



Straight Lines and Pair of Straight Lines

Question 1

Let A, B be points on the two half-lines $x - \sqrt{3}|y| = \alpha, \alpha > 0$ at a distance of α from their point of intersection P . The line segment AB meets the angle bisector of the given half-lines at the point Q . If $PQ = \frac{9}{2}$ and R is the radius of the circumcircle of $\triangle PAB$, then $\frac{\alpha^2}{R}$ is equal to _____

JEE Main 2026 (Online) 4th April Evening Shift

Answer: 9

Solution:

The two half-lines are given by

$$x - \sqrt{3}|y| = \alpha, \quad \alpha > 0$$

We will first understand their geometry.

1. Find the two half-lines and their point of intersection

Since $|y|$ is present, we split into two cases:

Case 1: $y \geq 0$

$$x - \sqrt{3}y = \alpha$$

Case 2: $y \leq 0$

$$x + \sqrt{3}y = \alpha$$

These are two lines meeting at the point P .

To find their intersection, put $y = 0$ in either equation:

$$x = \alpha$$

So,

$$P = (\alpha, 0)$$

Thus the two half-lines start from P and go outward along these two lines.

2. Find points A and B

Points A and B lie on these two half-lines and are each at distance α from P .

Let A be on the upper line $x - \sqrt{3}y = \alpha$ and B on the lower line $x + \sqrt{3}y = \alpha$.

The slope of the upper line is

$$y = \frac{x - \alpha}{\sqrt{3}} \Rightarrow m = \frac{1}{\sqrt{3}}$$

So this line makes angle 30° with the positive x -axis.

Similarly, the lower line makes angle -30° with the positive x -axis.

Hence the angle between the two half-lines is

$$60^\circ$$

Since $PA = \alpha$ and $PB = \alpha$, the points are obtained by moving distance α from $P = (\alpha, 0)$ along directions $\pm 30^\circ$.

Therefore,

$$A = (\alpha + \alpha \cos 30^\circ, \alpha \sin 30^\circ)$$

$$B = (\alpha + \alpha \cos 30^\circ, -\alpha \sin 30^\circ)$$

Using $\cos 30^\circ = \frac{\sqrt{3}}{2}$ and $\sin 30^\circ = \frac{1}{2}$,

$$A = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, \frac{\alpha}{2} \right)$$

$$B = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, -\frac{\alpha}{2} \right)$$

3. Find the angle bisector and point Q

The figure is symmetric about the x -axis, so the angle bisector of the two half-lines is the positive x -axis.

Also, A and B have the same x -coordinate, so AB is a vertical segment.

Hence AB meets the x -axis at

$$Q = \left(\alpha + \frac{\sqrt{3}\alpha}{2}, 0 \right)$$

So,

$$PQ = \left(\alpha + \frac{\sqrt{3}\alpha}{2} \right) - \alpha = \frac{\sqrt{3}\alpha}{2}$$

Given that

$$PQ = \frac{9}{2}$$

therefore,

$$\frac{\sqrt{3}\alpha}{2} = \frac{9}{2}$$

So,

$$\sqrt{3}\alpha = 9$$

$$\alpha = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

Hence,

$$\alpha^2 = (3\sqrt{3})^2 = 27$$

4. Find circumradius R of $\triangle PAB$

In $\triangle PAB$,

$$PA = PB = \alpha$$

and the included angle

$$\angle APB = 60^\circ$$

So the triangle is actually equilateral, because

$$AB^2 = \alpha^2 + \alpha^2 - 2\alpha^2 \cos 60^\circ = 2\alpha^2 - \alpha^2 = \alpha^2$$

Thus,

$$AB = \alpha$$

Hence $\triangle PAB$ is equilateral with side α .

For an equilateral triangle of side a , circumradius is

$$R = \frac{a}{\sqrt{3}}$$

So here,

$$R = \frac{\alpha}{\sqrt{3}}$$

Using $\alpha = 3\sqrt{3}$,

$$R = \frac{3\sqrt{3}}{\sqrt{3}} = 3$$

5. Compute $\frac{\alpha^2}{R}$

$$\frac{\alpha^2}{R} = \frac{27}{3} = 9$$

Therefore, the required value is

9

Question2

From the point $(-1, -1)$, two rays are sent making angles of 45° with the line $x + y = 0$. These rays get reflected from the mirror $x + 2y = 1$. If the equations of the reflected rays are $ax + by = 9$ and $cx + dy = 7$, $a, b, c, d \in \mathbf{Z}$, then the value of $ad + bc$ is ____ .

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Answer: 7

Solution:

Line $x + y = 0$ has slope -1 . If a ray has slope m and makes an angle 45° with this line, then by the angle formula between two lines we use:

$$\tan 45^\circ = \left| \frac{m+1}{1-m} \right|$$

Now $\tan 45^\circ = 1$, so we get:

$$\Rightarrow \pm 1 = \frac{m+1}{1-m}$$

Solving this gives two possible directions. One solution gives a horizontal line ($m = 0$), and the other corresponds to a vertical line (perpendicular to the x -axis).

$\rightarrow m = 0$ & one line is perpendicular to x axis.

Both rays start from $(-1, -1)$, so the two incident rays are:

$$L_1: x = -1, L_2: y = -1$$

Next, each ray reflects from the mirror line $x + 2y = 1$. To get the reflected ray, we take the mirror image of the incident line in the mirror.

First, take the mirror image of $x + 1 = 0$ in $x + 2y = 1$.

Now taking mirror image of $x + 1 = 0$ in $x + 2y = 1$

$$A' : \frac{x+1}{1} = \frac{y+1}{2} = \frac{-2(-1-2-1)}{1^2+2^2}$$

From this, we get the image point:

$$x = \frac{3}{5}, y = \frac{11}{5} \quad A' \left(\frac{3}{5}, \frac{11}{5} \right)$$

So the corresponding reflected line is written as:

$$\text{line} = 3x - 4y + 7 = 0$$

Similarly, take the mirror image of $y + 1 = 0$ in $x + 2y = 1$.

Now taking mirror image of $y + 1 = 0$ in $x + 2y = 1$

$$A'' : \frac{x+1}{1} = \frac{y+1}{2} = \frac{-2(-1-2-1)}{1^2+2^2}$$

This again gives:

$$x = \frac{3}{5}, y = \frac{11}{5}$$

$$A'' \left(\frac{3}{5}, \frac{11}{5} \right)$$

Now write the equation of the reflected line using point-slope form:

line :

$$(y + 1) = \frac{\left(\frac{11}{5} + 1\right)}{\left(\frac{3}{5} - 3\right)}(x - 3)$$

Simplifying gives one reflected ray as:

$$4x + 3y = 9$$

Compare $4x + 3y = 9$ with $ax + by = 9$. Then:

comparing $4x + 3y = 9$ with $ax + by = 9$ &

$$a = 4, b = 3$$

The other reflected ray is written as:

by line

$$-3x + 4y = 7$$

Compare $-3x + 4y = 7$ with $cx + dy = 7$. Then:

comparing with

$$cx + dy = 7$$

$$\therefore c = -3, d = 4$$

Now calculate $ad + bc$:

$$\text{So, } ad + bc$$

$$= 4 \times 4 + 3 \times (-3)$$

$$= 16 - 9 = 7$$

Question3

Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2 , respectively, is :

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Options:

A.

$$21\sqrt{3}$$

B.

$$12\sqrt{2}$$

C.

$$15\sqrt{6}$$

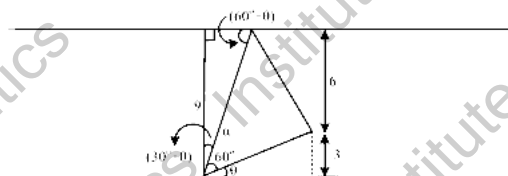
D.

$$27$$

Answer: A

Solution:

Let a be the side of $\triangle ABC$



$$\sin \theta = \frac{3}{a}$$

$$\sin (60^\circ + \theta) = \frac{9}{a}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \times \sqrt{1 - \frac{9}{a^2}} + \frac{3}{a} \times \frac{1}{2} = \frac{9}{a}$$

$$\Rightarrow \sqrt{3} \left(\sqrt{a^2 - 9} \right) + 3 = 18$$

$$\Rightarrow 3(a^2 - 9) = 15^2 \Rightarrow a^2 - 9 = 15 \times 5$$

$$\Rightarrow a^2 = 84$$

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 = 21\sqrt{3}$$

Question4

A rectangle is formed by the lines $x = 0, y = 0, x = 3$ and $y = 4$. Let the line L be perpendicular to $3x + y + 6 = 0$ and divide the area of the rectangle into two equal parts. Then the distance of the point $(\frac{1}{2}, -5)$ from the line L is equal to :

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Options:

A.

$$\sqrt{10}$$

B.

$$2\sqrt{5}$$

C.

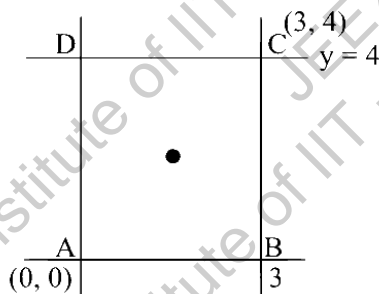
$$2\sqrt{10}$$

D.

$$3\sqrt{10}$$

Answer: C

Solution:



For L to bisect area, line must pass through the centre of rectangle $(\frac{3}{2}, 2)$

$$\Rightarrow L \equiv (y - 2) = \frac{1}{3} \left(x - \frac{3}{2} \right)$$

$$\Rightarrow 3y - 6 = x - \frac{3}{2} \Rightarrow 6y - 12 = 2x - 3$$

$$6y - 2x - 9 = 0$$

The distance from L of $(\frac{1}{2}, -5)$ is

$$\left| \frac{6(-5) - 2(\frac{1}{2}) - 9}{\sqrt{40}} \right| = \sqrt{40}$$
$$\Rightarrow 2\sqrt{10}$$

Question5

Let $A(1, 2)$ and $C(-3, -6)$ be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x - y = 14$. If $B(\alpha, \beta)$ and $D(\gamma, \delta)$ are the other two vertices, then $|\alpha + \beta + \gamma + \delta|$ is equal to :

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Options:

A.

3

B.

6

C.

1

D.

9

Answer: B

Solution:

Let $A = (1, 2)$ and $C = (-3, -6)$ be diagonally opposite vertices of a rhombus.

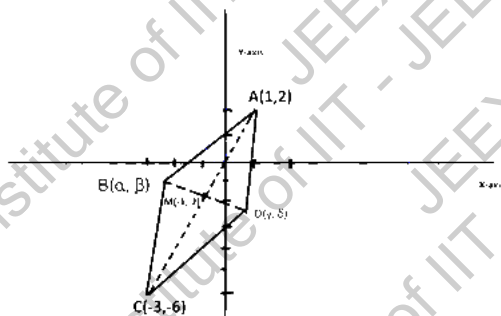
In a rhombus, the diagonals bisect each other at a common point M .

Calculation of Midpoint M :

$$M = \left(\frac{1 + (-3)}{2}, \frac{2 + (-6)}{2} \right)$$

$$M = \left(\frac{-2}{2}, \frac{-4}{2} \right)$$

$$M = (-1, -2)$$



Since M is also the midpoint of the other diagonal BD where $B = (\alpha, \beta)$ and $D = (\gamma, \delta)$:

$$\frac{\alpha + \gamma}{2} = -1 \Rightarrow \alpha + \gamma = -2$$

$$\frac{\beta + \delta}{2} = -2 \Rightarrow \beta + \delta = -4$$

$$|\alpha + \beta + \gamma + \delta| = |(\alpha + \gamma) + (\beta + \delta)|$$

$$|\alpha + \beta + \gamma + \delta| = |(-2) + (-4)|$$

$$|\alpha + \beta + \gamma + \delta| = |-6|$$

$$|\alpha + \beta + \gamma + \delta| = 6$$

Option (B) is correct.

Question 6

Let $A(1, 0)$, $B(2, -1)$ and $C\left(\frac{7}{3}, \frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x + \beta y = 5$, then the value of $\alpha^2 + \beta^2$ is

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Options:

A.

5

B.

10

C.

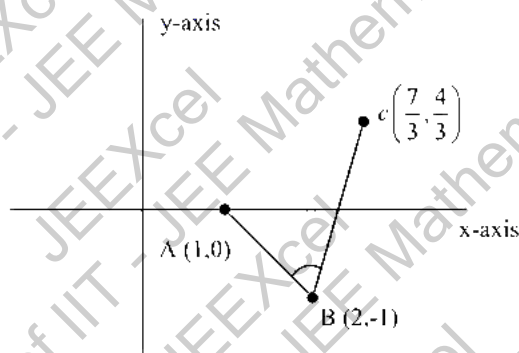
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D.

13

Answer: B

Solution:



solution : $A(1, 0)$, $B(2, -1)$ and $C(\frac{7}{3}, \frac{4}{3})$

equation of line AB using two point form

$$y - 0 = \frac{0 - (-1)}{1 - 2}(x - 1)$$

$$y = -x + 1$$

$$L_1: x + y - 1 = 0$$

similarly equation of line BC.

$$y + 1 = \frac{\frac{4}{3} - (-1)}{\frac{7}{3} - 2}(x - 2)$$

$$y + 1 = \frac{7/3}{1/3}(x - 2)$$

$$y = 7x - 14 - 1$$

$$L_2: 7x - y - 15 = 0$$

from graph $(2, 0)$ lies in the region of $\angle ABC$.

calculating $L_1(2, 0) \cdot L_2(2, 0)$

$$(2 + 0 - 1) \cdot (7 \times 2 - 0 - 15) = 1 \cdot (-1) = -1$$

equation of angular bisector of $\angle ABC$ is given as

$$\frac{x + y - 1}{\sqrt{1^2 + 1^2}} = \pm \frac{7x - y - 15}{\sqrt{7^2 + (-1)^2}}$$

$$\frac{x + y - 1}{\sqrt{2}} = \pm \frac{7x - y - 15}{\sqrt{50}}$$

$$5x + 5y - 5 = \pm(7x - y - 15)$$

since $L_1(2, 0) \cdot L_2(2, 0)$ is negative so equation of $\angle ABC$ bisector in the region $(2, 0)$ can be

find using negative sign from equation (1)

$$5x + 5y - 5 = -(7x - y - 15)$$

$$5x + 5y - 5 + 7x - y - 15 = 0$$

$$12x + 4y - 20 = 0$$

$$3x + y = 5 \text{ comparing with bisector } \alpha x + \beta y = 5.$$

$$\alpha = 3 \text{ and } \beta = 1$$

$$\alpha^2 + \beta^2 = 9 + 1 = 10.$$

Question 7

Let the angles made with the positive x -axis by two straight lines drawn from the point $P(2, 3)$ and meeting the line $x + y = 6$ at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2 . Then the value of $(\theta_1 + \theta_2)$ is:

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Options:

A.

$$\frac{\pi}{2}$$

B.

$$\frac{\pi}{3}$$

C.

$$\frac{\pi}{12}$$

D.

$$\frac{\pi}{6}$$

Answer: A

Solution:

Let any point $Q(t, 6 - t)$ on line $x + y = 6$ such that.

$$\text{distance } PQ = \sqrt{\frac{2}{3}} \Rightarrow PQ^2 = \frac{2}{3}.$$

using distance formula for points $P(2, 3)$ and $Q(t, 6 - t)$.

$$(2 - t)^2 + (3 - 6 + t)^2 = \frac{2}{3}$$

$$(2 - t)^2 + (t - 3)^2 = \frac{2}{3}$$

$$4 + t^2 - 4t + t^2 + 9 - 6t = \frac{2}{3}$$

$$2t^2 - 10t + 13 = \frac{2}{3}$$

$$6t^2 - 30t + 39 - 2 = 0$$

$$6t^2 - 30t + 37 = 0$$

let t_1 & t_2 are roots of this equation.

$$t_1 + t_2 = \frac{30}{6} = 5 \text{ and } t_1 t_2 = \frac{37}{6}.$$

So, there are two point $Q_1(t_1, 6 - t_1)$ & $Q_2(t_2, 6 - t_2)$.

It is given that angle made with positive x -axis by straight lines from $P(2, 3)$ to Q_1 & Q_2 is θ_1 & θ_2 respectively.

This means $\tan \theta_1 = \text{slope of line } PQ_1$.

$$\tan \theta_1 = \frac{3-(6-t_1)}{2-t_1} \Rightarrow \theta_1 = \tan^{-1} \left(\frac{t_1-3}{2-t_1} \right)$$

similarly, $\tan \theta_2 =$ slope of line PQ_2

$$\tan \theta_2 = \frac{t_2-3}{2-t_2} \Rightarrow \theta_2 = \tan^{-1} \left(\frac{t_2-3}{2-t_2} \right)$$

adding equation (1) & (2)

$$\begin{aligned} \theta_1 + \theta_2 &= \tan^{-1} \left(\frac{t_1-3}{2-t_1} \right) + \tan^{-1} \left(\frac{t_2-3}{2-t_2} \right) \\ &= \tan^{-1} \left(\frac{\frac{t_1-3}{2-t_1} + \frac{t_2-3}{2-t_2}}{1 - \left(\frac{t_1-3}{2-t_1} \right) \left(\frac{t_2-3}{2-t_2} \right)} \right), \text{ use } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \\ &= \tan^{-1} \frac{2t_1 - 6 - t_1t_2 + 3t_1 + 2t_2 - 6 - t_1t_2 + 3t_1}{4 - 2(t_1+t_2) + t_1t_2 - (9-3(t_1+t_2) + t_1t_2)} \\ \theta_1 + \theta_2 &= \tan^{-1} \frac{5(t_1+t_2) - 2t_1t_2 - 12}{(t_1+t_2) - 5} \end{aligned}$$

substitute value of $t_1 + t_2 = 5$ & $t_1t_2 = \frac{37}{6}$

$$\theta_1 + \theta_2 = \tan^{-1} \frac{5 \times 5 - 2 \times \frac{37}{6} - 12}{5 - 5}$$

$$\theta_1 + \theta_2 = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Question 8

Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x + 2\sqrt{2}y = 4$. If the co-ordinates of the vertex A are (α, β) , then the greatest integer less than or equal to $|\alpha + \sqrt{2}\beta|$ is

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Options:

A.

5

B.

4

C.

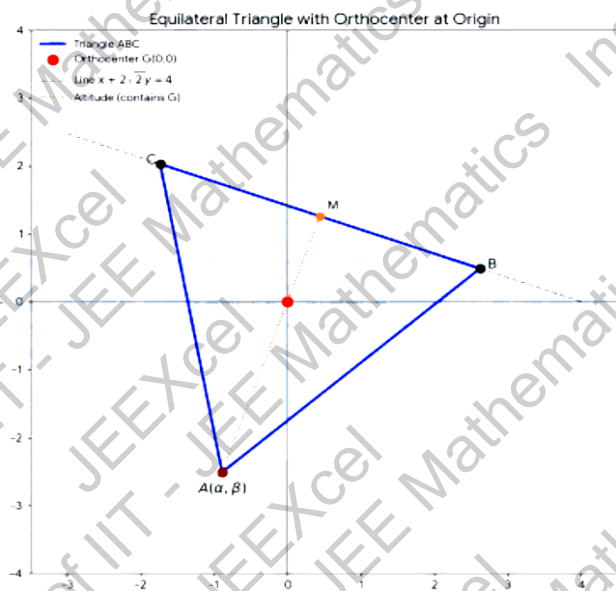
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D.

3

Answer: B

Solution:



Orthocenter (O) : $(0, 0)$. Since it's an equilateral triangle, this is also the Centroid.

- Side BC Line : $x + 2\sqrt{2}y = 4$.
- Vertex A : (α, β) .

1. Find the distance from the Centroid to side BC

The side BC lies on the line $x + 2\sqrt{2}y - 4 = 0$. The distance from the origin $G(0, 0)$ to this line is the length of the "lower" part of the median (r).

Using the point-to-line distance formula :

$$r = \frac{|0 + 2\sqrt{2}(0) - 4|}{\sqrt{1^2 + (2\sqrt{2})^2}} = \frac{|-4|}{\sqrt{1+8}} = \frac{4}{3}$$

2. Find the coordinates of Vertex $A(\alpha, \beta)$

The centroid G divides the segment AD (where D is the foot of the altitude on BC) in the ratio $2 : 1$. Therefore, the distance from A to G is $2r$.

$$AG = 2 \times \frac{4}{3} = \frac{8}{3}$$

The line AG is perpendicular to side BC . The slope of BC is $m = -\frac{1}{2\sqrt{2}}$, so the slope of the line AG (the altitude) is the negative reciprocal:

$$m_{\perp} = 2\sqrt{2}$$

Since A lies on the line passing through the origin with slope $2\sqrt{2}$, its coordinates can be written as:

$$\alpha = k, \quad \beta = 2\sqrt{2}k$$

Because A and the side BC are on opposite sides of the origin (the centroid), and the constant term in our line equation is negative, A must satisfy the condition that its position yields an opposite sign. Substituting these into the distance formula $AG = \frac{8}{3}$:

$$\sqrt{k^2 + (2\sqrt{2}k)^2} = \frac{8}{3} \implies \sqrt{9k^2} = \frac{8}{3} \implies 3|k| = \frac{8}{3} \implies |k| = \frac{8}{9}$$

3. Calculate $|\alpha + \sqrt{2}\beta|$

Substitute $\alpha = k$ and $\beta = 2\sqrt{2}k$:

$$|\alpha + \sqrt{2}\beta| = |k + \sqrt{2}(2\sqrt{2}k)|$$

$$= |k + 4k| = |5k|$$

$$= 5 \times \frac{8}{9} = \frac{40}{9} \approx 4.44$$

greatest integer less than or equal to $|\alpha + \sqrt{2}\beta| = [4.44] = 4$.

Question9

Let the mid points of the sides of a triangle ABC be $(\frac{5}{2}, 7)$, $(\frac{5}{2}, 3)$ and $(4, 5)$. If its incentre is (h, k) , then $3h + k$ is equal to :

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Options:

A.

11

B.

12

C.

13

D.

14

Answer: C

Solution:

Let the midpoints of the sides of triangle ABC be

$$D\left(\frac{5}{2}, 7\right), \quad E\left(\frac{5}{2}, 3\right), \quad F(4, 5)$$

We have to find the incentre (h, k) of triangle ABC .

Step 1: Use the fact about the medial triangle

The triangle formed by joining the midpoints of the sides of a triangle is called the medial triangle.

If the midpoints are known, then the original triangle can be reconstructed using:

$$A = E + F - D, \quad B = F + D - E, \quad C = D + E - F$$

Now calculate each vertex.

