



Sets

Question1

Let S be the set of the first 11 natural numbers. Then the number of elements in $A = \{B \subseteq S : n(B) \geq 2 \text{ and the product of all elements of } B \text{ is even}\}$ is _____.

JEE Main 2026 (Online) 22nd January Evening Shift

Answer: 1979

Solution:

Let $S = \{1, 2, \dots, 11\}$. The product of all elements of B is even iff B contains at least one even number.

- Evens in S : 5 (2, 4, 6, 8, 10)
- Odds in S : 6

Number of subsets containing at least one even:

$$2^{11} - 2^6 = 2048 - 64 = 1984$$

(we subtract the subsets made only of odds).

Now enforce $n(B) \geq 2$: subtract the 5 singleton subsets $\{2\}, \{4\}, \{6\}, \{8\}, \{10\}$.

$$1984 - 5 = 1979$$

1979

Question2

Let $A = \{2, 3, 4, 5, 6\}$. Let R be a relation on the set $A \times A$ given by $(x, y)R(z, w)$ if and only if x divides z and $y \leq w$. Then the number of elements in R is _____.

JEE Main 2026 (Online) 2nd April Evening Shift

Answer: 120

Solution:

We are given the set $A = \{2, 3, 4, 5, 6\}$.

The relation R is defined on the Cartesian product $A \times A$, meaning its elements are ordered pairs of the form $((x, y), (z, w))$ where all $x, y, z, w \in A$.

The condition for the relation $(x, y)R(z, w)$ is:

1. x divides z
2. $y \leq w$

Notice that the condition on x and z is completely independent of the condition on y and w . Therefore, by the Fundamental Principle of Counting, the total number of elements in R will be the number of possible pairs for (x, z) multiplied by the number of possible pairs for (y, w) .

Find the number of pairs (x, z) where x divides z

Let us list down the possible values for $z \in A$ for each $x \in A$.

- If $x = 2$, then z can be 2, 4, or 6. (This gives 3 pairs)
- If $x = 3$, then z can be 3 or 6. (This gives 2 pairs)
- If $x = 4$, then z can only be 4. (This gives 1 pair)
- If $x = 5$, then z can only be 5. (This gives 1 pair)
- If $x = 6$, then z can only be 6. (This gives 1 pair)

Adding these up, the total number of pairs for (x, z) is:

$$3 + 2 + 1 + 1 + 1 = 8$$

Find the number of pairs (y, w) where $y \leq w$

Let us list down the possible values for $w \in A$ for each $y \in A$.

- If $y = 2$, then w can be 2, 3, 4, 5, or 6. (This gives 5 pairs)
- If $y = 3$, then w can be 3, 4, 5, or 6. (This gives 4 pairs)
- If $y = 4$, then w can be 4, 5, or 6. (This gives 3 pairs)
- If $y = 5$, then w can be 5 or 6. (This gives 2 pairs)
- If $y = 6$, then w can only be 6. (This gives 1 pair)

Adding these up, the total number of pairs for (y, w) is:

$$5 + 4 + 3 + 2 + 1 = 15$$

Calculate the total number of elements in R

Now, we multiply the number of possibilities for (x, z) and (y, w) together:

$$\text{Total elements in } R = 8 \times 15 = 120$$

Therefore, the number of elements in R is 120.

Question3

Let $A = \{1, 4, 7\}$ and $B = \{2, 3, 8\}$. Then the number of elements, in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in ((A \times B) \times (A \times B)) : a_1 + b_2 \text{ divides } a_2 + b_1\}$ is ____.

JEE Main 2026 (Online) 5th April Evening Shift

Answer: 18

Solution:

We need to count the number of ordered pairs

$$((a_1, b_1), (a_2, b_2)) \in (A \times B) \times (A \times B)$$

such that

$$a_1 + b_2 \text{ divides } a_2 + b_1.$$

Given

$$A = \{1, 4, 7\}, \quad B = \{2, 3, 8\}$$

first note that

$$A \times B = \{(1, 2), (1, 3), (1, 8), (4, 2), (4, 3), (4, 8), (7, 2), (7, 3), (7, 8)\}$$

So there are 9 elements in $A \times B$, and hence $9 \times 9 = 81$ ordered pairs in $(A \times B) \times (A \times B)$.

