

Three Dimensional Geometry

Question 1

Let a line L pass through the origin and be perpendicular to the lines

$$L_2 : \vec{r} = (3 + t)\hat{i} + (2t - 1)\hat{j} + (2t + 4)\hat{k} \text{ and}$$

$$L_3 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, t, s \in \mathbb{R}.$$

, $a \in \mathbb{Z}$, is the point on L_3 at a distance of $\sqrt{17}$ from the L and L_2 , then $(a + b + c)^2$ is equal to _____

JEE Main 2026 (Online) 8th April Evening Shift

Answer: 4

Solution:

A pair of lines in 3D have the vector equations :

L_2	L_3	(t, s)
$\vec{r} = (3 + t)\hat{i} + (2t - 1)\hat{j} + (2t + 4)\hat{k}$	$\vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$	$\in \mathbb{R}$

A third line L_1 passes through the origin and is $\perp$ to both the lines L_2 and L_3 defined just above. The point of intersection of L_1 and L_2 is at a distance $\sqrt{17}$ units from another point with coordinates (a, b, c) and lying on L_3 where $a \in \mathbb{Z}$

$$L_2 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = t$$

$$L_3 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$$

vector $\perp$ to L_2 & L_3

$$\begin{aligned} & \vec{r} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j} + \hat{k}) \\ \Rightarrow & 1 - 2 = \hat{i}(-2) - \hat{j}(-3) + \hat{k}(-2) \\ & = -2\hat{i} + 3\hat{j} - 2\hat{k} = -(2\hat{i} - 3\hat{j} + 2\hat{k}) \end{aligned}$$

$$\text{Now, } L_1 : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \ell$$

intersection of L_1 & L_2

$$\Rightarrow t + 3 = 2\ell, 2t - 1 = -3\ell, 2t + 4 = 2\ell$$

$$\Rightarrow t + 3 = 2t + 4$$

$$t = -1$$

$$\Rightarrow P \equiv (2, -3, 2)$$

A point on L_3 is $Q(2s + 3, 2s + 3, s + 2)$

$$\Rightarrow PQ^2 = (2s + 1)^2 + (2s + 6)^2 + s^2 = 17$$

$$\Rightarrow 9s^2 + 28s + 20 = 0$$

$$\Rightarrow (9s + 10)(s + 2) = 0 \Rightarrow s = -2$$

$$\therefore Q \equiv (-1, -1, 0) \equiv (a, b, c) \Rightarrow (a + b + c)^2 = 4$$

Question2

Let a line L passing through the point $P(1, 1, 1)$ be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the yz -plane at the point Q . Another line parallel to L and passing through the point $S(1, 0, -1)$ intersects the yz -plane at the point R . Then the square of the area of the parallelogram $PQRS$ is equal to _____.

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Answer: 6

Solution:

$$\text{let } L_1 : \frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1} = \lambda_1$$

$$L_2 : \frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0} = \lambda_2$$

given that L is passing through $P(1, 1, 1)$

and perpendicular to L_1 and L_2 .

dr's of $L = \text{dr's of } L_1 \times \text{dr's of } L_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(0-1) + \hat{k}(4-1)$$

$$= -\hat{i} + \hat{j} + 3\hat{k}$$

$$\text{Equation of line } L : \frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-1}{3} = \lambda$$

It is given that L intersect yz plane at Q for Q on $y-z$ plane $x = 0$

$$\lambda = \frac{0-1}{-1} = 1$$

$$\frac{y-1}{1} = \lambda \Rightarrow y-1 = 1 \Rightarrow y = 2$$

$$\frac{z-1}{3} = \lambda \Rightarrow \frac{z-1}{3} = 1 \Rightarrow z = 4$$

Point Q is $(0, 2, 4)$

Another line parallel to L let L' passing through $S(1, 0, -1)$.

The area of parallelogram $PQRS$

$$= |\overrightarrow{PQ} \times \overrightarrow{PS}|$$

$$\overrightarrow{PQ} = Q - P = (0, 2, 4) - (1, 1, 1) = (-1, 1, 3)$$

$$\overrightarrow{PS} = S - P = (1, 0, -1) - (1, 1, 1) = (0, -1, -2)$$

$$= \hat{i}(-2+3) - \hat{j}(2-0) + \hat{k}(1-0)$$

$$= \hat{i} - 2\hat{j} + \hat{k}$$

$$\text{Area} = |\overrightarrow{PQ} \times \overrightarrow{PS}| = \sqrt{1^2 + (-2)^2 + (1)^2} = \sqrt{6}$$

Square of area = 6

Question3

Let the image of the point $P(0, -5, 0)$ in the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$ be the point R and the image of the point $Q(0, \frac{-1}{2}, 0)$ in the line $\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1}$ be the point S . Then the square of the area of the parallelogram $PQRS$ is _____.

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Answer: 162

Solution:

Let us find the reflected points R and S , and then use vectors to get the area of parallelogram $PQRS$.

1. Reflection of $P(0, -5, 0)$ in the line

$$\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$$

Write the line in parametric form:

$$x = 1 + 2t, \quad y = t, \quad z = -1 - 2t$$

So, a point on the line is

$$A(1, 0, -1)$$

and its direction vector is

$$\vec{d}_1 = (2, 1, -2)$$

To reflect point P in the line, first find the foot of perpendicular from P to the line.

Let the foot be

$$M = A + t\vec{d}_1 = (1 + 2t, t, -1 - 2t)$$

Since $PM \perp \vec{d}_1$,

$$(\vec{PM}) \cdot \vec{d}_1 = 0$$

Now,

$$\vec{PM} = M - P = (1 + 2t, t + 5, -1 - 2t)$$

So,

$$(1 + 2t, t + 5, -1 - 2t) \cdot (2, 1, -2) = 0$$

$$2(1 + 2t) + (t + 5) + (-2)(-1 - 2t) = 0$$

$$2 + 4t + t + 5 + 2 + 4t = 0$$

$$9 + 9t = 0$$

$$t = -1$$

Hence,

$$M = (1 + 2(-1), -1, -1 - 2(-1)) = (-1, -1, 1)$$

Since M is the midpoint of PR ,

$$R = 2M - P$$

$$R = 2(-1, -1, 1) - (0, -5, 0) = (-2, 3, 2)$$

So,

$$R = (-2, 3, 2)$$

2. Reflection of $Q(0, -\frac{1}{2}, 0)$ in the line

$$\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1}$$

Write the line in parametric form:

$$x = 1 - u, \quad y = -9 + 4u, \quad z = -1 + u$$

So, a point on the line is

$$B(1, -9, -1)$$

and direction vector is

$$\vec{d}_2 = (-1, 4, 1)$$

Let the foot of perpendicular from Q to this line be

$$N = B + u\vec{d}_2 = (1 - u, -9 + 4u, -1 + u)$$

Since $QN \perp \vec{d}_2$,

$$(\vec{QN}) \cdot \vec{d}_2 = 0$$

Now,

$$\vec{QN} = N - Q = (1 - u, -9 + 4u + \frac{1}{2}, -1 + u) = (1 - u, -\frac{17}{2} + 4u, -1 + u)$$

So,

$$(1-u, -\frac{17}{2}+4u, -1+u) \cdot (-1, 4, 1) = 0$$

$$-(1-u) + 4(-\frac{17}{2}+4u) + (-1+u) = 0$$

$$-1+u-34+16u-1+u=0$$

$$18u-36=0$$

$$u=2$$

Hence,

$$N = (1-2, -9+8, -1+2) = (-1, -1, 1)$$

Again, N is the midpoint of QS , so

$$S = 2N - Q$$

$$S = 2(-1, -1, 1) - (0, -\frac{1}{2}, 0) = (-2, -2 + \frac{1}{2}, 2) = (-2, -\frac{3}{2}, 2)$$

So,

$$S = (-2, -\frac{3}{2}, 2)$$

3. Check the parallelogram sides

Now,

$$P = (0, -5, 0), \quad Q = (0, -\frac{1}{2}, 0), \quad R = (-2, 3, 2), \quad S = (-2, -\frac{3}{2}, 2)$$

Take adjacent sides from P :

$$\vec{PQ} = Q - P = (0, \frac{9}{2}, 0)$$

and

$$\vec{PR} = R - P = (-2, 8, 2)$$

Also,

$$\vec{QS} = S - Q = (-2, -1, 2)$$

which is not equal to $\vec{PR}$, so let us check the correct order of parallelogram.

Observe:

$$\vec{PS} = S - P = (-2, \frac{7}{2}, 2)$$

and

$$\vec{QR} = R - Q = (-2, \frac{7}{2}, 2)$$

Thus, $PS \parallel QR$.

Also,

$$\vec{PQ} = (0, \frac{9}{2}, 0), \quad \vec{SR} = R - S = (0, \frac{9}{2}, 0)$$

So the correct parallelogram order is $PQRS$, with adjacent sides

$$\vec{PQ} \quad \text{and} \quad \vec{PS}$$

Hence area of parallelogram is

$$|\vec{PQ} \times \vec{PS}|$$

4. Compute cross product

$$\vec{PQ} = (0, \frac{9}{2}, 0), \quad \vec{PS} = (-2, \frac{7}{2}, 2)$$

$$\vec{PQ} \times \vec{PS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \frac{9}{2} & 0 \\ -2 & \frac{7}{2} & 2 \end{vmatrix}$$

$$= \hat{i} \left(\frac{9}{2} \cdot 2 - 0 \cdot \frac{7}{2} \right) - \hat{j} (0 \cdot 2 - 0 \cdot (-2)) + \hat{k} \left(0 \cdot \frac{7}{2} - \frac{9}{2} \cdot (-2) \right)$$

$$= 9\hat{i} + 0\hat{j} + 9\hat{k}$$

So,

$$|\vec{PQ} \times \vec{PS}| = \sqrt{9^2 + 9^2} = \sqrt{162} = 9\sqrt{2}$$

Therefore, the square of the area is

$$(9\sqrt{2})^2 = 162$$

$$\boxed{162}$$

Question4

Let a line L_1 pass through the origin and be perpendicular to the lines

$$L_2 : \vec{r} = (3 + t)\hat{i} + (2t - 1)\hat{j} + (2t + 4)\hat{k} \text{ and}$$

$$L_3 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, t, s \in \mathbf{R}.$$

If (a, b, c) , $a \in \mathbf{Z}$, is the point on L_3 at a distance of $\sqrt{17}$ from the point of intersection of L_1 and L_2 , then $(a + b + c)^2$ is equal to _____

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Answer: 4

Solution:

A pair of lines in 3D have the vector equations :

L_2	L_3	(t, s)
$\vec{r} = (3 + t)\hat{i} + (2t - 1)\hat{j} + (2t + 4)\hat{k}$	$\vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$	$\in \mathbb{R}$

A third line L_1 passes through the origin and is $\perp$ to both the lines L_2 and L_3 defined just above

The point of intersection of L_1 and L_2 is at a distance $\sqrt{17}$ units from another point with coordinates (a, b, c) and lying on L_3 where $a \in \mathbb{Z}$

$$L_2 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = t$$

$$L_3 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$$

vector $\perp$ to L_2 & L_3

$$\begin{aligned} & \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-3) + \hat{k}(-2) \\ & = -2\hat{i} + 3\hat{j} - 2\hat{k} \\ & = -(2\hat{i} - 3\hat{j} + 2\hat{k}) \end{aligned}$$

$$\text{Now, } L_1 : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \ell$$

intersection of L_1 & L_2

$$\Rightarrow t + 3 = 2\ell, 2t - 1 = -3\ell, 2t + 4 = 2\ell$$

$$\Rightarrow t + 3 = 2t + 4$$

$$t = -1$$

$$\Rightarrow P \equiv (2, -3, 2)$$

A point on L_3 is $Q(2s + 3, 2s + 3, s + 2)$

$$\Rightarrow PQ^2 = (2s + 1)^2 + (2s + 6)^2 + s^2 = 17$$

$$\Rightarrow 9s^2 + 28s + 20 = 0$$

$$\Rightarrow (9s + 10)(s + 2) = 0 \Rightarrow s = -2$$

$$\therefore Q \equiv (-1, -1, 0) \equiv (a, b, c) \Rightarrow (a + b + c)^2 = 4$$

Question5

Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$, and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D , respectively, then $\left| \overrightarrow{CD} \right|^2$ is equal to:

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Options:

A.

290

B.

171

C.

89

D.

312

Answer: A

Solution:

$$L_1 : \frac{x-2}{-3} = \frac{y-6}{2} = \frac{z-7}{4} = t$$

$$L_2 : \frac{x-4}{2} = \frac{y-3}{1} = \frac{z-5}{3} = s$$

Any point on L_1 is $C = (-3t + 2, 2t + 6, 4t + 7)$

Any point on L_2 is $D = (2s + 4, s + 3, 3s + 5)$

$$\text{d.r's of } \overrightarrow{CD} = (2s + 3t + 2, s - 2t - 3, 3s - 4t - 2)$$

CD is parallel to $-3i + 5j + 16k$

$$\frac{2s + 3t + 2}{-3} = \frac{s - 2t - 3}{5} = \frac{3s - 4t - 2}{16}$$

$$13s + 9t + 1 = 0 \dots (i)$$

$$41s + 36t + 26 = 0 \dots (2)$$

Solve (i) \& (2) $s = 2; t = -3$

$$C = (11, 0, -5)$$

$$D = (8, 5, 11)$$

$$CD^2 = 290$$

Question6

Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is :

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Options:

A.

16

B.

12

C.

6

D.

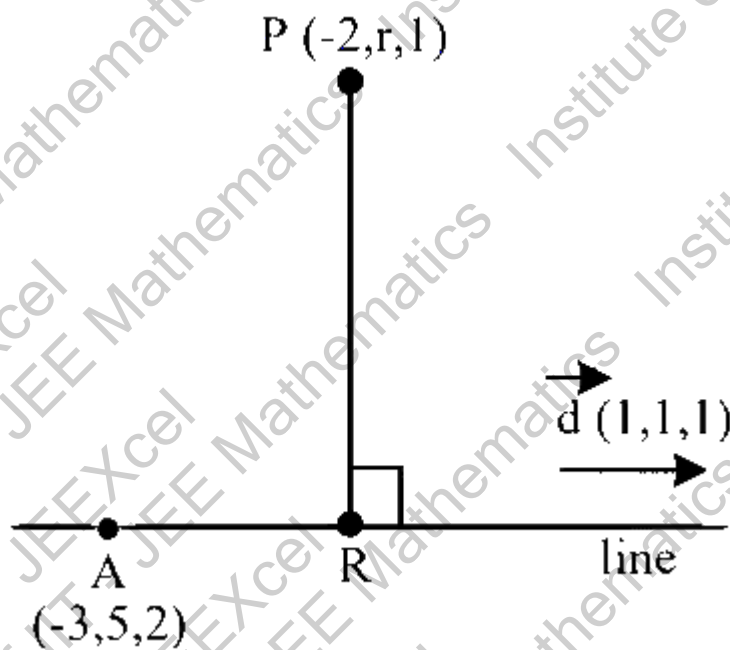
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Answer: D

Solution:

$$\text{Equation line is : } \frac{x+3}{1} = \frac{y-5}{1} = \frac{z-2}{1} = \lambda$$

$\therefore$ General point R on line is $r(\lambda - 3, \lambda + 5, \lambda + 2)$



$$\overrightarrow{PR} = (\lambda - 1, \lambda + 5 - r, \lambda + 1)$$

$$\text{Now } \overrightarrow{PR} = \vec{d} = 0$$

$$\Rightarrow (\lambda - 1)1 + (\lambda + 5 - r)1 + (\lambda + 1)1 = 0$$

$$\Rightarrow 3\lambda + 5 = 0$$

$$\Rightarrow \lambda = \frac{r - 5}{3}$$

$$\therefore R \equiv \left(\frac{r - 5}{3} - 3, \frac{r - 5}{3} + 5, \frac{r - 5}{3} + 2 \right)$$

$$\therefore R \equiv \left(\frac{r - 14}{3}, \frac{r + 10}{3}, \frac{r + 1}{3} \right)$$

$$\text{Now } PR = \sqrt{\frac{14}{3}} \Rightarrow (PR)^2 = \frac{14}{3}$$

$$\Rightarrow \left(\frac{r - 14}{3} + 2 \right)^2 + \left(\frac{r + 10}{3} - r \right)^2 + \left(\frac{r + 1}{3} - 1 \right)^2 = \frac{14}{3}$$

$$\Rightarrow \frac{(r - 8)^2}{9} + \frac{(10 - 2r)^2}{9} + \frac{(r - 2)^2}{9} = \frac{14}{3}$$

$$\Rightarrow (r^2 - 16r + 64) + (100 + 4r^2 - 40r) + (r^2 - 4r + 4) = 42$$

$$\Rightarrow 6r^2 - 60r + 126 = 0$$

$$\Rightarrow r^2 - 10r + 21 = 0$$

$$\Rightarrow r = 3, 7$$

Sum of possible value of r is $= 10$

Question 7

If the image of the point $P(1, 2, a)$ in the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{7-z}{2}$ is $Q(5, b, c)$, then $a^2 + b^2 + c^2$ is equal to

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Options:

A.

298

B.

264

C.

293

D.

283

Answer: A

Solution:

$$P(1, 2, a), Q(5, b, c) \quad \frac{x-6}{3} = \frac{y-7}{2} = \frac{7-z}{2}$$

$$PQ \perp \text{line}$$

$$PQ(4, b-2, c-a)$$

$$3(4) + 2(b-2) - 2(c-a) = 0$$

$$a - c = -12$$

$$a = 3, c = 15, b = 8$$

$$a^2 + b^2 + c^2 = 9 + 64 + 225 = 298$$

Question 8

Let $P(\alpha, \beta, \gamma)$ be the point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ and nearer to the origin. Then the

shortest distance, between the lines $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ and

$\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$, is equal to

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Options:

A.

$$4\sqrt{\frac{7}{5}}$$

B.

$$7\sqrt{\frac{5}{4}}$$

C.

$$4\sqrt{\frac{5}{7}}$$

D.

$$2\sqrt{\frac{7}{4}}$$

Answer: A

Solution:

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z}{1} = t$$

$$(1+2t, -1-3t, t)$$

$$4t^2 + 9t^2 + t^2 = 16 \times 14$$

$$t = \pm 4$$

$$\text{Points are } (9, -13, 4), (-7, 11, -4)$$

$$(\alpha, \beta, \gamma) = (-7, 11, -4)$$

$$\vec{a} = (-7, 11, -4), \vec{c} = (-5, 10, 3)$$

$$\vec{c} - \vec{a} = (2, -1, 7)$$

$$b \times d = (-1, 5, -3)$$

$$S.D = \frac{\left| \begin{matrix} -2 & -5 & -21 \\ \sqrt{1+25+9} \end{matrix} \right|}{\sqrt{35}}$$

Question9

Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L , whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c) \in S} (a + b + c)$ is equal to :

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Options:

A.

28

B.

6

C.

40

D.

34

Answer: D

Solution:

M is the point of intersection of L_1 & L_2

$$\Rightarrow 2\lambda - 1 = 2\mu - 1, 3\lambda - 1 = 3\mu - 1, 6\lambda - 3 = 9 \Rightarrow \lambda = 2 = \mu$$
$$M(3, 5, 9)$$

Now let point P be $(2K - 1, 3K - 1, 6K - 3)$ on L_2

Such that $PM = 7$

$$\Rightarrow \sqrt{(2K-4)^2 + (3K-6) + (6K-12)^2} = 7$$

$$\Rightarrow 49K^2 + 196 - 196K = 49$$

$$\Rightarrow K^2 + 4 - 4K = 1 \Rightarrow K^2 - 4K + 3 = 0 \Rightarrow K = 1, 3$$

So points P&Q are (1, 2, 3)&(5, 8, 15) So sum of all co-ordinates of P&Q = 34

Question 10

The vertices B and C of a triangle ABC lie on the line

$\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are (1, 6, 3) and (4, 9, α) respectively and C is at a distance of 10 units from B. The area (in sq. units) of $\triangle ABC$ is :

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Options:

A.

$$20\sqrt{13}$$

B.

$$5\sqrt{13}$$

C.

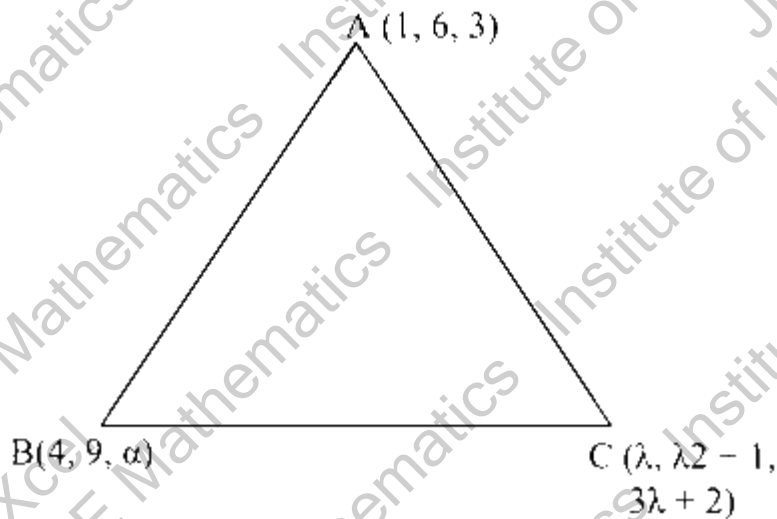
$$15\sqrt{13}$$

D.

$$10\sqrt{13}$$

Answer: B

Solution:



$$L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

Any point $(\lambda, 2\lambda + 1, 3\lambda + 2)$

Also B lies on L

$$\Rightarrow \frac{4}{1} = \frac{\alpha - 1}{2} \Rightarrow \alpha = 9 \therefore B(4, 9, 14)$$

$$C \equiv \langle 4, 9, 14 \rangle + \frac{10}{\sqrt{14}} \langle 1, 2, 3 \rangle$$

$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} = \langle 3, 3, 11 \rangle$$

$$\vec{AC} = \vec{AB} + \frac{10}{\sqrt{14}} \langle 1, 2, 3 \rangle$$

Cross product of $\vec{AB}$ to eliminate parallel component

$$\text{So, area} = \frac{1}{2} \times \frac{10}{\sqrt{14}} |\vec{AB} \times \langle 1, 2, 3 \rangle|$$

$$\vec{AB} \times \langle 1, 2, 3 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 11 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \langle -13, 2, 3 \rangle$$

$$\text{Area} = \frac{1}{2} \times \frac{10}{\sqrt{14}} \cdot \sqrt{182} = 5\sqrt{13}$$

Question 11

Let the direction cosines of two lines satisfy the equations :

$$4l + m - n = 0 \text{ and } 2mn + 10nl + 3lm = 0.$$

Then the cosine of the acute angle between these lines is :

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Options:

A.

$$\frac{10}{7\sqrt{38}}$$

B.

$$\frac{10}{\sqrt{38}}$$

C.

$$\frac{10}{3\sqrt{38}}$$

D.

$$\frac{20}{3\sqrt{38}}$$

Answer: C

Solution:

$$n = 4l + m$$

$$n(2m + 10l) + 3lm = 0$$

$$\Rightarrow 2(4l + m)(m + 5l) + 3lm = 0$$

$$\Rightarrow 8lm + 2m^2 + 10ml + 40l^2 + 3lm = 0$$

$$\Rightarrow 40l^2 + 21ml + 2m^2 = 0$$

$$\Rightarrow 40l^2 + 16ml + 5ml + 2m^2 = 0$$

$$\Rightarrow (8l + m)(5l + 2m) = 0$$

$$m = -8l; 2m = -5l$$

$$n = -4l \Rightarrow m = \frac{-5l}{2}$$

$$\begin{aligned}
 D \cdot R_1 &= (l_1, m_1, n_1) \\
 &= (-1, -8, -4) \\
 DR_2 &= (l_2, m_2, n_2) \\
 &= \left(l_2, \frac{-5l_2}{2}, \frac{3l_2}{2} \right) \\
 &= (2, -5, 3) \\
 \cos \theta &= \left| \frac{2 + 40 - 12}{9 \cdot \sqrt{38}} \right| \\
 \cos \theta &= \frac{10}{3\sqrt{38}} \\
 \text{i.e. option (1) is correct}
 \end{aligned}$$

Question 12

The sum of all values of α , for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is

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Options:

A.

-6

B.

-8

C.

8

D.

6

Answer: A

Solution:

Lines into vector form

$$L_1 : \vec{r} = -\hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\alpha\hat{i} - \hat{j} - \alpha\hat{k})$$

$$L_2 : \vec{r} = \hat{j} + \hat{k} + \mu(\alpha\hat{i} + 2\hat{j} + 2\alpha\hat{k})$$

Lines are not parallel because direction ratios are not proportional

$$\frac{\alpha}{\alpha} \neq \frac{-1}{2}$$

So lines are skew lines.

Minimum distance between two skew lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \mu\vec{d}$ is given as

$$d = \left| \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$$

$$\vec{a} = -\hat{i} + 2\hat{j} + 4\hat{k}, \quad \vec{c} = \hat{j} + \hat{k}$$

$$\vec{b} = \alpha\hat{i} - \hat{j} - \alpha\hat{k}, \quad \vec{d} = \alpha\hat{i} + 2\hat{j} + 2\alpha\hat{k}$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}$$

$$= \hat{i}(-2\alpha + 2\alpha) - \hat{j}(2\alpha^2 + \alpha^2) + \hat{k}(2\alpha + \alpha)$$

$$\therefore \vec{b} \times \vec{d} = -3\alpha^2\hat{j} + 3\alpha\hat{k}$$

$$\Rightarrow |\vec{b} \times \vec{d}| = \sqrt{9\alpha^4 + 9\alpha^2}$$

$$\Rightarrow \vec{a} - \vec{c} = -\hat{i} + \hat{j} + 3\hat{k}$$

$$\Rightarrow \left| \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right| = \sqrt{2}$$

$$\Rightarrow \left| \frac{(-\hat{i} + \hat{j} + 3\hat{k}) \cdot (-3\alpha^2\hat{j} + 3\alpha\hat{k})}{\sqrt{9\alpha^4 + 9\alpha^2}} \right| = \sqrt{2}$$

$$\Rightarrow \left| \frac{-3\alpha^2 + 9\alpha}{3\sqrt{\alpha^4 + \alpha^2}} \right| = \sqrt{2}$$

$$\Rightarrow \left| \frac{-3\alpha(\alpha - 3)}{3\alpha\sqrt{\alpha^2 + 1}} \right| = \sqrt{2}$$

$$\Rightarrow \left| \frac{\alpha - 3}{\sqrt{1 + \alpha^2}} \right| = \sqrt{2}$$

squaring

$$\Rightarrow \frac{(\alpha - 3)^2}{1 + \alpha^2} = 2$$

$$\Rightarrow \alpha^2 + 9 - 6\alpha = 2 + 2\alpha^2$$

$$\Rightarrow \alpha^2 + 6\alpha - 7 = 0$$

This equation has two roots say α_1 and α_2

So, sum of roots = $-\frac{\text{coefficient of } \alpha}{\text{coefficient of } \alpha^2}$

$$\therefore \alpha_1 + \alpha_2 = -6$$

Question 13

If the distances of the point $(1, 2, a)$ from the line $L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b}$ and $L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c}$ are equal, then $a + b + c$ is equal to

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Options:

A.

4

B.

6

C.

7

D.

5

Answer: C

Solution:

Let line $L: \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$, point $P(1, 2, a)$.

$$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda_1,$$

$$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \lambda_2.$$

Any point Q on L is $(t+1, 2t, t+1)$.

Let $Q_1(t_1 + 1, 2t_1, t_1 + 1)$ be point on L and d_1 be the distance measured from $P(1, 2, a)$ along L_1 such that $|PQ_1| = d_1$.

So, direction ratio of $\overrightarrow{PQ_1}$ and line L_1 are proportional.

Direction ratio of $\overrightarrow{PQ_1}$

$$= Q_1 - P = (t_1 + 1, 2t_1, t_1 + 1) - (1, 2, a) = (t_1, 2t_1 - 2, t_1 + 1 - a)$$

Direction ratio of $L_1 = (3, 4, b)$.

So,

$$\frac{t_1}{3} = \frac{2t_1 - 2}{4} = \frac{t_1 - a + 1}{b}$$

Equating

$$\frac{t_1}{3} = \frac{2t_1 - 2}{4} \quad \text{and} \quad \frac{t_1}{3} = \frac{t_1 - a + 1}{b}$$

$$4t_1 = 6t_1 - 6 \quad \text{and put } t_1 = 3$$

$$t_1 = 3 \quad \text{and} \quad \frac{3}{3} = \frac{3 - a + 1}{b} \Rightarrow a + b = 4 \dots \dots (1)$$

Similarly, let $Q_2(t_2 + 1, 2t_2, t_2 + 1)$ be point on L and d_2 be the distance measured from $P(1, 2, a)$ along L_2 such that $|PQ_2| = d_2$.

so, direction ratio of $\overrightarrow{PQ_2}$ and d_2 are proportional.

direction ratio of $\overrightarrow{PQ_2}$,

$$= Q_2 - P = (t_2 + 1, 2t_2, t_2 + 1) - (1, 2, a)$$

$$= (t_2, 2t_2 - 2, t_2 + 1 - a)$$

Direction ratio of $L_2 = (1, 4, c)$.

$$\frac{t_2}{1} = \frac{2t_2 - 2}{4} = \frac{t_2 + 1 - a}{c}$$

Equating

$$\frac{t_2}{1} = \frac{2t_2 - 2}{4} \quad \text{and} \quad \frac{t_2}{1} = \frac{t_2 + 1 - a}{c}$$

$$4t_2 = 2t_2 - 2 \quad \text{and put } t_2 = -1$$

$$2t_2 = -2 \quad \text{and} \quad \frac{-1}{1} = \frac{-1 + 1 - a}{c}$$

$$\Rightarrow -1 = \frac{-a}{c} \Rightarrow a = c \dots \dots (2)$$

Now, calculating

$$d_1^2 = |PQ_1|^2 = t_1^2 + (2t_1 - 2)^2 + (t_1 + 1 - a)^2, \text{ use } t_1 = 3$$

$$\Rightarrow d_1^2 = (3)^2 + (2 \times 3 - 2)^2 + (3 + 1 - a)^2$$

$$= 9 + 16 + (4 - a)^2 = 25 + 16 + a^2 - 8a$$

$$= a^2 - 8a + 41$$

Similarly,

$$d_2^2 = |PQ_2|^2 = t_2^2 + (2t_2 - 2)^2 + (t_2 + 1 - a)^2$$

$$\Rightarrow d_2^2 = (-1)^2 + (2 \times (-1) - 2)^2 + (-1 + 1 - a)^2 \{ \text{use } t_2 = -1 \}$$

$$\Rightarrow d_2^2 = 1 + 16 + a^2 = 17 + a^2$$

$$\text{It is given that } |PQ_1| = |PQ_2| \Rightarrow d_1^2 = d_2^2$$

$$\Rightarrow a^2 - 8a + 41 = 17 + a^2$$

$$\Rightarrow -8a + 41 = 17$$

$$\Rightarrow 8a = 41 - 17 = 24 \Rightarrow a = 3$$

from (1) & (2)

$$b = 4 - a = 4 - 3 = 1 \quad \text{and} \quad c = a = 3$$

$$\text{So value of } a + b + c = 3 + 1 + 3 = 7$$

Question 14

Let Q(a, b, c) be the image of the point P(3, 2, 1) in the line

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}. \text{ Then the distance of Q from the line}$$

$$\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2} \text{ is}$$

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Options:

A.

8

B.

7

C.

6

D.

Answer: B

Solution:

Point $P(3, 2, 1)$

Let Line $L_1 : \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$ and $L_2 : \frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$

Direction vector of $L_1 : (1, 2, 1)$

Let the first line be $L_1 : \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1} = \lambda$.

Any general point M on this line is $M(\lambda + 1, 2\lambda, \lambda + 1)$.

If M is the foot of the perpendicular from $P(3, 2, 1)$ to L_1 , the direction ratio of PM is $(\lambda - 2, 2\lambda - 2, \lambda)$.

Since $PM \perp L_1$: Since $PM \perp L_1$:

$$1(\lambda - 2) + 2(2\lambda - 2) + 1(\lambda) = 0$$

$$\Rightarrow \lambda - 2 + 4\lambda - 4 + \lambda = 0 \Rightarrow 6\lambda = 6 \Rightarrow \lambda = 1$$

The foot of the perpendicular M is $(2, 2, 2)$. Since M is the midpoint of PQ , where Q is (a, b, c) :

$$\bullet \frac{a+3}{2} = 2 \Rightarrow a = 1$$

$$\bullet \frac{b+2}{2} = 2 \Rightarrow b = 2$$

$$\bullet \frac{c+1}{2} = 2 \Rightarrow c = 3$$

So, $Q = (1, 2, 3)$.

Find the distance of $Q(1, 2, 3)$ from the second line

The second line is $L_2 : \frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2} = \mu$.

Let $A(9, 9, 5)$ be a point on L_2 and $d = 3\hat{i} + 2\hat{j} - 2\hat{k}$ be its direction vector.

The vector $AQ = (1 - 9)\hat{i} + (2 - 9)\hat{j} + (3 - 5)\hat{k} = -8\hat{i} - 7\hat{j} - 2\hat{k}$.

The distance D of a point from a line is given by:

Distance of point from line :

$$D = \frac{|\vec{AQ} \times \vec{d}|}{|\vec{d}|}$$

Cross product

$$\vec{AQ} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & -7 & -2 \\ 3 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}[(-7)(-2) - (-2)(2)] - \hat{j}[(-8)(-2) - (-2)(3)] + \hat{k}[(-8)(2) - (-7)(3)]$$

$$= \hat{i}(14 + 4) - \hat{j}(16 + 6) + \hat{k}(-16 + 21)$$

$$\vec{AQ} \times \vec{d} = 18\hat{i} - 22\hat{j} + 5\hat{k}$$

Magnitudes

$$|\vec{AQ} \times \vec{d}| = \sqrt{18^2 + (-22)^2 + 5^2} = \sqrt{833}$$

$$|\vec{d}| = \sqrt{3^2 + 2^2 + (-2)^2} = \sqrt{17}$$

Distance of point from line

$$D = \frac{|\vec{AQ} \times \vec{d}|}{|\vec{d}|} = \sqrt{\frac{833}{17}} = \sqrt{49} = 7$$

Question 15

Let a line **L** passing through the point **(1, 1, 1)** be perpendicular to both the vectors $2\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$. If $P(a, b, c)$ is the foot of perpendicular from the origin on the line **L**, then the value of $34(a + b + c)$ is :

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Options:

A.

50

B.

80

C.

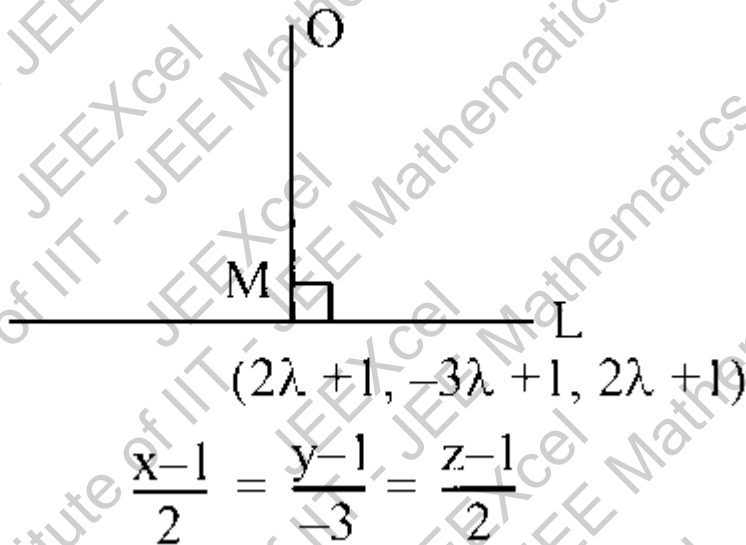
100

D.

Answer: C**Solution:**

Direction vector of line is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$



So equation of line is

$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{2} = \lambda \text{ let}$$

let $(2\lambda + 1, -3\lambda + 1, 2\lambda + 1)$ be the foot of perpendicular from $(0, 0, 0)$ upon given line

$$\begin{aligned} \Rightarrow \vec{OM} \cdot (2\hat{i} - 3\hat{j} + 2\hat{k}) &= 0 \\ \Rightarrow 2(2\lambda + 1) - 3(-3\lambda + 1) + 2(2\lambda + 1) &= 0 \\ \Rightarrow \lambda &= -\frac{1}{17} \end{aligned}$$

$$\text{So } (a, b, c) \equiv \left(\frac{-2}{17} + 1, \frac{3}{17} + 1, \frac{-2}{17} + 1\right)$$

$$\Rightarrow 34(a + b + c) = 100$$

Question 16

If the point of intersection of the lines $\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7}$ and **$\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7}$ lies on xy-plane, then the value of $a + b$ is:**

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Options:

A.

2

B.

5

C.

7

D.

9

Answer: C

Solution:

Let the common point of intersection of the two lines be P .

Since the point lies on the xy -plane, its z -coordinate must be 0.

We write both lines in parametric form.

For the first line,

$$\frac{x+1}{3} = \frac{y+a}{5} = \frac{z+b+1}{7} = \lambda$$

So,

$$x = 3\lambda - 1, \quad y = 5\lambda - a, \quad z = 7\lambda - b - 1$$

For the second line,

$$\frac{x-2}{1} = \frac{y-b}{4} = \frac{z-2a}{7} = \mu$$

So,

$$x = \mu + 2, \quad y = 4\mu + b, \quad z = 7\mu + 2a$$

At the point of intersection, coordinates are equal.

So we get:

$$3\lambda - 1 = \mu + 2$$

$$5\lambda - a = 4\mu + b$$

$$7\lambda - b - 1 = 7\mu + 2a$$

Also, since the point lies on the xy -plane,

$$z = 0$$

Hence from both lines:

$$7\lambda - b - 1 = 0$$

and

$$7\mu + 2a = 0$$

Now solve step by step.

From

$$3\lambda - 1 = \mu + 2$$

we get

$$3\lambda - \mu = 3 \quad \Rightarrow \quad \mu = 3\lambda - 3$$

From $z = 0$ in the first line,

$$b = 7\lambda - 1$$

From $z = 0$ in the second line,

$$2a = -7\mu \quad \Rightarrow \quad a = -\frac{7\mu}{2}$$

Now use

$$5\lambda - a = 4\mu + b$$

Substitute $\mu = 3\lambda - 3$, $b = 7\lambda - 1$:

$$5\lambda - a = 4(3\lambda - 3) + (7\lambda - 1)$$

$$5\lambda - a = 12\lambda - 12 + 7\lambda - 1$$

$$5\lambda - a = 19\lambda - 13$$

$$-a = 14\lambda - 13$$

$$a = 13 - 14\lambda$$

Also,

$$a = -\frac{7\mu}{2} = -\frac{7(3\lambda-3)}{2}$$

$$a = \frac{-21\lambda+21}{2}$$

Equate the two values of a :

$$13 - 14\lambda = \frac{-21\lambda+21}{2}$$

Multiply by 2:

$$26 - 28\lambda = -21\lambda + 21$$

$$5 = 7\lambda$$

$$\lambda = \frac{5}{7}$$

Then

$$\mu = 3\lambda - 3 = 3 \cdot \frac{5}{7} - 3 = \frac{15}{7} - \frac{21}{7} = -\frac{6}{7}$$

Now find a and b .

$$b = 7\lambda - 1 = 5 - 1 = 4$$

$$a = -\frac{7\mu}{2} = -\frac{7}{2} \left(-\frac{6}{7}\right) = 3$$

Therefore,

$$a + b = 3 + 4 = 7$$

So the correct answer is:

7

Option C

Question 17

Let the point A be the foot of perpendicular drawn from the point P $(a, b, 0)$ on the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3}.$$

If the midpoint of the line segment PA is $\left(0, \frac{3}{4}, -\frac{1}{4}\right)$, then the value of $a^2 + b^2 + \alpha^2$ is equal to :

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Options:

A.

1

B.

2

C.

6

D.

9

Answer: A**Solution:**

Let the given line be written in parametric form by taking

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-\alpha}{3} = r.$$

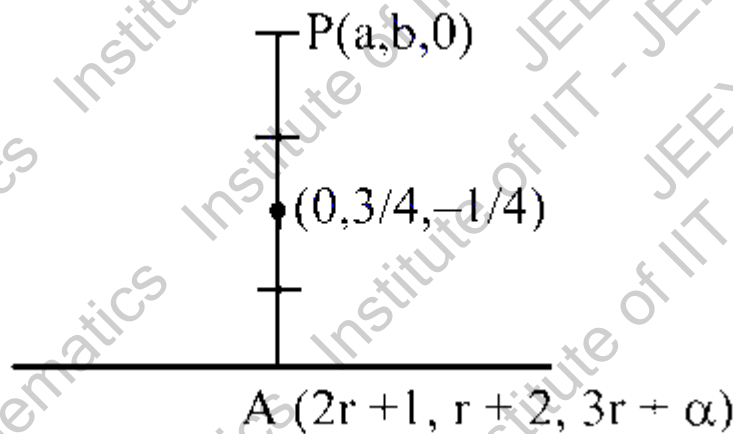
Then a general point A on the line is

$$A(1+2r, 2+r, \alpha+3r).$$

Given $P(a, b, 0)$ and the midpoint of PA is $(0, \frac{3}{4}, -\frac{1}{4})$. Using midpoint formula coordinate-wise:

$$\frac{2r+1+a}{2} = 0 \Rightarrow 2r+a = -1$$

$$\frac{r+2+b}{2} = \frac{3}{4} \Rightarrow r+b = -\frac{1}{2}$$



Similarly, from the z -coordinate of the midpoint:

$$\frac{3r+\alpha}{2} = -\frac{1}{4} \Rightarrow 3r+\alpha = -\frac{1}{2}$$

Now, A is the foot of the perpendicular from P to the line, so $\overrightarrow{PA}$ is perpendicular to the direction vector of the line.

The direction ratios of the line are $(2, 1, 3)$, so we use the dot product condition:

$$2(x_A - a) + 1(y_A - b) + 3(z_A - 0) = 0.$$

Using the midpoint point $(0, \frac{3}{4}, -\frac{1}{4})$, we can write $\overrightarrow{PA} = 2((0, \frac{3}{4}, -\frac{1}{4}) - P)$, so its components are proportional to $(-a, \frac{3}{4} - b, -\frac{1}{4})$. Hence the perpendicularity condition becomes:

$$2 \cdot a + 1 \cdot (b - 3/4) + 3 \cdot \frac{1}{4} = 0 \Rightarrow a + b = 0$$

From $2r + a = -1$, we get $a = -1 - 2r$.

From $r + b = -\frac{1}{2}$, we get $b = -\frac{1}{2} - r$.

Substitute in $a + b = 0$:

$$\Rightarrow 2(-1 - 2r) + (-r - 1/2) = 0$$

$$\Rightarrow -3/2 = 3r = 1 \Rightarrow r = -1/2, a = 0, b = 0, \alpha = 1$$

$$\therefore a^2 + b^2 + \alpha^2 = 1$$

Question 18

The square of the distance of the point $(-2, -8, 6)$ from the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{-1}$ along the line $\frac{x+5}{1} = \frac{y+5}{-1} = \frac{z}{2}$ is equal to:

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Options:

A.

3

B.

6

C.

8

D.

12

Answer: B

Solution:

Let the given point be $P(-2, -8, 6)$. We are looking for the distance of point P from the first line, measured parallel to the second line.

Let the first line be L_1 :

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1}$$

And the second line be L_2 :

$$\frac{x+5}{1} = \frac{y+5}{-1} = \frac{z}{2}$$

The direction ratios of the line L_2 are $(1, -1, 2)$. Since we need to measure the distance parallel to L_2 , we will form the equation of a new line, say L_3 , which passes through the point $P(-2, -8, 6)$ and is parallel to L_2 .

The direction ratios of L_3 will be the same as those of L_2 . Using the point-direction form from 3D geometry, the equation of the line L_3 is:

$$\frac{x-(-2)}{1} = \frac{y-(-8)}{-1} = \frac{z-6}{2}$$

$$\frac{x+2}{1} = \frac{y+8}{-1} = \frac{z-6}{2}$$

Let the line L_3 intersect the line L_1 at a point Q . To find the coordinates of Q , we can take a general point on L_1 .

Let:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1} = \lambda$$

From this, the coordinates of any general point on L_1 can be expressed in terms of λ :

$$x = \lambda + 1$$

$$y = 2\lambda + 1$$

$$z = -\lambda$$

So, we can assume the intersection point is $Q(\lambda + 1, 2\lambda + 1, -\lambda)$.

Since Q also lies on the line L_3 , its coordinates must satisfy the equation of L_3 . Substituting the coordinates of Q into the equation of L_3 , we get:

$$\frac{(\lambda+1)+2}{1} = \frac{(2\lambda+1)+8}{-1} = \frac{-\lambda-6}{2}$$

$$\frac{\lambda+3}{1} = \frac{2\lambda+9}{-1} = \frac{-\lambda-6}{2}$$

Now, let us solve for λ by equating the first two expressions:

$$\lambda + 3 = -(2\lambda + 9)$$

$$\lambda + 3 = -2\lambda - 9$$

$$3\lambda = -12$$

$$\lambda = -4$$

We can verify this value by substituting $\lambda = -4$ into the third expression:

$$\frac{-(-4)-6}{2} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

This matches the first part $(-4) + 3 = -1$. Thus, $\lambda = -4$ is correct.

