



Statistics

Question1

A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}$, $\frac{8a-1}{30}$, $\frac{4a+1}{30}$, b respectively, where $a, b \in \mathbb{R}$.

Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$.

Then $\frac{a}{b}$ is equal to:

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Options:

A.

12

B.

60

C.

30

D.

3

Answer: B

Solution:

X	0	1	2	3
$p(n)$	$\frac{2a+1}{30}$	$\frac{8a-1}{30}$	$\frac{4a+1}{30}$	b

$$\mu = \Sigma x \times p(n) = 0 + \frac{8a-1}{30} + 2 \times \frac{4a+1}{30} + 3b = \frac{16a+90b+1}{30}$$

$$\sigma^2 = \Sigma x^2 p(n) - \mu^2 \Rightarrow \sigma^2 + \mu^2 = \Sigma x^2 p(n) = 2$$

$$\frac{8a-1}{30} + \frac{4(4a+1)}{30} + 9b = 2$$

$$8a + 90b - 19 = 0 \dots\dots\dots (i)$$

$$\Sigma p(n) = 1 \Rightarrow 14a + 1 + 30b = 30$$

$$14a + 30b - 29 = 0 \dots\dots\dots (ii)$$

From (i) and (ii)

$$a = 2 \quad b = \frac{1}{30} = \frac{a}{b} = 60$$

Question2

If the mean deviation about the median of the numbers $k, 2k, 3k, \dots, 1000k$ is 500, then k^2 is equal to :

JEE Main 2026 (Online) 22nd January Evening Shift

Options:

A.

1

B.

16

C.

9

D.

4

Answer: D

Solution:

$$\therefore \text{median} = \frac{1001k}{2} = X_M$$

$$\therefore \text{mean deviation about median} = \frac{\sum |X_i - X_M|}{n}$$

$$= \frac{2 \left(\frac{k}{2} + \frac{3k}{2} + \frac{5k}{2} + \dots + 500 \text{ terms} \right)}{1000} = \frac{2 \cdot \frac{k}{2} (500)^2}{1000} = \frac{500k}{2} = 500 \text{ (given)}$$

$$\therefore k = 2$$

$$\therefore k^2 = 4$$

Question3

Let the mean and variance of 8 numbers $-10, -7, -1, x, y, 9, 2, 16$ be $\frac{7}{2}$ and $\frac{293}{4}$, respectively.

Then the mean of 4 numbers $x, y, x + y + 1, |x - y|$ is :

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Options:

A.

11

B.

9

C.

10

D.

12

Answer: A

Solution:

$$\begin{aligned}\text{Mean} &= \frac{7}{2} \\ \frac{-10 - 7 - 1 + x + y + 16 + 2 + 9}{8} &= \frac{7}{2} \\ \Rightarrow x + y + 9 &= 28 \\ \Rightarrow x + y &= 19 \\ \text{Variance} &= \frac{293}{4} \\ \frac{100 + 49 + 1 + x^2 + y^2 + 256 + 4 + 81}{8} - \frac{49}{4} &= \frac{293}{4} \\ \frac{491 + x^2 + y^2}{8} &= \frac{342}{4} \\ x^2 + y^2 &= 193\end{aligned}$$

$$\begin{aligned}\Rightarrow (x, y) &= (7, 12) \text{ or } (12, 7) \\ \text{Mean} &= \frac{x + y + x + y + 1 + (x - y)}{4} \\ &= \frac{19 + 20 + 5}{4} \\ &= 11\end{aligned}$$

Question4

If the mean and the variance of the data

Class	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is :

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Options:

A.

21

B.

19

C.

18

D.

20

Answer: B

Solution:

Class	f_i	x_i	$f_i \cdot x_i$	$f_i \cdot x_i^2$
4-8	3	6	18	108
8-12	λ	10	10λ	100λ
12-16	4	14	56	784
16-20	7	18	126	2268

Total	$14 + \lambda$	$200 + 10\lambda$	$3160 + 100\lambda$
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$$\sum f_i = 14 + \lambda$$

$$\sum f_i x_i = 18 + 10\lambda + 56 + 126 = 200 + 10\lambda$$

$$\text{Mean } \mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{200 + 10\lambda}{14 + \lambda}$$

$$\sum f_i x_i^2 = 3(6^2) + \lambda(10^2) + 4(14^2) + 7(18^2) = 108 + 100\lambda + 784 + 2268 = 3160 + 100\lambda$$

$$\text{Variance} = \frac{\sum f_i x_i^2}{\sum f_i} - \mu^2 = 19$$

$$19 = \frac{3160 + 100\lambda}{14 + \lambda} - \left(\frac{200 + 10\lambda}{14 + \lambda} \right)^2$$

$$19(14 + \lambda)^2 = (3160 + 100\lambda)(14 + \lambda) - (200 + 10\lambda)^2$$

$$19(196 + 28\lambda + \lambda^2) = 44240 + 4560\lambda + 100\lambda^2 - 40000 - 4000\lambda - 100\lambda^2$$

$$3724 + 532\lambda + 19\lambda^2 = 4240 + 560\lambda$$

$$19\lambda^2 - 28\lambda - 516 = 0$$

Solving for λ using the quadratic formula:

$$\lambda = \frac{28 \pm \sqrt{28^2 - 4(19)(-516)}}{38} = \frac{28 + 200}{38}$$

$$\text{Taking the positive value, } \lambda = \frac{228}{38} = 6$$

Finding the mean μ :

$$\mu = \frac{200 + 10(6)}{14 + 6} = \frac{260}{20} = 13$$

The value of $\lambda + \mu$ is:

$$6 + 13 = 19$$

Question5

The mean and variance of a data of 10 observations are 10 and 2 , respectively. If an observations α in this data is replaced by β , then the mean and variance become 10.1 and 1.99 , respectively. Then $\alpha + \beta$ equals

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Options:

A.

15

B.

10

C.

5

D.

20

Answer: D

Solution:

Given, total observation $n = 10$

Initial mean $\bar{x}_1 = 10$

$$\Rightarrow \frac{\sum_{i=1}^{10} x_i}{10} = 10$$

Sum of 10 observation $\sum_{i=1}^{10} x_i = 10 \times 10 = 100$

Initial variance $\sigma_1^2 = 2$

$$\Rightarrow \frac{\sum_{i=1}^{10} x_i^2}{n} - (\bar{x}_1)^2 = 2$$

$$\Rightarrow \sum_{i=1}^{10} x_i^2 = n [2 + (\bar{x}_1)^2] = 10 [2 + (10)^2] = 1020$$

It is given that if α is replace by β

then, new mean $(\bar{x}_2) = 10.1$

$$\bar{x}_2 = \frac{100 - \alpha + \beta}{10} = 10.1$$

$$\Rightarrow 100 - \alpha + \beta = 101$$

$$\Rightarrow \beta - \alpha = 1 \dots \dots (1)$$

Also new variance $\sigma_2^2 = 1.99$ (given)

$$\Rightarrow \sigma_2^2 = \frac{1020 - \alpha^2 + \beta^2}{10} - (10.1)^2 = 1.99$$

$$\Rightarrow 1020 - \alpha^2 + \beta^2 = 10[1.99 + 102.01] = 10 \times 104 = 1040$$

$$\Rightarrow \beta^2 - \alpha^2 = 20$$

$$\Rightarrow (\beta + \alpha)(\beta - \alpha) = 20$$

$$\Rightarrow (\beta + \alpha)(1) = 20 \text{ (substitute } \beta - \alpha = 1)$$

$$\Rightarrow \alpha + \beta = 20$$

Question 6

Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 19\}$ and for some $a, b \in \mathbb{R}$, $Y = \{ax + b : x \in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is

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Options:

A.

20

B.

100

C.

80

D.

60

Answer: D

Solution:

$$Y = \{ax + b : x \in X\}. \text{ and } X = \{x \in \mathbb{N} : 1 \leq x \leq 19\}$$

mean for Y is 30.

$$\therefore (\bar{Y} = 30)$$

$$\bar{Y} = \frac{\sum_{i=1}^{19} Y_i}{19}$$

$$\Rightarrow \frac{\sum_{i=1}^{19} ax_i + b}{19} = 30$$

$$\Rightarrow \sum_{i=1}^{19} ax_i + \sum_{i=1}^{19} b = 19 \times 30$$

$$\Rightarrow a(1 + 2 + \dots + 19) + 19b = 19 \times 30$$

$$\Rightarrow a \times \frac{19 \times 20}{2} + 19b = 570$$

$$\Rightarrow 10a + b = 30 \Rightarrow a = \frac{30-b}{10} \dots\dots(1)$$

Variance for Y is 750

$$\sigma^2 = \frac{\sum_{i=1}^{19} Y_i^2}{19} - (\bar{Y})^2 = 750$$

$$\Rightarrow \frac{\sum_{i=1}^{19} (ax_i + b)^2}{19} - (30)^2 = 750$$

$$\Rightarrow \sum_{i=1}^{19} a^2 x_i^2 + b^2 + 2ax_i = (750 + 900)19$$

$$\Rightarrow a^2 \sum_{i=1}^{19} \left(x_i^2 + b^2 + 2a \sum_{i=1}^{19} x_i \right) = 1650 \times 19$$

$$\Rightarrow a^2 (1^2 + 2^2 + \dots + 19^2) + \sum_{i=1}^{19} b^2 + 2ab(1 + 2 + \dots + 19) = 1650 \times 19$$

$$\Rightarrow a^2 \cdot \frac{19 \times 20 \times 39}{6} + 19b^2 + \frac{2ab \times 19 \times 20}{2} = 1650 \times 19$$

$$\Rightarrow 130a^2 + b^2 + 20ab = 1650 \dots\dots(2)$$

Substitute value of a from (1) in equation (2)

$$130 \left(\frac{30-b}{10} \right)^2 + b^2 + 20b \left(\frac{30-b}{10} \right) = 1650$$

$$\Rightarrow \frac{13}{10} (900 + b^2 - 60b) + b^2 + 60b - 2b^2 = 1650$$

$$\Rightarrow 1170 + 1.3b^2 - 78b + 10b^2 + 600b - 20b^2 = 1650$$

$$\Rightarrow 3b^2 - 180b - 4800 = 0$$

$$\Rightarrow b^2 - 60b - 1600 = 0$$

Let roots of quadratic equation is b_1, b_2

So sum of possible values of b = sum of roots

$$b_1 + b_2 = \frac{-\text{coefficient of } b}{\text{coefficient of } b^2} = 60$$

Question7

The mean and variance of 10 observations are 9 and 34.2 , respectively. If 8 of these observations are 2, 3, 5, 10, 11, 13, 15, 21, then the mean deviation about the median of all the 10 observations is

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Options:

A.

B.

4

C.

5

D.

6

Answer: C

Solution:

Let the missing observations be x and y .

$$\text{Mean } (\bar{x}) = 9 : \frac{2+3+5+10+11+13+15+21+x+y}{10} = 9$$

$$80 + x + y = 90 \Rightarrow x + y = 10$$

$$\text{Variance } (\sigma^2) = 34.2 : \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$
$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+x^2+y^2}{10} - 9^2 = 34.2$$

$$\frac{4+9+25+100+121+169+225+441+x^2+y^2}{10} - 81 = 34.2$$

$$\frac{1094+x^2+y^2}{10} = 115.2$$

$$1094 + x^2 + y^2 = 1152 \Rightarrow x^2 + y^2 = 58$$

Solving $x + y = 10$ and $x^2 + y^2 = 58$

To solve the system $x + y = 10$ and $x^2 + y^2 = 58$ by finding $x - y$ use the formula $x - y = \sqrt{2(x^2 + y^2) - (x + y)^2}$:

$$x - y = \sqrt{2(x^2 + y^2) - (x + y)^2}$$

$$x - y = \sqrt{2(58) - (10)^2} = \sqrt{116 - 100} = \sqrt{16}$$

$$x - y = 4$$

Now add these equations $x + y = 10$ & $x - y = 4$

$$2x = 14 \Rightarrow x = 7$$

Put $x = 7$ in $x + y = 10$

$$7 + y = 10 \Rightarrow y = 3$$

missing numbers are 3 and 7 .

Find the Median

The 10 observations in ascending order are: 2, 3, 3, 5, 7, 10, 11, 13, 15, 21.

- Since $n = 10$, the median is the average of the 5th and 6th terms :

$$\text{Median} = \frac{7+10}{2} = 8.5$$

Calculate Mean Deviation about Median

$$\text{M.D. (median)} = \frac{\sum |x_i - \text{Median}|}{n}$$
$$= \frac{|2 - 8.5| + |3 - 8.5| + |3 - 8.5| + |5 - 8.5| + |7 - 8.5| + |10 - 8.5| + |11 - 8.5| + |13 - 8.5| + |15 - 8.5| + |21 - 8.5|}{10}$$
$$= \frac{6.5 + 5.5 + 5.5 + 3.5 + 1.5 + 1.5 + 2.5 + 4.5 + 6.5 + 12.5}{10} = \frac{50}{10} = 5$$

Question 8

If the mean of the data

Class	5-10	10-15	15-20	20-25	25-30	30-35
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Frequency	2	k	28	54	k+1	5
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is 21, then k is one of the roots of the equation:

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Options:

A.

$$2x^2 - 23x - 10 = 0$$

B.

$$4x^2 - 35x + 24 = 0$$

C.

$$2x^2 - 19x - 10 = 0$$

D.

$$2x^2 - 35x + 98 = 0$$

Answer: C

Solution:

For the grouped data, we first find the class marks of each class interval.

The classes are:

$$5 - 10, 10 - 15, 15 - 20, 20 - 25, 25 - 30, 30 - 35$$

So their class marks are:

$$7.5, 12.5, 17.5, 22.5, 27.5, 32.5$$

The frequencies are:

$$2, k, 28, 54, k+1, 5$$

Now use the formula for mean of grouped data:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Given mean = 21.

Now calculate $\sum f_i$:

$$\sum f_i = 2 + k + 28 + 54 + (k+1) + 5$$

$$\sum f_i = 2k + 90$$

Now calculate $\sum f_i x_i$:

$$\sum f_i x_i = 2(7.5) + k(12.5) + 28(17.5) + 54(22.5) + (k+1)(27.5) + 5(32.5)$$

Now simplify step by step:

$$2(7.5) = 15$$

$$28(17.5) = 490$$

$$54(22.5) = 1215$$

$$5(32.5) = 162.5$$

$$(k+1)(27.5) = 27.5k + 27.5$$

So,

$$\sum f_i x_i = 15 + 12.5k + 490 + 1215 + 27.5k + 27.5 + 162.5$$

$$\sum f_i x_i = 40k + 1910$$

Now apply the mean formula:

$$21 = \frac{40k + 1910}{2k + 90}$$

Cross-multiply:

$$21(2k + 90) = 40k + 1910$$

$$42k + 1890 = 40k + 1910$$

$$2k = 20$$

$$k = 10$$

Now we check which equation has $k = 10$ as a root.

Option A:

$$2x^2 - 23x - 10 = 0$$

Put $x = 10$:

$$2(10)^2 - 23(10) - 10 = 200 - 230 - 10 = -40 \neq 0$$

Option B:

$$4x^2 - 35x + 24 = 0$$

Put $x = 10$:

$$4(10)^2 - 35(10) + 24 = 400 - 350 + 24 = 74 \neq 0$$

Option C:

$$2x^2 - 19x - 10 = 0$$

Put $x = 10$:

$$2(10)^2 - 19(10) - 10 = 200 - 190 - 10 = 0$$

So 10 is a root.