We now count only those for which

$$a_1 + b_2 \mid a_2 + b_1.$$

Let

$$x = a_1 + b_2, \quad y = a_2 + b_1.$$

Since $a_1, a_2 \in \{1, 4, 7\}$ and $b_1, b_2 \in \{2, 3, 8\}$, possible values of such sums are:

$$1 + 2 = 3, 1 + 3 = 4, 1 + 8 = 9,$$

$$4 + 2 = 6, 4 + 3 = 7, 4 + 8 = 12,$$

$$7 + 2 = 9, 7 + 3 = 10, 7 + 8 = 15.$$

So possible values are

$$3, 4, 6, 7, 9, 10, 12, 15.$$

Now we count how many times each value occurs as a sum of one element from A and one from B :

- 3 occurs once: (1, 2)
- 4 occurs once: (1, 3)
- 6 occurs once: (4, 2)
- 7 occurs once: (4, 3)
- 9 occurs twice: (1, 8), (7, 2)
- 10 occurs once: (7, 3)
- 12 occurs once: (4, 8)
- 15 occurs once: (7, 8)

Let this frequency be $f(s)$.

Now for each possible value of $x = a_1 + b_2$, we count possible $y = a_2 + b_1$ such that $x \mid y$.

Also, number of ways to get a given x is $f(x)$, and number of ways to get a given y is $f(y)$.

Hence total number required is

$$\sum_{x|y} f(x)f(y)$$

where x, y run over the above sums.

Now check each x :

1. For $x = 3$

Multiples among the possible sums:

$$3, 6, 9, 12, 15$$

So contribution is

$$f(3)(f(3) + f(6) + f(9) + f(12) + f(15))$$

$$= 1(1 + 1 + 2 + 1 + 1) = 6$$

2. For $x = 4$

Multiples among possible sums:

$$4, 12$$

Contribution:

$$1(1 + 1) = 2$$

3. For $x = 6$

Multiples among possible sums:

$$6, 12$$

Contribution:

$$1(1 + 1) = 2$$

4. For $x = 7$

Multiples among possible sums:

$$7$$

Contribution:

$$1 \cdot 1 = 1$$

5. For $x = 9$

Multiples among possible sums:

$$9$$

Contribution:

$$2 \cdot 2 = 4$$

6. For $x = 10$

Multiples among possible sums:

$$10$$

Contribution:

$$1 \cdot 1 = 1$$

7. For $x = 12$

Multiples among possible sums:

$$12$$

Contribution:

$$1 \cdot 1 = 1$$

8. For $x = 15$

Multiples among possible sums:

$$15$$

Contribution:

$$1 \cdot 1 = 1$$

Now add all contributions:

$$6 + 2 + 2 + 1 + 4 + 1 + 1 + 1 = 18$$

Therefore, the number of elements in the relation R is

18

Question4

Let $R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : \log_e(x + y) \leq 2\}$. Then the minimum number of elements, required to be added in R to make it a transitive relation, is ____ .

JEE Main 2026 (Online) 6th April Evening Shift

Answer: 15

Solution:

We have

$$R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : \ln(x + y) \leq 2\}.$$

To make the relation transitive, we first find all pairs already in R .

Since

$$\ln(x + y) \leq 2 \iff x + y \leq e^2.$$

Now,

$$e^2 \approx 7.389.$$

So, for natural numbers x, y , we must have

$$x + y \leq 7.$$

Hence,

$$R = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x + y \leq 7\}.$$

Now list all such pairs:

- For $x = 1$: $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$
- For $x = 2$: $(2, 1), (2, 2), (2, 3), (2, 4), (2, 5)$
- For $x = 3$: $(3, 1), (3, 2), (3, 3), (3, 4)$
- For $x = 4$: $(4, 1), (4, 2), (4, 3)$
- For $x = 5$: $(5, 1), (5, 2)$
- For $x = 6$: $(6, 1)$

So these are the elements of R .