Vertex A

$$A = \left(\frac{5}{2}, 3\right) + (4, 5) - \left(\frac{5}{2}, 7\right)$$

$$A = (4, 1)$$

Vertex B

$$B = (4, 5) + \left(\frac{5}{2}, 7\right) - \left(\frac{5}{2}, 3\right)$$

$$B = (4, 9)$$

Vertex C

$$C = \left(\frac{5}{2}, 7\right) + \left(\frac{5}{2}, 3\right) - (4, 5)$$

$$C = (1, 5)$$

So the triangle has vertices

$$A(4, 1), \quad B(4, 9), \quad C(1, 5)$$

Step 2: Find the side lengths

Using distance formula:

Side opposite A, i.e. BC

$$BC = \sqrt{(4-1)^2 + (9-5)^2} = \sqrt{3^2 + 4^2} = 5$$

Side opposite B, i.e. CA

$$CA = \sqrt{(4-1)^2 + (1-5)^2} = \sqrt{3^2 + (-4)^2} = 5$$

Side opposite C, i.e. AB

$$AB = \sqrt{(4-4)^2 + (9-1)^2} = 8$$

Thus,

$$a = 5, \quad b = 5, \quad c = 8$$

Step 3: Use incentre formula

The incentre of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

where

- $a = BC$
- $b = CA$
- $c = AB$

Substitute the values:

$$h = \frac{5 \cdot 4 + 5 \cdot 4 + 8 \cdot 1}{5 + 5 + 8} = \frac{20 + 20 + 8}{18} = \frac{48}{18} = \frac{8}{3}$$

$$k = \frac{5 \cdot 1 + 5 \cdot 9 + 8 \cdot 5}{18} = \frac{5 + 45 + 40}{18} = \frac{90}{18} = 5$$

So the incentre is

$$\left(\frac{8}{3}, 5 \right)$$

Step 4: Find $3h + k$

$$3h + k = 3 \cdot \frac{8}{3} + 5 = 8 + 5 = 13$$

Therefore, the answer is

13

So, the correct option is **Option C**.

Question 10

Let the vertex A of a triangle ABC be $(1, 2)$, and the mid-point of the side AB be $(5, -1)$. If the centroid of this triangle is $(3, 4)$ and its circumcenter is (α, β) , then $21(\alpha + \beta)$ is equal to :

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Options:

A.

309

B.

403

C.

497

D.

524

Answer: C

Solution:

Let the coordinates of vertex B be (x_B, y_B) . We are given that vertex A is $(1, 2)$ and the mid-point of side AB is $(5, -1)$.

Using the mid-point formula, the coordinates of the mid-point are given by:

$$\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = (5, -1)$$

By equating the x and y coordinates, we get:

$$\frac{1+x_B}{2} = 5 \implies 1 + x_B = 10 \implies x_B = 9$$

$$\frac{2+y_B}{2} = -1 \implies 2 + y_B = -2 \implies y_B = -4$$

So, the coordinates of vertex B are (9, -4).

Let the coordinates of vertex C be (x_C, y_C) . The centroid of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is given by the formula:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

We are given the centroid of triangle ABC is (3, 4). Using the coordinates of vertices A, B, and C:

$$\frac{1+9+x_C}{3} = 3 \implies 10 + x_C = 9 \implies x_C = -1$$

$$\frac{2+(-4)+y_C}{3} = 4 \implies -2 + y_C = 12 \implies y_C = 14$$

So, the coordinates of vertex C are (-1, 14).

The circumcenter of a triangle is a point that is equidistant from all three of its vertices. Let the given circumcenter be marked as $O(\alpha, \beta)$. Therefore, we have:

$$OA = OB = OC \implies OA^2 = OB^2 = OC^2$$

Using the distance formula, we will write the equations for the squares of the distances:

$$OA^2 = (\alpha - 1)^2 + (\beta - 2)^2 = \alpha^2 + \beta^2 - 2\alpha - 4\beta + 5$$

$$OB^2 = (\alpha - 9)^2 + (\beta + 4)^2 = \alpha^2 + \beta^2 - 18\alpha + 8\beta + 97$$

$$OC^2 = (\alpha + 1)^2 + (\beta - 14)^2 = \alpha^2 + \beta^2 + 2\alpha - 28\beta + 197$$

Now, equating $OA^2 = OB^2$:

$$\alpha^2 + \beta^2 - 2\alpha - 4\beta + 5 = \alpha^2 + \beta^2 - 18\alpha + 8\beta + 97$$

$$16\alpha - 12\beta = 92$$

Dividing the entire equation by 4 gives our first linear equation:

$$4\alpha - 3\beta = 23 \quad \text{--- (Equation 1)}$$

Next, equating $OA^2 = OC^2$:

$$\alpha^2 + \beta^2 - 2\alpha - 4\beta + 5 = \alpha^2 + \beta^2 + 2\alpha - 28\beta + 197$$

$$-4\alpha + 24\beta = 192$$

Dividing this equation by -4 gives our second linear equation:

$$\alpha - 6\beta = -48 \implies \alpha = 6\beta - 48 \quad \text{--- (Equation 2)}$$

Substitute the expression for α from Equation 2 into Equation 1:

$$4(6\beta - 48) - 3\beta = 23$$

$$24\beta - 192 - 3\beta = 23$$

$$21\beta - 192 = 23$$

$$21\beta = 215 \implies \beta = \frac{215}{21}$$

Now, substitute the value of β back into Equation 2 to find α :

$$\alpha = 6 \left(\frac{215}{21} \right) - 48$$

$$\alpha = \frac{1290}{21} - \frac{1008}{21}$$

$$\alpha = \frac{282}{21}$$

The problem asks for the value of $21(\alpha + \beta)$. We can simply plug in our values:

$$21(\alpha + \beta) = 21 \left(\frac{282}{21} + \frac{215}{21} \right)$$

$$21(\alpha + \beta) = 282 + 215 = 497$$

Option C is the correct answer.

Question 11

Let the line $L_1 : x + 3 = 0$ intersect the lines $L_2 : x - y = 0$ and $L_3 : 3x + y = 0$ at the points A and B, respectively. Let the bisector of the obtuse angle between the lines L_2 and L_3 intersect the line L_1 at the point C. Then $BC^2 : AC^2$ is equal to:

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Options:

A.

5:1

B.

1:5

C.

2:3

D.

3:2

Answer: A

Solution:

Given:

- $L_1 : x + 3 = 0 \Rightarrow x = -3$
- $L_2 : x - y = 0 \Rightarrow y = x$
- $L_3 : 3x + y = 0 \Rightarrow y = -3x$

We first find points A and B .

1. Coordinates of A

Point A is the intersection of L_1 and L_2 .

Since $x = -3$ and $y = x$, we get

$$y = -3$$

So,

$$A = (-3, -3)$$

2. Coordinates of B

Point B is the intersection of L_1 and L_3 .

Since $x = -3$ and $y = -3x$, we get

$$y = -3(-3) = 9$$

So,

$$B = (-3, 9)$$

3. Equation of angle bisectors of L_2 and L_3

The lines are:

$$L_2 : x - y = 0$$

$$L_3 : 3x + y = 0$$

Angle bisectors are given by

$$\frac{x-y}{\sqrt{1^2+(-1)^2}} = \pm \frac{3x+y}{\sqrt{3^2+1^2}}$$

That is,

$$\frac{x-y}{\sqrt{2}} = \pm \frac{3x+y}{\sqrt{10}}$$

We need the bisector of the obtuse angle.

Let us find its slope.

Since both lines pass through the origin, we can use the formula for slope of angle bisectors.

The slopes of L_2 and L_3 are:

- for $L_2, m_1 = 1$
- for $L_3, m_2 = -3$

The acute angle bisector lies between the directions of these lines, and the obtuse angle bisector is the other one.

Now let us find the bisector which intersects $x = -3$ between $y = -3$ and $y = 9$ or outside, by solving directly.

Substitute $x = -3$ into angle bisector equations,

4. Find point C

From

$$\frac{x-y}{\sqrt{2}} = \frac{3x+y}{\sqrt{10}}$$

put $x = -3$:

$$\frac{-3-y}{\sqrt{2}} = \frac{-9+y}{\sqrt{10}}$$

Cross-multiplying:

$$\sqrt{10}(-3-y) = \sqrt{2}(-9+y)$$

This gives one point on L_1 .

From

$$\frac{x-y}{\sqrt{2}} = -\frac{3x+y}{\sqrt{10}}$$

put $x = -3$:

$$\frac{-3-y}{\sqrt{2}} = -\frac{-9+y}{\sqrt{10}}$$

Instead of solving both, it is easier to use the fact that any point on the angle bisector is equidistant from the two lines.

Let $C = (-3, k)$ on L_1 .

Distance of C from $L_2 : x - y = 0$ is

$$\frac{|x-y|}{\sqrt{2}} = \frac{|-3-k|}{\sqrt{2}} = \frac{|k+3|}{\sqrt{2}}$$

Distance of C from $L_3 : 3x + y = 0$ is

$$\frac{|3x+y|}{\sqrt{10}} = \frac{|-9+k|}{\sqrt{10}} = \frac{|k-9|}{\sqrt{10}}$$

So,

$$\frac{|k+3|}{\sqrt{2}} = \frac{|k-9|}{\sqrt{10}}$$

Squaring:

$$\frac{(k+3)^2}{2} = \frac{(k-9)^2}{10}$$

$$5(k+3)^2 = (k-9)^2$$

Expand:

$$5(k^2 + 6k + 9) = k^2 - 18k + 81$$

$$5k^2 + 30k + 45 = k^2 - 18k + 81$$

$$4k^2 + 48k - 36 = 0$$

$$k^2 + 12k - 9 = 0$$

$$k = -6 \pm 3\sqrt{5}$$

So the two bisectors meet L_1 at points with $y = -6 + 3\sqrt{5}$ and $y = -6 - 3\sqrt{5}$.

Now $A = (-3, -3)$ and $B = (-3, 9)$.

The obtuse angle is the larger angle between L_2 and L_3 .

Its bisector lies inside that larger region, which is the bisector with slope closer to negative small value, and it cuts $x = -3$ at the point between A and B .

Since

$$-6 + 3\sqrt{5} \approx 0.708$$

this lies between -3 and 9 .

Hence,

$$C = (-3, -6 + 3\sqrt{5})$$

5. Find AC^2 and BC^2

Since all three points have the same x -coordinate, distances depend only on y -coordinates.

AC

$$AC = |-6 + 3\sqrt{5} - (-3)| = |-3 + 3\sqrt{5}| = 3(\sqrt{5} - 1)$$

So,

$$AC^2 = 9(\sqrt{5} - 1)^2$$

BC

$$BC = |9 - (-6 + 3\sqrt{5})| = |15 - 3\sqrt{5}| = 3(5 - \sqrt{5})$$

So,

$$BC^2 = 9(5 - \sqrt{5})^2$$

Therefore,

$$BC^2 : AC^2 = (5 - \sqrt{5})^2 : (\sqrt{5} - 1)^2$$

Now,

$$(5 - \sqrt{5})^2 = 25 + 5 - 10\sqrt{5} = 30 - 10\sqrt{5} = 10(3 - \sqrt{5})$$

and

$$(\sqrt{5} - 1)^2 = 5 + 1 - 2\sqrt{5} = 6 - 2\sqrt{5} = 2(3 - \sqrt{5})$$

Hence,

$$BC^2 : AC^2 = 10(3 - \sqrt{5}) : 2(3 - \sqrt{5}) = 5 : 1$$

So the required ratio is

$$\boxed{5 : 1}$$

Hence, the correct option is **A**.

Question 12

In an equilateral triangle PQR , let the vertex P be at $(3, 5)$ and the side QR be along the line $x + y = 4$. If the orthocentre of the triangle PQR is (α, β) , then $9(\alpha + \beta)$ is equal to:

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Options:

A.

16

B.

27

C.

36

D.

48

Answer: D

Solution:

Since QR lies on the line

$$x + y = 4,$$

and $P = (3, 5)$, first find the altitude from P to the side QR .

In an equilateral triangle, all altitudes, medians and angle bisectors coincide. Also, the orthocentre lies on the altitude from each vertex and divides the altitude in the ratio $2 : 1$ from the vertex.

So let us proceed step by step.

The line QR has equation

$$x + y - 4 = 0.$$

Distance of point $P(3, 5)$ from this line is

$$d = \frac{|3+5-4|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

This is the altitude of the equilateral triangle from vertex P .

Now, the foot of the perpendicular from P to the line $x + y = 4$ lies on the altitude. Since in an equilateral triangle the orthocentre lies at $\frac{2}{3}$ of the altitude from the vertex, we can find it along the perpendicular direction.

The slope of $x + y = 4$ is -1 , so the perpendicular slope is 1 .

Hence the altitude through $P(3, 5)$ is

$$y - 5 = 1(x - 3)$$

or

$$y = x + 2.$$

The orthocentre lies on this line.

Now find the foot of perpendicular from P to $x + y = 4$ by solving

$$y = x + 2$$

and

$$x + y = 4.$$

Substitute $y = x + 2$ into $x + y = 4$:

$$x + (x + 2) = 4$$

$$2x = 2$$

$$x = 1, \quad y = 3.$$

So the foot is $(1, 3)$.

Thus the altitude segment is from $P(3, 5)$ to $(1, 3)$.

The orthocentre divides this altitude in the ratio $2 : 1$ from vertex P . Therefore,

$$H = \left(\frac{2 \cdot 1 + 1 \cdot 3}{3}, \frac{2 \cdot 3 + 1 \cdot 5}{3} \right)$$

This is incorrect ratio placement, so let us use the correct section carefully:

If H divides the segment from P to foot D in the ratio $2 : 1$, then

$$H = P + \frac{2}{3}(D - P).$$

Here,

$$D - P = (1 - 3, 3 - 5) = (-2, -2).$$

So

$$H = (3, 5) + \frac{2}{3}(-2, -2) = \left(3 - \frac{4}{3}, 5 - \frac{4}{3} \right) = \left(\frac{5}{3}, \frac{11}{3} \right).$$

Thus,

$$\alpha = \frac{5}{3}, \quad \beta = \frac{11}{3}.$$

So,

$$\alpha + \beta = \frac{5}{3} + \frac{11}{3} = \frac{16}{3}.$$

Therefore,

$$9(\alpha + \beta) = 9 \cdot \frac{16}{3} = 48.$$

Hence the correct answer is

48

So, Option D is correct.

Question 13

If a straight line drawn through the point of intersection of the lines $4x + 3y - 1 = 0$ and $3x + 4y - 1 = 0$, meets the co-ordinate axes at the points P and Q, then the locus of the mid point of PQ is :

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Options:

A.

$$x + y - 7 = 0$$

B.

$$x + y - 14xy = 0$$

C.

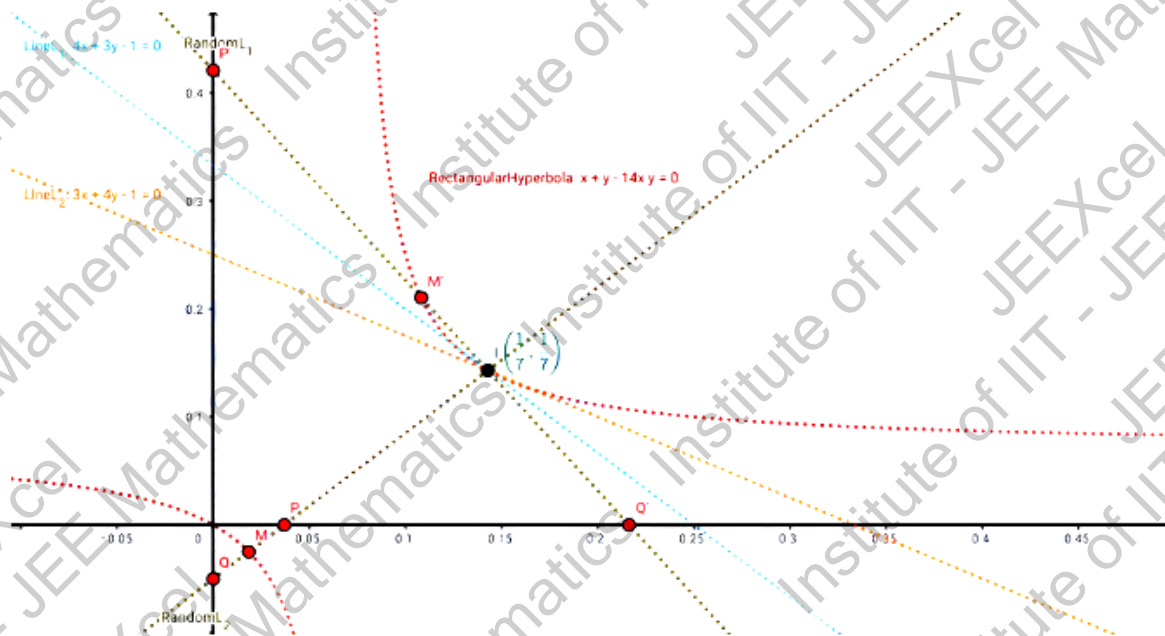
$$2x + y + 14xy = 0$$

D.

$$x + 2y - 14xy = 0$$

Answer: B

Solution:



1. A straight line L passes through the point of intersection of 2 straight lines:

$$\begin{cases} L_1 : 4x + 3y - 1 = 0 \\ L_2 : 3x + 4y - 1 = 0 \end{cases}$$

2. The straight line L cuts the axes at P and Q

3. To identify the locus of the mid-point of the segment PQ

The point of intersection of L_1 and L_2 is $I(\alpha, \beta)$

The mid-point of the segment PQ is $M(h, k)$

Analytical Study :

We first solve the pair of straight lines L_1 and L_2 to locate the point of intersection :

$$\begin{aligned} 4x + 3y - 1 &= 0 \Rightarrow 12x + 9y = 3 \\ 3x + 4y - 1 &= 0 \Rightarrow 12x + 16y = 4 \end{aligned} \Rightarrow \beta = \frac{1}{7} \Rightarrow \alpha = \frac{1}{7} \Rightarrow I\left(\frac{1}{7}, \frac{1}{7}\right) \dots (i)$$

The straight line L passes through the point I identified by equation (i)

The equation of the straight line L in point slope form is written below :

$$L: \left(y - \frac{1}{7}\right) = m \left(x - \frac{1}{7}\right) \equiv L: 7mx - 7y = m - 1 \equiv L: \frac{7mx}{m-1} + \frac{7y}{1-m} = 1 \dots (ii)$$

The equation (ii) gives the straight line L in intercept form where the X and Y intercepts are observed below :

P	Q
$\left(\frac{m-1}{7m}, 0\right)$	$\left(0, \frac{1-m}{7}\right)$

..... Chart 1

From Chart 1, the mid-point of the section PQ is derived applying mid-point formulae :

$$M(h, k) = M\left(\frac{m-1}{14m}, \frac{1-m}{14}\right) \equiv \begin{array}{|c|c|c|} \hline \text{Point} & h & k \\ \hline M(h, k) & \frac{m-1}{14m} & \frac{1-m}{14} \\ \hline \end{array} \dots \text{Chart 2}$$

Chart 2 has the X and Y coordinate each as a function of m where we can eliminate m and mathematically trace the locus :

$$k = \frac{1-m}{14} \Rightarrow m = 1 - 14k \Rightarrow h = \frac{1-14k-1}{14(1-14k)} = \frac{14k}{14(14k-1)} = \frac{k}{(14k-1)} \Rightarrow 14hk - h = k \Rightarrow h + k - 14hk = 0$$

The locus of the mid-point of the section PQ after replacement of the dummy variables is found to be the rectangular hyperbola $x + y - 14xy = 0$

Question 14

Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _____.

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Answer: 28

Solution:

We set up a coordinate system so that the two parallel lines are given by

$$y = 0 \quad \text{and} \quad y = 5,$$

since their distance is 5 units. Choose point

$$P = (0, 1)$$

so that the distance from P to the line $y = 0$ is 1 unit (and its distance to the line $y = 5$ is 4 units).