Now, substitute $\lambda = -4$ back into our assumption to find the exact coordinates of Q :

$$x = (-4) + 1 = -3$$

$$y = 2(-4) + 1 = -7$$

$$z = -(-4) = 4$$

So, the point of intersection is $Q(-3, -7, 4)$.

We are asked to find the square of the distance between point P and the line L_1 measured along L_3 . This naturally corresponds to the square of the distance between points $P(-2, -8, 6)$ and $Q(-3, -7, 4)$.

Using the distance formula $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$:

$$PQ^2 = (-3 - (-2))^2 + (-7 - (-8))^2 + (4 - 6)^2$$

$$PQ^2 = (-3 + 2)^2 + (-7 + 8)^2 + (-2)^2$$

$$PQ^2 = (-1)^2 + (1)^2 + (-2)^2$$

$$PQ^2 = 1 + 1 + 4$$

$$PQ^2 = 6$$

The square of the required distance is 6.

Option B

Question 19

A line with direction ratios $1, -1, 2$ intersects the lines $\frac{x}{2} = \frac{y}{3} = \frac{z+1}{3}$ and $\frac{x+1}{-1} = \frac{y-2}{1} = \frac{z}{4}$ at the points P and Q , respectively. If the length of the line segment PQ is α , then $225\alpha^2$ is equal to:

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Options:

A.

1024

B.

1014

C.

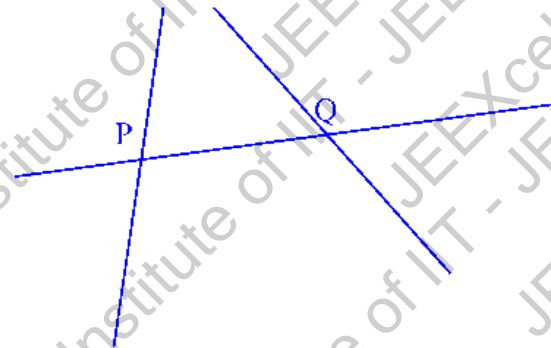
1104

D.

1204

Answer: B

Solution:



$P(2\lambda, 3\lambda, 3\lambda - 1)$ and $Q(-\mu - 1, \mu + 2, 4\mu)$

D.R of PQ $2\lambda + \mu + 1, 3\lambda - \mu - 2, 3\lambda - 4\mu - 1,$

$$\frac{2\lambda + \mu + 1}{1} = \frac{3\lambda - \mu - 2}{-1} = \frac{3\lambda - 4\mu - 1}{2}$$

Solving $\lambda = \frac{1}{5}$ and $\mu = -\frac{8}{15}$

$P\left(\frac{2}{5}, \frac{3}{5}, -\frac{2}{5}\right)$ & $Q\left(-\frac{7}{15}, \frac{22}{15}, -\frac{32}{15}\right)$

$$PQ^2 = \left(\frac{13}{15}\right)^2 + \left(\frac{13}{15}\right)^2 + \left(\frac{26}{15}\right)^2$$

$$\alpha^2 = \frac{1014}{225} \therefore 225\alpha^2 = 1014$$

Question 20

If $(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha-1}{2})$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the possible value(s) of α is (are)

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Options:

A.

Only 3

B.

Only 3 and -1

C.

Only 3, $\frac{1}{4}$ and -1

D.

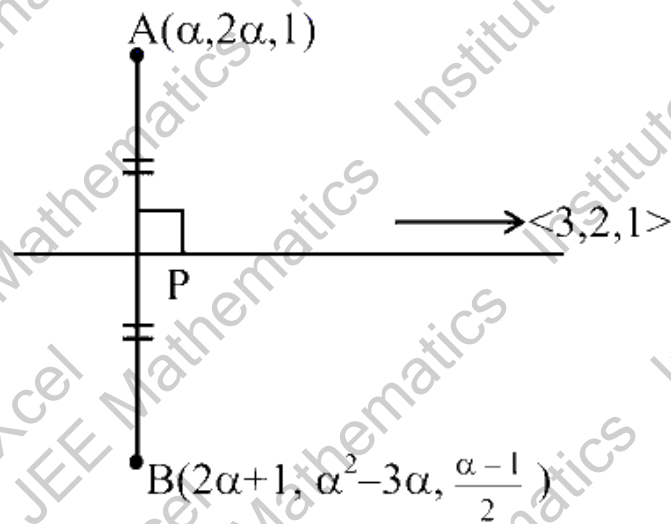
Only 3 and $\frac{1}{4}$

Answer: A

Solution:

$\therefore$ P is mid point of AB and lies on the given line

$$\Rightarrow P\left(\frac{3\alpha+1}{2}, \frac{\alpha^2-\alpha}{2}, \frac{\alpha+1}{4}\right)$$



Now put coordinates of P in the line

$$\begin{aligned} \Rightarrow \frac{\frac{3\alpha+1}{2} - 2}{3} &= \frac{\frac{\alpha^2-\alpha}{2} - 1}{2} = \frac{\alpha+1}{4} \\ \Rightarrow \frac{\alpha-1}{2} &= \frac{\alpha+1}{4} = \frac{\alpha^2-\alpha-2}{4} \\ \Rightarrow \alpha &= 3 \end{aligned}$$

Now for $\alpha = 3$, clearly AB is perpendicular to $3\hat{i} + 2\hat{j} + \hat{k}$

for $\alpha = 3$ point B is image of point A

Question 21

The shortest distance between the lines

$$\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k} \right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$$

$$\text{and } \vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k} \right) + \mu(\hat{j} - \hat{k}), \lambda, \mu \in \mathbb{R}, \text{ is:}$$

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Options:

A.

$$\sqrt{5}$$

B.

$$3$$

C.

$$2\sqrt{3}$$

D.

$$\sqrt{15}$$

Answer: B

Solution:

For two skew lines,

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad \text{and} \quad \vec{r} = \vec{a}_2 + \mu \vec{b}_2,$$

the shortest distance is

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Now compare with the given lines.

First line:

$$\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k} \right) + \lambda(2\hat{i} - 5\hat{j} + 6\hat{k})$$

So,

$$\vec{a}_1 = \left(\frac{1}{3}, 2, \frac{8}{3} \right), \quad \vec{b}_1 = (2, -5, 6)$$

Second line:

$$\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k} \right) + \mu(\hat{j} - \hat{k})$$

So,

$$\vec{a}_2 = \left(-\frac{2}{3}, 0, -\frac{1}{3} \right), \quad \vec{b}_2 = (0, 1, -1)$$

Now find

$$\vec{a}_2 - \vec{a}_1 = \left(-\frac{2}{3} - \frac{1}{3}, 0 - 2, -\frac{1}{3} - \frac{8}{3} \right) = (-1, -2, -3)$$

Next, calculate $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}[(-5)(-1) - 6(1)] - \hat{j}[2(-1) - 6(0)] + \hat{k}[2(1) - (-5)(0)]$$

$$= \hat{i}(5 - 6) - \hat{j}(-2) + \hat{k}(2) = -\hat{i} + 2\hat{j} + 2\hat{k}$$

So,

$$\vec{b}_1 \times \vec{b}_2 = (-1, 2, 2)$$

Its magnitude is

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = 3$$

Now find the scalar triple product:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-1, -2, -3) \cdot (-1, 2, 2)$$

$$= (-1)(-1) + (-2)(2) + (-3)(2) = 1 - 4 - 6 = -9$$

Therefore,

$$d = \frac{|-9|}{3} = 3$$

Hence, the shortest distance is

$$\boxed{3}$$

So, the correct option is **Option B**.

Question22

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(a\hat{i} - \hat{j})$, $a \neq 0$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + a\hat{k})$
from the origin is :

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Options:

A.

5

B.

10

C.

17

D.

26

Answer: C**Solution:**

For the first line, the equation is:

$$\vec{r}_1 = (\hat{i} + \hat{j} - \hat{k}) + \lambda(a\hat{i} - \hat{j})$$

We can group the components along the $\hat{i}$, $\hat{j}$, and $\hat{k}$ directions:

$$\vec{r}_1 = (1 + a\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k}$$

For the second line, the equation is:

$$\vec{r}_2 = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + a\hat{k})$$

Grouping the components similarly:

$$\vec{r}_2 = (4 + 2\mu)\hat{i} + 0\hat{j} + (a\mu - 1)\hat{k}$$

If the two lines intersect, there must be a common point. This means that for some value of λ and μ , their position vectors will be exactly the same. We can find this by equating the corresponding components of $\vec{r}_1$ and $\vec{r}_2$:

1. Equating the $\hat{j}$ components:

$$1 - \lambda = 0$$

$$\lambda = 1$$

2. Equating the $\hat{k}$ components:

$$-1 = a\mu - 1$$

$$a\mu = 0$$

Since the question specifies that $a \neq 0$, it must be true that $\mu = 0$.

3. Equating the $\hat{i}$ components:

$$1 + a\lambda = 4 + 2\mu$$

Now, substitute the values of $\lambda = 1$ and $\mu = 0$ into this equation:

$$1 + a(1) = 4 + 2(0)$$

$$1 + a = 4$$

$$a = 3$$

Now that we have $\mu = 0$ (and $\lambda = 1$), we can substitute either of these back into the respective line equations to find the position vector of the point of intersection. Let us use the second line's equation by substituting $\mu = 0$:

$$\vec{r} = (4 + 2(0))\hat{i} + (3(0) - 1)\hat{k}$$

$$\vec{r} = 4\hat{i} - \hat{k}$$

So, the coordinates of the point of intersection are $(4, 0, -1)$.

Next, we calculate the distance d of this point of intersection from the origin $(0, 0, 0)$ using the distance formula:

$$d = \sqrt{x^2 + y^2 + z^2}$$

$$d = \sqrt{(4)^2 + (0)^2 + (-1)^2}$$

$$d = \sqrt{16 + 0 + 1}$$

$$d = \sqrt{17}$$

Looking at the given options, the problem asks for the square of the distance from the origin.

The square of the distance is:

$$d^2 = (\sqrt{17})^2 = 17$$

Therefore, the correct choice is Option C.

Question 23

The square of the distance of the point $P(5, 6, 7)$ from the line

$$\frac{x-2}{2} = \frac{y-5}{3} = \frac{z-2}{4} \text{ is equal to:}$$

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Options:

A.

3

B.

5

C.

6

D.

8

Answer: C**Solution:**

Write the given line in vector form.

From

$$\frac{x-2}{2} = \frac{y-5}{3} = \frac{z-2}{4},$$

a point on the line is

$$A(2, 5, 2)$$

and its direction vector is

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}.$$

Given point is

$$P(5, 6, 7).$$

Now,

$$\overrightarrow{AP} = (5-2)\hat{i} + (6-5)\hat{j} + (7-2)\hat{k} = 3\hat{i} + \hat{j} + 5\hat{k}.$$

The distance d of point P from the line is given by

$$d = \frac{|\overrightarrow{AP} \times \vec{b}|}{|\vec{b}|}.$$

Since we need the square of the distance,

$$d^2 = \frac{|\overrightarrow{AP} \times \vec{b}|^2}{|\vec{b}|^2}.$$

Now calculate the cross product:

$$\overrightarrow{AP} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 5 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(1 \cdot 4 - 5 \cdot 3) - \hat{j}(3 \cdot 4 - 5 \cdot 2) + \hat{k}(3 \cdot 3 - 1 \cdot 2)$$

$$= \hat{i}(4 - 15) - \hat{j}(12 - 10) + \hat{k}(9 - 2)$$

$$= -11\hat{i} - 2\hat{j} + 7\hat{k}.$$

So,

$$|\vec{AP} \times \vec{b}|^2 = (-11)^2 + (-2)^2 + 7^2 = 121 + 4 + 49 = 174.$$

Also,

$$|\vec{b}|^2 = 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29.$$

Therefore,

$$d^2 = \frac{174}{29} = 6.$$

So, the square of the distance is

$$\boxed{6}$$

Hence, the correct option is **Option C**.

Question 24

If the distance of the point $(a, 2, 5)$ from the image of the point $(1, 2, 7)$ in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ is 4, then the sum of all possible values of a is equal to :

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Options:

A.

11

B.

9

C.

6

D.

Answer: C

Solution:

Let the point $P(1, 2, 7)$.

We need the image of P in the line

$$\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$$

This means the line passes through the point

$$A(0, 1, 2)$$

and has direction ratios

$$\vec{d} = (1, 1, 2).$$

If P' is the image of P in this line, then the given line is the perpendicular bisector of the segment joining P and P' in the sense of reflection about a line in 3D. So first we find the foot of perpendicular from P to the line.

Let the foot be M .

A general point on the line is

$$M(t) = (t, 1+t, 2+2t).$$

Now,

$$\overrightarrow{PM} = (t-1, t-1, 2t-5).$$

Since PM is perpendicular to the line, we must have

$$\overrightarrow{PM} \cdot \vec{d} = 0$$

So,

$$(t-1)(1) + (t-1)(1) + (2t-5)(2) = 0$$

$$t-1+t-1+4t-10=0$$

$$6t-12=0$$

$$t=2$$

Hence,

$$M = (2, 3, 6).$$

Since M is the midpoint of $P(1, 2, 7)$ and its image P' , we get

$$P' = 2M - P$$

$$P' = (4, 6, 12) - (1, 2, 7) = (3, 4, 5).$$

So the image of $(1, 2, 7)$ in the given line is

$(3, 4, 5)$.

Now the distance of $(a, 2, 5)$ from this image is 4:

$$\sqrt{(a-3)^2 + (2-4)^2 + (5-5)^2} = 4$$

$$\sqrt{(a-3)^2 + 4} = 4$$

Squaring,

$$(a-3)^2 + 4 = 16$$

$$(a-3)^2 = 12$$

$$a-3 = \pm 2\sqrt{3}$$

So,

$$a = 3 \pm 2\sqrt{3}.$$

Therefore, the sum of all possible values of a is

$$(3 + 2\sqrt{3}) + (3 - 2\sqrt{3}) = 6.$$

Hence, the correct option is

6

So, option C is correct.

Question 25

Let a triangle PQR be such that P and Q lie on the line

$\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ and are at a distance of 6 units from $R(1, 2, 3)$. If (α, β, γ) is the centroid of ΔPQR , then $\alpha + \beta + \gamma$ is equal to :

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Options:

A.

4

B.

5

C.

6

D.

8

Answer: C**Solution:**

Write the given line in parametric form.

$$\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2} = t$$

So any point on the line is

$$P(t) = (-3 + 8t, 4 + 2t, -1 + 2t)$$

Since both P and Q lie on this line and are at a distance 6 from $R(1, 2, 3)$, we find the points on the line whose distance from R is 6.

Distance condition:

$$\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = 6$$

Substitute $x = -3 + 8t$, $y = 4 + 2t$, $z = -1 + 2t$:

$$(-3 + 8t - 1)^2 + (4 + 2t - 2)^2 + (-1 + 2t - 3)^2 = 36$$

$$(8t - 4)^2 + (2t + 2)^2 + (2t - 4)^2 = 36$$

Now expand:

$$64t^2 - 64t + 16 + 4t^2 + 8t + 4 + 4t^2 - 16t + 16 = 36$$

$$72t^2 - 72t + 36 = 36$$

$$72t^2 - 72t = 0$$

$$72t(t - 1) = 0$$

So,

$$t = 0 \quad \text{or} \quad t = 1$$

Hence the two points are:

For $t = 0$,

$$P = (-3, 4, -1)$$

For $t = 1$,

$$Q = (5, 6, 1)$$

Now centroid of triangle PQR is

$$(\alpha, \beta, \gamma) = \left(\frac{x_P + x_Q + x_R}{3}, \frac{y_P + y_Q + y_R}{3}, \frac{z_P + z_Q + z_R}{3} \right)$$

Substitute $P = (-3, 4, -1)$, $Q = (5, 6, 1)$, $R = (1, 2, 3)$:

$$\alpha = \frac{-3+5+1}{3} = \frac{3}{3} = 1$$

$$\beta = \frac{4+6+2}{3} = \frac{12}{3} = 4$$

$$\gamma = \frac{-1+1+3}{3} = \frac{3}{3} = 1$$

Therefore,

$$\alpha + \beta + \gamma = 1 + 4 + 1 = 6$$

So the correct answer is:

6

Hence, Option C is correct.

Question 26

Let a line L be perpendicular to both the lines

$$L_1 : \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ and } L_2 : \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}.$$

If θ is the acute angle between the lines L and $L_3 : \frac{x-\frac{8}{7}}{2} = \frac{y-\frac{4}{7}}{1} = \frac{z}{2}$, then $\tan \theta$ is equal to:

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Options:

A.

$$\frac{3}{2}\sqrt{2}$$

B.

$$\frac{5}{2}\sqrt{2}$$

C.

$$\frac{5}{3}\sqrt{2}$$

D.

$$\frac{4}{3}\sqrt{2}$$

Answer: B

Solution:

For a line L to be perpendicular to both lines L_1 and L_2 , its direction ratios must be perpendicular to the direction ratios of both L_1 and L_2 .

So first let us find the direction vectors.

For L_1 :

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$$

Direction ratios of L_1 are

$$\vec{d}_1 = (3, 5, 7)$$

For L_2 :

$$\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$$

Direction ratios of L_2 are

$$\vec{d}_2 = (1, 4, 7)$$

Since line L is perpendicular to both, its direction vector will be parallel to

$$\vec{d} = \vec{d}_1 \times \vec{d}_2$$

Now,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$= \hat{i}(35 - 28) - \hat{j}(21 - 7) + \hat{k}(12 - 5)$$

$$= 7\hat{i} - 14\hat{j} + 7\hat{k}$$

$$= 7(1, -2, 1)$$

So direction ratios of line L can be taken as

$$(1, -2, 1)$$

Now for L_3 :

$$\frac{x-\frac{8}{7}}{2} = \frac{y-\frac{4}{7}}{1} = \frac{z}{2}$$

Direction ratios of L_3 are

$$(2, 1, 2)$$

Let θ be the acute angle between L and L_3 .

Using the formula

$$\tan \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|}$$

where

$$\vec{a} = (1, -2, 1), \quad \vec{b} = (2, 1, 2)$$

First find dot product:

$$\begin{aligned} \vec{a} \cdot \vec{b} &= 1 \cdot 2 + (-2) \cdot 1 + 1 \cdot 2 \\ &= 2 - 2 + 2 = 2 \end{aligned}$$

Now find cross product:

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & 2 \end{vmatrix} \\ &= \hat{i}((-2)(2) - 1 \cdot 1) - \hat{j}(1 \cdot 2 - 1 \cdot 2) + \hat{k}(1 \cdot 1 - (-2) \cdot 2) \\ &= \hat{i}(-4 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 4) \\ &= -5\hat{i} + 0\hat{j} + 5\hat{k} \end{aligned}$$

So,

$$|\vec{a} \times \vec{b}| = \sqrt{(-5)^2 + 0^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

Hence,

$$\tan \theta = \frac{5\sqrt{2}}{2}$$

Therefore,

$$\tan \theta = \frac{5}{2}\sqrt{2}$$

So the correct option is:

Option B

Question 27

Let the image of the point $P(1, 6, a)$ in the line

$L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-a+1}{b}, b > 0$, be $(\frac{a}{3}, 0, a+c)$. If $S(\alpha, \beta, \gamma), \alpha > 0$, is the point on L such that the distance of S from the foot of perpendicular from the point P on L is $2\sqrt{14}$, then $\alpha + \beta + \gamma$ is equal to:

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Options:

A.

19

B.

20

C.

21

D.

22

Answer: C

Solution:

Let the line be written in parametric form as

$$L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-a+1}{b} = t$$

So a general point on L is

$$Q(t) = (t, 1+2t, a-1+bt).$$

We are given:

- Point $P = (1, 6, a)$
- Its image in the line L is $(\frac{a}{3}, 0, a+c)$

For reflection in a line in 3D, the line is the perpendicular bisector of the segment joining the point and its image.

So if image of P is $P' = \left(\frac{a}{3}, 0, a+c\right)$, then:

1. Midpoint of PP' lies on L
2. The line L is perpendicular to $\overrightarrow{PP'}$

1. Midpoint of P and P'

Midpoint M is

$$M = \left(\frac{1+\frac{a}{3}}{2}, \frac{6+0}{2}, \frac{a+(a+c)}{2}\right) = \left(\frac{a+3}{6}, 3, \frac{2a+c}{2}\right)$$

Since M lies on L , for some parameter t ,

$$(t, 1+2t, a-1+bt) = \left(\frac{a+3}{6}, 3, \frac{2a+c}{2}\right)$$

From the y -coordinate,

$$1+2t = 3 \implies 2t = 2 \implies t = 1$$

Then from the x -coordinate,

$$t = \frac{a+3}{6}$$

So

$$1 = \frac{a+3}{6} \implies a+3 = 6 \implies a = 3$$

Now from the z -coordinate,

$$a-1+b = \frac{2a+c}{2}$$

Substitute $a = 3$:

$$3-1+b = \frac{6+c}{2}$$

$$2+b = \frac{6+c}{2}$$

$$4+2b = 6+c$$

$$c = 2b-2$$

2. Perpendicularity condition

Direction vector of line L is

$$\vec{d} = (1, 2, b)$$

Now

$$\overrightarrow{PP'} = \left(\frac{a}{3} - 1, 0 - 6, a + c - a\right) = \left(\frac{a}{3} - 1, -6, c\right)$$

Since $a = 3$,

$$\overrightarrow{PP'} = (0, -6, c)$$

This must be perpendicular to $\vec{d} = (1, 2, b)$, so

$$(0, -6, c) \cdot (1, 2, b) = 0$$

$$0 - 12 + bc = 0$$

$$bc = 12$$

But we already have

$$c = 2b - 2$$

So

$$b(2b - 2) = 12$$

$$2b^2 - 2b - 12 = 0$$

$$b^2 - b - 6 = 0$$

$$(b - 3)(b + 2) = 0$$

Given $b > 0$, so

$$b = 3$$

Then

$$c = 2b - 2 = 6 - 2 = 4$$

Thus the line is

$$L : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

and the image point is

$$P' = (1, 0, 7).$$

3. Foot of perpendicular from P to L

Let the foot be F . Since midpoint of P and its image lies on the mirror line, F is exactly the midpoint of P and P' :

$$F = \left(\frac{1+1}{2}, \frac{6+0}{2}, \frac{3+7}{2} \right) = (1, 3, 5)$$

Check that this lies on L :

For $t = 1$,

$$Q(1) = (1, 3, 5)$$

Correct.

4. Point $S(\alpha, \beta, \gamma)$ on L such that $SF = 2\sqrt{14}$

A general point on L is

$$S(t) = (t, 1 + 2t, 2 + 3t)$$

Since $F = (1, 3, 5)$ corresponds to $t = 1$, we get

$$\overrightarrow{FS} = (t - 1, 2t - 2, 3t - 3) = (t - 1)(1, 2, 3)$$

Hence

$$\begin{aligned} SF &= \sqrt{(t - 1)^2 + (2t - 2)^2 + (3t - 3)^2} \\ &= |t - 1|\sqrt{1 + 4 + 9} = |t - 1|\sqrt{14} \end{aligned}$$

Given

$$SF = 2\sqrt{14}$$

So

$$|t - 1| = 2$$

Thus

$$t = 3 \quad \text{or} \quad t = -1$$

Since $\alpha > 0$ and $\alpha = x = t$, we need

$$t > 0 \implies t = 3$$

Therefore

$$S = (3, 7, 11)$$

So

$$\alpha + \beta + \gamma = 3 + 7 + 11 = 21$$

Hence the correct answer is

21

So, Option C is correct.

Question 28

The shortest distance between the lines $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$ and $\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5}$ is:

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Options:

A.

$$\frac{5\sqrt{6}}{6}$$

B.

$$2\sqrt{5}$$

C.

$$3\sqrt{5}$$

D.

$$4\sqrt{5}$$

Answer: C

Solution:

For the two lines

$$\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$$

and

$$\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5},$$

we use the NCERT formula for the shortest distance between two skew lines:

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

where:

- $\vec{a}_1$ is a point on the first line
- $\vec{a}_2$ is a point on the second line
- $\vec{b}_1, \vec{b}_2$ are direction vectors of the two lines

Step 1: Identify points and direction vectors

From the first line,

$$\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$$

a point on it is

$$A(4, 3, 2)$$

and its direction vector is

$$\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$$

From the second line,

$$\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5}$$

a point on it is

$$B(-2, 6, 5)$$

and its direction vector is

$$\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

Step 2: Find $\vec{AB}$

$$\vec{AB} = B - A = (-2 - 4, 6 - 3, 5 - 2) = (-6, 3, 3)$$

So,

$$\vec{AB} = -6\hat{i} + 3\hat{j} + 3\hat{k}$$

Step 3: Find $\vec{b}_1 \times \vec{b}_2$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

Expanding,

$$= \hat{i}(2(-5) - (-3)(4)) - \hat{j}(1(-5) - (-3)(2)) + \hat{k}(1 \cdot 4 - 2 \cdot 2)$$

$$= \hat{i}(-10 + 12) - \hat{j}(-5 + 6) + \hat{k}(4 - 4)$$

$$= 2\hat{i} - \hat{j} + 0\hat{k}$$

So,

$$\vec{b}_1 \times \vec{b}_2 = (2, -1, 0)$$

Its magnitude is

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{2^2 + (-1)^2 + 0^2} = \sqrt{5}$$

Step 4: Find scalar triple product

$$\vec{AB} \cdot (\vec{b}_1 \times \vec{b}_2) = (-6, 3, 3) \cdot (2, -1, 0)$$

$$= (-6)(2) + (3)(-1) + (3)(0)$$

$$= -12 - 3 = -15$$

So,

$$|\vec{AB} \cdot (\vec{b}_1 \times \vec{b}_2)| = 15$$

Step 5: Find shortest distance

$$d = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

Hence, the shortest distance is

$$3\sqrt{5}$$

So, the correct option is:

Option C

Question 29

Let the foot of perpendicular from the point $(\lambda, 2, 3)$ on the line $\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$ be the point $(1, \mu, 2)$. Then the distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-\lambda}{2} = \frac{y-\mu}{3} = \frac{z+5}{6}$ is equal to :

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Options:

A.

$$\frac{12}{7}$$

B.

$$\frac{\sqrt{145}}{7}$$

C.

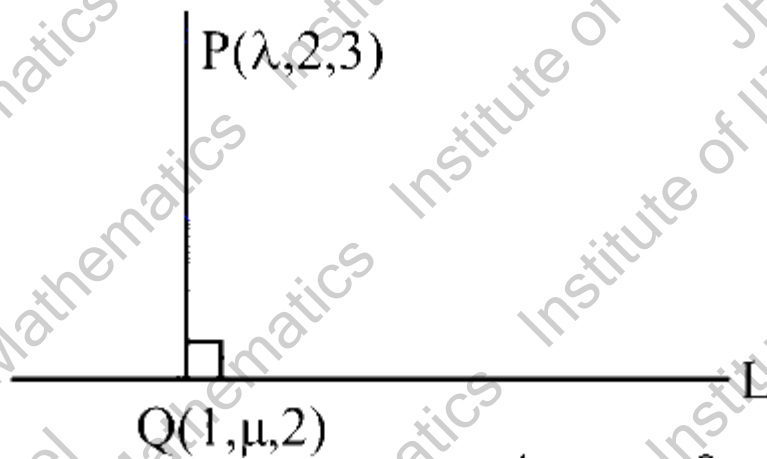
$$\frac{\sqrt{146}}{7}$$

D.

$$\frac{\sqrt{143}}{7}$$

Answer: C

Solution:



$$\frac{x-4}{1} = \frac{y-9}{2} = \frac{z-5}{1}$$

$Q(1, \mu, 2)$ satisfies the line L

$$\therefore \frac{1-4}{1} = \frac{\mu-9}{2} = \frac{2-5}{1}$$

$$\Rightarrow -3 = \frac{\mu-9}{2} = -3 \Rightarrow \mu = 3$$

D.R. of PQ : $\lambda - 1, 2 - \mu, 1$

Since $PQ \perp L \Rightarrow (\lambda - 1).1 + 2(2 - \mu) + 1.1 = 0$

$$\Rightarrow \lambda - 2\mu + 4 = 0 \therefore \lambda = 2$$

Now lines : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ &

$$\frac{x-2}{2} = \frac{y-3}{3} = \frac{z+5}{6}$$

$$\text{i.e. } \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + t_1(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\& \vec{r} = (2\hat{i} + 3\hat{j} - 5\hat{k}) + t_2(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Distance

$$\begin{aligned} &= \frac{|((\hat{i} + 2\hat{j} - 4\hat{k}) - (2\hat{i} + 3\hat{j} - 5\hat{k})) \times (2\hat{i} + 3\hat{j} + 6\hat{k})|}{|2\hat{i} + 3\hat{j} + 6\hat{k}|} \\ &= \frac{|-9\hat{i} + 8\hat{j} - \hat{k}|}{7} = \frac{\sqrt{146}}{7} \end{aligned}$$

Question 30

Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}$, $\alpha \in \mathbf{R}$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point A(1, 1, -1) on L_2 , then the value of $26\alpha(\text{PB})^2$ is

_____.

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Answer: 216

Solution:

To find the value of $26\alpha(\text{PB})^2$, we proceed as follows:

Intersection Point B of L_1 and L_2

We are given the equations of the lines:

$$L_1 : \frac{x-1}{3} = \frac{y+1}{-1} = \frac{z+1}{0}$$

$$L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}$$

For L_1 , $z+1=0$, which implies $z=-1$.

For L_2 , since $y=0$, the direction ratios can be matched using the parameter μ :

$$\begin{aligned} x &= 3\lambda + 1, & y &= -\lambda + 1, & z &= -1, \\ x &= 2\mu + 2, & y &= 0, & z &= \alpha\mu - 4. \end{aligned}$$

Setting the coordinates equal for intersection:

$$\begin{aligned} 3\lambda + 1 &= 2\mu + 2, \\ -\lambda + 1 &= 0, \\ -1 &= \alpha\mu - 4. \end{aligned}$$

From $-\lambda + 1 = 0$, we find $\lambda = 1$.

Substituting $\lambda = 1$ into $3\lambda + 1 = 2\mu + 2$ gives:

$$3(1) + 1 = 2\mu + 2 \implies \mu = 1.$$

Also, substituting $\mu = 1$ into $-1 = \alpha\mu - 4$ gives:

$$-1 = \alpha(1) - 4 \implies \alpha = 3.$$

So, the intersection point B is:

$$B(4, 0, -1).$$

Foot of Perpendicular from A to L_2

The point P on L_2 is given by:

$$P(2\delta + 2, 0, 3\delta - 4).$$

For vector $\overrightarrow{AP} = (2\delta + 1, -1, 3\delta - 3)$, since AP is perpendicular to L_2 , the dot product should be zero:

$$(2\delta + 1) \cdot 2 + (-1) \cdot 0 + (3\delta - 3) \cdot 3 = 0.$$

Simplifying gives:

$$4\delta + 2 + 9\delta - 9 = 0 \implies 13\delta = 7 \implies \delta = \frac{7}{13}.$$

So, the coordinates of point P are:

$$P\left(\frac{40}{13}, 0, \frac{-31}{13}\right).$$

Distance PB and Calculation

The vector $\overrightarrow{PB}$ is:

$$\overrightarrow{PB} = \left(4 - \frac{40}{13}, 0 - 0, -1 - \left(\frac{-31}{13}\right)\right).$$

Calculating the components:

$$\overrightarrow{PB} = \left(\frac{12}{13}, 0, \frac{18}{13}\right).$$

The square of the distance $(PB)^2$ is:

$$\left(\frac{12}{13}\right)^2 + (0)^2 + \left(\frac{18}{13}\right)^2 = \frac{144}{169} + \frac{324}{169} = \frac{468}{169}.$$

Thus, $26\alpha(PB)^2$ is:

$$26 \times 3 \times \frac{468}{169} = 216.$$

So, the final value is 216.

Question 31

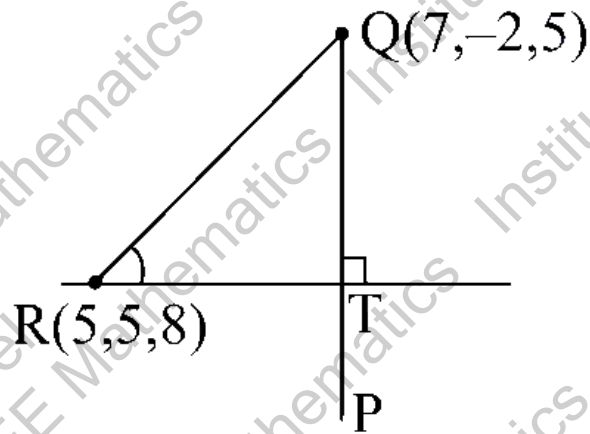
Let P be the image of the point $Q(7, -2, 5)$ in the line

$L: \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and $R(5, p, q)$ be a point on L . Then the square of the area of $\triangle PQR$ is _____.

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Answer: 957

Solution:



$$\text{Let } R(2\lambda + 1, 3\lambda - 1, 4\lambda)$$

$$2\lambda + 1 = 5$$

$$\lambda = 2$$

$$R(5, 5, 8)$$

$$\text{let } T(2\lambda + 1, 3\lambda - 1, 4\lambda)$$

$$\vec{QT} = (2\lambda - 6)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda - 5)\hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{QT} \cdot \vec{b} = 0$$

$$4\lambda - 12 + 9\lambda + 3 + 16\lambda - 20 = 0$$

$$\lambda = 1$$

$$T(3, 2, 4)$$

$$QT = \sqrt{33} \quad RT = \sqrt{29}$$

$$(\text{area of } \triangle PQR)^2 = \left(\frac{1}{2} \sqrt{29} \cdot 2\sqrt{33} \right)^2$$

$$= 957$$

Question 32

Let the area of the triangle formed by the lines

$$x + 2 = y - 1 = z, \quad \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} \quad \text{and} \quad \frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1} \quad \text{be } A.$$

Then A^2 is equal to _____.

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Answer: 56

Solution:

$$L_1 : x + 2 = y - 1 = z = \ell$$

$$L_2 : \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = m$$

$$L_3 : \frac{x}{-3} = \frac{y-3}{5} = \frac{z-2}{1} = n$$

Point of intersection of L_1 and L_2

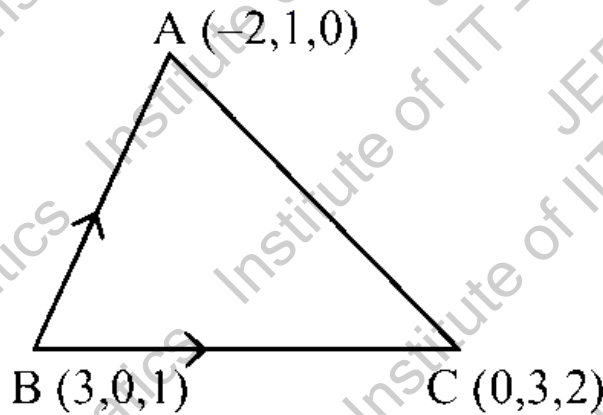
$$\left. \begin{aligned} \ell - 2 &= 5m + 3 \\ \ell + 1 &= -m \\ \ell &= m + 1 \end{aligned} \right\} \ell = 0, m = -1 \quad A(-2, 1, 0)$$

Point of intersection of L_2 and L_3

$$\left. \begin{aligned} 5m + 3 &= -3n \\ -m &= 3n + 3 \\ m + 1 &= n + 2 \end{aligned} \right\} m = 0, n = -1, B(3, 0, 1)$$

Point of intersection L_3 and L_4

$$\left. \begin{aligned} -3n &= \ell - 2 \\ 3n + 3 &= \ell + 1 \\ n + 2 &= \ell \end{aligned} \right\} \ell = 2, n = 0, C(0, 3, 2)$$



$$\text{Ar}(\triangle ABC) = \left| \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{vmatrix} \right|$$

$$A = \frac{1}{2} |\hat{i}(4) - \hat{j}(-8) + \hat{k}(-12)|$$

$$A = \frac{1}{2} \sqrt{16 + 64 + 144} = \sqrt{56}$$

$$A^2 = 56$$

Question33

Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

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Options:

A. $(\frac{14}{3}, -3, \frac{22}{3})$

B. $(2, 3, \frac{1}{3})$

C. $(\frac{8}{3}, -1, \frac{1}{3})$

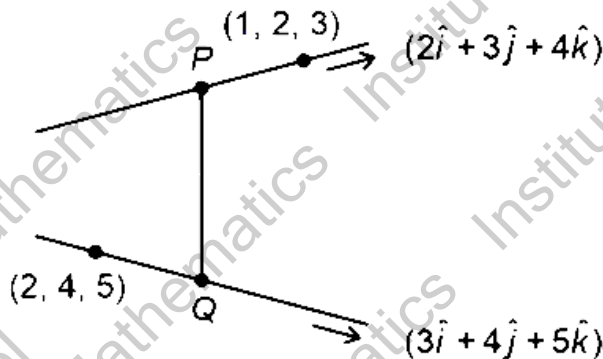
D. $(-\frac{5}{3}, -7, 1)$

Answer: A

Solution:

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$



$$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$$

$$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$$

$$\text{Dir's of } PQ < 2\lambda - 3\mu - 1, 3\lambda - 4\mu - 2,$$

$$4\lambda - 5\mu - 2 >$$

$$PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\Rightarrow \frac{2\lambda - 3\mu - 1}{-1} = \frac{3\lambda - 4\mu - 2}{2} = \frac{4\lambda - 5\mu - 2}{-1}$$

$$\Rightarrow \lambda = \frac{1}{3}\mu = \frac{-1}{6}$$

$$\Rightarrow P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$$

Dr's $PQ(1, -2, 1)$

$\therefore$ Line

$$\frac{y-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{y-\frac{13}{3}}{1}$$

Question34

The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point $P(2, -10, 1)$, is :

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Options:

A. 6

B. $4\sqrt{3}$

C. $3\sqrt{5}$

D. $5\sqrt{2}$

Answer: C

Solution:

To find the perpendicular distance from the point $P(2, -10, 1)$ to the line given by

$$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2},$$

follow these steps:

Parametrize the Line:

From the given symmetric equations, set the common parameter as t :

$$x = 1 + 2t$$

$$y = -2 - t$$

$$z = -3 + 2t$$

This shows that:

A point on the line is $A(1, -2, -3)$ (when $t = 0$).

The direction vector is $\vec{d} = \langle 2, -1, 2 \rangle$.

Determine the Vector from Point A to P:

Calculate

$$\vec{AP} = P - A = \langle 2 - 1, -10 - (-2), 1 - (-3) \rangle = \langle 1, -8, 4 \rangle.$$

Compute the Cross Product:

The formula for the perpendicular distance from a point to a line in 3D is:

$$d = \frac{\|\vec{AP} \times \vec{d}\|}{\|\vec{d}\|}.$$

First, find the cross product $\vec{AP} \times \vec{d}$:

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -8 & 4 \\ 2 & -1 & 2 \end{vmatrix} = \langle (-8)(2) - (4)(-1), (4)(2) - (1)(2), (1)(-1) - (-8)(2) \rangle.$$

Evaluate each component:

$$\text{First component: } (-8 \times 2) - (4 \times -1) = -16 + 4 = -12.$$

$$\text{Second component: } (4 \times 2) - (1 \times 2) = 8 - 2 = 6.$$

$$\text{Third component: } (1 \times -1) - (-8 \times 2) = -1 + 16 = 15.$$

So,

$$\vec{AP} \times \vec{d} = \langle -12, 6, 15 \rangle.$$

Calculate the Magnitudes:

For the cross product:

$$\|\vec{AP} \times \vec{d}\| = \sqrt{(-12)^2 + 6^2 + 15^2} = \sqrt{144 + 36 + 225} = \sqrt{405} = 9\sqrt{5}.$$

For the direction vector:

$$\|\vec{d}\| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

Compute the Distance:

Substitute the magnitudes into the distance formula:

$$d = \frac{9\sqrt{5}}{3} = 3\sqrt{5}.$$

Thus, the perpendicular distance from the point $P(2, -10, 1)$ to the line is $3\sqrt{5}$.

Comparing with the options given, the correct answer is:

Option C: $3\sqrt{5}$.

Question 35

Let a line pass through two distinct points $P(-2, -1, 3)$ and Q , and be parallel to the vector $3\hat{i} + 2\hat{j} + 2\hat{k}$. If the distance of the point Q from the point $R(1, 3, 3)$ is 5, then the square of the area of $\triangle PQR$ is equal to :

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Options:

A. 148

B. 144

C. 136

D. 140

Answer: C

Solution:

$$P = (-2, -1, 3)$$

Since the line through P is parallel to the vector

$$\vec{d} = (3, 2, 2),$$

any point Q on this line can be expressed as:

$$Q = P + t\vec{d} = (-2 + 3t, -1 + 2t, 3 + 2t),$$

where t is a real number. Given that P and Q must be distinct, we require $t \neq 0$.

The point R is given by:

$$R = (1, 3, 3).$$

The distance from Q to R is 5, so we have:

$$[(-2 + 3t - 1)^2 + (-1 + 2t - 3)^2 + (3 + 2t - 3)^2] = 5^2.$$

Simplify each coordinate difference:

For the x -coordinate:

$$-2 + 3t - 1 = 3t - 3 = 3(t - 1).$$

For the y -coordinate:

$$-1 + 2t - 3 = 2t - 4 = 2(t - 2).$$

For the z -coordinate:

$$3 + 2t - 3 = 2t.$$

So the equation becomes:

$$[3(t - 1)]^2 + [2(t - 2)]^2 + (2t)^2 = 25.$$

Expanding the squares:

$$9(t - 1)^2 + 4(t - 2)^2 + 4t^2 = 25.$$

Expand each term:

$$9(t^2 - 2t + 1) + 4(t^2 - 4t + 4) + 4t^2 = 25,$$

which simplifies to:

$$9t^2 - 18t + 9 + 4t^2 - 16t + 16 + 4t^2 = 25.$$

Combine like terms:

$$(9t^2 + 4t^2 + 4t^2) - (18t + 16t) + (9 + 16) = 25,$$

$$17t^2 - 34t + 25 = 25.$$

Subtract 25 from both sides:

$$17t^2 - 34t = 0.$$

Factor out $17t$:

$$17t(t - 2) = 0.$$

Since $t \neq 0$, we must have:

$$t = 2.$$

Substitute $t = 2$ back into the equation for Q :

$$Q = (-2 + 3(2), -1 + 2(2), 3 + 2(2)) = (4, 3, 7).$$

Next, to find the square of the area of triangle PQR , first compute the vectors:

$$\vec{PQ} = Q - P = (4 - (-2), 3 - (-1), 7 - 3) = (6, 4, 4),$$

$$\vec{PR} = R - P = (1 - (-2), 3 - (-1), 3 - 3) = (3, 4, 0).$$

The area of the triangle is given by:

$$\text{Area} = \frac{1}{2} \|\vec{PQ} \times \vec{PR}\|.$$

Calculate the cross product:

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}.$$

Expanding the determinant:

$\hat{i}$ -component:

$$4 \times 0 - 4 \times 4 = -16,$$

$\hat{j}$ -component:

$$-(6 \times 0 - 4 \times 3) = 12,$$

$\hat{k}$ -component:

$$6 \times 4 - 4 \times 3 = 24 - 12 = 12.$$

Thus,

$$\vec{PQ} \times \vec{PR} = (-16, 12, 12).$$

Find the magnitude squared of the cross product:

$$\|\vec{PQ} \times \vec{PR}\|^2 = (-16)^2 + 12^2 + 12^2 = 256 + 144 + 144 = 544.$$

The square of the area of triangle PQR is:

$$(\text{Area})^2 = \left(\frac{1}{2}\right)^2 \|\vec{PQ} \times \vec{PR}\|^2 = \frac{1}{4} \times 544 = 136.$$

Thus, the square of the area of $\triangle PQR$ is 136.

