sin\left(\frac{6\pi}{8}\right) \sin\left(\frac{7\pi}{8}\right)$ is
[26 Aug 2021 Shift 2]

Options:

A. $\frac{1}{4\sqrt{2}}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{8\sqrt{2}}$

Answer: C

Solution:

$$\begin{aligned} & 2 \sin\left(\frac{\pi}{8}\right) \sin\left(\frac{2\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right) \sin\left(\frac{5\pi}{8}\right) \sin\left(\frac{6\pi}{8}\right) \sin\left(\frac{7\pi}{8}\right) \\ &= 2 \sin\frac{\pi}{8} \sin\frac{2\pi}{8} \sin\frac{3\pi}{8} \sin\left(\pi - \frac{3\pi}{8}\right) \sin\left(\pi - \frac{2\pi}{8}\right) \sin\left(\pi - \frac{\pi}{8}\right) \\ &= 2 \sin\frac{\pi}{8} \sin\frac{2\pi}{8} \sin\frac{3\pi}{8} \sin\frac{3\pi}{8} \sin\frac{2\pi}{8} \sin\frac{\pi}{8} \\ &= 2\sin^2\left(\frac{\pi}{8}\right) \sin^2\left(\frac{2\pi}{8}\right) \sin^2\left(\frac{3\pi}{8}\right) 2\sin^2\frac{2\pi}{8} \left(\frac{1}{\sqrt{2}}\right)^2 \sin^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right) \\ &= 2\sin^2\frac{2\pi}{8} \times \frac{1}{2} \times \cos^2\frac{2\pi}{8} = \sin^2\left(\frac{\pi}{8}\right) \cos^2\left(\frac{\pi}{8}\right) \\ &= \frac{1}{4} \left(2 \sin\frac{\pi}{8} \cos\frac{\pi}{8}\right)^2 \frac{1}{4} \sin^2\left(\frac{\pi}{4}\right) = \frac{1}{4} \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{8} \end{aligned}$$

Question 58

If $\sin \theta + \cos \theta = \frac{1}{2}$, then $16 (\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to:
[27 Jul 2021 Shift 1]

Options:

A. 23

B. -27

C. -23

D. 27

Answer: C

Solution:

$$\sin \theta + \cos \theta = \frac{1}{2}$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{4}$$

$$\sin 2\theta = -\frac{3}{4}$$

Now :

$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$= 1 - 2\left(-\frac{3}{4}\right)^2$$

$$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$$

$$\sin 6\theta = 3 \sin 2\theta - 4 \sin^3 2\theta$$

$$= (3 - 4 \sin^2 2\theta) \cdot \sin 2\theta$$

$$= \left[3 - 4\left(\frac{9}{16}\right)\right] \cdot \left(-\frac{3}{4}\right)$$

$$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$$

$$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$$

$$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$$

Question 59

The value of $\cot \frac{\pi}{24}$ is:
[25 Jul 2021 Shift 2]

Options:

A. $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

B. $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

C. $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

D. $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

Answer: B

Solution:

$$\cot \theta = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$\theta = \frac{\pi}{24}$$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2} + \sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$$

Question60

If $15\sin^4 \alpha + 10\cos^4 \alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6 \alpha + 8\operatorname{cosec}^6 \alpha$ is equal to
[18 Mar 2021 Shift 2]

Options:

- A. 350
- B. 500
- C. 400
- D. 250

Answer: D

Solution:

$$\text{Given, } 15\sin^4 \alpha + 10\cos^4 \alpha = 6$$

$$\Rightarrow 15\sin^4 \alpha + 10\cos^4 \alpha = 6(\sin^2 \alpha + \cos^2 \alpha)^2$$

$$\Rightarrow 15\sin^4 \alpha + 10\cos^4 \alpha = 6(\sin^4 \alpha + \cos^4 \alpha + 2\sin^2 \alpha \cos^2 \alpha)$$