Let point

$$Q = (a, 0)$$

be on the line $y = 0$, and let point

$$R = (b, 5)$$

be on the line $y = 5$. Since triangle PQR is equilateral with side length s , we require:

$$PQ = PR = QR = s.$$

A convenient method is to "rotate" Q about P by an angle of 60° to obtain R . In complex-number (or vector) terms, if we translate so that P is at the origin, then the rotation is given by

$$e^{i60^\circ} = \cos 60^\circ + i \sin 60^\circ = \frac{1}{2} + i \frac{\sqrt{3}}{2}.$$

Thus, writing Q in vector form relative to P , we have

$$Q - P = (a, -1).$$

Rotating this by 60° gives

$$R - P = (a \cos 60^\circ - (-1) \sin 60^\circ, a \sin 60^\circ + (-1) \cos 60^\circ).$$

Substituting the values $\cos 60^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$, we obtain

$$R - P = \left(\frac{a}{2} + \frac{\sqrt{3}}{2}, \frac{a\sqrt{3}}{2} - \frac{1}{2} \right),$$

$$\text{so } R = \left(\frac{a + \sqrt{3}}{2}, 1 + \frac{a\sqrt{3}}{2} - \frac{1}{2} \right) = \left(\frac{a + \sqrt{3}}{2}, \frac{a\sqrt{3} + 1}{2} \right).$$

Since R lies on $y = 5$, its y -coordinate must equal 5:

$$\frac{a\sqrt{3} + 1}{2} = 5.$$

Solve for a :

$$a\sqrt{3} + 1 = 10,$$

$$a\sqrt{3} = 9,$$

$$a = \frac{9}{\sqrt{3}} = 3\sqrt{3}.$$

Now, the side length s (which is the distance PQ) is given by

$$\begin{aligned} s^2 &= PQ^2 = (3\sqrt{3} - 0)^2 + (0 - 1)^2 \\ &= (3\sqrt{3})^2 + 1^2 \\ &= 27 + 1 \\ &= 28. \end{aligned}$$

Thus, the square of side QR is

$$(QR)^2 = s^2 = 28.$$

Question 15

Let the triangle PQR be the image of the triangle with vertices $(1, 3)$, $(3, 1)$ and $(2, 4)$ in the line $x + 2y = 2$. If the centroid of $\triangle PQR$ is the point (α, β) , then $15(\alpha - \beta)$ is equal to :

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Options:

A. 21

B. 19

C. 22

D. 24

Answer: C

Solution:

Let 'G' be the centroid of Δ formed by $(1, 3)$, $(3, 1)$ & $(2, 4)$

$$G \cong \left(2, \frac{8}{3} \right)$$

Image of G w.r.t. $x + 2y - 2 = 0$

$$\begin{aligned}\frac{\alpha - 2}{1} &= \frac{\beta - \frac{8}{3}}{2} = -2 \frac{(2 + \frac{16}{3} - 2)}{1 + 4} \\ &= \frac{-2}{5} \left(\frac{16}{3} \right) \\ \Rightarrow \alpha &= \frac{-32}{15} + 2 = \frac{-2}{15}, \beta = \frac{-32 \times 2}{15} + \frac{8}{3} = \frac{-24}{15} \\ 15(\alpha - \beta) &= -2 + 24 = 22\end{aligned}$$

Question 16

A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P , that divides the rod AB internally in the ratio $2 : 1$ is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to :

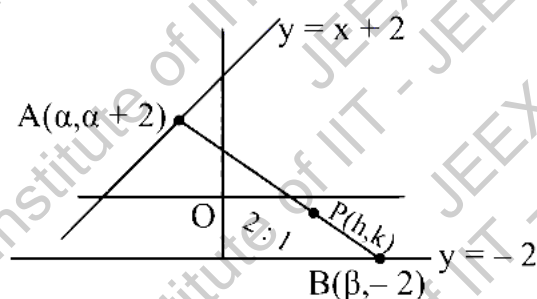
JEE Main 2025 (Online) 23rd January Evening Shift

Options:

- A. 24
- B. 22
- C. 21
- D. 23

Answer: D

Solution:



$$\begin{aligned}h &= \frac{3\beta + \alpha}{3} \\ k &= \frac{-4 + \alpha + 2}{3} \\ \alpha &= 3k + 2 \\ 2\beta &= 3h - \alpha = 3h - 3k - 2 \\ \text{so } AB &= 8 \\ (\alpha - \beta)^2 + (\alpha + 4)^2 &= 64 \\ \left(3k + 2 - \left(\frac{3h - 3k - 2}{2} \right) \right)^2 + (3k + 2 + 4)^2 &= 64 \\ \frac{(9k - 3h + 6)^2}{4} + (3k + 6)^2 &= 64 \\ 9[(3k - h + 2)^2 + 4(k + 2)^2] &= 64 \times 4 \\ 9(x^2 + 13y^2 - 6xy - 4x + 28y) &= 76 \\ \alpha - \beta - \gamma &= 13 + 6 + 4 = 23\end{aligned}$$

Question 17

Let the lines $3x - 4y - \alpha = 0$, $8x - 11y - 33 = 0$, and $2x - 3y + \lambda = 0$ be concurrent. If the image of the point $(1, 2)$ in the line $2x - 3y + \lambda = 0$ is $(\frac{57}{13}, \frac{-40}{13})$, then $|\alpha\lambda|$ is equal to

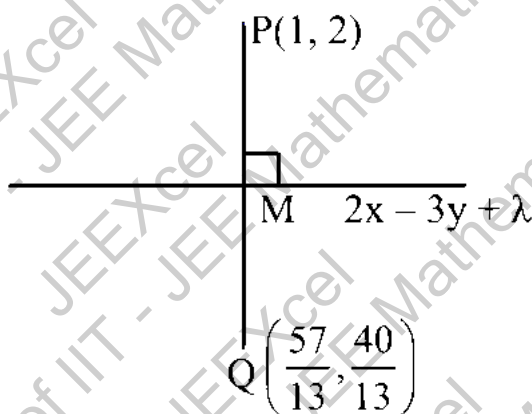
JEE Main 2025 (Online) 24th January Morning Shift

Options:

- A. 91
- B. 113
- C. 101
- D. 84

Answer: A

Solution:



$$\because PM = QM$$

$$\text{So, } M\left(\frac{\frac{57}{13}+1}{2}, \frac{\frac{40}{13}+2}{2}\right)$$

$$= \left(\frac{35}{13}, \frac{7}{13}\right)$$

$\because$ M lies on the line

$$2x - 3y + \lambda = 0$$

$$2\left(\frac{35}{13}\right) - 3\left(\frac{7}{13}\right) + \lambda = 0$$

$$\lambda = -\frac{70}{13} + \frac{21}{13}$$

$$= \frac{-91}{13} = -7$$

$$\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 3(-11\lambda - 99) + 4(8\lambda + 66) - \alpha(-24 + 22) = 0$$

$$\Rightarrow 33\lambda - 297 + 32\lambda + 264 + 24\alpha - 22\alpha = 0$$

$$\Rightarrow -\lambda + 2\alpha - 33 = 0 \quad \dots (1)$$

$$\therefore \lambda = -7$$

$$-(-7) + 2\alpha - 33 = 0$$

$$2\alpha = 26$$

$$\alpha = 13$$

$$\therefore |\alpha\lambda| = |13 \times (-7)|$$

$$= 91$$

Question 18

Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$. Then the product of the smallest and the largest values of α is equal to :

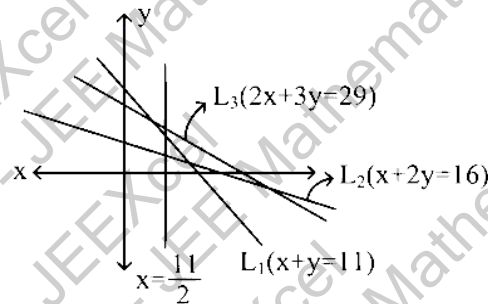
JEE Main 2025 (Online) 24th January Evening Shift

Options:

- A. 22
- B. 33
- C. 55
- D. 44

Answer: B

Solution:



Point of intersection of $x = \frac{11}{2}$ with L_1 & L_3 gives,

$$\alpha_{\min} = \frac{11}{2}$$

$$\text{and } \alpha_{\max} = 6$$

$$\therefore \alpha_{\min} \cdot \alpha_{\max} = \frac{11}{2} \times 6 = 33$$

Question19

If A and B are the points of intersection of the circle $x^2 + y^2 - 8x = 0$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and a point P moves on the line $2x - 3y + 4 = 0$, then the centroid of $\triangle PAB$ lies on the line :

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Options:

- A.
 $x + 9y = 36$
- B.
 $9x - 9y = 32$
- C.
 $4x - 9y = 12$
- D.
 $6x - 9y = 20$

Answer: D

Solution:

$$x^2 + y^2 - 8x = 0, \frac{x^2}{9} - \frac{y^2}{4} = 1 \quad \dots (1)$$

$$4x^2 - 9y^2 = 36 \quad \dots (2)$$

Solve (1)&(2)

$$4x^2 - 9(8x - x^2) = 36$$

$$13x^2 - 72x - 36 = 0$$

$$(13x + 6)(x - 6) = 0$$

$$x = \frac{-6}{13}, x = 6$$

$$x = \frac{-6}{13} \text{ (rejected)}$$

$y \rightarrow$ Imaginary

$$n = 6, \frac{36}{9} - \frac{y^2}{4} = 1$$

$$y^2 = 12, y = I\sqrt{12}$$

$$A(6, \sqrt{12}), B(6, -\sqrt{12})$$

$$P\left(\alpha, \frac{2\alpha + 4}{3}\right) \text{ P lies on}$$

$$\text{centroid (h, k)} \quad 2x - 3y + y = 0$$

$$h = \frac{12 + \alpha}{3}, \alpha = 3h - 12$$

$$k = \frac{\frac{2\alpha - 3y}{3}}{3} \Rightarrow 2\alpha + 4 = 9y$$

$$\alpha = \frac{9k - 4}{2}$$

$$6h - 2y = 9k - 4$$

$$6x - 9y = 20$$

Question 20

Two equal sides of an isosceles triangle are along $-x + 2y = 4$ and $x + y = 4$. If m is the slope of its third side, then the sum, of all possible distinct values of m , is:

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Options:

A.

$$-2\sqrt{10}$$

B.

$$12$$

C.

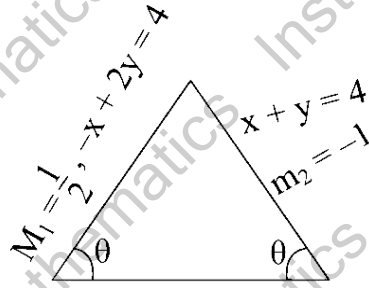
$$-6$$

D.

$$6$$

Answer: D

Solution:



$$\tan \theta = \frac{m - \frac{1}{2}}{1 + \frac{1}{2} \cdot m} = \frac{-1 - m}{1 - m} = \frac{m + 1}{m - 1}$$

$$\frac{2m - 1}{2 + m} = \frac{m + 1}{m - 1}$$

$$2m^2 - 3m + 1 = m^2 + 3m + 2$$

$$m^2 - 6m - 1 = 0$$

sum of root = 6
sum is 6

Question 21

Let the line $x + y = 1$ meet the axes of x and y at A and B , respectively. A right angled triangle AMN is inscribed in the triangle OAB , where O is the origin and the points M and N lie on the lines OB and AB , respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $AN : NB = \lambda : 1$, then the sum of all possible value(s) of λ is:

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Options:

A.

$$\frac{1}{2}$$

B.

$$\frac{5}{2}$$

C.

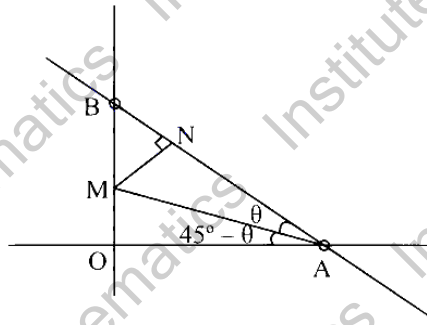
$$2$$

D.

$$\frac{13}{6}$$

Answer: C

Solution:



$$\text{Area of } \triangle AOB = \frac{1}{2}$$

$$\text{Area of } \triangle AMN = \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$$

$$\text{Equation of AB is } x + y = 1$$

$$OA = 1, AM = \sec(45^\circ - \theta)$$

$$AN = \sec(45^\circ - \theta) \cos \theta$$

$$MN = \sec(45^\circ - \theta) \sin \theta$$

$$\text{Ar}(\triangle AMN) = \frac{1}{2} \times \sec^2(45^\circ - \theta) \sin \theta \cdot \cos \theta = \frac{2}{9}$$

$$\Rightarrow \tan \theta = 2, \frac{1}{2}$$

$\tan \theta = 2$ is rejected

$$\frac{AN}{NB} = \frac{\lambda}{1} = \cot \theta = 2$$

Question22

Let the area of the triangle formed by a straight line $L : x + by + c = 0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x -axis, then the value of $b^2 + c^2$ is :

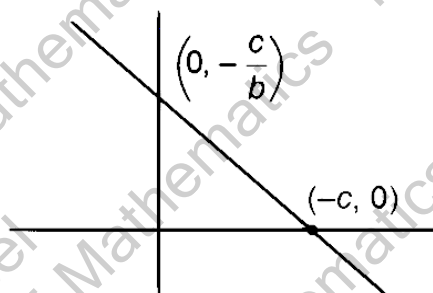
JEE Main 2025 (Online) 2nd April Evening Shift

Options:

- A. 90
- B. 83
- C. 93
- D. 97

Answer: D

Solution:



$$L: x + by + c = 0$$

$$\therefore \frac{1}{2} \left| (-c) \cdot \left(\frac{-c}{b} \right) \right| = 48$$

$$\therefore \left| \frac{c^2}{b} \right| = 96 \dots\dots (i)$$

$$\text{Slope of line } L = -\frac{1}{b}$$

$\therefore$ Slope of line perpendicular to L is b .

$$\therefore b = 1$$

$$\therefore c^2 = 96$$

$$\therefore b^2 + c^2 = 97$$

Question 23

A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1: 2x + y + 6 = 0$ and $L_2: 4x + 2y - p = 0, p > 0$, at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

A. 5

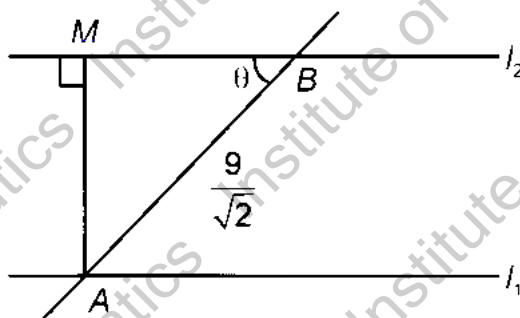
B. 3

C. 2

D. 4

Answer: B

Solution:



$$\tan \theta = \left| \frac{-2-1}{1+(-2)(1)} \right| = \frac{3}{1}$$

$$\sin \theta = \frac{3}{\sqrt{10}}$$

$$AM = \frac{3}{\sqrt{10}} \times \frac{9}{\sqrt{2}} = \frac{27}{2\sqrt{5}}$$

$$\text{Dist. between } l_1 \text{ \& } l_2, \quad AM = \left| \frac{p+6}{\sqrt{5}} \right| = \frac{27}{2\sqrt{5}}$$

$$\frac{P}{2} + 6 = \pm \frac{27}{2}$$

$$\frac{P}{2} = \frac{27}{2} - 6 \Rightarrow P = 15, \text{ As } P > 0$$

$$BM = \sqrt{\frac{81}{2} - \left(\frac{27}{2\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{810 - 729}{20}} = \sqrt{\frac{81}{20}} = \frac{9}{2\sqrt{5}}$$

$$\text{Now, } \frac{AM}{BM} = \frac{\frac{27}{2\sqrt{5}}}{\frac{9}{2\sqrt{5}}} = 3$$

Question 24

Let $\triangle ABC$ be a triangle formed by the lines $7x - 6y + 3 = 0$, $x + 2y - 31 = 0$ and $9x - 2y - 19 = 0$. Let the point (h, k) be the image of the centroid of $\triangle ABC$ in the line $3x + 6y - 53 = 0$. Then $h^2 + k^2 + hk$ is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

47

B.

37

C.

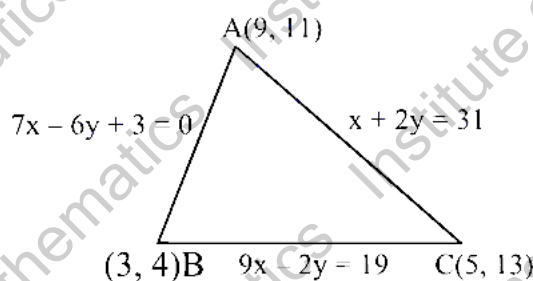
40

D.

36

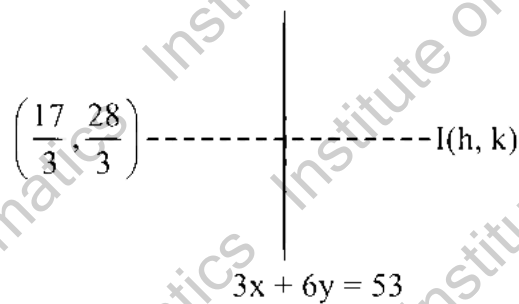
Answer: B

Solution:



$$\therefore \text{centroid of } \triangle ABC = \left(\frac{9 + 3 + 5}{3}, \frac{11 + 4 + 13}{3} \right)$$

$$= \left(\frac{17}{3}, \frac{28}{3} \right)$$



Let image of centroid with respect to line mirror is (h, k)

$$\therefore \left(\frac{k - \frac{28}{3}}{h - \frac{17}{3}} \right) \left(-\frac{1}{2} \right) = -1$$

$$\&3 \left(\frac{h + \frac{17}{3}}{2} \right) + 6 \left(\frac{k + \frac{28}{3}}{2} \right) = 53$$

Solving (1) & (2) we get $h = 3, k = 4$

$$\therefore h^2 + k^2 + hk = 37$$

Question 25

Consider the lines $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$, λ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point $(3, 6)$ is d , then the value of d^2 is

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

- A. 10
- B. 20
- C. 15
- D. 30

Answer: B

Solution:

$$x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$$

$$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$$

$$L_1 + \lambda L_2 = 0$$

$\Rightarrow$ P is intersection of L_1 & L_2 i.e. $(1, 2)$

$$y - 2 = m(x - 1)$$

$$mx - y + 2 - m = 0$$

$$\text{Distance from origin} = \left| \frac{2 - m}{\sqrt{1 + m^2}} \right| = \max$$

$$2y - 4 = -x + 1$$

$$\text{For } m = -\frac{1}{2}$$

$$\therefore L = y - 2 = -\frac{1}{2}(x - 1)$$

$$L : x + 2y - 5 = 0$$

$$\text{Now, } d = \left| \frac{3 + 12 - 5}{\sqrt{5}} \right| = \left| \frac{10}{\sqrt{5}} \right|$$

$$d^2 = \frac{100}{5} = 20$$

Question 26

Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0$, $x + y = 5$ and $7x + 4y = 15$. Then the distance of its orthocentre from the orthocentre of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 1$ is

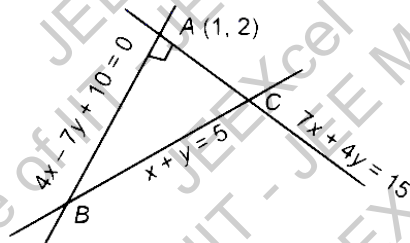
JEE Main 2025 (Online) 4th April Morning Shift

Options:

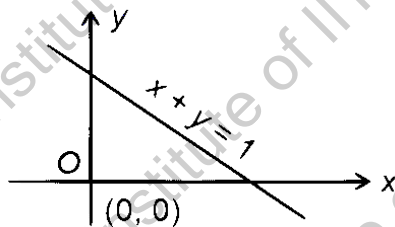
- A. $\sqrt{20}$
- B. 20
- C. $\sqrt{5}$
- D. 5

Answer: C

Solution:



A is orthocentre of above Δ .



O is orthocentre of above Δ .

$$OA = \sqrt{5}$$

Question 27

Let ABC be the triangle such that the equations of lines AB and AC be $3y - x = 2$ and $x + y = 2$, respectively, and the points B and C lie on x -axis. If P is the orthocentre of the triangle ABC, then the area of the triangle PBC is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

- A. 8

- B. 4
C. 10
D. 6

Answer: D

Solution:

Equation of line AB is $3y - x = 2$

And AC is $x + y = 2$

In line AB ,

When $y = 0, x = -2$

$\therefore B(-2, 0)$

In line AC ,

When $y = 0, x = 2$

$\therefore C(2, 0)$

Equation of altitude of BC ,

$Y = x + 2$

Similarly, equation of altitude of AB ,

$y = -3x + 6$

$\therefore$ On solving, orthocentre $P(1, 3)$

$\therefore \ar(\triangle PBC) = 6$

Question 28

If the orthocenter of the triangle formed by the lines $y = x + 1$, $y = 4x - 8$ and $y = mx + c$ is at $(3, -1)$, then $m - c$ is :

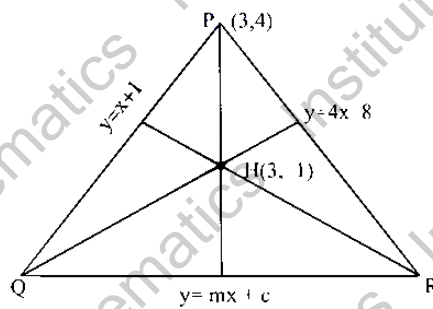
JEE Main 2025 (Online) 7th April Evening Shift

Options:

- A.
0
B.
2
C.
-2
D.
4

Answer: A

Solution:



Solve line PQ & QR

$$\text{Point Q} \left(\frac{1-c}{m-1}, \frac{1-c}{m-1} + 1 \right)$$

$$m_{2H} = \frac{\frac{1-c}{m-1} + 2}{\frac{1-c}{m-1} - 3} = \frac{1-c+2m-2}{1-c-3m+3} = \frac{1}{4} \dots (1)$$

$$\therefore m_{PH} = \frac{5}{0} \rightarrow \infty$$

$\Rightarrow$ Slope of line QR (m) = 0

Put value of m in equation (1)

$$\frac{1-c-2}{1-c+3} = -\frac{1}{4} \Rightarrow c = 0$$

so m - c = 0 Ans.

Question 29

A line passing through the point $P(a, 0)$ makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clockwise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2 \tan^2 \alpha - 2\sqrt{3}$ is :

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

8

B.

4

C.

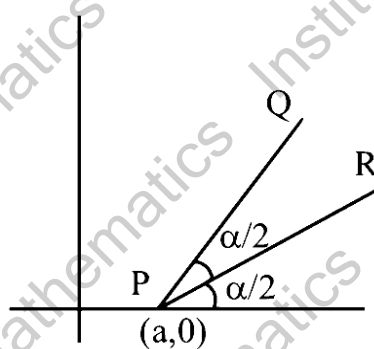
5

D.

6

Answer: B

Solution:



$$\tan \angle PR = 2 - \sqrt{3} = \tan 15^\circ$$

$$\therefore \frac{\alpha}{2} = 15^\circ$$

$$\Rightarrow \alpha = 30^\circ$$

equation of PR:

$$y = \tan 15^\circ (x - a)$$

$$y = (2 - \sqrt{3})(x - a)$$

$$\perp \text{ distance from origin} = \frac{1}{\sqrt{2}}$$

$$\left| \frac{\sqrt{3}a - 2a}{\sqrt{4 + 3 - 4\sqrt{3} + 1}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{|a|(2 - \sqrt{3})}{2\sqrt{(2 - \sqrt{3})}} = \frac{1}{\sqrt{2}}$$

$$|a| = \frac{\sqrt{2}}{\sqrt{2 - \sqrt{3}}} = \sqrt{2}(\sqrt{2 + \sqrt{3}})$$

$$a^2 = 2(2 + \sqrt{3})$$

$$3a^2 \tan^2 \alpha - 2\sqrt{3}$$

$$3 \times (4 + 2\sqrt{3}) \cdot \frac{1}{3} - 2\sqrt{3} = 4$$

Question 30

Let a be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle α with the positive x-axis and the equations of its diagonals are $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 0$ and $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y + 8\sqrt{3} = 0$. Then a^2 is equal to :

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

48

B.

16

C.

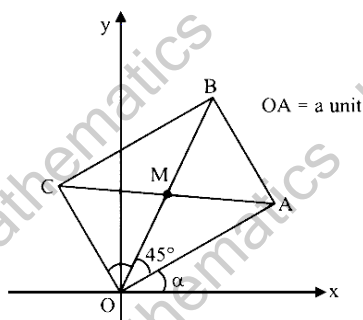
24

D.

32

Answer: A

Solution:



$$\text{Slope of diagonal OB} = \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$$= \tan 105^\circ$$

$$\therefore \alpha = 60^\circ$$

$$\therefore A (\cos 60^\circ, \sin 60^\circ)$$

$$\therefore A \left(\frac{a}{2}, \frac{\sqrt{3}a}{2} \right)$$

A Lies on other diagonal

$$\therefore \left(\frac{\sqrt{3}-1}{2} \right) a - \left(\frac{\sqrt{3}+1}{2} \right) \cdot \sqrt{3}a + 8\sqrt{3} = 0$$

$$a \left[\frac{\sqrt{3}-1-3-\sqrt{3}}{2} \right] = 8\sqrt{3}$$

$$a = 4\sqrt{3}$$

$$\therefore a^2 = 48$$

Question31

The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

[27-Jan-2024 Shift 1]

Options:

A.

$\frac{8}{5}$

B.

$\frac{25}{41}$

C.

$\frac{2}{5}$

D.