Question 36

Let P be the foot of the perpendicular from the point $Q(10, -3, -1)$ on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}$. Then the area of the right angled triangle PQR , where R is the point $(3, -2, 1)$, is

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Options:

A. $\sqrt{30}$

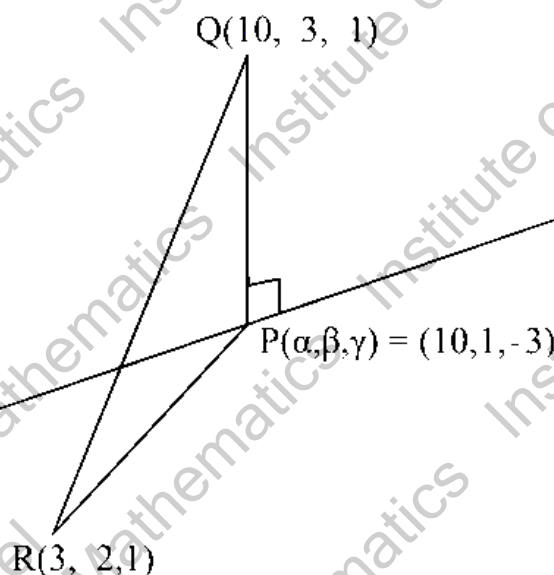
B. $9\sqrt{15}$

C. $3\sqrt{30}$

D. $8\sqrt{15}$

Answer: C

Solution:



$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2} = \lambda$$

$$\Rightarrow 7\lambda + 3, -\lambda + 2, -2\lambda - 1$$

dr's of QP $\Rightarrow$

$$7\lambda - 7, -\lambda + 5, -2\lambda$$

Now

$$(7\lambda - 7) \cdot 7 - (-\lambda + 5) + (2\lambda) \cdot 2 = 0$$

$$54\lambda - 54 = 0 \Rightarrow \lambda = 1$$

$$\therefore P = (10, 1, -3)$$

$$\vec{PQ} = -4\hat{j} + 2\hat{k}$$

$$\vec{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\text{Area} = \frac{1}{2} \begin{vmatrix} i & j & k \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{vmatrix} = 3\sqrt{30}$$

Question 37

The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point $(1, 4, 0)$ along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

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Options:

A. $\sqrt{17}$

B. $\sqrt{13}$

C. $\sqrt{15}$

D. $\sqrt{14}$

Answer: D

Solution:

To find the distance from the line

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

to the point $(1, 4, 0)$ along the line

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3},$$

we first consider a parallel line passing through the point $(1, 4, 0)$. The equation of this parallel line is:

$$\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3}.$$

Next, we find the point of intersection between the given line and the parallel line, which can be expressed in parameter form as:

$$(\lambda + 1, 2\lambda + 4, 3\lambda) = (2t + 2, 3t + 6, 4t + 3).$$

Solving for λ in terms of t , from the first coordinate:

$$\lambda = 2t + 1.$$

Using the second coordinates, we equate:

$$2\lambda + 4 = 3t + 6 \Rightarrow 2(2t + 1) + 4 = 3t + 6.$$

Simplifying gives:

$$4t + 2 + 4 = 3t + 6 \Rightarrow 4t + 6 = 3t + 6 \Rightarrow t = 0.$$

Substituting $t = 0$ into either line equation yields the point of intersection (POI) as:

$$(2, 6, 3).$$

The distance from the point $(1, 4, 0)$ to the POI $(2, 6, 3)$ is computed as:

$$\sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}.$$

Question 38

If the square of the shortest distance between the lines

$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m + n$ is equal to :

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Options:

A. 14

B. 6

C. 21

D. 9

Answer: D

Solution:

$$\vec{a} = (2, 1, -3)$$

$$\vec{b} = (-1, -3, -5)$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$$= 2\hat{i} - \hat{j}$$

$$\vec{b} - \vec{a} = -3\hat{i} - 4\hat{j} - 2\hat{k}$$

$$S_d = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$$

$$= \frac{2}{\sqrt{5}}$$

$$(S_d)^2 = \frac{4}{5}$$

$$m = 4, n = 5 \Rightarrow m + n = 9$$

Question 39

Let in a $\triangle ABC$, the length of the side AC be 6, the vertex B be $(1, 2, 3)$ and the vertices A, C lie on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle ABC$ is:

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Options:

A. 42

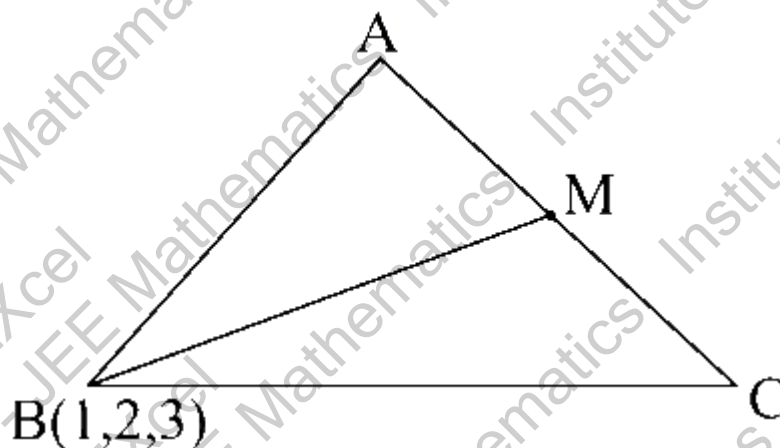
B. 17

C. 56

D. 21

Answer: D

Solution:



Let $M(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

$$\overrightarrow{BM} = (3\lambda + 5)\hat{i} + (2\lambda + 5)\hat{j} + (-2\lambda + 4)\hat{k}$$

$$\overrightarrow{AC} \cdot \overrightarrow{BM} = 0 = 3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4)$$

$$\overrightarrow{BM} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$|\overrightarrow{BM}| = 7$$

$$\text{Area} = \frac{1}{2} \times 6 \times 7 = 21$$

Question 40

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P . Then the distance of P from the point $Q(4, -5, 1)$ is

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Options:

A. $5\sqrt{6}$

B. 5

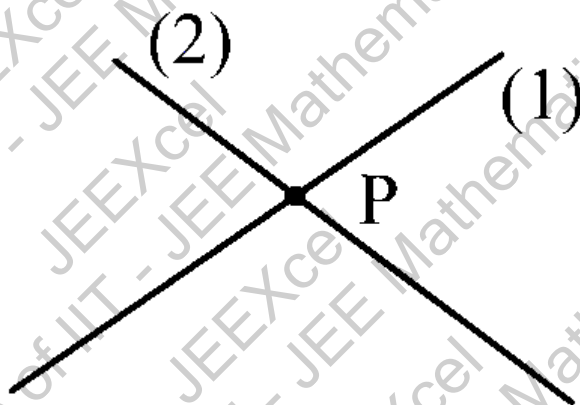
C. $5\sqrt{5}$

D. 10

Answer: C

Solution:

Equation of line through point $(-1, 2, 1)$ is $\rightarrow$



$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = (2) = \lambda$$

$$\text{So, } \begin{cases} x = 2\lambda - 1 \\ y = 3\lambda + 2 \\ z = 4\lambda + 1 \end{cases}$$

$$\text{By (1)} \rightarrow \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu \text{ (Let)}$$

$$\text{So, } \begin{cases} x = 3\mu - 2 \\ y = 2\mu + 3 \\ z = \mu + 4 \end{cases}$$

For intersection point 'P'

$$x = 2\lambda - 1 = 3\mu - 2$$

$$y = 3\lambda + 2 = 2\mu + 3 \quad \begin{cases} \lambda = 1 \\ \mu = 1 \end{cases}$$

$$z = 4\lambda + 1 = \mu + 4$$

$$\text{So, point } P(x, y, z) = (1, 5, 5)$$

$$\& Q(4, -5, 1)$$

$$\begin{aligned} \therefore PQ &= \sqrt{9 + 100 + 16} \\ &= \sqrt{125} = 5\sqrt{5} \end{aligned}$$

Question 41

If the image of the point $(4, 4, 3)$ in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to

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Options:

A. 12

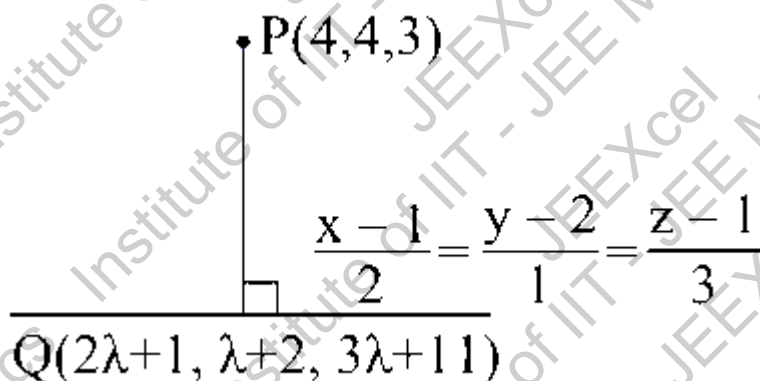
B. 9

C. 7

D. 8

Answer: B

Solution:



$$\begin{aligned} \overrightarrow{PQ} &\perp (2\hat{i} + \hat{j} + 3\hat{k}) \\ \Rightarrow 2(2\lambda - 3) + 1(\lambda - 2) + 3(3\lambda - 2) &= 0 \\ \Rightarrow 14\lambda - 14 &= 0, \lambda = 1 \\ \text{So, } Q(3, 3, 4) \\ \text{Let image in } R(\alpha, \beta, \gamma) \\ \frac{\alpha + 4}{2} = 3, \frac{\beta + 4}{2} = 3, \frac{\gamma + 3}{2} &= 4 \\ (\alpha, \beta, \gamma) &= (2, 2, 5) \\ \Rightarrow \alpha + \beta + \gamma &= 9 \end{aligned}$$

Question 42

Let $A(x, y, z)$ be a point in xy -plane, which is equidistant from three points $(0, 3, 2)$, $(2, 0, 3)$ and $(0, 0, 1)$.

Let $B = (1, 4, -1)$ and $C = (2, 0, -2)$. Then among the statements

(S1) : $\triangle ABC$ is an isosceles right angled triangle, and

(S2) : the area of $\triangle ABC$ is $\frac{9\sqrt{2}}{2}$,

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Options:

- A. both are false
- B. only (S2) is true
- C. only (S1) is true
- D. both are true

Answer: C

Solution:

$A(x, y, z)$ Let $P(0, 3, 2)$, $Q(2, 0, 3)$, $R(0, 0, 1)$

$$AP = AQ = AR$$

$$x^2 + (y - 3)^2 + (z - 2)^2 = (x - 2)^2 + y^2 + (z - 3)^2 = x^2 + y^2 + (z - 1)^2$$

In xy plane $z = 0$

$$\text{So, } x^2 - 4x + 4 + y^2 + 9 = x^2 + y^2 + 1$$

$$\Rightarrow y = 2$$

$$x = 3$$

$$9 + y^2 - 6y + 9 + 4 = x^2 + y^2 + 1$$

So, $A(3, 2, 0)$ also $B(1, 4, -1)$ & $C(2, 0, -2)$

$$\text{Now } AB = \sqrt{4 + 4 + 1} = 3$$

$$AC = \sqrt{1+4+4} = 3$$

$$BC = \sqrt{1+16+1} = \sqrt{18}$$

$$AB = AC$$

$$\text{isosceles } \Delta \& AB^2 + AC^2 = BC^2$$

right angle Δ

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \cdot \text{height}$$

$$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$$

So only S_1 is true

Question43

The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ from the line

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ in the direction of the vector } \hat{i} + 4\hat{j} + 7\hat{k} \text{ is:}$$

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Options:

A.

66

B.

54

C.

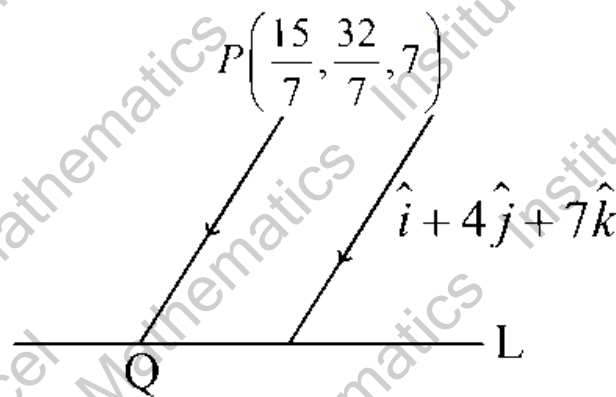
41

D.

44

Answer: A

Solution:



$$L = \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$$

$$PQ = \frac{x - \frac{15}{7}}{1} = \frac{y - \frac{32}{7}}{4} = \frac{z - 7}{7} = \lambda$$

$$\Rightarrow Q \left(\lambda + \frac{15}{7}, 4\lambda + \frac{32}{7}, 7\lambda + 7 \right)$$

Since Q lies on line L

$$\text{So, } \frac{\lambda + \frac{15}{7} + 1}{3} = \frac{7\lambda + 7 + 5}{7}$$

$$\Rightarrow 7\lambda + 22 = 21\lambda + 36$$

$$\Rightarrow \lambda = -1$$

$$\therefore \text{Point } Q \left(\frac{8}{7}, \frac{4}{7}, 0 \right)$$

$$PQ = \sqrt{\left(\frac{15}{7} - \frac{8}{7} \right)^2 + \left(\frac{32}{7} - \frac{4}{7} \right)^2 + (7 - 0)^2}$$

$$PQ = \sqrt{66}$$

$$\Rightarrow (PQ)^2 = 66$$

Question 44

Let $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and $L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2 . If L_3 intersects L_1 , then $|5\alpha - 11\beta - 8\gamma|$ equals :

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Options:

A.

25

B.

20

C.

16

D.

18

Answer: A

Solution:

Step 1. Identify the Given Lines

The line

$$L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$$

passes through the point

$$P_1 = (1, 2, 1)$$

with direction vector

$$\mathbf{u} = (1, -1, 2).$$

Similarly, the line

$$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$$

passes through the point

$$P_2 = (-1, 2, 0)$$

with direction vector

$$\mathbf{v} = (-1, 2, 1).$$

Step 2. Determine the Direction of L_3

Since L_3 is perpendicular to both L_1 and L_2 , its direction vector must be parallel to the cross product of $\mathbf{u}$ and $\mathbf{v}$. Compute:

$$\mathbf{d} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix}.$$

Using the determinant, we obtain:

$$d_x = (-1)(1) - (2)(2) = -1 - 4 = -5,$$

$$d_y = (2)(-1) - (1)(1) = -2 - 1 = -3,$$

$$d_z = (1)(2) - (-1)(-1) = 2 - 1 = 1.$$

Thus, a valid direction vector is

$$\mathbf{d} = (-5, -3, 1).$$

Step 3. Express L_3 in Terms of Its Intersection with L_1

Since L_3 intersects L_1 , let the intersection point (on L_1) be expressed using a parameter t as:

$$A(t) = (1 + t, 2 - t, 1 + 2t).$$

Since L_3 passes through $A(t)$ and also through the given point

$$Q = (\alpha, \beta, \gamma),$$

its parametric form can be written as:

$$(\alpha, \beta, \gamma) = A(t) + s\mathbf{d} = (1 + t, 2 - t, 1 + 2t) + s(-5, -3, 1)$$

for some parameters s and t .

Step 4. Express α, β, γ in Terms of t and s

Comparing coordinates, we have:

$$\begin{cases} \alpha = 1 + t - 5s, \\ \beta = 2 - t - 3s, \\ \gamma = 1 + 2t + s. \end{cases}$$

Step 5. Compute $5\alpha - 11\beta - 8\gamma$

Substitute the expressions for α, β , and γ :

$$\begin{aligned} 5\alpha - 11\beta - 8\gamma &= 5(1 + t - 5s) - 11(2 - t - 3s) - 8(1 + 2t + s) \\ &= (5 + 5t - 25s) - (22 - 11t - 33s) - (8 + 16t + 8s) \\ &= (5 - 22 - 8) + (5t + 11t - 16t) + (-25s + 33s - 8s) \\ &= -25 + 0t + 0s \\ &= -25. \end{aligned}$$

Taking the absolute value yields:

$$|5\alpha - 11\beta - 8\gamma| = 25.$$

Conclusion

The value of

$|5\alpha - 11\beta - 8\gamma|$ is $\boxed{25}$.

Question 45

Let P be the foot of the perpendicular from the point $(1, 2, 2)$ on the line L : $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$.

Let the line $\vec{r} = (-\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}), \lambda \in \mathbf{R}$, intersect the line L at Q. Then $2(PQ)^2$ is equal to :

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Options:

A.

25

B.

27

C.

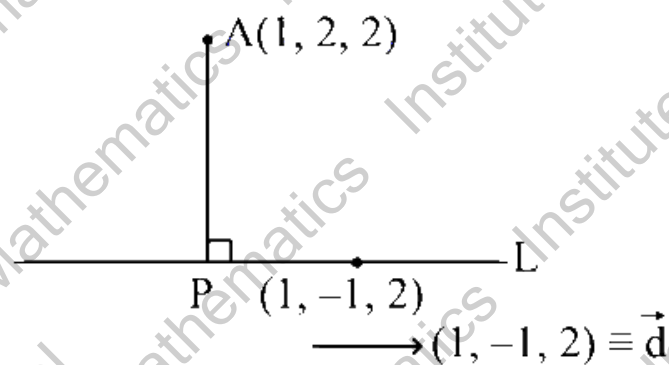
19

D.

29

Answer: B

Solution:



$$L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2} = \mu$$

$$P(\mu+1, -\mu-1, 2\mu+2)$$

$$\vec{AP} \cdot \vec{d} = 0 \Rightarrow (\mu, -\mu-3, 2\mu) \cdot (1, -1, 2) = 0$$

$$\Rightarrow \mu + \mu + 3 + 4\mu = 0 \Rightarrow \mu = -\frac{1}{2}$$

$$\therefore P\left(\frac{-1}{2} + 1, +\frac{1}{2} - 1, 2\left(\frac{-1}{2}\right) + 2\right)$$

$$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$$

$$\left. \begin{array}{l} \mu + 1 = -1 + \lambda \\ \mu = \lambda - 2 \end{array} \right| \left. \begin{array}{l} -\mu - 1 = 1 - \lambda \\ \mu = \lambda - 2 \end{array} \right| \begin{array}{l} 2\mu + 2 = -2 + \lambda \\ \downarrow \end{array}$$

$$2(\lambda - 2) + 2 = -2 + \lambda$$

$$2\lambda - 4 + 2 = -2 + \lambda$$

$$\therefore \mu = -2, \lambda = 0$$

$$\therefore Q \equiv (-1, 1, -2)$$

$$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right) \text{ and } Q(-1, 1, -2)$$

$$PQ = \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{-1}{2} - 1\right)^2 + (1 + 2)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{9}{4} + 9} = \sqrt{\frac{54}{4}}$$

$$\therefore 2(PQ)^2 = 2\left(\frac{54}{4}\right) = 27$$

Question 46

Let a straight line L pass through the point $P(2, -1, 3)$ and be

perpendicular to the lines $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$ and $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z+2}{4}$.

If the line L intersects the yz -plane at the point Q , then the distance between the points P and Q is:

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Options:

A.

$$\sqrt{10}$$

B.

$$2$$

C.

$$2\sqrt{3}$$

D.

$$3$$

Answer: D

Solution:

To find the distance between the points P and Q , let's consider the following steps:

Determine the direction vector of line L :

For the line L to be perpendicular to both given lines, we find the direction vector of L using the cross product of the direction vectors of the given lines.

The first line has direction vector $\langle 2, 1, -2 \rangle$ and the second line has direction vector $\langle 1, 3, 4 \rangle$. The cross product is:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 10\hat{i} - 10\hat{j} + 5\hat{k} = 5(2\hat{i} - 2\hat{j} + \hat{k})$$

Thus, the direction vector of line L is $\langle 2, -2, 1 \rangle$.

Equation of line L :

Since L passes through point $P(2, -1, 3)$ with direction vector $\langle 2, -2, 1 \rangle$, its parametric form is:

$$\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-3}{1} = \lambda$$

Find the intersection with the yz -plane:

On the yz -plane, $x = 0$. Substitute into the line equation to find λ :

$$2\lambda + 2 = 0 \implies \lambda = -1$$

Substitute $\lambda = -1$ back into the parametric equations to find Q :

$$x = 2(-1) + 2 = 0, \quad y = -2(-1) - 1 = 1, \quad z = (-1) + 3 = 2$$

Therefore, the intersection point Q is $(0, 1, 2)$.

Calculate the distance between P and Q :

$$d(P, Q) = \sqrt{(2-0)^2 + (-1-1)^2 + (3-2)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

Thus, the distance between points P and Q is 3.

Question 47

Let $ABCD$ be a tetrahedron such that the edges AB , AC and AD are mutually perpendicular. Let the areas of the triangles ABC , ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the $\triangle BCD$ is equal to :

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Options:

A. $\sqrt{110}$

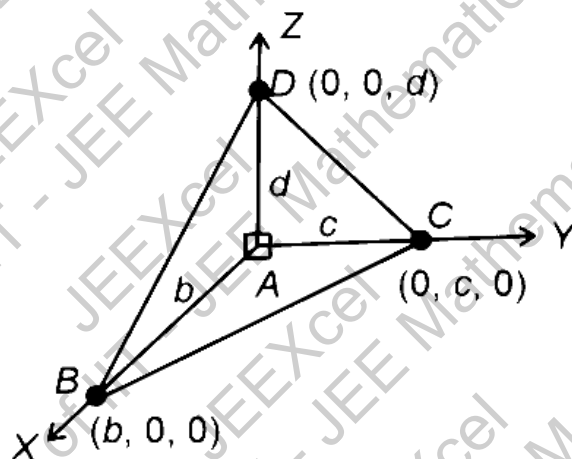
B. 12

C. $\sqrt{340}$

D. $7\sqrt{3}$

Answer: A

Solution:



$$\text{ar}(\triangle ABC) = 5$$

$$\frac{1}{2} \times bc = 5$$

$$\Rightarrow bc = 10$$

$$\text{ar}(\triangle ACD) = 6$$

$$\frac{1}{2} \times cd = 6$$

$$\Rightarrow cd = 12$$

$$\text{ar}(\triangle ABD) = 7$$

$$bd = 14$$

$$\text{area}(\triangle BCD) = \frac{1}{2} |\vec{BC} \times \vec{BD}|$$

$$\vec{BC} = \langle -b, c, 0 \rangle$$

$$\vec{BD} = \langle -b, 0, d \rangle$$

$$|\vec{BC} \times \vec{BD}| = \sqrt{c^2 d^2 + b^2 d^2 + b^2 c^2}$$

$$= \sqrt{12^2 + 14^2 + 10^2}$$

$$= \sqrt{440}$$

$$\text{ar}(\triangle BCD) = \sqrt{110}$$

Question 48

Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, $QR = 5$ and the coordinates of the point P be $(0, 2, 3)$. If the area of the triangle PQR is $\frac{m}{n}$ then :

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Options:

A. $2m - 5\sqrt{21}n = 0$

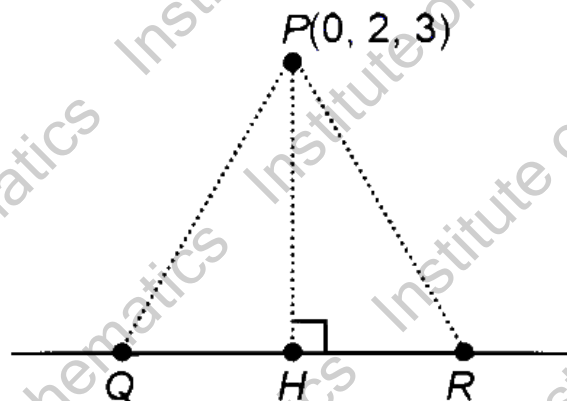
B. $m - 5\sqrt{21}n = 0$

C. $5m - 21\sqrt{2}n = 0$

D. $5m - 2\sqrt{21}n = 0$

Answer: A

Solution:



$$H : (5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$$

$$\langle \text{DR of PH} \rangle$$

$$\langle 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 \rangle$$

$$\begin{aligned}
 \vec{PH} \cdot \vec{QR} &= 0 \\
 \Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 &= 0 \\
 \Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 &= 0 \\
 \Rightarrow 38\lambda &= 38 \\
 \Rightarrow \lambda &= 1 \\
 \therefore H(2, 3, -1) \\
 PH &= \sqrt{4 + 1 + 16} = \sqrt{21} \\
 \therefore \text{Area} &= \frac{1}{2} \times PH \times QR \\
 &= \frac{1}{2} \times \sqrt{21} \times 5 = \frac{5\sqrt{21}}{2} = \frac{m}{n} \\
 2m - 5\sqrt{21}n &= 0
 \end{aligned}$$

Question 49

The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$ and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is :

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Options:

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{38}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{57}}$

Answer: A

Solution:

The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$. The line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$.

Equations of the Lines:

Equation of L_1 :

$$\mathbf{r} = 7\hat{i} + 6\hat{j} + 2\hat{k} + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$$

Equation of L_2 :

$$\mathbf{r} = 5\hat{i} + 3\hat{j} + 4\hat{k} + \mu(2\hat{i} + \hat{j} + 3\hat{k})$$

Shortest Distance Between L_1 and L_2 :

The formula for the shortest distance between two skew lines is given by:

$$\text{Distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Calculations:

Vector between the points:

$$\vec{a}_2 - \vec{a}_1 = (5\hat{i} + 3\hat{j} + 4\hat{k}) - (7\hat{i} + 6\hat{j} + 2\hat{k}) = -2\hat{i} - 3\hat{j} + 2\hat{k}$$

Cross product of direction vectors:

$$\vec{b}_1 = -3\hat{i} + 2\hat{j} + 4\hat{k}, \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} + 17\hat{j} - 7\hat{k}$$

Magnitude of the cross product:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{2^2 + 17^2 + (-7)^2} = \sqrt{342}$$

Dot product:

$$\begin{aligned} (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (-2\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k}) \\ &= (-2)(2) + (-3)(17) + (2)(-7) = -4 - 51 - 14 = -69 \end{aligned}$$

Shortest Distance:

$$\text{Distance} = \frac{|-69|}{\sqrt{342}} = \frac{23}{\sqrt{38}}$$

Question 50

If the image of the point $P(1, 0, 3)$ in the line joining the points $A(4, 7, 1)$ and $B(3, 5, 3)$ is $Q(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to :

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Options:

A. $\frac{46}{3}$

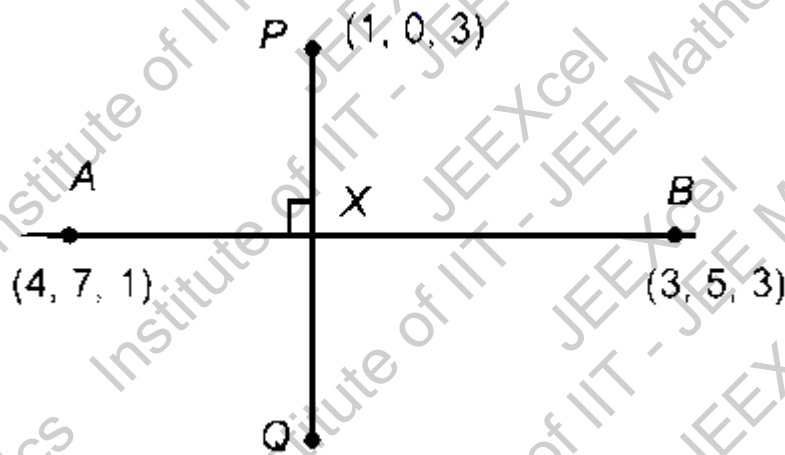
B. 18

C. 13

D. $\frac{47}{3}$

Answer: A

Solution:



Let X be mid point of P and Q , which would be also feet of perpendicular.

Let X divides A and B in $\lambda : 1, \lambda \neq -1$

$$X = \left(\frac{3\lambda + 4}{\lambda + 1}, \frac{5\lambda + 7}{\lambda + 1}, \frac{3\lambda + 1}{\lambda + 1} \right)$$

$$\text{Now } PX \perp AB \Rightarrow \vec{PX} \cdot \vec{AB} = 0$$

$$\left(\frac{3\lambda + 4}{\lambda + 1} - 1 \right) \cdot (4 - 3) + \left(\frac{5\lambda + 7}{\lambda + 1} - 0 \right) (7 - 5) + \left(\frac{3\lambda + 1}{\lambda + 1} - 3 \right) \cdot (1 - 3) = 0$$

$$\frac{2\lambda + 3}{\lambda + 1} + \frac{10\lambda + 14}{\lambda + 1} + \frac{4}{\lambda + 1} = 0$$

$$\Rightarrow \frac{12\lambda + 21}{\lambda + 1} = 0 \Rightarrow \lambda = \frac{-7}{4}$$

$$X = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3} \right)$$

X is mid point of PQ

$$Q \equiv \left(2 \cdot \frac{5}{3} - 1, 2 \cdot \frac{7}{3} - 0, 2 \cdot \frac{17}{3} - 3 \right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{2(5 + 7 + 17)}{3} - 4 = \frac{58}{3} - 4 = \frac{46}{3}$$

Question51

Line L_1 passes through the point $(1, 2, 3)$ and is parallel to z -axis.

Line L_2 passes through the point $(\lambda, 5, 6)$ and is parallel to y -axis.

Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line L_1 is

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Options:

A. 25

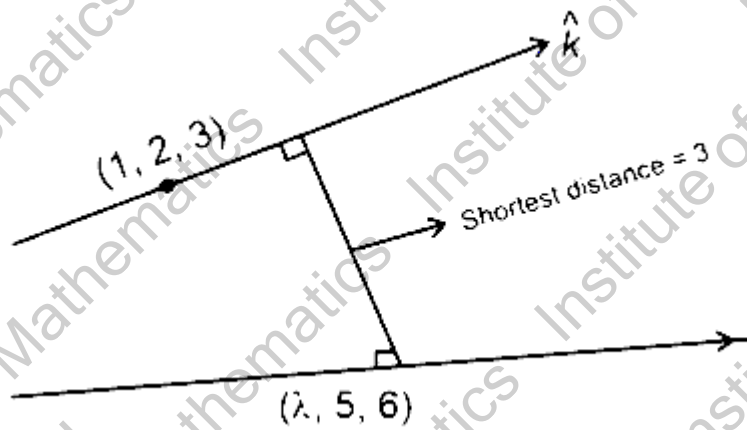
B. 32

C. 40

D. 37

Answer: A

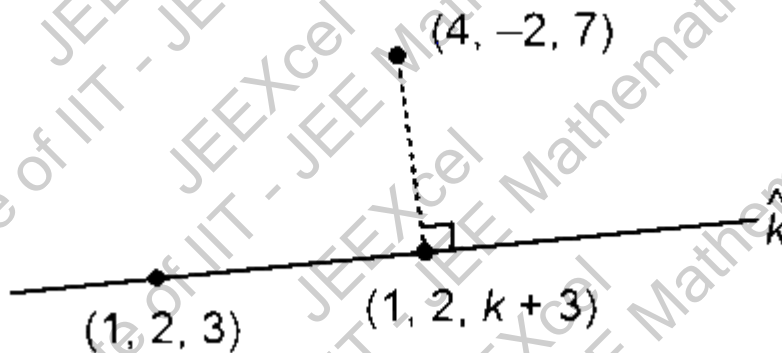
Solution:



$$3 = \frac{|[(\lambda - 1)\hat{i} + 3\hat{j} + 3\hat{k}] \cdot (\hat{k} \times \hat{j})|}{|\hat{k} \times \hat{j}|} = |\lambda - 1| = 3$$

$$\Rightarrow \lambda = 4 \text{ or } -2, \lambda_2 < \lambda_1 \Rightarrow \lambda_2 = -2$$

$$\lambda_1 = 4$$



$$(3, -4, 4 - k) \perp \hat{k}$$

$$4 - k = 0 \Rightarrow k = 4$$

$$\Rightarrow \text{Distance between } (4, -2, 7) \text{ \& } (1, 2, 7)$$

$$\Rightarrow \sqrt{3^2 + 4^2} = 5$$

Question 52

Let a line passing through the point $(4, 1, 0)$ intersect the line

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ at the point } A(\alpha, \beta, \gamma) \text{ and the line}$$

$$L_2 : x - 6 = y = -z + 4 \text{ at the point } B(a, b, c). \text{ Then } \begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix} \text{ is}$$

equal to

Options:

A. 16

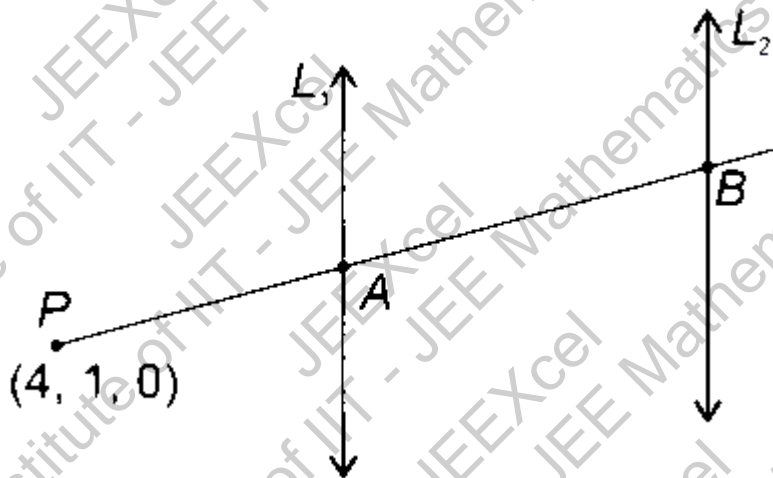
B. 6

C. 8

D. 12

Answer: C

Solution:



$$A = (\alpha, \beta, 4) = (2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$$

$$\text{and } B = (a, b, c) = (\mu + 6, \mu, -\mu + 4)$$

$$\therefore \frac{\mu + 2}{2\lambda - 3} = \frac{\mu - 1}{3\lambda + 1} = \frac{-\mu + 4}{4\lambda + 3}$$

$$\therefore \lambda\mu + 8\lambda + 4\mu = 1 \quad \dots (1)$$

$$\text{and } 7\lambda\mu - 16\lambda + 4\mu = 7 \quad \dots (2)$$

from equation (1) and (2)

$$\lambda = -1 \text{ and } \mu = 3$$

$$\text{Or } \lambda = -\frac{1}{3} \text{ and } \mu = 1$$

By taking, $\lambda = -1$ and $\mu = 3$, we get

$$\therefore A(\alpha, \beta, 4) = (-1, -1, -1)$$

$$\text{and } B(a, b, c) = (9, 3, 1)$$

$$\therefore \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 8$$

Question 53

The distance of the point $(7, 10, 11)$ from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

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Options:

A. 16

B. 12

C. 18

D. 14

Answer: D

Solution:

Equation of line passing through $P(7, 10, 11)$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

$$\frac{x-7}{2} = \frac{y-10}{3} = \frac{z-11}{6} = \lambda$$

Let the point on the line is

$$Q(2\lambda + 7, 3\lambda + 10, 6\lambda + 11)$$

$$Q \text{ lies on line } \frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$$

$$3\lambda + 10 = 4 \Rightarrow \lambda = -2$$

$$\therefore Q(3, 4, -1)$$

$$PQ = \sqrt{16 + 36 + 144} = 14$$

Question 54

Each of the angles β and γ that a given line makes with the positive y - and z -axes, respectively, is half of the angle that this line makes with the positive x -axis. Then the sum of all possible values of the angle β is

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Options:

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{3\pi}{2}$

Answer: C

Solution:

Given:

Each of the angles β and γ is half of the angle that the line makes with the positive x -axis, i.e., $\beta = \gamma = \frac{\alpha}{2}$.

The equation for the direction cosines of angles a line makes with the coordinate axes is given by:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Since $\beta = \gamma$, we substitute to get:

$$\cos^2 \alpha + 2 \cos^2 \beta = 1$$

Substitute $\cos \beta = \cos \gamma$:

$$\cos \alpha = 2 \cos^2 \beta - 1$$

Replacing back into the equation:

$$(2 \cos^2 \beta - 1)^2 + 2 \cos^2 \beta = 1$$

Simplify:

$$(2 \cos^2 \beta - 1)(2 \cos^2 \beta + 1) = 0$$

Solving for $\cos^2 \beta$, we have:

$$2 \cos^2 \beta - 1 = 0 \quad \text{or} \quad 2 \cos^2 \beta + 1 = 0$$

The latter gives no real solutions, thus:

$$\cos^2 \beta = \frac{1}{2}$$

Therefore, $\beta = \frac{\pi}{4}$ or $\beta = \frac{\pi}{2}$.

Thus, the sum of all possible values of β is:

$$\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

Question 55

Let A and B be two distinct points on the line $L : \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1, 2, 3)$ on the line L . If O is the origin, then

$\vec{OA} \cdot \vec{OB}$ is equal to

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Options:

A. 49

B. 21

C. 47

D. 62

Answer: C

Solution:

$$L : \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$$

$$\text{Point } A(3\lambda + 6, 2\lambda + 7, 7 - 2\lambda)$$

$$B(3\mu + 6, 2\mu + 7, 7 - 2\mu)$$

Let $P(3k + 6, 2k + 7, 7 - 2k)$ be foot of perpendicular from $P'(1, 2, 3)$

$$\therefore \vec{PP'} \cdot \langle 3, 2, -2 \rangle = 0$$

$$3(3k + 5) + (2k + 5)2 + (4 - 2k)(-2) = 0$$

$$9k + 15 + 4k + 10 - 8 + 4k = 0$$

$$17k + 17 = 0$$

$$\Rightarrow k = -1 \quad \therefore P(3, 5, 9)$$

$$|\vec{AP}| = 2\sqrt{17}$$

$$\begin{aligned}
 &\Rightarrow (3\lambda + 3)^2 + (2\lambda + 2)^2 + (-2 - 2\lambda)^2 = 17 \times 4 \\
 &= 17(\lambda + 1)^2 = 17 \times 4 \\
 &\Rightarrow \lambda + 1 = \pm 2 \Rightarrow \lambda = 1 \text{ or } \lambda = -3 \\
 &\therefore A(9, 9, 5), B(-3, 1, 13) \\
 &\overrightarrow{OA} \cdot \overrightarrow{OB} = (9\hat{i} + 9\hat{j} + 5\hat{k}) \cdot (-3\hat{i} + \hat{j} + 13\hat{k}) \\
 &= -27 + 9 + 65 = 47
 \end{aligned}$$

Question 56

Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

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Options:

- A. 42
- B. 40
- C. 48
- D. 46

Answer: D

Solution:

$$L_1 : \frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$$

$$L_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$$

$$a_1 : (3, \alpha, 3) \quad a_2 = (-3, -7, \beta)$$

$$\vec{b}_1 = 3\hat{i} - \hat{j} + \hat{k} \quad \vec{b}_2 = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = 3\sqrt{30}$$

$$\begin{aligned}
 &= \frac{\begin{vmatrix} 6 & \alpha+7 & 3-\beta \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{6^2 + 15^2 + 3^2}} = 3\sqrt{30}
 \end{aligned}$$

$$\Rightarrow |-15\alpha - 3\beta - 132| = 270$$

$$|5\alpha + \beta + 44| = 90$$

$$\Rightarrow 5\alpha + \beta = 90 - 44 = 46$$

Question 57

Let the values of p , for which the shortest distance between the lines

$$\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5} \text{ and } \vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ is } \frac{1}{\sqrt{6}}, \text{ be}$$

$a, b, (a < b)$. Then the length of the latus rectum of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is :}$$

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Options:

A. $\frac{3}{2}$

B. 9

C. 18

D. $\frac{2}{3}$

Answer: D

Solution:

$$\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}; (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$d = \frac{|(\vec{a} - \vec{b}) \cdot (\vec{p}_1 \times \vec{p}_2)|}{|\vec{p}_1 \times \vec{p}_2|} = \frac{1}{\sqrt{6}}$$

$$\vec{a} - \vec{b} = (p+1)\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\frac{1}{\sqrt{6}} = \frac{|(p+1) - 4 + 1|}{\sqrt{6}}$$

$$= |p - 2| = 1 \Rightarrow p = 3, 1$$

$$a = 1, b = 3$$

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

$$\text{Length of } LR = \frac{2a^2}{b} = \frac{2}{3}$$

Question 58

Let A be the point of intersection of the lines $L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$ and $L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is :

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Options:

A. 63

B. 57

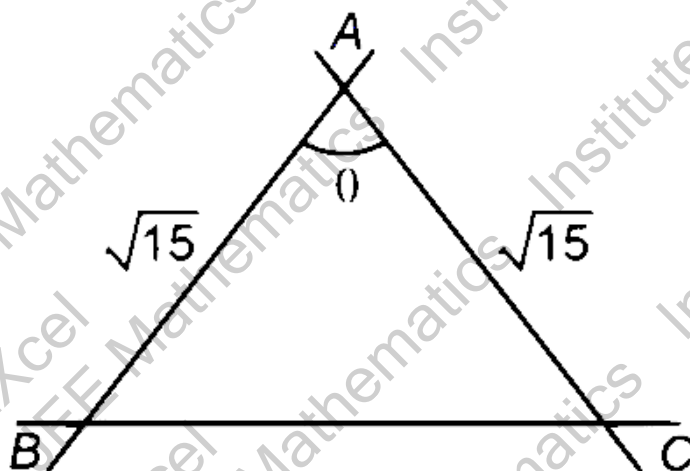
C. 60

D. 54

Answer: D

Solution:

$$L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}; L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$$



$$\cos \theta = \left| \frac{3 + 0 - 5}{\sqrt{2} \times \sqrt{50}} \right|$$

$$= \frac{2}{10} = \frac{1}{5}$$

$$\therefore \sin \theta = \frac{2\sqrt{6}}{5}$$

$$\text{Area} = \frac{1}{2} ab \sin \theta$$

$$= \frac{1}{2} \times \sqrt{15} \times \sqrt{15} \times \frac{2\sqrt{6}}{5}$$

$$= 3\sqrt{6}$$

$$(\text{Area})^2 = 9 \times 6 = 54$$

Question 59

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

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Options:

A. $\frac{3}{2}$

B. 3

C. -3

D. $-\frac{3}{2}$

Answer: C

Solution:

$$d = \frac{|(\vec{a} - \vec{b}) \cdot \vec{p}_1 \times \vec{p}_2|}{|\vec{p}_1 \times \vec{p}_2|} = \frac{5}{\sqrt{6}}$$

$$\vec{a} - \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & \alpha & 1 \end{vmatrix}$$

$$= \hat{i}(3 - 4\alpha) - \hat{j}(-2) + \hat{k}(2\alpha - 3)$$

$$(\vec{a} - \vec{b}) \cdot \vec{p}_1 \times \vec{p}_2 = 3 - 4\alpha + 4 - 4\alpha + 6$$

$$= 13 - 8\alpha$$

$$\left| \frac{13 - 8\alpha}{\sqrt{(3 - 4\alpha)^2 + 4 + (2\alpha - 3)^2}} \right| = \frac{5}{\sqrt{6}}$$

$$\left| \frac{13 - 8\alpha}{\sqrt{20\alpha^2 - 36\alpha + 22}} \right| = \frac{5}{\sqrt{6}}$$

$$= 6(13 - 8\alpha)^2 = 25(20\alpha^2 - 36\alpha + 22)$$

$$= 116\alpha^2 + 348\alpha - 464 = 0$$

$$\text{Sum of roots} = -3$$

Question60

Let the line L pass through $(1, 1, 1)$ and intersect the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z}{1}$. Then, which of the following points lies on the line L ?

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Options:

A. $(7, 15, 13)$

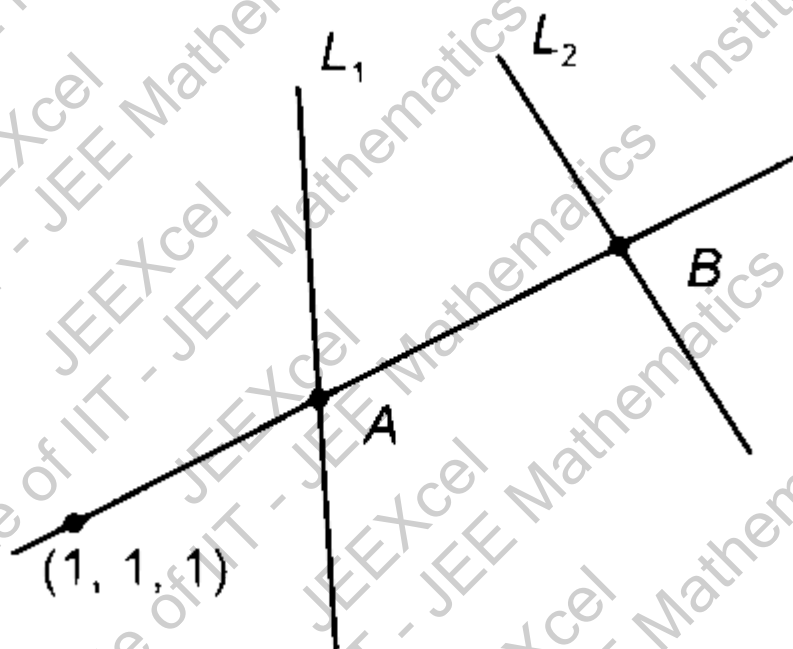
B. $(4, 22, 7)$

C. $(10, -29, -50)$

D. $(5, 4, 3)$

Answer: A

Solution:



$$L : \frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$$

$$L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda \text{ (say)}$$

Any point on L_1 be $A(2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$

$$L_2 : \frac{x-3}{1} = \frac{y-4}{2} = \frac{z-1}{1} = \mu \text{ (say)}$$

Any point on L_2 be $B(\mu + 3, 2\mu - 4, \mu)$

D of L be : $\langle 2\lambda, 3\lambda - 2, 4\lambda \rangle$ or $\langle \mu + 2, 2\mu + 3, \mu - 1 \rangle$

$$\text{Now } \frac{2\lambda}{\mu+2} = \frac{3\lambda-2}{2\mu+3} = \frac{4\lambda}{\mu-1}$$

$$\Rightarrow \lambda = \frac{-6}{5} \quad \mu = -5$$

$$\therefore \langle a, b, c \rangle \equiv \langle -3, -7, -6 \rangle \text{ or } \langle 3, 7, 6 \rangle$$

$$\therefore L : \frac{x-1}{3} = \frac{y-1}{7} = \frac{z-1}{6}$$

$(7, 15, 13)$ lies on the line.

Question 61

Consider the lines $L_1: x - 1 = y - 2 = z$ and $L_2: x - 2 = y = z - 1$. Let the feet of the perpendiculars from the point $P(5, 1, -3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A , then $4A^2$ is equal to :

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Options:

A.

151

B.

147

C.

139

D.