$$\Rightarrow 9\sin^4 \alpha + 4\cos^4 \alpha - 12\sin^2 \alpha \cos^2 \alpha = 0$$

$$\Rightarrow (3\sin^2 \alpha - 2\cos^2 \alpha)^2 = 0$$

$$\Rightarrow 3\sin^2 \alpha - 2\cos^2 \alpha = 0$$

$$\Rightarrow 3\sin^2 \alpha = 2\cos^2 \alpha$$

$$\Rightarrow \tan^2 \alpha = \frac{2}{3}$$

$$\therefore \cot^2 \alpha = 3/2$$

Now,

$$27\sec^6 \alpha + 8\operatorname{cosec}^6 \alpha = 27(\sec^2 \alpha)^3 + 8(\operatorname{cosec}^2 \alpha)^3$$

$$= 27(1 + \tan^2 \alpha)^3 + 8(\cot^2 \alpha)^3$$

$$= 27\left(1 + \frac{2}{3}\right)^3 + 8\left(1 + \frac{3}{2}\right)^3 = 250$$

Question 61

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to
[16 Mar 2021 Shift 1]

Options:

- A. 20
- B. 12
- C. 9
- D. 16

Answer: B

Solution:

$$\log_{10} \sin x + \log_{10} \cos x = -1, x \in (0, \pi/2)$$

$$\log_{10}(\sin x \cos x) = -1$$

$$\Rightarrow \sin x \cos x = 10^{-1} = 1/10$$

$$\log_{10}(\sin x + \cos x) = 1/2(\log_{10} n - 1), n > 0$$

$$2\log_{10}(\sin x + \cos x) = (\log_{10} n - \log_{10} 10)$$

$$\Rightarrow \log_{10}(\sin x + \cos x)^2 = \log_{10}(n/10)$$

$$\Rightarrow (\sin x + \cos x)^2 = n/10$$

$$\Rightarrow \sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{n}{10}$$

$$\Rightarrow 1 + 2(1/10) = n/10 \Rightarrow 12/10 = n/10$$

$$\therefore n = 12$$

Question 62

If $0 < a, b < 1$ and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + \dots \text{ is}$$

[26 Feb 2021 Shift 2]

Options:

A. $\log_e 2$

B. $e^2 - 1$

C. e

D. $\log_e \left(\frac{e}{2} \right)$

Answer: A

Solution:

Given, $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$

$\Rightarrow \tan^{-1} \left[\frac{a+b}{1-ab} \right] = \frac{\pi}{4}$

$\Rightarrow \frac{a+b}{1-ab} = \tan \frac{\pi}{4} = 1$

$\Rightarrow a+b = 1-ab$

$\Rightarrow (1+a)(1+b) = 2 \dots (i)$

Now, $(a+b) - \left(\frac{a^2+b^2}{2} \right) + \left(\frac{a^3+b^3}{3} \right) - \left(\frac{a^4+b^4}{4} \right) + \dots$

$= \left[a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \dots \right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} + \dots \right]$

$= \log_e(1+a) + \log_e(1+b)$ [Using expansion of $\log_e(1+x)$]

$= \log_e(1+a)(1+b)$ [$\because \log a + \log b = \log ab$]

$= \log_e 2$ [use Eq. (i)]

Question 63

All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in
[25 Feb 2021 Shift 1]

Options:

A. $\left(0, \frac{\pi}{2} \right) \cup \left(\pi, \frac{3\pi}{2} \right)$

B. $\left(0, \frac{\pi}{2} \right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4} \right) \cup \left(\pi, \frac{7\pi}{6} \right)$

C. $\left(0, \frac{\pi}{4} \right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4} \right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6} \right)$

D. $\left(0, \frac{\pi}{4} \right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4} \right) \cup \left(\pi, \frac{5\pi}{4} \right) \cup$