Hence, the correct option is:

Option C $2x^2 - 19x - 10 = 0$

Question9

The mean and variance of n observations are 8 and 16, respectively. If the sum of the first $(n - 1)$ observations is 48 and the sum of squares of the first $(n - 1)$ observations is 496, then the value of n is :

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Options:

A.

21

B.

16

C.

13

D.

7

Answer: D

Solution:

$$X_1 + X_2 + \dots + X_{n-1} + X_n = 8n$$

$$48 + x_n = 8n$$

$$\Rightarrow x_n = 8n - 48 \dots (1)$$

$$\Rightarrow 16 = \frac{496 + x_n^2}{n} - (8)^2$$

$$\Rightarrow 80n = 496 + x_n^2$$

$$\Rightarrow x_n^2 = 80n - 496 \dots (2)$$

$$\Rightarrow (8n - 48)^2 = 80n - 496$$

$$\Rightarrow 64(n - 6)^2 = 8(10n - 62)$$

$$\Rightarrow 8(n - 6)^2 = 10n - 62$$

$$\Rightarrow 4(n - 6)^2 = 5n - 31$$

$$\Rightarrow 4(n^2 - 12n + 36) = 5n - 31$$

$$\Rightarrow 4n^2 + 144 - 48n = 5n - 31$$

$$\Rightarrow 4n^2 - 53n + 175 = 0$$

$$\Rightarrow 4n^2 - 28n - 25n + 175 = 0$$

$$\Rightarrow 4n(n - 7) - 25(n - 7) = 0$$

$$n = 7$$

Question 10

Suppose that the mean and median of the non-negative numbers

21, 8, 17, a , 51, 103, b , 13, 67, ($a > b$), are 40 and 21, respectively. If the mean deviation about the median is 26, then $2a$ is equal to :

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Options:

A.

109

B.

117

C.

161

D.

131

Answer: D

Solution:

The given data set has $n = 9$ observations: 21, 8, 17, a , 51, 103, b , 13, 67.

The mean ($\bar{x}$) of these numbers is given as 40.

The formula for the mean is:

$$\text{Mean} = \frac{\text{Sum of all observations}}{n}$$

Substitute the given values into the formula:

$$40 = \frac{21+8+17+a+51+103+b+13+67}{9}$$

Multiply both sides by 9:

$$360 = 280 + a + b$$

$$a + b = 360 - 280$$

$$a + b = 80 \quad \text{--- (Equation 1)}$$

The median (M) of the data is 21. For $n = 9$ (an odd number), the median is the $\left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ observation when aligned in ascending order.

Let's arrange the known numbers in ascending order:

$$8, 13, 17, 21, 51, 67, 103$$

Here, there are exactly three numbers less than 21 (8, 13, 17) and three numbers greater than 21 (51, 67, 103).

For 21 to remain the middle (5th) observation among all 9 numbers, one of the unknown variables (a or b) must be less than or equal to 21, and the other must be greater than or equal to 21.

Since it is given that $a > b$, we can conclude:

$$a \geq 21 \text{ and } b \leq 21$$

The formula for mean deviation about the median is:

$$\text{M.D. (M)} = \frac{\sum_{i=1}^n |x_i - M|}{n}$$

We are given that M.D. (M) = 26.

Substitute the values:

$$26 = \frac{|21-21|+|8-21|+|17-21|+|a-21|+|51-21|+|103-21|+|b-21|+|13-21|+|67-21|}{9}$$

Multiply by 9 and simplify the absolute values for the known numbers:

$$234 = 0 + 13 + 4 + |a - 21| + 30 + 82 + |b - 21| + 8 + 46$$

$$234 = 183 + |a - 21| + |b - 21|$$

$$|a - 21| + |b - 21| = 234 - 183$$

$$|a - 21| + |b - 21| = 51$$

From Step 2, we know that $a \geq 21 \implies |a - 21| = a - 21$ and $b \leq 21 \implies |b - 21| = 21 - b$. Replacing these in the equation:

$$(a - 21) + (21 - b) = 51$$

$$a - b = 51 \quad \text{--- (Equation 2)}$$

Now we have a system of two linear equations:

$$1) a + b = 80$$

$$2) a - b = 51$$

To find $2a$, simply add Equation 1 and Equation 2:

$$(a + b) + (a - b) = 80 + 51$$

$$2a = 131$$

Therefore, the value of $2a$ is 131.

Option D is the correct answer.

Question 11

For 10 observations $x_1, x_2, \dots, x_{10}$, if $\sum_{i=1}^{10} (x_i + 2)^2 = 180$ and $\sum_{i=1}^{10} (x_i - 1)^2 = 90$, then their standard deviation is :

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Options:

A.

2

B.

$\sqrt{3}$

C.

$2\sqrt{2}$

D.

3

Answer: D

Solution:

$$\sum_{i=1}^{10} (x_i + 2)^2 = 180$$

$$\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 4 = 180$$

$$\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i = 180 - 40 \quad \dots (1)$$

$$\text{Also } \sum_{i=1}^{10} (x_i - 1)^2 = 90$$

$$\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 1 = 90$$

$$\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i = 90 - 10 \quad \dots (2)$$

$$\sum_{i=1}^{10} x_i^2 + 4 \sum_{i=1}^{10} x_i = 140$$

$$\sum_{i=1}^{10} x_i^2 - 2 \sum_{i=1}^{10} x_i = 80$$

$$\sum_{i=1}^{10} x_i^2 = 100 \quad \text{and} \quad \sum_{i=1}^{10} x_i = 10$$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{N} - \left(\frac{\sum_{i=1}^{10} x_i}{N} \right)^2$$

$$= \frac{100}{10} - \left(\frac{10}{10} \right)^2$$

$$\sigma^2 = 10 - 1 = 9$$

$$\Rightarrow \sigma = 3$$

Question 12

The mean deviation about the mean for the data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

is equal to:

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Options:

A.

$$\frac{40}{13}$$

B.

$$\frac{42}{13}$$

C.

$$\frac{44}{13}$$

D.

$$\frac{46}{13}$$

Answer: C

Solution:

For a discrete frequency distribution, the mean deviation about the mean is

$$\text{M.D. about mean} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

First, find the mean $\bar{x}$.

Step 1: Calculate $\sum f_i$ and $\sum f_i x_i$

x_i	f_i	$f_i x_i$
5	8	40
7	6	42
9	2	18
10	2	20
12	2	24
15	6	90

Now,

$$\sum f_i = 8 + 6 + 2 + 2 + 2 + 6 = 26$$

and

$$\sum f_i x_i = 40 + 42 + 18 + 20 + 24 + 90 = 234$$

So, the mean is

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{234}{26} = 9$$

Step 2: Calculate $\sum f_i |x_i - \bar{x}|$

Since $\bar{x} = 9$, we find $|x_i - 9|$:

x_i	f_i	$ x_i - 9 $	$f_i x_i - 9 $
5	8	4	32
7	6	2	12
9	2	0	0
10	2	1	2
12	2	3	6
15	6	6	36

Now,

$$\sum f_i|x_i - \bar{x}| = 32 + 12 + 0 + 2 + 6 + 36 = 88$$

Step 3: Find mean deviation about the mean

$$\text{M.D. about mean} = \frac{88}{26} = \frac{44}{13}$$

So, the correct answer is

$$\frac{44}{13}$$

Hence, Option C is correct.

Question 13

A variable X takes values $0, 0, 2, 6, 12, 20, \dots, n(n-1)$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, {}^nC_4, {}^nC_5, \dots, {}^nC_n$, respectively. If the mean of this data is 60, then its median is :

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Options:

A.

56

B.

42

C.

72

D.

90

Answer: A

Solution:

The given variable X takes values

$$0, 0, 2, 6, 12, 20, \dots, n(n-1)$$

with corresponding frequencies

$${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, {}^nC_4, {}^nC_5, \dots, {}^nC_n$$

We notice that the values of X follow the pattern

$$X = r(r-1), \quad r = 0, 1, 2, \dots, n$$

and the corresponding frequency is

$${}^nC_r$$

So this is a discrete frequency distribution with:

- variate value $x_r = r(r-1)$
- frequency $f_r = {}^nC_r$

Step 1: Use the formula for mean

Mean is given by

$$\bar{X} = \frac{\sum f_r x_r}{\sum f_r}$$

So,

$$\bar{X} = \frac{\sum_{r=0}^n {}^nC_r r(r-1)}{\sum_{r=0}^n {}^nC_r}$$

Given mean = 60, so

$$\frac{\sum_{r=0}^n {}^nC_r r(r-1)}{\sum_{r=0}^n {}^nC_r} = 60$$

Now use standard binomial identities:

$$\sum_{r=0}^n {}^nC_r = 2^n$$

and

$$\sum_{r=0}^n r(r-1) {}^nC_r = n(n-1)2^{n-2}$$

Hence,

$$\bar{X} = \frac{n(n-1)2^{n-2}}{2^n} = \frac{n(n-1)}{4}$$

Given this is 60:

$$\frac{n(n-1)}{4} = 60$$

$$n(n-1) = 240$$

$$n^2 - n - 240 = 0$$

$$(n-16)(n+15) = 0$$

So,

$$n = 16$$

Step 2: Write the values and frequencies for $n = 16$

Now total frequency is

$$N = \sum_{r=0}^{16} {}^{16}C_r = 2^{16} = 65536$$

To find the median, we need the value of X whose cumulative frequency just exceeds $\frac{N}{2}$.

$$\frac{N}{2} = 32768$$

Now frequencies are:

$${}^{16}C_0, {}^{16}C_1, {}^{16}C_2, \dots, {}^{16}C_{16}$$

Using symmetry of binomial coefficients:

$${}^{16}C_0 + {}^{16}C_1 + \dots + {}^{16}C_7 = \frac{1}{2}(2^{16} - {}^{16}C_8)$$

Since

$${}^{16}C_8 = 12870$$

we get

$${}^{16}C_0 + {}^{16}C_1 + \dots + {}^{16}C_7 = \frac{65536 - 12870}{2} = 26333$$

Now add ${}^{16}C_8$:

$$26333 + 12870 = 39203$$

So the cumulative frequency crosses 32768 at $r = 8$.

Hence the median corresponds to

$$X = 8(8 - 1) = 56$$

Final Answer

56

So, the correct option is A.

Question 14

A data consists of 20 observations $x_1, x_2, \dots, x_{20}$. If $\sum_{i=1}^{20} (x_i + 5)^2 = 2500$ and $\sum_{i=1}^{20} (x_i - 5)^2 = 100$, then the ratio of mean to standard deviation of this data is :

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A.

2 : 1

B.

3 : 1

C.

3 : 2

D.

4 : 1

Answer: B

Solution:

Let the mean of the 20 observations be $\bar{x}$ and the standard deviation be σ .

We are given:

$$\sum_{i=1}^{20} (x_i + 5)^2 = 2500$$

and

$$\sum_{i=1}^{20} (x_i - 5)^2 = 100$$

We will expand both expressions.

First,

$$\begin{aligned} \sum_{i=1}^{20} (x_i + 5)^2 &= \sum_{i=1}^{20} (x_i^2 + 10x_i + 25) \\ &= \sum x_i^2 + 10 \sum x_i + 20 \cdot 25 \\ &= \sum x_i^2 + 10 \sum x_i + 500 \end{aligned}$$

So,

$$\begin{aligned} \sum x_i^2 + 10 \sum x_i + 500 &= 2500 \\ \sum x_i^2 + 10 \sum x_i &= 2000 \quad \dots (1) \end{aligned}$$

Now expand the second one:

$$\begin{aligned} \sum_{i=1}^{20} (x_i - 5)^2 &= \sum_{i=1}^{20} (x_i^2 - 10x_i + 25) \\ &= \sum x_i^2 - 10 \sum x_i + 500 \end{aligned}$$

So,

$$\sum x_i^2 - 10 \sum x_i + 500 = 100$$

$$\sum x_i^2 - 10 \sum x_i = -400 \quad \dots (2)$$

Now add (1) and (2):

$$2 \sum x_i^2 = 1600$$

$$\sum x_i^2 = 800$$

Now subtract (2) from (1):

$$20 \sum x_i = 2400$$

$$\sum x_i = 120$$

Hence mean:

$$\bar{x} = \frac{\sum x_i}{20} = \frac{120}{20} = 6$$

Now use the formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum x_i^2}{20} - \bar{x}^2}$$

$$= \sqrt{\frac{800}{20} - 6^2}$$

$$= \sqrt{40 - 36}$$

$$= \sqrt{4} = 2$$

Therefore, the ratio of mean to standard deviation is

$$\bar{x} : \sigma = 6 : 2 = 3 : 1$$

So, the correct option is:

3 : 1

Option B

Question 15

Let the mean and the variance of seven observations $2, 4, \alpha, 8, \beta, 12, 14$, $\alpha < \beta$, be 8 and 16 respectively. Then the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$ is :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

$$x^2 - 35x + 306 = 0$$

B.

$$x^2 - 41x + 420 = 0$$

C.

$$x^2 - 45x + 506 = 0$$

D.

$$x^2 - 37x + 342 = 0$$

Answer: B

Solution:

To find the required quadratic equation, we must first find the values of α and β .

Given that there are 7 observations: 2, 4, α , 8, β , 12, 14. Let $n = 7$.

The mean of these observations ($\bar{x}$) is 8.

According to the formula for the mean:

$$\bar{x} = \frac{\sum x_i}{n}$$

Substituting the given values:

$$8 = \frac{2+4+\alpha+8+\beta+12+14}{7}$$

$$56 = 40 + \alpha + \beta$$

$$\alpha + \beta = 16 \quad \text{--- (1)}$$

Next, the variance (σ^2) is given as 16. The formula for variance is:

$$\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

Substituting the known values:

$$16 = \frac{2^2+4^2+\alpha^2+8^2+\beta^2+12^2+14^2}{7} - (8)^2$$

$$16 = \frac{4+16+\alpha^2+64+\beta^2+144+196}{7} - 64$$

$$16 + 64 = \frac{424+\alpha^2+\beta^2}{7}$$

$$80 \times 7 = 424 + \alpha^2 + \beta^2$$

$$560 = 424 + \alpha^2 + \beta^2$$

$$\alpha^2 + \beta^2 = 136 \quad \text{--- (2)}$$

Now, using the algebraic identity $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$, we can find the product $\alpha\beta$:

$$(16)^2 = 136 + 2\alpha\beta$$

$$256 = 136 + 2\alpha\beta$$

$$2\alpha\beta = 120$$

$$\alpha\beta = 60$$

The numbers α and β are the roots of the quadratic equation $t^2 - (\alpha + \beta)t + \alpha\beta = 0$:

$$t^2 - 16t + 60 = 0$$

$$t^2 - 10t - 6t + 60 = 0$$

$$t(t - 10) - 6(t - 10) = 0$$

$$(t - 6)(t - 10) = 0$$

Thus, the values are 6 and 10. Since we are given that $\alpha < \beta$, we have:

$$\alpha = 6$$

$$\beta = 10$$

We are asked to find the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$. Let's evaluate these roots:

$$\text{First root} = 3(6) + 2 = 18 + 2 = 20$$

$$\text{Second root} = 2(10) + 1 = 20 + 1 = 21$$

Now, let's find the sum and the product of the roots for our new quadratic equation:

$$\text{Sum of roots} = 20 + 21 = 41$$

$$\text{Product of roots} = 20 \times 21 = 420$$

The standard form of a quadratic equation is $x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$.