A relation is transitive if whenever

$$(a, b) \in R \quad \text{and} \quad (b, c) \in R,$$

then

$$(a, c) \in R.$$

We need to find all pairs (a, c) that must be present because of transitivity.

Step 1: Find which middle elements are possible

If $(a, b) \in R$, then

$$a + b \leq 7.$$

If $(b, c) \in R$, then

$$b + c \leq 7.$$

For a fixed b , this gives

$$a \leq 7 - b, \quad c \leq 7 - b.$$

So for each b , transitivity demands that all pairs

$$(a, c) \quad \text{with } 1 \leq a \leq 7 - b, \quad 1 \leq c \leq 7 - b$$

must be in the relation.

Taking union over all possible $b \in \mathbb{N}$ with at least one such pair, we get:

- For $b = 1$: required pairs are all

$$(a, c), \quad 1 \leq a \leq 6, \quad 1 \leq c \leq 6.$$

This is already the largest possible set forced by transitivity.

So, if relation is to be transitive, it must contain **every pair**

$$(a, c) \in \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}.$$

That means the transitive closure must at least contain all 36 pairs

$$(a, c), \quad 1 \leq a, c \leq 6.$$

Step 2: Count how many of these are already in R

Among these 36 pairs, R already contains those with

$$a + c \leq 7.$$

Count them:

- $a = 1$: $c = 1$ to $6 \Rightarrow 6$
- $a = 2$: $c = 1$ to $5 \Rightarrow 5$
- $a = 3$: $c = 1$ to $4 \Rightarrow 4$
- $a = 4$: $c = 1$ to $3 \Rightarrow 3$
- $a = 5$: $c = 1$ to $2 \Rightarrow 2$
- $a = 6$: $c = 1 \Rightarrow 1$

Total:

$$6 + 5 + 4 + 3 + 2 + 1 = 21.$$

So the number of additional pairs needed is

$$36 - 21 = 15.$$

Step 3: Check that this is sufficient

If we add all missing pairs among

$$\{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\},$$

then the relation becomes

$$\{(a, c) : 1 \leq a \leq 6, \quad 1 \leq c \leq 6\}.$$

This is clearly transitive, because whenever

$$(a, b) \in R \quad \text{and} \quad (b, c) \in R,$$

all three numbers lie in $\{1, 2, 3, 4, 5, 6\}$, so (a, c) is also in the relation.

Therefore, the minimum number of elements to be added is

15

Question5

The number of relations, defined on the set $\{a, b, c, d\}$, which are both reflexive and symmetric, is equal to:

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Options:

A.

16

B.

64

C.

256

D.

1024

Answer: B

Solution:

Number of such relations:

From $A \rightarrow A$ such that

$$n(A) = N \text{ is}$$
$$\Rightarrow 2^{\left(\frac{N^2-N}{2}\right)}$$

$$\text{for } N = 4 \Rightarrow 2^6 = 64$$

Alter :

$$\begin{bmatrix} 0 & - & - & - \\ \Theta & 0 & - & - \\ \Theta & \Theta & 0 & - \\ \Theta & \Theta & \Theta & 0 \end{bmatrix} \Rightarrow \text{each } \Theta \text{ has two choices}$$
$$\Rightarrow 2^6 = 64$$

Question6

Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by xRy if and only if $2x \leq 3y$. Let l be the number of elements in R , and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $l + m$ is equal to:

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Options:

A.

21

B.

25

C.

23

D.