Answer: D

Solution:

$$\sin 2\theta + \tan 2\theta > 0$$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\Rightarrow \sin 2\theta \left(1 + \frac{1}{\cos 2\theta}\right) > 0$$

$$\Rightarrow \sin 2\theta \left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right) > 0$$

$$\Rightarrow \tan 2\theta (2\cos^2 \theta) > 0$$

$$\cos 2\theta \neq 0$$

$$\Rightarrow 1 - 2\sin^2 \theta \neq 0$$

$$\sin \theta \neq \pm \frac{1}{\sqrt{2}}$$

$$\text{Now, } \tan 2\theta (1 + \cos 2\theta) > 0$$

$$\Rightarrow \tan 2\theta > 0 \text{ as } 1 + \cos 2\theta > 0$$

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\therefore \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

$$\text{Since, } \sin \theta \neq \pm \frac{1}{\sqrt{2}} \Big]$$

Question 64

If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to
[25 Feb 2021 Shift 2]

Options:

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1-\sqrt{3}}{2}$

D. $\frac{1+\sqrt{3}}{2}$

Answer: D

Solution:

$$\text{Given, } \cos x + \cos y - \cos(x+y) = \frac{3}{2}$$

$$\Rightarrow 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - \left[2 \cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2} \text{ [Use formula,}$$

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)]$$

$$\begin{aligned}
 \cos 2x &= 2\cos^2 x - 1 \\
 \Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) &= \frac{3}{2} - 1 = \frac{1}{2} \\
 \Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 4\cos^2\left(\frac{x+y}{2}\right) &= 1 \\
 = \frac{1}{2} \times 2 = 1 &= \cos^2\left(\frac{x-y}{2}\right) + \sin^2\left(\frac{x-y}{2}\right) \\
 \Rightarrow \left[\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right)\right]^2 + \sin^2\left(\frac{x-y}{2}\right) &= 0 \\
 \Rightarrow \sin\left(\frac{x-y}{2}\right) = 0 \text{ and } \cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right) &= 0 \\
 \Rightarrow x = y \text{ and } \cos 0 - 2\cos x = 0 \\
 \text{Gives, } \cos x = \frac{1}{2} = \cos y \\
 \therefore \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \\
 \therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1 + \sqrt{3}}{2}
 \end{aligned}$$

Question 65

If n is the number of solutions of the equation

$2 \cos x \left(4 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) - 1 \right) = 1$, $x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is
[1 Sep 2021 Shift 2]

Options:

A. $\left(3, \frac{13\pi}{9} \right)$

B. $\left(2, \frac{2\pi}{3} \right)$

C. $\left(2, \frac{8\pi}{9} \right)$

D. $\left(3, \frac{5\pi}{3} \right)$

Answer: A

Solution:

$$\begin{aligned}
 2 \cos x \left(4 \sin \left(\frac{\pi}{4} + x \right) \sin \left(\frac{\pi}{4} - x \right) - 1 \right) &= 1 \\
 \Rightarrow 2 \cos x \left(2 \cos(2x) - 2 \cos \left(\frac{\pi}{2} \right) - 1 \right) &= 1
 \end{aligned}$$

$$\Rightarrow 2 \cos x (4 \cos^2 x - 3) = 1$$

$$\Rightarrow \cos 3x = \frac{1}{2}$$

$$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

$$\text{Number of solutions} = n = 3$$

$$\text{Sum of solutions} = S = \frac{13\pi}{9}$$

Question 66

The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \leq x \leq \frac{\pi}{4}$ is
[31 Aug 2021 Shift 2]

Options:

A. 3

B. 1

C. 0

D. 2

Answer: B

Solution:

$$32^{\tan^2 x} + 32^{\sec^2 x} = 81$$

$$\Rightarrow 32^{\tan^2 x} + 32^{1 + \tan^2 x} = 81$$

$$\Rightarrow 33 \times 32^{\tan^2 x} = 81$$

$$\Rightarrow 32^{\tan^2 x} = \frac{27}{11}$$

$$\Rightarrow \tan^2 x = \ln_{32} \left(\frac{27}{11} \right)$$

$$\tan x = \sqrt{\ln_{32} \left(\frac{27}{11} \right)} \in (0, 1)$$

$$\Rightarrow \text{One solution in } \left[0, \frac{\pi}{4} \right]$$

Question 67

Section B : Numerical Type Questions

Let S be the sum of all solutions (in radians) of the equation

$\sin^4 \theta + \cos^4 \theta - \sin \theta \cos \theta = 0$ in $[0, 4\pi]$. Then, $\frac{85}{\pi}$ is equal to
[27 Aug 2021 Shift 2]