Substituting the values:

$$x^2 - 41x + 420 = 0$$

Hence, the correct option is Option B.

Question 16

A set of four observations has mean 1 and variance 13. Another set of six observations has mean 2 and variance 1. Then, the variance of all these 10 observations is equal to :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

5.96

B.

6.14

C.

6.04

D.

6.24

Answer: C

Solution:

Observation	x_1	x_2	x_3	x_4	n	Observation	y_1	y_2	y_3	y_4	y_5	y_6	n
Set	1	1	1	1	4	Set	2	2	2	2	2	2	6

$$\text{Set 1} \Rightarrow \mu_x = \frac{\sum_{i=1}^4 x_i}{4} = 1 \dots (i)$$

$$\sigma_x^2 = \left[\left(\frac{\sum_{i=1}^4 x_i^2}{4} \right) - \mu_x^2 \right] = 13 \dots (ii)$$

$$\text{Set 2} \Rightarrow \mu_y = \frac{\sum_{j=1}^6 y_j}{6} = 2 \dots (iii)$$

$$\sigma_y^2 = \left[\left(\frac{\sum_{j=1}^6 y_j^2}{6} \right) - \mu_y^2 \right] = 1 \dots (iv)$$

We mathematically extract from equations (i) and (ii) :

$$\sum_{i=1}^4 x_i = 4 \dots (v) \quad \sum_{i=1}^4 x_i^2 = 4 (\mu_x^2 + 13) = 4(1 + 13) = 56 \dots (vi)$$

We mathematically extract from equations (iii) and (iv) :

$$\sum_{j=1}^6 y_j = 12 \dots (vii) \quad \sum_{j=1}^6 y_j^2 = 6 (\mu_y^2 + 1) = 6(4 + 1) = 30 \dots (viii)$$

We have mathematically compiled a summary of all the information in equations (v) through (viii) to compute the mean and the variance for the 10 datapoints in set 1 $\cup$ set 2 :

$$\mu_t = \frac{\sum_{i=1}^4 x_i + \sum_{j=1}^6 y_j}{4+6} = \frac{4+12}{10} = 1.6 \dots (ix)$$

$$\sigma_t^2 = \left[\left(\frac{\sum_{i=1}^4 x_i^2 + \sum_{j=1}^6 y_j^2}{4+6} \right) - \mu_t^2 \right]$$

$$= \frac{56+30}{4+6} - (1.6)^2$$

$$= 8.6 - 2.56 = 6.04$$

Question 17

The variance of the numbers 8, 21, 34, 47, ..., 320 is _____.

JEE Main 2025 (Online) 23rd January Evening Shift

Answer: 8788

Solution:

$$\text{Var}(8, 21, 34, 47, \dots, 320)$$

$$\text{Var}(0, 13, 26, 39, \dots, 312)$$

$$13^2 \cdot \text{Var}(0, 1, 2, \dots, 24)$$

$$13^2 \cdot \text{Var}(1, 2, 3, \dots, 25)$$

$$\text{So, } \sigma^2 = 13^2 \times \left(\frac{25^2 - 1}{12} \right) = 8788$$

Alternate solution

$$8 + (n - 1)13 = 320$$

$$13n = 325$$

$$n = 25$$

$$\text{no. of terms} = 25$$

$$\text{mean} = \frac{\sum x_i}{n} = \frac{8 + 21 + \dots + 320}{25} = \frac{\frac{25}{2}(8 + 320)}{25}$$

$$\text{variance } \sigma^2 = \frac{\sum x_i^2}{n} - (\text{mean})^2$$

$$= \frac{8^2 + 21^2 + \dots + 320^2}{25} - (164)^2$$

$$= 8788$$

Question 18

Marks obtained by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is :

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. 52

B. 44

C. 40

D. 48

Answer: B

Solution:

The median for grouped data is given by:

$$\text{Median} = L + \left(\frac{\frac{n}{2} - CF}{f} \right) \times h$$

where

L is the lower limit (or boundary) of the median class.

CF is the cumulative frequency of all classes preceding the median class.

f is the frequency of the median class.

h is the class width.

n is the total number of students.

Given:

$$\text{Median} = 14$$

Median class interval is $12 - 18$, so $L = 12$ and the class width $h = 18 - 12 = 6$.

Frequency of median class $f = 12$.

Cumulative frequency below the median class $= 18$ (i.e., $CF = 18$).

Plugging these into the formula:

$$14 = 12 + \left(\frac{\frac{n}{2} - 18}{12} \right) \times 6$$

Step 1: Subtract 12 from both sides:

$$2 = \left(\frac{\frac{n}{2} - 18}{12} \right) \times 6$$

Step 2: Simplify the multiplication factor:

$$\left(\frac{6}{12} \right) = \frac{1}{2}$$

So the equation becomes:

$$2 = \frac{1}{2} \left(\frac{n}{2} - 18 \right)$$

Step 3: Multiply both sides by 2 to remove the fraction:

$$4 = \frac{n}{2} - 18$$

Step 4: Solve for $\frac{n}{2}$:

$$\frac{n}{2} = 4 + 18 = 22$$

Step 5: Multiply both sides by 2 to find n :

$$n = 44$$

Thus, the total number of students is 44.

Question19

For a statistical data $x_1, x_2, \dots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$. He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

JEE Main 2025 (Online) 24th January Morning Shift

Options:

- A. 5
- B. 7
- C. 9
- D. 4

Answer: B

Solution:

$$\text{Mean } \bar{x} = 5.5$$

$$= \sum_{i=1}^{10} x_i = 5.5 \times 10 = 55$$

$$= \sum_{i=1}^{10} x_i^2 = 371$$

$$\left(\sum x_i \right)_{\text{new}} = 55 - (4 + 5) + (6 + 8) = 60$$

$$\left(\sum x_i^2 \right)_{\text{new}} = 371 - (4^2 + 5^2) + (6^2 + 8^2) = 430$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10} \right)^2$$

$$\sigma^2 = \frac{430}{10} - \left(\frac{60}{10} \right)^2$$

$$\sigma^2 = 43 - 36$$

$$\sigma^2 = 7$$

Question 20

Let $x_1, x_2, \dots, x_{10}$ be ten observations such that $\sum_{i=1}^{10} (x_i - 2) = 30$, $\sum_{i=1}^{10} (x_i - \beta)^2 = 98$, $\beta > 2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \dots, 2(x_{10} - 1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

100

B.

90

C.

120

D.

110

Answer: A

Solution:

$$\sum_{i=1}^{10} x_i = 50, \therefore \text{mean} = 5$$

$$\text{Variance} = \frac{4}{5} = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10} \right)^2$$

$$\frac{4}{5} = \frac{\sum x_i^2}{10} - 25$$

$$\Rightarrow \sum x_i^2 = 258 \quad \dots (1)$$

$$\text{Now } \sum_{i=1}^{10} (x_i - \beta)^2 = 98$$

$$\sum_{i=1}^{10} (x_i^2 - 2\beta \cdot x_i + \beta^2) = 98$$

$$258 - 2\beta(50) + 10\beta^2 = 98$$

$$(\beta - 8)(\beta - 2) = 0$$

$$\beta = \text{ or } \beta = 2 \quad (\text{as } \beta > 2)$$

$$\therefore \beta = 8 \quad \dots (2)$$

Now as per the question

$$2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \dots, 2(x_{10} - 1) + 4\beta$$

can be simplified to

$$2x_1 + 30, 2x_2 + 30, \dots, 2x_{10} + 30 \text{ using eq. (2)}$$

$$\mu = 2(5) + 30 = 40 \quad \dots (3)$$

$$\sigma^2 = 2^2 \left(\frac{4}{5} \right) = \frac{16}{5}$$

$$\therefore \frac{\beta\mu}{\sigma^2} = \frac{8 \times 40}{16/5} = 100$$

Question21

If the mean and the variance of 6, 4, a , 8, b , 12, 10, 13 are 9 and 9.25 respectively, then $a + b + ab$ is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 103

B. 106

C. 100

D. 105

Answer: A

Solution:

Let's set up two equations from the given mean and variance:

$$\text{Mean} = 9 \Rightarrow \text{total sum} = 8 \cdot 9 = 72$$

Known values sum to $6+4+8+12+10+13 = 53$, so

$$a + b = 72 - 53 = 19.$$

$$\text{Population variance} = 9.25 \Rightarrow$$

$$\frac{\sum x_i^2}{8} - 9^2 = 9.25 \implies \sum x_i^2 = 8(81 + 9.25) = 722.$$

Known squares sum to $6^2+4^2+8^2+12^2+10^2+13^2 = 529$, so

$$a^2 + b^2 = 722 - 529 = 193.$$

Now use

$$(a + b)^2 = a^2 + 2ab + b^2 \implies 19^2 = 193 + 2ab$$

so

$$361 = 193 + 2ab \implies ab = 84.$$

Finally,

$$a + b + ab = 19 + 84 = 103.$$

Answer: 103 (Option A).

Question 22

Let the Mean and Variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7$ and $x_5 = b, a > b$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n, n = 1, 2, \dots, 5$ is

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. 17

B. 16

C. 16.4

D. 17.4

Answer: B

Solution:

First, find the values of a and b using the given conditions:

Mean Condition:

$$5 = \frac{1+3+a+7+b}{5}$$

This implies:

$$a + b = 14$$

Variance Condition:

$$\frac{1+9+a^2+49+b^2}{5} - (5)^2 = 10$$

Simplifying, we get:

$$a^2 + b^2 = 116$$

Using $a + b = 14$, we have:

$$b = 14 - a$$

Substitute b into the equation for $a^2 + b^2$:

$$a^2 + (14 - a)^2 = 116$$

Expand and simplify:

$$a^2 + 196 - 28a + a^2 = 116$$

$$2a^2 - 28a + 80 = 0$$

Simplify further:

$$a^2 - 14a + 40 = 0$$

Solve the quadratic equation:

$$(a - 10)(a - 4) = 0$$

Thus, $a = 10$ and $b = 4$ (since $a > b$).

New Observations:

The transformed observations are:

$$1 + 1 = 2, \quad 3 + 2 = 5, \quad 10 + 3 = 13, \quad 7 + 4 = 11, \quad 4 + 5 = 9$$

Calculate the Variance of the New Observations:

First, find the mean of the new data:

$$\text{Mean} = \frac{2+5+13+11+9}{5} = 8$$

Calculate the variance:

$$\begin{aligned}\text{Variance} &= \frac{(2-8)^2 + (5-8)^2 + (13-8)^2 + (11-8)^2 + (9-8)^2}{5} \\ &= \frac{36+9+25+9+1}{5} \\ &= \frac{80}{5} = 16\end{aligned}$$

Therefore, the variance of the transformed observations is 16.

Question23

Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a , b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

- A. $\frac{1}{2}$
- B. $\frac{3}{4}$
- C. 1
- D. 2

Answer: C

Solution:

$$\begin{aligned}\frac{2+3+3+4+5+7+a+b}{8} &= 4 \\ \Rightarrow a+b &= 8 \\ (\sqrt{2})^2 &= \frac{2^2+3^2+3^2+4^2+5^2+7^2+a^2+b^2}{8} - 16 \\ 112+a^2+b^2 &= 18 \times 8 \\ \Rightarrow a^2+b^2 &= 32 \\ \Rightarrow a=b &= 4 \\ \text{Now numbers be} & \\ 2, 3, 3, 4, 4, 4, 5, 7 &\end{aligned}$$

Mode = 4

Mean deviation about mode :

$$\begin{aligned}&\frac{|2-4|+|3-4|+|3-4|+|4-4|+|4-4|+|4-4|+|5-4|+|7-4|}{8} \\ &= \frac{2+1+1+1+3}{8} = \frac{8}{8} = 1\end{aligned}$$

Question24

The mean and standard deviation of 100 observations are 40 and 5.1 , respectively. By mistake one observation is taken as 50 instead of 40 . If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu + \sigma)$ is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

- A. 447
- B. 445
- C. 449
- D. 451

Answer: C

Solution:

Let the observations be $x_1, x_2, \dots, x_{99}, 50$

$$\text{Mean} = \frac{x_1 + x_2 + \dots + x_9 + 50}{100} = 40$$

$$\Rightarrow x_1 + x_2 + \dots + x_{99} = 4000 - 50$$

$$\Rightarrow x_1 + x_2 + \dots + x_{99} = 3950$$

$$\text{Current Mean} = \frac{3950 + 40}{100}$$

$$\mu = \frac{399}{10} = 39.9$$

$$(\text{S.D})^2 = \sum_{i=1}^{99} \frac{(x_i)^2 + 2500}{100} - (40)^2$$

$$\sum_{i=1}^{99} x_i^2 = 160101$$

$$(\text{Correct S.D})^2 = \frac{160101 + 1600}{100} - \left(\frac{399}{10}\right)^2$$

$$\sigma = 5$$

$$10(\mu + \sigma) = 10(39.9 + 5) = 449$$

Question 25

Let $a_1, a_2, \dots, a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k < j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \dots, a_{10}$ is equal to :

[27-Jan-2024 Shift 1]

Options:

- A. 5
- B. $\sqrt{5}$

C.

10

D.

$\sqrt{115}$

Answer: B

Solution:

$$\sum_{k=1}^{10} a_k = 50$$

$$a_1 + a_2 + \dots + a_{10} = 50 \dots\dots\dots(i)$$

$$\sum_{\forall k < j} a_k a_j = 1100 \dots\dots\dots(ii)$$

$$\text{If } a_1 + a_2 + \dots + a_{10} = 50 .$$

$$(a_1 + a_2 + \dots + a_{10})^2 = 2500$$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2 \sum_{k < j} a_k a_j = 2500$$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$$

$$\sum_{i=1}^{10} a_i^2 = 300, \text{ Standard deviation ' } \sigma \text{ '}$$

$$= \sqrt{\frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$$

$$= \sqrt{30 - 25} = \sqrt{5}$$

Question26

The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12 . If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to ____

[27-Jan-2024 Shift 2]

Answer: 2521

Solution:

Let the incorrect mean be μ' and standard deviation be σ'

We have

$$\mu' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$$

As per given information correct $\Sigma x_i = 180 - 10 + 12$

$$\Rightarrow \mu(\text{correct mean}) = \frac{182}{15}$$

Also

$$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$$

Correct $\Sigma x_i^2 = 2295 - 100 + 144 = 2339$

$$\sigma^2(\text{correct variance}) = \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}$$

Required value

$$= 15(\mu + \mu^2 + \sigma^2)$$

$$= 15 \left(\frac{182}{15} + \frac{182 \times 182}{15 \times 15} + \frac{2339}{15} - \frac{182 \times 182}{15 \times 15} \right)$$

$$= 15 \left(\frac{182}{15} + \frac{2339}{15} \right)$$

$$= 2521$$

Question 27

If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α , β , 60 where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

[29-Jan-2024 Shift 1]

Answer: 6344

Solution:

$$\bar{x} = 56$$

$$\sigma^2 = 66.2$$

$$\Rightarrow \frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$$

$$\therefore \alpha^2 + \beta^2 = 6344$$

Question 28

If the mean and variance of five observations are $24/5$ and $194/25$ respectively and the mean of first four observations is $7/2$, then the variance of the first four observations is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$4/5$$

B.

$$77/12$$

C.

$$5/4$$

D.

$$105/4$$

Answer: C

Solution:

$$\bar{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$$

Let first four observation be x_1, x_2, x_3, x_4

$$\text{Here, } \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5} \dots (1)$$

$$\text{Also, } \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14$$

Now from eqn -1

$$x_5 = 10$$

$$\text{Now, } \sigma^2 = \frac{194}{25}$$

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$$

Now, variance of first 4 observations

$$\begin{aligned} \text{Var} &= \frac{\sum_{i=1}^4 x_i^2}{4} - \left(\frac{\sum_{i=1}^4 x_i}{4} \right)^2 \\ &= \frac{54}{4} - \frac{49}{4} = \frac{5}{4} \end{aligned}$$

Question 29

Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then $20M$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

416

B.