27

Answer: B

Solution:

For $x = 2$

$$4 \leq 3y \quad y \in \{2, 3, 5, 7, 9\}$$
$$(2, 2)(2, 3)(2, 5)(2, 7)(2, 9)$$

For $x = 3$

$$6 \leq 3y \quad y \in \{2, 3, 5, 7, 9\}$$
$$(3, 2)(3, 3)(3, 5)(3, 7)(3, 9)$$

For $x = 5$

$$10 \leq 3y \quad y \in \{5, 7, 9\}$$

$$(5, 5)(5, 7)(5, 9)$$

For $x = 7$

$$14 \leq 3y \quad y \in \{5, 7, 9\}$$

$$(7, 5)(7, 7)(7, 9)$$

For $x = 9$

$$18 \leq 3y \quad y \in \{7, 9\}$$

$$(9, 7)(9, 9)$$

$$R = 5 + 5 + 3 + 3 + 2 = 18$$

$$(2, 3) \in R \text{ but } (3, 2) \notin R \text{ (symmetric)}$$

$$(2, 5) \in R \text{ but } (5, 2) \notin R \text{ Add}(5, 2)$$

$$(2, 7) \in R \text{ but } (7, 2) \notin R \text{ Add}(7, 2)$$

$$(2, 9) \in R \text{ but } (9, 2) \notin R \text{ Add}(9, 2)$$

$$(3, 5) \in R \text{ but } (5, 3) \notin R \text{ Add}(5, 3)$$

$$(3, 7) \in R \text{ but } (7, 3) \notin R \text{ Add}(7, 3)$$

$$(3, 9) \in R \text{ but } (9, 3) \notin R \text{ Add}(9, 3)$$

$$(5, 7) \in R \text{ but } (7, 5) \in R$$

$$(5, 9) \in R \text{ but } (9, 5) \notin R \text{ Add}(9, 5)$$

$$(7, 9) \in R \text{ but } (9, 7) \in R$$

$$18 + 7 = 25$$

Question 7

Let $A = \{x : |x^2 - 10| \leq 6\}$ and $B = \{x : |x - 2| > 1\}$. Then

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Options:

A.

$$A \cup B = (-\infty, 1] \cup (2, \infty)$$

B.

$$B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$$

C.

$$A - B = [2, 3)$$

D.

$$A \cap B = [-4, -2] \cup [3, 4]$$

Answer: B

Solution:

For set A , we have the inequality:

$$|x^2 - 10| \leq 6$$

This means that the expression inside the modulus lies between -6 and 6 :

$$-6 \leq x^2 - 10 \leq 6$$

By adding 10 to all parts of the inequality, we get:

$$4 \leq x^2 \leq 16$$

Taking the square root gives us the range for $|x|$:

$$2 \leq |x| \leq 4$$

This tells us that the value of x can be positive or negative, giving us the intervals:

$$A = [-4, -2] \cup [2, 4]$$

Now, let's find the elements of set B from its inequality:

$$|x - 2| > 1$$

This breaks down into two conditions:

$$x - 2 < -1 \quad \text{or} \quad x - 2 > 1$$

Solving these, we get:

$$x < 1 \quad \text{or} \quad x > 3$$

Writing this in interval notation, we have:

$$B = (-\infty, 1) \cup (3, \infty)$$

Now we check the options by performing the set operations. Let's find $B - A$, which represents the elements present in B but not in A .

We take the set B and remove any overlapping parts from A .

Looking at the first part of B , which is $(-\infty, 1)$, we need to remove the interval $[-4, -2]$ that belongs to A . Since the endpoints -4 and -2 are in A , they will be removed, leaving open brackets. This breaks the remaining interval into:

$$(-\infty, -4) \cup (-2, 1)$$

Next, looking at the second part of B , which is $(3, \infty)$, we need to remove the elements from A that overlap here. The set A contains $[2, 4]$. Removing this portion from $(3, \infty)$ cuts off everything up to 4. Since 4 is in A , it is removed, so we use an open bracket starting from 4:

$$(4, \infty)$$

Combining these remaining parts together, we get:

$$B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$$

This exactly matches Option B.

Option B is the correct choice.

Question8

Let the relation R on the set $M = \{1, 2, 3, \dots, 16\}$ be given by $R = \{(x, y) : 4y = 5x - 3, x, y \in M\}$.