Answer: 56

Solution:

$$\sin^4 \theta + \cos^4 \theta - \sin \theta \cos \theta = 0 \text{ in } [0, 4\pi]$$

$$\Rightarrow 1 - 2\sin^2 \theta \cos^2 \theta - \sin \theta \cos \theta = 0$$

$$\Rightarrow 2 - \sin^2 2\theta - \sin 2\theta = 0$$

$$\Rightarrow \sin^2 2\theta + \sin 2\theta - 2 = 0$$

$$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$$

$$\Rightarrow \sin 2\theta = 1, 2\theta \in [0, 8\pi]$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\text{Sum of solutions } S = \frac{28\pi}{4}$$

$$\text{Then, } \frac{8S}{\pi} = \frac{8}{\pi} \times \frac{28\pi}{4} = 56$$

Question 68

The sum of solutions of the equation $\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$

is

[26 Aug 2021 Shift 1]

Options:

A. $-\frac{11\pi}{30}$

B. $\frac{\pi}{10}$

C. $-\frac{7\pi}{30}$

D. $\frac{\pi}{15}$

Answer: A

Solution:

$$\text{We have, } \frac{\cos x}{1} = \left| \tan 2x \right|$$

$$\Rightarrow \frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\cos\frac{x}{2}\sin\frac{x}{2}} = \left| \tan 2x \right|$$

$$\Rightarrow \frac{\left[\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right] \left[\cos\frac{x}{2} + \sin\frac{x}{2}\right]}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} = \left| \tan 2x \right|$$

$$\Rightarrow \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} = \left| \tan 2x \right|$$

$$\Rightarrow \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = \left| \tan 2x \right|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$$

$$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\Rightarrow 2x = n\pi + \frac{\pi}{4} - \frac{x}{2}$$

$$\text{or } 2x = n\pi - \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\Rightarrow \frac{5x}{2} = \left(n + \frac{1}{4}\right)\pi$$

$$\text{or } \frac{3x}{2} = \left(n - \frac{1}{4}\right)\pi$$

$$\Rightarrow \frac{-\pi}{2} < \frac{2}{5}\left(n + \frac{1}{4}\right)\pi < \frac{\pi}{2}$$

$$\text{or } \frac{-\pi}{2} < \frac{2}{3}\left(n - \frac{1}{4}\right)\pi < \frac{\pi}{2}$$

$$\Rightarrow -\frac{5}{4} < n + \frac{1}{4} < \frac{5}{4}$$

$$\text{or } \frac{-3}{4} < n - \frac{1}{4} < \frac{3}{4}$$

$$\Rightarrow \frac{-6}{4} < n < 1$$

$$\text{or } \frac{-1}{2} < n < 1$$

$$\Rightarrow n = -1, 0$$

$$\text{or } n = 0$$

$$\text{When } n = -1, x = \left(\frac{-3}{10}\right)\pi$$

$$\text{or when } n = 0, x = -\frac{\pi}{6}$$

$$n = 0, x = \left(\frac{1}{10}\right)\pi$$

$$\therefore \text{Required sum} = \left(\frac{-3}{10}\right)\pi + \left(\frac{1}{10}\right)\pi + \left(\frac{-1}{6}\right)\pi = \left(\frac{-11}{30}\right)\pi$$

Question 69

The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to
[22 Jul 2021 Shift 2]

Options:

- A. 11
- B. 7
- C. 5
- D. 9

Answer: C

Solution:

$$\sin^7 x \leq \sin^2 x \leq \lambda 1 \dots\dots(1)$$

$$\text{and } \cos^7 x \leq \cos^2 x \leq 1 \dots\dots(2)$$

$$\text{also } \sin^2 x + \cos^2 x = 1$$

$\Rightarrow$ equality must hold for (1) & (2)

$$\Rightarrow \sin^7 x = \sin^2 x \text{ \& } \cos^7 x = \cos^2 x$$

$$\Rightarrow \sin x = 0 \text{ \& } \cos x = 1$$

or

$$\cos x = 0 \text{ \& } \sin x = 1$$

$$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$$

$\Rightarrow$ 5 solutions

Question 70

Let $\alpha = \max_{x \in \mathbb{R}} \{8^{2 \sin 3x} \cdot 4^{4 \cos 3x}\}$ and $\beta = \min_{x \in \mathbb{R}} \{8^{2 \sin 3x} \cdot 4^{4 \cos 3x}\}$. If $8x^2 + bx + c = 0$

is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :

[27 Jul 2021 Shift 2]

Options:

- A. 42
- B. 47
- C. 43
- D. 50

Answer: A

Solution:

$$\begin{aligned} \alpha &= \max \{8^{2 \sin 3x} \cdot 4^{4 \cos 3x}\} \\ &= \max \{2^{6 \sin 3x} \cdot 2^{8 \cos 3x}\} \end{aligned}$$

$$= \max\{2^{6\sin 3x + 8\cos 3x}\}$$

$$\text{and } \beta = \min\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\} = \min\{2^{6\sin 3x + 8\cos 3x}\}$$

Now range of $6\sin 3x + 8\cos 3x$

$$= [-\sqrt{6^2 + 8^2}, +\sqrt{6^2 + 8^2}] = [-10, 10]$$

$$\alpha = 2^{10} \text{ \& } \beta = 2^{-10}$$

$$\text{So, } \alpha^{1/5} = 2^2 = 4$$

$$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$$

$$\text{quadratic } 8x^2 + bx + c = 0, c - b =$$

$$8 \times [(\text{product of roots}) + (\text{sum of roots})]$$

$$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4} \right] = 8 \times \left[\frac{21}{4} \right] = 42$$

Question71

The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is
[18 Mar 2021 Shift 1]

Answer: 1

Solution:

$$\text{Given, } |\cot x| = \cot x + \frac{1}{\sin x} \dots (i)$$

$$\text{If } \cot x > 0, \text{ then } |\cot x| = \cot x$$

$$\text{From Eq. (i), } \cot x = \cot x + \frac{1}{\sin x}$$

$$\Rightarrow \frac{1}{\sin x} = 0 \text{ (not possible)}$$

$$\text{If } \cot x < 0, \text{ then } |\cot x| = -\cot x$$

$$\text{From Eq. (ii), } -\cot x = \cot x + \frac{1}{\sin x}$$

$$\Rightarrow 2\cot x + \frac{1}{\sin x} = 0 \Rightarrow 2\cos x = -1$$

$$\Rightarrow x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

Here, $x = \frac{4\pi}{3}$ rejected because $\frac{4\pi}{3} \in$ third quadrant and in third quadrant $\cot x$ is positive. Since, we considered $\cot x < 0$. $\therefore x = 2\pi/3$ is the only one solution.

Question72

The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is
[17 Mar 2021 Shift 2]

Options:

- A. 3
- B. 4
- C. 2
- D. 5

Answer: A

Solution:

$$\text{Given, } x + 2 \tan x = \frac{\pi}{2}$$

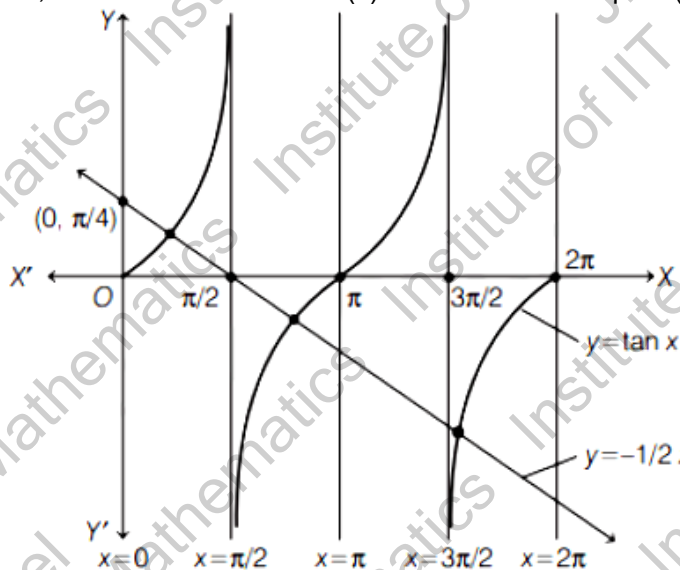
$$\Rightarrow 2 \tan x = \frac{\pi}{2} - x \Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \tan x = \left(-\frac{1}{2}\right)x + \frac{\pi}{4} \dots (i)$$

Approach In this type of problem solving, graphical approach is best because we have to find only number of solutions, not the solution (i.e. not the value(s) of x).

Concept To find the number of solution(s) for Eq. (i), first of all, let $y = \tan x \dots (ii)$ and $y = \left(-\frac{1}{2}\right)x + \frac{\pi}{4} \dots (iii)$ and then draw the graph of Eqs. (ii) and (iii).

Now, total number of solution(s) = Total number of point(s) of intersection of the graph (ii) and (iii).



$y = -\frac{1}{2}x + \frac{\pi}{4}$ intersects $y = \tan x$ at three distinct points in $[0, 2\pi]$.

$\therefore$ Total number of solutions = 3

Question 73

The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to
[16 Mar 2021 Shift 1]

Options:

- A. 3
- B. 4
- C. 8
- D. 2

Answer: B

Solution:

$$\text{Given, } 81^{\sin^2 x} + 81^{\cos^2 x} = 30$$

$$\Rightarrow 81^{\sin^2 x} + 81^{(1 - \sin^2 x)} = 30$$

$$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$$

$$\text{Let } 81^{\sin^2 x} = y.$$

$$\therefore y + \frac{81}{y} = 30$$

$$\Rightarrow y^2 - 30y + 81 = 0$$

$$\Rightarrow (y - 27)(y - 3) = 0$$

$$\Rightarrow y = 3 \text{ or } y = 27$$

$$81^{\sin^2 x} = 3 \text{ or } 81^{\sin^2 x} = 27$$

$$3^{4\sin^2 x} = 3 \text{ or } 3^{4\sin^2 x} = 3^3$$

$$\Rightarrow 4\sin^2 x = 1 \text{ or } 4\sin^2 x = 3$$

$$\Rightarrow \sin^2 x = 1/4 \text{ or } \sin^2 x = 3/4$$

$$\Rightarrow \sin^2 x = \sin^2(\pi/6) \text{ or } \sin^2 x = \sin^2(\pi/3)$$

$$\Rightarrow x = n\pi \pm \pi/6 \text{ or } x = n\pi \pm \pi/3$$

From $[0, \pi]$,

$$x = \pi/6, 5\pi/6 \text{ or } x = \pi/3, 2\pi/3$$

Hence, the total number of solutions = 4

Question 74

If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation

when $x \in \left[0, \frac{\pi}{2}\right]$ is

[26 Feb 2021 Shift 1]

Answer: 1

Solution:

Given, $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1, x \in [0, \pi/2]$