143

Answer: B

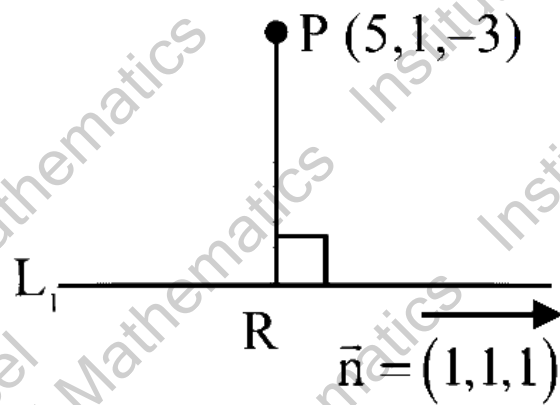
Solution:

$$L_1: \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-0}{2}$$

$$\text{Let } Q(\lambda + 1, \lambda + 2, \lambda)$$

$$\overrightarrow{PQ} = (\lambda - 4, \lambda - 1, \lambda + 3)$$

$$\overrightarrow{PQ} \cdot \vec{m} = 0$$



$$\Rightarrow \lambda - 4 + \lambda + 1, \lambda + 3 = 0$$

$$\Rightarrow 3\lambda = 0$$

$$\Rightarrow \lambda = 0$$

$$\Rightarrow Q(1, 2, 0)$$

$$L_2 : \frac{x-2}{1} = \frac{y-0}{1} = \frac{z-1}{2}$$

$$\text{Let } R(\mu + 2, \mu, \mu + 1) \quad \overrightarrow{PR} = (\mu - 3, \mu - 1, \mu + 4)$$

$$\overrightarrow{PR} \cdot \vec{n} = 0$$

$$\mu - 3 + \mu - 1 + \mu + 4 = 0$$

$$\Rightarrow \mu = 0$$

$$R(2, 0, 1)$$

$$\text{Area of } \triangle PQR (A) = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$A = \frac{1}{2} |(-4\hat{i} + \hat{j} + 3\hat{k}) \times (-3\hat{i} + \hat{j} + 4\hat{k})|$$

$$A = \frac{1}{2} |7\hat{i} + \hat{j} + \hat{k}|$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

$$= 7\hat{i} + 7\hat{j} + 7\hat{k}$$

$$4A^2 = 49 \times 3 = 147$$

Question 62

If the equation of the line passing through the point $(0, -\frac{1}{2}, 0)$ and perpendicular to the lines $\vec{r} = \lambda (\hat{i} + a\hat{j} + b\hat{k})$ and

$\vec{r} = (\hat{i} - \hat{j} - 6\hat{k}) + \mu(-b\hat{i} + a\hat{j} + 5\hat{k})$ is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then $a + b + c + d$ is equal to :

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Options:

A.

13

B.

14

C.

12

D.

10

Answer: B

Solution:

The line we need to find should be orthogonal to two given lines, meaning it will be parallel to the cross product of their direction vectors:

$$(\hat{i} + a\hat{j} + b\hat{k}) \times (-b\hat{i} + a\hat{j} + 5\hat{k})$$

Calculating the cross product results in the vector:

$$\hat{i}(5a - ab) - \hat{j}(b^2 + 5) + \hat{k}(a + ab)$$

This shows that the direction ratios of the required line are proportional to:

$$\alpha(5a - ab), -(b^2 + 5), (a + ab)$$

Since the line is also expressed as $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, the direction ratios based on this equation are:

$$\alpha(-2), d, -4$$

Thus, we have the equation:

$$\frac{5a-ab}{-2} = \frac{-(b^2+5)}{d} = \frac{a+ab}{-4}$$

The point $(0, -\frac{1}{2}, 0)$ lies on the line, which implies:

$$\frac{0-1}{-2} = \frac{-\frac{1}{2}+4}{d} = \frac{0-c}{-4}$$

Simplifying these gives $d = 7$ and $c = 2$.

Substitute into the main equation:

$$\frac{5a-ab}{-2} = \frac{a+ab}{-4}$$

$$\frac{-b^2-5}{7} = \frac{a+ab}{-4}$$

Solving these yields:

$$\begin{array}{l|l} -20a + 4ab = -2a - 2ab & 4b^2 + 20 = 70 + 7ab \\ 18a = 6ab & 36 + 20 = 70 + 21a \\ b = 3 & 56 = 28a \Rightarrow a = 2 \end{array}$$

Substituting these into the variables, we find:

$$a + b + c + d = 2 + 3 + 2 + 7 = 14$$

Question63

Let the values of λ for which the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $\frac{x-\lambda}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2 . Then the radius of the circle passing through the points $(0, 0)$, (λ_1, λ_2) and (λ_2, λ_1) is

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Options:

A.

3

B.

$$\frac{5\sqrt{2}}{3}$$

C.

$$\frac{\sqrt{2}}{3}$$

D.

4

Answer: B

Solution:

$$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\Rightarrow \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$A \equiv (1, 2, 3) B \equiv (\lambda, 4, 5)$$

$$\text{Shortest Distance} = \frac{|\vec{AB} \cdot (\vec{P} \times \vec{Q})|}{|\vec{P} \times \vec{Q}|}$$

$$\frac{1}{\sqrt{6}} = \left| \frac{(\lambda - 1)\hat{i} + 2\hat{j} + 2\hat{k} \cdot (-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}} \right|$$

$$\Rightarrow |-\lambda + 1 + 4 - 2| = 1 \Rightarrow |\lambda - 3| = 1$$

$$\Rightarrow \lambda = 3 \pm 1 = 4, 2$$

Radius of circle passing through points

$(0, 0), (4, 2) \& (2, 4)$

$$= \frac{abc}{4\Delta} = \frac{\sqrt{20} \times \sqrt{20} \times \sqrt{8}}{4 \times \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{vmatrix}} = \frac{20 \times 2\sqrt{2}}{2 \times 12}$$

$$= \frac{5\sqrt{2}}{3}$$

Question 64

If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and

$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

[27-Jan-2024 Shift 1]

Options:

A. 5

B. 8

C. 7

D. 10

Answer: B

Solution:

$$\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$$

$$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$$

the shortest distance between the lines

$$= \left| \frac{(\vec{a} - \vec{b}) \cdot (\vec{d}_1 \times \vec{d}_2)}{|\vec{d}_1 \times \vec{d}_2|} \right|$$

$$= \left| \frac{\begin{vmatrix} \lambda-4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}} \right|$$

$$= \left| \frac{(\lambda-4)(-10+12) - 0 + 2(4-4)}{12\hat{i} - 1\hat{j} + 0\hat{k}} \right|$$

$$\frac{6}{\sqrt{5}} = \left| \frac{2(\lambda-4)}{\sqrt{5}} \right|$$

$$3 = |\lambda - 4|$$

$$\lambda - 4 = \pm 3$$

$$\lambda = 7, 1$$

Sum of all possible values of λ is = 8

Question 65

The distance, of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :

[27-Jan-2024 Shift 1]

Options:

A. 12

B. 14

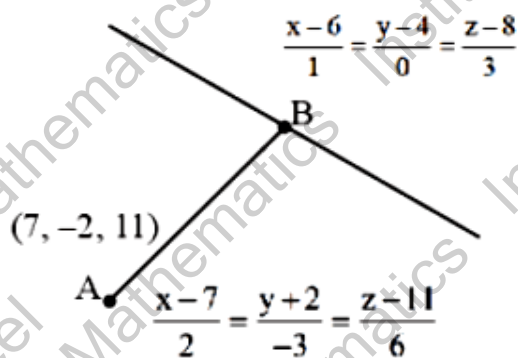
C. 18

D. 21

Answer: B

Solution:

$$B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$$

$$-3\lambda - 6 = 0$$

$$\lambda = -2$$

$$B \Rightarrow (3, 4, -1)$$

$$\begin{aligned} AB &= \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2} \\ &= \sqrt{16+36+144} \\ &= \sqrt{196} = 14 \end{aligned}$$

Question 66

The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :
[27-Jan-2024 Shift 2]

Options:

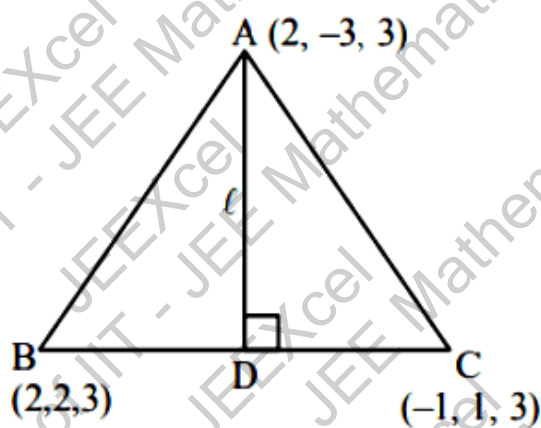
- A. 49
- B. 42
- C. 50
- D. 45

Answer: D

Solution:

$$AB = 5$$

$$AC = 5$$



$\therefore D$ is midpoint of BC

$$D\left(\frac{1}{2}, \frac{3}{2}, 3\right)$$

$$\therefore l = \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-3 - \frac{3}{2}\right)^2 + (3 - 3)^2}$$

$$l = \sqrt{\frac{45}{2}}$$

$$\therefore 2l^2 = 45$$

Question 67

Let the position vectors of the vertices A , B and C of a triangle be $2\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Let l_1 , l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB , BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals :

[27-Jan-2024 Shift 2]

Options:

A. $\frac{1}{5}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

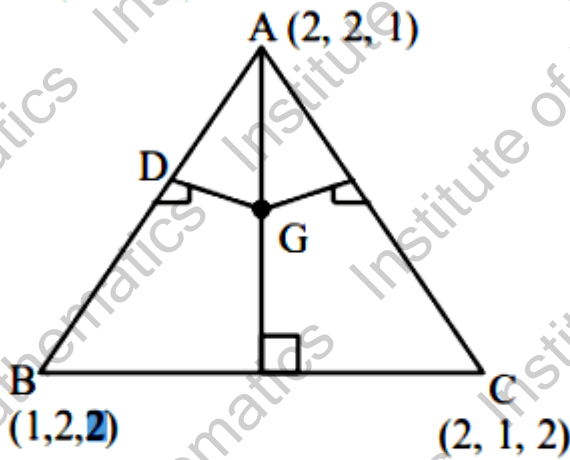
Answer: B

Solution:

$\triangle ABC$ is equilateral

Orthocentre and centroid will be same

$$G\left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3}\right)$$



Mid-point of AB is $D\left(\frac{3}{2}, 2, \frac{3}{2}\right)$

$$\therefore \ell_1 = \sqrt{\frac{1}{36} + \frac{1}{9} + \frac{1}{36}}$$

$$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$$

$$\therefore \ell_1^2 + \ell_2^2 + \ell_3^2 = \frac{1}{2}$$

Question 68

Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ) . Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?
[27-Jan-2024 Shift 2]

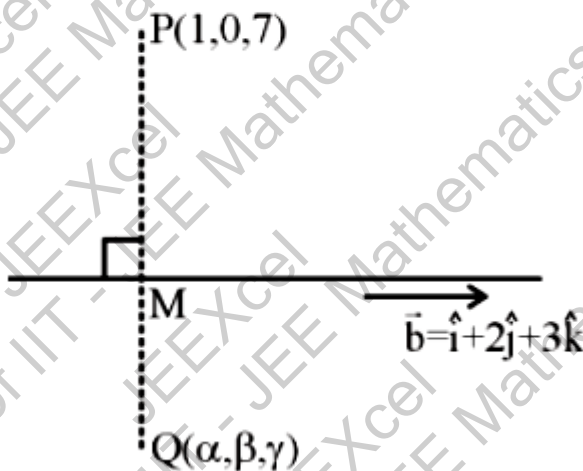
Options:

- A. $(1, -2, 1 + \sqrt{2})$
- B. $(1, 2, 1 - \sqrt{2})$
- C. $(3, 4, 3 - 2\sqrt{2})$
- D. $(3, -4, 3 + 2\sqrt{2})$

Answer: C

Solution:

$$L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$



$$M(\lambda, 1+2\lambda, 2+3\lambda)$$

$$\vec{PM} = (\lambda - 1)\hat{i} + (1 + 2\lambda)\hat{j} + (3\lambda - 5)\hat{k}$$

$\vec{PM}$ is perpendicular to line L_1

$$\vec{PM} \cdot \vec{b} = 0 \quad (\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k})$$

$$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$$

$$14\lambda = 14 \Rightarrow \lambda = 1$$

$$\therefore M = (1, 3, 5)$$

$$\vec{Q} = 2\vec{M} - \vec{P} \quad [M \text{ is midpoint of } \vec{P} \text{ \& } \vec{Q}]$$

$$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$$

$$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$$

$$\therefore (a, b, c) = (1, 6, 3)$$

Required line having direction cosine (l, m, n)

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$l^2 = \frac{1}{4}$$

$$\therefore l = \frac{1}{2} \quad [\text{Line make acute angle with x-axis}]$$

Equation of line passing through $(1, 6, 3)$ will be

$$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu \left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k} \right)$$

Option (3) satisfying for $\mu = 4$

Question 69

The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is 1, then $14l^2$ is equal to.....
[27-Jan-2024 Shift 2]

Answer: 108

Solution:

$$\frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda$$

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k$$

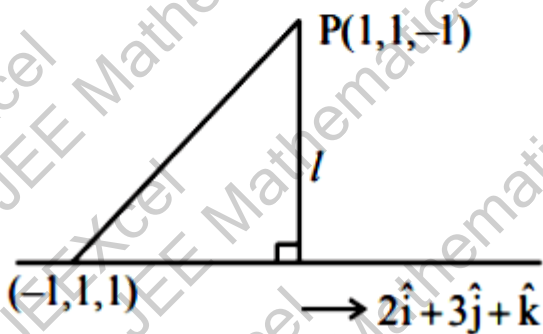
$$\Rightarrow \lambda + 2 = 4k - 3$$

$$-\lambda = 3k - 2$$

$$\Rightarrow k = 1, \lambda = -1$$

$$8\lambda + 7 = k - 2$$

$$\therefore P = (1, 1, -1)$$



Projection of $2\hat{i} - 2\hat{k}$ on $2\hat{i} + 3\hat{j} + \hat{k}$ is

$$= \frac{4 - 2}{\sqrt{4 + 9 + 1}} = \frac{2}{\sqrt{14}}$$

$$\therefore l^2 = 8 - \frac{4}{14} = \frac{108}{14}$$

$$\Rightarrow 14l^2 = 108$$

Question 70

Let O be the origin and the position vector of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is
[29-Jan-2024 Shift 1]

Options:

A. $\frac{2}{3}\sqrt{31}$

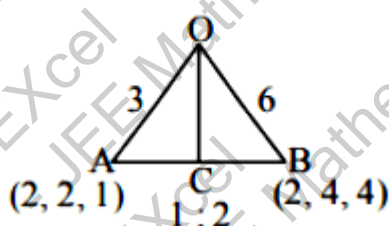
B. $\frac{2}{3}\sqrt{34}$

C. $\frac{3}{4}\sqrt{34}$

D. $\frac{3}{2}\sqrt{31}$

Answer: B

Solution:



$$\text{length of } OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$$

Question 71

Let PQR be a triangle with R(-1, 4, 2). Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of $\triangle PQR$ from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

[29-Jan-2024 Shift 1]

Options:

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Answer: C

Solution:

Centroid G divides MR in 1 : 2

G(1, 2, 2)

Point of intersection A of given lines is (2, -6, 0)

$$AG = \sqrt{69}$$

Question 72

A line with direction ratios 2, 1, 2 meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is 1, then l^2 is [29-Jan-2024 Shift 1]

Answer: 65

Solution:

Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$$\Rightarrow t=6 \text{ and } s=2$$

$$\Rightarrow P(6, 4, 6) \text{ and } Q(2, 2, 2)$$

$$PQ: \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

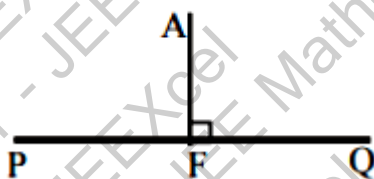
Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1, 2, 12)$

$$\vec{AF} \cdot \vec{PQ} = 0$$

$$\therefore \lambda = 2$$

So $F(6, 4, 6)$ and $AF = \sqrt{65}$



Question 73

Let $P(3, 2, 3)$, $Q(4, 6, 2)$ and $R(7, 3, 2)$ be the vertices of $\triangle PQR$.
Then, the angle $\angle QPR$ is
[29-Jan-2024 Shift 2]

Options:

A. $\frac{\pi}{6}$

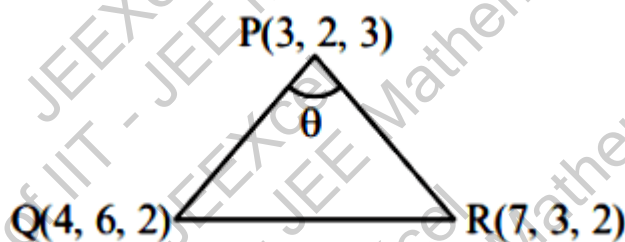
B. $\cos^{-1}\left(\frac{7}{18}\right)$

C. $\cos^{-1}\left(\frac{1}{18}\right)$

D. $\frac{\pi}{3}$

Answer: D

Solution:



Direction ratio of PR = (4, 1, -1)

Direction ratio of PQ = (1, 4, -1)

$$\text{Now, } \cos \theta = \left| \frac{4 + 4 + 1}{\sqrt{18} \cdot \sqrt{18}} \right|$$

$$\theta = \frac{\pi}{3}$$

Question 74

Let O be the origin, and M and N be the points on the lines

$\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\vec{OM} \cdot \vec{ON}$ is equal to [29-Jan-2024 Shift 2]

Answer: 9

Solution:

$$L_1: \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda \quad \text{drs}(4, 1, 3) = b_1$$

$$M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$$

$$L_2: \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$$

$$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$$

$$MN = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16) \dots\dots\dots(1)$$

Now

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \dots\dots\dots(2)$$

Equation (1) and (2)

$$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$$

I and II

$$\lambda - 5\mu + 6 = 0 \dots\dots\dots(3)$$

I and III

$$\lambda - 3\mu + 4 = 0 \dots\dots\dots(4)$$

Solve (3) and (4) we get

$$\lambda = -1, \mu = 1$$

$$\therefore M(1, 3, 2)$$

$$N(4, 3, -2)$$

$$\therefore \vec{OM} \cdot \vec{ON} = 4 + 9 - 4 = 9$$

Question 75

Let (α, β, γ) be the foot of perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. then $19(\alpha + \beta + \gamma)$ is equal to:

[30-Jan-2024 Shift 1]

Options:

A. 102

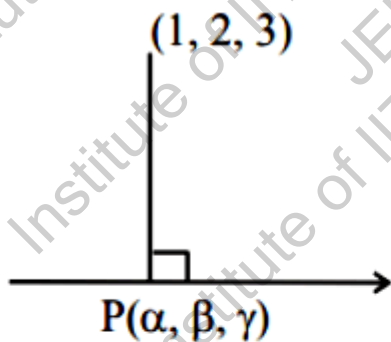
B. 101

C. 99

D. 100

Answer: B

Solution:



Let foot $P(5k-3, 2k+1, 3k-4)$

DR's $\rightarrow$ AP: $5k-4, 2k-1, 3k-7$

DR's $\rightarrow$ Line: $5, 2, 3$

Condition of perpendicular lines $(25k-20) + (4k-2) + (9k-21) = 0$

Then $k = \frac{43}{38}$

Then $19(\alpha + \beta + \gamma) = 101$

Question 76

If d_1 is the shortest distance between the lines

$x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance

between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$, $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$, then the value

of $\frac{32\sqrt{3}d_1}{d_2}$ is : —

[30-Jan-2024 Shift 1]

Answer: 16

Solution:

$$L_1: \frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}, L_2: \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

d_1 = shortest distance between L_1 & L_2

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d_1 = 2$$

$$L_3: \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, L_4: \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

d_2 = shortest distance between L_3 & L_4

$$d_2 = \frac{12}{\sqrt{3}} \text{ Hence}$$

$$= \frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \times 2}{\frac{12}{\sqrt{3}}} = 16$$

Question 77

Let $L_1: \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$

$L_2: \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + \hat{k}), \mu \in \mathbb{R}$ and

$$\mathbf{L}_3: \vec{r} = \delta(\hat{i} + m\hat{j} + n\hat{k}) \quad \delta \in \mathbb{R}$$

Be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2 . Then the point which lies on L_3 is
[30-Jan-2024 Shift 2]

Options:

A. $(-1, 7, 4)$

B. $(-1, -7, 4)$

C. $(1, 7, -4)$

D. $(1, -7, 4)$

Answer: A

Solution:

$$L_1 \perp L_2$$

$$L_3 \perp L_1, L_2$$

$$3 - 1 + 2P = 0$$

$$P = -1$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$$\therefore (-\delta, 7\delta, 4\delta) \text{ will lie on } L_3$$

For $\delta = 1$ the point will be $(-1, 7, 4)$

Question 78

Let a line passing through the point $(-1, 2, 3)$ intersect the lines

$$L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2} \text{ at } M(\alpha, \beta, \gamma) \text{ and } L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4} \text{ at}$$

$N(a, b, c)$. Then the value of $\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2}$ equals

[30-Jan-2024 Shift 2]

Answer: 196

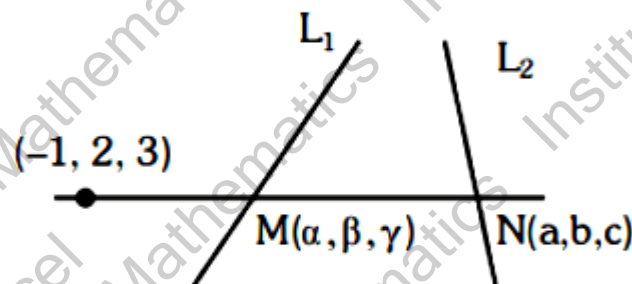
Solution:

$$M(3\lambda + 1, 2\lambda + 2, -2\lambda - 1)$$

$$\therefore \alpha + \beta + \gamma = 3\lambda + 2$$

$$N(-3\mu - 2, -2\mu + 2, 4\mu + 1)$$

$$\therefore a + b + c = -\mu + 1$$



$$\frac{3\lambda + 2}{-3\mu - 1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda - 4}{4\mu - 2}$$

$$3\lambda\mu + 2\mu = 3\lambda\mu + \lambda$$

$$2\mu = \lambda$$

$$2\lambda\mu - \lambda = \lambda\mu + 2\mu$$

$$\lambda\mu = \lambda + 2\mu$$

$$\Rightarrow \lambda\mu = 2\lambda$$

$$\Rightarrow \mu = 2 \quad (\lambda \neq 0)$$

$$\therefore \lambda = 4$$

$$\alpha + \beta + \gamma = 14$$

$$a + b + c = -1$$

$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2} = 196$$

Question 79

The distance of the point $Q(0, 2, -2)$ from the line passing through the point $P(5, -4, 3)$ and perpendicular to the lines $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k})$, $\lambda \in \mathbb{R}$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k})$, $\mu \in \mathbb{R}$ is

[31-Jan-2024 Shift 1]

Options:

A. $\sqrt{86}$

B. $\sqrt{20}$

C. $\sqrt{54}$

D. $\sqrt{74}$

Answer: D

Solution:

A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

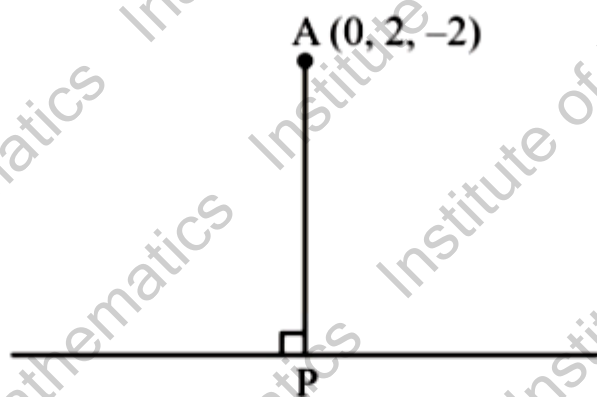
$$= -9\hat{i} - 9\hat{j} + 9\hat{k}$$

Required line,

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$$

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

Now distance of $(0, 2, -2)$



$$\text{P.V. of P} \equiv (5 + \lambda)\hat{i} + (\lambda - 4)\hat{j} + (3 - \lambda)\hat{k}$$

$$\vec{AP} = (5 + \lambda)\hat{i} + (\lambda - 6)\hat{j} + (5 - \lambda)\hat{k}$$

$$\vec{AP} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$$

$$\lambda = 2$$

$$|\vec{AP}| = \sqrt{49 + 16 + 9}$$

$$|\vec{AP}| = \sqrt{74}$$

Question 80

Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines $x = y, z = 1$ and $x = -y, z = -1$ respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____
[31-Jan-2024 Shift 1]

Answer: 12

Solution:

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \rightarrow Q(r, r, 1)$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \rightarrow R(k, -k, -1)$$

$$\vec{PQ} = (a-r)\hat{i} + (a-r)\hat{j} + (a-1)\hat{k}$$

$$a-r+a-r=0$$

$$2a=2r \rightarrow a=r$$

$$\vec{PR} = (a-k)\hat{i} + (a+k)\hat{j} + (a+1)\hat{k}$$

$$a-k-a-k=0 \Rightarrow k=0$$

$$\text{As, } PQ \perp PR$$

$$(a-r)(a-k) + (a-r)(a+k) + (a-1)(a+1) = 0$$

$$a=1 \text{ or } -1$$

$$12a^2 = 12$$

Question 81

Let (α, β, γ) be mirror image of the point $(2, 3, 5)$ in the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}.$$

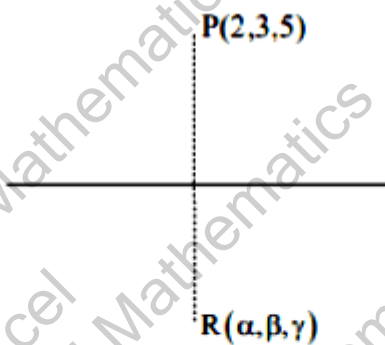
Then $2\alpha + 3\beta + 4\gamma$ is equal to
[31-Jan-2024 Shift 2]

Options:

- A. 32
- B. 33
- C. 31
- D. 34

Answer: B

Solution:



$$\therefore \vec{PR} \perp (2, 3, 4)$$

$$\therefore \vec{PR} \cdot (2, 3, 4) = 0$$

$$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$$

$$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$$

Question 82

The shortest distance between lines L_1 and L_2 , where

$L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points

$A(-4, 4, 3)$ and $B(-1, 6, 3)$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$,
is

[31-Jan-2024 Shift 2]

Options:

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Answer: C

Solution:

$$L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\therefore \text{S.D} = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$\begin{aligned}
 &= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|} \\
 &= \frac{141}{| -4\hat{i} + 6\hat{j} + 13\hat{k} |} \\
 &= \frac{141}{\sqrt{16 + 36 + 169}} \\
 &= \frac{141}{\sqrt{221}}
 \end{aligned}$$

Question 83

A line passes through A(4, -6, -2) and B(16, -2, 4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, -12, 3) is equal to [31-Jan-2024 Shift 2]

Answer: 22

Solution:

$$\frac{x-4}{12} = \frac{y+6}{4} = \frac{z+2}{6}$$

$$\frac{x-4}{6} = \frac{y+6}{2} = \frac{z+2}{3} = 21$$

$$\left(21 \times \frac{6}{7} + 4, \frac{2}{7} \times 21 - 6, \frac{3}{7} \times 21 - 2\right)$$

$$= (22, 0, 7) = (a, b, c)$$

$$\therefore \sqrt{324 + 144 + 16} = 22$$

Question 84

If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and

$\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is

[1-Feb-2024 Shift 1]

Options:

A. 0

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. $-2\sqrt{3}$

Answer: B

Solution:

Passing points of lines L_1 & L_2 are

$(\lambda, 2, 1)$ & $(\sqrt{3}, 1, 2)$

$$\text{S.D} = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left| \frac{\sqrt{3}-\lambda}{\sqrt{3}} \right|$$

$$\lambda = 0, \lambda = 2\sqrt{3}$$

Question 85

Let the line of the shortest distance between the lines

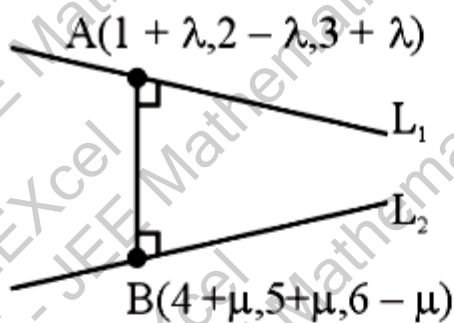
$$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to _____
[1-Feb-2024 Shift 1]

Answer: 21

Solution:



$$\vec{b} = \hat{i} - \hat{j} + \hat{k} \text{ (DR's of } L_1)$$

$$\vec{d} = \hat{i} + \hat{j} - \hat{k} \text{ (DR's of } L_2)$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 0\hat{i} + 2\hat{j} + 2\hat{k} \text{ (DR's of Line perpendicular to } L_1 \text{ and } L_2)$$

DR of AB line

$$= (0, 2, 2) = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda)$$

$$\frac{3 + \mu - \lambda}{0} = \frac{3 + \mu + \lambda}{2} = \frac{3 - \mu - \lambda}{2}$$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

$$\text{point A} = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2} \right)$$

$$\text{B} = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2} \right)$$

$$\text{Point of AB} = \left(\frac{5}{2}, 2, 6 \right) = (\alpha, \beta, \gamma)$$

$$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$$

Question 86

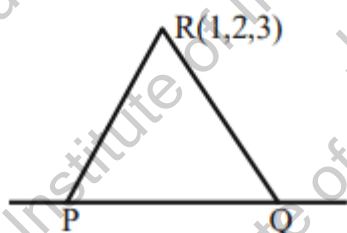
Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ) , then $\alpha^2 + \beta^2 + \gamma^2$ is:
[1-Feb-2024 Shift 2]

Options:

- A. 26
- B. 36
- C. 18
- D. 24

Answer: C

Solution:



$$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$$

$$PR = 6$$

$$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$$

$$\lambda = 0, 1$$

$$\text{Hence } P(-3, 4, -1) \text{ \& } Q(5, 6, 1)$$

$$\text{Centroid of } \Delta PQR = (1, 4, 1) \equiv (\alpha, \beta, \gamma)$$

$$\alpha^2 + \beta^2 + \gamma^2 = 18$$

Question87

If the mirror image of the point P(3, 4, 9) in the line

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} \text{ is } (\alpha, \beta, \gamma), \text{ then } 14(\alpha + \beta + \gamma) \text{ is :}$$

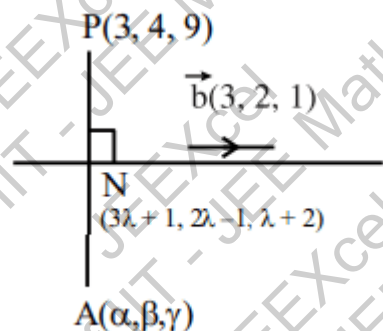
[1-Feb-2024 Shift 2]

Options:

- A. 102
- B. 138
- C. 108
- D. 132

Answer: C

Solution:



$$\vec{PN} \cdot \vec{b} = 0?$$

$$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$$

$$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$$

$$N\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$$

$$\therefore \frac{\alpha + 3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$$

$$\frac{\beta + 4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$$

$$\frac{\gamma + 9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$$

$$\text{Ans. } 14(\alpha + \beta + \gamma) = 108$$

Question 88

The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

[24-Jan-2023 Shift 1]

Options:

A. $13\sqrt{2}$

B. 31

C. 26

D. $20\sqrt{2}$

Answer: C

Solution:

Equation of line

$$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$$

G.P on line $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$

point of intersection of line & plane

$$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$$

$$\lambda = 2$$

Point $(5, 1, 8)$

$$\text{Distance} = \sqrt{36 + 64 + 576} = 26$$

Question 89

The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1)$, $(-1, 1, -2)$ and $(3, -4, 2)$ is :

[24-Jan-2023 Shift 1]

Options:

A. 4

B. 5

C. $5\sqrt{2}$

D. $4\sqrt{2}$

Answer: C

Solution:

Equation of Plane is

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$$

$$x - z - 1 = 0$$

Distance of $P(7, -3, -4)$ from Plane is

$$d = \left| \frac{7 + 4 - 1}{\sqrt{2}} \right| = 5\sqrt{2}$$

Question90

The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and

$\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to

[24-Jan-2023 Shift 1]

Answer: 14

Solution:

$$= \begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}$$

$$= \frac{16 + 12 + 168}{-4\hat{i} + 6\hat{j} - 12\hat{k}} = \frac{196}{14} = 14$$

Question91

If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes

$x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to

[24-Jan-2023 Shift 2]

Options:

A. -1

B. 3

C. 1

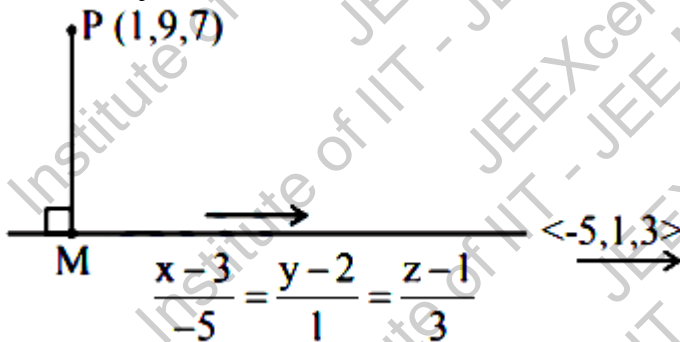
D. 5

Answer: D

Solution:

$$\text{Direction ratio of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3) \\ = -5\hat{i} + \hat{j} + 3\hat{k}$$



$$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$$

$$\vec{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$$

$$-5(-5\lambda + 2) + (\lambda + 2) + 3(3\lambda + 1) = 0$$

$$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$$

$$\Rightarrow \lambda = 1$$

$$\text{Point } M = (-2, 3, 4) = (\alpha, \beta, \gamma)$$

$$\alpha + \beta + \gamma = 5$$

Question 92

Let the plane containing the line of intersection of the planes

$$P_1 : x + (\lambda + 4)y + z = 1 \text{ and}$$

$$P_2 : 2x + y + z = 2 \text{ pass through the points } (0, 1, 0) \text{ and } (1, 0, 1).$$

Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is

[24-Jan-2023 Shift 2]

Options:

A. $5\sqrt{6}$

B. $4\sqrt{6}$

C. $2\sqrt{6}$

D. $3\sqrt{6}$

Answer: D

Solution:

Equation of plane passing through point of intersection of P1 and P2

$$P = P_1 + kP_2$$

$$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$$

Passing through (0, 1, 0) and (1, 0, 1)

$$(\lambda + 4 - 1) + k(1 - 2) = 0$$

$$(\lambda + 3) - k = 0 \dots (1)$$

Also passing (1, 0, 1)

$$(1 + 1 - 1) + k(2 + 1 - 2) = 0$$

$$1 + k = 0$$

$$k = -1$$

put in (1)

$$\lambda + 3 + 1 = 0$$

$$\lambda = -4$$

Then point $(2\lambda, \lambda, -\lambda)$

$$d = \left| \frac{-16 - 4, -4, 4}{\sqrt{6}} \right|$$

$$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$$

Question93

If the shortest between the lines $\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}$ and

$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is

[24-Jan-2023 Shift 2]

Answer: 384

Solution:

Shortest distance between the lines

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4} \quad \frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{2+2\sqrt{6}}{5} \text{ is } 6$$

Vector along line of shortest distance

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}, \Rightarrow -\hat{i} + 2\hat{j} - \hat{k} \text{ (its magnitude is } \sqrt{6} \text{)}$$

$$\text{Now } \frac{1}{\sqrt{6}} \begin{vmatrix} \sqrt{6} + \lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \pm 6$$

$$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$$

So, square of sum of these values is 384 .

Question94

Consider the lines L_1 and L_2 given by

$$L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

A line L_3 having direction ratios 1, -1, -2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is
[25-Jan-2023 Shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{2}$

C. $4\sqrt{3}$

D. 4

Answer: A

Solution:

$$\text{Let } P = (2\lambda + 1, \lambda + 3, 2\lambda + 2)$$

$$\text{Let } Q = (\mu + 2, 2\mu + 2, 3\mu + 3)$$

$$\Rightarrow \frac{2\lambda - \mu - 1}{1} = \frac{\lambda - 2\mu + 1}{-1} = \frac{2\lambda - 3\mu - 1}{-2}$$

$$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8) \text{ and } Q(5, 8, 12)$$

$$PQ = 2\sqrt{6}$$

Question 95

The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to :

[25-Jan-2023 Shift 1]

Options:

A. 3

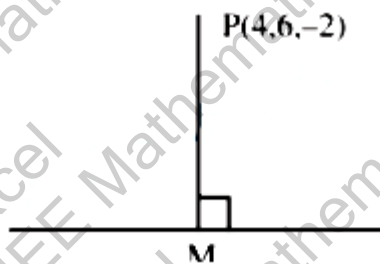
B. $\sqrt{6}$

C. $2\sqrt{3}$

D. $\sqrt{14}$

Answer: D

Solution:



$$\text{Equation of line is } \frac{x+3}{3} = \frac{y-2}{3} = \frac{z-3}{-1} = \lambda$$

$$M(3\lambda - 3, 3\lambda + 2, 3 - \lambda)$$

$$\text{D.R of PM } (3\lambda - 7, 3\lambda - 4, 5 - \lambda)$$

Since PM is perpendicular to line

$$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$$

$$\Rightarrow \lambda = 2$$

$$\Rightarrow M(3, 8, 1) \Rightarrow PM = \sqrt{14}$$

Question96

Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is

[25-Jan-2023 Shift 1]

Answer: 9

Solution:

Equation of plane is

$$(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow b = 12$$

$$\therefore \text{plane is } 13x + 10y + 35z = 65$$

$$\text{Distance from given point to plane} = 9$$

Question97

The foot of perpendicular of the point $(2, 0, 5)$ on the line

$\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ) . Then. Which of the following is NOT correct?

[25-Jan-2023 Shift 2]

Options:

A. $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

B. $\frac{\alpha}{\beta} = -8$

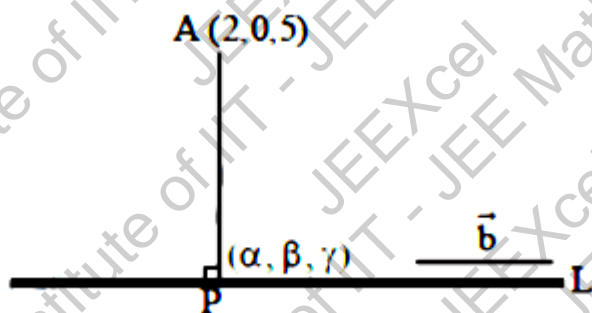
C. $\frac{\beta}{\gamma} = -5$

D. $\frac{\gamma}{\alpha} = \frac{5}{8}$

Answer: C

Solution:

$$L: \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda \text{ (let)}$$



Let foot of perpendicular is

$$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$$

$$\vec{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$$

$$\text{Direction ratio of line} \Rightarrow \vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$$

$$\text{Now, } \Rightarrow \vec{PA} \cdot \vec{b} = 0$$

$$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$$

$$\Rightarrow \lambda = -\frac{1}{6}$$

$$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \alpha = -\frac{4}{3}$$

$$\Rightarrow \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \beta = \frac{1}{6}$$

$$\Rightarrow \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \gamma = -\frac{5}{6}$$

$\therefore$ Check options

Question 98

The shortest distance between the lines $x + 1 = 2y = -12z$ and

$$x = y + 2 = 6z - 6 \text{ is}$$

[25-Jan-2023 Shift 2]

Options:

A. 2

B. 3

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

$$\frac{x+1}{1} = \frac{y}{1} = \frac{z}{12} \quad \text{and} \quad \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{6}$$

$$\Rightarrow \text{Shortest distance} = \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}$$

$$\text{S.D.} = (-\hat{i} + 2\hat{j} - \hat{k}) \cdot \frac{(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}$$

$$\left\{ \vec{p} \times \vec{q} \equiv \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{matrix} \right| = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \text{ or } 2\hat{i} - 3\hat{j} + 6\hat{k} \right\}$$

$$\text{S.D.} = \frac{(-\hat{i} + 2\hat{j} - \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{2^2 + 3^2 + 6^2}} = \left| \frac{-14}{7} \right| = 2$$

Question99

If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α , then $28\alpha^2$ is equal to

[25-Jan-2023 Shift 2]

Answer: 18

Solution:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) \quad \vec{r} = \vec{a} + \lambda\vec{p}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j}) \quad \vec{r} = \vec{b} + \mu\vec{q}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$$d = \left| \frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}} \right|$$

$$= \left| \frac{-6 + 3}{\sqrt{14}} \right| = \frac{3}{\sqrt{14}}$$

$$\alpha = \frac{3}{\sqrt{14}}$$

$$\text{Now, } 28\alpha^2 = 2/2 \times \frac{9}{14} = 18$$

Question100

Let the co-ordinates of one vertex of $\triangle ABC$ be $A(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of $\triangle ABC$ is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____.

[29-Jan-2023 Shift 1]

Answer: 9

Solution:

A. $(0, 2, \alpha)$

$$\begin{array}{ccc} \bullet & \bullet & \bullet \\ (-\alpha, 1, -4) & B & C (5\alpha + 2, 5\alpha + 3, 5\alpha + 4) \end{array}$$

$$\sqrt{(2\alpha + 5)^2 + (2\alpha + 20)^2 + (2\alpha - 5)^2} = \sqrt{21} \sqrt{38}$$

$$\Rightarrow 12\alpha^2 + 80\alpha + 450 = 798$$

$$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$$

$$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$$

Question101

Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a + b)$ and the distance of the plane P from the point $(1, 27, 7)$ be c . Then $a^2 + b^2 + c^2$ is equal to _____.
[29-Jan-2023 Shift 1]

Answer: 355

Solution:

The line $\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$ have a point $(-10, 8, 0)$ with d. r. $(1, -2, 1)$

$\therefore$ the plane $ax + by + 3z = 2(a + b)$

$\Rightarrow b = 2a$

& dot product of d.r.'s is zero

$\therefore a - 2b + 3 = 0$

$\therefore a = 1 \& b = 2$

Distance from $(1, 27, 7)$ is

$$c = \frac{1 + 54 + 21 - 6}{\sqrt{14}} = \frac{70}{\sqrt{14}} = 5\sqrt{14}$$

$$\therefore a^2 + b^2 + c^2 = 1 + 4 + 350 = 355$$

Question102

The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2 : 1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to:
[29-Jan-2023 Shift 2]

Options:

A. $\frac{17}{3}$

B. $\frac{16}{3}$

C. $\frac{73}{3}$

D. $\frac{97}{3}$

Answer: A

Solution:

$$A(a, -2, 4), B(2, b, -3)$$

$$AC : CB = 2 : 1$$

$$\Rightarrow C \equiv \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3} \right)$$

$$C \text{ lies on } 2x - y + 2 = 4$$

$$\Rightarrow \frac{2a+8}{3} - \frac{2b-2}{3} - \frac{2}{3} = 4$$

$$\Rightarrow a - b = 2 \dots (1)$$

$$\text{Also } OC = \sqrt{5}$$

$$\Rightarrow \left(\frac{a+4}{3} \right)^2 + \left(\frac{2b-2}{3} \right)^2 + \frac{4}{9} = 5 \dots (2)$$

Solving, (1) and (2)

$$(b+6)^2 + (2b-2)^2 = 41$$

$$\Rightarrow 5b^2 + 4b - 1 = 0$$

$$\Rightarrow b = -1 \text{ or } \frac{1}{5}$$

$$\Rightarrow a = 1 \text{ or } \frac{11}{5}$$

$$\text{But } ab < 0 \Rightarrow (a, b) = (1, -1)$$

$$C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3} \right), P \equiv (2, -1, -3)$$

$$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$$

Question 103

Shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ and

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3} \text{ is}$$

[29-Jan-2023 Shift 2]

Options:

A. $2\sqrt{3}$

B. $4\sqrt{3}$

C. $3\sqrt{3}$

D. $5\sqrt{3}$

Answer: B

Solution:

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \quad \vec{a} = \hat{i} - 8\hat{j} + 4\hat{k}$$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3} \quad \vec{b} = \hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{p} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \vec{q} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$$

$$= 16(\hat{i} + \hat{j} + \hat{k})$$

$$= \left| \frac{-12}{\sqrt{3}} \right| = 4\sqrt{3}$$

Question 104

If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z = a$ is :
[29-Jan-2023 Shift 2]

Options:

A. 16

B. 28

C. 10

D. 22

Answer: B

Solution:

Point on $L_1 \equiv (\lambda + 1, 2\lambda + 2, \lambda - 3)$

Point on $L_2 \equiv (2\mu + a, 3\mu - 2, \mu + 3)$

$$\lambda - 3 = \mu + 3 \Rightarrow \lambda = \mu + 6 \dots (1)$$

$$2\lambda + 2 = 3\mu - 2 \Rightarrow 2\lambda = 3\mu - 4 \dots (2)$$

Solving, (1) and (2)

$$\Rightarrow \lambda = 22 \text{ \& } \mu = 16$$

$$\Rightarrow P \equiv (23, 46, 19)$$

$$\Rightarrow a = -9$$

Distance of P from $z = -9$ is 28

Question 105

Let a unit vector $\hat{OP}$ make angle α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$ $\hat{OP}$ is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true ?
[30-Jan-2023 Shift 1]

Options:

A. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

B. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

C. $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

D. $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Answer: A

Solution:

Equation of plane :-

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow [x-1] - 4[y-2] + 3[z-3] = 0$$

$$\Rightarrow x - 4y + 3z = 2$$

D.R's of normal of plane $\langle 1, -4, 3 \rangle$

$$\text{D.C's of } \left\langle \pm \frac{1}{\sqrt{26}}, \mp \frac{4}{\sqrt{26}}, \pm \frac{3}{\sqrt{26}} \right\rangle$$

$$\cos \beta = \frac{4}{\sqrt{26}}$$

$$\cos \alpha = \frac{-1}{\sqrt{26}} \quad \frac{\pi}{2} < \alpha < \pi$$

$$\cos \gamma = \frac{-3}{\sqrt{26}} \quad \frac{\pi}{2} < \gamma < \pi$$

Question 106

The line l_1 passes through the point $(2, 6, 2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

[30-Jan-2023 Shift 1]

Options:

A. 7

B. $\frac{19}{3}$

C. $\frac{19}{3}$

D. 9

Answer: D

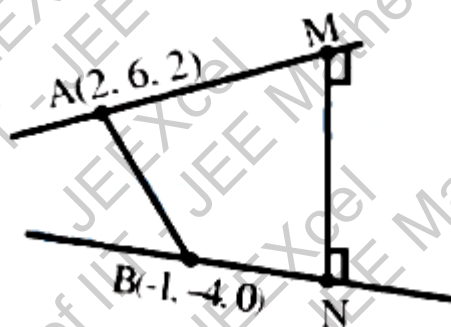
Solution:

Line ℓ , is given by

$$L_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$$

Given,

$$L_2: \frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$$



$$\text{Shortest distance} = \left| \vec{AB} \cdot \frac{\vec{MN}}{|\vec{MN}|} \right|$$

$$\vec{AB} = 3\hat{i} + 10\hat{j} + 2\hat{k}$$

$$\vec{MN} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$$

$$|\vec{MN}| = \sqrt{16 + 64 + 64} = 12$$

$$\therefore \text{Shortest distance} = \left| \frac{-12 - 80 - 16}{12} \right| = 9$$

$\therefore$ Option (4) is correct.