Then the minimum number of elements required to be added in R , in order to make the relation symmetric, is equal to

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Options:

A.

4

B.

3

C.

1

D.

2

Answer: D

Solution:

$$M = \{1, 2, 3, \dots, 16\}$$

$$R = \{(x, y) : 4y = 5x - 3, x, y \in N\}$$

$$4y = 5x - 3$$

$$y = \frac{5x - 3}{4}$$

$$\text{No of order pairs} = 4$$

Question9

The number of elements in the relation $R = \{(x, y) : 4x^2 + y^2 < 52, x, y \in \mathbf{Z}\}$ is

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Options:

A.

86

B.

67

C.

89

D.

77

Answer: D

Solution:

$$4x^2 + y^2 < 52, x, y \in \mathbf{Z}$$

$$0 \quad 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7 \rightarrow 1 \times 15 = 15$$

$$\pm 1 \quad 0, \pm 1, \pm 2, \pm 3 \dots \dots \dots \pm 6 \rightarrow 2 \times 13 = 26$$

$$\pm 2 \quad 0, \pm 1, \pm 2, \pm 3 \dots \dots \dots \pm 5 \rightarrow 2 \times 11 = 22$$

$$\pm 3 \quad 0, \pm 1, \pm 2, \pm 3 \rightarrow 2 \times 7 = 14$$

$$\text{Number of elements} = 77$$

Question 10

Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Let R be a relation on A defined by xRy if and only if $2x + y \leq 2$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then $l + m + n$ is equal to :

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Options:

A.

34

B.

32

C.

Answer: C**Solution:**

$\frac{x}{y}$	(-2)	(-1)	0	1	2	3	4
-2	P	P	P	P	P	A	A
-1	P	P	P	P	A	A	A
0	P	P	P	P	A		
1	P	P	P	A			
2	P	P	P		A		
3	P	P				A	
4	P	P					A

Here P implies, A implies absent

 $\Rightarrow$ 10 elements to be added as $A = 10, P = 23$ $\Rightarrow A + P = 33$

Question 11

Let $A = \{0, 1, 2, \dots, 9\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $|x - y|$ is a multiple of 3.

Given below are two statements :

Statement I : $n(R) = 36$.

Statement II : R is an equivalence relation.

In the light of the above statements, choose the *correct* answer from the options given below :

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Options:

A.

Statement I is correct but Statement II is incorrect

B.

Both Statement I and Statement II are correct

C.

Both Statement I and Statement II are incorrect

D.

Statement I is incorrect but Statement II is correct

Answer: D

Solution:

$$A = \{0, 1, 2, \dots, 9\}$$

A relation R on A

$R = \{x - y \mid x - y \text{ is multiple of } 3\}$

0, 3, 6, 9 are $3n$ type numbers

1, 4, 7 are $3n + 1$ type numbers

2, 5, 8 are $3n + 2$ type numbers

Case 1 : Relation R is valid if x & y both are $3n$ type.

$$= 4 \times 4 = 16 \text{ pairs}$$

Case 2 : Relation R is valid if x is $3n + 1$ type & y is $3n + 2$ type. $= 3 \times 3 = 9$ pairs

Case 3 : similarly relation R is valid if x is $3n + 2$ type & y is $3n + 1$ type. $= 3 \times 3 = 9$ pairs

$$n(R) = 16 + 9 + 9 = 34 \neq 36$$

statement I is false.

A relation is an equivalence relation if it is reflexive, symmetric, and transitive.

Reflexive: For any $x \in A$, $|x - x| = 0$. Since 0 is a multiple of 3, $(x, x) \in R$.

Symmetric: If $(x, y) \in R$, then $|x - y| = 3k$. Since $|y - x| = |x - y|$, then $|y - x| = 3k$, so $(y, x) \in R$.

Transitive: If $|x - y|$ and $|y - z|$ are multiples of 3, then $x - y = 3k$ and $y - z = 3m$.

Adding these gives $x - z = 3(k + m)$, so $|x - z|$ is a multiple of 3.