$\frac{30}{41}$

Answer: D

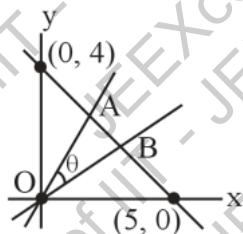
Solution:

Co-ordinates of A = $\left(\frac{5}{3}, \frac{8}{3}\right)$

Co-ordinates of B = $\left(\frac{10}{3}, \frac{4}{3}\right)$

Slope of OA = $m_1 = \frac{8}{5}$

Slope of OB = $m_2 = \frac{2}{5}$



$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \frac{\frac{6}{5}}{1 + \frac{16}{25}} = \frac{30}{41}$$

$$\tan \theta = \frac{30}{41}$$

Question 32

Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is :

[27-Jan-2024 Shift 2]

Options:

A.

$$(-3, -1) \cup \left(-\frac{1}{3}, 1\right)$$

B.

$$(-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

C.

$$(-3, 0) \cup \left(\frac{2}{3}, 1\right)$$

D.

$$(-3, -1) \cup \left(\frac{1}{3}, 1\right)$$

Answer: B

Solution:

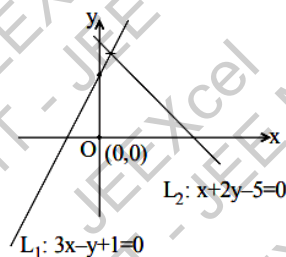
$$P(a^2, a+1)$$

$$L_1: 3x - y + 1 = 0$$

Origin and P lies same side w.r.t. L_1

$$\Rightarrow L_1(0) \cdot L_1(P) > 0$$

$$\therefore 3(a^2) - (a+1) + 1 > 0$$



$$2L_1: 3x - y + 1 = 0$$

$$\Rightarrow 3a^2 - a > 0$$

$$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \dots \dots \dots (1)$$

$$\text{Let } L_2: x + 2y - 5 = 0$$

Origin and P lies same side w.r.t. L_2

$$\Rightarrow L_2(0) \cdot L_2(P) > 0$$

$$\Rightarrow a^2 + 2(a+1) - 5 < 0$$

$$\Rightarrow a^2 + 2a - 3 < 0$$

$$\Rightarrow (a+3)(a-1) < 0$$

$$\therefore a \in (-3, 1) \dots \dots \dots (2)$$

Intersection of (1) and (2)

$$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

Question33

Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m,n) from the point Q(-2, -3) is

[27-Jan-2024 Shift 2]

Options:

A.

10

B.

6

C.

4

D.

8

Answer: A

Solution:

$$2^m - 2^n = 56$$

$$2^n(2^{m-n} - 1) = 2^3 \times 7$$

$$2^n = 2^3 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow n = 3 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow n = 3 \text{ and } m - n = 3$$

$$\Rightarrow n = 3 \text{ and } m = 6$$

$$P(6, 3) \text{ and } Q(-2, -3)$$

$$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Hence option (1) is correct

Question34

If the sum of squares of all real values of α , for which the lines $2x - y + 3 = 0$, $6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is p , then the greatest integer less than or equal to p is

[27-Jan-2024 Shift 2]

Answer: 32

Solution:

$$2x - y + 3 = 0$$

$$6x + 3y + 1 = 0$$

$$\alpha x + 2y - 2 = 0$$

Will not form a Δ if $\alpha x + 2y - 2 = 0$ is concurrent with $2x - y + 3 = 0$ and $6x + 3y + 1 = 0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2} = \frac{-6}{3} \text{ or } -\frac{\alpha}{2} = 2$$

$$\Rightarrow \alpha = 4 \text{ or } \alpha = -4$$

$$P = 16 + 16 + \frac{16}{25}$$

$$[P] = \left[32 + \frac{16}{25} \right] = 32$$

Question35

In a $\triangle ABC$, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the point A and B are respectively $(4,6)$ and (α, β) , then $\alpha + 2\beta$ is equal to
[29-Jan-2024 Shift 1]

Options:

A.

42

B.

39

C.

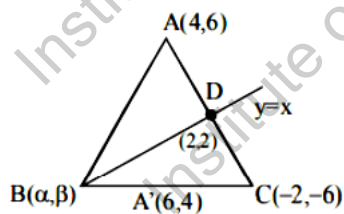
48

D.

45

Answer: A

Solution:



$$AD : DC = 1 : 2$$

$$\frac{4 - \alpha}{6 - \alpha} = \frac{10}{8}$$

$$\alpha = \beta$$

$$\alpha = 14 \text{ and } \beta = 14$$

Question36

Let $(5, a/4)$, be the circumcenter of a triangle with vertices $A(a, -2)$, $B(a, 6)$ and $C(a/4, -2)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha + \beta + \gamma$ is

[29-Jan-2024 Shift 1]

Options:

A.

60

B.

53

C.

62

D.

30

Answer: B

Solution:

$$A(a, -2), B(a, 6), C\left(\frac{a}{4}, -2\right), O\left(5, \frac{a}{4}\right)$$

$$AO = BO$$

$$(a-5)^2 + \left(\frac{a}{4} + 2\right)^2 = (a-5)^2 + \left(\frac{a}{4} - 6\right)^2$$

$$a = 8$$

$$AB = 8, AC = 6, BC = 10$$

$$\alpha = 5, \beta = 24, \gamma = 24$$

Question 37

The distance of the point $(2, 3)$ from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$4\sqrt{2}$$

B.

$$6\sqrt{3}$$

C.

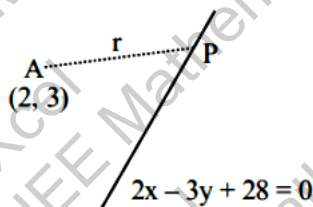
$$3 + 4\sqrt{2}$$

D.

$$4 + 6\sqrt{3}$$

Answer: D

Solution:



Writing P in terms of parametric co-ordinates $2 + r \cos \theta, 3 + r \sin \theta$ as $\tan \theta = \sqrt{3}$

$$\cos \theta, 3 + r \sin \theta \text{ as } \tan \theta = \sqrt{3}$$

$$P\left(2 + \frac{r}{2}, 3 + \frac{\sqrt{3}r}{2}\right)$$

P must satisfy $2x - 3y + 28 = 0$

$$\text{So, } 2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{\sqrt{3}r}{2}\right) + 28 = 0$$

$$\text{We find } r = 4 + 6\sqrt{3}$$

Question38

Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8$, $6x + y = 5$. The distance of the point $P(5, -2)$ from the line AB is

[29-Jan-2024 Shift 2]

Options:

A.

$13/2$

B.

8

C.

$5/2$

D.

6

Answer: D

Solution:

Solving lines $L_1 (3x + 2y = 14)$ and $L_2 (5x - y = 6)$ to get $A(2, 4)$ and solving lines $L_3 (4x + 3y = 8)$ and $L_4 (6x + y = 5)$ to get $B(1/2, 2)$.

Finding Eqn. of AB : $4x - 3y + 4 = 0$

Calculate distance PM

$$\Rightarrow \left| \frac{4(5) - 3(-2) + 4}{5} \right| = 6$$

Question39

A line passing through the point $A(9, 0)$ makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is

[30-Jan-2024 Shift 1]

Options:

A.

$$\frac{y}{\sqrt{3}-2} + x = 9$$

B.

$$\frac{x}{\sqrt{3}-2} + y = 9$$

C.

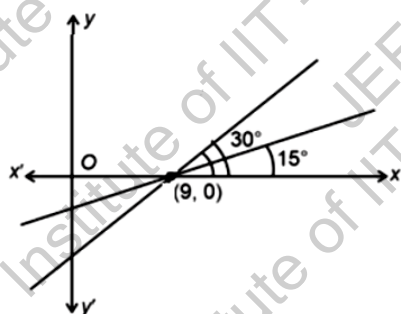
$$\frac{x}{\sqrt{3}+2} + y = 9$$

D.

$$\frac{y}{\sqrt{3}+2} + x = 9$$

Answer: A

Solution:



$$\text{Eq}^n : y - 0 = \tan 15^\circ (x - 9) \Rightarrow y = (2 - \sqrt{3})(x - 9)$$

Question40

If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

[30-Jan-2024 Shift 2]

Options:

A.

14

B.

6

C.

8

D.

29

Answer: A

Solution:

Cocus of point P(x, y) whose distance from

Gives

$x+2y+7=0$ & $2x-y+8=0$ are equal is

$$\frac{x+2y+7}{\sqrt{5}} = \pm \frac{2x-y+8}{\sqrt{5}}$$

$$(x+2y+7)^2 - (2x-y+8)^2 = 0$$

Combined equation of lines

$$(x-3y+1)(3x+y+15)=0$$

$$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$$

$$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$$

$$x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$$

$$h = \frac{4}{3}, g = 3, f = -\frac{22}{3}, c = 5$$

$$g+c+h-f = 3+5 - \frac{4}{3} + \frac{22}{3} = 8+6 = 14$$

Question41

Let $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ and let $A(\alpha, \beta)$, $B(1, 0)$, $C(\gamma, \delta)$ and $D(1, 2)$ be the vertices of a parallelogram ABCD. If $AB = \sqrt{10}$ and the points A and C lie on the line $3y = 2x + 1$, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

[31-Jan-2024 Shift 1]

Options:

A.

10

B.

5

C.

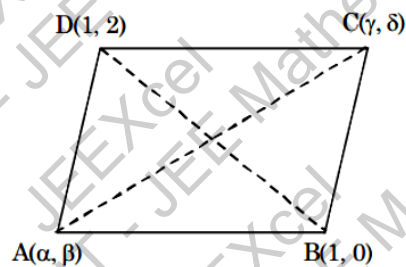
12

D.

8

Answer: D

Solution:



Let E is mid point of diagonals

$$\frac{\alpha+\gamma}{2} = \frac{1+1}{2} \text{ \& \& } \frac{\beta+\delta}{2} = \frac{2+0}{2}$$

$$\alpha+\gamma=2 \quad \beta+\delta=2$$

$$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$$

Question 42

Let $A(a, b)$, $B(3, 4)$ and $(-6, -8)$ respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point $P(2a + 3, 7b + 5)$ from the line $2x + 3y - 4 = 0$ measured parallel to the line $x - 2y - 1 = 0$ is

[31-Jan-2024 Shift 2]

Options:

A.

$$\frac{15\sqrt{5}}{7}$$

B.

$$\frac{17\sqrt{5}}{6}$$

C.

$$\frac{17\sqrt{5}}{7}$$

D.

$$\frac{\sqrt{5}}{17}$$

Answer: C

Solution:

$$A(a, b), B(3, 4), C(-6, -8)$$

$$\begin{array}{ccc} & 2:1 & \\ \hline C & A & B \\ (-6, -8) & (a, b) & (3, 4) \end{array}$$

$$\Rightarrow a = 0, b = 0 \Rightarrow P(3, 5)$$

Distance from P measured along $x - 2y - 1 = 0$

$$\Rightarrow x = 3 + r \cos \theta, y = 5 + r \sin \theta$$

$$\text{Where } \tan \theta = \frac{1}{2}$$

$$r(2 \cos \theta + 3 \sin \theta) = -17$$

$$\Rightarrow r = \left| \frac{-17\sqrt{5}}{7} \right| = \frac{17\sqrt{5}}{7}$$

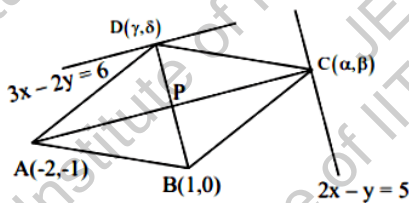
Question43

Let $A(-2, -1)$, $B(1, 0)$, $C(\alpha, \beta)$ and $D(\gamma, \delta)$ be the vertices of a parallelogram ABCD. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha + \beta + \gamma + \delta|$ is equal to _____

[31-Jan-2024 Shift 2]

Answer: 32

Solution:



$$P \equiv \left(\frac{\alpha - 2}{2}, \frac{\beta - 1}{2} \right) \equiv \left(\frac{\gamma + 1}{2}, \frac{\delta}{2} \right)$$

$$\frac{\alpha - 2}{2} = \frac{\gamma + 1}{2} \text{ and } \frac{\beta - 1}{2} = \frac{\delta}{2}$$

$$\Rightarrow \alpha - \gamma = 3 \dots (1), \beta - \delta = 1 \dots (2)$$

Also, (γ, δ) lies on $3x - 2y = 6$

$$3\gamma - 2\delta = 6 \dots (3)$$

and (α, β) lies on $2x - y = 5$

$$\Rightarrow 2\alpha - \beta = 5 \dots (4)$$

Solving (1), (2), (3), (4)

$$\alpha = -3, \beta = -11, \gamma = -6, \delta = -12$$

$$|\alpha + \beta + \gamma + \delta| = 32$$

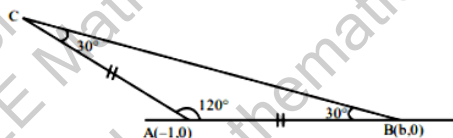
Question44

Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = 2\pi/3$, $AB = AC$ and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β) , then β^4/α^2 is:

[1-Feb-2024 Shift 2]

Answer: 36

Solution:



$$\frac{c}{\sin 30^\circ} = \frac{4\sqrt{3}}{\sin 120^\circ} \text{ [By sine rule]}$$

$$2c = 8 \Rightarrow c = 4$$

$$AB = |(b+1)| = 4$$

$$b = 3, m_{AB} = 0$$

$$m_{BC} = \frac{-1}{\sqrt{3}}$$

$$BC : -y = \frac{-1}{\sqrt{3}}(x-3)$$

$$\sqrt{3}y + x = 3$$

$$\text{Point of intersection : } y = x+3, \sqrt{3}y + x = 3$$

$$(\sqrt{3}+1)y = 6$$

$$y = \frac{6}{\sqrt{3}+1}$$

$$x = \frac{6}{\sqrt{3}+1} - 3$$

$$= \frac{6-3\sqrt{3}-3}{\sqrt{3}+1}$$

$$= 3 \frac{(1-\sqrt{3})}{(1+\sqrt{3})} = \frac{-6}{(1+\sqrt{3})^2}$$

$$\frac{\beta^4}{\alpha^2} = 36$$

Question 45

Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that

$\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)}$ is equal to

[24-Jan-2023 Shift 1]

Options:

A. 4

B. 3

C. 2

D. —

Answer: B

Solution:

Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of $\triangle PQR$ is $= \frac{1}{2} |\vec{q} \times \vec{r}|$

Area of $\triangle ABC$ is $\frac{1}{2} |\vec{AB} \times \vec{AC}|$

$\vec{AB} = \vec{r} - 2\frac{\vec{q}}{3}$, $\vec{AC} = -\vec{r} - \frac{\vec{q}}{3}$

$$\text{Area of } \triangle ABC = \frac{1}{6} |\vec{q} \times \vec{r}|$$

$$\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)} = 3$$

Question 46

The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is [24-Jan-2023 Shift 2]

Options:

- A. $\sqrt{6}$
- B. $2\sqrt{2}$
- C. 2
- D. 4

Answer: B

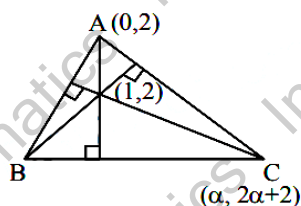
Solution:

$$AB : (\lambda + 1)x + \lambda y = 4$$

$$AC : \lambda x + (1 - \lambda)y + \lambda = 0$$

Vertex A is on y-axis

$$\Rightarrow x = 0$$



$$\text{So } y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \lambda = 2$$

$$AB : 3x + 2y = 4$$

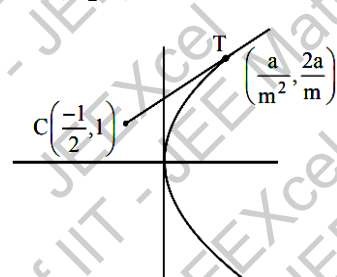
$$AC : 2x - y + 2 = 0$$

$$\Rightarrow A(0, 2) \text{ Let } C(\alpha, 2\alpha + 2)$$

$$\text{Now (Slope of Altitude through C)} \left(-\frac{3}{2}\right) = -1$$

$$\left(\frac{2\alpha}{\alpha - 1}\right) \left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$$

$$\text{So } C\left(-\frac{1}{2}, 1\right)$$



$$\text{Let Equation of tangent be } y = mx + \frac{3}{2m}$$

$$m^2 + 2m - 3 = 0$$

$$\Rightarrow m = 1, -3$$

So tangent which touches in first quadrant at T is

$$\begin{aligned} T &= \left(\frac{a}{m^2}, \frac{2a}{m} \right) \\ &= \left(\frac{3}{2}, 3 \right) \\ \Rightarrow CT &= \sqrt{4+4} = 2\sqrt{2} \end{aligned}$$

Question47

The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is:
[25-Jan-2023 Shift 2]

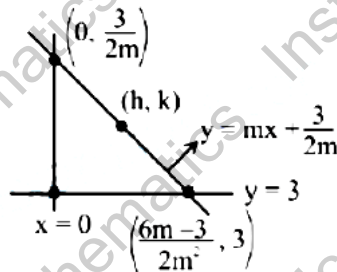
Options:

- A. $4y^2 - 18y - 3x - 18 = 0$
- B. $4y^2 + 18y + 3x + 18 = 0$
- C. $4y^2 - 18y + 3x + 18 = 0$
- D. $4y^2 - 18y - 3x + 18 = 0$

Answer: C

Solution:

$$\begin{aligned} y^2 &= 6x \Rightarrow y^2 = 4ax \\ \Rightarrow 4a &= 6 \Rightarrow a = \frac{3}{2} \end{aligned}$$



$$y = mx + \frac{3}{2m}; (m \neq 0)$$

$$h = \frac{6m-3}{4m^2}, k = \frac{6m+3}{4m}, \text{ Now eliminating } m \text{ and we get}$$

$$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$$

$$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$$

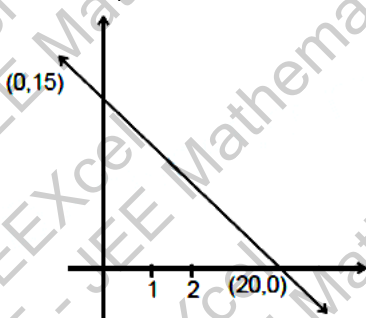
Question48

A triangle is formed by X - axis, Y - axis and the line $3x + 4y = 60$. Then the number of points P(a, b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is _____.
[25-Jan-2023 Shift 2]

Answer: 31

Solution:

If $x = 1, y = \frac{57}{4} = 14.25$



$(1, 1)(1, 2) - (1, 14) \Rightarrow 14$ pts.

If $x = 2, y = \frac{27}{2} = 13.5$

$(2, 2)(2, 4) \dots (2, 12) \Rightarrow 6$ pts.

If $x = 3, y = \frac{51}{4} = 12.75$

$(3, 3)(3, 6) - (3, 12) \Rightarrow 4$ pts.

If $x = 4, y = 12$

$(4, 4)(4, 8) \Rightarrow 2$ pts.

If $x = 5, y = \frac{45}{4} = 11.25$

$(5, 5), (5, 10) \Rightarrow 2$ pts.

If $x = 6, y = \frac{21}{2} = 10.5$

$(6, 6) \Rightarrow 1$ pt

If $x = 7, y = \frac{39}{4} = 9.75$

$(7, 7) \Rightarrow 1$ pt.

If $x = 8, y = 9$

$(8, 8) \Rightarrow 1$ pt.

If $x = 9, y = \frac{33}{4} = 8.25 \Rightarrow$ no pt.

Total = 31 pts.