Question 107

If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line $x - 3y + 2z - 1 = 0$ $4x - y + z$ is

$Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to _____.

[30-Jan-2023 Shift 1]

Answer: 15

Solution:

$$x - 3y + 2z - 1 = 0$$

$$4x - y + z = 0$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix}$$

$$= -\hat{i} + 7\hat{j} + 11\hat{k}$$

Direction of normal to the plane is $-1, 7, 11$

Equation of plane :

$$-1(x-1) + 7(y-1) + 11(z-2) = 0$$

$$-x + 7y + 11z = 28$$

$$\frac{-1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$$

$$Ax + By + Cz = 1$$

$$140(C - B + A) = 140 \left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28} \right)$$

$$= 140 \times \frac{3}{28} = 15$$

Question 108

If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes

$$P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7 \text{ and } P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9 \text{ is } \sin^{-1} \left(\frac{2\sqrt{6}}{5} \right),$$

then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is _____.

[30-Jan-2023 Shift 1]

Answer: 315

Solution:

$$P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$$

$$P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$$

$$\theta = \sin^{-1} \left(\frac{2\sqrt{6}}{5} \right)$$

$$\Rightarrow \sin \theta = \frac{2\sqrt{6}}{5}$$

$$\therefore \cos \theta = \frac{1}{5}$$

$$\cos \theta = \frac{\vec{r} \cdot \vec{r}}{|\vec{r}_1| |\vec{r}_2|}$$

$$= \frac{(3i - 5j + K)(\lambda i + j - 3K)}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}}$$

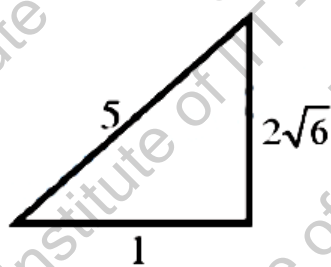
$$\frac{1}{5} = \left| \frac{3\lambda - 8}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}} \right|$$

$$\text{Square} \Rightarrow \frac{1}{25} = \frac{9\lambda^2 + 64 - 48\lambda}{35(\lambda^2 + 10)}$$

$$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$$

$$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$$

$$\Rightarrow x = 5, \frac{25}{19}$$



Perpendicular distance of point $(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$ from plane P_1

$$= \frac{|3 \times 50 - 5 \times 50 + 2 - 7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$$

$$\text{Square} = \frac{105 \times 105}{35} = 315$$

Question 109

A vector $\vec{v}$ in the first octant is inclined to the x axis at 60° , to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c) , is normal to $\vec{v}$, then
[30-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}a + b + c = 1$

B. $a + b + \sqrt{2}c = 1$

C. $a + \sqrt{2}b + c = 1$

D. $\sqrt{2}a - b + c = 1$

Answer: C

Solution:

$$\begin{aligned}\hat{v} &= \cos 60^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos \gamma \hat{k} \\ \Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \gamma &= 1 \quad (\gamma \rightarrow \text{Acute}) \\ \Rightarrow \cos \gamma &= \frac{1}{2}\end{aligned}$$

$$\Rightarrow \gamma = 60^\circ$$

Equation of plane is

$$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y + 1) + \frac{1}{2}(z - 1) = 0$$

$$\Rightarrow x + \sqrt{2}y + z = 1$$

(a, b, c) lies on it.

$$\Rightarrow a + \sqrt{2}b + c = 1$$

Question 110

If a plane passes through the points $(-1, k, 0)$, $(2, k, -1)$, $(1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is [30-Jan-2023 Shift 2]

Options:

A. $\frac{17}{5}$

B. $\frac{5}{17}$

C. $\frac{6}{13}$

D. $\frac{13}{6}$

Answer: D

Solution:

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$$

$$\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$$

Points : A (-1, k, 0), B(2, k, -1), C(1, 1, 2)

$$\vec{CA} = -2\hat{i} + (k-1)\hat{j} - 2\hat{k}$$

$$\vec{CB} = \hat{i} + (k-1)\hat{j} - 3\hat{k}$$

$$\vec{CA} \times \vec{CB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & k-1 & -2 \\ 1 & k-1 & -3 \end{vmatrix}$$

$$= \hat{i}(-3k+3+2k-2) - \hat{j}(6+2) + \hat{k}(-2k+2-k+1)$$

$$= (1-k)\hat{i} - 8\hat{j} + (3-3k)\hat{k}$$

The line $\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$ is perpendicular to normal vector.

$$\therefore 1 \cdot (1-k) + 1(-8) + (-1)(3-3k) = 0$$

$$\Rightarrow 1-k-8-3+3k=0$$

$$\Rightarrow 2k=10 \Rightarrow k=5$$

$$\therefore \frac{k^2+1}{(k-1)(k-2)} = \frac{26}{4 \cdot 3} = \frac{13}{6}$$

Question111

Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point (5, 3, 8) is α , then $3\alpha^2$ is equal to _____.

[30-Jan-2023 Shift 2]

Answer: 158

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\therefore \text{Equation of line is } \frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, R $\equiv$ (k+2, -k+3, -k+1)

DR of QR are (k-3, -k, -k-7)

$$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$$

$$\Rightarrow k = -\frac{4}{3}$$

$$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$$

$$\therefore 3\alpha^2 = 158$$

Question112

Let the shortest distance between the lines

$$L : \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0 \text{ and } L_1 : x+1 = y-1 = 4-z \text{ be } 2\sqrt{6}. \text{ If}$$

(α, β, γ) lies on L, then which of the following is NOT possible?

[31-Jan-2023 Shift 1]

Options:

A. $\alpha + 2\gamma = 24$

B. $2\alpha + \gamma = 7$

C. $2\alpha - \gamma = 9$

D. $\alpha - 2\gamma = 19$

Answer: A

Solution:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$$

$$a_2 - a_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$$

$$2\sqrt{6} = \left| \frac{-6 - \lambda + 1 + 2\lambda + 8}{\sqrt{1+1+4}} \right|$$

$$|\lambda + 3| = 12 \Rightarrow \lambda = 9, -15$$

$$\alpha = -2k + 5, \gamma = k - \lambda \text{ where } k \in \mathbb{R}$$

$$\Rightarrow \alpha + 2\gamma = 5 - 2\lambda = -13, 35$$

Question 113

Let θ be the angle between the planes $P_1 = \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and

$$P_2 = \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15.$$

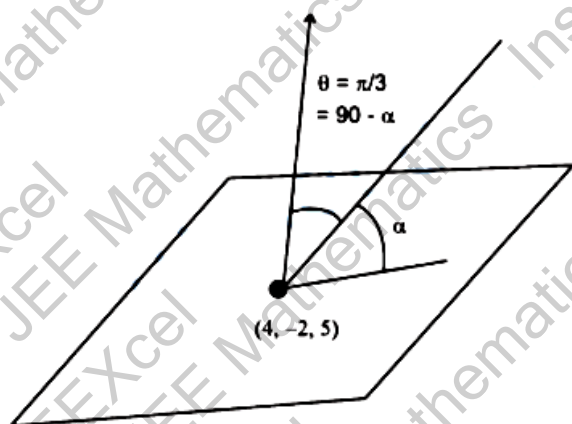
Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2 . If α is the angle between L and P_2

then $(\tan^2 \theta)(\cot^2 \alpha)$ is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 9

Solution:



$$\cos \theta = \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})}{6} = \frac{2 - 1 + 2}{6} = \frac{1}{2}$$

$$\theta = \pi/3 \quad \alpha = \pi/6$$

$$\frac{(\tan^2 \theta)(\cot^2 \alpha)}{(3)(3)} = 9$$

Question 114

Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point R(1, -1, -3) on the line L. If α is the area of triangle PQR, then α^2 is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 180

Solution:

Any point on L $((2\lambda + 1), (-\lambda - 1), (\lambda + 3))$

$$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$$

$$6\lambda + 10 = 16 \Rightarrow \lambda = 1$$

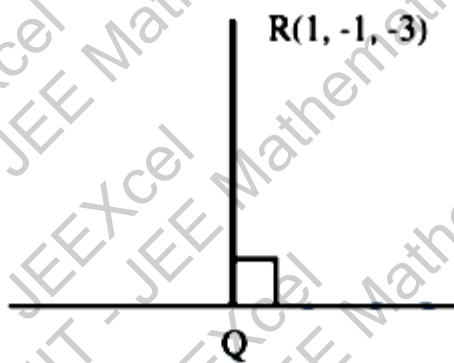
$$\therefore P = (3, -2, 4)$$

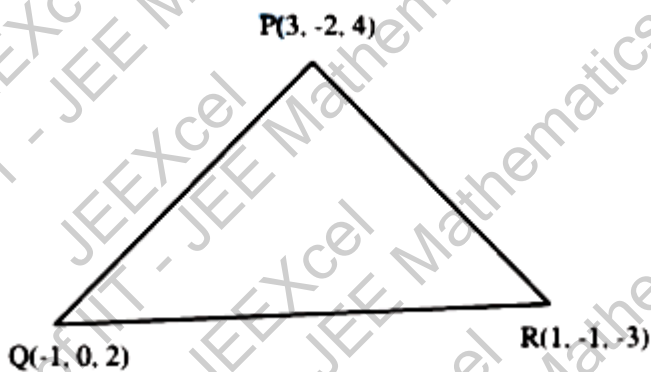
$$\text{DR of QR} = \langle 2\lambda, -\lambda, \lambda + 6 \rangle$$

$$\text{DR of L} = \langle 2, -1, 1 \rangle$$

$$4\lambda + \lambda + \lambda + 6 = 0 \quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$$

$$Q = (-1, 0, 2)$$





$$\vec{QR} = 2\hat{i} - \hat{j} - 5\hat{k} \quad \vec{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{QR} \times \vec{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$$

Question 115

If a point $P(\alpha, \beta, \gamma)$ satisfying $(\alpha\beta\gamma) \begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0 \ 0 \ 0)$ lies on

the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to:

[31-Jan-2023 Shift 2]

Options:

A. -1

B. $\frac{11}{5}$

C. $\frac{5}{4}$

D. 11

Answer: D

Solution:

$$2\alpha + 4\beta + 3\gamma = 5 \dots (1)$$

$$2\alpha + 9\beta + 8\gamma = 0 \dots (2)$$

$$10\alpha + 3\beta + 4\gamma = 0 \dots (3)$$

$$8\alpha + 8\beta + 8\gamma = 0 \dots (4)$$

Subtract (4) from (2)

$$-6\alpha + \beta = 0$$

$$\beta = 6\alpha \dots (5)$$

From equation (4)

$$8\alpha + 48\alpha + 8\gamma = 0$$

$$\gamma = -7\alpha \dots (6)$$

From equation (1)

$$2\alpha + 24\alpha - 21\alpha = 5$$

$$5\alpha = 5$$

$$\alpha = 1$$

$$\beta = 6, \gamma = -7$$

$$\therefore 6\alpha + 9\beta + 7\gamma$$

$$= 6 + 54 - 49$$

$$= 11$$

Question 116

Let the plane $P : 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line

$L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is :

[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{14}$

B. $\frac{6}{\sqrt{14}}$

C. $\sqrt{\frac{2}{7}}$

D. $\sqrt{\frac{7}{2}}$

Answer: A

Solution:

$$P: 8x + \alpha_1 y + \alpha_2 z + 12 = 0$$

$$L: \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$$

$\therefore P$ is parallel to L

$$\Rightarrow 8(2) + \alpha_1(3) + 5(\alpha_2) = 0$$

$$\Rightarrow 3\alpha_1 + 5(\alpha_2) = -16$$

Also y-intercept of plane P is 1

$$\Rightarrow \alpha_1 = -12$$

$$\text{And } \alpha_2 = 4$$

$\Rightarrow$ Equation of plane P is $2x - 3y + z + 3 = 0$

$\Rightarrow$ Distance of line L from Plane P is

$$= \left| \frac{0 - 3(6) + 1 + 3}{\sqrt{4 + 9 + 1}} \right|$$

$$= \sqrt{14}$$

Question117

Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$.

Then the distance of P from the point $(3, -2, 2)$ is

[31-Jan-2023 Shift 2]

Options:

A. 6

B. 4

C. 5

D. 7

Answer: C

Solution:

Equation of Plane :

$$2(x-1) - 3(y+1) - 6(z+5) = 0$$

$$\text{Or } 2x - 3y - 6z = 35$$

$$\Rightarrow \text{Required distance} = \frac{|2(3) - 3(-2) - 6(2) - 35|}{\sqrt{4 + 9 + 36}} = 5$$

Question 118

The shortest distance between the lines $\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and

$\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is
[1-Feb-2023 Shift 1]

Options:

A. $7\sqrt{3}$

B. $5\sqrt{3}$

C. $6\sqrt{3}$

D. $4\sqrt{3}$

Answer: C

Solution:

Shortest distance between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3} \text{ \& }$$

$$\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3} \text{ is given as}$$

$$\frac{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{(a_1 b_3 - a_3 b_2)^2 + (a_1 b_3 - a_3 b_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

$$\frac{\begin{vmatrix} 5 - (-3) & 2 - (-5) & 4 - 1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(-10 + 12)^2 + (-5 + 3)^2 + (4 - 2)^2}}$$

$$\begin{aligned}
 & \sqrt{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}^2 + (2)^2 + (2)^2} \\
 &= \frac{|8(-10+12) - 7(-5+3) + 3(4-2)|}{\sqrt{4+4+4}} \\
 &= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}} \\
 &= \frac{18}{\sqrt{3}} = 6\sqrt{3}
 \end{aligned}$$

Question 119

Let the image of the point $P(2, -1, 3)$ in the plane $x + 2y - z = 0$ be Q . Then the distance of the plane $3x + 2y + z + 29 = 0$ from the point Q is
[1-Feb-2023 Shift 1]

Options:

A. $\frac{22\sqrt{2}}{7}$

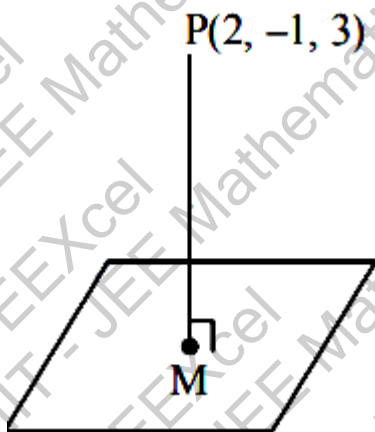
B. $\frac{24\sqrt{2}}{7}$

C. $2\sqrt{14}$

D. $3\sqrt{14}$

Answer: D

Solution:



$$\text{eq. of line PM } \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \lambda$$

$$\text{any point on line } = (\lambda+2, 2\lambda-1, -\lambda+3)$$

$$\text{for point 'm' } (\lambda+2) + 2(2\lambda-1) - (3-\lambda) = 0$$

$$\lambda = \frac{1}{2}$$

$$\text{Point m} \left(\frac{1}{2} + 2, 2 \times \frac{1}{2} - 1, -\frac{1}{2} + 3 \right)$$

$$= \left(\frac{5}{2}, 0, \frac{5}{2} \right)$$

For Image Q(α, β, γ)

$$\frac{\alpha+2}{2} = \frac{5}{2}, \quad \frac{\beta-1}{2} = 0,$$

$$\frac{\gamma+3}{2} = \frac{5}{2}$$

$$Q : (3, 1, 2)$$

$$d = \left| \frac{3(3) + 2(1) + 2 + 29}{\sqrt{3^2 + 2^2 + 1^2}} \right|$$

$$d = \frac{42}{\sqrt{14}} = 3\sqrt{14}$$

Question 120

Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to :

[1-Feb-2023 Shift 2]

Options:

A. $\frac{250}{83}$

B. $\frac{15}{53}$

C. $\frac{25}{83}$

D. $\frac{250}{82}$

Answer: A

Solution:

$$P \equiv P_1 + \lambda P_2 = 0$$

$$(2 + \lambda)x + (3 + 2\lambda)y + (3\lambda - 1)z - 2 - 6\lambda = 0$$

Plane P is perpendicular to $P_3 \therefore \vec{n} \cdot \vec{n}_3 = 0$

$$2(\lambda + 2) + (2\lambda + 3) - (3\lambda - 1) = 0$$

$$\lambda = -8$$

$$P \equiv -6x - 13y - 25z + 46 = 0$$

$$6x + 13y + 25z - 46 = 0$$

Dist from $(-7, 1, 1)$

$$d = \left| \frac{-42 + 13 + 25 - 46}{\sqrt{36 + 169 + 625}} \right| = \frac{50}{\sqrt{830}}$$

$$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$$

Question121

The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $A(-3, -6, 1)$ and $B(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c ($|a|, |b|, |c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to _____.

[1-Feb-2023 Shift 2]

Answer: 10

Solution:

Plane: $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

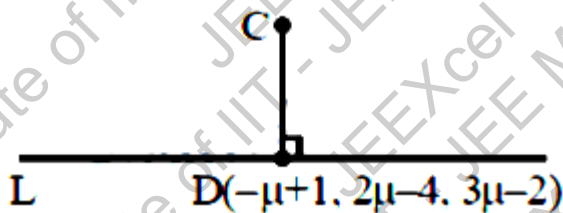
Point of intersection of line and plane

$$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$$

$$\lambda = -\frac{1}{3}$$

$$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$$

$$L: \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$$



$$\vec{CD} = \left(-\mu + \frac{2}{3}\right) \hat{i} + \left(2\mu - \frac{14}{3}\right) \hat{j} + \left(3\mu - \frac{1}{3}\right) \hat{k}$$

$$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$$

$$\mu = \frac{11}{14}$$

$$\vec{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$$

Direction ratios $\rightarrow (-1, -26, 17)$

$$|a + b + c| = 10$$

Question 122

Let $ax + \beta y + \gamma z = 1$ be the equation of a plane passing through the point $(3, -2, 5)$ and perpendicular to the line joining the points $(1, 2, 3)$ and $(-2, 3, 5)$. Then the value of $\alpha\beta\gamma$ is equal to _____.

[1-Feb-2023 Shift 2]

Answer: 6

Solution:

Given Equation is not equation of plane as γz is present. If we consider y is γ then answer would be 6.

Normal vector of plane $\hat{n} = 3\hat{i} - \hat{j} - 2\hat{k}$

Plane : $3x - y - 2z + \lambda = 0$

Point $(3, -2, 5)$ satisfies the plane

$$\lambda = -1$$

$$3x - y - 2z = 1$$

$$6 - 2 - 10 = -6$$

Question 123

One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :
[6-Apr-2023 shift 1]

Options:

A. $\frac{12}{5\sqrt{5}}$

B. $12\sqrt{5}$

C. $\frac{12}{5}$

D. $\frac{12}{\sqrt{5}}$

Answer: C

Solution:

Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$$a_1 = (0, 0, 0) \quad a_2 = (3, 0, 5)$$

$$b_1 = (3, 4, 5) \quad b_2 = (0, 0, 1)$$

Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$S.D = \frac{(\vec{a}_2 \cdot \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\frac{\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{3(4)}{|4\hat{i} - 3\hat{j}|} = \frac{12}{5}$$

Question 124

If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to :

[6-Apr-2023 shift 1]

Options:

- A. 15
- B. 14
- C. 13
- D. 12

Answer: B

Solution:

Using family of planer

$$P : P_1 + \lambda P_2 = 0 \Rightarrow P(2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z = (3 - 9\lambda)$$

$$P \text{ is } \parallel \text{ to } \frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$$

$$\text{Then for } \lambda : \vec{n}_p \cdot \vec{v}_L = 0$$

$$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$$

$$-3 + 5\lambda = 0 \Rightarrow \lambda = \frac{3}{5}$$

$$\text{Hence : } P : 22x - 14y + 20z = -12$$

$$P : 11x - 7y + 10z + 6 = 0$$

$$\Rightarrow a = 11$$

$$b = -7$$

$$c = 10$$

$$\Rightarrow a + b + c = 14$$

Question 125

Let the image of the point $P(1, 2, 3)$ in the plane $2x - y + z = 9$ be Q . If the coordinates of the point R are $(6, 10, 7)$, then the square of the area of the triangle PQR is _____.

[6-Apr-2023 shift 1]

Answer: 594

Solution:

Let $Q(\alpha, \beta, \gamma)$ be the image of P , about the plane

$$2x - y + z = 9$$

$$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$$

$$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$$

Then area of triangle PQR is $= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

$$= \left| -12\hat{i} - 3\hat{j} + 21\hat{k} \right| = \sqrt{144 + 9 + 441} = \sqrt{594}$$

$$\text{Square of area} = 594$$

Question 126

Let the line L pass through the point $(0, 1, 2)$, intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point $P(1, -9, 2)$ from the line L is :

[6-Apr-2023 shift 2]

Options:

A. 9

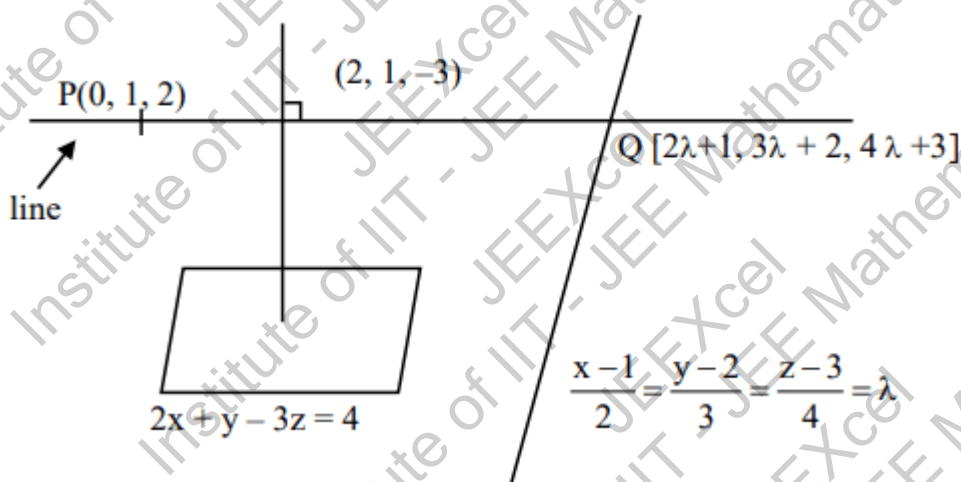
B. $\sqrt{54}$

C. $\sqrt{69}$

D. $\sqrt{74}$

Answer: D

Solution:



$$\vec{PQ} = (2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$$

$$\vec{PQ} \cdot \vec{n} = 0 \Rightarrow (2\lambda + 1) \cdot (2) + (3\lambda + 1)(1) + (4\lambda + 1)(-3) = 0$$

$$\Rightarrow -5\lambda = 0$$

$$\Rightarrow \lambda = 0$$

$$Q = (1, 2, 3)$$

$$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \mu$$

distance of line from $(1, -9, 2)$

$$(\vec{PQ}) \cdot (1, 1, 1) = 0$$

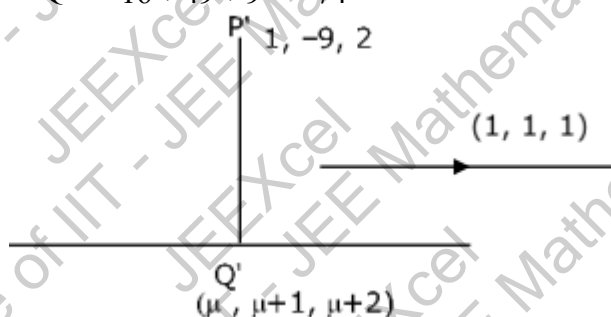
$$\Rightarrow [\mu - 1, \mu + 10, \mu] \cdot [1, 1, 1] = 0$$

$$\Rightarrow \mu - 1 + \mu + 10 + \mu = 0$$

$$\mu = -3$$

$$Q' = (-3, -2, 1)$$

$$PQ' = \sqrt{16 + 49 + 9} = \sqrt{74}$$



Question 127

A plane P contains the line of intersection of the plane

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point $(0, 2, -2)$, then the square of distance of the point $(12, 12, 18)$ from the plane P is :
[6-Apr-2023 shift 2]

Options:

- A. 620
- B. 1240
- C. 310
- D. 155

Answer: A

Solution:

eqⁿ of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$

pass th. $(0, 2, -2)$

$(-6) + \lambda(6 - 8 + 5) = 0$

$(-6) + \lambda[3] = 0 \Rightarrow \lambda = 2$

eqⁿ of plane

$5x + 7y + 9z + 4 = 0$

distance from $(12, 12, 18)$

$$d = \left| \frac{60 + 84 + 162 + 4}{\sqrt{25 + 49 + 81}} \right|$$

$$d = \frac{310}{\sqrt{155}}$$

$$d^2 = \frac{310 \times 310}{155}$$

$$d^2 = 620$$

Ans. Option 1

Question 128

If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____.

[6-Apr-2023 shift 2]

Answer: 18

Solution:

If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect Point of first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i} + \hat{j} + 3\hat{k}$

Now vector along second line is $2\hat{i} + 3\hat{j} + \alpha\hat{k}$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix}$$

$$\Rightarrow 2(6 - \beta) - 3(15 + 3\beta) + \alpha(11) = 0$$

$$\Rightarrow \alpha - \beta = 3$$

$$\text{Now } \alpha = 3 + \beta$$

$$\text{Given expression } 8(3 + \beta) \cdot \beta = 8(\beta^2 + 3\beta)$$

$$= 8\left(\beta^2 + 3\beta + \frac{9}{4} - \frac{9}{4}\right) = 8\left(\beta + \frac{3}{2}\right)^2 - 18$$

So magnitude of minimum value = 18

Question 129

The shortest distance between the lines $\frac{x-4}{4} = \frac{y+2}{5} = \frac{z+3}{3}$ and

$\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{2}$ is

[8-Apr-2023 shift 1]

Options:

A. $2\sqrt{6}$

B. $3\sqrt{6}$

C. $6\sqrt{3}$

D. $6\sqrt{2}$

Answer: B

Solution:

$$S_d = \left| \frac{(\vec{a} - \vec{b}) \times (\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|} \right|$$

$$\vec{a} = (4, -2, -3)$$

$$\vec{b} = (1, 3, 4)$$

$$\vec{n}_1 = (4, 5, 3)$$

$$\vec{n}_2 = (3, 4, 2)$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2) - \hat{j}(-1) + \hat{k}(1) = (-2, 1, 1)$$

$$S_d = \frac{(3, -5, -7) \cdot (-2, 1, 1)}{\sqrt{6}} = \left| \frac{-6 - 5 - 7}{\sqrt{6}} \right| = 3\sqrt{6}$$

Question130

If the equation of the plane containing the line $x + 2y + 3z - 4 = 0$ and perpendicular to the plane

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is $ax + by + cz = 4$, then

$(a - b + c)$ is equal to

[8-Apr-2023 shift 1]

Options:

A. 22

B. 24

C. 20

D. 18

Answer: A

Solution:

$$\text{D.R's of line } \vec{n}_1 = -5\hat{i} + 7\hat{j} - 3\hat{k}$$

D.R's of normal of second plane

$$\vec{n}_2 = 5\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\vec{n}_1 \times \vec{n}_2 = -27\hat{i} - 30\hat{j} - 25\hat{k}$$

A point on the required plane is $\left(0, -\frac{11}{5}, \frac{14}{5}\right)$

The equation of required plane is

$$27x + 30y + 25z = 4$$

$$\therefore a - b + c = 22$$

Question131

Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line

$$\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2} \text{ is } \underline{\hspace{2cm}}.$$

[8-Apr-2023 shift 1]

Answer: 9

Solution:

$$2x + 3y - 6z + 7 = 0 \left(\frac{5}{2}, 1, \lambda \right), (-2, 0, 1)$$

$$d_1 = \left| \frac{5+3-6\lambda+7}{7} \right| = d_2 = \left| \frac{-4-6+7}{7} \right|$$

$$\Rightarrow |15-6\lambda| = |3|$$

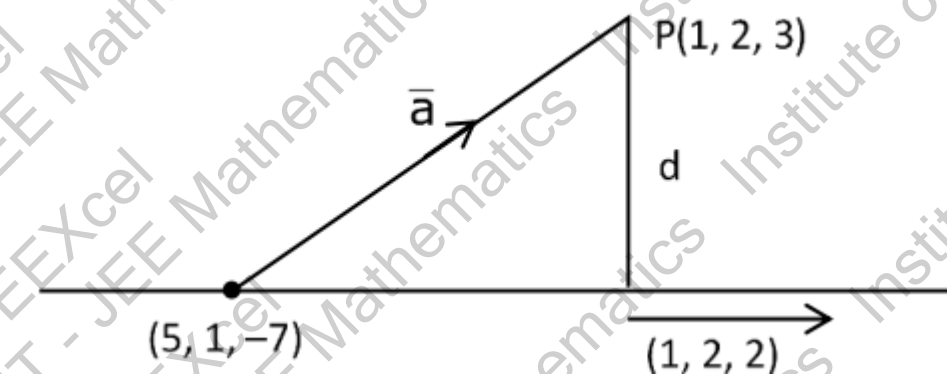
$$15-6\lambda = 3 \text{ or } 15-6\lambda = -3$$

$$6\lambda = 12 \quad 6\lambda = 18$$

$$\lambda = 2 \quad \lambda = 3$$

$$\lambda_1 = 3, \quad \lambda_2 = 2$$

$$\therefore P(1, 2, 3) \quad \frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$$



$$d = \left| \frac{(4, -1, -10) \times (1, 2, 2)}{3} \right| = \left| \frac{18\hat{i} - 18\hat{j} + 9\hat{k}}{3} \right| = 9$$

Question 132

For $a, b \in \mathbb{Z}$ and $|a - b| \leq 10$, let the angle between the plane

$P : ax + y - z = b$ and the line $l : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If

the distance of the point $(6, -6, 4)$ from the plane P is $3\sqrt{6}$, then

$a^4 + b^2$ is equal to

[8-Apr-2023 shift 2]

Options:

A. 85

B. 48

C. 25

D. 32

Answer: D

Solution:

$$\theta = \cos^{-1}\frac{1}{3} \therefore \sin \theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin \theta = \frac{a \cdot 1 + 1(-1) + (-1) \cdot 1}{\sqrt{a^2 + 1 + 1} \cdot \sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \{3(a - 2)\}^2 = 24(a^2 + 2)$$

$$\Rightarrow 3(a^2 - 4a + 4) = 8a^2 + 16$$

$$\Rightarrow 5a^2 + 12a + 4 = 0$$

$$\Rightarrow 5a^2 + 10a + 2a + 4 = 0$$

$$\therefore a = -2, -\frac{2}{5} \therefore a \in \mathbb{Z}$$

$$\therefore a = -2$$

Distance of $(6, -6, 4)$ from

$-2x + y - z - b = 0$ is $3\sqrt{6}$

$$\therefore \left| \frac{-12 - 6 - 4 - b}{\sqrt{4 + 1 + 1}} \right| = 3\sqrt{6}$$

$$\Rightarrow |b + 22| = 18 \therefore b = -40, -4$$

$$\therefore |a - b| \leq 10$$

$$\therefore b = -4$$

$$\therefore a^4 + b^2$$

$$= 32 \text{ Ans.}$$

Question 133

Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to _____.

[8-Apr-2023 shift 2]

Answer: 11

Solution:

P_2 is given by

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$$x - y - z = 3$$

DR of line intersection of P_1 & P_2

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$$+ 6\hat{i} + 4\hat{j} + 2\hat{k}$$

Let

$$z = 0, \quad x - y = 3$$

$$3x - y = 11, 2x = 8$$

$$x = 4$$

$$y = 1$$

So Line is

$$\frac{x-4}{6} = \frac{y-1}{4} = \frac{z-0}{2} = r$$

$$(\alpha, \beta, \gamma) = (6r+4, 4r+1, 2r)$$

$$6(\alpha-7) + 4(\beta-4) + 2(\gamma+1) = 0$$

$$6\alpha - 42 + 4\beta - 16 + 2\gamma + 2 = 0$$

$$36r + 24 + 16r + 4 + 4r - 56 = 0$$

$$56r = 28$$

$$r = \frac{1}{2} \quad \alpha + \beta + \gamma = 12r + 5$$

$$= 6 + 5 = 11$$

Question 134

Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point $(2, 4, -3)$. If the image of the point $(-1, 3, 4)$ in the plane P is (α, β, γ) then $\alpha + \beta + \gamma$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 12

B. 9

C. 10

D. 11

Answer: C

Solution:

Equation of plane is given by

$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$

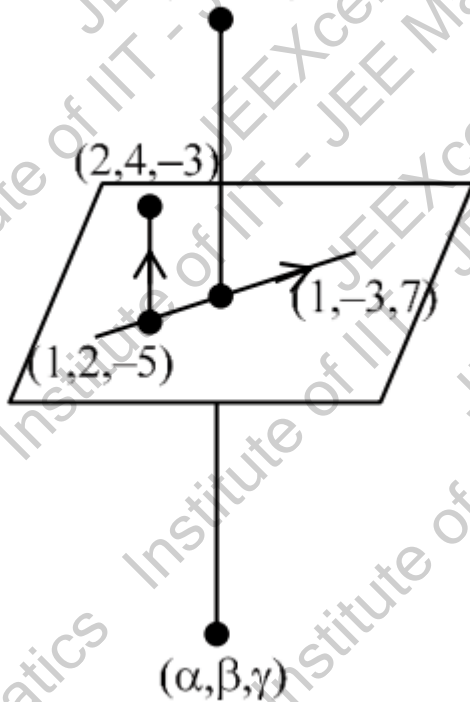
$$4x - y - z = 7$$

$$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$$\alpha = 7, \beta = 1, \gamma = 2$$

$$\alpha + \beta + \gamma = 10 \text{ (Option 3)}$$

$$(-1, 3, 4)$$



Question 135

Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A , B and C are $-2\hat{i} + \hat{j} - 3\hat{k}$, $2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\vec{OP}$ on a vector perpendicular to the vectors $\vec{AB}$ and $\vec{AC}$ is :

[10-Apr-2023 shift 1]

Options:

A. $\frac{10}{3}$

B. $\frac{8}{3}$

C. $\frac{7}{3}$

D. 3

Answer: D

Solution:

Position vector of the point P(-1, -2, 3), A(-2, 1, -3), B(2, 4, -2), and C(-4, 2, -1)

Then $\vec{OP} \cdot \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|}$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(5) - \hat{j}(8+2) + \hat{k}(4+6)$$

$$= 5\hat{i} - 10\hat{j} + 10\hat{k}$$

Now

$$\begin{aligned} \vec{OP} \cdot \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} &= (-\hat{i} - 2\hat{j} + 3\hat{k}) \cdot \frac{(5\hat{i} - 10\hat{j} + 10\hat{k})}{\sqrt{(5)^2 + (-10)^2 + (10)^2}} \\ &= \frac{-5 + 20 + 30}{\sqrt{25 + 100 + 100}} \\ &= \frac{45}{\sqrt{225}} = \frac{45}{15} = 3 \end{aligned}$$

Question136

Let two vertices of a triangle ABC be (2, 4, 6) and (0, -2, -5), and its centroid be (2, 1, -1). If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ) , then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. 76

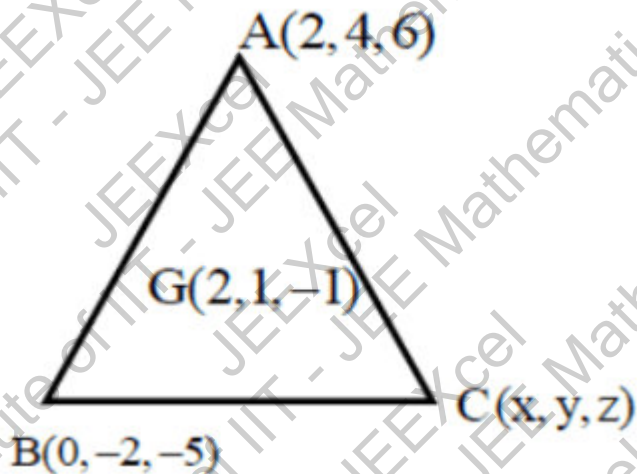
B. 74

C. 70

D. 72

Answer: B

Solution:



Given Two vertices of Triangle $A(2, 4, 6)$ and $B(0, -2, -5)$ and if centroid $G(2, 1, -1)$

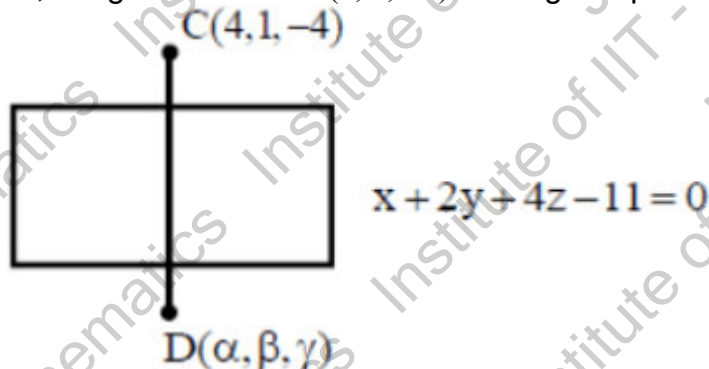
Let Third vertices be (x, y, z)

$$\text{Now } \frac{2+0+x}{3} = 2, \quad \frac{4-2+y}{3} = 1, \quad \frac{6-5+z}{3} = -1$$

$$x = 4, y = 1, z = -4$$

Third vertices $C(4, 1, -4)$

Now, Image of vertices $C(4, 1, -4)$ in the given plane is $D(\alpha, \beta, \gamma)$



Now

$$\frac{\alpha - 4}{1} = \frac{\beta - 1}{2} = \frac{\gamma + 4}{4} = -2 \frac{(4 + 2 - 16 - 11)}{1 + 4 + 16}$$

$$\frac{\alpha - 4}{1} = \frac{\beta - 1}{2} = \frac{\gamma + 4}{4} = \frac{42}{21} \Rightarrow 2$$

$$\alpha = 6, \beta = 5, \gamma = 4$$

$$\text{Then } \alpha\beta + \beta\gamma + \gamma\alpha$$

$$= (6 \times 5) + (5 \times 4) + (4 \times 6)$$

$$= 30 + 20 + 24$$

$$= 74$$

Question137

The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and

$$\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0} \text{ is :}$$

[10-Apr-2023 shift 1]

Options:

A. 8

B. 7

C. 6

D. 9

Answer: D

Solution:

$$\begin{aligned} \frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2} \text{ and } \frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0} \\ = \frac{|\vec{r_1} - \vec{r_2}|}{|\vec{d_1} \times \vec{d_2}|} \\ = \frac{54}{\sqrt{16+4+16}} \\ = \frac{54}{6} \\ = 9 \end{aligned}$$

Question 138

Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the

plane $x + y + z = 2$. If the distance of the point P from the plane

$3x - 4y + 12z = 32$ is q, then q and 2q are the roots of the equation :

[10-Apr-2023 shift 1]

Options:

A. $x^2 + 18x - 72 = 0$

B. $x^2 + 18x + 72 = 0$

C. $x^2 - 18x - 72 = 0$

D. $x^2 - 18x + 72 = 0$

Answer: D

Solution:

$$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$$

$$x = 3\lambda - 3, y = \lambda - 2, z = 1 - 2\lambda$$

$P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of plane $x + y + z = 2$.

$$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$$

$$2\lambda - 4 = 2$$

$$\lambda = 3$$

$$P(6, 1, -5)$$

Perpendicular distance of P from plane $3x - 4y + 12z - 32 = 0$ is

$$q = \left| \frac{3(6) - 4(1) + 12(-5) - 32}{\sqrt{9 + 16 + 144}} \right|$$

$$q = 6$$

$$2q = 12$$

$$\text{Sum of roots} = 6 + 12 = 18$$

$$\text{Product of roots} = 6(12) = 72$$

$\therefore$ Quadratic equation having q and 2q as roots is $x^2 - 18x + 72$.

Question 139

Let the image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be Q(α , β , γ). Then

($\alpha^2 + \beta^2 + \gamma^2$) is equal to

[10-Apr-2023 shift 2]

Options:

A. 70

B. 76

C. 62

D. 65

Answer: D

Solution:

$$\text{Equation of plane } A(x-1) + B(y-2) + C(z-0) = 0$$

$$\text{Put } (1, 4, 1) \Rightarrow 2B + C = 0$$

$$\text{Put } (0, 5, 1) \Rightarrow -A + 3B + C = 0$$

$$\text{Sub : } B - A = 0 \Rightarrow A = B, C = -2B$$

$$1(x-1) + 1(y-2) - 2(z-0) = 0$$

$$x + y - 2z - 3 = 0$$

$$\text{Image is } (\alpha, \beta, \gamma) \text{pt} \equiv (1, 2, 6)$$

$$\frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2} = \frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2} = 4$$

$$\alpha = 5, \beta = 6, \gamma = -2 \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 25 + 36 + 4 = 65$$

Question 140

Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is
[10-Apr-2023 shift 2]

Options:

A. 3

B. $\frac{10}{3}$

C. 4

D. $\frac{11}{3}$

Answer: C

Solution:

$$\frac{x}{1} = \frac{y-6}{-2} = \frac{z+8}{5} = \lambda \dots (1)$$

$$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} = \mu \dots (2)$$

$$\frac{x+3}{6} = \frac{y-3}{-3} = \frac{z-6}{1} = \gamma \dots (3)$$

Intersection of (1)&(2) "A"

$$(\lambda, -2\lambda+6, 5\lambda-8) & (4\mu+5, 3\mu+7, \mu-2)$$

$$\lambda = 1, \mu = -1$$

$$A(1, 4, -3)$$

Intersection (1) & (3) B"

$$(\lambda, -2\lambda+6, 5\lambda-8) & (6\gamma-3, -3\gamma+3, \gamma+6)$$

$$\lambda = 3$$

$$\gamma = 1$$

$$B(3, 0, 7)$$

$$\text{Mid point of A \& B} \Rightarrow (2, 2, 2)$$

Perpendicular distance from the plane

$$2x - 2y + z = 14$$

$$\frac{2(2) - 2(2) + 2 - 14}{\sqrt{4 + 4 + 1}} = 4$$

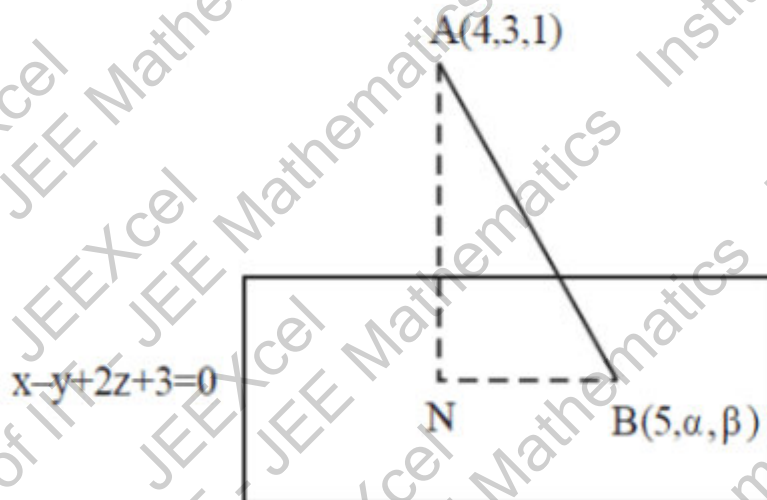
Question141

Let the foot of perpendicular from the point A(4, 3, 1) on the plane $P : x - y + 2z + 3 = 0$ be N. If B(5, α , β), $\alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _____.