Statement II is correct.

Question 12

Consider two sets $A = \{x \in \mathbb{Z} : (|x - 3| - 3)| \leq 1\}$ and

$B = \left\{x \in \mathbb{R} - \{1, 2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0\right\}$. Then the number of

onto functions $f : A \rightarrow B$ is equal to :

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Options:

A.

32

B.

81

C.

79

D.

62

Answer: D**Solution:**

$$\text{Set } A = \{x \in \mathbb{Z} : |(|x-3|-3)| \leq 1\}$$

$$|(|x-3|-3)| \leq 1 \quad \{ |x| \leq a \Rightarrow x \in [-a, a] \}$$

$$\Rightarrow (|x-3|-3) \in [-1, 1]$$

$$\Rightarrow |x-3| \in [2, 4]$$

$$\text{Using } |x| \in [\alpha, \beta] \Rightarrow x \in [-\beta, -\alpha] \cup [\alpha, \beta]$$

$$x-3 \in [-4, -2] \cup [2, 4]$$

$$\Rightarrow x \in [-1, 1] \cup [5, 7]$$

Since $x \in \mathbb{Z}$ (integers), $x = -1, 0, 1, 5, 6, 7$

So set $A = \{-1, 0, 1, 5, 6, 7\}$

$$\text{Set } B = \left\{ x \in \mathbb{R} - \{1, 2\}, \frac{(x-2)(x-4)}{(x-1)} \log_e(|x-2|) = 0 \right\}$$

$$\frac{(x-2)(x-4)}{(x-1)} \log_e(|x-2|) = 0$$

$$\Rightarrow x-2 = 0 \Rightarrow x = 2 \text{ but } x \neq 2$$

$$\Rightarrow x-4 = 0 \Rightarrow x = 4$$

$$\Rightarrow \log_e |x-2| = 0 \Rightarrow |x-2| = 1$$

$$x-2 = \pm 1 \Rightarrow x = 3, 1 \text{ but } x \neq 1$$

So set $B = \{3, 4\}$

Let $n(A)$ & $n(B)$ be number of elements in set A and B respectively

$$n(A) = 6, n(B) = 2$$

Number of onto function $f: A \rightarrow B$ is

= Total function - Into function

$$\text{Total function} = (n(B))^{n(A)} = 2^6$$

number of Into function can be find using inclusion-exclusion principle

$$\text{Number of into function} = {}^2C_1(1)^6$$

$${}^2C_1(1)^6 = \text{only one image or range left from set } B.$$

$$\text{Number of onto function} = 2^6 - {}^2C_1(1)^6$$

$$= 64 - 2 = 62$$

Question 13

Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by

$$R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}.$$

Then the number of elements in R is

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Options:

A.

6

B.

15

C.

12

D.

18

Answer: C

Solution:

$$\text{let } A = \{1, 2, 3, 4\}$$

R is defined on $A \times A$ such that

$$R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}$$

since $a, b, c, d \in A$

Let possible values of $2a + 3b = L1$ and possible values of $3c + 4d = L2$

a	b	$2a + 3b$	$3c + 4d$
1	1	5	7
1	2	8	11
1	3	11	15
1	4	14	19
2	1	7	10
2	2	10	14
2	3	13	18
2	4	16	22
3	1	9	13
3	2	12	17
3	3	15	21
3	4	18	25
4	1	11	16
4	2	14	20
4	3	17	24
4	4	20	28

Common values of $2a + 3b = 3c + 4d$

7, 10, 11, 13, 14, 15, 16, 17, 18, 20

value 7, 10, 13, 15, 16, 17, 18, 20

occurs 1 time in $L1$ and 1 time in $L2$

e.g value 7 occurs 1 time in L1 & 1 time in L2 total = $1 \times 1 = 1$

so, total number of these types of elements

in R = 8--- (1)

value 11 and 14 occurs 2 times in L1

and 1 time in L2

e.g value 11 occurs 2 time in L1 & 1 time in L2 total

elements in R is $2 \times 1 = 2$

so total number these type of elements = $2 + 2 = 4$ --- (2)

total number of elements in R = $8 + 4 = 12$

Question 14

Consider the relation R on the set $\{-2, -1, 0, 1, 2\}$ defined by $(a, b) \in R$ if and only if $1 + ab > 0$. Then, among the statements :

I. The number of elements in R is 17

II. R is an equivalence relation

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Options:

A.