Question 49

A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is
[29-Jan-2023 Shift 1]

Options:

A. $\frac{2}{(\sqrt{3}-1)}$

B. $\frac{2}{3+\sqrt{3}}$

C. $\frac{2}{3-\sqrt{3}}$

D. $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

Answer: B

Solution:

Slope of reflected ray $= \tan 60^\circ = \sqrt{3}$

Line $y = \frac{x}{\sqrt{3}}$ intersect $y + x = 1$ at $\left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1} \right)$

Equation of reflected ray is

$y - \frac{1}{\sqrt{3}+1} = \sqrt{3} \left(x - \frac{\sqrt{3}}{\sqrt{3}+1} \right)$

Put $y = 0 \Rightarrow x = \frac{2}{3+\sqrt{3}}$

Question50

Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is

[29-Jan-2023 Shift 1]

Options:

A. $3\sqrt{3}$

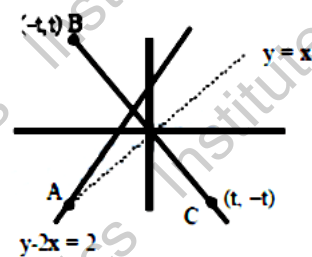
B. $2\sqrt{3}$

C. $\frac{8}{\sqrt{3}}$

D. $\frac{10}{\sqrt{3}}$

Answer: C

Solution:



At A $x = y$

$y - 2x = 2$

$(-2, -2)$

Height from line $x + y = 0$

$$h = \frac{4}{\sqrt{2}}$$

$$\text{Area of } \Delta = \frac{\sqrt{3}}{4} \cdot \frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$$

Question51

A triangle is formed by the tangents at the point $(2, 2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 10

Solution:

$$S_1 : y^2 = 2x \quad S_2 : x^2 + y^2 = 4x$$

$P(2, 2)$ is common point on S_1 & S_2

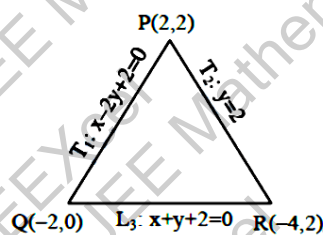
T_1 is tangent to S_1 at $P \Rightarrow T_1 : y \cdot 2 = x + 2$

$$\Rightarrow T_1 : x - 2y + 2 = 0$$

T_2 is tangent to S_2 at $P \Rightarrow T_2 : x \cdot 2 + y \cdot 2 = 2(x + 2)$

$$\Rightarrow T_2 : y = 2$$

$L_3 : x + y + 2 = 0$ is third line



$$PQ = a = \sqrt{20}$$

$$QR = b = \sqrt{8}$$

$$RP = c = 6$$

$$\text{Area } (\Delta PQR) = \Delta = \frac{1}{2} \times 6 \times 2 = 6$$

$$\therefore r = \frac{abc}{4\Delta} = \frac{\sqrt{160}}{4} = \sqrt{10} \Rightarrow r^2 = 10$$

Question 52

A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of x -axis and y -axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y -axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:
[30-Jan-2023 Shift 1]

Options:

A. $\frac{392}{3}$

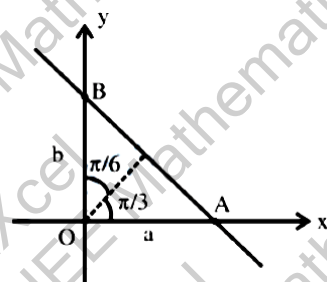
B. 196

C. $\frac{196}{3}$

D. 98

Answer: A

Solution:



$$\text{Equation of straight line : } \frac{x}{a} + \frac{y}{b} = 1$$

$$\text{Or } x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} = p$$

$$\frac{x}{2} + \frac{y\sqrt{3}}{2} = p$$

$$\frac{x}{3p} + \frac{y}{2p} = 1$$

$$\text{Comparing both : } a = 3p, b = \frac{2p}{\sqrt{3}}$$

$$\text{Now area of } \Delta OAB = \frac{1}{2} \cdot ab = \frac{98}{3} \cdot \sqrt{3}$$

$$\frac{1}{2} \cdot 2p \cdot \frac{2p}{\sqrt{3}} = \frac{98}{3} \cdot \sqrt{3}$$

$$p^2 = 49$$

$$a^2 - b^2 = 4p^2 - \frac{4p^2}{3} = \frac{2}{3} 4p^2$$

$$= \frac{8}{3} \cdot 49 = \frac{392}{3}$$

Question 53

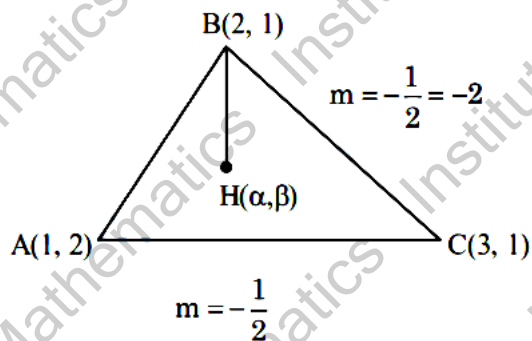
If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α , β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is
[1-Feb-2023 Shift 1]

Options:

- A. $x^2 - 19x + 90 = 0$
- B. $x^2 - 18x + 80 = 0$
- C. $x^2 - 22x + 120 = 0$
- D. $x^2 - 20x + 99 = 0$

Answer: D

Solution:



Here $m_{BH} \times m_{AC} = -1$

$$\left(\frac{\beta - 1}{\alpha - 2} \right) \left(\frac{1}{-2} \right) = -1$$

$$\beta - 1 = 2\alpha - 4$$

$$\beta = 2\alpha - 3$$

$$m_{AH} \times m_{BC} = -1$$

$$\Rightarrow \left(\frac{\beta - 2}{\alpha - 1} \right) (-2) = -1$$

$$\Rightarrow 2\beta - 4 = \alpha - 1$$

$$\Rightarrow 2(2\alpha - 3) = \alpha - 1$$

$$\Rightarrow 3\alpha = 5$$

$$\alpha = \frac{5}{3}, \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$$

$$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$$

$$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$$

$$x^2 - 20x + 99 = 0$$

Question54

The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$

The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is [1-Feb-2023 Shift 1]

Options:

A. $3x^2 + 5xy + 2y^2 = 0$

B. $x^2 - y^2 + 10xy = 0$

C. $3x^2 + xy - 2y^2 = 0$

D. $x^2 - y^2 - 10xy = 0$

Answer: D

Solution:

Equation of the pair of angle bisector for the homogenous equation $ax^2 + 2hxy + by^2 = 0$ is given as

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Here $a = 2$, $h = 1/2$ & $b = -3$

Equation will become

$$\frac{x^2 - y^2}{2 - (-3)} = \frac{xy}{1/2}$$

$$x^2 - y^2 = 10xy$$

$$x^2 - y^2 - 10xy = 0$$

Question55

The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line $L : 9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2 , then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on.

[6-Apr-2023 shift 1]

Options:

A. $6x + y = 10$

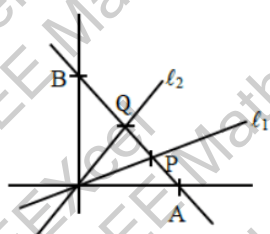
B. $6x - y = 15$

C. $y - 2x = 5$

D. $y - x = 5$

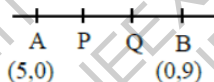
Answer: D

Solution:



$$L : 9x + 5y = 45$$

$$A(5,0), B(0,9)$$



$$\rightarrow P_x = \frac{2 \times 5 + 1 \times 0}{1+2} = \frac{10}{3}$$

$$P_y = \frac{0 \times 2 + 9 \times 1}{1+2} = 3$$

$$P: \left(\frac{10}{3}, 3\right)$$

$$\text{Similarly } \rightarrow Q_x = \frac{1 \times 5 + 2 \times 0}{1+2} = \frac{5}{3}$$

$$Q_y = \frac{1 \times 0 + 2 \times 9}{1+2} = 6$$

$$Q: \left(\frac{5}{3}, 6\right)$$

$$\text{Now } m_1 = \frac{\frac{3}{10} - 0}{\frac{10}{3} - 0} = \frac{9}{10}$$

$$m_2 = \frac{\frac{6}{5} - 0}{\frac{5}{3} - 0} = \frac{18}{5}$$

from (1) & (2)

$$9x + 5y = 45$$

$$9x - 2y = 0$$

$$\begin{array}{r} -+ \\ 7y = 45 \end{array} \Rightarrow y = \frac{45}{7}$$

$$\Rightarrow x = \frac{10}{7}$$

which satisfy $y - x = 5$ Ans. 4

Question 56

Let $A(0, 1)$, $B(1, 1)$ and $C(1, 0)$ be the mid-points of the sides of a triangle with incentre at the point D . If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{3}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

[8-Apr-2023 shift 2]

Options:

A. 6

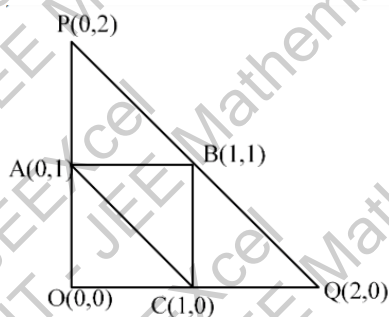
B. 8

C. $\frac{9}{2}$

D. 12

Answer: B

Solution:



$$a = OP = 2 \quad b = OQ = 2 \quad c = PQ = 2\sqrt{2}$$

$$D\left(\frac{4}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right) \equiv D\left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$$

$$y^2 = 4ax \Rightarrow \left(\frac{2}{2+\sqrt{2}}\right)^2 = 4a \cdot \left(\frac{2}{2+\sqrt{2}}\right)$$

$$\therefore 4a = \frac{2}{2+\sqrt{2}} \therefore a = \frac{1}{2} \cdot \frac{2-\sqrt{2}}{4-2} = \frac{1}{4}(2-\sqrt{2})$$

$$\therefore \alpha = \frac{2}{4} = \frac{1}{2} \quad \beta = \frac{-1}{4}$$

$$\therefore \frac{\alpha}{\beta^2} = 8 \quad \text{Ans.}$$

Question 57

The area of the quadrilateral ABCD with vertices A(2, 1, 1), B(1, 2, 5), C(-2, -3, 5) and D(1, -6, -7) is equal to
[8-Apr-2023 shift 2]

Options:

A. 54

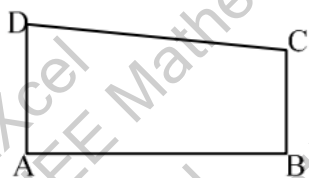
B. $9\sqrt{38}$

C. 48

D. $8\sqrt{38}$

Answer: D

Solution:



$$\text{Vector Area} = \vec{v} \\ = \frac{1}{2} \vec{AB} \times \vec{AC} + \frac{1}{2} \vec{AC} \times \vec{AD}$$

$$= \frac{1}{2} (\vec{AB} - \vec{AD}) \times \vec{AC} \quad \begin{pmatrix} \vec{AB} = \hat{i} + \hat{j} + 4\hat{k} \\ \vec{AD} = -\hat{i} - 7\hat{j} - 8\hat{k} \\ \vec{AC} = -4\hat{i} - 4\hat{j} + 4\hat{k} \end{pmatrix}$$

$$= \frac{1}{2} (8\hat{j} + 12\hat{k}) \times (-4)(\hat{i} + \hat{j} - \hat{k})$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 12 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\begin{aligned}
 &= (-2)(-20\hat{i} + 12\hat{j} - 8\hat{k}) \\
 &= 8(5\hat{i} - 3\hat{j} + 2\hat{k}) \\
 \therefore \text{Area} &= |\vec{v}| = 8\sqrt{25 + 9 + 4} = 8\sqrt{38} \text{ Ans.}
 \end{aligned}$$

Question 58

A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ . Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

[10-Apr-2023 shift 1]

Options:

A. $\frac{2}{3}\lambda$

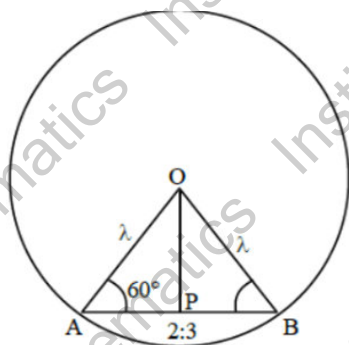
B. $\frac{\sqrt{19}}{7}\lambda$

C. $\frac{3}{5}\lambda$

D. $\frac{\sqrt{19}}{5}\lambda$

Answer: D

Solution:



Since OAB form equilateral Δ

$$\therefore \angle OAP = 60^\circ$$

$$AP = \frac{2\lambda}{5}$$

$$\cos 60^\circ = \frac{OA^2 + AP^2 - OP^2}{2OA \cdot AP}$$

$$\Rightarrow \frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{2\lambda \left(\frac{2\lambda}{5} \right)}$$

$$\Rightarrow \frac{2\lambda^2}{5} = \lambda^2 + \frac{4\lambda^2}{25} - OP^2$$

$$\Rightarrow OP^2 = \frac{19\lambda^2}{25}$$

$$\Rightarrow OP = \frac{\sqrt{19}\lambda}{5}$$

Therefore, locus of point P is $\frac{\sqrt{19}\lambda}{5}$

Question 59

Let A be the point (1, 2) and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C(α , β) then the length of the line segment AC is
[10-Apr-2023 shift 2]

Options:

A. $\frac{6\sqrt{5}}{5}$

B. $\frac{2\sqrt{5}}{5}$

C. $\frac{3\sqrt{5}}{5}$

D. $\frac{4\sqrt{5}}{5}$

Answer: C

Solution:



$$\frac{12 \cos \theta + 2}{5} = h \Rightarrow 12 \cos \theta = 5h - 2$$

sq & add

$$144 = (5h - 2)^2 + (5k - 4)^2$$

$$\left(x - \frac{2}{5}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{144}{25}$$

$$\text{Centre} \equiv \left(\frac{2}{5}, \frac{4}{5}\right) \equiv (\alpha, \beta)$$

$$AC = \sqrt{\left(1 - \frac{2}{5}\right)^2 + \left(2 - \frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

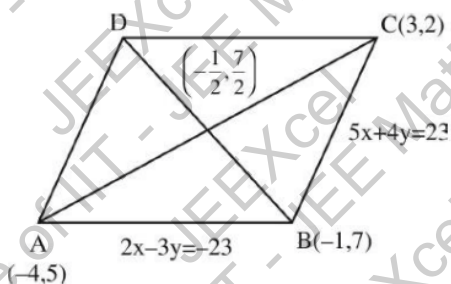
Question 60

Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50d^2$ is equal to _____.

[10-Apr-2023 shift 2]

Answer: 529

Solution:



A & C point will be $(-4, 5)$ & $(3, 2)$

mid point of AC will be $\left(-\frac{1}{2}, \frac{7}{2}\right)$

equation of diagonal BD is

$$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}} \left(x + \frac{1}{2}\right)$$

$$\Rightarrow 7x + y = 0$$

Distance of A from diagonal BD

$$= d = \frac{23}{\sqrt{50}}$$

$$\Rightarrow 50d^2 = (23)^2$$

$$50d^2 = 529$$

Question 61

Let R be a rectangle given by the line $x = 0$, $x = 2$, $y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the midpoint of AB lies on a :

[11-Apr-2023 shift 1]

Options:

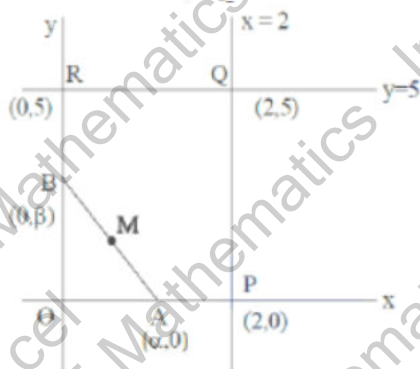
- A. straight line
- B. parabola
- C. circle
- D. hyperbola

Answer: D

Solution:

$$\frac{\text{ar(OPQR)}}{\text{ar(OAB)}} = \frac{4}{1}$$

Let M be the mid-point of AB.



$$M(h, k) = \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

$$\Rightarrow \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$$

$$\Rightarrow \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$$

$$\Rightarrow (2h)(2k) = 4$$

$\therefore$ Locus of M is $xy = 1$

Which is a hyperbola.

Question 62

If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the line $l_2 : x - y + 1 = 0$ and $l_3 : ax + by + 17$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _____.
[11-Apr-2023 shift 2]

Answer: 348

Solution:

Point of intersection of $l_1 : 3y - 2x = 3$

$l_2 : x - y + 1 = 0$ is $P \equiv (0, 1)$

Which lies on $l_3 : ax - by + 17 = 0$,

$\Rightarrow \beta = -17$

Consider a random point $Q \equiv (-1, 0)$

on $l_2 : x - y + 1 = 0$, image of Q about

$l_2 : x - y + 1 = 0$, is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13} \right)$ which is calculated by formulae

$$\frac{x - (-1)}{2} = \frac{y - 0}{-3} = 2 \left(\frac{-2 + 3}{13} \right)$$

Now, Q' lies in $l_3 : ax + by + 17 = 0$

$\Rightarrow \alpha = 7$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

Question 63

If the point $\left(\alpha, \frac{7\sqrt{3}}{3} \right)$ lies on the curve traced by the mid-points of the line segments of the lines

$x \cos \theta + y \sin \theta = 7$, $\theta \in \left(0, \frac{\pi}{2} \right)$ between the co-ordinates axes, then α is equal to

[12-Apr-2023 shift 1]

Options:

A. $7\sqrt{3}$

B. -7

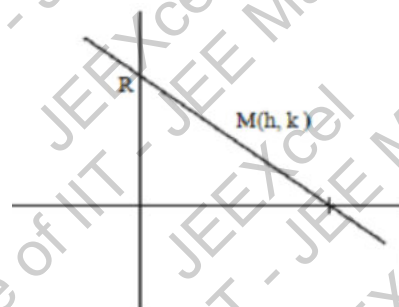
C. 7

D. $-7\sqrt{3}$

Answer: C

Solution:

pt $\left(\alpha, \frac{7\sqrt{3}}{3} \right)$



$$x \cos \theta + y \sin \theta = 7$$

$$x - \text{intercept} = \frac{7}{\cos \theta}$$

$$y\text{-intercept} = \frac{7}{\sin \theta}$$

$$A : \left(\frac{7}{\cos \theta}, 0 \right) B : \left(0, \frac{7}{\sin \theta} \right)$$

Locus of mid pt M : (h, k)

$$h = \frac{7}{2 \cos \theta}, k = \frac{7}{2 \sin \theta}$$

$$\frac{7}{2 \sin \theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\alpha = \frac{7}{2 \cos \theta} = 7$$

Question64

Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation [13-Apr-2023 shift 2]

Options:

A. $x^2 - 13x + 42 = 0$

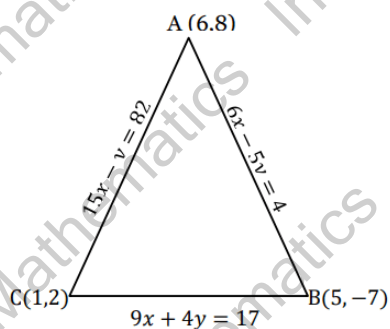
B. $x^2 - 10x + 25 = 0$

C. $x^2 - 7x + 12 = 0$

D. $x^2 - 14x + 48 = 0$

Answer: A

Solution:



$$\text{Centroid } (\alpha, \beta) = \left(\frac{6+1+5}{3}, \frac{8-7+2}{3} \right) = (4, 1)$$

$$\alpha + 2\beta = 4 + 2 = 6$$

$$2\alpha - \beta = 8 - 1 = 7$$

Quadratic equation

$$x^2 - (6+7)x + (6 \times 7) = 0$$

$$\Rightarrow x^2 - 13x + 42 = 0$$

Question65

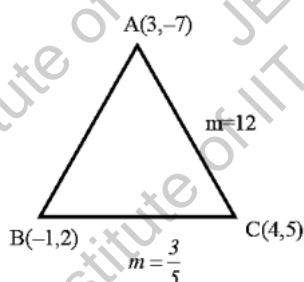
If (α, β) is the orthocenter of the triangle ABC with vertices A(3, -7), B(-1, 2) and C(4, 5), then $9\alpha - 6\beta + 60$ is equal to [15-Apr-2023 shift 1]

Options:

- A. 30
- B. 35
- C. 40
- D. 25

Answer: D

Solution:



Altitude of BC : $y + 7 = \frac{-5}{3}(x - 3)$

$$3y + 21 = -5x + 15$$
$$5x + 3y + 6 = 0$$

Altitude of AC: $y - 2 = \frac{1}{12}(x + 1)$

$$12y - 24 = x - 1$$

$$x + 12y = 23$$

$$\alpha = \frac{-47}{19}, \quad \beta = \frac{121}{57}$$

$$9\alpha - 6\beta + 60 = 25$$

Question 66

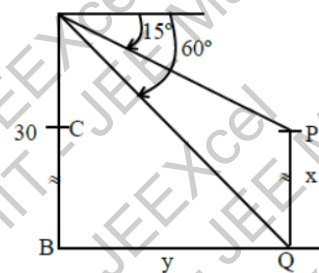
From the top A of a vertical wall AB of height 30m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB such that $CB = PQ$, then the area (in m^2) of the quadrilateral BCPQ is equal to :
[6-Apr-2023 shift 1]

Options:

- A. $200(3 - \sqrt{3})$
- B. $300(\sqrt{3} + 1)$
- C. $300(\sqrt{3} - 1)$
- D. $600(\sqrt{3} - 1)$

Answer: D

Solution:



$$\frac{AB}{BQ} = \tan 60^\circ$$

$$BQ = \frac{30}{\sqrt{3}} = 10\sqrt{3} = y$$

& $\triangle ACP$

$$\frac{AC}{CP} = \tan 15^\circ \Rightarrow \frac{(30-x)}{y} = (2-\sqrt{3})$$

$$30-x = 10\sqrt{3}(2-\sqrt{3})$$

$$30-x = 20\sqrt{3} - 30$$

$$x = 60 - 20\sqrt{3}$$

$$\text{Area} = x \cdot y = 20(3-\sqrt{3}) \cdot 10\sqrt{3}$$

$$= 600(\sqrt{3}-1)$$

Question67

The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30° . If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to
[11-Apr-2023 shift 2]

Options:

A. 10

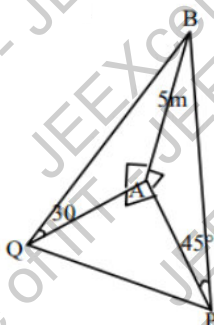
B. $5\sqrt{5}$

C. $\frac{5}{2}\sqrt{5}$

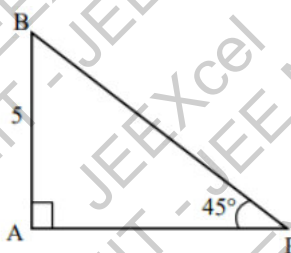
D. 5

Answer: A

Solution:



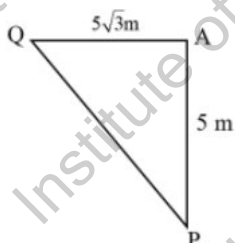
Tower AB = 5m
 $\angle APB = 45^\circ$
 $\angle PAB = 90^\circ$



$$\tan 45^\circ = \frac{AB}{AP}$$

$$1 = \frac{AB}{AP}$$

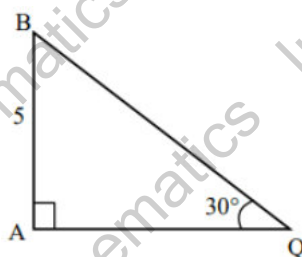
$$AP = 5\text{m}$$



$$\tan 30^\circ = \frac{AP}{AQ}$$

$$\frac{1}{\sqrt{3}} = \frac{5}{AQ}$$

$$AQ = 5\sqrt{3}$$



$$AP^2 + AQ^2 = PQ^2$$

$$PQ^2 = 5^2 + (5\sqrt{3})^2$$

$$PQ^2 = 25 + 75 = 100$$

$$PQ = 10\text{ cm}$$

Option (A) 10 cm correct.