Let $\cos x = t$, then

$$\sqrt{3}t^2 = (\sqrt{3} - 1)t + 1$$

$$\Rightarrow \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$$

$$\Rightarrow (\sqrt{3}t^2 - \sqrt{3}t) + (t - 1) = 0$$

$$\Rightarrow \sqrt{3}t(t - 1) + 1(t - 1) = 0$$

$$\Rightarrow (t - 1)(\sqrt{3}t + 1) = 0$$

This gives $t = 1$ and $t = \frac{-1}{\sqrt{3}}$

Put, $t = \cos x$, then

$$\cos x = 1 \text{ and } \cos x = \frac{-1}{\sqrt{3}}$$

$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi/2]$

$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi]$

$\therefore \cos x = 1$

Since, $x \in [0, \frac{\pi}{2}]$, then $\cos x = \cos 0$

This gives $x = 0$ is only solution.

Therefore, number of solution when $x \in [0, \pi/2]$ is 1.

Question 75

The number of integral values of k for which the equation $3 \sin x + 4 \cos x = k + 1$ has a solution, $k \in \mathbb{R}$ is
[26 Feb 2021 Shift 1]

Answer: 11

Solution:

Given, $3 \sin x + 4 \cos x = k + 1 \dots (i)$

Multiply and divide LHS of Eq. (i) by $\sqrt{3^2 + 4^2} = 5$

$$\text{i.e. } 5 \left(\frac{3}{5} \sin x + \frac{4}{5} \cos x \right) = k + 1$$

$$\Rightarrow 5(\cos \alpha \sin x + \sin \alpha \cos x) = k + 1$$

$$[\text{Let } \cos \alpha = 3/5 \text{ then } \sin \alpha = \sqrt{1 - (3/5)^2} = \frac{4}{5}]$$

$$\Rightarrow 5 \sin(x + \alpha) = k + 1 \quad [\text{Use } \sin(a + b) = \sin a \cos b + \cos a \sin b]$$

$$\Rightarrow \sin(x + \alpha) = \frac{k + 1}{5}$$

Let $x + \alpha = \theta$

$$\text{Then, } \sin \theta = \frac{k + 1}{5}$$

$$\therefore -1 \leq \sin \theta \leq 1$$

$$\Rightarrow -1 \leq \frac{k + 1}{5} \leq 1$$

$$\Rightarrow -5 \leq k + 1 \leq 5$$

$$\Rightarrow -6 \leq k \leq 4$$

$\therefore$ Possible integral values of k are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3$ and 4 . i.e.
Total 11 integral values of k are possible for which Eq. (i) has solution.

Question 76

The value of $\cos^3\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{3\pi}{8}\right)$ is

[Jan. 9, 2020 (I)]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

$$\begin{aligned} & \cos^3\frac{\pi}{8} \left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8} \right] + \sin^3\frac{\pi}{8} \left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8} \right] \\ &= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8} \\ &= 4 \left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} \right) \right] \\ & \quad \left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8} \right) \right]; \\ &= 3 \left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} \right) \left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8} \right) \right]; \\ &= \cos\frac{\pi}{4} \left[4 \left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8} \right) - 3 \right] \\ &= \frac{1}{\sqrt{2}} \left[1 - \frac{1}{2} \right] = \frac{1}{2\sqrt{2}}. \end{aligned}$$

Question 77

If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

[Sep. 05, 2020 (II)]

Options:

$$\text{A. } L = -\frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

$$\text{B. } L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$$

$$\text{C. } M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$$

$$\text{D. } M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

Answer: D

Solution:

$$L + M = 1 - 2\sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \dots\dots (i)$$

$$\text{and } L - M = -\cos \frac{\pi}{8} \dots\dots (ii)$$

$$\text{From equations (i) and (ii), } L = \frac{1}{2} \left(\frac{1}{\sqrt{2}} - \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8} \text{ and } M = \frac{1}{2} \left(\frac{1}{\sqrt{2}} + \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

Question 78

If $\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to

[Jan. 8, 2020 (II)]