[10-Apr-2023 shift 2]

Answer: 7

Solution:



$$AN = \sqrt{6}$$

$$5 - \alpha + 2\beta + 3 = 0$$

$$\Rightarrow \alpha = 8 + 2\beta$$

N is given by

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$$

$$x = 3, y = 4, z = -1$$

N

$$(3, 4, -1)$$

$$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$$

$$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$$

$$\text{Area of } \triangle ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$$

$$\frac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2}$$

$$BN = 2\sqrt{3}$$

$$4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$$

$$(2\beta + 4)^2 + (\beta + 1)^2 - 8 = 0$$

$$5\beta^2 + 18\beta + 9 = 0$$

$$(5\beta + 3)(\beta + 3) = 0$$

$$\beta = -3$$

$$\alpha = 2$$

$$\alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$$

Question 142

Let (α, β, γ) be the image of the point $P(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to :
[11-Apr-2023 shift 1]

Options:

A. 5

B. 9

C. 10

D. 12

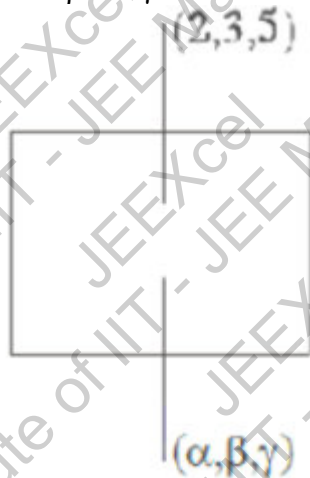
Answer: C

Solution:

$$\frac{\alpha-2}{2} = \frac{\beta-3}{1} = \frac{\gamma-5}{-3} = -2 \left(\frac{2 \times 2 + 3 - 3 \times 5 - 6}{2^2 + 1^2 + 1 - 3^2} \right) = 2$$

$$\frac{\alpha-2}{2} = 2 \quad \beta-3 = 2 \quad \gamma-5 = -6$$

$$\alpha = 6 \quad \beta = 5 \quad \gamma = -1$$



$$\alpha + \beta + \gamma = 10$$

Question 143

If the equation of the plane that contains the point $(-2, 3, 5)$ and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$:
[11-Apr-2023 shift 1]

Options:

A. 15

B. 18

C. 17

D. 16

Answer: A

Solution:

The equation of plane through $(-2, 3, 5)$ is

$$a(x+2) + b(y-3) + c(z-5) = 0$$

it is perpendicular to $2x + 4y + 5z = 8$ & $3x - 2y + 3z = 5$

$$\therefore 2a + 4b + 5c = 0$$

$$3a - 2b + 3c = 0$$

$$\therefore \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$$

$\therefore$ Equation of plane is

$$22(x+2) + 9(y-3) - 16(z-5) = 0$$

$$\Rightarrow 22x + 9y - 16z + 97 = 0$$

$$\text{Comparing with } \alpha x + \beta y + \gamma z + 97 = 0$$

$$\text{We get } \alpha + \beta + \gamma = 22 + 9 - 16 = 15$$

Question 144

Let a line l pass through the origin and be perpendicular to the lines

$$l_1 : \vec{r} = \hat{i} - 11\hat{j} - 7\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k}, \lambda \in \mathbb{R} \text{ and}$$

$$l_2 : \vec{r} = -\hat{i} + \hat{k} + \mu 2\hat{i} + 2\hat{j} + \hat{k}, \mu \in \mathbb{R}.$$

If P is the point of intersection of l and l_1 , and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2 , then $9(\alpha + \beta + \gamma)$ is equal to _____.

[11-Apr-2023 shift 1]

Answer: 5

Solution:

$$\text{Let } \ell = 0\hat{i} + 0\hat{j} + 0\hat{k} + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$$

$$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$$

$$= -4\hat{i} - 5\hat{j} - 2\hat{k}$$

$$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$$

P is intersection of ℓ and ℓ_1

$$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$$

By solving these equation $\gamma = -1, P(4, -5, 2)$

$$\text{Let } Q(-1+2\mu, 2\mu, 1+\mu)$$

$$\vec{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$$

$$-2 + 4\mu + 4\mu + 1 + \mu = 0$$

$$9\mu = 1$$

$$\mu = \frac{1}{9}$$

$$Q\left(-\frac{7}{9}, \frac{2}{9}, \frac{10}{9}\right)$$

$$9(\alpha + \beta + \gamma) = 9\left(-\frac{7}{9} + \frac{2}{9} + \frac{10}{9}\right)$$

$$= 5$$

Question 145

Let P be the plane passing through the points (5, 3, 0), (13, 3, -2) and (1, 6, 2). For $\alpha \in \mathbb{N}$, if the distances of the points A(3, 4, α) and B(2, α , a) from the plane P are 2 and 3 respectively, then the positive value of a is

[11-Apr-2023 shift 2]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: C

Solution:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

$$\text{Normal of the plane} = 3\hat{i} - 4\hat{j} + 12\hat{k}$$

$$\text{Plane : } 3x - 4y + 12z = 3$$

$$\text{Distance from A(3, 4, } \alpha)$$

$$\left| \frac{9 - 16 + 12\alpha - 3}{13} \right| = 2$$

$$\alpha = 3$$

$$\alpha = -8 \text{ (rejected)}$$

Distance from B(2, 3, a)

$$\left| \frac{6 - 12 + 12a - 3}{13} \right| = 3$$

$$a = 4$$

Question 146

Let the line passing through the point P(2, -1, 2) and Q(5, 3, 4) meet the plane $x - y + z = 4$ at the point T. Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line

$$\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1} \text{ is equal to}$$

[11-Apr-2023 shift 2]

Options:

A. 3

B. $\sqrt{61}$

C. $\sqrt{31}$

D. $\sqrt{189}$

Answer: A

Solution:

$$\text{Line: } \frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$$

$$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$$

$$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$$

$$\lambda + 6 = 4 \therefore \lambda = -2$$

$$\therefore R \equiv (-1, -5, 0)$$

$$\text{Line: } \frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$$

$$\text{Point T} = (2\mu - 1, 2\mu - 5, \mu)$$

It lies on plane

$$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$$

$$\mu = 1$$

$$\therefore T = (1, -3, 1)$$

$$\therefore RT = 3$$

Question 147

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ) . If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to _____.

[11-Apr-2023 shift 2]

Answer: 11

Solution:

$$\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$$

Dr's of line $\ell(1, 2, \lambda)$

Dr's of normal vector of plane $P : x + 2y + 3z = 4$ are $(1, 2, 3)$

Now, angle between line ℓ and plane P is given by

$$\sin \theta = \left| \frac{1+4+3\lambda}{\sqrt{5+\lambda^2} \cdot \sqrt{14}} \right| = \frac{3}{\sqrt{14}} \left(\text{given } \cos \theta = \sqrt{\frac{5}{14}} \right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$

line of plane P .

$$\Rightarrow t = -1$$

$$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha + 2\beta + 6\gamma = 11$$

Question 148

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and

$l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point

$P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 10

B. 8

C. 12

D. 14

Answer: A

Solution:

$$(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$$

$$3(3 + \mu) + 1 \cdot (2 - 3\mu) - 2(1 + 2\mu) = 0$$

$$9 - 4\mu = 0$$

$$\mu = \frac{9}{4}$$

$$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$$

$$-100 + 4\alpha - 54 + 18\alpha = 0$$

$$\Rightarrow \alpha = 7$$

$$\text{Let } P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$$

$$\text{Direction ratio of PQ } (3\lambda - 1, \lambda - 1, -2\lambda + 5)$$

$$\text{But } PQ \perp \ell_1$$

$$\Rightarrow 3(3\lambda - 1) + 1 \cdot (\lambda - 1) - 2(-2\lambda + 5) = 0$$

$$\Rightarrow \lambda = 1$$

$$P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$$

Question 149

Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to

[12-Apr-2023 shift 1]

Options:

A. 90

B. 105

C. 85

D. 126

Answer: D

Solution:

Let equation in new position is

$$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$$

$$4(4 + \lambda) - 1 \cdot (-1 + \lambda) + 1 \cdot (1 - \lambda) = 0$$

$$\Rightarrow \lambda = -9$$

So equation in new position is

$$-5x - 10y + 10z + 26 = 0$$

$$\Rightarrow \alpha = \frac{54}{15}$$

Question 150

Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$.

For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$,

then $\frac{a+b}{c}$ is equal to

[13-Apr-2023 shift 1]

Options:

A. -4

B. 2

C. -2

D. 4

Answer: A

Solution:

$$(x + 2y + az - 2) + \lambda(x - y + z - 3) = 0$$

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$\lambda = -\frac{7}{2}, a = 3, b = 1$$

$$\frac{2}{\sqrt{a}} = \left| \frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}} \right|$$

$$c = -1$$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

Question 151

The distance of the point $(-1, 2, 3)$ from the plane

$\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ parallel to the line of the shortest distance

between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is

[13-Apr-2023 shift 1]

Options:

A. $2\sqrt{5}$

B. $3\sqrt{5}$

C. $3\sqrt{6}$

D. $2\sqrt{6}$

Answer: D

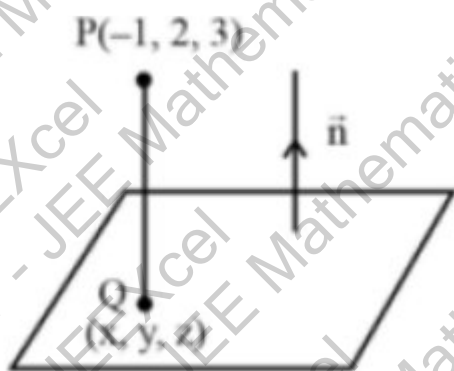
Solution:

Let $L_1: \vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$

$L_2: \vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{n} = \hat{i} - \hat{j} - 2\hat{k}$$



Equation of line along shortest distance of L_1 and L_2

$$\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = r$$

$$\Rightarrow (x, y, z) \equiv (r-1, 2-r, 3-2r)$$

$$\Rightarrow (r-1) - 2(2-r) + 3(3-2r) = 10$$

$$\Rightarrow r = -2$$

$$\Rightarrow Q(x, y, z) \equiv (-3, 4, 7)$$

$$\Rightarrow PQ = \sqrt{4+4+16} = 2\sqrt{6}$$

Question152

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$

be P. If the distance of the point $Q(6, -2, \alpha)$, $\alpha > 0$, from P is 13,

then α is equal to _____.

[13-Apr-2023 shift 1]

Answer: 15

Solution:

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$$\frac{x - \frac{5}{3}}{1} = \frac{y - \frac{5}{3}}{-2} = \frac{z - \frac{8}{3}}{1} = \frac{-2\left(1 \times \frac{5}{3} + (-2) \times \frac{8}{3} + 1 \times \frac{8}{3} - 2\right)}{1^2 + 2^2 + 1^2}$$

$$= \frac{1}{3}$$

$$\therefore x = 2, y = 1, z = 3$$

$$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$$

$$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$$

Question153

The plane, passing through the points $(0, -1, 2)$ and $(-1, 2, 1)$ and parallel to the line passing through $(5, 1, -7)$ and $(1, -1, -1)$, also passes through the point
[13-Apr-2023 shift 2]

Options:

- A. $(0, 5, -2)$
- B. $(-2, 5, 0)$
- C. $(2, 0, 1)$
- D. $(1, -2, 1)$

Answer: B

Solution:

Plane passing through $(0, -1, 0)$ and $(-1, 2, 1)$

Then vector in plane $\langle -1, 3, -1 \rangle$ vector parallel to plane is $\langle 4, 2, -6 \rangle$

$$\text{Normal vector to plane } (\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$$

$$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$$

$$\vec{n} = \langle 8, 5, 7 \rangle$$

Equation of plane

$$8(x-0) + 5(y+1) + 7(z-2) = 0$$

$$\Rightarrow 8x + 5y + 7z = 9$$

From given options point $(-2, 5, 0)$ lies on plane.

Question154

The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is
[13-Apr-2023 shift 2]

Options:

$$A. \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$$

$$B. \frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$$

$$C. \frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$$

$$D. \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$$

Answer: A

Solution:

Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now. Solving options

Point $(-3, 1, 5)$ & point $(-1, 2, 5)$

$$\begin{aligned} (1) \quad & \begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix} \\ & = -3(5) - (10) + 5(-1 + 4) \\ & = -15 - 10 + 15 = -10 \end{aligned}$$

(2) point $(-1, 2, 5)$

$$\begin{aligned} & \begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix} \\ & = 3(5) - (10) + 5(1 + 4) \\ & = -25 + 5 = 0 \end{aligned}$$

(3) point $(-1, 2, 5)$

$$\begin{aligned} & \begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix} \\ & = -3(4) - (8) + 5(1 + 4) \\ & = -12 - 8 + 25 = 5 \end{aligned}$$

(4) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$$-3(-5) - (-20) + 5(-1 - 8) \\ 15 + 20 - 45 = -10$$

Question 155

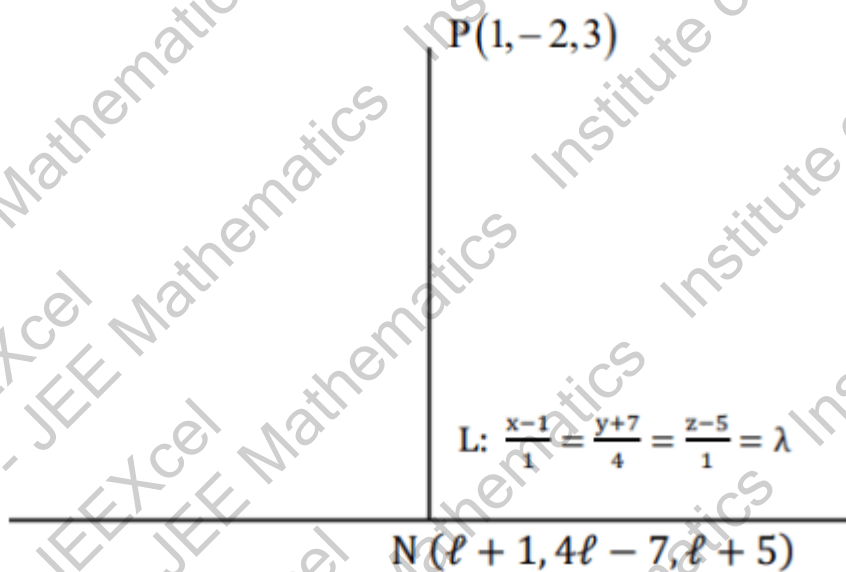
Let N be the foot of perpendicular from the point P(1, -2, 3) on the line passing through the points (4, 5, 8) and (1, -7.5). Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is
[13-Apr-2023 shift 2]

Options:

- A. 6
- B. 7
- C. 9
- D. 8

Answer: B

Solution:



$$\vec{PN} = \langle \lambda, 4\lambda - 5, \lambda + 2 \rangle$$

$$\vec{PN} \cdot \langle 1, 4, 1 \rangle = 0$$

$$\Rightarrow \lambda + 16\lambda - 20 + \lambda + 2 = 0$$

$$\Rightarrow \lambda = 1$$

$$N(2, -3, 6)$$

Distance of N from $2x - 2y + z + 5 = 0$ is

$$d = \left| \frac{2(2) - 2(-3) + 6 + 5}{\sqrt{2^2 + (-2)^2 + (1)^2}} \right|$$

$$= \left| \frac{21}{3} \right| = 7$$

Question 156

Let the foot of perpendicular of the point $P(3, -2, -9)$ on the plane passing through the points $(-1, -2, -3)$, $(9, 3, 4)$, $(9, -2, 1)$ be $Q(\alpha, \beta, \gamma)$. Then the distance of Q from the origin is [15-Apr-2023 shift 1]

Options:

A. $\sqrt{29}$

B. $\sqrt{38}$

C. $\sqrt{42}$

D. $\sqrt{35}$

Answer: C

Solution:

$P(3, -2, -9)$

Equation of plane through A, B, C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$$2x + 3y - 5z - 7 = 0$$

Foot of I^r of $P(3, -2, -9)$ is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(6-6+45-7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$$Q(1, -5, -4) \equiv (\alpha, \beta, \gamma)$$

$$OQ = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{42}$$

Question 157

Let S be the set of all values of λ , for which the shortest distance between the lines $\frac{x-\lambda}{0} = \frac{y-3}{4} = \frac{z+6}{1}$ and $\frac{x+\lambda}{3} = \frac{y}{-4} = \frac{z-6}{0}$ is 13. Then

$8 \left| \sum_{\lambda \in S} \lambda \right|$ is equal to

[15-Apr-2023 shift 1]

Options:

A. 302

B. 306

C. 304

D. 308

Answer: B

Solution:

$$\text{Shortest distance} = \frac{\begin{vmatrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2\lambda & 3 & -12 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}}$$

$$13 = \frac{|153 + 8\lambda|}{|4\hat{i} + 3\hat{j} - 12\hat{k}|}$$

$$= \frac{|153 + 8\lambda|}{13}$$

$$|153 + 8\lambda| = 169$$

$$153 + 8\lambda = 169, -169$$

$$\lambda = \frac{16}{8}, \frac{-322}{8}$$

$$8 \left| \sum_{\lambda \in S} \lambda \right| = 306$$

Question 158

If the line $x = y = z$ intersects the line

$x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$,
where A, B, C are the angles of a triangle ABC , then

$80 \left(\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right)$ is equal to _____

[15-Apr-2023 shift 1]

Answer: 5

Solution:

$$\sin A + \sin B + \sin C = \frac{18}{x}$$

$$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$$

$$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$$

$$4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = 2(4 \sin A \sin B \sin C)$$

$$16 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = 1$$

$$\text{Hence Ans.} = 5.$$

Question 159

Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$
and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$ Then the distance of the
point $A(8, -1, -19)$ from the plane P measured parallel to the line

$\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _____

[15-Apr-2023 shift 1]

Answer: 26

Solution:

$$\text{Plane} \equiv P_1 = \lambda P_2 = 0$$

$$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9 = 0$$

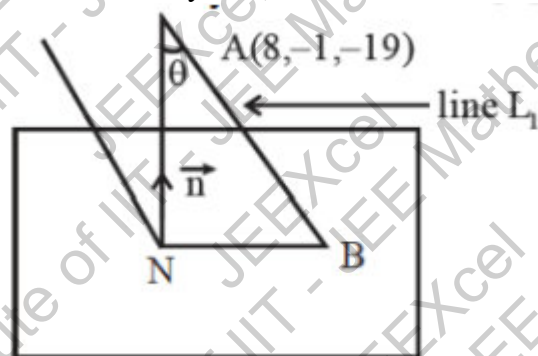
$$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$$

$$\vec{n} \cdot \vec{b} = 0 \text{ where } \vec{b} (2, 4, 5)$$

$$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$$

$$\lambda = -\frac{1}{6}$$

$$\text{Plane } 7x + 9y - 10z - 27 = 0$$



Equation of line AB is

$$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$$

Let $B = (8 - 3\lambda, -1 + 4\lambda, -19 + 12\lambda)$ lies on plane P

$$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$$

$$\lambda = 2$$

$$\therefore \text{Point } B = (2, 7, 5)$$

$$AB = \sqrt{6^2 + 8^2 + 24^2} = 26$$

Question160

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and

$\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible value of λ is :

[24-Jun-2022-Shift-2]

Options:

A. 16

B. 6

C. 12

D. 15

Answer: A

Solution:

$$\text{Let } \vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}, \vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$$

$$\therefore \vec{p} \times \vec{q} = (15 - 4\lambda)\hat{i} - (10 - \lambda)\hat{j} + 5\hat{k}$$

$$\vec{a_2} - \vec{a_1} = \hat{i} + 2\hat{j} + 2\hat{k}$$

$\therefore$ Shortest distance

$$= \left| \frac{(15 - 4\lambda) - 2(10 - \lambda) + 10}{\sqrt{(15 - 4\lambda)^2 + (10 - \lambda)^2 + 25}} \right| = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 3(5 - 2\lambda)^2 = (15 - 4\lambda)^2 + (10 - \lambda)^2 + 25$$

$$\Rightarrow 5\lambda^2 - 80\lambda + 275 = 0$$

$$\therefore \text{Sum of values of } \lambda = \frac{80}{5} = 16$$

Question 161

Let the points on the plane P be equidistant from the points $(-4, 2, 1)$ and $(2, -2, 3)$. Then the acute angle between the plane P and the plane $2x + y + 3z = 1$ is
[24-Jun-2022-Shift-2]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: C

Solution:

Let $P(x, y, z)$ be any point on plane P_1

$$\text{Then } (x-4)^2 + (y+2)^2 + (z-1)^2 = (x-2)^2 + (y+2)^2 + (z-3)^2$$

$$\Rightarrow 12x - 8y + 4z + 4 = 0$$

$$\Rightarrow 3x - 2y + z + 1 = 0$$

$$\text{And } P_2 : 2x + y + 3z = 0$$

$\therefore$ angle between P_1 and P_2

$$\cos \theta = \left| \frac{6 - 2 + 3}{14} \right| \Rightarrow \theta = \frac{\pi}{3}$$

Question 162

Let Q be the mirror image of the point $P(1, 0, 1)$ with respect to the plane $S : x + y + z = 5$. If a line L passing through $(1, -1, -1)$, parallel to the line PQ meets the plane S at R , then QR^2 is equal to :

[25-Jun-2022-Shift-1]

Options:

A. 2

B. 5

C. 7

D. 11

Answer: B

Solution:

As L is parallel to PQ d.r.s of S is $(1, 1, 1)$

$$\therefore L \equiv \frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$$

Point of intersection of L and S be λ

$$\Rightarrow (\lambda+1) + (\lambda-1) + (\lambda-1) = 5$$

$$\Rightarrow \lambda = 2$$

$$\therefore R \equiv (3, 1, 1)$$

Let $Q(\alpha, \beta, \gamma)$

$$\Rightarrow \frac{\alpha-1}{1} = \frac{\beta}{1} = \frac{\gamma-1}{1} = \frac{-2(-3)}{3}$$

$$\Rightarrow \alpha = 3, \beta = 2, \gamma = 3$$

$$\Rightarrow Q \equiv (3, 2, 3)$$

$$(QR)^2 = 0^2 + (1)^2 + (2)^2 = 5$$

Question 163

Let the lines

$$L_1 : \vec{r} = \lambda (\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = (\hat{i} + 3\hat{j} + \hat{k}) + \mu (\hat{i} + \hat{j} + 5\hat{k}); \mu \in \mathbb{R}$$

intersect at the point S . If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2 , then the value of

$a + b + d$ is equal to

[25-Jun-2022-Shift-1]

Answer: 5

Solution:

As plane is parallel to both the lines we have d.r's of normal to the plane as $\langle 7, -2, -1 \rangle$

$$\left(\text{from } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - \hat{j}(2) + \hat{k}(-1) \right)$$

Also point of intersection of lines is $2\hat{i} + 4\hat{j} + 6\hat{k}$

∴ Equation of plane is

$$7(x-2) - 2(y-4) - 1(z-6) = 0$$

$$\Rightarrow 7x - 2y - z = 0$$

$$a + b + c = 7 - 2 + 0 = 5$$

Question 164

Let p be the plane passing through the intersection of the planes

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 5 \text{ and } \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3, \text{ and the point } (2, 1, -2). \text{ Let}$$

the position vectors of the points X and Y be $\hat{i} - 2\hat{j} + 4\hat{k}$ and

$5\hat{i} - \hat{j} + 2\hat{k}$ respectively. Then the points

[25-Jun-2022-Shift-2]

Options:

- A. X and $X + Y$ are on the same side of P
- B. Y and $Y - X$ are on the opposite sides of P
- C. X and Y are on the opposite sides of P
- D. $X + Y$ and $X - Y$ are on the same side of P

Answer: C

Solution:

Let the equation of required plane

$$\pi : (x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$$

$$\because (2, 1, -2) \text{ lies on it so, } 2 + \lambda(-2) = 0$$

$$\Rightarrow \lambda = 1$$

$$\text{Hence, } \pi : 3x + 2y - 8 = 0$$

$$\because \pi_x = -9, \pi_y = 5, \pi_{x+y} = 4$$

$$\pi_{x-y} = -22 \text{ and } \pi_{y-x} = 6$$

Clearly X and Y are on opposite sides of plane π

Question 165

Let I_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and I_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line I_1 and I_2 , then d^{-2} is equal to _____
[25-Jun-2022-Shift-2]

Answer: 51

Solution:

$$\frac{x - \frac{1}{8}}{\frac{1}{8}} = \frac{y}{-\frac{1}{4\sqrt{2}}} = \frac{z}{0} \quad L_1$$

$$\text{or } \frac{x - \frac{1}{8}}{1} = \frac{y}{-\sqrt{2}} = \frac{z}{0} \dots\dots (i)$$

Equation of L_2

$$\frac{x + \frac{1}{8}}{-6\sqrt{3}} = \frac{y}{0} = \frac{z}{8} \dots\dots (ii)$$

$$d = \left| \frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|} \right|$$

$$= \frac{\left(\frac{1}{4}\hat{i}\right) \cdot \left(4\sqrt{2}\hat{i} + 4\hat{j} + 3\sqrt{6}\hat{k}\right)}{\sqrt{(4\sqrt{2})^2 + 4^2 + (3\sqrt{6})^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{32 + 16 + 54}} = \frac{1}{\sqrt{51}}$$

$$d^{-2} = 51$$

Question 166

If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z=2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ are perpendicular, then an angle between the lines l_2 and

$l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

[26-Jun-2022-Shift-1]

Options:

A. $\cos^{-1}\left(\frac{29}{4}\right)$

B. $\sec^{-1}\left(\frac{29}{4}\right)$

C. $\cos^{-1}\left(\frac{2}{29}\right)$

D. $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Answer: B

Solution:

$\because l_1$ and l_2 are perpendicular, so

$$3 \times 1 + (-2) \left(\frac{\alpha}{2}\right) + 0 \times 2 = 0$$

$$\Rightarrow \alpha = 3$$

Now angle between l_2 and l_3 ,

$$\cos \theta = \frac{1(-3) + \frac{\alpha}{2}(-2) + 2(4)}{\sqrt{1 - \frac{\alpha^2}{4} + 4\sqrt{9+4+16}}}$$

$$\Rightarrow \cos \theta = \frac{2}{29} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{29}\right) = \sec^{-1}\left(\frac{29}{2}\right)$$

Question 167

Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is

B(a, b, c), then :

[26-Jun-2022-Shift-1]

Options:

A. $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

B. $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$

C. $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$

D. $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

Answer: A

Solution:

Consider the equation of plane,

$$P : (2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$$

$$P : (2 + \lambda)x + 3(3 - 3\lambda)y - 1(1 + 5\lambda)z + (20 - 8\lambda) = 0$$

$$\therefore \text{Plane P is perpendicular to } 2x + 3y + z + 20 = 0$$

$$\text{So, } 4 - 2\lambda + 9 - 9\lambda + 1 + 5\lambda = 0$$

$$\Rightarrow \lambda = 7$$

$$P : 9x - 18y + 36z - 36 = 0$$

$$\text{or } P : x - 2y + 4z = 4$$

If image of $\left(2, -\frac{1}{2}, 2\right)$ in plane P is (a, b, c) then

$$\frac{a-2}{1} = \frac{b+\frac{1}{2}}{-2} = \frac{c-2}{4}$$

$$\text{and } \left(\frac{a+2}{2}\right) - 2\left(\frac{b-\frac{1}{2}}{2}\right) - 4\left(\frac{c-2}{2}\right) = 4$$

$$\text{Clearly } a = \frac{4}{3}, b = \frac{5}{6} \text{ and } c = -\frac{2}{3}$$

$$\text{So, } a : b : c = 8 : 5 : -4$$

Question 168

If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :
[26-Jun-2022-Shift-2]

Options:

A. $(2, -2, 0)$

B. $(-2, 2, 0)$

C. $(1, 0, 2)$

D. $(-1, 0, -2)$

Answer: C

Solution:

$$P_1 : 2x + y - 5z = 0, P_2 : 3x - y + 4z - 7 = 0$$

Family of planes P_1 and P_2

$$P : P_1 + \lambda P_2$$

$$\therefore P : (2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$$

$$\therefore P \perp P_1$$

$$\therefore 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$$

$$\lambda = 2$$

$$\therefore P : 8x - y + 3z - 14 = 0$$

It passes through the point $(1, 0, 2)$

Question 169

If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ are co-planar, then the distance of the plane containing these two lines from the point $(\alpha, 0, 0)$ is :
[26-Jun-2022-Shift-2]

Options:

A. $\frac{2}{9}$

B. $\frac{2}{11}$

C. $\frac{4}{11}$

D. 2

Answer: B

Solution:

∴ Both lines are coplanar, so

$$\begin{vmatrix} \alpha - 1 & 0 & -1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} =$$

$$\Rightarrow \alpha = \frac{5}{3}$$

Equation of plane containing both lines

$$\begin{vmatrix} x - 1 & y + 1 & z - 1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 9x + 2y + 6z = 13$$

So, distance of $\left(\frac{5}{3}, 0, 0\right)$ from this plane

$$= \frac{2}{\sqrt{81 + 4 + 36}} = \frac{2}{11}$$

Question 170

If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :
[27-Jun-2022-Shift-1]

Options:

A. 6

B. 4

C. 3

D. 2

Answer: A

Solution:

$$l + m - n = 0 \Rightarrow n = l + m$$

$$3l^2 + m^2 + cnl = 0$$

$$\begin{aligned}
 3l^2 + m^2 + cl(1+m) &= 0 \\
 &= (3+c)l^2 + clm + m^2 = 0 \\
 &= (3+c)\left(\frac{1}{m}\right)^2 + c\left(\frac{1}{m}\right) + 1 = 0
 \end{aligned}$$

$\therefore$ Lines are parallel

$$D = 0$$

$$c^2 - 4(3+c) = 0$$

$$c^2 - 4c - 12 = 0$$

$$(c-4)(c+3) = 0$$

$$c = 4 \text{ (as } c > 0 \text{)}$$

Question 171

Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to
[27-Jun-2022-Shift-1]

Answer: 137

Solution:

$$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-2(3a-4b+12c+19)}{3^2+(-4)^2+12^2}$$

$$\frac{x-a}{3} = \frac{y-b}{-4} = \frac{z-c}{12} = \frac{-6a+8b-24c-38}{169}$$

$$(x, y, z) \equiv (a-6, \beta, \gamma)$$

$$\frac{(a-6)-a}{3} = \frac{\beta-b}{-4} = \frac{\gamma-c}{12} = \frac{-6a+8b-24c-38}{169}$$

$$\frac{\beta-b}{-4} = -2 \Rightarrow \beta = 8+b$$

$$\frac{\gamma-c}{12} = -2 \Rightarrow \gamma = -24+c$$

$$\frac{-6a+8b-24c-38}{169} = -2$$

$$\Rightarrow 3a-4b+12c = 150$$

$$a+b+c = 5$$

$$3a + 3b + 3c = 15$$

Applying (1) - (2)

$$-7b + 9c = 135$$

$$7b - 9c = -135$$

$$7b - 9c = 7(8 + b) - 9(-24 + c)$$

$$= 56 + 21b + 7b - 9c$$

$$= 56 + 21b - 135 = 137$$

Question 172

Let the foot of the perpendicular from the point (1, 2, 4) on the line

$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane

$$3x + 4y + 12z + 23 = 0 \text{ is}$$

[27-Jun-2022-Shift-2]

Options:

A. 5

B. $\frac{50}{13}$

C. 4

D. $\frac{63}{13}$

Answer: A

Solution:

$$L : \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = t$$

$$\text{Let } P = (4t - 2, 2t + 1, 3t - 1)$$

∵ P is the foot of perpendicular of (1, 2, 4)

$$\therefore 4(4t - 3) + 2(2t - 1) + 3(3t - 5) = 0$$

$$\Rightarrow 29t = 29 \Rightarrow t = 1$$

$$\therefore P = (2, 3, 2)$$

Now, distance of P from the plane

$$3x + 4y + 12z + 23 = 0, \text{ is}$$

$$\left| \frac{6 + 12 + 24 + 23}{\sqrt{9 + 16 + 144}} \right| = \frac{65}{13} = 5$$

Question 173

The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$, is

[27-Jun-2022-Shift-2]

Options:

A. $\frac{18}{\sqrt{5}}$

B. $\frac{22}{3\sqrt{5}}$

C. $\frac{46}{3\sqrt{5}}$

D. $6\sqrt{3}$

Answer: A

Solution:

$$L_1: \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

$$L_2: \frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

$$\text{Now, } \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

$$= 10\hat{i} - 8\hat{j} - 4\hat{k}$$

$$\text{and } \vec{a}_2 - \vec{a}_1 = 6\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\therefore \text{S.D.} = \frac{|60 + 32 + 16|}{\sqrt{100 + 64 + 16}} = \frac{108}{\sqrt{180}} = \frac{18}{\sqrt{5}}$$

Question 174

If two distinct point Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is (1, -2, 3), then the area of the triangle PQR is equal to

[28-Jun-2022-Shift-1]

Options:

A. $\frac{2}{3}\sqrt{38}$

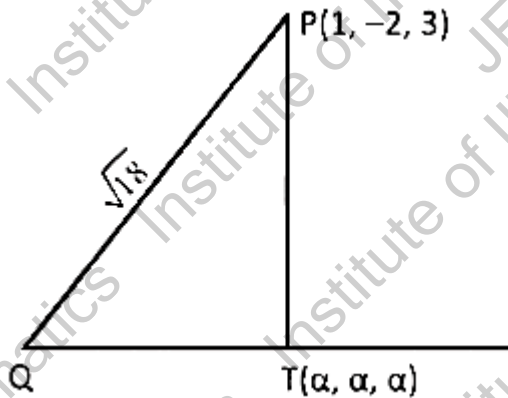
B. $\frac{4}{3}\sqrt{38}$

C. $\frac{8}{3}\sqrt{38}$

D. $\sqrt{\frac{152}{3}}$

Answer: B

Solution:



Line L is $x = y = z$

$$\vec{PQ} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\Rightarrow (\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$$

$$\Rightarrow \alpha = \frac{2}{3} \text{ so, } T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3} \right)$$

$$PT = \sqrt{\frac{38}{3}}$$

$$\Rightarrow QT = \frac{4}{\sqrt{3}}$$

$$\text{So, Area} = \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}} \right) \cdot 2$$

$$= \frac{4\sqrt{38}}{3} \text{ sq. units}$$

Question 175

The acute angle between the planes P_1 and P_2 , when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points $(2, 1, 3)$ and $(0, 1, 2)$, respectively, is
[28-Jun-2022-Shift-1]

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer: A

Solution:

Family of Plane's equation can be given by
 $(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z - (29 + 20\lambda) = 0$

P_1 passes through $(2, 1, 3)$

$$\Rightarrow (10 + 16\lambda) + (8 - 7\lambda) + (39 + 3\lambda) - (29 + 20\lambda) = 0$$

$$\Rightarrow -8\lambda + 28 = 0 \Rightarrow \lambda = \frac{7}{2}$$

d.r, s of normal to P_1

$$\left\langle 33, \frac{-33}{2}, \frac{33}{2} \right\rangle \text{ or } \left\langle 1, -\frac{1}{2}, \frac{1}{2} \right\rangle$$

P_2 passes through $(0, 1, 2)$

$$\Rightarrow 8 - 7\lambda + 26 + 2\lambda - (29 + 20\lambda) = 0$$

$$\Rightarrow 5 - 25\lambda = 0$$

$$\Rightarrow \lambda = \frac{1}{5}$$

d.r, s of normal to P_2

$$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5} \right\rangle \text{ or } \langle 1, 1, 2 \rangle$$

Angle between normals

$$\frac{\left(\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} \right) \cdot \left(\hat{i} + \hat{j} + 2\hat{k} \right)}{\frac{\sqrt{3}}{2}}$$

$$\cos \theta = \frac{1 - \frac{1}{2} + 1}{3} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Question 176

Let the plane $P : \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$. If the plane P passes through the point $(2, 3, \frac{1}{2})$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

[28-Jun-2022-Shift-1]

Options:

- A. 90
- B. 93
- C. 95
- D. 97

Answer: B

Solution:

$$P_1 : x + 3y - z = 6$$

$$P_2 : -6x + 5y - z = 7$$

Family of planes passing through line of intersection of P_1 and P_2 is given by

$$x(1 - 6\lambda) + y(3 + 5\lambda) + z(-1 - \lambda) - (6 + 7\lambda) = 0$$

It passes through $(2, 3, \frac{1}{2})$

$$\text{So, } 2(1 - 6\lambda) + 3(3 + 5\lambda) + \frac{1}{2}(-1 - \lambda) - (6 + 7\lambda) = 0$$

$$\Rightarrow 2 - 12\lambda + 9 + 15\lambda - \frac{1}{2} - \frac{\lambda}{2} - 6 - 7\lambda = 0$$

$$\Rightarrow \frac{9}{2} - \frac{9\lambda}{2} = 0 \Rightarrow \lambda = 1$$

Required plane is

$$-5x + 8y - 2z - 13 = 0$$

$$\text{Or } \vec{r} \cdot (-5\hat{i} + 8\hat{j} - 2\hat{k}) = 13$$

$$\frac{|13\vec{a}|^2}{|d|^2} = \frac{13^2}{(13)^2} \cdot |\vec{a}|^2 = 93$$

Question 177

Let the plane $ax + by + cz = d$ pass through $(2, 3, -5)$ and is perpendicular to the planes

$2x + y - 5z = 10$ and $3x + 5y - 7z = 12$. If a, b, c, d are integers $d > 0$ and $\gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to :

[28-Jun-2022-Shift-2]

Options:

A. 18

B. 20

C. 24

D. 22

Answer: D

Solution:

Equation of plane through point $(2, 3, -5)$ and perpendicular to planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$\begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 0$$

$\therefore$ Equation of plane is $(x-2)(-7+25) - (y-3)$

$$(-14+15) + (z+5) \cdot 7 = 0$$

$$\therefore 18x - y + 7z + 2 = 0$$

$$\Rightarrow 18x - y + 7z = -2$$

$$\therefore -18x + y - 7z = 2$$

On comparing with $ax + by + cz = d$ where $d > 0$ is $a = -18, b = 1, c = -7, d = 2$

$$\therefore a + 7b + c + 20d = 22$$

Question178

Let the image of the point $P(1, 2, 3)$ in the line $L : \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q . Let $R(\alpha, \beta, \gamma)$ be a point that divides internally the line segment PQ in the ratio $1:3$. Then the value of $22(\alpha + \beta + \gamma)$ is equal to _____
[28-Jun-2022-Shift-2]

Answer: 125

Solution:

The point dividing PQ in the ratio $1:3$ will be mid-point of P & foot of perpendicular from P on the line.

$\therefore$ Let a point on line be λ

$$\Rightarrow \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$$

$$\Rightarrow P'(3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

as P' is foot of perpendicular

$$(3\lambda + 5)3 + (2\lambda - 1)2 + (3\lambda - 1)3 = 0$$

$$\Rightarrow 22\lambda + 15 - 2 - 3 = 0$$

$$\Rightarrow \lambda = \frac{-5}{11}$$

$$\therefore P' \left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11} \right)$$

$$\text{Mid-point of } PP' = \left(\frac{\frac{51}{11} + 1}{2}, \frac{\frac{1}{11} + 2}{2}, \frac{\frac{7}{11} + 3}{2} \right)$$

$$= \left(\frac{62}{22}, \frac{23}{22}, \frac{40}{22} \right) = (\alpha, \beta, \gamma)$$

$$\Rightarrow 22(\alpha, \beta, \gamma) = 62 + 23 + 40 = 125$$

Question179

If the mirror image of the point $(2, 4, 7)$ in the plane $3x - y + 4z = 2$ is (a, b, c) , then $2a + b + 2c$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 54

B. 50

C. -6

D. -42

Answer: C

Solution:

We know mirror image of point (x_1, y_1, z_1) in the plane $ax + by + cz = d$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 - d)}{a^2 + b^2 + c^2}$$

Here given point $(2, 4, 7)$ and plane $3x - y + 4z = 2$ then mirror image is

$$\frac{x - 2}{3} = \frac{y - 4}{-1} = \frac{z - 7}{4} = \frac{-2(6 - 4 + 28 - 2)}{9 + 1 + 16}$$

$$\Rightarrow \frac{x - 2}{3} = \frac{y - 4}{-1} = \frac{z - 7}{4} = -\frac{28}{13}$$

$$\therefore x = -\frac{58}{13} = a$$

$$y = \frac{80}{13} = b$$

$$z = -\frac{21}{13} = c$$

$$\therefore 2a + b + 2c$$

$$= 2\left(-\frac{58}{13}\right) + \frac{80}{13} + 2\left(-\frac{21}{13}\right)$$

$$= \frac{-116 + 80 - 42}{13} = \frac{-78}{13} = -6$$

Question 180

Let d be the distance between the foot of perpendiculars of the points $P(1, 2, -1)$ and $Q(2, -1, 3)$ on the plane $-x + y + z = 1$. Then d^2 is equal to ____
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Foot of perpendicular from P

$$\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z+1}{1} = \frac{(-1+2-1-1)}{3}$$

$$\Rightarrow P' \equiv \left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3} \right)$$

and foot of perpendicular from Q

$$\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z-3}{1} = \frac{-(-2-1+3-1)}{3}$$

$$\Rightarrow Q' \equiv \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3} \right)$$

$$PQ' = \sqrt{(1)^2 + (3)^2 + (4)^2} = d = \sqrt{26}$$

$$\Rightarrow d^2 = 26$$

Question 181

Let $P_1 : \vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points $(2, -3, 2)$, $(2, -2, -3)$ and $(1, -4, 2)$.

If the direction ratios of the line of intersection of P_1 and P_2 be

$16, \alpha, \beta$, then the value of $\alpha + \beta$ is equal to _____

[29-Jun-2022-Shift-1]

Answer: 28

Solution:

Direction ratio of normal to $P_1 \equiv \langle 2, 1, -3 \rangle$

$$\text{and that of } P_2 \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -5 \\ -1 & -2 & 5 \end{vmatrix} = -5\hat{i} - \hat{j}(-5) + \hat{k}(1)$$

i.e. $\langle -5, 5, 1 \rangle$

d.r's of line of intersection are along vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = \hat{i}(16) - \hat{j}(-13) + \hat{k}(15)$$

i.e. $\langle 16, 13, 15 \rangle$

$$\therefore \alpha + \beta = 13 + 15 = 28$$

Question 182

Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in \mathbb{R}$. The shortest distance of the plane from the origin is :
[29-Jun-2022-Shift-2]

Options:

A. $\sqrt{\frac{3}{109}}$

B. $\sqrt{\frac{5}{142}}$

C. $\frac{5}{\sqrt{71}}$

D. $\frac{1}{\sqrt{142}}$

Answer: B

Solution:

$(2, -1, -3)$ satisfy the given plane.

So $2p + q = 8$

Also given line is perpendicular to normal plane so $3p + 2q - 1 = 0$

$\Rightarrow p = 15, q = -22$

Eq. of plane $15x - 22y + z - 5 = 0$

its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

Question 183

Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$, $\lambda \in \mathbb{R}$. Then, which of the following points lies on T ?

[29-Jun-2022-Shift-2]

Options:

A. (2, 1, 0)

B. (1, 2, 1)

C. (1, 2, 2)

D. (1, 3, 2)

Answer: B

Solution:

Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$

is given by Q(4, 8, 7)

$$\text{Eq. of plane T} = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.

Question 184

Let a line having direction ratios 1, -4, 2 intersect the lines

$\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to

[24-Jun-2022-Shift-1]

Answer: 84

Solution:

Let $A(3\lambda + 7, -\lambda + 1, \lambda - 2)$ and $B(2\mu, 3\mu + 7, \mu)$

So, DR's of $AB \propto 3\lambda - 2\mu + 7, -(\lambda + 3\mu + 6), \lambda - \mu - 2$

$$\text{Clearly } \frac{3\lambda - 2\mu + 7}{1} = \frac{\lambda + 3\mu + 6}{4} = \frac{\lambda - \mu - 2}{2}$$

$$\Rightarrow 5\lambda - 3\mu = -16$$

$$\text{And } \lambda - 5\mu = 10$$

From (i) and (ii) we get $\lambda = -5, \mu = -3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$$AB = \sqrt{4 + 64 + 16} \Rightarrow (AB)^2 = 84$$

Question 185

Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then, the value of $\sin^4 \alpha + \cos^4 \alpha$ is

[25 Feb 2021 Shift 1]

Options:

A. $\frac{3}{4}$

B. $\frac{3}{8}$

C. $\frac{5}{8}$

D. $\frac{1}{2}$

Answer: C

Solution:

Given, $1 + m - n = 0$. . . (i)

and $1^2 + m^2 - n^2 = 0$... (ii)

On squaring Eq. (i), we get

$$(1 + m)^2 = n^2$$

$$\Rightarrow 1^2 + m^2 + 21m = n^2 \dots (iii)$$

From Eqs. (ii) and (iii),

$$1^2 + m^2 - n^2 = 0$$

$$1^2 + m^2 + 21m = n^2$$

$$\frac{-n^2 - 21m = -n^2}{-n^2 - 21m = -n^2}$$

$$\Rightarrow 21m = 0 \Rightarrow 1m = 0$$

$$\Rightarrow 1 = 0 \text{ or } m = 0$$

Case I When $1 = 0$

$$\Rightarrow 0 + m - n = 0$$

$$\Rightarrow m = n$$

$$\text{and } 1^2 + m^2 + n^2 = 1$$

$$\Rightarrow m^2 + m^2 = 1 \quad [\because n = m \text{ and } 1 = 0]$$

$$\Rightarrow m^2 = \frac{1}{2}$$

$$m = \pm \frac{1}{\sqrt{2}} = n$$

$$\therefore (1, m, n) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \text{ or } \left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

Case II When $m = 0$

$$\text{then, } 1 + m - n = 0$$

$$\Rightarrow 1 = n \text{ and } 1^2 + m^2 + n^2 = 1$$

$$[\because n = 1 \text{ and } m = 0]$$

$$\Rightarrow 1^2 + 0 + 1^2 = 1$$

$$1 = \pm \frac{1}{\sqrt{2}} \quad [\because n = 1 \text{ and } m = 0]$$

$$\Rightarrow \therefore (1, m, n) = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \text{ or } \left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$$\Rightarrow a = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \text{ and } b = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$\text{Then } \cos \alpha = \frac{a \cdot b}{|a||b|} = \frac{1}{2}$$

$$\therefore \sin \alpha = \pm \frac{\sqrt{3}}{2}$$

$$\text{Now, } \cos^4 \alpha + \sin^4 \alpha = \frac{1}{16} + \frac{9}{16} = \frac{10}{16} = \frac{5}{8}$$

Question 186

A line 'l' passing through origin is perpendicular to the lines

$$l_1 : r = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$$

$$l_2 : r(3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$$

If the coordinates of the point in the first octant on ' L_2 ' at a distance of $\sqrt{17}$ from the point of intersection of ' L_1 ' and ' L_2 ' are (a, b, c), then $18(a + b + c)$ is equal to
[25 Feb 2021 Shift 2]

Answer: 44

Solution:

$$\text{Let } L_1 \Rightarrow \frac{x-3}{1} = \frac{y-3}{2} = \frac{z-4}{2} = u \text{ (say)}$$

$$\Rightarrow \text{Direction ratios of } L_1 = 1, 2, 2$$

$$L_2 \Rightarrow \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = v \text{ (say)}$$

$$\text{Direction ratios of } L_2 = 2, 2, 1$$

Line L passing through origin is perpendicular to L_1 and L_2 .