Question68

Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, $a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3 \cos \theta, a \sin \theta)$ is a point in the fourth quadrant such that the maximum area of $\triangle ACD$ is 12 square units, then a is equal to _____
 [24-Jun-2022-Shift-1]

Answer: 8

Solution:

Clearly B is $\left(-\frac{3}{\sqrt{a}}, +\sqrt{a}\right)$ and C is $\left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

$$\text{Area of } \triangle ACD = \frac{1}{2} \begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = |3\sqrt{a} \sin \theta + 3\sqrt{a} \cos \theta| = 3\sqrt{a} |\sin \theta + \cos \theta|$$

$$\Rightarrow \Delta_{\max} = 3\sqrt{a} \cdot \sqrt{2} = 12 \Rightarrow a = (2\sqrt{2})^2 = 8$$

Question 69

Let the area of the triangle with vertices $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ be 4 sq. units. If the points $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear, then β is equal to
[24-Jun-2022-Shift-2]

Options:

- A. 64
- B. -8
- C. -64
- D. 512

Answer: C

Solution:

$\therefore A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ are the vertices of $\triangle ABC$ and area of $\triangle ABC = 4$

$$\therefore \left| \frac{1}{2} \begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 0 & 1 \\ 0 & \alpha & 1 \end{vmatrix} \right| = 4$$

$$\Rightarrow |1(1 - \alpha) - \alpha(\alpha)| = 8$$

$$\Rightarrow \alpha = \pm 8$$

Now, $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear

$$\therefore \begin{vmatrix} 8 & -8 & 1 \\ -8 & 8 & 1 \\ 64 & \beta & 1 \end{vmatrix} = 0 = \begin{vmatrix} -8 & 8 & 1 \\ 8 & -8 & 1 \\ 64 & \beta & 1 \end{vmatrix}$$

$$\Rightarrow 8(8 - \beta) + 8(-8 - 64) + 1(-8\beta - 8 \times 64) = 0$$

$$\Rightarrow 8 - \beta - 72 - \beta - 64 = 0$$

$$\Rightarrow \beta = -64$$

Question 70

Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle. Then the area of $\triangle PQR$ is :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{25}{4\sqrt{3}}$

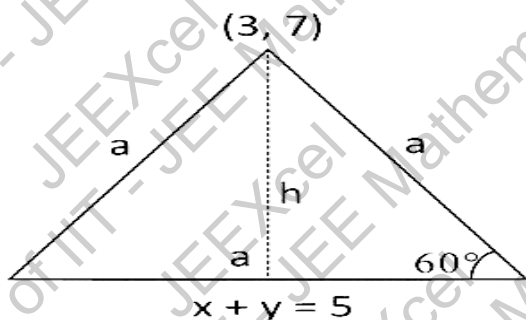
B. $\frac{25\sqrt{3}}{2}$

C. $\frac{25}{\sqrt{3}}$

D. $\frac{25}{2\sqrt{3}}$

Answer: D

Solution:



Let, side of triangle = a .

$$h = \frac{|3 + 7 - 5|}{\sqrt{1^2 + 1^2}} \\ = \frac{5}{\sqrt{2}}$$

From figure, $h = a \sin 60^\circ$

$$\Rightarrow a = \frac{2h}{\sqrt{3}} \\ = \frac{2}{\sqrt{3}} \times \frac{5}{\sqrt{2}} \\ = \frac{10}{\sqrt{6}}$$

$$\therefore \text{Area} = \frac{3}{4} \left(\frac{10}{\sqrt{6}} \right)^2 \\ = \frac{25}{2\sqrt{3}}$$

Question 71

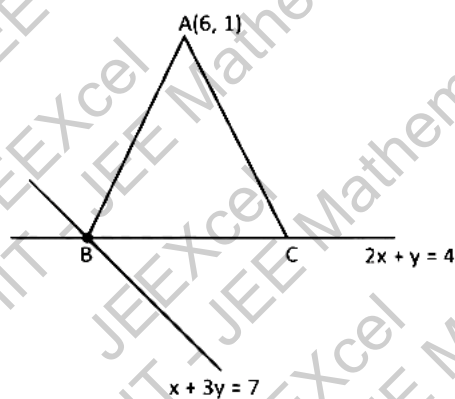
In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of $\triangle ABC$, then $15(\alpha + \beta)$ is equal to:
[27-Jun-2022-Shift-1]

Options:

- A. 39
- B. 41
- C. 51
- D. 63

Answer: C

Solution:



$$\begin{cases} 2x + y = 4 \\ 2x + 6y = 14 \end{cases} \Rightarrow y = 2, x = 3$$

B(1, 2)

Let C(k, 4 - 2k)

Now $AB^2 = AC^2$

$$5^2 + (-1)^2 = (6 - k)^2 + (-3 + 2k)^2$$

$$\Rightarrow 5k^2 - 24k + 19 = 0$$

$$(5k - 19)(k - 1) = 0 \Rightarrow k = \frac{19}{5}$$

$$C\left(\frac{19}{5}, -\frac{18}{5}\right)$$

Centroid (α , β)

$$\alpha = \frac{6 + 1 + \frac{19}{5}}{3} = \frac{18}{5}$$

$$\beta = \frac{1 + 2 - \frac{18}{5}}{3} = -\frac{1}{5}$$

Now $15(\alpha + \beta)$

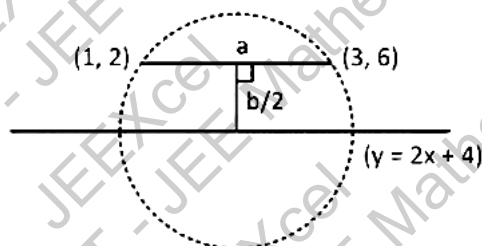
$$15\left(\frac{17}{5}\right) = 51$$

Question 72

A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is [27-Jun-2022-Shift-1]

Answer: 16

Solution:



As slope of line joining (1, 2) and (3, 6) is 2 given diameter is parallel to side

$$\therefore a = \sqrt{(3-1)^2 + (6-2)^2} = \sqrt{20}$$

$$\text{and } b/2 = \frac{4}{\sqrt{5}} \Rightarrow b = \frac{8}{\sqrt{5}}$$

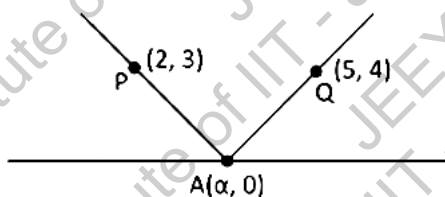
$$\text{Area} = ab = 2\sqrt{5} \cdot \frac{8}{\sqrt{5}} = 16$$

Question 73

A ray of light passing through the point $P(2, 3)$ reflects on the x -axis at point A and the reflected ray passes through the point $Q(5, 4)$. Let R be the point that divides the line segment AQ internally into the ratio $2 : 1$. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β) . Then, the value of $7\alpha + 3\beta$ is equal to
[28-Jun-2022-Shift-1]

Answer: 31

Solution:



$$\frac{4}{5-\alpha} = \frac{3}{\alpha-2} \Rightarrow 4\alpha - 8 = 15 - 3\alpha$$
$$\alpha = \frac{23}{7}$$

$$A = \left(\frac{23}{7}, 0 \right) \quad Q = (5, 4)$$

$$R = \left(\frac{10 + \frac{23}{7}}{3}, \frac{8}{3} \right)$$
$$= \left(\frac{31}{7}, \frac{8}{3} \right)$$

$$\text{Bisector of angle PAQ is } X = \frac{23}{7}$$

$$\Rightarrow M = \left(\frac{23}{7}, \frac{8}{3} \right)$$

$$\text{So, } 7\alpha + 3\beta = 31$$

Question 74

Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3 , which passes through the point $P(2, 3)$, intersects L_2 at A and L_1 at B . If the point P divides the line-segment AB , internally in the ratio $1 : 3$, then the area of the triangle is equal to :
[28-Jun-2022-Shift-2]

Options:

A. $\frac{110}{13}$

B. $\frac{132}{13}$

C. $\frac{142}{13}$

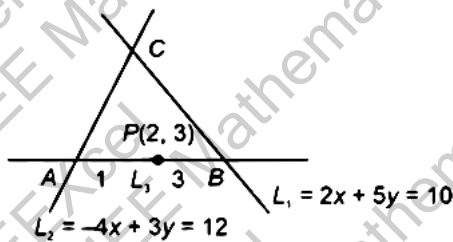
D. $\frac{151}{13}$

Answer: B

Solution:

$$L_1 : 2x + 5y = 10$$

$$L_2 : -4x + 3y = 12$$



Solving L_1 and L_2 we get

$$C \equiv \left(\frac{-15}{13}, \frac{32}{13} \right)$$

Now, Let $A \left(x_1, \frac{1}{3}(12 + 4x_1) \right)$ and

$$B \left(x_2, \frac{1}{5}(10 - 2x_2) \right)$$

$$\therefore \frac{3x_1 + x_2}{4} = 2$$

$$\text{and } \frac{(12 + 4x_1) + \frac{10 - 2x_2}{5}}{4} = 3$$

$$\text{So, } 3x_1 + x_2 = 8 \text{ and } 10x_1 - x_2 = -5$$

$$\text{So, } (x_1, x_2) = \left(\frac{3}{13}, \frac{95}{13} \right)$$

$$A = \left(\frac{3}{13}, \frac{56}{13} \right) \text{ and } B = \left(\frac{95}{13}, \frac{-12}{13} \right)$$

$$= \left| \frac{1}{2} \left(\frac{3}{13} \left(\frac{-44}{13} \right) - \frac{56}{13} \left(\frac{110}{13} \right) + 1 \left(\frac{2860}{169} \right) \right) \right|$$

$$= \frac{132}{13} \text{ sq. units}$$

Question 75

The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and A'B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to:

[29-Jun-2022-Shift-1]

Options:

A. 10

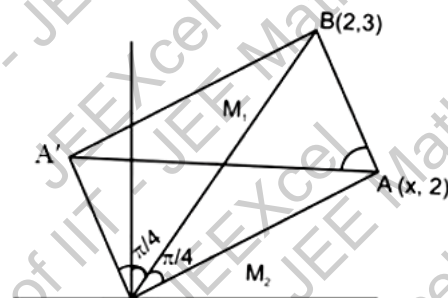
B. $\frac{48}{5}$

C. $\frac{52}{5}$

D. 3

Answer: C

Solution:



$$M_1 = 3/2 \quad M_2 = 2/x$$

$$\tan \pi/4 = \left| \frac{3/2 - 2/x}{1 + 6/2x} \right| = 1$$

$$\Rightarrow x_1 = 10, x_2 = -2/5$$

$$\Rightarrow AA^1 = 52/5$$

Question 76

The distance of the origin from the centroid of the triangle whose two sides have the equations

$x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is :

[29-Jun-2022-Shift-2]

Options:

A. $\sqrt{2}$

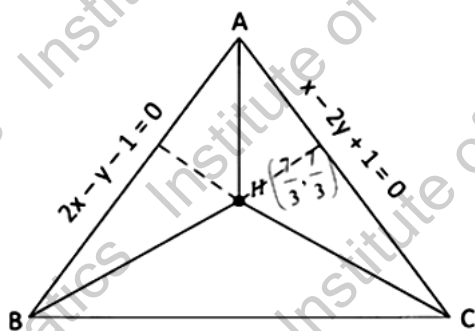
B. 2

C. $2\sqrt{2}$

D. 4

Answer: C

Solution:



For point A,

$$2x - y - 1 = 0$$

$$x - 2y + 1 = 0$$

Solving (1) and (2), we get

$$x = 1, y = 1$$

$$\therefore \text{Point A} = (1, 1)$$

Altitude from B to line AC is perpendicular to line AC.

$\therefore$ Equation of altitude BH is

$$2x + y + \lambda = 0$$

It passes through point H $\left(\frac{7}{3}, \frac{7}{3}\right)$ so it satisfy the equation (3).

$$\frac{14}{3} + \frac{7}{3} + \lambda = 0$$

$$\Rightarrow \lambda = -7$$

$$\therefore \text{Altitude BH} = 2x + y - 7 = 0$$

Solving equation (1) and (4), we get

$$x = 2, y = 3$$

$$\therefore \text{Point B} = (2, 3)$$

Altitude from C to line AB is perpendicular to line AB.

$\therefore$ Equation of altitude CH is

$$x + 2y + \lambda = 0$$

It passes through point H $\left(\frac{7}{3}, \frac{7}{3}\right)$ so it satisfy equation (5).

$$\frac{7}{3} + \frac{14}{3} + \lambda = 0$$

$$\Rightarrow \lambda = -7$$

$$\therefore \text{Altitude CH} = x + 2y - 7 = 0$$

Solving equation (2) and (6), we get

$$x = 3, y = 2$$

$$\therefore \text{Point C} = (3, 2)$$

Centroid G(x, y) of triangle A(1, 1), B(2, 3) and C(3, 2) is

$$x = \frac{1+2+3}{3} = 2, y = \frac{1+2+3}{3} = 2$$

Now d Distance of point G(2, 2) from center O(0, 0) is

$$OG = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

Question 77

Let AB and PQ be two vertical poles, 160m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2 \theta$ is equal to [28-Jun-2022-Shift-1]

Options:

A. $\frac{3-2\sqrt{2}}{2}$

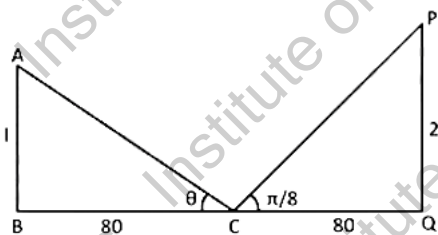
B. $\frac{3+\sqrt{2}}{2}$

C. $\frac{3-2\sqrt{2}}{4}$

D. $\frac{3-\sqrt{2}}{4}$

Answer: C

Solution:



$$\frac{1}{80} = \tan \theta \dots (i)$$

$$\frac{21}{80} = \tan \frac{\pi}{8} \dots (ii)$$

From (i) and (ii)

$$\frac{1}{2} = \frac{\tan \theta}{\tan \frac{\pi}{8}} \Rightarrow \tan^2 \theta = \frac{1}{4} \tan^2 \frac{\pi}{8}$$

$$\Rightarrow \tan^2 \theta = \frac{\sqrt{2}-1}{4(\sqrt{2}+1)} = \frac{3-2\sqrt{2}}{4}$$

Question 78

From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60° . The pole subtends an angle 30° at the top of the tower. Then the height of the tower is [29-Jun-2022-Shift-2]

Options:

A. $15\sqrt{3}$

B. $20\sqrt{3}$

C. $20 + 10\sqrt{3}$

D. 30

Answer: D

Solution:

Here AB is a tower and CD is a pole.

$$\text{In triangle ABC, } \tan 60^\circ = \frac{AB}{AC} = \frac{20+h}{x} \dots\dots (1)$$

$$\text{In triangle BED, } \tan 30^\circ = \frac{h}{x} \dots\dots (2)$$

Divide equation (1) by equation (2), we get

$$\frac{\tan 60^\circ}{\tan 30^\circ} = \frac{20+h}{x} \times \frac{x}{h}$$

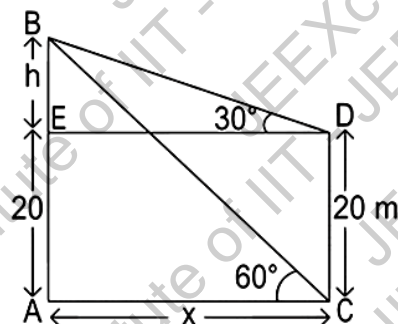
$$\Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{20+h}{h}$$

$$\Rightarrow 3 = \frac{20+h}{h}$$

$$\Rightarrow 3h = 20 + h$$

$$\Rightarrow h = 10\text{m}$$

$$\therefore \text{Height of tower} = 20 + 10 = 30\text{m}$$



Question79

A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line:
[25-Jul-2022-Shift-1]

Options:

A. $2x + y = 9$

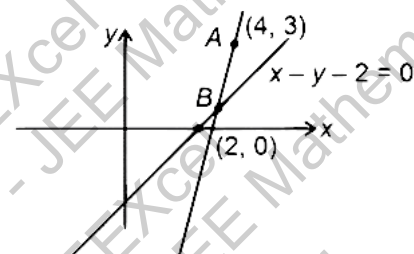
B. $3x - 2y = 7$

C. $x + 2y = 6$

D. $2x - 3y = 3$

Answer: C

Solution:



Let inclination of required line is θ ,

So the coordinates of point B can be assumed as

$$\left(4 - \frac{\sqrt{29}}{3} \cos \theta, 3 - \frac{\sqrt{29}}{3} \sin \theta \right)$$

Which satisfies $x - y - 2 = 0$

$$4 - \frac{\sqrt{29}}{3} \cos \theta - 3 + \frac{\sqrt{29}}{3} \sin \theta - 2 = 0$$

$$\sin \theta - \cos \theta = \frac{3}{\sqrt{29}}$$

By squaring

$$\sin 2\theta = \frac{20}{29} = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$\tan \theta = \frac{5}{2}$ only (because slope is greater than 1)

$$\sin \theta = \frac{5}{\sqrt{29}}, \cos \theta = \frac{2}{\sqrt{29}}$$

Point B : $\left(\frac{10}{3}, \frac{4}{3}\right)$

Which also satisfies $x + 2y = 6$

Question80

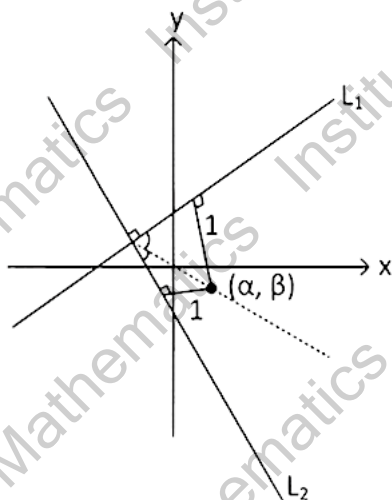
Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2 , then $100(\alpha + \beta)$ is equal to [25-Jul-2022-Shift-2]

Options:

- A. -14
- B. 42
- C. -22
- D. 14

Answer: D

Solution:



$$L_1 : 3x - 4y + 12 = 0$$

$$L_2 : 8x + 6y + 11 = 0$$

Equation of angle bisector of L_1 and L_2 of angle containing origin

$$2(3x - 4y + 12) = 8x + 6y + 11$$

$$2x + 14y - 13 = 0 \dots (i)$$

$$\frac{3\alpha - 4\beta + 12}{5} = 1$$

$$\Rightarrow 3\alpha - 4\beta + 7 = 0 \dots (ii)$$

Solution of $2x + 14y - 13 = 0$ and $3x - 4y + 7 = 0$ gives the required point $P(\alpha, \beta)$, $\alpha = \frac{-23}{25}$, $\beta = \frac{53}{50}$ $100(\alpha + \beta) = 14$

Question81

A point P moves so that the sum of squares of its distances from the points $(1, 2)$ and $(-2, 1)$ is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral ACBD is equal to : [26-Jul-2022-Shift-1]

Options:

A. $\frac{9}{2}$

B. $\frac{3\sqrt{17}}{2}$

C. $\frac{3\sqrt{17}}{4}$

D. 9

Answer: B

Solution:

Let point P : (h, k)

$$(h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 14$$

$$2h^2 + 2k^2 + 2h - 6k - 4 = 0$$

$$\text{Locus of P : } x^2 + y^2 + x - 3y - 2 = 0$$

Intersection with x-axis,

$$x^2 + x - 2 = 0$$

$$\Rightarrow x = -2, 1$$

Intersection with y-axis,

$$y^2 - 3y - 2 = 0$$

$$\Rightarrow y = \frac{3 \pm \sqrt{17}}{2}$$

Area of the quadrilateral ACBD is

$$= \frac{1}{2}(|x_1| + |x_2|)(|y_1| + |y_2|)$$

$$= \frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}$$

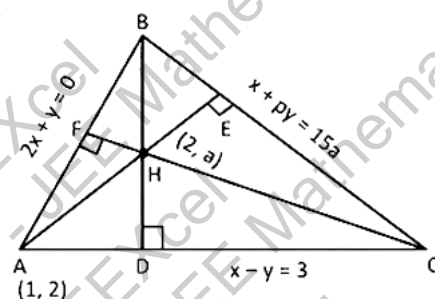
Question 82

The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is $(2, a)$, $-\frac{1}{2} < a < 2$, then p is equal to _____.

[26-Jul-2022-Shift-1]

Answer: 3

Solution:



$$\text{Slope of AH} = \frac{a-2}{1}$$

$$\text{Slope of BC} = -\frac{1}{p}$$

$$\therefore p = a - 2 \dots (i)$$

$$\text{Coordinate of C} = \left(\frac{18p-30}{p+1}, \frac{15p-33}{p+1} \right)$$

$$\begin{aligned}\text{Slope of HC} &= \frac{\frac{15p-33}{p+3} - a}{\frac{18p-33}{p+1} - 2} = \frac{15p-33 - (p-2)(p+1)}{18p-30-2p-2} \\ &= \frac{16p-p^2-31}{16p-32} \\ \therefore \frac{16p-p^2-31}{16p-32} \times -2 &= -1 \\ \therefore p^2 - 8p + 15 &= 0 \\ \therefore p &= 3 \text{ or } 5 \\ \text{But if } p &= 5 \text{ then } a = 3 \text{ not acceptable} \\ \therefore p &= 3\end{aligned}$$

Question83

Let A(1, 1), B(-4, 3), C(-2, -5) be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangles APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is [27-Jul-2022-Shift-1]

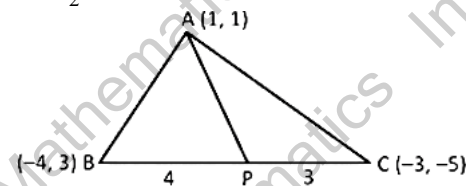
Options:

- A. $\frac{1}{4}$
- B. $\frac{3}{4}$
- C. $\frac{1}{2}$
- D. 1

Answer: C

Solution:

$$\frac{\Delta_1}{\Delta_2} = \frac{\frac{1}{2} \times BP \times AH}{\frac{1}{2} \times BC \times AH} = \frac{4}{7}$$



$$P\left(\frac{-20}{7}, \frac{-11}{7}\right)$$

$$\text{Line AC: } y - 1 = 2(x - 1)$$

$$\text{Intersection with x-axis} = \left(\frac{1}{2}, 0\right)$$

$$\text{Line AP: } y - 1 = \frac{2}{3}(x - 1)$$

$$\text{Intersection with x-axis} = \left(\frac{-1}{2}, 0\right)$$

$$\text{Vertices are } (1, 1), \left(\frac{1}{2}, 0\right) \text{ and } \left(\frac{-1}{2}, 0\right)$$

$$\text{Area} = \frac{1}{2} \text{ sq. unit}$$

Question84

The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and P(2, 3) is its circumcentre. Then which of the following is NOT true? [27-Jul-2022-Shift-2]

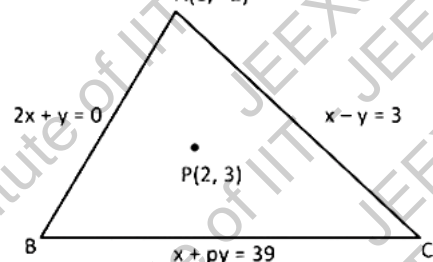
Options:

- A. $(AC)^2 = 9p$
- B. $(AC)^2 + p^2 = 136$
- C. $32 < \text{area}(\triangle ABC) < 36$
- D. $34 < \text{area}(\triangle ABC) < 38$

Answer: D

Solution:

Intersection of $2x + y = 0$ and $x - y = 3$: $A(1, -2)$



Equation of perpendicular bisector of AB is

$$x - 2y = -4$$

Equation of perpendicular bisector of AC is

$$x + y = 5$$

Point B is the image of A in line $x - 2y + 4 = 0$ which is obtained as $B\left(\frac{-13}{5}, \frac{26}{5}\right)$

Similarly vertex C : (7, 4)

Equation of line BC : $x + 8y = 39$

So, $p = 8$

$$AC = \sqrt{(7-1)^2 + (4+2)^2} = 6\sqrt{2}$$

Area of triangle ABC = 32.4

Question85

For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t)$, $B(\cos t, \sin t)$ and $C(a, b)$

such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to:

[28-Jul-2022-Shift-1]

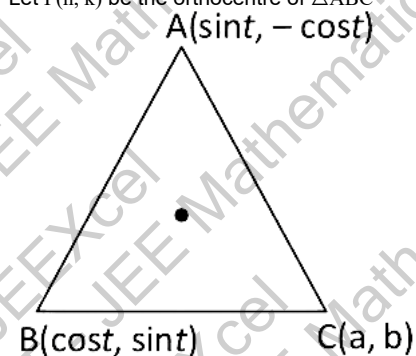
Options:

- A. $\frac{8}{3}$
- B. 8
- C. $\frac{77}{9}$
- D. $\frac{80}{9}$

Answer: B

Solution:

Let $P(h, k)$ be the orthocentre of $\triangle ABC$



Then

$$h = \frac{\sin t + \cos t + a}{3}, k = \frac{-\cos t + \sin t + b}{3}$$

(orthocentre coincide with centroid)

$$\therefore (3h - a)^2 + (3k - b)^2 = 0$$

$$\therefore \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = 0$$

$\therefore$ orthocentre lies on circle with centre $\left(\frac{a}{3}, \frac{b}{3}\right)$

$$\therefore a = 3, b = 1$$

$$\therefore a^2 - b^2 = 8$$

Question 86

Let the circumcentre of a triangle with vertices $A(a, 3)$, $B(b, 5)$ and $C(a, b)$, $ab > 0$ be $P(1, 1)$. If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :

[29-Jul-2022-Shift-1]

Options:

A. 2

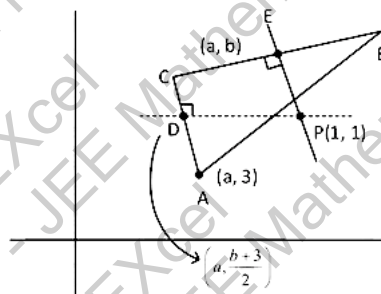
B. $\frac{4}{7}$

C. $\frac{2}{7}$

D. 4

Answer: B

Solution:



Let D be mid-point of AC , then

$$\frac{b+3}{2} = 1 \Rightarrow b = -1$$

Let E be mid-point of BC ,

$$\frac{5-b}{2} = \frac{a+b-1}{2} = -1$$

On putting $b = -1$, we get $a = 5$ or -3

But $a = 5$ is rejected as $ab > 0$

$A(-3, 3)$, $B(-1, 5)$, $C(-3, -1)$, $P(1, 1)$

Line $BC \Rightarrow y = 3x + 8$

$$\text{Line AP} \Rightarrow y = \frac{3-x}{2}$$

$$\text{Point of intersection} \left(\frac{-13}{7}, \frac{17}{7} \right)$$

Question 87

Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that

$a^2 + 11a + 3(m_1^2 + m_2^2) = 220$. If one vertex of the square is $(10(\cos \alpha - \sin \alpha), 10(\sin \alpha + \cos \alpha))$, where

$\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos \alpha - \sin \alpha)x + (\sin \alpha + \cos \alpha)y - 10 = 0$, then

$72(\sin^4 \alpha + \cos^4 \alpha) + a^2 - 3a + 13$ is equal to:

[29-Jul-2022-Shift-2]

Options:

A. 119

B. 128

C. 145

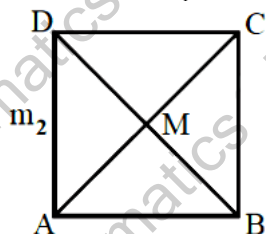
D. 155

Answer: B

Solution:

$$m_1 m_2 = -1$$

$$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$



Eq. of AC

$$AC = (\cos \alpha - \sin \alpha)x + (\sin \alpha + \cos \alpha)y = 10$$

$$BD = (\sin \alpha - \cos \alpha)x + (\sin \alpha - \cos \alpha)y = 0$$

$$(10(\cos \alpha - \sin \alpha), 10(\sin \alpha - \cos \alpha))$$

$$\text{Slope of AC} = \left(\frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \right) = \tan \theta = M$$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan \frac{\pi}{4}}{1 \pm m \tan \frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} + 1}{1 - \left(\frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \right)} = \frac{\frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} - 1}{1 + \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}}$$

$$m_1, m_2 = \tan \alpha, \cot \alpha$$

mid point of AC & BD

$$= M(5(\cos \alpha - \sin \alpha), 5(\cos \alpha + \sin \alpha))$$

$$B(10(\cos \alpha - \sin \alpha), 10(\cos \alpha + \sin \alpha))$$

$$a = AB = \sqrt{2} \text{ BM} = \sqrt{2}(5\sqrt{2}) = 10$$

$$a = 10$$

$$\therefore a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$

$$100 + 110 + 3(\tan^2 \alpha + \cot^2 \alpha) = 220$$

$$\text{Hence } \tan^2 \alpha = 3, \tan^2 \alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3} \text{ or } \frac{\pi}{6}$$

$$\begin{aligned}
 &\text{Now } 72(\sin^4 \alpha + \cos^4 \alpha) + a^2 - 3a + 13 \\
 &= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13 \\
 &= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128
 \end{aligned}$$

Question88

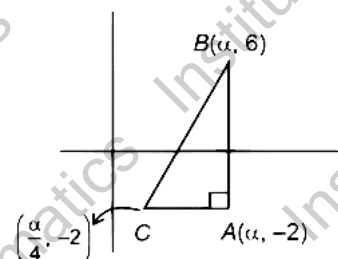
Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a $\triangle ABC$. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of $\triangle ABC$, then which of the following is NOT correct about $\triangle ABC$?
[29-Jul-2022-Shift-2]

Options:

- A. area is 24
- B. perimeter is 25
- C. circumradius is 5
- D. inradius is 2

Answer: B

Solution:



Circumcentre of $\triangle ABC$

$$\begin{aligned}
 &= \left(\frac{\alpha + \frac{\alpha}{4}}{2}, \frac{6 - 2}{2}\right) \\
 &= \left(\frac{5\alpha}{8}, 2\right) \\
 &= \left(5, \frac{\alpha}{4}\right) \\
 &\Rightarrow \alpha = 8
 \end{aligned}$$

$$\text{area}(\triangle ABC) = \frac{1}{2} \cdot \frac{3\alpha}{4} \times 8 = 24 \text{ sq. units}$$

$$\text{Perimeter} = 8 + \frac{3\alpha}{4} + \sqrt{8^2 + \left(\frac{3\alpha}{4}\right)^2} = 8 + 6 + 10 = 24$$

$$\text{Circumradius} = \frac{10}{2} = 5$$

$$r = \frac{\Delta}{s} = \frac{24}{12} = 2$$

Question89

A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that $QR = 15\text{m}$. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :
[25-Jul-2022-Shift-1]

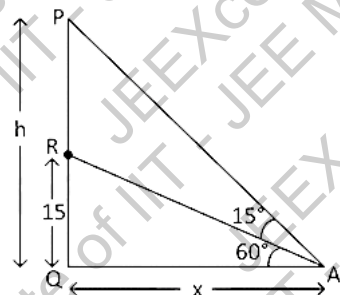
Options:

- A. $5(2\sqrt{3} + 3)\text{m}$

- B. $5(\sqrt{3} + 3)\text{m}$
 C. $10(\sqrt{3} + 1)\text{m}$
 D. $10(2\sqrt{3} + 1)\text{m}$

Answer: A

Solution:



For $\triangle AQR$,
 $\tan 60^\circ = \frac{15}{x} \dots (1)$

From $\triangle AQP$,
 $\tan 75^\circ = \frac{h}{x}$

$$\Rightarrow (2 + \sqrt{3}) = \frac{h}{x} [\because \tan 75^\circ = 2 + \sqrt{3}]$$

$$\Rightarrow h = (2 + \sqrt{3})x$$

$$= (2 + \sqrt{3}) \frac{15}{\sqrt{3}} \text{ [From (1)]}$$

$$= (2 + \sqrt{3}) \times \frac{15\sqrt{3}}{3}$$

$$= (2 + \sqrt{3}) \times 5\sqrt{3}$$

$$= 5(2\sqrt{3} + 3)\text{m}$$

Question90

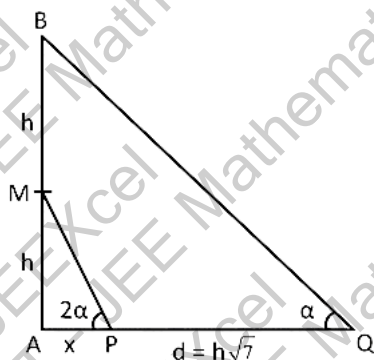
Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α . When from P, he moves a distance d in the direction of $\vec{AP}$, he can see the top B of the tower with an angle of elevation α . If $d = \sqrt{7}h$, then $\tan \alpha$ is equal to
 [27-Jul-2022-Shift-1]

Options:

- A. $\sqrt{5} - 2$
 B. $\sqrt{3} - 1$
 C. $\sqrt{7} - 2$
 D. $\sqrt{7} - \sqrt{3}$

Answer: C

Solution:



$\triangle APM$ gives

$$\tan 2\alpha = \frac{h}{x} \dots (i)$$

$\triangle AQB$ gives

$$\tan 2\alpha = \frac{2h}{x+d} = \frac{2h}{x+h\sqrt{7}} \dots (ii)$$

From (i) and (ii)

$$\tan 2\alpha = \frac{2 \cdot \tan 2\alpha}{1 + \sqrt{7} \cdot \tan 2\alpha}$$

Let $t = \tan \alpha$

$$\Rightarrow t = \frac{2 - \frac{2t}{1-t^2}}{1 + \sqrt{7} \cdot \frac{2t}{1-t^2}}$$

$$\Rightarrow t^2 - 2\sqrt{7}t + 3 = 0$$

$$t = \sqrt{7} - 2$$

Question 91

The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45° . Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60° . If $\angle BAQ = 30^\circ$, $AB = d$ and the area of the trapezium PQRB is α , then the ordered pair (d, α) is :
[27-Jul-2022-Shift-2]

Options:

A. $(10(\sqrt{3} - 1), 25)$

B. $\left(10(\sqrt{3} - 1), \frac{25}{2}\right)$

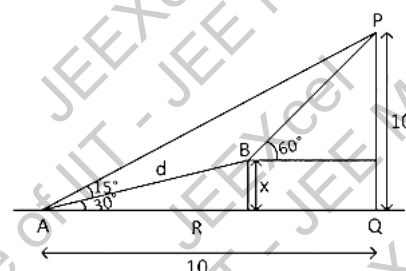
C. $(10(\sqrt{3} + 1), 25)$

D. $\left(10(\sqrt{3} + 1), \frac{25}{2}\right)$

Answer: A

Solution:

Let $BR = x$



$$\frac{x}{d} = \frac{1}{2} \Rightarrow x = \frac{d}{2}$$

$$\frac{10-x}{10-x\sqrt{3}} = \sqrt{3} \Rightarrow 10-x = 10\sqrt{3}-3x$$

$$\begin{aligned}
 2x &= 10(\sqrt{3} - 1) \\
 x &= 5(\sqrt{3} - 1) \\
 d &= 2x = 10(\sqrt{3} - 1) \\
 \alpha &= \frac{1}{2}(x + 10)(10 - x\sqrt{3}) = \text{Area(PQRB)} \\
 &= \frac{1}{2}(5\sqrt{3} - 5 + 10)(10 - 5\sqrt{3}(\sqrt{3} - 1)) \\
 &= \frac{1}{2}(5\sqrt{3} + 5)(10 - 15 + 5\sqrt{3}) - \frac{1}{2}(75 - 25) = 25
 \end{aligned}$$

Question92

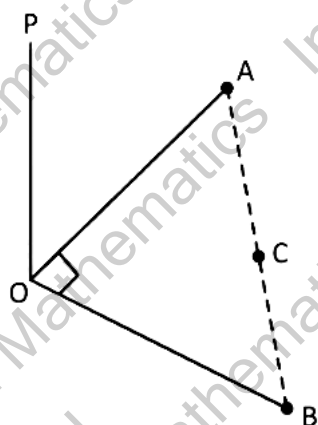
A horizontal park is in the shape of a triangle OAB with $AB = 16$. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15^\circ$ and $\angle PCO = 45^\circ$, where C is the midpoint of AB. Then $(OP)^2$ is equal to
[28-Jul-2022-Shift-2]

Options:

- A. $\frac{32}{\sqrt{3}}(\sqrt{3} - 1)$
- B. $\frac{32}{\sqrt{3}}(2 - \sqrt{3})$
- C. $\frac{16}{\sqrt{3}}(\sqrt{3} - 1)$
- D. $\frac{16}{\sqrt{3}}(2 - \sqrt{3})$

Answer: B

Solution:



$$\begin{aligned}
 OP &= OA \tan 15 = OB \tan 15 \dots\dots (i) \\
 OP &= OC \tan 45 \Rightarrow OP = OC \dots\dots (ii) \\
 OA &= OB \dots\dots (iii) \\
 OC^2 + 8^2 &= OA^2 \\
 OP^2 + 64 &= OP^2 \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \right)^2 \\
 64 &= OP^2 \left[\frac{(\sqrt{3} + 1)^2 - (\sqrt{3} - 1)^2}{(\sqrt{3} - 1)^2} \right] \\
 &= OP^2 \left(\frac{4\sqrt{3}}{(\sqrt{3} - 1)^2} \right) \\
 OP^2 &= \frac{64(\sqrt{3} - 1)^2}{4\sqrt{3}} = \frac{32}{\sqrt{3}}(2 - \sqrt{3})
 \end{aligned}$$

Question93

The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot \alpha$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $\frac{6}{5}$

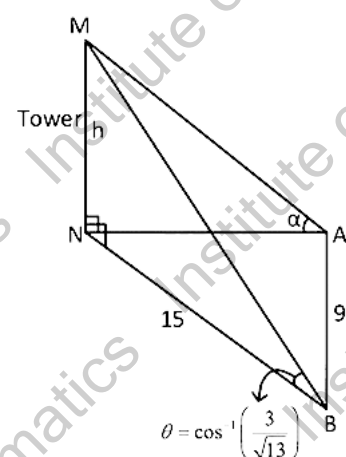
B. $\frac{9}{5}$

C. $\frac{4}{3}$

D. $\frac{7}{3}$

Answer: A

Solution:



$$NA = \sqrt{15^2 - 9^2} = 12$$

$$\frac{h}{15} = \tan \theta = \frac{2}{3}$$

$$h = 10 \text{ units}$$

$$\cot \alpha = \frac{12}{10} = \frac{6}{5}$$

Question94

The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a
[2021, 26 Feb. Shift-1]

Options:

A. right angled triangle

B. equilateral triangle

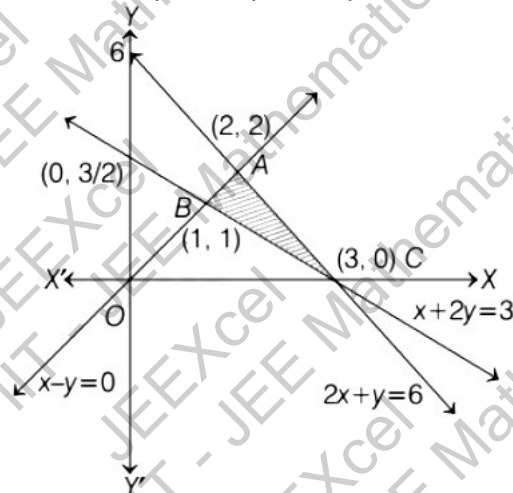
C. isosceles triangle

D. None of these

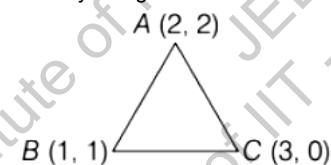
Answer: C

Solution:

Given lines, $x - y = 0$, $x + 2y = 3$, $2x + y = 6$



The only triangle which include all three lines is $\triangle ABC$.



$$\text{Now, } AB = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$$

$$AC = \sqrt{(2-3)^2 + (2-0)^2} = \sqrt{5}$$

$$BC = \sqrt{(3-1)^2 + (0-1)^2} = \sqrt{5}$$

$$\Rightarrow AC = BC \text{ (two sides are equal)}$$

$$\Rightarrow \triangle ABC \text{ is isosceles triangle.}$$

Question 95

If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is
[2021, 25 Feb. Shift-II]

Options:

A. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

B. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

C. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

D. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Answer: A

Solution:

Curve $x^2 + 2y^2 = 2$ intersect the line $x + y = 1$ at points P and Q. First we have to find any common relation between these two curves.

Use substitution for the same as follows,

$$x^2 + 2y^2 = 2 \quad \dots\dots (i)$$

$$x + y = 1, \text{ then } (x + y)^2 = 1^2 \\ \Rightarrow x^2 + y^2 + 2xy = 1 \quad \dots\dots (ii)$$

We can write Eq. (i) as,

$$x^2 + 2y^2 - 2(1)^2 = 0$$

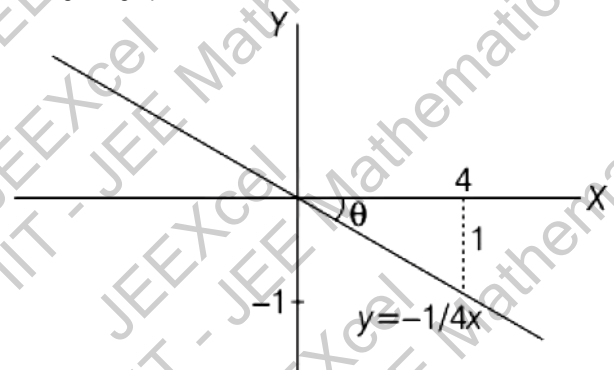
$$\Rightarrow x^2 + 2y^2 - 2(x + y)^2 = 0 \quad [\text{using Eq. (ii) in Eq. (i)}]$$

$$\Rightarrow x^2 + 2y^2 - 2x^2 - 2y^2 - 4xy = 0$$

$$\Rightarrow -x^2 - 4xy = 0 \Rightarrow -x(x + 4y) = 0$$

Given, $x = 0$ and $x + 4y = 0$ or $y = -\frac{1}{4}x$

Draw the line $y = -\frac{1}{4}x$ on graph and take arbitrary point (any one) as follows,
From given graph,



$$\tan \theta = \frac{1}{4} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{4} \right)$$

We have two lines, $y = -\frac{1}{4}x$ and $x = 0$ (i.e. Y-axis).

Thus, any line joining these two curves makes an angle $\frac{\pi}{2} + \theta$ at origin.

$\therefore$ Answer is $\frac{\pi}{2} + \tan^{-1} \left(\frac{1}{4} \right)$.

Question96

The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on
[2021, 25 Feb. Shift-1]

Options:

- A. $(x-2)^2 + (y-2)^2 = 12$
- B. $(x-4)^2 + (y+2)^2 = 16$
- C. $(x-4)^2 + (y-4)^2 = 8$
- D. $(x-2)^2 + (y-4)^2 = 4$

Answer: D

Solution:

Image of P(3, 5) on the line $x - y + 1 = 0$ is

$$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2}$$

$$\Rightarrow \frac{x-3}{1} = \frac{y-5}{-1} = 1$$

$$\Rightarrow \frac{x-3}{1} = 1 \text{ and } \frac{y-5}{-1} = 1$$

$$x = 4, y = 4$$

$\therefore$ Required image is at (4, 4).

Clearly, this point lies on

$$(x-2)^2 + (y-4)^2 = 4 \text{ as}$$

(4, 4) satisfies this equation.

Question97

A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4)

respectively. Then which of these stones is/are on the path of the man?
24 Feb 2021 Shift 1

Options:

- A. A only
- B. C only
- C. All the three
- D. B only

Answer: D

Solution:

Let the line be $y = mx + c$

$\therefore$ x-intercept $:- \frac{c}{m}$

y-intercept $: c$

A.M. of reciprocals of the intercepts :

$$\frac{\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1 - m) = c$$

Line : $y = mx + 2(1 - m) = c$

$\Rightarrow (y - 2) - m(x - 2) = 0$

$\Rightarrow$ line always passes through (2, 2)

Question 98

Let A(-1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$ intersects lines AC and BC at point P and Q_1 respectively. Let A_1 and A_2 be the areas of $\triangle ABC$ and $\triangle PQC_1$ respectively, such that $A_1 = 3A_2$, then the value of m is equal to
[2021, 16 March Shift-II]

Options:

- A. $\frac{4}{15}$
- B. 1
- C. 2
- D. 3

Answer: B

Solution:

Given, points A(-1, 1), B(3, 4), C(2, 0)

$$\text{Equation of AC} = \frac{y-1}{x+1} = \frac{0-1}{2+1} = -\frac{1}{3}$$

$$\Rightarrow 3y - 3 = -x - 1 \Rightarrow x + 3y = 2 \quad \dots\dots (i)$$

On solving Eq. (i) and $y = mx$, we get

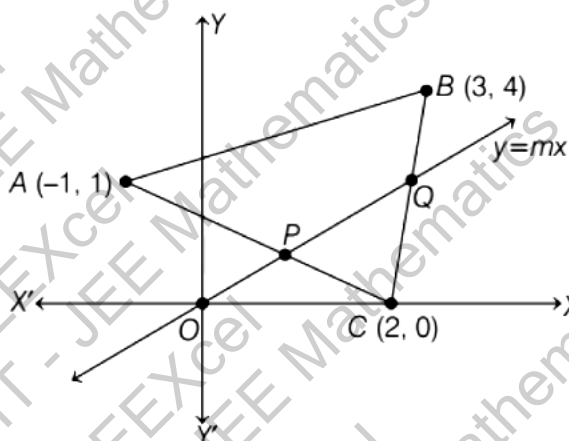
$$P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$$

$$\text{Equation of BC} = \frac{y-0}{x-2} = \frac{4-0}{3-2}$$

$$\Rightarrow y = 4x - 8 \quad \dots\dots (ii)$$

Similarly, on solving Eq. (ii) and $y = mx$,

$$\text{we get } Q\left(\frac{8}{4-m}, \frac{8m}{4-m}\right)$$



Area of $\triangle ABC = 3$ Area of $\triangle PQC$ (given)

$$\frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 3 \times \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{8}{4-m} & \frac{8m}{2} & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \end{vmatrix}$$

$$\Rightarrow 13 = 3 \left(\frac{1}{4-m} \right) \left(\frac{1}{3m+1} \right) \begin{vmatrix} 2 & 0 & 1 \\ 8 & 8m & 4-m \\ 2 & 2m & 3m+1 \end{vmatrix}$$

$$\Rightarrow 13 = \frac{3}{4+11m-3m^2} \times (52m^2)$$

$$\Rightarrow 15m^2 - 11m - 4 = 0$$

$$\Rightarrow m = 1, -\frac{4}{15} \text{ [but } m > 0 \text{]}$$

$$\Rightarrow m = 1$$

Question 99

In a $\triangle PQR$, the coordinates of the points P and Q are $(-2, 4)$ and $(4, -2)$, respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the $\triangle PQR$ is [2021, 17 March Shift-1]

Options:

A. $(-1, 0)$

B. $(-2, -2)$

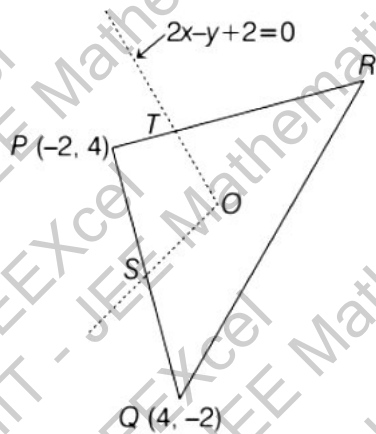
C. $(0, 2)$

D. $(1, 4)$

Answer: B

Solution:

Let O be the centre of the circumcircle.