Only I is true

B.

Only II is true

C.

Both I and II are true

D.

Neither I nor II is true

Answer: A

Solution:

We first observe the relation R defined on

$S \times S$ (a, b) is an ordered pair $\in R$ iff $(1 + ab) > 0 = ab > -1 \dots (i)$

Refer the chart below :

a	b	ab $\in R$
-2	-2	4 Yes
-1	-1	2 Yes

0 0 0 Yes
 1 1 2 Yes
 2 2 4 Yes
 -2 -1 2 Yes
 Yes
 Yes

We also observe the mathematical relationship from (i) :

$$ab > -1 \Rightarrow ba > -1 \equiv (a, b) \in R \Rightarrow (b, a) \in R \dots \dots (ii) \leftarrow \text{symmetric}$$

We take 2 ordered pair entries from R :

Ordered pair (a, b)	$(2, 0)$	$ab > -1$	$(0, -2)$	$ab > -1$	$(2, -2)$	$ab \not> -1$	Transitive
ab	0	$0 > -1$	0	$0 > -1$	-4	$-4 < -1$	
$\in R$	Yes		Yes		No		No

Relation	Reflexive	Symmetric	Transitive
R	Yes	Yes	No

Now let us focus on the cardinality of

$$R \in S \times S; (ab > -1)$$

S	R	a	b	No of entries	Cumulative Total
$\{-2, -1, 0, 1, 2\}$	$ab > -1$	-2	$\in \{-2, -1, 0\}$	3	3
$\{-2, -1, 0, 1, 2\}$	$ab > -1$	-1	$\in \{-2, -1, 0\}$	3	6
$\{-2, -1, 0, 1, 2\}$	$ab > -1$	0	$\in S$	5	11
$\{-2, -1, 0, 1, 2\}$	$ab > -1$	1	$\in \{0, 1, 2\}$	3	14
$\{-2, -1, 0, 1, 2\}$	$ab > -1$	2	$\in \{0, 1, 2\}$	3	17

I	The number of elements in R is 17	Valid	Option
II	R is an equivalence relation	Invalid	A

Question 15

Let $A = \{1, 2, 3\}$. The number of relations on A, containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _____.

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Answer: 3

Solution:

Transitivity

$$(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$$

For reflexive $(1, 1), (2, 2), (3, 3) \in R$

Now $(2, 1), (3, 2), (3, 1)$

$(3, 1)$ cannot be taken

(1) $(2, 1)$ taken and $(3, 2)$ not taken

(2) (3, 2) taken and (2, 1) not taken

(3) Both not taken

therefore 3 relations are possible.

Question 16

Let $S = \{p_1, p_2, \dots, p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S . Then the number of all ordered pairs (x, y) , $x \in S$, $y \in A$, such that x divides y , is _____.

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Answer: 5120

Solution:

$$\begin{aligned}\text{Let } \frac{y}{x} &= \lambda \\ y &= \lambda x \\ &= 10 \times ({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + \dots + {}^9C_9) \\ &= 10 \times (2^9) \\ &= 10 \times 512 \\ &= 5120\end{aligned}$$

Question 17

For $n \geq 2$, let S_n denote the set of all subsets of $\{1, 2, \dots, n\}$ with no two consecutive numbers. For example $\{1, 3, 5\} \in S_6$, but $\{1, 2, 4\} \notin S_6$. Then $n(S_5)$ is equal to _____

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Answer: 13

Solution:

To find $n(S_5)$, which is the number of subsets of $\{1, 2, 3, 4, 5\}$ with no consecutive numbers, we start by enumerating these subsets.