Hence, direction ratios of L is parallel to $(L_1 \times L_2)$.

$$\Rightarrow (-2, 3, -2)$$

$$\text{Equation of L} \Rightarrow \frac{x}{-2} = \frac{y}{3} = \frac{z}{-2} = \lambda \text{ (say)}$$

Solve L and L_1 , we get

$$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + 4)$$

$$\text{Gives, } \lambda = 1, \mu = -1$$

So, intersection point P(2, -3, 2).

Let Q(2v + 3, 2v + 3, v + 2) be required point on L_2 .

$$\text{Now, } PQ = \sqrt{17} \text{ (given)}$$

$$\text{Now, } PQ = \sqrt{17} \text{ (given)}$$

$$PQ = \sqrt{(2v+1)^2 + (2v+6)^2 + (v)^2}$$

$$= \sqrt{17}$$

$$\Rightarrow (2v+1)^2 + (2v+6)^2 + v^2 = 17 \quad (\text{squaring on both sides})$$

$$\Rightarrow 9v^2 + 28v + 20 = 0$$

$$\text{On solving, we get } v = -2 \text{ (rejected), } \frac{-10}{9} \text{ (accepted)}$$

$$\therefore Q \text{ is } \left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9} \right)$$

$$\therefore 18(a + b + c) = 18 \left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9} \right) = 44$$

Question 187

The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is
[25 Feb 2021 Shift 1]

Options:

A. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

B. $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

C. $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

D. $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Answer: D

Solution:

Given, line $\Rightarrow \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$ (let)

Any point on this line is B(2λ + 1, 3λ - 1, -2λ + 1) and direction ratios of this line = (2, 3, -2) = d₁

Let given point be A(0, 1, 2).

Then direction ratio of

AB = (2λ + 1, 3λ - 2, -2λ - 1) = d₂

∴ Both lines are perpendicular to each other.

∴ d₁ · d₂ = 0

2(2λ + 1) + 3(3λ - 2) - 2(-2λ - 1) = 0

⇒ 4λ + 2 + 9λ - 6 + 4λ + 2 = 0

⇒ 17λ = 2

λ = 2/17

∴ Direction ratio of required line d₂ = (21, -28, -21)

= (3, -4, -3) = (-3, 4, 3)

This line passes through A(0, 1, 2).

∴ Required equation of line $\Rightarrow \frac{x-0}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Question 188

Let λ be an integer. If the shortest distance between the lines

$x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of |λ|

is

[24 Feb 2021 Shift 2]

Answer: 1

Solution:

Given, equation of line $\Rightarrow x - \lambda = 2y - 1 =$

$$\Rightarrow \frac{x - \lambda}{1} = \frac{y - 1/2}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$$

$$\text{or } \frac{x - \lambda}{2} = \frac{y - 1/2}{1} = \frac{z}{-1}$$

Point on this line through which it passes is $(\lambda, \frac{1}{2}, 0)$.

Equation of another line $\Rightarrow x = y + 2\lambda = z - \lambda$

$$\Rightarrow \frac{x}{1} = \frac{y - (-2\lambda)}{1} = \frac{z - \lambda}{1} \dots (ii)$$

A point through which this line passes is $(0, -2\lambda, \lambda)$.

Now, distance between two skew lines

$$= \frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$$

$$\text{According to the question, } \frac{|-5\lambda - \frac{3}{2}|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow |10\lambda + 3| = 7$$

$$10\lambda + 3 = \pm 7$$

$$\Rightarrow 10\lambda = 4, -10$$

$$\Rightarrow \lambda = \frac{2}{5} \text{ and } \lambda = -1$$

$$\therefore \lambda = -1$$

$$(\lambda = \frac{2}{5} \text{ is not possible as } \lambda \text{ is an integer})$$

$$\therefore$$

$$\text{Hence, } |\lambda| = |-1| = 1$$

Question 189

Let $a, b \in \mathbb{R}$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to

[24 Feb 2021 Shift 2]

Options:

- A. 88
- B. 86
- C. 84
- D. 90

Answer: A

Solution:

Given, $P(a, 6, 9)$

$$\text{Equation of line } \frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$$

Image of point P with respect to line is point $Q(20, b, -a-9)$

$$\text{Mid-point of P and Q} = \left(\frac{a+20}{2}, \frac{6+b}{2}, \frac{-a}{2} \right)$$

This point lies on line

$$\therefore \frac{\frac{a+20}{2}-3}{7} = \frac{\frac{6+b}{2}-2}{5} = \frac{\frac{-a}{2}-1}{-9}$$

$$\Rightarrow \frac{a+14}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$$\Rightarrow \frac{a+14}{14} = \frac{a+2}{18} \text{ and } \frac{b+2}{10} = \frac{a+2}{18}$$

Solving, we get $a = -56, b = -32$

$$\therefore |a+b| = |-56-32| = 88$$

Question190

Consider the three planes $P_1 : 3x + 15y + 21z = 9$, $P_2 : x - 3y - z = 5$ and $P_3 : 2x + 10y + 14z = 5$ Then, which one of the following is true?

[26 Feb 2021 Shift 1]

Options:

- A. P_1 and P_2 are parallel
- B. P_1 and P_3 are parallel
- C. P_2 and P_3 are parallel

D. P_1 , P_2 and P_3 all are parallel

Answer: B

Solution:

$$\text{Given, } P_1 \Rightarrow 3x + 15y + 21z = 9$$

$$P_2 \Rightarrow x - 3y - z = 5$$

$$P_3 \Rightarrow 2x + 10y + 14z = 5$$

Consider plane P_1 , it can be written as

$$3x + 15y + 21z = 9 \text{ or } x + 5y + 7z = 3$$

Again, consider plane P_3 , it can be written as,

$$2x + 10y + 14z = 5 \text{ or } x + 5y + 7z = 5/2$$

Hence, P_1 and P_3 are parallel.

Question 191

If the mirror image of the point $(1, 3, 5)$ with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ) , then $5(\alpha + \beta + \gamma)$ equals
[26 Feb 2021 Shift 2]

Options:

A. 47

B. 43

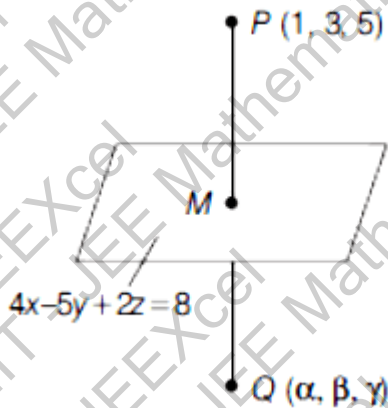
C. 39

D. 41

Answer: A

Solution:

Given, point $P(1, 3, 5)$ has the mirror image $Q(\alpha, \beta, \gamma)$ with respect to plane $4x - 5y + 2z = 8$



Then, M be the mid-point of line joining point P and Q . and M lies on given plane.

Coordinates of $M(a, b, c)$ are as follows

$$a = \frac{\alpha+1}{2}, b = \frac{\beta+3}{2}, c = \frac{\gamma+5}{2}$$

$\therefore M$ lies on plane, then

$$4a - 5b + 2c = 8$$

$$4\left(\frac{\alpha+1}{2}\right) - 5\left(\frac{\beta+3}{2}\right) + 2\left(\frac{\gamma+5}{2}\right) = 8 \dots (i)$$

Also, PQ is perpendicular to plane

$$\Rightarrow \frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = \lambda \text{ (say)}$$

$$\Rightarrow \alpha = 4\lambda + 1, \beta = 3 - 5\lambda, \gamma = 2\lambda + 5 \dots (ii)$$

Use Eq. (ii) in Eq. (i), we get

$$2(4\lambda + 2) - 5\left(\frac{6 - 5\lambda}{2}\right) + 2\lambda + 10 = 8$$

$$\Rightarrow 8\lambda + 4 - 15 + \frac{25\lambda}{2} + 2\lambda + 10 = 8 \Rightarrow \lambda = \frac{2}{5}$$

$$\therefore \alpha = 4\lambda + 1 = 4\left(\frac{2}{5}\right) + 1 = \frac{13}{5}$$

$$\beta = 3 - 5\left(\frac{2}{5}\right) = \frac{5}{5} = 1$$

$$\text{and } \gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$$

$$\therefore 5(\alpha + \beta + \gamma) = 5\left(\frac{13}{5} + 1 + \frac{29}{5}\right) = 47$$

Question 192

Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L , then the value of $21(\alpha + \beta + \gamma)$ equals

[26 Feb 2021 Shift 2]

Options:

A. 142

B. 68

C. 136

D. 102

Answer: D

Solution:

Given, $x + 2y + z = 6 \dots (i)$

and $y + 2z = 4 \dots (ii)$

Put $y = 4 - 2z$ from Eq. (ii) Eq. in (i), we get

$$x + 8 - 4z + z = 6$$

$$\Rightarrow x = -2 + 3z$$

$$\Rightarrow \frac{x+2}{3} = z \dots (iii)$$

$$y = 4 - 2z$$

$$\Rightarrow \frac{y-4}{-2} = z \dots (iv)$$

From Eqs. (iii) and (iv), line of intersection of two planes is

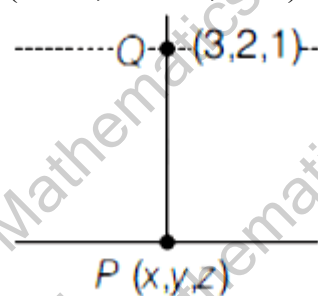
$$\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1} = \lambda$$

Then, Direction ratios of PQ is

$$x - 3\lambda - 4 - 2\lambda, z = \lambda$$

$$(3\lambda - 5, -2\lambda + 2, \lambda - 1)$$

$$(3\lambda - 5, -2\lambda + 2, \lambda - 1)$$



Since, PQ is perpendicular to the line, then

$$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$$

$$\therefore \lambda = \frac{10}{7}$$

$$\therefore P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$$

$$\text{Then, } 21(\alpha + \beta + \gamma) = 21\left(\frac{16}{7} + \frac{8}{7} + \frac{10}{7}\right)$$

$$= 21\left(\frac{34}{7}\right) = 3 \times 34 = 102$$

Question 193

Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points

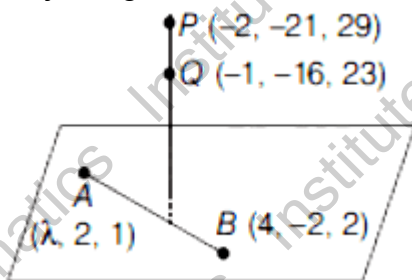
$(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is

[26 Feb 2021 Shift 1]

Answer: 8

Solution:

Given $(\lambda, 2, 1)$ be point on the plane which passes through $(4, -2, 2)$ and plane is perpendicular to line joining P and Q.



Given, AB is perpendicular to PQ i.e., $\vec{AB} \cdot \vec{PQ} = 0$

$$\text{Now, } \vec{AB} = (4 - \lambda)\hat{i} + (-2 - 2)\hat{j} + (2 - 1)\hat{k}$$

$$= (4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}$$

$$\vec{PQ} = (-1 + 2)\hat{i} + (-16 + 21)\hat{j} + (23 - 29)\hat{k}$$

$$= \hat{i} + 5\hat{j} - 6\hat{k}$$

$$= (4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}$$

$$\vec{PQ} = (-1 + 2)\hat{i} + (-16 + 21)\hat{j} + (23 - 29)\hat{k}$$

$$= \hat{i} + 5\hat{j} - 6\hat{k}$$

Hence, $\vec{AB} \cdot \vec{PQ} = 0$

$$\Rightarrow (4 - \lambda)(1) + (-4)(5) + (1)(-6) = 0$$

$$\Rightarrow 4 - \lambda - 20 - 6 = 0$$

$$\Rightarrow \lambda = -22$$

$$\text{Then, } \left(\frac{\lambda}{11}\right)^2 - \left(\frac{4\lambda}{11}\right) - 4 = \left(\frac{-22}{11}\right)^2 - \left(\frac{4 \times (-22)}{11}\right) - 4$$

$$= 4 - (-8) - 4 = 8$$

Question194

A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of OP on this plane is of length
[2021 25 Feb Shift 2]

Options:

A. $\sqrt{\frac{2}{3}}$

B. $\sqrt{\frac{2}{11}}$

C. $\sqrt{\frac{2}{7}}$

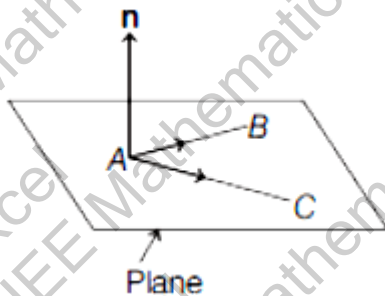
D. $\sqrt{\frac{2}{5}}$

Answer: B

Solution:

Refer diagram, the normal vector be $\vec{n}$ and it is perpendicular to both AB and AC

$$\vec{AB} \times \vec{AC} = \vec{n}$$



Now, A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2)

$$\text{Then, } \vec{AB} = (2-1)\hat{i} + (3-2)\hat{j} + (1-3)\hat{k}$$

$$= \hat{i} + \hat{j} - 2\hat{k}$$

$$= (2-1)\hat{i} + (4-2)\hat{j} + (2-3)\hat{k}$$

$$\vec{AC} = (2-1)\hat{i} + (4$$

$$= \hat{i} + 2\hat{j} - \hat{k}$$

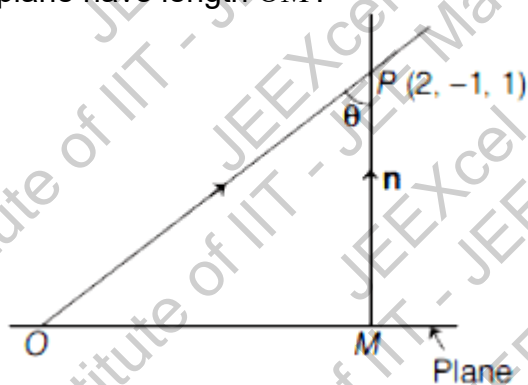
$$\text{Now, } \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \hat{i}(-1+4) - \hat{j}(-1+2) + \hat{k}(2-1)$$

$$= 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{n} = 3\hat{i} - \hat{j} + \hat{k}$$

Let P be any point on normal vector and O be origin. Then refer the diagram, projection of OP on plane have length OM.



$$\vec{OP} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{n} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{OP} \cdot \vec{n} = |\vec{OP}| |\vec{n}| \cos \theta$$

$$(6+1+1) = \sqrt{4+1+1}(\sqrt{9+1+1}) \cos \theta$$

$$8 = \sqrt{6}\sqrt{11} \cos \theta \Rightarrow \cos \theta = \frac{8}{\sqrt{66}}$$

$$\text{Again, } \sin \theta = \frac{|\vec{OM}|}{|\vec{OP}|}, \text{ gives } |\vec{OM}| = \sin \theta \cdot |\vec{OP}|$$

$$\Rightarrow |\vec{OM}| = \sqrt{1 - \cos^2 \theta} |\vec{OP}|$$

$$= \sqrt{1 - \frac{64}{66}} \sqrt{4+1+1} \quad (\text{use } \cos \theta = \frac{8}{\sqrt{66}})$$

$$= \sqrt{\frac{2}{66}} \cdot \sqrt{6}$$

$$\therefore |\vec{OM}| = \sqrt{\frac{2}{11}}$$

Question 195

The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$ and the point $(1, 0, 2)$ is

[24 Feb 2021 Shift 2]

Options:

$$\text{A. } \mathbf{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$$

$$\text{B. } \mathbf{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

$$\text{C. } \mathbf{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

$$\text{D. } \mathbf{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$$

Answer: C

Solution:

Given, point (1, 0, 2)

Equation of plane =

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \text{ and } \mathbf{r} \cdot (\hat{i} - 2\hat{j}) = -2$$

Equation of plane passing through the intersection of given planes is

$$[\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda [\mathbf{r} \cdot (\hat{i} - 2\hat{j}) + 2] = 0$$

$\therefore$ This plane passes through point (1, 0, 2) i.e.,

$$\text{vector}(\hat{i} + 2\hat{k})$$

$$\therefore [(\hat{i} + 2\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda [(\hat{i} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j}) + 2] = 0$$

$$\Rightarrow (3 - 1) + \lambda(1 + 2) = 0$$

$$\Rightarrow 2 + \lambda \times 3 = 0$$

$$\Rightarrow \lambda = -2/3$$

Hence, equation of required plane is

$$[\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \left(-\frac{2}{3}\right) [\mathbf{r} \cdot (\hat{i} - 2\hat{j}) + 2] = 0$$

$$\text{or } 3[\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] - 2[\mathbf{r} \cdot (\hat{i} - 2\hat{j}) + 2] = 0$$

$$\text{or } \mathbf{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

Question 196

The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is:

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Options:

A. $2\sqrt{19}$

B. $19\sqrt{2}$

C. 38

D. $\sqrt{38}$

Answer: D

Solution:

$$\text{Let } \frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = t$$

$$\Rightarrow x = 3+t, y = 2t+4, z = 2t+5$$

$$3+t+2t+4+2t+5 = 17$$

$$\Rightarrow 5t = 5 \Rightarrow t = 1$$

$$\Rightarrow \text{Point of intersection is } (4, 6, 7)$$

Distance between $(1, 1, 9)$ and $(4, 6, 7)$ is

$$\sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2}$$

$$= \sqrt{9+25+4} = \sqrt{38}.$$

Question 197

The equation of the plane passing through the point $(1, 2, -3)$ and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is
24 Feb 2021 Shift 1

Options:

A. $3x - 10y - 2z + 11 = 0$

B. $6x - 5y - 2z - 2 = 0$

C. $11x + y + 17z + 38 = 0$

D. $6x - 5y + 2z + 10 = 0$

Answer: C

Solution:

Normal vector:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$$

So, direction ratios of normal to the required plane are $\langle 11, 1, 17 \rangle$

Plane passes through $(1, 2, -3)$

So, equation of plane :

$$11(x-1) + 1(y-2) + 17(z+3) = 0$$

$$\Rightarrow 11x + y + 17z + 38 = 0$$

Question 198

Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T. If the vector TA is perpendicular to both PR and QS and the length of vector TA is $\sqrt{5}$ units, then the modulus of a position vector of A is
[16 Mar 2021 Shift 1]

Options:

A. $\sqrt{482}$

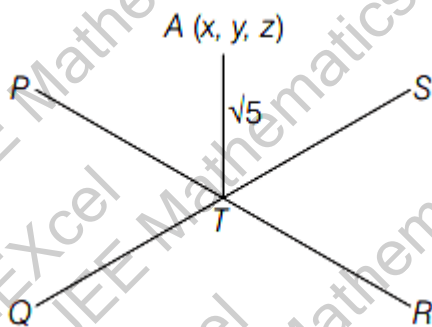
B. $\sqrt{171}$

C. $\sqrt{5}$

D. $\sqrt{227}$

Answer: B

Solution:



$$\mathbf{P} = (3\hat{i} - \hat{j} + 2\hat{k}) \text{ and } \mathbf{Q} = (\hat{i} + 2\hat{j} - 4\hat{k})$$

$$\mathbf{V}_{PR} = (4, -1, 2) \text{ and } \mathbf{V}_{QS} = (-2, 1, -2)$$

$$\text{Equation of line PR} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(4\hat{i} - \hat{j} + 2\hat{k})$$

$$\text{Equation of line QS} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(-2\hat{i} + \hat{j} - 2\hat{k})$$

Let T be the point of intersection.

$$\mathbf{T} = (3 + 4\lambda, -1 - \lambda, 2 + 2\lambda)$$

$$\mathbf{T} = (1 - 2\mu, 2 + \mu, -4 - 2\mu)$$

$$3 + 4\lambda = 1 - 2\mu$$

$$\Rightarrow 2\lambda + \mu = -1 \dots\dots (i)$$

$$-1 - \lambda = 2 + \mu$$

$$\Rightarrow \lambda + \mu = -3 \dots\dots (ii)$$

From Eqs. (i) and (ii),

$$\lambda = 2 \text{ and } \mu = -5$$

$$\mathbf{T} = [3 + 4(2), -1 - (2), 2 + 2(2)] = (11, -3, 6)$$

Now, DC of TA will be $\mathbf{V}_{PR} \times \mathbf{V}_{QS}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -2 \\ 4 & -1 & 2 \end{vmatrix} = 0\hat{i} - 4\hat{j} - 2\hat{k}$$

$$\mathbf{L}_{TA} \Rightarrow (11\hat{i} - 3\hat{j} + 6\hat{k}) + x(-4\hat{j} - 2\hat{k})$$

$$\text{Let } \mathbf{A} = (11, -3 - 4x, 6 - 2x)$$

$$TA = \sqrt{5}$$

$$\Rightarrow \sqrt{(11 - 11)^2 + (-3 - 4x + 3)^2 + (6 - 2x - 6)^2} = \sqrt{5}$$

$$\Rightarrow (4x)^2 + (2x)^2 = 5 \Rightarrow 20x^2 = 5$$

$$\Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$$

$$\mathbf{A} = [11, -3 - 4(1/2), 6 - 2(1/2)]$$

$$\mathbf{A} = (11, -5, 5)$$

Or

$$\mathbf{A} = [11, -3 + 4(1/2), 6 + 2(1/2)]$$

$$\mathbf{A} = (11, -1, 7)$$

$$\therefore |\mathbf{A}| = \sqrt{11^2 + 5^2 + 5^2} \text{ or}$$

$$\Rightarrow |\mathbf{A}| = \sqrt{11^2 + 1^2 + 7^2}$$

$$\Rightarrow |\mathbf{A}| = \sqrt{171} \text{ or } \sqrt{171}$$

$$\therefore |\mathbf{A}| = \sqrt{171}$$

Question 199

If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}$, $I \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to
[16 Mar 2021 Shift 2]

Options:

A. $1/2$

B. $1/\sqrt{6}$

C. $\sqrt{2/3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

$$L_1 \Rightarrow \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}$$

Foot of perpendicular from A(4, 3, 8) to L_1 is B(3, 5, 7).

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (3\hat{i} + 5\hat{j} + 7\hat{k}) - (4\hat{i} + 3\hat{j} + 8\hat{k})$$

$$= -\hat{i} + 2\hat{j} - \hat{k}$$

Now, AB is perpendicular to direction cosine of L_1 ,

$$\text{So, } (-\hat{i} + 2\hat{j} - \hat{k}) \cdot (\hat{i} + 3\hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow -1 + 6 - 4 = 0 \Rightarrow I = 2$$

As, (3, 5, 7) lies on L_1 ,

$$\frac{3-a}{2} = \frac{5-2}{3} = \frac{7-b}{4}$$

$$3-a=2$$

$$\text{So, } a=1, 7-b=4$$

$$\text{So, } b=3$$

$$L_1 \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\text{and } L_2 = \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

$\therefore$ Shortest distance will be along common normal.

$$\text{So, common normal} = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{matrix} \right|$$

$$\Rightarrow n = -\hat{i} + 2\hat{j} - \hat{k} \Rightarrow \hat{n} = \frac{1}{\sqrt{6}}(-\hat{i} + 2\hat{j} - \hat{k})$$

Shortest distance will be the projection of $(2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}$ or $\hat{i} + 2\hat{j} + 2\hat{k}$ along $\hat{n}$

$$\Rightarrow (\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \frac{(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}} = \frac{-1+4-2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

Question 200

Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to
[17 Mar 2021 Shift 2]

Answer: 0

Solution:

Let $P = (\alpha, \beta, \gamma)$

Distance of point P from the plane $x + y + z = 0$ is

$$= \frac{\alpha + \beta + \gamma}{\sqrt{3}}$$

Distance of point P from the plane $lx - nz = 0$ is

$$= \frac{l\alpha - n\gamma}{\sqrt{l^2 + n^2}}$$

and distance of point P from the plane $x - 2y + z = 0$ is

$$= \frac{\alpha - 2\beta + \gamma}{\sqrt{6}}$$

According to the question,

$$\left(\frac{\alpha + \beta + \gamma}{\sqrt{3}} \right)^2 + \left(\frac{l\alpha - n\gamma}{\sqrt{l^2 + n^2}} \right)^2 + \left(\frac{\alpha - 2\beta + \gamma}{\sqrt{6}} \right)^2 = 9$$

$\therefore$ Locus is

$$\frac{(x+y+z)^2}{3} + \frac{(lx-nz)^2}{l^2+n^2} + \frac{(x-2y+z)^2}{6} = 9$$

$$\Rightarrow x^2 \left(\frac{1}{2} + \frac{l^2}{l^2+n^2} \right) + y^2 + z^2 \left(\frac{1}{2} + \frac{n^2}{l^2+n^2} \right) + xz \left(1 - \frac{2ln}{l^2+n^2} \right) = 0$$

Comparing it with the given equation of locus, we get

$$2ln = l^2 + n^2$$

$$\Rightarrow (l - n)^2 = 0$$

$$\Rightarrow l - n = 0$$

Question201

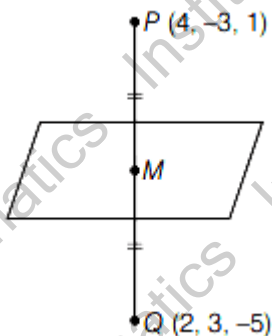
Let the plane, $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is
[18 Mar 2021 Shift 1]

Answer: 28

Solution:

Let $P \equiv (4, -3, 1)$

and $Q \equiv (2, 3, -5)$



$$\therefore M = \frac{P+Q}{2}$$

$$\Rightarrow M \equiv \left(\frac{4+2}{2}, \frac{-3+3}{2}, \frac{1-5}{2} \right)$$

$$\Rightarrow M \equiv (3, 0, -2)$$

Also, direction ratios of $PQ = \{4-2, -3-3, 1+5\}$

$$= \{2, -6, 6\}$$

$\Rightarrow$ Direction ratios of $PQ = \{1, -3, 3\} =$ direction ratios of normal to the plane.

$\therefore$ Equation of the plane is

$$1(x-3) - 3(y-0) + 3(z+2) = 0$$

$$\Rightarrow x - 3y + 3z + 3 = 0$$

Comparing this to $ax + by + cz + d = 0$, we get

$$a = 1, b = -3, c = 3, d = 3$$

$$\therefore \text{Minimum value of } (a^2 + b^2 + c^2 + d^2) = 28$$

Question202

The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

[18 Mar 2021 Shift 1]

Answer: 4

Solution:

Equation of any plane parallel to the plane $x - 2y + 2z - 3 = 0$ is

$x - 2y + 2z + \lambda = 0 \dots (i)$

Given, distance from $(1, 2, 3)$ is 1.

$$\therefore \left| \frac{1 - 2 \times 2 + 2 \times 3 + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} \right| = 1$$

$$\Rightarrow |\lambda + 3| = 3$$

$$\Rightarrow \lambda + 3 = \pm 3$$

$$\Rightarrow \lambda = 0, -6$$

Consider, $\lambda = -6$

$\therefore$ Equation of required plane is

$$x - 2y + 2z - 6 = 0$$

On comparing this equation to

$ax + by + cz + d = 0$, we get

$$a = 1, b = -2, c = 2 \text{ and } d = -6$$

$$\therefore (b - d) = k(c - a)$$

$$\therefore 4 = k \times (1)$$

$$\Rightarrow k = 4$$

Question 203

The equation of the plane which contains the Y -axis and passes through the point $(1, 2, 3)$ is

[17 Mar 2021 Shift 1]

Options:

A. $x + 3z = 10$

B. $x + 3z = 0$

C. $3x + z = 6$

D. $3x - z = 0$

Answer: D

Solution:

Equation of plane passing through a point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$

So, $a(x - 1) + b(y - 2) + c(z - 3) = 0$

Now, Y-axis lies on it.

Direction ratio of Y-axis is $(0, 1, 0)$.

Normal vector to the plane $= a\hat{i} + b\hat{j} + c\hat{k}$

So, the normal vector of the plane will be perpendicular to direction ratio of Y-axis.

$$a \cdot 0 + b \cdot 1 + c \cdot 0 = 0 \Rightarrow b = 0$$

Equation of plane becomes

$$a(x - 1) + c(z - 3) = 0$$

Now, $x = 0, z = 0$ also satisfies the equation.

$$a(0 - 1) + c(0 - 3) = 0$$

$$\Rightarrow -a - 3c = 0 \Rightarrow a = -3c$$

So, $-3c(x - 1) + c(z - 3) = 0$

$$-3x + 3 + z - 3 = 0$$

$$[as, C \neq 0]$$

$$\Rightarrow 3x - z = 0$$

Question 204

If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then, the length of line segment CD is equal to
[16 Mar 2021 Shift 1]

Options:

A. $\sqrt{31}$

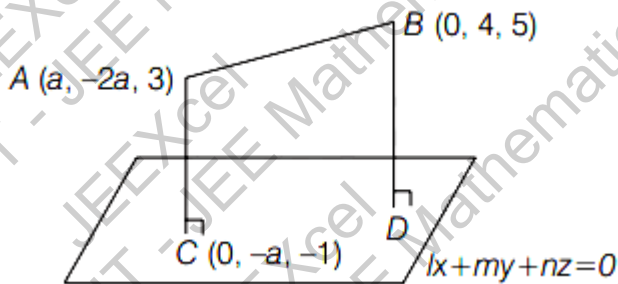
B. $\sqrt{41}$

C. $\sqrt{55}$

D. $\sqrt{66}$

Answer: D

Solution:



Given, A $\Rightarrow$ (a, -2a, 3)

B $\Rightarrow$ (0, 4, 5)

C $\Rightarrow$ (0, -a, -1)

Equation of plane P $\Rightarrow$ lx + my + nz = 0

As, C is foot of perpendicular from A to plane P. So, CA $\perp$ N, where N is the normal vector to the plane.

$$\begin{aligned} \vec{CA} &= (a-0)\hat{i} + (-2a+a)\hat{j} + (3+1)\hat{k} \\ &= a\hat{i} - a\hat{j} + 4\hat{k} \end{aligned}$$

Now, CA $\perp$ N

$$\text{So, } \frac{a}{l} = \frac{-a}{m} = \frac{4}{n} = \lambda$$

where λ is any real number.

$$P \Rightarrow \left(\frac{a}{\lambda}\right)x - \left(\frac{a}{\lambda}\right)y + \left(\frac{4}{\lambda}\right)z = 0$$

$$P \Rightarrow ax - ay + 4z = 0$$

C lies on plane.

$$\text{So, } a \cdot 0 - a(-a) + 4(-1) = 0$$

$$a^2 - 4 = 0 \Rightarrow a = \pm 2$$

As per the question, $a > 0$, so $a = 2$

$$\text{So, equation of plane P} \Rightarrow 2x - 2y + 4z = 0$$

$$P \Rightarrow x - y + 2z = 0$$

Coordinates of D

$$\frac{x-0}{1} = \frac{y-4}{-1} = \frac{z-5}{2} = \frac{-(0-4+10)}{[1^2 + (-1)^2 + 2^2]}$$

If (x, y, z) be the foot of perpendicular drawn from (x₁, y₁, z₁) to the plane ax + by + cz + d = 0.

Then,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Here, (x, y, z) = (0, 4, 5)

$$\Rightarrow x-0 = -(y-4) = \frac{z-5}{2} = \frac{-6}{6}$$

$$\therefore x = -1, y = 5, z = 3$$

$$C = (0, -2, -1) \Rightarrow D = (-1, 5, 3)$$

$$\begin{aligned} \therefore CD &= \sqrt{(0+1)^2 + (-2-5)^2 + (-1-3)^2} \\ &= \sqrt{1+49+16} \end{aligned}$$

$$CD = \sqrt{66}$$

Question 205

If (x, y, z) be an arbitrary point lying on a plane P, which passes through the points $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to

[16 Mar 2021 Shift 2]

Options:

A. 0

B. 3

C. 39

D. -45

Answer: B

Solution:

Equation of plane passing through $A(42, 0, 0)$, $B(0, 42, 0)$ and $C(0, 0, 42)$ will be

$$\frac{x}{42} + \frac{y}{42} + \frac{z}{42} = 1$$

$$\Rightarrow x + y + z = 42$$

$$(x-11) + (y-19) + (z-12) = 0$$

Now,

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} + \frac{y-19}{(x-11)^2(z-12)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{3(x-11)^2(y-19)^2(z-12)^2 + (x-11)^3 + (y-19)^3 + (z-12)^3}{(x-11)^2(y-19)^2(z-12)^2} - \frac{42}{14(x-11)(y-19)(z-12)}$$

$$\Rightarrow \frac{(x-11)^3 + (y-19)^3 + (z-12)^3 - 3(x-11)(y-19)(z-12) + 3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

$$\Rightarrow \text{If } A + B + C = 0$$

$$\text{Then, } A^3 + B^3 + C^3 = 3ABC$$

$$\Rightarrow (x-11)^3 + (y-19)^3 + (z-12)^3 = 3(x-11)(y-19)(z-12)$$

$$\Rightarrow \frac{3(x-11)(y-19)(z-12) - 3(x-11)(y-19)(z-12) + 3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2}$$

$$\Rightarrow \frac{3(x-11)^2(y-19)^2(z-12)^2}{(x-11)^2(y-19)^2(z-12)^2} = 3$$

Question 206

If the equation of the plane passing through the line of intersection of the planes

$$2x - 7y + 4z - 3 = 0, 3x - 5y + 4z + 11 = 0$$

and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is

[17 Mar 2021 Shift 1]

Answer: 4

Solution:

Equation of the plane passing through the line of intersections of planes $2x - 7y + 4z - 3 = 0$ and $3x - 5y + 4z + 11 = 0$ is

$$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$$

Since this plane passes through the point $(-2, 1, 3)$

$$\therefore (-4 - 7 + 12 - 3) + \lambda(-6 - 5 + 12 + 11) = 0$$

$$-2 + 12\lambda = 0 \Rightarrow \lambda = 1/6$$

$\therefore$ Equation of plane is

$$(2x - 7y + 4z - 3) + \frac{1}{6}(3x - 5y + 4z + 11) = 0$$

$$15x - 47y + 28z - 7 = 0$$

$$\therefore a = 15, b = -47, c = 28$$

$$2a + b + c - 7 = 30 - 47 + 28 - 7 = 4$$

Question 207

Let the mirror image of the point $(1, 3, a)$ with respect to the plane

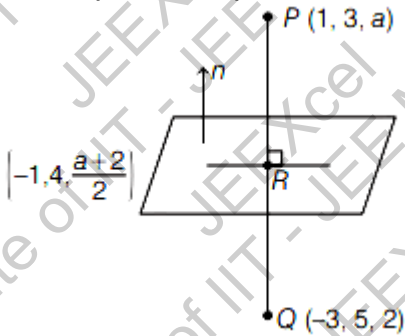
$$\mathbf{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0 \text{ be } (-3, 5, 2). \text{ Then the value of } |a + b| \text{ is equal to}$$

[18 Mar 2021 Shift 2]

Answer: 1

Solution:

Given equation of plane in vector form is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$



Its Cartesian form will be

$$2x - y + z = b \dots (i)$$

$\therefore R$ is the mid-point of PQ .

$$\therefore R \equiv \frac{P+Q}{2} \Rightarrow R \equiv \left(-1, 4, \frac{a+2}{2}\right)$$

$\therefore R$ lies on the plane (i).

$$\therefore -2 - 4 + \frac{a+2}{2} = b \Rightarrow a + 2 = 2b + 12$$

$$\Rightarrow a = 2b + 10 \dots (ii)$$

$\therefore$ Direction ratio's of QP is $(1 - (-3), 3 - 5, a - 2)$

i.e. $(4, -2, a - 2)$

and direction ratios of normal to the given plane are $(2, -1, 1)$

$\therefore n$ and QP are parallel.

$$\frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$$

$$\therefore a - 2 = 2 \Rightarrow a = 4$$

From Eq. (ii), $b = -3$

$$\therefore |a + b| = |4 - 3| = |1| = 1$$

Question 208

Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to

[18 Mar 2021 Shift 2]

Answer: 38

Solution:

Equation of required plane is
$$\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

Since, $(1, -1, \infty)$ lies on it,

So, replace x by 1 , y by (-1) and z and α

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 5\alpha + 38 = 0 \Rightarrow 5\alpha = -38$$

$$\therefore |5\alpha| = |-38| = 38$$

Question 209

Let P be a plane $lx + my + nz = 0$ containing the line,

$$\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}. \text{ If plane } P \text{ divides the line segment } AB \text{ joining}$$

points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$, then the value of k is equal to

[16 Mar 2021 Shift 1]

Options:

A. 1.5

B. 3

C. 2

D. 4

Answer: C

Solution:

$$P \Rightarrow lx + my + nz = 0$$

P contains L_1

$$L_1 \Rightarrow \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$$

So, (1, -4, -2) lies on plane.

$$1 - 4m - 2n = 0 \dots (i)$$

And (-1, 2, 3) will be perpendicular to (1, m, n).

$$-1 + 2m + 3n = 0 \dots (ii)$$

Adding Eqs. (i) and (ii),

$$-2m + n = 0$$

$$n = 2m$$

$$1 - 4m - 4m = 0$$

$$1 = 8m$$

$$\text{So, } 1 = 8m \text{ and } n = 2m$$

$$\text{Plane} \Rightarrow 8x + y + 2z = 0$$

$$\text{Now, } A(-3, -6, 1) \text{ and } B(2, 4, -3)$$

Plane P divides AB in the ratio of k : 1.

Let plane P intersect the line AB at point O.

$$\text{So, } O = \left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1} \right)$$

And O lies on plane P,

$$\text{So, } 8(2k-3) + (4k-6) + 2(-3k+1) = 0$$

$$\Rightarrow 14k - 28 = 0$$

$$\therefore k = 2$$

Question 210

If the distance of the point (1, -2, 3) from the plane

$x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is

$\sqrt{\frac{7}{2}}$, then the value of |m| is equal to.....

[16 Mar 2021 Shift 2]

Answer: 2

Solution:

$$\text{Given, point } A \Rightarrow (1, -2, 3)$$

$$\text{Plane} \Rightarrow x + 2y - 3z + 10 = 0$$

Distance of point from plane along the vector $(3\hat{i} - m\hat{j} + \hat{k})$ is $\sqrt{7/2}$.

Line passing through (1, -2, 3) in the direction of

$$(3\hat{i} - m\hat{j} + \hat{k}) \text{ is } \frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$$

Any general point B will be $(3\lambda + 1, -m\lambda - 2, \lambda + 3)$

Now, this point B lies on plane

So, $x + 2y - 3z + 10 = 0$

$$(3\lambda + 1) + 2(-m\lambda - 2) - 3(\lambda + 3) + 10 = 0$$

$$= (3 - 2m - 3)\lambda = 2$$

$$\Rightarrow \lambda = -1/m$$

Now, $A = (1, -2, 3)$

$$B = (3\lambda + 1, -m\lambda - 2, \lambda + 3)$$

$$|AB|^2 = (3\lambda + 1 - 1)^2 + (-m\lambda - 2 + 2)^2 + (\lambda + 3 - 3)^2$$

$$\Rightarrow 7/2 = 9\lambda^2 + m^2\lambda^2 + \lambda^2$$

$$\Rightarrow 7/2 = 10\lambda^2 + 1 \quad [\because m\lambda = -1]$$

$$\Rightarrow 10\lambda^2 = 5/2 \Rightarrow \lambda^2 = 1/4$$

$$\Rightarrow \lambda = \pm 1/2 \Rightarrow m = -1/\lambda$$

$$\therefore m = \pm 2$$

$$\text{and } |m| = 2$$

Question 211

If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is _____.
[25 Jul 2021 Shift 2]

Answer: 1

Solution:

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$$(k+1)[2-6] - 4[1-9] + 6[2-6] = 0$$

$$k = 1$$

Question 212

If the shortest distance between the straight lines

$$3(x-1) = 6(y-2) = 2(z-1) \text{ and } 4(x-2) = 2(y-\lambda) = (z-3), \lambda \in \mathbb{R} \text{ is}$$

$\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

[22 Jul 2021 Shift 2]

Options:

A. 3

B. 2

C. 5

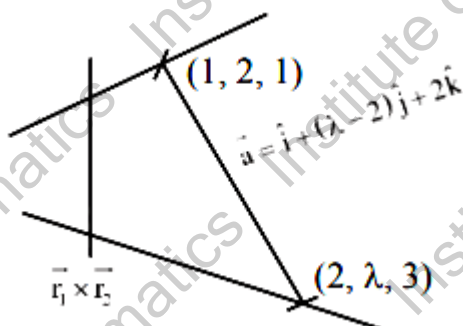
D. -1

Answer: A

Solution:

$$L_1 : \frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-1)}{3} \quad \vec{r}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$L_2 : \frac{(x-2)}{1} = \frac{(y-\lambda)}{2} = \frac{(z-3)}{4} \quad \vec{r}_2 = \hat{i} + 2\hat{j} + 4\hat{k}$$



Shortest distance = Projection of $\vec{a}$ on $\vec{r}_1 \times \vec{r}_2$

$$= \frac{|\vec{a} \cdot (\vec{r}_1 \times \vec{r}_2)|}{|\vec{r}_1 \times \vec{r}_2|}$$

$$|\vec{a} \cdot (\vec{r}_1 \times \vec{r}_2)| = \begin{vmatrix} 1 & \lambda - 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$$

$$|\vec{r}_1 \times \vec{r}_2| = \sqrt{38}$$

$$\therefore \frac{1}{\sqrt{38}} = \frac{|14 - 5\lambda|}{\sqrt{38}}$$

$$\Rightarrow |14 - 5\lambda| = 1$$

$$\Rightarrow 14 - 5\lambda = 1 \text{ or } 14 - 5\lambda = -1$$

$$\Rightarrow \lambda = \frac{13}{5} \text{ or } 3$$

$$\therefore \text{Integral value of } \lambda = 3$$

Question 213

If the shortest distance between the lines

$$\vec{r}_1 = \alpha \hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R}, \alpha > 0 \text{ and}$$

overrightarrow{\mathrm{r}}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in \mathbb{R} \text{ is 9, then } \alpha \text{ is equal to } \underline{\hspace{2cm}}.

[20 Jul 2021 Shift 1]

Answer: 6

Solution:

If $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{c} + \lambda \vec{d}$
then shortest distance between two lines is

$$L = \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}$$

$$\therefore \vec{a} - \vec{c} = ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\frac{\vec{b} \times \vec{d}}{|\vec{b} \times \vec{d}|} = \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3}$$

$$\therefore ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}) \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = 9$$

$$\text{or } \alpha = 6$$

Question 214

The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, ($ab \neq 0$) are coplanar, if :

[20 Jul 2021 Shift 2]

Options:

A. $b = 1, a \in \mathbb{R} - \{0\}$

B. $a = 1, b \in \mathbb{R} - \{0\}$

C. $a = 2, b = 2$

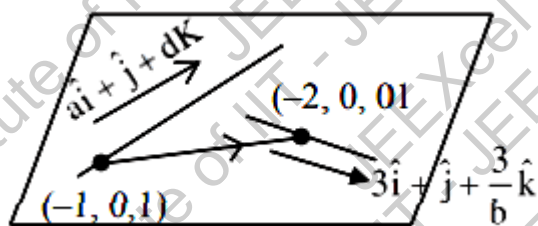
D. $a = 2, b = 3$

Answer: A

Solution:

$$\frac{x+1}{a} = y = \frac{z-1}{a}$$

$$\frac{x+2}{3} = y = \frac{z}{3/b}$$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a - 3) = 0$$

$$a - \frac{3}{b} - a + 3 = 0$$

$$-b = 1, a \in \mathbb{R} - \{0\}$$

Question 215

Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to:
[27 Jul 2021 Shift 1]

Options:

A. 3

B. 8

C. 5

D. 4

Answer: D

Solution:

Normal of req. plane $(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} - \hat{j} - \hat{k})$

$$= -2\hat{i} + \hat{j} - 3\hat{k}$$

Equation of plane

$$-2(x+1) + 1(y-0) - 3(z+2) = 0$$

$$-2x + y - 3z - 8 = 0$$

$$2x - y + 3z + 8 = 0$$

$$a + b + c = 4$$

Question 216

Let a , b and c be distinct positive numbers. If the vectors

$a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to:

[25 Jul 2021 Shift 2]

Options:

A. $\frac{2}{\frac{1}{a} + \frac{1}{b}}$

B. $\frac{a+b}{2}$

C. $\frac{1}{a} + \frac{1}{b}$

D. $\sqrt{ab}$

Answer: D

Solution:

Because vectors are coplanar

$$\text{Hence } \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

Question 217

Let a plane P pass through the point $(3, 7, -7)$ and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then

d^2 is equal to _____.