104

C.

52

D.

208

Answer: D

Solution:

Class	Frequency	Cumulativefrequency
0-4	3	3
4-8	9	12
8-12	10	22
12-16	8	30
16-20	6	36

$$M = 1 + \left(\frac{\frac{N}{2} - C}{f} \right) h$$

$$M = 8 + \frac{18 - 12}{10} \times 4$$

$$M = 10.4$$

$$20M = 208$$

Question30

The variance σ^2 of the data

xi	0	1	5	6	10	12	17
fi	3	2	3	2	6	3	3

Is

[30-Jan-2024 Shift 2]

Answer: 29

Solution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_i x_i^2 = 2048$

$$\therefore \Sigma f_i x_i = 176$$

$$\text{So } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{176}{22} = 8$$

$$\text{for } \sigma^2 = \frac{1}{N} \Sigma f_i x_i^2 - (\bar{x})^2$$

$$= \frac{1}{22} \times 2048 - (8)^2$$

$$= 93.090964$$

$$= 29.0909$$

Question 31

Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if a > b, then a + 3b is

[31-Jan-2024 Shift 2]

Options:

A.

200

B.

190

C.

180

D.

210

Answer: C

Solution:

a, b, 68, 44, 48, 60

$$\text{Mean} = 55 \quad a > b$$

$$\text{Variance} = 194 \quad a + 3b$$

$$\frac{a + b + 68 + 44 + 48 + 60}{6} = 55$$

$$\Rightarrow 220 + a + b = 330$$

$$\therefore a + b = 110 \dots (1)$$

Also,

$$\sum \frac{(x_i - \bar{x})^2}{n} = 194$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 + (68 - 55)^2 + (44 - 55)^2$$

$$+ (48 - 55)^2 + (60 - 55)^2 = 194 \times 6$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 + 169 + 121 + 49 + 25 = 1164$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 = 1164 - 364 = 800$$

$$a^2 + 3025 - 110a + b^2 + 3025 - 110b = 800$$

$$\Rightarrow a^2 + b^2 = 800 - 6050 + 12100$$

$$a^2 + b^2 = 6850 \dots (2)$$

Solve (1) & (2);

$$a = 75, b = 35$$

$$\therefore a + 3b = 75 + 3(35) = 75 + 105 = 180$$

Question 32

Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $205/7$ respectively. Then the mean deviation about the mean of these 7 observations is :

[1-Feb-2024 Shift 1]

Options:

A.

31

B.

28

C.

30

D.

32

Answer: D

Solution:

Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Mean deviation about

Median =

$$\frac{0 + 45 + 60 + 20 + 40 + 170 - a + 170 - b}{7} = \frac{205}{7}$$

$$\Rightarrow a + b = 300$$

$$\text{Mean} = \frac{170 + 125 + 230 + 190 + 210 + a + b}{7} = 175$$

Mean deviation

About mean =

$$\frac{50 + 175 - a + 175 - b + 5 + 15 + 35 + 55}{7} = 30$$

Question33

Consider 10 observation $x_1, x_2, \dots, x_{10}$ such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to:

[1-Feb-2024 Shift 2]

Options:

A.

2

B.

$\frac{3}{2}$

C.

$\frac{5}{2}$

D.

1

Answer: A

Solution:

$$x_1, x_2, \dots, x_{10}$$

$$\sum_{i=1}^{10} (x_i - \alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \mu = \frac{6}{5} = \frac{\sum x_i}{10}$$

$$\therefore \sum x_i = 12$$

$$10\alpha + 2 = 12 \quad \therefore \alpha = 1$$

$$\text{Now } \sum_{i=1}^{10} (x_i - \beta)^2 = 40 \text{ Let } y_i = x_i - \beta$$

$$\therefore \sigma_y^2 = \frac{1}{10} \sum y_i^2 - (\bar{y})^2$$

$$\sigma_x^2 = \frac{1}{10} \sum (x_i - \beta)^2 - \left(\frac{\sum (x_i - \beta)}{10} \right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12 - 10\beta}{10} \right)^2$$

$$\therefore \left(\frac{6 - 5\beta}{5} \right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5} \text{ (not possible) or } \beta = 2$$

$$\text{Hence } \frac{\beta}{\alpha} = 2$$

Question34

Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to

[24-Jan-2023 Shift 2]

Options:

A. 220

B. 210

C. 200

D. 105

Answer: B

Solution:

$$a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$$

$$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$$

$$\Rightarrow a_1 + a_6 = 19$$

$$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5$$

$$\Rightarrow a_1 = 2, d = 3$$

$$\text{Numbers : } 2, 5, 8, 11, 14, 17$$

$$\text{Variance} = \sigma^2 = \text{mean of squares} - \text{square of mean}$$

$$\begin{aligned}
 &= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2 \\
 &= \frac{699}{6} - \frac{361}{4} = \frac{105}{4} \\
 8\sigma^2 &= 210
 \end{aligned}$$

Question35

The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12 . If the new mean of the marks is 10.2. then their new variance is equal to:

[25-Jan-2023 Shift 1]

Options:

- A. 4.04
- B. 4.08
- C. 3.96
- D. 3.92

Answer: C

Solution:

$$\sum_{i=1}^n x_i = 10n$$

$$\sum_{i=1}^n x_i - 8 + 12 = (10.2)n \quad \therefore n = 20$$

$$\text{Now } \frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$$

$$\begin{aligned}
 &\frac{\sum_{i=1}^{20} x_i^2 - 8^2 + 12^2}{20} - (10.2)^2 \\
 &= 108 - 104.04 = 3.96
 \end{aligned}$$

Question36

There rotten apples are mixed accidentally with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X , respectively, then $10(\mu^2 + \sigma^2)$ is equal to

[29-Jan-2023 Shift 1]

Options:

- A. 20
- B. 250
- C. 25
- D. 30

Answer: A

Solution:

$$\sum xP(x) = \frac{6}{2} = \mu$$

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$

$$\sigma^2 + \mu^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$$

$$10(\sigma^2 + \mu^2) = 20 \text{ Ans.}$$

Question37

Let $X = \{11, 12, 13, \dots, 40, 41\}$ and $Y = \{61, 62, 63, \dots, 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 603

Solution:

$$\bar{x} = \frac{\sum_{i=1}^{41} i}{31} = \frac{11+41}{2} = 26 \text{ (31 elements)}$$

$$\bar{y} = \frac{\sum_{i=61}^{91} i}{31} = \frac{61+91}{2} = 76 \text{ (31 elements)}$$

$$\text{Combined mean, } \mu = \frac{31 \times 26 + 31 \times 76}{31 + 31}$$

$$= \frac{26 + 76}{2} = 51$$

$$\sigma^2 = \frac{1}{62} \times \left(\sum_{i=1}^{31} (x_i - \mu)^2 + \sum_{i=1}^{31} (y_i - \mu)^2 \right) = 705$$

Since, $x_i \in X$ are in A.P. with 31 elements & common difference 1, same is $y_i \in Y$, when written in increasing order.

$$\therefore \sum_{i=1}^{31} (x_i - \mu)^2 = \sum_{i=1}^{31} (y_i - \mu)^2$$

$$= 10^2 + 11^2 + \dots + 40^2$$

$$= \frac{40 \times 41 \times 81}{6} - \frac{9 \times 10 \times 19}{6} = 21855$$

$$\therefore |\bar{x} + \bar{y} - \sigma^2| = |26 + 76 - 705| = 603$$

Question38

The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observations, then $a + 3b - 5$ is equal to _____.

[30-Jan-2023 Shift 1]

Answer: 37

Solution:

$$\begin{aligned}
 \frac{x_1 + x_2 + \dots + x_7}{7} &= 8 \\
 \frac{x_1 + x_2 + x_3 + \dots + x_6 + 14}{7} &= 8 \\
 \Rightarrow x_1 + x_2 + \dots + x_6 &= 42 \\
 \therefore \frac{x_1 + x_2 + \dots + x_6}{6} &= \frac{42}{6} = 7 = a \\
 \frac{\sum x_i^2}{7} - 8^2 &= 16 \\
 \sum x_i^2 &= 560 \\
 \Rightarrow x_1^2 + x_2^2 + \dots + x_6^2 &= 364 \\
 b &= \frac{x_1^2 + x_2^2 + \dots + x_6^2}{6} - 7^2 \\
 &= \frac{364}{6} - 49 \\
 b &= \frac{70}{6} \\
 a + 3b - 5 &= 7 + 3 \times \frac{70}{6} - 5 \\
 &= 37
 \end{aligned}$$

Question 39

Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \dots, a_{100}$ is 25. Then S is
[30-Jan-2023 Shift 2]

Options:

- A. ϕ
- B. $\{99\}$
- C. $\mathbb{N}$
- D. $\{9\}$

Answer: C

Solution:

let a_1 be any natural number

$a_1, a_1 + 1, a_1 + 2, \dots, a_1 + 99$ are values of a_i 's

$$\begin{aligned}
 \bar{x} &= \frac{a_1 + (a_1 + 1) + (a_1 + 2) + \dots + a_1 + 99}{100} \\
 &= \frac{100a_1 + (1 + 2 + \dots + 99)}{100} = a_1 + \frac{99 \times 100}{2 \times 100} \\
 &= a_1 + \frac{99}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{Mean deviation about mean} &= \frac{\sum_{i=1}^{100} |x_i - \bar{x}|}{100} \\
 &= \frac{2 \left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + \dots + \frac{1}{2} \right)}{100} \\
 &= \frac{1 + 3 + \dots + 99}{100} \\
 &= \frac{\frac{50}{2} [1 + 99]}{100} \\
 &= 25
 \end{aligned}$$

So, it is true for every natural no. ' a_1 '

Question40

If the variance of the frequency distribution

X_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

[31-Jan-2023 Shift 1]

Answer: 5

Solution:

$$\begin{aligned}\sigma_x^2 &= \sigma_d^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2 \\ &= \frac{150}{45 + \alpha} - 0 = 3 \\ \Rightarrow 150 &= 135 + 3\alpha \\ \Rightarrow 3\alpha &= 15 \Rightarrow \alpha = 5\end{aligned}$$

Question41

Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha (>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of $100 + n$ students are respectively 50 and 350, then the sum of variances of classes A and B is:

[31-Jan-2023 Shift 2]

Options:

- A. 500
- B. 650
- C. 450
- D. 900

Answer: 0

Solution:

$$\begin{aligned}\bar{x} &= \frac{100 \times 40 + 55n}{100 + n} \\ 5000 + 55n &= 4000 + 55n \\ 1000 &= 5n \\ n &= 200 \\ \sigma_1^2 &= \frac{\sum x_i^2}{100} - 40^2 \\ \sigma_2^2 &= \frac{\sum x_i^2}{100} - 55^2 \\ 350 &= \sigma^2 = \frac{\sum x_i^2 + \sum x_i^2}{300} - (\bar{x})^2 \\ 2850 &= \frac{(1600 + \alpha^2) \times 100 + [(30 - \alpha)^2 + 3025] \times 200}{300} - (50)^2 \\ 8550 &= \alpha^2 + 2(30 - \alpha)^2 + 7650 \\ \alpha^2 + 2(30 - \alpha)^2 &= 900\end{aligned}$$

$$\alpha^2 - 40\alpha + 300 = 0$$

$$\alpha = 10, 30$$

$$\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$$

Question 42

The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is
[1-Feb-2023 Shift 1]

Options:

- A. 1072
- B. 1792
- C. 1216
- D. 1456

Answer: A

Solution:

$$\frac{1+3+5+a+b}{5} = 5$$

$$a+b = 16 \dots (1)$$

$$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{\sum x}{5} \right)^2$$

$$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$$

$$a^2+b^2 = 130 \dots (2)$$

by (1), (2)

$$a = 7, b = 9$$

or $a = 9, b = 7$

Question 43

Let $9 = x_1 < x_2 < \dots < x_7$ be in an A.P. with common difference d . If the standard deviation of $x_1, x_2, \dots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to :
[1-Feb-2023 Shift 2]

Options:

- A. $18 \left(1 + \frac{1}{\sqrt{3}} \right)$
- B. 34
- C. $2 \left(9 + \frac{8}{\sqrt{7}} \right)$
- D. 25

Answer: B

Solution:

$$\begin{aligned}
 9 &= x_1 < x_2 < \dots < x_7 \\
 9, 9+d, 9+2d, \dots, 9+6d \\
 0, d, 2d, \dots, 6d \\
 \bar{x}_{\text{new}} &= \frac{21d}{7} = 3d \\
 16 &= \frac{1}{7}(0^2 + 1^2 + \dots + 6^2)d^2 - 9d^2 \\
 &= \frac{1}{\text{not}} \left(\frac{\text{not } 6 \times \text{not } t \times 13}{\text{not } 6} \right) d^2 - 9d^2 \\
 16 &= 4d^2 \\
 d^2 &= 4 \\
 d &= 2 \\
 x + x_6 &= 6 + 9 + 10 + 9
 \end{aligned}$$

Question44

The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to :
[6-Apr-2023 shift 1]

Options:

- A. 12
- B. 10
- C. 11
- D. 9

Answer: B

Solution:

$$\begin{aligned}
 \text{Combine var.} &= \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2(m_1 - m_2)^2}{(n_1 + n_2)^2} \\
 13 &= \frac{15 \cdot 14 + 15 \cdot \sigma^2}{30} + \frac{15 \cdot 15(12 - 14)^2}{30 \times 30} \\
 13 &= \frac{14 + \sigma^2}{2} + \frac{4}{4} \\
 \sigma^2 &= 10
 \end{aligned}$$

Question45

If the mean and variance of the frequency distribution.

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _____ :
[6-Apr-2023 shift 2]

Answer: 25

Solution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
2	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$$N = \sum f_i = 40 + \alpha + \beta$$

$$\sum f_i x_i = 360 + 6\alpha + 12\beta$$

$$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 9$$

$$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$$

$$3\alpha = 3\beta \Rightarrow \alpha = \beta$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2$$

$$\Rightarrow \frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$$

$$\Rightarrow \frac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$$

$$\Rightarrow \alpha = 5$$

$$\text{Now, } \alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$$

Question 46

Let the mean and variance of 8 numbers $x, y, 10, 12, 6, 12, 4, 8$ be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to _____.