And T be the mid-point of PR .

So, equation of OT is given as

$$2x - y + 2 = 0$$

Let S be the mid-point of PQ .

Now, S will be

$$\left(\frac{-2+4}{2}, \frac{4-2}{2} \right) = (1, 1)$$

$$\text{Equation of OS} = \frac{y-1}{x-1} = \frac{-1}{m_{PO}}$$

$$m_{PO} = \frac{-2-4}{4+2} = -1$$

$$\therefore OS = y - 1 = 1(x - 1)$$

$$y = x$$

Now, coordinates of O will be the intersection of lines OS and OT.

$$\begin{cases} y = x \\ 2x - y + 2 = 0 \end{cases}$$

$$\Rightarrow 2x - x + 2 = 0 \Rightarrow x = -2$$

$$\therefore y = -2 \Rightarrow O = (-2, -2)$$

Question 100

The number of integral values of m , so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is
[2021, 18 March Shift-1]

Options:

- A. 1
- B. 2
- C. 3
- D. 0

Answer: B

Solution:

Given, $y = mx + 1$

and $3x + 4y = 9$

From Eqs. (i) and (ii),

$$3x + 4(mx + 1) = 9$$

$$\Rightarrow 3x + 4mx + 4 = 9$$

$$\Rightarrow x(3 + 4m) = 5$$

$$\Rightarrow x = \frac{5}{3 + 4m}$$

Given, that the abscissa of point of intersection of Eqs. (i) and (ii) i.e. $x = \frac{5}{3 + 4m}$ is an integer.

$\therefore$ Possible values of x are

$$x = 1, -1, 5, -5$$

$$\text{i.e. } \frac{5}{4m+3} = 1 \text{ or } \frac{5}{4m+3} = -1$$

$$\text{or } \frac{5}{4m+3} = 5 \text{ or } \frac{5}{4m+3} = -5$$

$$\Rightarrow 4m = 2 \text{ or } -4m = 8$$

$$\text{or } 4m = -2 \text{ or } -4m = 4$$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

$$\therefore \left\{ -\frac{1}{2}, \frac{1}{2} \right\} \notin 1$$

$$\therefore m = \{-1, -2\} \in 1$$

$\therefore$ Number of integral values of m are 2.

Question 101

The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}(\sqrt{2})$ with the straight line, $y + 1 = 3\sqrt{2}x$ is
[2021, 18 March Shift-1]

Options:

A. $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$

B. $5\sqrt{2}x + 4y - (15 + 4\sqrt{2}) = 0$

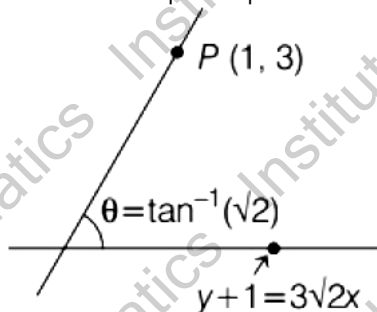
C. $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

D. $4\sqrt{2}x - 5y - (5 + 4\sqrt{2}) = 0$

Answer: A

Solution:

Method 1 Let m = Slope of required line



$\therefore$ Equation of required line

$$y - 3 = m(x - 1)$$

Given, equation of line is

$$3\sqrt{2}x - y - 1 = 0$$

Since, angle θ between Eqs. (i) and (ii) is $\tan^{-1}(\sqrt{2})$

$$\text{i.e. } \tan \theta = \sqrt{2}$$

$$\Rightarrow \left| \frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m} \right| = \sqrt{2}$$

($\because$ Slope of Eq. (i) = m and slope of Eq. (ii) = $3\sqrt{2}$)

Squaring on both sides,

$$m^2 - 6\sqrt{2}m + 18 = 2(1 + 18m^2 + 6\sqrt{2}m)$$

$$\Rightarrow 35m^2 + 18\sqrt{2}m - 16 = 0$$

$$\therefore m = \frac{-18\sqrt{2} \pm \sqrt{648 + 2240}}{70}$$

$$= \frac{-18\sqrt{2} \pm 38\sqrt{2}}{70}$$

$$\Rightarrow m = \frac{2\sqrt{2}}{7}, -\frac{4}{5}\sqrt{2}$$

For $m = \frac{2\sqrt{2}}{7}$, equation of required line

will be

$$y - 3 = \frac{2\sqrt{2}}{7}(x - 1)$$

$$\Rightarrow 2\sqrt{2}x - 7y + 21 - 2\sqrt{2} = 0$$

(options are not matching so, neglect this)

$$\text{For } m = -\frac{4\sqrt{2}}{5}, \text{ equation of required line}$$

will be

$$y - 3 = -\frac{4\sqrt{2}}{5}(x - 1)$$

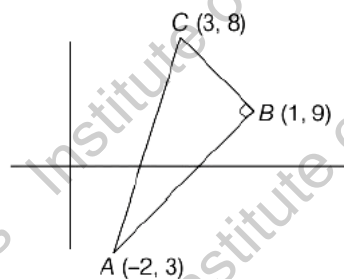
$$\begin{aligned}\Rightarrow 5y - 15 &= -4\sqrt{2}x + 4\sqrt{2} \\ \Rightarrow 4\sqrt{2}x + 5y - 15 - 4\sqrt{2} &= 0 \\ \Rightarrow 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) &= 0\end{aligned}$$

Question 102

Consider a triangle having vertices $A(-2, 3)$, $B(1, 9)$ and $C(3, 8)$. If a line L passing through the circumcentre of $\triangle ABC$, bisects line BC , and intersects Y -axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is
[2021, 20 July Shift-II]

Answer: 9

Solution:



$$AB = \sqrt{(1+2)^2 + (9-3)^2} = \sqrt{45}$$

$$BC = \sqrt{(3-1)^2 + (8-9)^2} = \sqrt{5}$$

$$AC = \sqrt{(3+2)^2 + (8-3)^2} = \sqrt{50}$$

$$\therefore (\sqrt{50})^2 = (\sqrt{45})^2 + (\sqrt{5})^2$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\angle B = 90^\circ$$

$\Rightarrow ABC$ is right angled triangle.

Circumcentre = Mid-point of hypotenuse

= Mid-point of AC

$$= \left(\frac{1}{2}, \frac{11}{2}\right)$$

$$\text{Mid-point of line } BC = \left(2, \frac{17}{2}\right)$$

Line passing through circumcentre and bisect line BC will be

$$y - \frac{11}{2} = \frac{\frac{17}{2} - \frac{11}{2}}{2 - \frac{1}{2}} \left(x - \frac{1}{2}\right)$$

$$\Rightarrow y - \frac{11}{2} = \frac{3 \times 2}{3} \left(x - \frac{1}{2}\right)$$

$$\Rightarrow y - \frac{11}{2} = 2 \left(x - \frac{1}{2}\right)$$

It passes through $\left(0, \frac{\alpha}{2}\right)$.

$$\therefore \frac{\alpha}{2} - \frac{11}{2} = 2 \left(0 - \frac{1}{2}\right) \Rightarrow \alpha - 11 = 4 \left(-\frac{1}{2}\right)$$

$$\Rightarrow \alpha = 11 - 2 = 9$$

$$\Rightarrow \alpha = 9$$

Question 103

Let the equation of the pair of lines, $y = px$ and $y = qx$ can be written as $(y - px)(y - qx) = 0$. Then, the equation of the pair of the angle bisectors of the line $x^2 - 4xy - 5y^2 = 0$ is
[2021, 25 July Shift-II]

Options:

A. $x^2 - 3xy + y^2 = 0$

B. $x^2 + 4xy - y^2 = 0$

C. $x^2 + 3xy - y^2 = 0$

D. $x^2 - 3xy - y^2 = 0$

Answer: C

Solution:

Equation of angle bisector of homogeneous equation of pair of straight line $ax^2 + 2hxy + by^2$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

For $x^2 - 4xy - 5y^2 = 0$

$a = 1, h = -2, b = -5$

So, equation of angle bisector is

$$\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2} \Rightarrow x^2 - y^2 = -3xy \Rightarrow x^2 + 3xy - y^2 = 0$$

So, combined equation of angle bisector is

$x^2 + 3xy - y^2 = 0$. is $x^2 + 3xy - y^2 = 0$.

Question 104

Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point [2021, 27 July Shift-II]

Options:

A. (1,2)

B. (2,2)

C. (2,1)

D. (1,3)

Answer: B

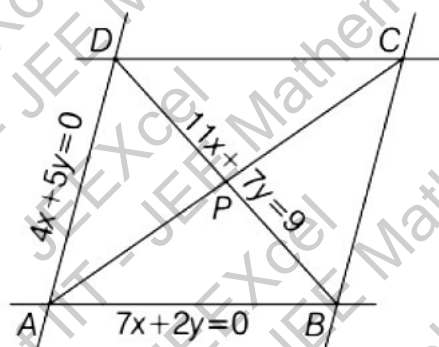
Solution:

Given, two sides of parallelogram are

$4x + 5y = 0$

$7x + 2y = 0$

and $7x + 2y = 0$



Both lines are passing through origin.

Thus, point A = (0, 0)

The equation of diagonal is $11x + 7y = 9$

Point D is the point of intersection of

$4x + 5y = 0$

and $11x + 7y = 9$

So, coordinate of D = $\left(\frac{5}{3}, -\frac{4}{3}\right)$

Also, point B is the point of intersection of $7x + 2y = 0$ and $11x + 7y = 9$

So, coordinate of point B = $\left(-\frac{2}{3}, \frac{7}{3}\right)$

We know that, diagonals of parallelogram bisect each other. Let P is the middle point of BD.

So, coordinate of

$$P = \left(\frac{\frac{5}{3} + \left(-\frac{2}{3}\right)}{2}, \frac{-\frac{4}{3} + \frac{7}{3}}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

Now, equation of diagonal AC

$$y - 0 = \frac{\frac{1}{2} - 0}{\frac{1}{2} - 0}(x - 0)$$

$$\Rightarrow y = \frac{\frac{1}{2}}{\frac{1}{2}}x \Rightarrow y = x$$

$\therefore$ Diagonal AC passes through (2, 2).

Question 105

Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the Y-axis at C.

The locus of the mid-point P of MC is
[2021, 27 Aug. Shift-1]

Options:

A. $3x^2 - 2y - 6 = 0$

B. $3x^2 + 2y - 6 = 0$

C. $2x^2 + 3y - 9 = 0$

D. $2x^2 - 3y + 9 = 0$

Answer: C

Solution:

Given, A(0, 6) and B(2t, 0)

Mid-point of AB = M(t, 3)

Equation of perpendicular bisector of AB passes through M

$$\therefore y - 3 = \frac{t}{3}(x - t) \dots\dots\dots (i)$$

So, C $\left(0, 3 - \frac{t^2}{3}\right)$

Intersection of Eq. (i) on Y-axis

$$C\left(0, 3 - \frac{t^2}{3}\right)$$

Let mid-point of MC is (h, k)

$$\text{Then, } (h, k) = \left(\frac{t}{2}, 3 - \frac{t^2}{6}\right)$$

$$\Rightarrow h = \frac{t}{2}, k = 3 - \frac{t^2}{6}$$

Eliminating t, we get

$$2h^2 = 3(3 - k)$$

Locus of (h, k)

$$2x^2 = 3(3 - y)$$

$$\Rightarrow 2x^2 + 3y - 9 = 0$$

Question 106

Let ABC be a triangle with A(-3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan \theta$ is equal to [2021, 26 Aug. Shift-I]

Options:

- A. $1/2$
- B. $3/4$
- C. $4/3$
- D. 2

Answer: C

Solution:

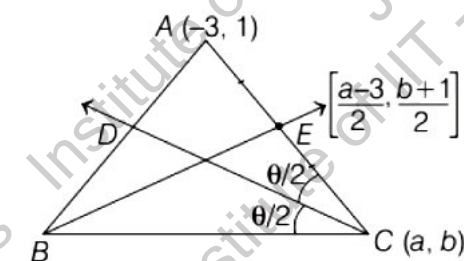
Given, the equation of median through B

i.e. BE : $2x + y - 3 = 0$

Equation, of angle bisector of C i.e.

CD : $7x - 4y = 1$

Since, E satisfies the equation of BE .



$$r_2 \left(\frac{a-3}{2} \right) + \left(\frac{b+1}{2} \right) - 3 = 0$$

$$2a - 6 + b + 1 - 6 = 0$$

$$2a + b = 11 \quad \dots \dots (i)$$

Since, C satisfies C(d)

$$\therefore 7a - 4b = 1 \quad \dots \dots (ii)$$

On solving Eqs. (i) and (ii), we get

$$a = 3, b = 5$$

$$\text{Slope of AC} = \frac{2}{3} \quad \text{Slope of CD} = \frac{7}{4}$$

$$\therefore \tan \left(\frac{\theta}{2} \right) = \left| \frac{\frac{2}{3} - \frac{7}{4}}{1 + \frac{14}{12}} \right| = \frac{1}{2}$$

$$\text{Now, } \tan \theta = \frac{2 \tan \left(\frac{\theta}{2} \right)}{1 - \tan^2 \left(\frac{\theta}{2} \right)} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

Question 107

If p and q are the lengths of the perpendiculars from the origin on the lines, $x \csc \alpha - y \sec \alpha = k \cot 2\alpha$ and $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$ respectively, then k^2 is equal to [2021, 31 Aug. Shift-1]

Options:

- A. $4p^2 + q^2$
- B. $2p^2 + q^2$
- C. $p^2 + 2q^2$
- D. $2p^2 + q^2$

Answer: A

Solution:

$$\begin{aligned}p &= \frac{k \cot 2\alpha}{\sqrt{\operatorname{cosec}^2 \alpha + \sec^2 \alpha}} \\ \Rightarrow q &= \frac{k \sin 2\alpha}{\sqrt{\sin^2 \alpha + \cos^2 \alpha}} \\ \Rightarrow p &= \frac{k \left(\frac{\cos 2\alpha}{\sin 2\alpha} \right)}{\sqrt{\frac{\sin^2 \alpha + \cos^2 \alpha}{\sin^2 \alpha \cos^2 \alpha}}} = \frac{k \cos 2\alpha}{\sin 2\alpha} \\ \Rightarrow p &= \left(\frac{k}{2} \right) \cos 2\alpha \\ \Rightarrow q &= k \sin 2\alpha \\ \Rightarrow \cos 2\alpha &= (2p/k) \\ \Rightarrow \sin 2\alpha &= (q/k) \\ \Rightarrow \sin^2 2\alpha + \cos^2 2\alpha &= 1 \\ \Rightarrow \frac{4p^2}{k^2} + \frac{q^2}{k^2} &= 1 \\ \Rightarrow 4p^2 + q^2 &= k^2\end{aligned}$$

Question 108

Let $A(1, 0)$, $B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC . If P is a point inside the triangle ABC such that the triangles APC , APB and BPC have equal areas, then the length of the line segment PQ , where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is _____.

[NA Jan. 7, 2020 (I)]

Answer: 5

Solution:

P will be centroid of $\triangle ABC$

$$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{(4)^2 + (3)^2} = 5$$

Question 109

The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is:

[Jan. 7, 2020 (II)]

Options:

A. $2x - 3y = 0$

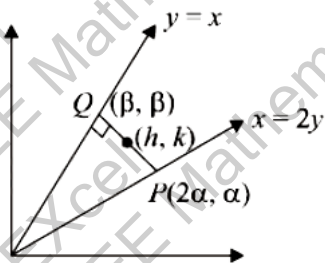
B. $5x - 7y = 0$

C. $3x - 2y = 0$

D. $7x - 5y = 0$

Answer: B

Solution:



Since, slope of PQ = $\frac{k-\alpha}{h-2\alpha} = -1$

$$\Rightarrow k - \alpha = -h + 2\alpha$$

$$\Rightarrow \alpha = \frac{h+k}{3}$$

$$\text{Also, } 2h = 2\alpha + \beta \text{ and}$$

$$2k = \alpha + \beta$$

$$\Rightarrow 2h = \alpha + 2k$$

$$\Rightarrow \alpha = 2h - 2k$$

From (i) and (ii), we have

$$\frac{h+k}{3} = 2(h-k)$$

$$\text{So, locus is } 6x - 6y = x + y$$

$$\Rightarrow 5x = 7y \Rightarrow 5x - 7y = 0$$

Question110

Let C be the centroid of the triangle with vertices (3,-1) (1,3) and (2,4) . Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point:

[Jan. 9, 2020 (I)]

Options:

A. (-9,-6)

B. (9,7)

C. (7,6)

D. (-9,-7)

Answer: A

Solution:

Coordinates of centroids

$$C = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \left(\frac{3+1+2}{3}, \frac{-1+3+4}{3} \right) = (2, 2)$$

The given equation of lines are

$$x + 3y - 1 = 0 \dots (i)$$

$$3x - y + 1 = 0 \dots (ii)$$

Then, from (i) and (ii)

$$\text{point of intersection } P \left(-\frac{1}{5}, \frac{2}{5} \right)$$

equation of line DP

$$8x - 11y + 6 = 0$$

Question111

If a ΔABC has vertices A(-1, 7), B(-7, 1) and C(5, -5), then its orthocentre has coordinates:

[Sep. 03, 2020 (II)]

Options:

A. $\left(-\frac{3}{5}, \frac{3}{5}\right)$

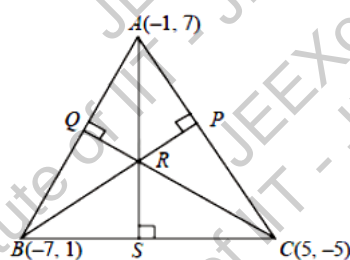
B. $(-3, 3)$

C. $\left(\frac{3}{5}, -\frac{3}{5}\right)$

D. $(3, -3)$

Answer: B

Solution:



$$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$$

$$\therefore \text{Equation of AS is } y - 7 = 2(x + 1)$$

$$y = 2x + 9 \therefore \text{ (i)}$$

$$m_{AC} = \frac{12}{-6} = -2$$

$$\therefore \text{Equation of BP is } y - 1 = \frac{1}{2}(x + 7)$$

$$y = \frac{x}{2} + \frac{9}{2} \therefore \text{ (ii)}$$

From equs. (i) and (ii),

$$2x + 9 = \frac{x + 9}{2}$$

$$\Rightarrow 4x + 18 = x + 9$$

$$\Rightarrow 3x = 9 \Rightarrow x = 3$$

$$\therefore y = 3$$

Question 112

A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^\circ$, and $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is:
[Sep. 04, 2020 (I)]

Options:

A. $1 + \sqrt{5}$

B. $1 + 2\sqrt{5}$

C. $2 + \sqrt{5}$

D. $2\sqrt{5} - 1$

Answer: B

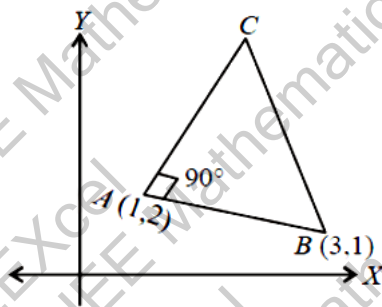
Solution:

Let ΔABC be in the first quadrant

$$\text{Slope of line AB} = -\frac{1}{2}$$

$$\text{Slope of line AC} = 2$$

$$\text{Length of AB} = \sqrt{5}$$



It is given that $\text{ar}(\triangle ABC) = 5\sqrt{5}$

$$\therefore \frac{1}{2} AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$$

$\therefore$ Coordinate of vertex $C = (1 + 10 \cos \theta, 2 + 10 \sin \theta)$

$$\because \tan \theta = 2 \Rightarrow \cos \theta = \frac{1}{\sqrt{5}}, \sin \theta = \frac{2}{\sqrt{5}}$$

$\therefore$ Coordinate of $C = (1 + 2\sqrt{5}, 2 + 4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1 + 2\sqrt{5}$.

Question 113

If the perpendicular bisector of the line segment joining the points $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to -4 then a value of k is :

[Sep. 04, 2020 (II)]

Options:

A. -2

B. -4

C. $\sqrt{14}$

D. $\sqrt{15}$

Answer: B

Solution:

Mid point of line segment PQ be $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Slope of perpendicular line passing through

$$(0, -4) \text{ and } \left(\frac{k+1}{2}, \frac{7}{2}\right) = \frac{\frac{7}{2} + 4}{\frac{k+1}{2} - 0} = \frac{15}{k+1}$$

$$\text{Slope of } PQ = \frac{4-3}{1-k} = \frac{1}{1-k}$$

$$\therefore \frac{15}{1+k} \times \frac{1}{1-k} = -1$$

$$1 - k^2 = -15 \Rightarrow k = \pm 4$$

Question 114

If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible value of α and β is _____.

[NA Sep. 05, 2020 (I)]

Answer: 30

Solution:

$$L_1 : 2x - y + 3 = 0$$

$$L_1 : 4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$$

$$L_1 : 6x - 3y + \beta = 0 \Rightarrow 2x - y + \frac{\beta}{3} = 0$$

Distance between L_1 and L_2

$$\left| \frac{\alpha - 6}{2\sqrt{5}} \right| = \frac{1}{\sqrt{5}} \Rightarrow |\alpha - 6| = 2$$

$$\Rightarrow \alpha = 4, 8$$

Distance between L_1 and L_3 :

$$\left| \frac{\beta - 9}{3\sqrt{5}} \right| = \frac{2}{\sqrt{5}} \Rightarrow |\beta - 9| = 6$$

$$\Rightarrow \beta = 15, 3$$

$$\text{Sum of all values} = 4 + 8 + 15 + 3 = 30$$

Question 115

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2 & x < 0 \\ 0 & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2 & x > 0. \end{cases}$$

The value of λ for which $f''(0)$ exists, is _____.

[NA Sep. 06, 2020 (I)]

Answer: 5

Solution:

$$f'(x) = \begin{cases} 5x^4 \cdot \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x & x < 0 \\ 0 & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x & x > 0. \end{cases}$$

$$f''(x) = \begin{cases} (20x^3 - x) \sin\left(\frac{1}{x}\right) - 8x^2 \cos\left(\frac{1}{x}\right) + 10 & x < 0 \\ 0 & x = 0 \\ (20x^3 - x) \cos\left(\frac{1}{x}\right) + 8x^2 \sin\left(\frac{1}{x}\right) + 2\lambda & x > 0. \end{cases}$$

$$\text{Now, } f''(0^+) = f''(0^-) \Rightarrow 2\lambda = 10 \Rightarrow \lambda = 5$$