Answer: 1

Solution:

$$\frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} = \frac{1}{7} \text{ and } \sqrt{\frac{1 - \cos^2 \beta}{2}} = \frac{1}{10}$$

$$\Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$$

$$\therefore \tan \alpha = \frac{1}{7} \text{ and } \sin \beta = \frac{1}{\sqrt{10}}$$

$$\tan \beta = \frac{1}{3}$$

$$\therefore \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\begin{aligned}\tan(\alpha + 2\beta) &= \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} \\ &= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1\end{aligned}$$

Question 79

The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x + y = 1$ is:
[Sep. 02, 2020 (II)]

Options:

- A. $\left(0, \frac{\pi}{2}\right)$
- B. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
- C. $\left(0, \frac{3\pi}{4}\right)$
- D. $\left(0, \frac{\pi}{4}\right)$

Answer: A

Solution:

Let $f(x, y) = x + y - 1$

Given $(1, 2)$ and $(\sin \theta, \cos \theta)$ are lies on same side.

$$\therefore f(1, 2) \cdot f(\sin \theta, \cos \theta) > 0$$

$$\Rightarrow 2[\sin \theta + \cos \theta - 1] > 0$$

$$\Rightarrow \sin \theta + \cos \theta > 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$$

Question 80

If the equation $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ has real solutions for θ , then λ lies in the interval :
[Sep. 02, 2020 (II)]

Options:

A. $\left(-\frac{5}{4}, -1\right)$

B. $\left[-1, -\frac{1}{2}\right]$

C. $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

D. $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

Answer: B

Solution:

Given equation is $\cos^4(\theta) + \sin^4(\theta) + \lambda = 0$

$$\Rightarrow \lambda = -[\cos^4(\theta) + \sin^4(\theta)]$$

$$\Rightarrow \lambda = -[(\cos^2(\theta) + \sin^2(\theta))^2 - 2\cos^2(\theta)\sin^2(\theta)]$$

$$[\because a^2 + b^2 = (a+b)^2 - 2ab]$$

$$\Rightarrow \lambda = -(1)^2 + \frac{1}{2}(2\cos(\theta)\sin(\theta))^2$$

$$[\because \cos^2(\theta) + \sin^2(\theta) = 1]$$

$$\Rightarrow \lambda = \frac{1}{2}\sin^2(2\theta) - 1$$

We know that $-1 \leq \sin(x) \leq 1$

Therefore $0 \leq \sin^2(x) \leq 1$

$$\text{So } \frac{1}{2}\sin^2(2\theta) \in \left[0, \frac{1}{2}\right]$$

$$\text{for } \frac{1}{2}\sin^2(2\theta) = 0 \Rightarrow \lambda = 0 - 1 = -1$$

$$\text{for } \frac{1}{2}\sin^2(2\theta) = \frac{1}{2} \Rightarrow \lambda = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\therefore \lambda \in \left[-1, -\frac{1}{2}\right]$$

Hence, Option(B) i.e. $\left[-1, -\frac{1}{2}\right]$ is correct

Question 81

The number of distinct solutions of the equation,

$\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$ in the interval $[0, 2\pi]$, is _____

[Jan. 9, 2020 (I)]

Answer: 8

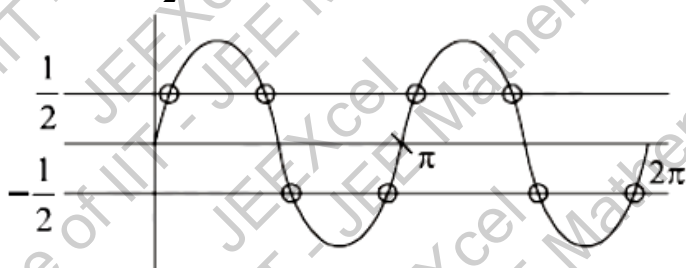
Solution:

$$\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$$

$$\Rightarrow \log_{1/2} |\sin x \cos x| = 2$$

$$\Rightarrow |\sin x \cos x| = \frac{1}{4}$$

$$\Rightarrow \sin 2x = \pm \frac{1}{2}$$



Hence, total number of solutions = 8.