Answer: 25

Solution:

$$\frac{x + y + 52}{8} = 9 \Rightarrow x + y = 20$$

For variance

$$\frac{x - 9, y - 9, 3, 3, 1, -5, -1, -3}{x = 0}$$

$$\therefore \frac{(x-9)^2 + (y-9)^2 + 54}{8} - 0^2 = 9.25$$

$$(x-9)^2 + (11-x)^2 = 20$$

$$x = 7 \text{ or } 13 \therefore y = 13, 7$$

$$3x - 2y = 3 \times 13 - 2 \times 7 = 25$$

Question47

Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to
[8-Apr-2023 shift 2]

Options:

- A. 316
- B. 317
- C. 315
- D. 314

Answer: B

Solution:

$$\begin{aligned}\frac{\sum x}{12} &= \frac{9}{2} \\ \sum x &= 54 \\ \frac{\sum x^2}{12} - \left(\frac{9}{2}\right)^2 &= 4 \\ \sum x^2 &= 291 \\ \sum x_{\text{new}} &= 54 - (9 + 10) + 7 + 14 = 56 \\ \sum x_{\text{new}}^2 &= 291 - (81 + 100) + 49 + 196 = 355 \\ \sigma_{\text{new}}^2 &= \frac{355}{12} - \left(\frac{56}{12}\right)^2 \\ \sigma_{\text{new}}^2 &= \frac{281}{36} = \frac{m}{n} \\ m + n &= 317 \text{ Option (2)}\end{aligned}$$

Question48

If the mean of the frequency distribution

Class:	0-10	10-20	20-30	30-40	40-50
Frequency:	2	3	x	5	4

is 28, then its variance is _____.
[10-Apr-2023 shift 1]

Answer: 151

Solution:

C.I.	f	x	$f_i x_i$	x_i^2
0-10	2	5	10	25
10-20	3	15	45	225
20-30	x	25	25x	625
30-40	5	35	175	1225
40-50	4	45	180	2025

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$28 = \frac{10 + 45 + 25x + 175 + 130}{14 + x}$$

$$28 \times 14 + 28x = 410 + 25x$$

$$\Rightarrow 3x = 410 - 392$$

$$\Rightarrow x = \frac{18}{3} = 6$$

$$\therefore \text{Variance} = \frac{1}{N} \sum f_i x_i^2 - (\bar{x})^2$$

$$= \frac{1}{20} 18700 - (28)^2$$

$$= 935 - 784 = 151$$

Question 49

Let m be the mean and sigma be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If $[x]$ denotes the greatest integer $\leq x$, then $[\mu^2 + \sigma^2]$ is equal to

[10-Apr-2023 shift 2]

Options:

A. 8

B. 7

C. 6

D. 9

Answer: A

Solution:

$$\sum f_i = 62$$

$$3k^2 + 16k - 12k - 64 = 0$$

$$k = 4 \text{ or } -\frac{16}{3} \text{ (rejected)}$$

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - (\sum f_i x_i)^2$$

$$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62} \right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62} \right)^2$$

$$\sigma^2 + \mu^2 = \frac{500}{62}$$

$$[\sigma^2 + \mu^2] = 8$$

Question50

Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is _____.

[11-Apr-2023 shift 1]

Options:

- A. 36
- B. 40
- C. 32
- D. 38

Answer: D

Solution:

$$\omega A = \{a_1, a_2, a_3, a_4, a_5\}$$

$$B = \{b_1, b_2, b_3, b_4, b_5\}$$

$$\frac{\sum_{i=1}^5 a_i^2}{5} - \left(\frac{\sum_{i=1}^5 a_i}{5} \right)^2 = 12, \quad \frac{\sum_{i=1}^5 b_i^2}{5} - \left(\frac{\sum_{i=1}^5 b_i}{5} \right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^5 a_i^2 = 185, \quad \sum_{i=1}^5 b_i^2 = 420$$

$$\text{Now, } C = \{C_1, C_2, \dots, C_{10}\}$$

$$\therefore \text{Mean of } C, \bar{C} = \frac{(\sum a_i - 15) + (\sum b_i - 10)}{10}$$

$$\bar{C} = \frac{10 + 50}{10} = 6$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} - (\bar{C})^2$$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i - 2)^2}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

$$= 32$$

$$\therefore \text{Mean} + \text{Variance} = \bar{C} + \sigma^2 = 6 + 32 = 38$$

Question51

Let the mean of 6 observations 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

[11-Apr-2023 shift 2]

Options:

- A. $\frac{7}{3}$

B. $\frac{10}{3}$

C. $\frac{8}{3}$

D. 3

Answer: C

Solution:

Mean of 1, 2, 4, 5, x, y is 5
and variance is 10

$$\Rightarrow \text{mean} \Rightarrow \frac{12+x+y}{6} = 5$$

$$12+x+y=30$$

$$x+y=18$$

$$\text{and by variance } \frac{x^2+y^2+46}{6} - 5^2 = 10$$

$$x^2+y^2=164$$

$$x=8 \quad y=10$$

$$\text{mean deviation} = \frac{|x-\bar{x}|}{6}$$

$$\Rightarrow \frac{4+3+1+0+3+5}{6} = \frac{16}{6} = \frac{8}{3}$$

Question52

Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocal is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then $m + n$ is equal to _____.
[12-Apr-2023 shift 1]

Answer: 211

Solution:

$$\text{Let } \frac{a}{r}, \frac{a}{r}, a, ar, ar^2$$

$$\text{Given } \frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 5 \times \frac{31}{10} \dots (1)$$

$$\text{And } \frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = 5 \times \frac{31}{40} \dots (2)$$

$$(1) \div (2) a^2 = 4 \Rightarrow a = 2$$

$$\therefore r + \frac{1}{r} = 5/2 \quad (a \neq -2)$$

$$\Rightarrow r = 2$$

$$\therefore \text{ Now } \frac{1}{2}, 1, 2, 4, 8$$

$$\therefore \text{ Now } \frac{1}{2}, 1, 2, 4, 8$$

$$\therefore \sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N} \right)^2$$

$$= \frac{186}{25} - \frac{M}{N} \Rightarrow 211 = m + n$$

Question53

Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	a	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3a}{m + \sigma^2}$ is equal to _____.

[13-Apr-2023 shift 1]

Answer: 8

Solution:

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4 + 72 + 140 + 7a + 72}{64 + a}$$

$$\Rightarrow 320 + 5a = 288 + 7a \Rightarrow 2a = 32 \Rightarrow a = 16$$

$$\text{M.D. } (\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} \text{ where } \sum f_i = 64 + 16 = 80$$

$$\text{M.D. } (\bar{x}) = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80} = \frac{8}{5}$$

$$\text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$= \frac{4 \times 16 + 24 \times 4 + 0 + 16 \times 4 + 8 \times 16}{80} = \frac{352}{80}$$

$$\therefore \frac{3a}{m + \sigma^2} = \frac{3 \times 16}{\frac{128}{80} + \frac{352}{80}} = 8$$

Question 54

The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _____.

[13-Apr-2023 shift 2]

Answer: 269

Solution:

$$\text{Mean} = \frac{\sum x_i}{10}$$

$$\Rightarrow 50 = \frac{\sum x_i}{10}$$

$$\Rightarrow \sum x_i = 500$$

$$\text{correct } \sum x_i = 500 - 45 - 50 + 20 + 25 = 450$$

$$\sigma^2 = \frac{\sum x_i^2}{10} - (\bar{x})^2$$

$$\Rightarrow 144 = \frac{\sum x_i^2}{10} - 2500$$

$$\Rightarrow \sum x_i^2 = 26440$$

$$\text{correct } \sum x_i^2 = 26440 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$= 26440 - 2025 - 2500 + 400 + 625$$

$$= 22940$$

$$\begin{aligned}\sigma^2 &= \frac{\text{correct } \sum x_i^2}{10} - \left(\frac{\text{correct } \sum x_i}{10} \right)^2 \\ &= \frac{22940}{10} - \left(\frac{450}{10} \right)^2 = 2294 - 2025 \\ &= 269\end{aligned}$$

Question 55

The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is [15-Apr-2023 shift 1]

Options:

- A. 14
- B. 11
- C. 12
- D. 13

Answer: D

Solution:

$$\begin{aligned}\mu &= 20, \sigma = 8 \\ \mu_{\text{Corrected}} &= \frac{200 - 50 + 40}{10} = 19 \\ \sigma^2 &= \frac{1}{10} \sum x_i^2 - 20^2 \\ (64 + 400)10 &= \sum x_i^2 \\ \sigma_{\text{Corrected}}^2 &= \frac{1}{10} [(64 + 400)10 - 2500 + 1600] - 19^2 \\ &= 374 - 361 = 13\end{aligned}$$

Question 56

Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is : [26-Jun-2022-Shift-1]

Options:

- A. $\frac{275}{6^5}$
- B. $\frac{36}{5^4}$
- C. $\frac{181}{5^5}$
- D. $\frac{46}{6^4}$

Answer: D

Solution:

Coin is tossed 5 times, so $n = 5$

Let, p = probability of getting heads

q = probability of getting tails.

$$\therefore p + q = 1 \dots \dots (1)$$

$\therefore$ Probability of getting 4 heads

$$= {}^5C_4 \cdot p^4 \cdot q$$

And probability of getting 5 heads

$$= {}^5C_5 \cdot p^5$$

$$\text{Given, } {}^5C_4 \cdot p^4 \cdot q = {}^5C_5 \cdot p^5$$

$$\Rightarrow 5q = p \dots \dots (2)$$

From equation (1) and (2), we get,

$$5q + q = 1$$

$$\Rightarrow 6q = 1$$

$$\Rightarrow q = \frac{1}{6}$$

$$\therefore p = 1 - \frac{1}{6} = \frac{5}{6}$$

Now, probability of getting atmost two heads

$$= p(x=0) + p(x=1) + p(x=2)$$

$$p(x=0) = \text{Getting zero head in 5 trials}$$

$$= {}^5C_0 \cdot p^0 \cdot q^5$$

$$p(x=1) = \text{Getting one head in 5 trials}$$

$$= {}^5C_1 \cdot p^1 \cdot q^4$$

$$p(x=2) = \text{Getting two heads in 5 trials}$$

$$= {}^5C_2 \cdot p^2 \cdot q^3$$

$$= {}^5C_0 \cdot q^5 + {}^5C_1 \cdot p q^4 + {}^5C_2 \cdot p^2 q^3$$

$$= \left(\frac{1}{6}\right)^5 + 5 \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^4 + 10 \cdot \left(\frac{5}{6}\right)^2 \cdot \left(\frac{1}{6}\right)^3$$

$$= \frac{1+25+250}{6^5} = \frac{276}{6^5} = \frac{46}{6^4}$$

Question57

If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p , then $96p$ is equal to
[26-Jun-2022-Shift-2]

Answer: 33

Solution:

Total number of numbers from given

$$\text{Condition} = n(s) = 2^6$$

Every required number is of the form

$$A = 7 \cdot (10^5 + 10^4 + 10^3 + \dots) + 111111$$

Here 111111 is always divisible by 21.

$\therefore$ If A is divisible by 21 then

$$10^5 + 10^4 + 10^3 + \dots \text{ must be divisible by 3.}$$

For this we have ${}^6C_0 + {}^6C_3 + {}^6C_6$ cases are there

$$\therefore n(E) = {}^6C_0 + {}^6C_3 + {}^6C_6 = 22$$

$$\therefore \text{Required probability} = \frac{22}{2^6} = p$$

$$\therefore \frac{11}{32} = p$$

$$\therefore 96p = 33$$

Question58

The mean of the numbers $a, b, 8, 5, 10$ is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then $25M$ is equal to :
[26-Jun-2022-Shift-1]

Options:

A. 60

B. 55

C. 50

D. 45

Answer: A

Solution:

$$\sigma^2 = \frac{\sum_{i=1}^5 (x_i - \bar{x})^2}{n}$$

$$\text{Mean} = 6$$

$$\frac{a+b+8+5+10}{5} = 6 \Rightarrow a+b = 7$$

$$a+b = 7$$

$$b = 7 - a$$

$$6.8 = \frac{(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2}{5}$$

$$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 16$$

$$a^2 - 7a + 12 = 0 \Rightarrow a = 4 \text{ or } a = 3$$

$$a = 4 \quad a = 3$$

$$b = 3 \quad b = 4$$

$$M = \frac{\sum_{i=1}^5 |x_i - \bar{x}|}{n}$$

$$M = \frac{|a-6| + |b-6| + |8-6| + |5-6| + |10-6|}{5}$$

$$\text{when } a = 3, b = 4 \quad \text{when } a = 4, b = 3$$

$$M = \frac{3+2+2+1+4}{5} \quad M = \frac{2+3+2+1+7}{5}$$

$$M = \frac{12}{5} \quad M = \frac{12}{5} \Rightarrow 25M = 25 \times \frac{12}{5} = 60$$

Question 59

The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70.

If the correct mean is 16, then the correct variance is equal to :

[26-Jun-2022-Shift-2]

Options:

A. 10

B. 36

C. 43

D. 60

Answer: C

Solution:

$$\text{Given } \bar{x} = 15, \sigma = 2 \Rightarrow \sigma^2 = 4$$

$$\therefore x_1 + x_2 + \dots + x_{50} = 15 \times 50 = 750$$

$$4 = \frac{x_1^2 + x_2^2 + \dots + x_{50}^2}{50} - 225$$

$$\therefore x_1^2 + x_2^2 + \dots + x_{50}^2 = 50 \times 229$$

Let a be the correct observation and b is the incorrect observation

$$\text{then } a + b = 70$$

$$\text{and } 16 = \frac{750 - b + a}{50}$$

$$\therefore a - b = 50 \Rightarrow a = 60, b = 10$$

$$\therefore \text{Correct variance} = \frac{50 \times 229 + 60^2 - 10^2}{50} - 256 = 43$$

Question 60

The mean and variance of the data 4, 5, 6, 6, 7, 8, x , y , where $x < y$, are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

[27-Jun-2022-Shift-2]

Options:

A. 162

B. 320

C. 674

D. 420

Answer: B

Solution:

$$\text{Mean} = \frac{4 + 5 + 6 + 6 + 7 + 8 + x + y}{8} = 6$$

$$\therefore x + y = 12 \dots (i)$$

And variance

$$= \frac{2^2 + 1^2 + 0^2 + 0^2 + 1^2 + 2^2 + (x-6)^2 + (y-6)^2}{8}$$

$$= \frac{9}{4}$$

$$\therefore (x-6)^2 + (y-6)^2 = 8 \dots (ii)$$

From (i) and (ii)

$$x = 4 \text{ and } y = 8$$

$$\therefore x^4 + y^2 = 320$$

Question61

The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _____

[28-Jun-2022-Shift-1]

Answer: 17

Solution:

$$\frac{\sum x_i^2}{15} - 8^2 = 9 \Rightarrow \sum x_i^2 = 15 \times 73 = 1095$$

Let $\bar{x}_c$ be corrected mean $\bar{x}_c = 9$

$$\sum x_c^2 = 1095 - 25 + 400 = 1470$$

$$\text{Correct variance} = \frac{1470}{15} - (9)^2 = 98 - 81 = 17$$

Question62

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to:

[29-Jun-2022-Shift-1]

Options:

A. 13

B. 15

C. 17

D. 18

Answer: B

Solution:

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$\text{Given, } \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 24$$

Now, Mean of first 4 observation

$$= \frac{x_1 + x_2 + x_3 + x_4}{4}$$

$$\text{Given, } = \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14$$

From equation (1) and (2), we get

$$14 + x_5 = 24$$

$$\Rightarrow x_5 = 10$$

Now, variance of first 5 observation

$$= \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \left(\frac{24}{5}\right)^2$$

Given,

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{24}$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 5 \left(\frac{194}{25} + \frac{576}{25} \right)$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 154$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + (10)^2 = 154$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$$

Now, variance of first 4 observation

$$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2$$

Given,

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2 = a$$

Given,

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2 = a$$

$$\Rightarrow \frac{54}{4} - \frac{49}{4} = a$$

$$\Rightarrow a = \frac{5}{4}$$

$$\therefore 4a + x_5$$

$$= 4 \times \frac{5}{4} + 10 = 15$$

Question63

The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12, a, $43 - a$ is a natural number is :
[29-Jun-2022-Shift-2]

Options:

A. 0

B. 2

C. 5

D. infinite

Answer: A

Solution:

$$\text{Mean} = 13$$

$$\text{Variance} = \frac{9 + 49 + 144 + a^2 + (43 - a)^2}{5} - 13^2 \in \mathbb{N}$$

$$\Rightarrow \frac{2a^2 - a + 1}{5} \in \mathbb{N}$$

$\Rightarrow 2a^2 - a + 1 - 5n = 0$ must have solution as natural numbers

its $D = 40n - 7$ always has 3 at unit place

$\Rightarrow D$ can't be perfect square

So, a can't be integer.