[27 Jul 2021 Shift 1]

Answer: 3

Solution:

$$\vec{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$$

$$\vec{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$$

$$\vec{BA} \times \vec{l} = \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$$

$$a\hat{i} + b\hat{j} + c\hat{k} = -14\hat{i} - \hat{j}(14) + \hat{k}(-14)$$

$$a = 1, b = 1, c = 1$$

$$\text{Plane is } (x-2) + (y-3) + (z+2) = 0$$

$$x + y + z - 3 = 0$$

$$d = \sqrt{3} \Rightarrow d^2 = 3$$

Question 218

For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$ lies on the plane

$x + 2y - z = 8$, then $\alpha - \beta$ is equal to :

[27 Jul 2021 Shift 2]

Options:

A. 5

B. 9

C. 3

D. 7

Answer: D

Solution:

First line is $(\phi + \alpha, 2\phi + 1, 3\phi + 1)$

and second line is $(q\beta + 4, 3q + 6, 3q + 7)$.

For intersection

$$\phi + \alpha = q\beta + 4 \dots (i)$$

$$2\phi + 1 = 3q + 6 \dots (ii)$$

$$3\phi + 1 = 3q + 7 \dots (iii)$$

for (ii) & (iii) $\phi = 1, q = -1$

So, from (i) $\alpha + \beta = 3$

Now, point of intersection is $(\alpha + 1, 3, 4)$

It lies on the plane.

Hence, $\alpha = 5$ & $\beta = -2$

Question 219

The distance of the point $P(3, 4, 4)$ from the point of intersection of the line joining the points $Q(3, -4, -5)$ and $R(2, -3, 1)$ and the plane $2x + y + z = 7$, is equal to _____.

[27 Jul 2021 Shift 2]

Answer: 7

Solution:

$$\vec{QR} : \frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$$

$$\Rightarrow (x, y, z) = (r+3, -r-4, -6r-5)$$

Now, satisfying it in the given plane.

We get $r = -2$.

so, required point of intersection is $T(1, -2, 7)$.

Hence, $PT = 7$.

Question 220

Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L : \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N .

Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q . If α is the acute angle between the lines PN and PQ , then $\cos \alpha$ is equal to _____.

[25 Jul 2021 Shift 1]

Options:

A. $\frac{1}{\sqrt{5}}$

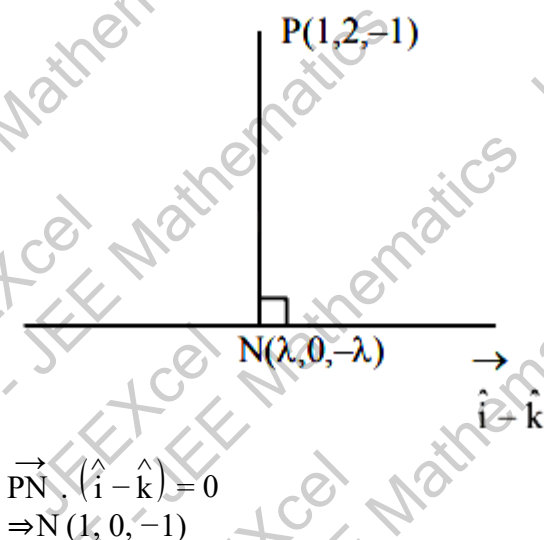
B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{\sqrt{3}}$

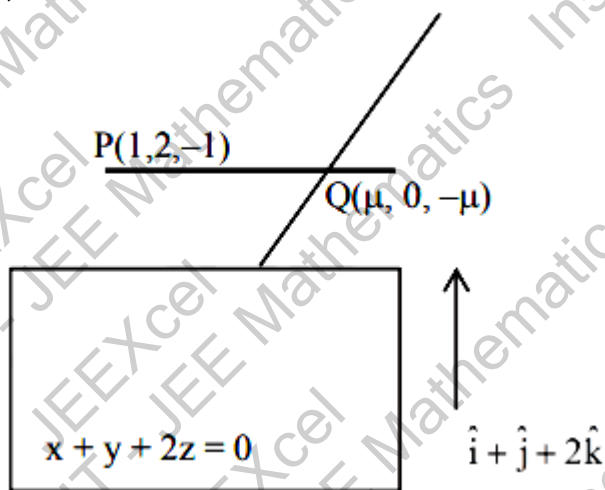
D. $\frac{1}{2\sqrt{3}}$

Answer: C

Solution:



Now,



$$\vec{PQ} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \mu = -1$$

$$\Rightarrow Q(-1, 0, 1)$$

$$\vec{PN} = 2\hat{j} \text{ and } \vec{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

Question 221

Let L be the line of intersection of planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 2$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point $(1, 2, 0)$, then the value of $35(\alpha + \beta + \gamma)$ is equal to :
[22 Jul 2021 Shift 2]

Options:

A. 101

B. 119

C. 143

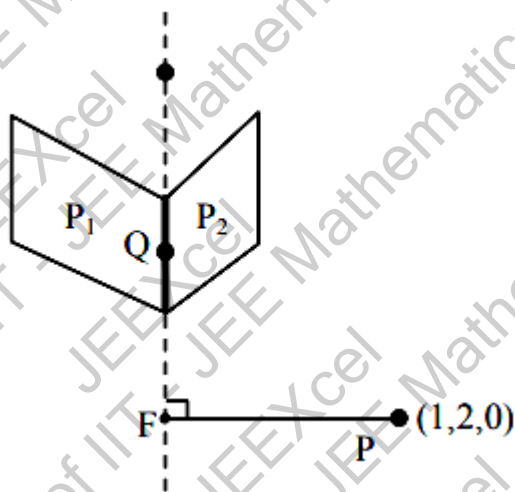
D. 134

Answer: B

Solution:

$$P_1 : x - y + 2z = 2$$

$$P_2 = 2x + y - 3 = 2$$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q .

$\Rightarrow$ z -coordinate of point Q is zero

$$\Rightarrow \left. \begin{array}{l} x - y = 2 \\ \text{and } 2x + y = 2 \end{array} \right\} \Rightarrow x = \frac{4}{3}, y = \frac{-2}{3}$$

$$\Rightarrow Q\left(\frac{4}{3}, \frac{-2}{3}, 0\right)$$

Vector parallel to the line of intersection

$$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} + 5\hat{j} + 3\hat{k}$$

Equation of Line of intersection

$$\frac{x - \frac{4}{3}}{-1} = \frac{y + \frac{2}{3}}{5} = \frac{z - 0}{3} = \lambda \text{ (say)}$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda + \frac{4}{3}, 5\lambda - \frac{2}{3}, 3\lambda\right)$$

$$\vec{PF} = \left(-\lambda + \frac{1}{3}\right)\hat{i} + \left(5\lambda - \frac{8}{3}\right)\hat{j} + (3\lambda)\hat{k}$$

$$\vec{PF} \cdot \vec{a} = 0$$

$$\Rightarrow \lambda - \frac{1}{3} + 25\lambda - \frac{40}{3} + 9\lambda = 0$$

$$\Rightarrow 35\lambda = \frac{41}{3} \Rightarrow \lambda = \frac{41}{105}$$

$$\text{Now, } \alpha = -\lambda + \frac{4}{3}, \beta = 5\lambda - \frac{2}{3}, \gamma = 3\lambda$$

$$\Rightarrow \alpha + \beta + \gamma = 7\lambda + \frac{2}{3}$$

$$= 7\left(\frac{41}{105}\right) + \frac{2}{3}$$

$$= \frac{51}{15}$$

$$\Rightarrow 35(\alpha + \beta + \gamma) = \frac{51}{15} \times 35 = 119$$

Question 222

Let P be a plane passing through the points (1, 0, 1), (1, -2, 1) and (0, 1, -2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____.

[20 Jul 2021 Shift 1]

Answer: 81

Solution:

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$$\Rightarrow 3x - z - 2 = 0$$

$$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel \text{to } 3x - z - 2 = 0$$

$$\Rightarrow 3\alpha - \gamma = 0 \dots\dots(1)$$

$$\vec{a} \perp \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\Rightarrow \alpha + 2\beta + 3\gamma = 0 \dots\dots(2)$$

$$\vec{a} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \alpha + \beta + 2\gamma = 2 \dots\dots(3)$$

on solving 1, 2 & 3

$$\alpha = 1, \beta = -5, \gamma = 3$$

$$\text{So } (\alpha - \beta + \gamma) = 81$$

Question 223

Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, -1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?

[20 Jul 2021 Shift 2]

Options:

A. $(-1, 1, 2)$

B. $(1, 1, 1)$

C. $(1, 1, 2)$

D. $(1, 2, 2)$

Answer: D

Solution:

Plane p is $\perp^r$ to line

$$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$$

& passes through pt. $(2, 3)$ equation of plane p

$$2(x-2) + 1(y-3) + 1(z+1) = 0$$

$$2x + y + z - 6 = 0$$

pt $(1, 2, 2)$ satisfies above equation

Question 224

The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is
[27 Aug 2021 Shift 2]

Options:

A. $\frac{\pi}{2}$

B. $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

C. $\cos^{-1}\left(\frac{8}{9}\right)$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Given,

$$2l + 2m - n = 0 \dots (i)$$

$$mn + nl + lm = 0 \dots (ii)$$

From Eq. (i), we get

$$n = 2l + 2m \dots (iii)$$

Substituting, $n = 2l + 2m$ in Eq. (ii), we have

$$m(2l + 2m) + l(2l + 2m) + lm = 0$$

$$\Rightarrow 2lm + 2m^2 + 2l^2 + 2lm + lm = 0$$

$$\Rightarrow 2l^2 + 4lm + lm + 2m^2 = 0$$

$$\Rightarrow 2l(l + 2m) + m(l + 2m) = 0$$

$$\Rightarrow (2l + m)(l + 2m) = 0$$

When

$$2l = -m$$

From Eq (iii),

$$n = m$$

$$\Rightarrow \frac{2l}{-1} = \frac{m}{1} = \frac{n}{1}$$

$$\Rightarrow \frac{l}{-\frac{1}{2}} = \frac{m}{1} = \frac{n}{1}$$

$$\text{or } \frac{l}{1} = \frac{m}{-2} = \frac{n}{-2}$$

$$\Rightarrow (l, m, n) = (1, -2, -2)$$

When

$$l = -2m$$

From Eq. (iii),

$$n = -2m$$

$$\Rightarrow l = -2m = n$$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-2}$$

$$\Rightarrow (l, m, n) = (-2, 1, -2)$$

$\therefore$ Angles between straight lines

$$\cos \theta = \frac{(\hat{i} - 2\hat{j} - 2\hat{k})(-2\hat{i} + \hat{j} - 2\hat{k})}{|\hat{i} - 2\hat{j} - 2\hat{k}| |-2\hat{i} + \hat{j} - 2\hat{k}|}$$

$$\cos \theta = \frac{-2 - 2 + 4}{9} = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Question 225

The square of the distance of the point of intersection of the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} \text{ and the plane } 2x - y + z = 6 \text{ from the point } (-1, -1, 2)$$

is

[31 Aug 2021 Shift 1]

Answer: 61

Solution:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$$

$$\begin{cases} x = 2\lambda + 1 \\ y = 3\lambda + 2 \\ z = 6\lambda - 1 \end{cases}$$

Equation of plane is $2x - y + z = 6$

$$\Rightarrow 2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$$

$$7\lambda = 7$$

$$\lambda = 1$$

$$P(3, 5, 5)$$

$$\begin{aligned} (\text{Distance})^2 &= (3+1)^2 + (5+1)^2 + (5-2)^2 \\ &= 16 + 36 + 9 = 61 \end{aligned}$$

Question 226

The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is
[31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{5}{2}$

C. $\frac{\sqrt{42}}{2}$

D. $\frac{\sqrt{34}}{2}$

Answer: D

Solution:

Let L be the line of intersection of $2x + 3y + 2z = 0$ and $x - 2y + z = 0$

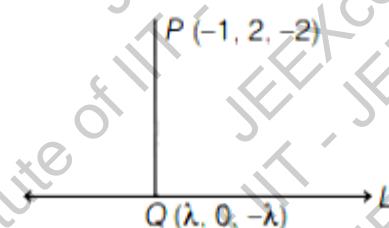
If $z = 0$, then $x = y = 0$

The line L is parallel to $\mathbf{r}_1 \times \mathbf{r}_2$, where $\mathbf{r}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\mathbf{r}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 7\hat{i} - 7\hat{k}$$

DR's of is $(1, 0, -1)$

and equation of L is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = \lambda$



Let PQ be the distance from the point $P(-1, 2, -2)$ to the line L.

DRs of $PQ = \lambda + 1, -2, 2 - \lambda$

$\therefore PQ \perp \mathbf{r}$

$$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$$

$$\Rightarrow \lambda + 1 - 2 + \lambda = 0$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Coordinate of Q is $\left(\frac{1}{2}, 0, -\frac{1}{2}\right)$

$$\begin{aligned} \Rightarrow PQ &= \sqrt{\left(-1 - \frac{1}{2}\right)^2 + (2 - 0)^2 + \left(-2 + \frac{1}{2}\right)^2} \\ &= \sqrt{\frac{9}{4} + 4 + \frac{9}{4}} = \frac{\sqrt{34}}{2} \end{aligned}$$

Question 227

If the equation of plane passing through the mirror image of a point $(2, 3, 1)$ with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line

$\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to

[17 Mar 2021 Shift 2]

Options:

A. 20

B. 19

C. 18

D. 21

Answer: B

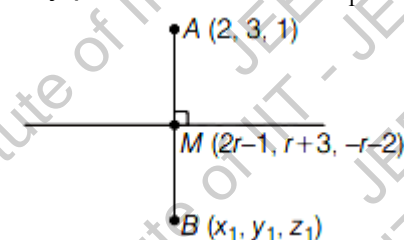
Solution:

Let $A = (2, 3, 1)$

$$L_1 \Rightarrow \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$$

$$L_2 \Rightarrow \frac{x-2}{3} = \frac{y-1}{-2} = \frac{z+1}{1}$$

Any point M taken on L_1 is $(2r-1, r+3, -r-2)$



$\therefore$ Direction ratios of AM are $(2r-3, r, -r-3)$

$\because AM \perp L_1$

$$\therefore 2(2r-3) + 1 \times r + (-1)(-r-3) = 0$$

$$\Rightarrow 4r - 6 + r + r + 3 = 0$$

$$\Rightarrow 6r = 3 \Rightarrow r = \frac{1}{2}$$

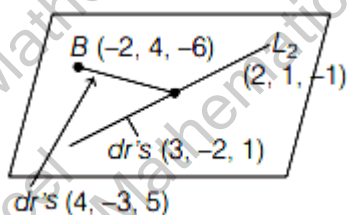
$$\therefore M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$$

$$\therefore M = \left(0, \frac{7}{2}, \frac{-5}{2}\right)$$

$$\therefore B \equiv \left((2 \times 0) - 2, \left(2 \times \frac{7}{2}\right) - 3, \left(2 \times \left(\frac{-5}{2}\right)\right) - 1\right)$$

$$\Rightarrow B = (-2, 4, -6)$$

Now, equation of plane containing $B(-2, 4, -6)$ and the line L_2 is



$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-9+8) = 0$$

$$\Rightarrow -7(x-2) - 11(y-1) - 1(z+1) = 0$$

$$\Rightarrow -7x - 11y - z = -14 - 11 + 1$$

$$\Rightarrow 7x + 11y + z = 24 \text{ comparing this to}$$

$$\alpha x + \beta y + \gamma z = 24$$

We get, $\alpha = 7, \beta = 11, \gamma = 1$

$$\therefore \alpha + \beta + \gamma = 7 + 11 + 1 = 19$$

Question 228

Suppose the line $\frac{x-2}{\alpha} + \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$.

Then, $(\alpha + \beta)$ is equal to

[31 Aug 2021 Shift 2]

Answer: 7

Solution:

Given equation of line

$$\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2} \dots (i)$$

and plane $x + 3y - 2z + \beta = 0 \dots (ii)$

Line (i) passes through $(2, 2, -2)$

which lies on plane (ii).

$$\therefore 2 + 6 + 4 + \beta = 0$$

$$\Rightarrow \beta = -12$$

Also, given line is perpendicular to normal of the plane

$$\alpha(1) - 5(3) + 2(-2) = 0$$

$$\Rightarrow \alpha = 19$$

$$\therefore \alpha + \beta = 7$$

Question 229

Let the equation of the plane, that passes through the point $(1, 4, -3)$ and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$ be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to

[31 Aug 2021 Shift 1]

Options:

A. -23

B. -15

C. 23

D. 15

Answer: A

Solution:

Equation of plane is

$$(3x - 2y + 4z - 7) + \lambda(x + 5y - 2z + 9) = 0$$

$$(\lambda + 3)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$$

Passing through $(1, 4, -3)$

$$(\lambda + 3) + 4(5\lambda - 2) - 3(4 - 2\lambda) + 9\lambda - 7 = 0$$

$$\Rightarrow 36\lambda - 24 = 0$$

$$\lambda = \frac{2}{3}$$

$\Rightarrow$ Equation of plane

$$\left(\frac{2}{3} + 3\right)x + \left(\frac{10}{3} - 2\right)y + \left(4 - \frac{4}{3}\right)z + 6 - 7 = 0$$

$$\Rightarrow 11x + 4y + 8z - 3 = 0$$

$$\alpha = -11, \beta = -4, \gamma = -8$$

$$\alpha + \beta + \gamma = -23$$

Question 230

The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is [27 Aug 2021 Shift 1]

Options:

A. 3

B. 5

C. 2

D. 1

Answer: D

Solution:

Let A be any point on the plane $x - y + z = 5$ and $B(1, -2, 3)$.

Then equation of the line AB whose direction ratios are $2, 3, -6$

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda \text{ (Let)}$$

$$\Rightarrow x = 1 + 2\lambda, y = -2 + 3\lambda, z = 3 - 6\lambda$$

$$A(1 + 2\lambda, -2 + 3\lambda, 3 - 6\lambda)$$

A lies on plane.

$$\text{Then, } 1 + 2\lambda - (-2 + 3\lambda) + 3 - 6\lambda = 5$$

$$\Rightarrow 1 + 2\lambda + 2 - 3\lambda + 3 - 6\lambda = 5$$

$$\Rightarrow \lambda = \frac{1}{7}$$

$$\therefore A\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$$

$$\text{Distance AB} = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

Question 231

Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which

contains the line of intersection of the planes $x - y - z - 1 = 0$ and

$2x + y - 3z + 4 = 0$, is

[27 Aug 2021 Shift 1]

Options:

A. $3x - y - 5z + 2 = 0$

B. $3x - 4z + 3 = 0$

C. $-x + 2y + 2z - 3 = 0$

D. $4x - y - 5z + 2 = 0$

Answer: D

Solution:

Given planes,

$$x - y - z - 1 = 0 \dots (i)$$

$$2x + y - 3z + 4 = 0 \dots (ii)$$

Equation of plane passing through line of intersection of planes (i) and (ii) is given by

$$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$$

$$\Rightarrow (2\lambda + 1)x + (\lambda - 1)y + (-3\lambda - 1)z + (4\lambda - 1) = 0 \dots (iii)$$

Distance of plane (iii) from origin $= \frac{2}{21}$ (given)

$$\Rightarrow \frac{|4\lambda - 1|}{\sqrt{(2\lambda + 1)^2 + (\lambda - 1)^2 + (-3\lambda - 1)^2}} = \sqrt{\frac{2}{21}}$$

Squaring both sides

$$\frac{(4\lambda - 1)^2}{(2\lambda + 1)^2 + (\lambda - 1)^2 + (3\lambda + 1)^2} = \frac{2}{21}$$
$$\Rightarrow 21(16\lambda^2 - 8\lambda + 1) = 2(14\lambda^2 + 8\lambda + 3)$$
$$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$$
$$308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$$
$$(2\lambda - 1)(154\lambda - 15) = 0$$
$$\Rightarrow \lambda = \frac{1}{2} \text{ or } \lambda = \frac{15}{154}$$

Putting $\lambda = \frac{1}{2}$ in Eq. (iii), we have

$$4x - y - 5z + 2 = 0$$

Question 232

The equation of the plane passing through the line of intersection of the planes $\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to the X-axis is
[27 Aug 2021 Shift 2]

Options:

A. $\mathbf{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$

B. $\mathbf{r} \cdot (\hat{i} + 3\hat{k}) + 6 = 0$

C. $\mathbf{r} \cdot (\hat{i} - 3\hat{k}) + 6 = 0$

D. $\mathbf{r} \cdot (\hat{j} - 3\hat{k}) - 6 = 0$

Answer: A

Solution:

Given, equation of planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \dots (i)$$

$$\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0 \dots (ii)$$

Equation of plane passing through the intersection of the planes Eqs. (i) and (ii) is given by

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

$$\text{or } (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (-1 + 4\lambda) = 0 \dots (iii)$$

Plane (iii) is parallel to X-axis

$$1 + 2\lambda = 0 \text{ [Coefficient of } x = 0]$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

$\therefore$ From Eq. (iii) becomes

$$y - 3z + 6 = 0$$

$$\text{or r. } (\hat{j} - 3\hat{k}) + 6 = 0$$

Question 233

Let S be the mirror image of the point Q (1, 3, 4) with respect to the plane $2x - y + z + 3 = 0$ and let R(3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is
[27 Aug 2021 Shift 2]

Answer: 72

Solution:

Let point S(a, b, c)

Then,

$$\frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = \frac{-2(2-3+4+3)}{4+1+1} = -2$$

$$\Rightarrow a = -3, b = 5, c = 2$$

$$\therefore S(-3, 5, 2)$$

and point R(3, 5, γ) lies on the plane $2x - y + z + 3 = 0$

$$\Rightarrow 6 - 5 + \gamma + 3 = 0$$

$$\Rightarrow \gamma = -4$$

$$\therefore R(3, 5, -4)$$

$$\begin{aligned} \text{Now, } SR^2 &= 6^2 + 0^2 + (6)^2 \\ &= 36 + 0 + 36 = 72 \end{aligned}$$

Question 234

A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$ and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P?
[26 Aug 2021 Shift 1]

Options:

- A. $(-1, 1, 2)$
- B. $(0, 1, 1)$
- C. $(1, 0, 1)$
- D. $(2, -1, 1)$

Answer: B

Solution:

Equation of plane containing the given planes is

$$(x + 2y + 3z + 1) + \lambda(x - y - z - 6) = 0$$

$$(1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + (1 - 6\lambda) = 0$$

This plane is perpendicular to the plane $-2x + y + z + 8 = 0$

$$\text{So, } -2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0$$

$$-2 - 2\lambda + 2 - \lambda + 3 - \lambda = 0$$

$$\lambda = \frac{3}{4}$$

So, equation of required plane is $7x + 5y + 9z = 14$

Now, $(0, 1, 1)$ satisfies the above plane.

Question 235

Let P be the plane passing through the point $(1, 2, 3)$ and the line of intersection of the planes $\mathbf{r} \cdot (\hat{i} + \hat{j} + 4\hat{k}) = 6$ and $\mathbf{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 6$. Then which of the following points does not lie on P?

[26 Aug 2021 Shift 2]

Options:

- A. $(3, 3, 2)$
- B. $(6, -6, 2)$
- C. $(4, 2, 2)$
- D. $(-8, 8, 6)$

Answer: C

Solution:

P is a plane passing through the intersection of P_1 and P_2 .

Equation of P : $P_1 + \lambda P_2 = 0$

$$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0 \dots(i)$$

Since plane P passes through (1, 2, 3), then

$$(1 + 2 + 12 - 16) + \lambda(-1 + 2 + 3 - 6) = 0$$

$$\Rightarrow -1 + \lambda(-2) = 0$$

$$\Rightarrow \lambda = \frac{-1}{-2}$$

On putting $\lambda = \frac{-1}{-2}$ in Eq. (i), we get

$$P : 3x + y + 7z - 26 = 0$$

Clearly (4, 2, 2) not lie on the plane.

Question 236

Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to
[26 Aug 2021 Shift 1]

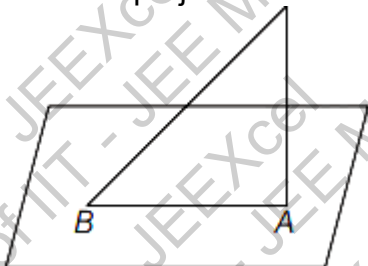
Answer: 26

Solution:

Given line, $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$

Given plane, $x - 2y - z = 3$

To find the projection let's find the foot of perpendicular from (1, 3, 4) to plane $x - 2y - z = 3$



$$\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-4}{-1} = \lambda_1$$

$$(\lambda_1 + 1) - 2(-2\lambda_1 + 3) - (-\lambda_1 + 4) = 3$$

$$\Rightarrow 6\lambda_1 = 12$$

$$\Rightarrow \lambda_1 = 2$$

So, foot of perpendicular from (1, 3, 4) to plane $x - 2y - z = 3$ is A (3, -1, 2).

Let us also find the intersection point of the plane and line

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2} = \lambda_2$$

$$(2\lambda_2 + 1) - 2(\lambda_2 + 3) - (2\lambda_2 + 4) = 3 - 2\lambda_2 = 12$$

$$\Rightarrow \lambda_2 = -6$$

The intersection point of the plane and line is B(-11, -3, -8)

Line passing through A and B is

$$\frac{x-3}{-14} = \frac{y+1}{-2} = \frac{z-2}{-10} = \mu$$

$$\frac{x-3}{7} = \frac{y+1}{1} = \frac{z-2}{5} = \mu$$

Now, let's find the distance from O(0, 0, 6) to this line L.

Let's say C(7μ + 3, μ - 1, 5μ + 2) is any point on L.

Then,

$$\{(7\mu + 3) - 0\}.7 + \{(\mu - 1) - 0\}.1 + \{(5\mu + 2) - 6\}.5 = 0$$

$$\Rightarrow 49\mu + 21 + \mu - 1 + 25\mu - 20 = 0$$

$$\Rightarrow \mu = 0$$

$$C(3, -1, 2)$$

$$\text{Distance} = \sqrt{(3-0)^2 + (-1-0)^2 + (2-6)^2} = \sqrt{26}$$

$$d^2 = 26$$

Question 237

Let Q be the foot of the perpendicular from the point P(7, -2, 13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x+1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$.

Then (PQ)² is equal to
[26 Aug 2021 Shift 2]

Answer: 96

Solution:

Plane containing the lines would be

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x+1)(49-40) - (y-1)(42-24) + (z-3)(30-21) = 0$$

$$\Rightarrow 9(x+1) - 18(y-1) + 9(z-3) = 0$$

$$\Rightarrow x - 2y + z = 0$$

Now, PQ will be equal to the perpendicular distance of the point P(7, -2, 13) from the plane

$$x - 2y + z = 0$$

$$\therefore PQ = \left| \frac{7 - 2(-2) + 13}{\sqrt{1^2 + (-2)^2 + (1)^2}} \right|$$

$$= \left| \frac{7 + 4 + 13}{\sqrt{1 + 4 + 1}} \right| = \left| \frac{24}{\sqrt{6}} \right| = 4\sqrt{6}$$

$$PQ^2 = (4\sqrt{6})^2 = 16 \times 6 = 96$$

Question 238

The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, -1, 6) is
[1 Sep 2021 Shift 2]

Options:

A. $\sqrt{26}$

B. $2\sqrt{5}$

C. $2\sqrt{6}$

D. $4\sqrt{2}$

Answer: C

Solution:

Equation of line

$$3y - 2z - 1 = 0 = 3x - z + 4$$

$$\Rightarrow \frac{3y-1}{2} = \frac{z-0}{1} = \frac{3x+4}{1}$$

$$\Rightarrow \frac{x + \frac{4}{3}}{\frac{1}{3}} = \frac{y - \frac{1}{3}}{\frac{2}{3}} = \frac{z - 0}{1}$$

$$PR = |PQ| \cos \theta = |PQ| \frac{PQ \cdot P}{|PQ| |P|} = \frac{PQ \cdot PQ}{|PR|}$$

$$PR = \left| \frac{\frac{1}{3}\left(2 + \frac{4}{3}\right) + \frac{2}{3}\left(-1 - \frac{1}{3}\right) + 1(6-0)}{\sqrt{\frac{1}{9} + \frac{4}{9} + 1}} \right| = 4\sqrt{\frac{14}{9}}$$

$$OR^2 = PQ^2 - PR^2$$

$$= \frac{100}{9} + \frac{16}{9} + 36 - \frac{224}{9}$$

$$= \frac{100}{9} + \frac{16}{9} + 36 - \frac{224}{9}$$

$$QR = \sqrt{24} = 2\sqrt{6}$$

Question 239

Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then, which of the following points lies on P ?
[1 Sep 2021 Shift 2]

Options:

A. $\left(3, 1, -\frac{1}{2}\right)$

B. $\left(-2, 0, -\frac{1}{2}\right)$

C. $(0, 2, -4)$

D. $(4, 0, -2)$

Answer: B

Solution:

Equation of angle bisectors

$$\frac{x - 2y - 2z + 1}{\sqrt{1 + 4 + 4}} = \pm \frac{2x - 3y - 6z + 1}{\sqrt{4 + 9 + 36}}$$

$$\Rightarrow x - 5y + 4z + 4 = 0 \text{ and } 13x - 23y - 32z + 10 = 0$$

$$\text{Then, } \cos \theta = \frac{1 + 10 - 8}{\sqrt{1 + 4 + 4}\sqrt{1 + 25 + 16}} = \frac{1}{\sqrt{42}}$$

$$\Rightarrow \tan \theta = \sqrt{41} > 1$$

$$\Rightarrow \theta > 45^\circ$$

Then, acute angle bisector in plane

$$P : 13x - 23y - 32z + 10 = 0$$

$\therefore$ Point $\left(-2, 0, \frac{-1}{2}\right)$ lies on the plane P.

Question 240

The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and

$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} \text{ is:}$$

[Jan. 08, 2020 (I)]

Options:

A. $2\sqrt{30}$

B. $\frac{7}{2}\sqrt{30}$

C. $3\sqrt{30}$

D. 3

Answer: C

Solution:

$$\vec{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$$

$$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$$

$$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

Shortest distance between the lines is

$$= \frac{|\vec{AB} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}$$

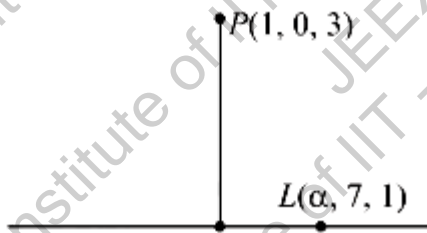
Question 241

If the foot of the perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha, 7, 1)$ is, then α is equal to _____.
[NAJan.07, 2020 (II)]

Answer: 4

Solution:

Since, PQ is perpendicular to L



$$\begin{aligned} \therefore \left(1 - \frac{5}{3}\right) \left(\alpha - \frac{5}{3}\right) + \left(\frac{-7}{3}\right) \left(7 - \frac{7}{3}\right) + \left(3 - \frac{17}{3}\right) \left(1 - \frac{17}{3}\right) &= 0 \\ \Rightarrow \frac{-2\alpha}{3} + \frac{10}{9} - \frac{98}{9} + \frac{112}{9} &= 0 \\ \Rightarrow \frac{2\alpha}{3} = \frac{24}{9} \Rightarrow \alpha &= 4 \end{aligned}$$

Question 242

If for some α and β in $\mathbb{R}$, the intersection of the following three planes

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

is a line in $\mathbb{R}^3$, then $\alpha + \beta$ is equal to:

[Jan. 9, 2020 (I)]

Options:

A. 0

B. 10

C. 2

D. -10

Answer: B

Solution:

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$$\Rightarrow (7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0$$

$$\Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

$$\text{Also, } D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0 \Rightarrow \beta = 13$$

$$\text{Hence, } \alpha + \beta = -3 + 13 = 10$$

Question 243

If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ ($\lambda \in \mathbb{R}$) is equal to $\frac{k}{\sqrt{633}}$, then k is equal to _____.

[NA Jan. 9, 2020 (II)]

Answer: 3

Solution:

Since, the line $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ contains the point $(-1, 3, -1)$ and line $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ contains the point $(-3, -2, 1)$

Then, the distance between the plane $23x - 10y - 2z + 48 = 0$ and the plane containing the lines = perpendicular distance of plane

$23x - 10y - 2z + 48 = 0$ either from $(-1, 3, -1)$ or $(-3, -2, 1)$

$$= \left| \frac{23(-1) - 10(3) - 2(-1)}{\sqrt{(23)^2 + (10)^2 + (-2)^2}} \right| = \frac{3}{\sqrt{633}}$$

It is given that distance between the planes

$$= \frac{k}{\sqrt{633}} \Rightarrow \frac{k}{\sqrt{633}} = \frac{3}{\sqrt{633}} \Rightarrow k = 3$$

Question 244

The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$.

Which of the following points lies on this plane?

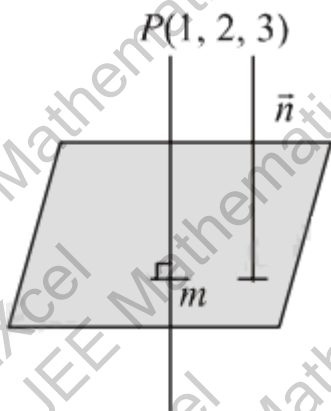
[Jan. 8, 2020 (II)]

Options:

- A. $(1, 1, 1)$
- B. $(1, -1, 1)$
- C. $(-1, -1, 1)$
- D. $(-1, -1, -1)$

Answer: B

Solution:



$$\vec{n} = \frac{-7}{3} - 1, \frac{-4}{3} - 2, \frac{-1}{3} - 3$$

$$\vec{n} = \frac{10}{3}, \frac{10}{3}, \frac{10}{3}$$

D. $\vec{r}$ of normal to the plane $(1, 1, 1)$

Midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$ Equation of required plane Q

$$\vec{r} \cdot \hat{n} = \hat{a} \cdot \hat{n}$$

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \frac{-2}{3} + \frac{1}{3} + \frac{4}{3}$$

$\therefore$ Equation of plane is $x + y + z = 1$

Question 245

Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:

[Jan. 7, 2020 (I)]

Options:

- A. (6, 5, 2)
- B. (6, 5, -2)
- C. (4, 3, 2)
- D. (3, 4, -2)

Answer: B

Solution:

$$\begin{aligned} \text{Equation of plane is } x + y - 2z &= 3 \\ \Rightarrow \frac{x-2}{1} &= \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{6} \\ \Rightarrow (x, y, z) &= (6, 5, -2) \end{aligned}$$

Question 246

A plane P meets the coordinate axes at A, B and C respectively. The centroid of $\triangle ABC$ is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is:

[Sep. 06, 2020 (II)]

Options:

A. $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

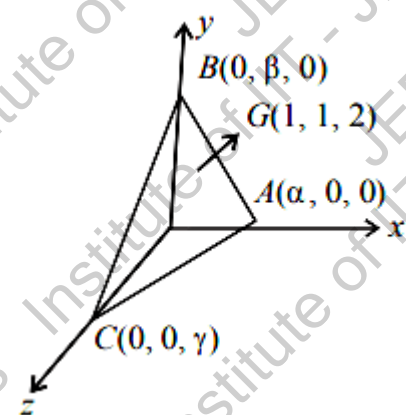
B. $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

C. $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

D. $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

Answer: C

Solution:



$\therefore \alpha = 3, \beta = 3$ and $\gamma = 6$ as G is centroid.

$\therefore$ The equation of plane is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

$$\Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1 \Rightarrow 2x + 2y + z = 6$$

$\therefore$ The required line is, $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Question 247

If (a, b, c) is the image of the point (1, 2, -3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$,

then a + b + c is equals to:

[Sep. 05, 2020 (I)]

Options:

A. 2

B. -1

C. 3

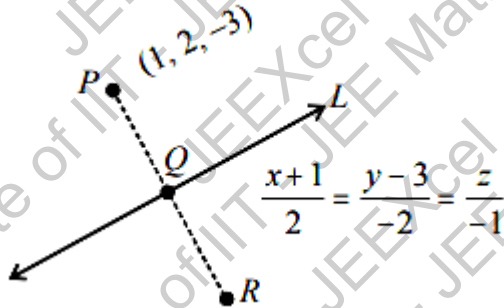
D. 1

Answer: A

Solution:

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$$

Any point on line = $Q(2\lambda - 1, -2\lambda + 3, -\lambda)$



$\therefore$ D.r. of $PQ = [2\lambda - 2, -2\lambda + 1, -\lambda + 3]$

D.r. of given line = $[2, -2, -1]$

$\therefore PQ$ is perpendicular to line L

$$\therefore 2(2\lambda - 2) - 2(-2\lambda + 1) + 1(-\lambda + 3) = 0$$

$$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$$

$$\Rightarrow 9\lambda - 9 = 0 \Rightarrow \lambda = 1$$

$\therefore Q$ is mid point of $PR = Q = (1, 1, -1)$

$\therefore$ Coordinate of image $R = (1, 0, 1) = (a, b, c)$

$$\therefore a + b + c = 2$$

Question 248

The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

[Sep. 03, 2020 (I)]

Options:

A. do not intersect for any values of l and m

B. intersect for all values of l and m

C. intersect when $l = 2$ and $m = \frac{1}{2}$

D. intersect when $l = 1$ and $m = 2$

Answer: A

Solution:

$$L_1 \equiv \vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$$

$$L_2 \equiv \vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$$

Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$$2l + 1 = m + 2 \dots\dots(i)$$

$$-1 = -1 + m \Rightarrow m = 0 \dots\dots(ii)$$

$$l = -m \dots\dots(iii)$$

$\Rightarrow m = 1 = 0$ which is not satisfy eqn. (i) hence lines do not intersect for any value of l and m .

Question 249

The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and

$x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :

[Sep. 06, 2020 (I)]

Options:

A. 1

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: B

Solution:

For line of intersection of planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$:

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Put $y = 0$, we get $x = -2$ and $z = 1$

$$L_2 : \vec{r} = (-2\hat{i} + \hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k}) \text{ and } L_1 : \vec{r} = (\hat{i} - \hat{j}) + \mu(-\hat{j} + \hat{k}) \text{ (Given)}$$

Now, $\vec{b}_1 \times \vec{b}_2 = -2[\hat{i} + \hat{j} + \hat{k}]$ and $\vec{a}_2 - \text{oc } a_1 = -3\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Shortest distance $= \frac{1}{\sqrt{3}}$

Question 250

If for some $\alpha \in \mathbb{R}$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

[Sep. 05, 2020 (II)]

Options:

A. (10, 2, 2)

B. (2, -10, -2)

C. (10, -2, -2)

D. (-2, 10, 2)

Answer: B

Solution:

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1(-1-5+\alpha) - 3(2-\alpha) + 2(10-2\alpha+\alpha) = 0$$

$$\therefore \alpha = -4$$

$$\therefore \text{Equation of } L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

$$\therefore \text{Point } (2, -10, -2) \text{ lies on line } L_2$$

Question 251

If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for

some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is _____.
[Sep. 04, 2020 (I)]

Answer: 3

Solution:

Equation of plane P is

$$\begin{aligned}(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) &= 0 \\ \Rightarrow x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + (7 - 8\lambda) &= 0 \\ \Rightarrow \frac{1 + 3\lambda}{a} = \frac{4 + \lambda}{b} = \frac{5\lambda - 1}{6} = \frac{7 - 8\lambda}{-15}\end{aligned}$$

From last two ratios, $\lambda = -1$

$$\Rightarrow \frac{-2}{a} = \frac{3}{b} = -1$$

$$\therefore a = 2, b = -3$$

$\therefore$ Equation of plane is, $2x - 3y + 6z - 15 = 0$

$$\text{Distance} = \frac{|6 - 6 - 6 - 15|}{7} = \frac{21}{7} = 3.$$

Question 252

The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:

[NA Sep. 04, 2020 (II)]

Options:

A. $\frac{7}{5}$

B. 1

C. $\frac{1}{7}$

D. 7

Answer: B

Solution:

Equation of line through point $P(1, -2, 3)$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

So, any point on line = $Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since, this point lies on plane $x - y + 2 = 5$

$$\therefore 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$$

$\therefore$ Point of intersection line and plane, $Q = \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance PQ

$$= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = 1$$

Question 253

The foot of the perpendicular drawn from the point $(4, 2, 3)$ to the line joining the points $(1, -2, 3)$ and $(1, 1, 0)$ lies on the plane:

[Sep. 03, 2020 (I)]

Options:

A. $2x + y - z = 1$

B. $x - y - 2z = 1$

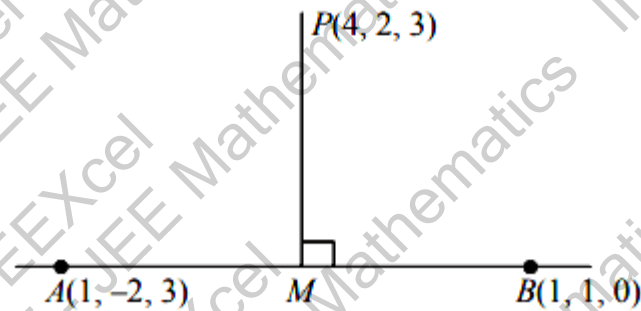
C. $x - 2y + z = 1$

D. $x + 2y - z = 1$

Answer: A

Solution:

Equation of line through points $(1, -2, 3)$ and $(1, 1, 0)$ is



$$\frac{x-1}{0} = \frac{y-1}{-3} = \frac{z-0}{3-0} (= \lambda \text{ say})$$

$$\therefore M(1, -\lambda + 1, \lambda)$$

$$\text{Direction ratios of PM} = [-3, -\lambda - 1, \lambda - 3]$$

$$\therefore \text{PM} \perp \text{AB}$$

$$\therefore (-3) \cdot 0 + (-1 - \lambda)(-1) + (\lambda - 3) \cdot 1 = 0$$

$$\therefore \lambda = 1$$

$$\therefore \text{Foot of perpendicular} = (1, 0, 1)$$

$$\text{This point satisfy the plane } 2x + y - z = 1$$

Question 254

The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point:
[Sep. 03, 2020 (II)]

Options:

A. $(4, 0, 1)$

B. $(0, -1, 1)$

C. $(4, 0, -1)$

D. $(0, 1, -1)$

Answer: C

Solution:

Direction ratios of normal to the plane are $\langle 1, -3, 2 \rangle$.

Plane passes through $(3, 1, 1)$.

Equation of plane is,

$$1(x - 3) - 3(y - 1) + 2(z - 1) = 0$$

$$\Rightarrow x - 3y + 2z - 2 = 0$$

Question 255

Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$, $\lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k})$, $\mu \in \mathbb{R}$. If Q(α , β , γ) is the foot of the perpendicular drawn from the point M (1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals

 .
[NA Sep. 03, 2020 (II)]

Answer: 5

Solution:

$$\text{Normal of plane} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$$

Direction ratios of normal to the plane = $\langle -1, 1, 1 \rangle$

Equation of plane

$$-1(x-1) + 1(y-0) + 1(z-0) = 0$$

$$\Rightarrow x - y - z - 1 = 0$$

If (x, y, z) is foot of perpendicular of M (1, 0, 1) on the plane then

$$\Rightarrow \frac{x-1}{-1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$$\therefore x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$$

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

$$\therefore 3(\alpha + \beta + \gamma) = 3 \times \frac{5}{3} = 5$$

Question 256

The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, $2x = 3y$, $z = 1$ also through the point :

[Sep. 02, 2020 (I)]

Options:

A. (0, 6, -2)

B. (-2, 0, 1)

C. (0, -6, 2)

D. (2, 0, -1)

Answer: B

Solution:

Let plane passes through (2, 1, 2) be

$$a(x-2) + b(y-1) + (z-2) = 0$$

It also passes through (1, 2, 1)

$$\therefore -a + b - c = 0 \Rightarrow a - b + c = 0$$

The given line is

$$\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0} \text{ is parallel to plane}$$

$$\therefore 3a + 2b + c(0) = 0$$

$$\Rightarrow \frac{a}{0-2} = \frac{b}{3-0} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{-5}$$

$$\therefore \text{plane is } 2x - 4 - 3y + 3 - 5z + 10 = 0$$

$$\Rightarrow 2x - 3y - 5z + 9 = 0$$

The plane satisfies the point (-2, 0, 1).

Question 257

A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, -2, 2 and 2, 3, -1 respectively. If this plane also passes through the point (α , -3, 5), then α is equal to:

[Sep. 02, 2020 (II)]

Options:

A. 5

B. -10

C. 10

D. -5

Answer: A

Solution:

∴ Plane contains two lines

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$= -4\hat{i} + 5\hat{j} + 7\hat{k}$$

So, equation of plane is

$$-4(x-3) + 5(y-1) + 7(z-1) = 0$$

$$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$$

$$\Rightarrow -4x + 5y + 7z = 0$$

This also passes through $(\alpha, -3, 5)$

$$\text{So, } -4\alpha - 15 + 35 = 0$$

$$\Rightarrow -4\alpha = -20 \Rightarrow \alpha = 5$$