Question64

If the mean deviation about the mean of the numbers 1, 2, 3, n , where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to

[25-Jun-2022-Shift-2]

Answer: 21

Solution:

$$\text{Mean} = \frac{n(n+1)}{n} = \frac{n+1}{2}$$

$$\text{M.D.} = \frac{2\left(\frac{n-1}{2} + \frac{n-3}{2} + \frac{n-5}{2} + \dots + 0\right)}{n} = \frac{5(n+1)}{n}$$

$$\Rightarrow ((n-1) + (n-3) + (n-5) + \dots + 0) = 5(n+1)$$

$$\Rightarrow \left(\frac{n+1}{4}\right) \cdot (n-1) = 5(n+1)$$

$$\text{So, } n = 21$$

Question65

If the mean deviation about median for the numbers 3, 5, 7, $2k$, 12, 16, 21, 24, arranged in the ascending order, is 6 then the median is

[25-Jul-2022-Shift-2]

Options:

A. 11.5

B. 10.5

C. 12

D. 11

Answer: D

Solution:

$$\text{Median} = \frac{2k+12}{2} = k+6$$

$$\text{Mean deviation} = \sum \frac{|x_i - M|}{n} = 6$$

$$\Rightarrow \frac{(k+3) + (k+1) + (k-1) + (6-k) + (6-k) + (10-k) + (15-k) + (18-k)}{8}$$

$$\therefore \frac{58-2k}{8} = 6$$

$$k = 5$$

$$\text{Median} = \frac{2 \times 5 + 12}{2} = 11$$

Question66

The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to _____.

[26-Jul-2022-Shift-2]

Answer: 238

Solution:

$$\mu = \frac{\sum x_i}{40} = 30 \Rightarrow \sum x_i = 1200$$

$$\sigma^2 = \frac{\sum x_i^2}{40} - (30)^2 = 25 \Rightarrow \sum x_i^2 = 37000$$

After omitting two wrong observations

$$\sum y_i = 1200 - 12 - 10 = 1178$$

$$\sum y_i^2 = 37000 - 144 - 100 = 36756$$

$$\text{Now } \sigma^2 = \frac{\sum y_i^2}{38} - \left(\frac{\sum y_i}{38} \right)^2$$

$$= \frac{36756}{38} - \left(\frac{1178}{38} \right)^2 = -31^2$$

$$= 38\sigma^2 = 36756 - 36518 = 238$$

Question67

The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____.

[27-Jul-2022-Shift-1]

Answer: 2

Solution:

$$\text{Given } \frac{\sum_{i=1}^{10} x_i}{10} = 15 \dots$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 150$$

$$\text{and } \frac{\sum_{i=1}^{10} x_i^2}{10} - 15^2 = 15$$

$$\Rightarrow \sum_{i=1}^{10} x_i^2 = 2400$$

Replacing 25 by 15 we get

$$\sum_{i=1}^9 x_i + 25 = 150$$

$$\Rightarrow \sum_{i=1}^9 x_i = 125$$

$$\therefore \text{Correct mean} = \frac{\sum_{i=1}^9 x_i + 15}{10} = \frac{125 + 15}{10} = 14$$

$$\text{Similarly, } \sum_{i=1}^9 x_i^2 + 25^2 = 1775$$

$$\therefore \text{Correct variance} = \frac{\sum_{i=1}^9 x_i^2 + 15^2}{10} - 14^2 = \frac{1775 + 225}{10} - 14^2 = 4$$

$$\therefore \text{Correct S.D} = \sqrt{4} = 2.$$

Question68

Let the mean and the variance of 20 observations $x_1, x_2, \dots, x_{20}$ be 15 and 9, respectively. For $\alpha \in \mathbb{R}$, if the mean of $(x_1 + \alpha)^2, (x_2 + \alpha)^2, \dots, (x_{20} + \alpha)^2$ is 178, then the square of the maximum value of α is equal to _____.

[29-Jul-2022-Shift-1]

Answer: 4

Solution:

$$\text{Given } \frac{\sum_{i=1}^{20} x_i}{20} = 15 \Rightarrow \sum_{i=1}^{20} x_i = 300$$

$$\text{and } \frac{\sum_{i=1}^{20} x_i^2}{20} - (\bar{x})^2 = 9 \Rightarrow \sum_{i=1}^{20} x_i^2 = 4680$$

$$\text{Mean} = \frac{(x_1 + \alpha)^2 + (x_2 + \alpha)^2 + \dots + (x_{20} + \alpha)^2}{20} = 178$$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i^2 + 2\alpha \sum_{i=1}^{20} x_i + 20\alpha^2}{20} = 178$$

$$\Rightarrow 4680 + 600\alpha + 20\alpha^2 = 3560$$

$$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$$

$$\Rightarrow \alpha^2 + 28\alpha + 2\alpha + 56 = 0$$

$$\Rightarrow (\alpha + 28)(\alpha + 2) = 0$$

$$\alpha_{\max} = -2 \Rightarrow \alpha_{\max}^2 = 4$$

Question69

If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is

[2021, 24 Feb. Shift-11]

Answer: 11

Solution:

Given, 10 natural numbers = 1, 1, 1, ..., 1, k

According to the question, variance

$$(\sigma^2) < 10$$

$$\therefore \sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$$

$$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10} \right)^2 < 10$$

$$\Rightarrow 10(9+k^2) - (81+k^2+18k) < 1000$$

$$\Rightarrow 90+10k^2-81-k^2-18k < 1000$$

$$\Rightarrow 9k^2-18k+9 < 1000$$

$$\Rightarrow (k-1)^2 < \frac{1000}{9}$$

$$\Rightarrow k-1 < \frac{10\sqrt{10}}{3}$$

$$\text{and } k-1 < \frac{-10\sqrt{10}}{3}$$

$$\Rightarrow k < \frac{10\sqrt{10}}{3} + 1$$

It is not possible because $k \in \mathbb{N}$.

$\therefore$ Maximum possible integral value of k is 11.

Question 70

Let $X_1, X_2, \dots, X_{18}$ be eighteen observations, such that $\sum_{i=1}^{18} (X_i - \alpha) = 36$ and $\sum_{i=1}^{18} (X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is
[2021, 26 Feb. Shift-II]

Answer: 4

Solution:

$$\text{Given, } \sum_{i=1}^{18} [x_i - \alpha] = 36$$

$$\text{and } \sum_{i=1}^{18} |x_i - \beta|^2 = 90$$

$$\text{Now, } \sum_{i=1}^{18} (x_i - \alpha) = 36$$

$$\sum_{i=1}^{18} x_i - 18\alpha = 36$$

$$\sum_{i=1}^{18} x_i = 36 + 18\alpha \dots (i)$$

Again,

$$\sum_{i=1}^{18} x_i^2 + 18\beta^2 - 2\beta \sum_{i=1}^{18} x_i = 90$$

$$\Rightarrow \sum_{i=1}^{18} x_i^2 + 18\beta^2 - 2\beta(36 + 18\alpha) = 90$$

[Using Eq. (i)]

$$\Rightarrow \sum_{i=1}^{18} x_i^2 = 90 - 18\beta^2 + 2\beta(36 + 18\alpha)$$

Given, standard deviation = 1

i.e $\sigma^2 = 1$

$$\begin{aligned}
 &\Rightarrow \frac{\sum_{i=1}^{18} x_i^2}{18} - \left(\frac{\sum_{i=1}^{18} x_i}{18} \right)^2 = 1 \\
 &\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta) \\
 &\quad - \left(\frac{36 + 18\alpha}{18} \right)^2 = 1 \\
 &\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (2 + \alpha)^2 = 1 \\
 &\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - 4 - \alpha^2 + (-4\alpha) = 1 \\
 &\Rightarrow -\beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha = 0 \\
 &\Rightarrow \alpha^2 + \beta^2 + (-2\alpha\beta) - 4\beta + 4\alpha = 0 \\
 &\Rightarrow (\alpha - \beta)^2 + 4(\alpha - \beta) = 0 \\
 &\Rightarrow (\alpha - \beta)(\alpha - \beta + 4) = 0 \\
 &\Rightarrow \alpha - \beta = -4 \quad (\because \alpha \neq \beta) \\
 &|\alpha - \beta| = |-4| = 4
 \end{aligned}$$

Question 71

The mean age of 25 teachers in a school is 40yr. A teacher retires at the age of 60yr and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39yr, then the age (in years) of the newly appointed teacher is

[2021, 18 March Shift-I]

Answer: 35

Solution:

$$\begin{aligned}
 &\text{Given, } n = 25, \bar{x} = 40 \\
 &\therefore \bar{x} = \frac{\sum x}{n} \\
 &\Rightarrow \sum x = 25 \times 40 = 1000 \\
 &\text{New sum} = 1000 - 60 + x \\
 &= 940 + x \\
 &\text{New average} = 39 \\
 &\therefore 39 = \frac{940 + x}{25} \\
 &\Rightarrow 975 = 940 + x \\
 &\Rightarrow x = 35
 \end{aligned}$$

Question 72

Let in a series of $2n$ observations, half of them are equal to a and remaining half are equal to $-a$. Also, by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then, the value of $a^2 + b^2$ is equal to

[2021, 18 March Shift-11]

Options:

- A. 425
- B. 650
- C. 250
- D. 925

Answer: A

Solution:

Let observations are denoted by x_i for $1 \leq i < 2n$.

$$\begin{aligned}\therefore \text{Mean}(\bar{x}) &= \frac{\sum x_i}{2n} \\&= \frac{(a+a+a+\dots+a) - (a+a+a+\dots+a)}{2n} \\&\Rightarrow \bar{x} = 0\end{aligned}$$

$$\begin{aligned}\text{and } \sigma_x^2 &= \frac{\sum x_i^2}{2n} - (\bar{x})^2 \\&= \frac{a^2 + a^2 + a^2 + \dots + a^{2n \text{ times}}}{2n} - 0 = a^2 \\&\Rightarrow \sigma_x^2 = a^2 \Rightarrow \sigma_x = a\end{aligned}$$

According to the question, adding a constant b , then new mean $(\bar{y}) = \bar{x} + b = 5$

$$\Rightarrow 0 + b = 5$$

$$\Rightarrow b = 5$$

and new SD $(\sigma_y) = \sigma_x$

$$\Rightarrow \sigma_y = \sigma_x = 20$$

$$\Rightarrow a = 20$$

$$\therefore a^2 + b^2 = 400 + 25 = 425$$

Question 73

Consider a set of $3n$ numbers having variance 4. In this set, the mean of first $2n$ numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first $2n$ numbers and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k , then $9k$ is equal to
[2021, 17 March Shift-II]

Answer: 68

Solution:

$$x : x_1, x_2, x_3, \dots, x_{n+1}$$

$$x_{n+2}, \dots, x_{2n}, x_{2n+1}, \dots, x_{3n}$$

$$\frac{1}{3n} \sum_{i=1}^{3n} x_i^2 - (\bar{x})^2 = 4$$

$$\left[\text{where, } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{2n(6) + n(3)}{3n} = 5 \right]$$

$$\frac{1}{3n} \sum_{i=1}^{3n} x_i^2 = 25 + 4 = 29$$

$$\Rightarrow \sum_{i=1}^{3n} x_i^2 = 87n$$

$$x' : x_1 + 1, x_2 + 1, \dots, x_{2n} + 1, x_{(2n+1)} - 1, \dots,$$

$$x_{3n} - 1$$

$$\bar{x}' = \frac{2n(6+1) + n(3-1)}{3n} = \frac{16}{3}$$

$$k = \frac{1}{3n} \sum_{i=1}^{3n} x_i'^2 - \frac{256}{9}$$

$$= \frac{1}{3n} \left[\sum_{i=1}^{3n} x_i^2 + 2 \sum_{i=1}^{2n} x_i' - 2 \sum_{i=2n+1}^{3n} x_i' + 3n \right]$$

$$- \frac{256}{9}$$

$$= \frac{1}{3n} [87n + 2(12n) - 2 \cdot 3n + 3n] - \frac{256}{9}$$

$$\Rightarrow k = 36 - \frac{256}{9}$$

$$\Rightarrow 9k = 324 - 256 = 68$$

Question 74

Consider three observations a , b and c , such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d , then which of the following is true?
[2021, 16 March Shift-1]

Options:

A. $b^2 = 3(a^2 + c^2) + 9d^2$

B. $b^2 = a^2 + c^2 + 3d^2$

C. $b^2 = 3(a^2 + c^2 + d^2)$

D. $b^2 = 3(a^2 + c^2) - 9d^2$

Answer: D

Solution:

Given, three observations = a, b, c

$b = a + c$

Standard deviation = σ

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

When $x_1 = a + 2, x_2 = b + 2, x_3 = c + 2$

$$\mu = \frac{a + 2 + b + 2 + c + 2}{3} = \frac{2b}{3} + 2$$

$$x_1 - \mu = (a + 2) - (2b/3 + 2) = a - 2b/3$$

$$x_2 - \mu = (b + 2) - (2b/3 + 2) = b - 2b/3$$

$$x_3 - \mu = (c + 2) - (2b/3 + 2) = c - 2b/3$$

$$d = \sqrt{\frac{(a - 2b/3)^2 + (b - 2b/3)^2 + (c - 2b/3)^2}{3}}$$

$$3d^2 = a^2 + b^2 + c^2 + \frac{12b^2}{9}$$

$$- \frac{4}{3}(ab + b^2 + bc)$$

$$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}b^2 - \frac{4}{3}$$

$$[b(a + c) + b^2]$$

$$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}$$

$$b^2 - \frac{4}{3}[b \cdot b + b^2]$$

$$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}b^2 - \frac{4}{3} \cdot 2b^2$$

$$\Rightarrow 3d^2 = a^2 + c^2 - \frac{b^2}{3}$$

$$\Rightarrow b^2 = 3a^2 + 3c^2 - 9d^2$$

Question 75

Consider the statistics of two sets of observations as follows Size Mean Variance

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is 17 then the value of n is equal to $\frac{17}{9}$,

then the value of n is equal to
[2021, 16 March Shift-1]

Answer: 5

Solution:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

Variance of combined set is $17/9$. Let the observations for group I be set A : $\{x_1, x_2, x_3, \dots, x_{10}\}$

$$\Rightarrow \frac{\sum x_i}{10} = 2$$

$$\Rightarrow \sum_{x_i} = 20$$

$$\text{and } \Sigma \left(\frac{x_i^2}{10} \right) - \left(\frac{\sum x_i}{10} \right)^2 = 2$$

$$\Rightarrow \frac{\sum x_i^2}{10} - \frac{400}{100} = 2$$

$$\Rightarrow \frac{\sum x_i^2}{10} = 6$$

$$\Rightarrow \sum x_i^2 = 60$$

Let the observation for group II be set B : $\{y_1, y_2, y_3, \dots, y_{10}\}$

$$\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$$

$$\text{and } \frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n} \right)^2 = 1$$

$$\frac{\sum y_i^2}{n} - 9 = 1 \Rightarrow \sum y_i^2 = 10n$$

Combined variance = $17/9$

$$\Rightarrow \frac{\sigma^2}{n+10} = \frac{\sum (x_i^2 + y_i^2)}{n+10} - \left[\sum \left(\frac{x_i + y_i}{n+10} \right) \right]^2$$

$$\Rightarrow \frac{17}{9} = \frac{60 + 10n}{n+10} - \left[\frac{20 + 3n}{n+10} \right]^2$$

$$\Rightarrow \frac{17}{9} = \frac{-(10n+60)(n+10) - (3n+20)^2}{(n+10)^2}$$

$$\Rightarrow \frac{17}{9} = \frac{n^2 + 40n + 200}{n^2 + 20n + 100}$$

$$\Rightarrow n^2 + 340n + 1700 = 9n^2 + 360n + 1800$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0$$

$$\Rightarrow 2n^2 - 10n + 5n - 25 = 0$$

$$\Rightarrow 2n(n-5) + 5(n-5) = 0$$

$$\Rightarrow (2n+5)(n-5) = 0$$

$$\therefore n = 5$$

Question 76

If the mean and variance of the following data :

6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to
[2021, 27 July Shift-1]

Options:

A. 24

B. 12

C. 32

D. 16

Answer: D

Solution:

Given, data = {6, 10, 7, 13, a, 12, b, 12}

Mean = 9, Variance = $\frac{37}{4}$

Now, mean

$$= \frac{\sum x_i}{n} = \frac{6+10+7+13+a+12+b+12}{8}$$

$$\text{or } 9 = \frac{60+a+b}{8}$$

$$\Rightarrow 72 = 60 + a + b$$

$$\Rightarrow a + b = 12 \dots (i)$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$\frac{37}{4} = \frac{6^2+10^2+7^2+13^2+a^2+12^2+b^2+12^2}{8}$$

$$\Rightarrow \frac{37}{4} = \frac{642+a^2+b^2}{8} - 81$$

$$\Rightarrow \frac{37}{4} = \frac{642+a^2+b^2-648}{8}$$

$$\Rightarrow 74 = a^2 + b^2 - 6$$

$$\Rightarrow a^2 + b^2 = 80 \dots (ii)$$

$$\Rightarrow (a+b)^2 = a^2 + b^2 + 2ab$$

Putting the values from Eqs. (i) and (ii), we get

$$2ab = (a+b)^2 - (a^2 + b^2)$$

$$\Rightarrow 2ab = 12^2 - 80 = 64$$

$$\text{Now, } (a-b)^2 = a^2 + b^2 - 2ab$$

$$= 80 - 64 = 16$$

Question 77

Let the mean and variance of the frequency distribution

x	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f	4	4	α	β

be 6 and 6.8, respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be
[2021, 27 July Shift-II]

Options:

A. 4

B. 5

C. $\frac{17}{3}$

D. $\frac{16}{3}$

Answer: C

Solution:

Given,

x	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f	4	4	α	β

Mean = 6

$$\Rightarrow \frac{8 + 24 + 8\alpha + 9\beta}{8 + \alpha + \beta} = 6$$

$$\Rightarrow \left(\because \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} \right)$$

$$\Rightarrow 2\alpha + 3\beta = 16 \dots (i)$$

Also, variance = 6.8

$$\frac{4 \times 16 + 4 \times \alpha + 9 \times \beta}{8 + \alpha + \beta} = 6.8$$

$$\left(\because \text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} \right)$$

$$\Rightarrow 64 + 4\alpha + 9\beta = (8 + \alpha + \beta)6.8$$

$\Rightarrow$ Multiply both sides by 10,

$$640 + 40\alpha + 90\beta = 544 + 68\alpha + 68\beta$$

$$\Rightarrow 28\alpha - 22\beta = 96$$

$$\Rightarrow 14\alpha - 11\beta = 48 \dots (ii)$$

From Eqs. (i) and (ii),

$$\alpha = 5 \text{ and } \beta = 2$$

If x_3 is changed from 8 to 7 then,

x	$x_1 = 2$	$x_2 = 6$	$x_3 = 7$	$x_4 = 9$
f	4	4	5	2

$$\text{New mean} = \frac{8 + 24 + 35 + 18}{15} = \frac{85}{15} = \frac{17}{3}$$

Question 78

10 Consider the following frequency distribution

Class	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to

[2021, 25 July Shift-1]

Answer: 164

Solution:

$$\text{Sum of frequencies} = 584$$

$$\alpha + \beta + 110 + 54 + 30 = 584$$

$$\alpha + \beta = 390$$

$$\text{Median is at } \frac{584}{2} = 292 \text{ th}$$

$$\therefore \text{Median} = 45 \text{ (lies in class 40 – 50)}$$

$$\Rightarrow \alpha + 110 + 54 + 15 = 292$$

$$\Rightarrow \alpha = 113$$

$$\Rightarrow \beta = 390 - 113 = 277$$

$$\Rightarrow \alpha - \beta = 113 - 277 = -164$$

$$\therefore |\alpha - \beta| = 164$$

Question 79

The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is
[2021, 25 July Shift-II]

Options:

- A. 8
- B. 6
- C. 4
- D. 5

Answer: C

Solution:

Combined mean = 15.6

$$\Rightarrow \frac{100 \times 15 + 150 \times \bar{x}_B}{250} = 15.6$$

$$\Rightarrow 1500 + 150\bar{x}_B = 3900$$

$$\Rightarrow 150\bar{x}_B = 2400$$

$$\Rightarrow \bar{x}_B = \frac{2400}{150} = 16$$

Combined SD = $\sqrt{13.44}$

$$\Rightarrow \text{Combined variance } (\sigma^2) = 13.44$$

$$\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow 13.44 = \frac{\sum x_i^2}{250} - (15.6)^2$$

$$\Rightarrow \sum \sum x_i^2 = 64200 \dots (i)$$

For sample A₁

$$\Rightarrow \sum \sum (x_i)_A^2 = 23400$$

$$\text{Now, } \sum (x_i)_B^2 = 64200 - 23400 = 40800$$

$$\text{SD of sample B} = \sqrt{\frac{\sum (x_i)_B^2}{n_B} - (\bar{x}_B)^2}$$

$$= \sqrt{\frac{40800}{150} - 256} = 4$$

Question 80

12 Consider the following frequency distribution

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	a	b	12	9	5

If mean = $\frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to
[2021, 22 July Shift-II]

Answer: 4

Solution:

$$\begin{aligned}\text{Mean} &= \frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n 1} = \frac{3a+9b+504}{a+b+26} = \frac{309}{22} \\ \Rightarrow 66a+198b+11088 &= 309a+309b+8034 \\ \Rightarrow 243a+11bb &= 3054 \\ \Rightarrow 81a+37b &= 1018 \\ \text{Median} &= 12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14 \\ \Rightarrow \frac{13}{2} - \left(\frac{a+b}{4}\right) &= 2 \Rightarrow \frac{a+b}{4} = \frac{9}{2} \\ \Rightarrow a+b &= 18 \\ \text{So, } 81a+37b &= 1018 \\ a+b &= 18 \\ a=8 \text{ and } b &= 10 \\ \therefore (a-b)^2 &= 4\end{aligned}$$

Question81

The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are [2021, 20 July Shift-1]

Options:

- A. 10, 11
- B. 3, 18
- C. 8, 13
- D. 1, 20

Answer: A

Solution:

$$\begin{aligned}\text{Mean} &= 6.5 \\ \therefore \text{Total} &= 6 \times 6.5 = 39 \\ \text{Variance} &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \\ \Rightarrow 10.25 &= \frac{\sum x_i^2}{6} - 6.5^2 \\ \therefore \sum x_i^2 &= 6(10.25 + 42.25) = 52.5 \times 6 \\ \Rightarrow 2+4+5+7+x_1+x_2 &= 39 \\ \Rightarrow x_1+x_2 &= 21 \\ \sum x_i^2 &= 52.5 \times 6 \\ 4+16+25+49+x_1^2+x_2^2 &= 315 \\ \Rightarrow x_1^2+x_2^2 &= 221 \\ \Rightarrow x_1^2+(21-x_1)^2 &= 221 \\ \Rightarrow 2x_1^2-42x_1+441 &= 221 \\ \Rightarrow x_1^2-21x_1+110 &= 0 \\ \Rightarrow (x_1-11)(x_1-10) &= 0 \\ \Rightarrow x_1 &= 10 \text{ or } 11\end{aligned}$$

Question82

If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to

[2021, 20 July Shift-II]

Options:

- A. 9
- B. 11
- C. 7
- D. 1

Answer: D

Solution:

Given, Mean = 10

$$\text{i.e. } \frac{7+10+11+15+a+b}{6} = 10$$

$$\Rightarrow a+b = 17 \dots (i)$$

$$\text{Also, variance} = \frac{20}{3}$$

$$\text{i.e. } \frac{(7)^2 + (10)^2 + (11)^2 + (15)^2 + a^2 + b^2}{6}$$

$$- (\text{Mean})^2 = \frac{20}{3}$$

$$\Rightarrow \frac{495 + a^2 + b^2}{6} - 100 = \frac{20}{3}$$

$$\Rightarrow a^2 + b^2 = 145 \dots (ii)$$

Using Eqs. (i) and (ii), put $b = 17 - a$

$$a^2 + (17 - a)^2 = 145$$

$$\Rightarrow a^2 + 289 + a^2 - 34a = 145$$

$$\Rightarrow 2a^2 - 34a + 144 = 0$$

$$\Rightarrow a^2 - 17a + 72 = 0$$

$$\Rightarrow (a - 9)(a - 8) = 0$$

$$\therefore a = 9 \text{ or } a = 8$$

Using $b = 17 - a$

$$\therefore b = 8 \text{ or } b = 9$$

$$\therefore |a - b| = 1$$

Question 83

The mean of 10 numbers $7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \dots$ is
[2021, 31 Aug. Shift-1]

Answer: 398

Solution:

$(7 \times 8), (10 \times 10), (13 \times 12), \dots$

$7, 10, 13, \dots$

$$a_n = 7 + (n - 1)3 = 3n + 4$$

$8, 10, 12, \dots$

$$b_n = 8 + (n - 1)2 = 2n + 6$$

$$\text{So, } T_n = (3n + 4)(2n + 6)$$

$$= 6n^2 + 26n + 24$$

$$\text{Sum } S_n = \sum T_n$$

$$= 6 \cdot \left[\frac{n(n+1)(2n+1)}{6} \right] + 26 \left[\frac{n(n+1)}{2} \right] + 24n$$

$$\text{Mean} = \frac{\text{sum}}{10} = \frac{10 \cdot 11 \cdot 21}{10} + \frac{13 \cdot 10 \cdot 11}{10} + \frac{24 \cdot 10}{10} = 398$$

Question84

The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is
[2021, 31 Aug. Shift-II]

Options:

A. $\frac{92}{5}$

B. $\frac{134}{5}$

C. $\frac{536}{25}$

D. $\frac{112}{5}$

Answer: C

Solution:

Let a, b, c, d and e are 5 remaining observations $n = 7$, Mean = 8, Variance = 16

Sum of observations = $7 \times 8 = 56$

Mean of remaining observations

$$= \frac{56 - 8 - 6}{5} = \frac{42}{5}$$

and Variance = $\frac{\sum x^2}{n} - (\bar{x})^2$

$$16 = \frac{\sum x^2}{7} - 64 \Rightarrow \sum x^2 = 560$$

$$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 = 560 - 8^2 - 6^2 = 460$$

Variance of 5 remaining observations

$$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{536}{25}$$

Question85

Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to
[2021, 27 Aug. Shift-1]

Answer: 13

Solution:

$$\text{Variance} = \frac{\sum x^2}{n} - (\bar{x})^2 \quad [\bar{x} = \text{means}]$$

$$14 = \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n}$$

$$- \left(\frac{1 + 2 + 3 + \dots + n}{n}\right)^2$$

$$\Rightarrow 14 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$\Rightarrow 14 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$\Rightarrow 14 = \left(\frac{n+1}{12}\right)[2(2n+1) - 3(n+1)]$$

$$\Rightarrow 168 = (n+1)(n-1)$$

$$\Rightarrow n^2 - 1 = 168$$

$$\Rightarrow n^2 = 169 \Rightarrow n = 13$$

Question86

An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to
[2021, 27 Aug. Shift-II]

Answer: 25

Solution:

Number of boys = 20

Number of girls = 30

$$\sigma_B^2 = 2, \bar{X}_B = 12, \sigma_G^2 = 2$$

$$\bar{X}_G = \frac{50 \times 15 - 20 \times 12}{30}$$

$$= \frac{510}{30} = 17 = \mu$$

Variance of 50 candidates

$$\sigma^2 = \frac{20\sigma_B^2 + 30\sigma_G^2}{50} + \frac{20 \cdot 30}{(20+30)^2}$$

$$(\bar{X}_B - \bar{X}_G)^2$$

$$= \frac{20 \times 2 + 30 \times 2}{50} + \frac{600}{2500} \times 25 = 8$$

$$\therefore \mu + \sigma^2 = 17 + 8 = 25$$

Question87

The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is
[2021, 26 Aug. Shift-1]

Options:

A. (11, 26)

B. (10.5, 25)

C. (11, 25)

D. (10.5, 26)

Answer: D

Solution:

$$\bar{x} = \frac{\sum x_i}{20} = 10$$

$$\Rightarrow \sum x_i = 200$$

$$\sigma^2 = \frac{\sum x_i^2}{20} - 100 = 6.25$$

$$\sum x_i^2 = 20 \times 106.25 = 2125$$

Now, replacing 25 with 35 as one data, then

$$\sum x_i - 25 + 35 = 210$$

$$\sum x_i^2 - 25^2 + 35^2 = 2725$$

$$\text{New mean} = \frac{210}{20} = 10.5 = (\alpha)$$

$$\text{New SD} = \sqrt{\frac{2725}{20} - (10.5)^2}$$

$$\text{SD} = \sqrt{136.25 - 110.25}$$

$$= \sqrt{26} = \sqrt{\beta}$$

$$(\alpha, \beta) = (10.5, 26)$$

Question 88

Let the mean and variance of four numbers 3, 7, x and y (x > y) be 5 and 10 respectively. Then, the mean of four numbers $3 + 2x$, $7 + 2y$, x + y and x - y is

[2021, 26 Aug. Shift-II]

Answer: 12

Solution:

Given, Mean of 3, 7x and y is 5

So, $3 + 7 + x + y = 20$

$$\Rightarrow x + y = 10 \dots (i)$$

and Variance of 3, 7, x and y is 10

$$\Rightarrow \frac{3^2 + 7^2 + x^2 + y^2}{4} = (5)^2$$

$$\Rightarrow 9 + 49 + x^2 + y^2 - 100 = 40$$

$$\Rightarrow x^2 + y^2 = 140 - 58$$

$$\Rightarrow x^2 + y^2 = 82$$

$$\Rightarrow x^2 + (10 - x)^2 = 82 \quad [\text{Using Eq. (i)}]$$

$$\Rightarrow x^2 + 100 + x^2 - 20x = 82 \quad (ii)$$

$$\Rightarrow 2x^2 - 20x + 18 = 0$$

$$\Rightarrow x^2 - 10x + 9 = 0$$

$$\Rightarrow (x - 9)(x - 1) = 0$$

$$\Rightarrow x = 1, 9 \quad (x, y) = (9, 1) \quad [\text{as } x > y]$$

$$\text{Mean} = \frac{3 + 2x + 7 + 2y + x + y + x - y}{4}$$

$$= \frac{10 + 4x + 2y}{4} = \frac{48}{4} = 12$$

Question 89

Let X be a random variable with distribution.

x	-2	-1	3	4	5
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2 , then $100\sigma^2$ is equal to
[2021, 01 Sep. Shift-II]

Answer: 781

Solution:

Given, mean $\mu = 2.3$

$$\Rightarrow \Sigma P \cdot x = 2 \cdot 3$$

$$\Rightarrow \frac{-2}{5} - a + 1 + \frac{4}{5} + 6b = 2 \cdot 3$$

$$\Rightarrow 6b - a = 0 \cdot 9 \dots (i)$$

Also, $\Sigma P = 1$

$$\frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$$

$$\Rightarrow a + b = \frac{4}{15} \dots (ii)$$

$\Rightarrow$ Solving Eqs. (i) and (ii), we get

$$a = \frac{1}{10}, b = \frac{1}{6}$$

Variance $\sigma^2 = \Sigma P x_i^2 - \mu^2$

$$= \frac{1}{5}(4) + \frac{1}{10}(1) + \frac{1}{3}(9) + \frac{1}{5}(16)$$

$$+ \frac{1}{6}(36) = 5.29$$

$$\sigma^2 = 7.81$$

$$\Rightarrow 100\sigma^2 = 781$$

Question90

Let the observations $x_i (1 \leq i \leq 10)$ satisfy the equations, $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \dots, x_{10} - 3$, then the ordered pair (μ, λ) is equal to:
[Jan. 9, 2020 (I)]

Options:

A. (3, 3)

B. (6, 3)

C. (6, 6)

D. (3, 6)

Answer: A

Solution:

$$\text{Mean of the observation } (x_i - 5) = \frac{\sum (x_i - 5)}{10} = 1$$

$$\therefore \lambda = \{\text{Mean}(x_i - 5)\}^2 + 2 = 3$$

Variance of the observation

$$\mu = \text{var}(x_i - 5) = \frac{\sum (x_i - 5)^2}{10} - \frac{\sum (x_i - 5)^2}{10} = 3$$

Question91

The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q , where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to:
[Jan. 8, 2020 (I)]

Options:

- A. -5
- B. 10
- C. -20
- D. -10

Answer: C

Solution:

Let $\bar{x}$ and σ be the mean and standard deviations of given observations.
If each observation is multiplied with p and then q is subtracted.

New mean $(\bar{x}_1) = p\bar{x} - q$

$$\Rightarrow 10 = p(20) - q \dots\dots (i)$$

and new standard deviations $\sigma_1 = |p| \sigma$

$$\Rightarrow 1 = |p|(2) \Rightarrow |p| = \frac{1}{2} \Rightarrow p = \pm \frac{1}{2}$$

If $p = \frac{1}{2}$, then $q = 0$ (from equation (i))

If $p = -\frac{1}{2}$, then $q = -20$

Question 92

The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:
[Jan. 8, 2020 (II)]

Options:

- A. 3.99
- B. 4.01
- C. 4.02
- D. 3.98

Answer: A

Solution:

Let $x_1, x_2, \dots, x_{20}$ be 20 observations, then

$$\text{Mean} = \frac{x_1 + x_2 + \dots + x_{20}}{20} = 10$$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i}{20} = 10 \dots\dots (i)$$

$$\text{Variance} = \sum x_i^2 n - (\bar{x})^2$$

$$\Rightarrow \frac{\sum x_i^2}{20} - 100 = 4 \dots\dots (ii)$$

$$\sum x_i^2 = 104 \times 20 = 2080$$

$$\text{Actual mean} = \frac{200 - 9 + 11}{20} = \frac{202}{20}$$

$$\begin{aligned}\text{Variance} &= \frac{2080 - 81 + 121}{20} - \left(\frac{202}{20}\right)^2 \\ &= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99\end{aligned}$$

Question93

If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to _____.
[7-Jan-2020 Shift 1]

Answer: 18

Solution:

Step -1: Find n using variance of first n natural number is .

$$\text{var}(1, 2, 3, \dots, n) = 10$$

Using formula for variance we have,

$$\Rightarrow \frac{1^2 + 2^2 + \dots + n^2}{n} - \left(\frac{1 + 2 + \dots + n}{n}\right)^2 = 10$$

$$\left[\text{Since, } \sigma^2 = \frac{\sum f.d}{N} - \left(\frac{\sum f.d}{N}\right)^2 \right]$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$$

$$\Rightarrow \frac{n^2 - 1}{12} = 10$$

$$\Rightarrow n^2 - 1 = 120$$

$$\Rightarrow n^2 = 121$$

$$\Rightarrow n = 11$$

Step -2: The variance of the first m even natural number:

$$\Rightarrow \text{var}(2, 4, 6 \dots 2m) = 16$$

$$\Rightarrow 2^2 \text{var}(1, 2, \dots, m) = 16$$

$$\Rightarrow \text{var}(1, 2, \dots, m) = \frac{16}{2^2} = 4$$

$$\Rightarrow \frac{m^2 - 1}{12} = 4$$

$$\Rightarrow m^2 - 1 = 48$$

$$\Rightarrow m^2 = 49$$

$$\Rightarrow m = 7$$

Step -3: Evaluate $m + n$.

$$m + n = 7 + 11$$

$$\Rightarrow m + n = 18.$$

The value of $m + n = 18$.

Question94

If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _____.
[NA Jan. 7, 2020 (II)]

Answer: 52

Solution:

$$\text{Mean} = \bar{x} = \frac{3+7+9+12+13+20+x+y}{8} = 10$$

$$\Rightarrow x+y = 16 \dots\dots(i)$$

$$\text{Variance} = \sigma^2 = \frac{\sum(x_i)^2}{8} - (\bar{x})^2 = 25$$

$$\sigma^2 = \frac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$$

$$\Rightarrow x^2+y^2 = 148$$

$$\text{From eqn. (i), } (x+y)^2 = (16)^2$$

$$\Rightarrow x^2+y^2+2xy = 256$$

$$\text{Using eqn. (ii), } 148+2xy = 256$$

$$\Rightarrow xy = 52$$

Question95

Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____.

[NA Sep. 06, 2020 (II)]

Answer: 6

Solution:

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{0 \cdot {}^nC_0 + 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \dots + 2^n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + \dots + {}^nC_n}$$

To find sum of numerator consider

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n \dots\dots(i)$$

$$\text{Put } x=2 \Rightarrow 3^n - 1 = 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \dots + 2^n \cdot {}^nC_n$$

To find sum of denominator, put $x=1$ in (i), we get

$$2^n = {}^nC_0 + {}^nC_1 + \dots + {}^nC_n$$

$$\therefore \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3^n = 729 \Rightarrow n = 6$$

Question96

The minimum value of $2^{\sin x} + 2^{\cos x}$ is:

[Sep. 04, 2020 (II)]

Options:

A. $2^{-1+\frac{1}{\sqrt{2}}}$

B. $2^{-1+\sqrt{2}}$

C. $2^{1-\sqrt{2}}$

D. $2^{1-\frac{1}{\sqrt{2}}}$

Answer: D

Solution:

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq (2^{\sin x + \cos x})^{\frac{1}{2}} \quad (\because \text{AM} \geq \text{GM})$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2 \cdot 2^{\frac{\sin x + \cos x}{2}}$$

$$\text{Since, } -2 \leq \sin x + \cos x \leq \sqrt{2}$$

$$\therefore \text{Minimum value of } 2^{\frac{\sin x + \cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{-\frac{1}{\sqrt{2}}}$$

Question 97

If $\sum_{i=1}^n (x_i - a) = n$ and $\sum_{i=1}^n (x_i - a)^2 = na$, ($n, a > 1$), then the standard deviation of n observations

$x_1, x_2, \dots, x_n$ is :

[Sep. 06, 2020 (I)]

Options:

A. $a - 1$

B. $n\sqrt{a - 1}$

C. $\sqrt{n(a - 1)}$

D. $\sqrt{a - 1}$

Answer: D

Solution:

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\frac{\sum_{i=1}^n (x_i - a)^2}{n} - \left(\frac{\sum_{i=1}^n (x_i - a)}{n} \right)^2} \quad [\because n, a > 1] \\ &= \sqrt{\frac{na}{n} - \left(\frac{n}{n} \right)^2} = \sqrt{a - 1} \end{aligned}$$

Question 98

The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :

[Sep. 05, 2020 (I)]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Let two remaining observations are x_1, x_2 .

$$\text{So, } \bar{x} = \frac{2+4+10+12+14+x_1+x_2}{7} = 8 \text{ (given)}$$

$$\Rightarrow x_1 + x_2 = 14 \dots\dots(i)$$

$$\text{Now, } \sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N} \right)^2 = 16 \text{ (given)}$$

$$= \frac{4+16+100+144+196+x_1^2+x_2^2}{7} - 64 = 16$$

$$\Rightarrow 460 + x_1^2 + x_2^2 = (16+64) \times 7$$

$$\Rightarrow x_1^2 + x_2^2 = 100 \dots\dots(ii)$$

$$\therefore (x+y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48 \dots\dots(iii)$$

$$\therefore (x-y)^2 = (x+y)^2 - 4xy = 196 - 192 = 4$$

$$\Rightarrow x-y=2 \Rightarrow |x-y| = 2$$

Question99

If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

[Sep. 05, 2020 (II)]

Options:

A. $x^2 - 10x + 18 = 0$

B. $2x^2 - 20x + 19 = 0$

C. $x^2 - 10x + 19 = 0$

D. $x^2 - 20x + 18 = 0$

Answer: C

Solution:

$$\text{Mean} = \frac{3+5+7+a+b}{5} = 5 \Rightarrow a+b = 10$$

$$\text{Variance} = \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$$

$$\Rightarrow a^2 + b^2 = 62$$

$$\Rightarrow (a+b)^2 - 2ab = 62$$

$$\Rightarrow ab = 19$$

Hence, a and b are the roots of the equation,

$$x^2 - 10x + 19 = 0$$

Question100

The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

[Sep. 04, 2020 (I)]

Options:

A. 9

B. 5

C. 3

D. 7

Answer: D

Solution:

Let the two remaining observations be x and y .

$$\therefore \bar{x} = \frac{5+7+10+12+14+15+x+y}{8}$$

$$\Rightarrow 10 = \frac{63+x+y}{8}$$

$$\Rightarrow x+y = 80-63$$

$$\Rightarrow x+y = 17 \dots\dots(i)$$

$$\therefore \text{var}(x) = 13.5$$

$$= \frac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$$

$$\Rightarrow x^2+y^2 = 169 \dots\dots(ii)$$

From (i) and (ii) we get

$$(x, y) = (12, 5) \text{ or } (5, 12)$$

$$\text{So, } |x-y| = 7$$

Question101

If a variance of the following frequency distribution :

Class	10-20	20-30	30-40
Frequency	2	x	2

is 50, then x is equal to _____.
[NA Sep. 04, 2020 (II)]

Answer: 4

Solution:

x_i	15	25	35
f_i	2	x	2

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{30+70+25x}{4+x} = 25$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$$

$$\Rightarrow 50 = \frac{450+625x+2450}{4+x} - 625$$

$$\Rightarrow 675 = \frac{2900+625x}{4+x} \Rightarrow 50x = 200$$

$$\therefore x = 4$$

Question102

For the frequency distribution :

Variate (x) :	x_1	x_2	$x_1 \dots x_{15}$
Frequency (f) :	f_1	f_2	$f_3 \dots f_{15}$

where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :
[Sep. 03, 2020 (I)]

Options:

- A. 4
- B. 1
- C. 6
- D. 2

Answer: C

Solution:

If variate varies from a to b then variance

$$\text{var}(x) \leq \left(\frac{b-a}{2}\right)^2$$

$$\Rightarrow \text{var}(x) < \left(\frac{10-0}{2}\right)^2$$

$$\Rightarrow \text{var}(x) < 25$$

$$\Rightarrow \text{standard deviation} < 5$$

It is clear that standard deviation can't be 6 .

Question103

Let $x_i (1 \leq i \leq 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$ where $0 \neq p \in \mathbb{R}$, then the standard deviation of these observations is :
[Sep. 03, 2020 (II)]

Options:

- A. $\sqrt{\frac{3}{5}}$
- B. $\frac{4}{5}$
- C. $\frac{9}{10}$
- D. $\frac{7}{10}$

Answer: C

Solution:

$$\begin{aligned} \text{S.D.} &= \sqrt{\frac{\sum_{i=1}^{10} (x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10} (x_i - p)}{10}\right)^2} \\ &= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10} \end{aligned}$$

Question104

Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in \mathbb{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to :

[Sep. 02, 2020 (I)]

Options:

- A. 7
- B. -7
- C. -27
- D. 9

Answer: B

Solution:

$$\bar{x} = \frac{1+2+3+\dots+17}{17} = \frac{17 \times 18}{17 \times 2} = 9$$

$$\bar{y} = \overline{ax + b} = \frac{a(1+2+3+\dots+17)}{17} + b = 17$$

$$\Rightarrow \frac{a \cdot (17 \cdot 18)}{17 \cdot 2} + b = 17 \Rightarrow 9a + b = 17 \dots (i)$$

$$\text{Var}(x) = \sigma_A^2 = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$= \frac{1^2 + 2^2 + \dots + 17^2}{17} - (9)^2$$

$$= \frac{17 \cdot 18 \cdot 35}{6 \cdot 17} - (9)^2 = 105 - 81 = 24$$

$$\text{Var}(y) = a^2 \text{Var}(x) = a^2 \cdot 24 = 216$$

$$a^2 = \frac{216}{24} = 9 \Rightarrow a = 3$$

$$\therefore \text{From (i), } b = 17 - 9a = 17 - 27 = -10$$

$$\therefore a + b = 3 + (-10) = -7$$

Question 105

If the variance of the terms in an increasing A.P. $b_1, b_2, b_3, \dots, b_{11}$ is 90, then the common difference of this A.P. is _____.

[NA Sep. 02, 2020 (II)]

Answer: 3

Solution:

$$\text{Variance} = \frac{\sum_{i=1}^{11} b_i^2}{11} - \left(\frac{\sum_{i=1}^{11} b_i}{11} \right)^2$$

Let common difference of A.P. be d

$$= \frac{\sum_{r=0}^{10} (b_1 + rd)^2}{11} - \left(\frac{\sum_{r=0}^{10} (b_1 + rd)}{11} \right)^2$$

$$= \frac{11b_1^2 + 2b_1d \left(\frac{10 \times 11}{2} \right) + d^2 \left(\frac{10 \times 11 \times 21}{6} \right)}{11} - \left(\frac{11b_1 + \frac{10 \times 11}{2}d}{11} \right)^2$$

$$= (b_1^2 + 10b_1d + 35d^2) - (b_1 + 5d)^2 = 10d^2$$

$$\therefore \text{Variance} = 90 \text{ (Given)}$$

$$\Rightarrow 10d^2 = 90 \Rightarrow d = 3$$