Let's denote the set $\{1, 2, 3, 4, 5\}$ as A . The subsets of A that meet the criteria are:

The empty set: $\{\}$

Single-element sets: $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}$

Two-element sets with no consecutive numbers: $\{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 4\}, \{2, 5\}, \{3, 5\}$

Three-element set with no consecutive numbers: $\{1, 3, 5\}$

Counting these subsets, we have:

1 subset with zero elements

5 subsets with one element

6 subsets with two elements

1 subset with three elements

Adding these counts, there are $1 + 5 + 6 + 1 = 13$ subsets in total.

Thus, $n(S_5) = 13$.

Question 18

The number of relations on the set $A = \{1, 2, 3\}$, containing at most 6 elements including $(1, 2)$, which are reflexive and transitive but not symmetric, is _____.

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Answer: 6

Solution:

Since relation needs to be reflexive the ordered pairs $(1, 1), (2, 2), (3, 3)$ need to be there and $(1, 2)$ is also to be included.

Let's call $R_0 = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$ the base relation.

$\therefore A \times A$ contain $3 \times 3 = 9$ ordered pairs, remaining 5 ordered are

$2, 1), (1, 3), (3, 1), (2, 3), (3, 2)$

We have to add at most two ordered pairs to R_0 such that resulting relation is reflexive, transitive but not symmetric.

Following are the only possibilities.

$R = R_0 \cup \{(1, 3)\}$

OR $R_0 \cup \{(3, 2)\}$

OR $R_0 \cup \{(1, 3), (3, 1)\}$

OR $R_0 \cup \{(1, 3), (3, 2)\}$

OR $R_0 \cup \{(3, 1), (3, 2)\}$

Question 19

The number of non-empty equivalence relations on the set $\{1, 2, 3\}$ is :

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Options:

- A. 7
- B. 4
- C. 5
- D. 6

Answer: C

Solution:

An equivalence relation on a finite set is uniquely determined by its partition into equivalence classes. Hence, counting the number of equivalence relations on a set is equivalent to counting the number of ways to partition that set.

Step: Counting partitions of $\{1, 2, 3\}$

We want all possible ways to split the set $\{1, 2, 3\}$ into nonempty subsets (its “blocks”).

3 blocks (each element in its own block)

$\{\{1\}, \{2\}, \{3\}\}$.

2 blocks

$\{\{1, 2\}, \{3\}\}$

$\{\{1, 3\}, \{2\}\}$

$\{\{2, 3\}, \{1\}\}$

1 block (all elements together)

$\{\{1, 2, 3\}\}$.

Counting these, there are a total of **5** distinct partitions, and thus **5** equivalence relations on the set $\{1, 2, 3\}$.

All equivalence relations are automatically nonempty (they include at least $(1, 1)$, $(2, 2)$, $(3, 3)$ because they are reflexive), so the answer to “the number of nonempty equivalence relations” is also **5**.

Answer: Option C (5)

Question 20

Let $A = \{1, 2, 3, \dots, 10\}$ and $B = \left\{ \frac{m}{n} : m, n \in A, m < n \text{ and } \gcd(m, n) = 1 \right\}$. Then $n(B)$ is equal to :

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Options:

- A. 29
- B. 31
- C. 37

D. 36

Answer: B

Solution:

To find the number of elements in set B , we consider pairs $\left(\frac{m}{n}\right)$ where $m, n \in A$ with $m < n$ and $\gcd(m, n) = 1$.

Here's the breakdown for each possible m :

For $m = 1$:

Possible values for n are 2, 3, 4, 5, 6, 7, 8, 9, 10.

Total pairs: 9.

For $m = 2$:

Possible values for n are 3, 5, 7, 9 (since these have $\gcd(2, n) = 1$).

Total pairs: 4.

For $m = 3$:

Possible values for n are 4, 5, 7, 8, 10.

Total pairs: 5.

For $m = 4$:

Possible values for n are 5, 7, 9.

Total pairs: 3.

For $m = 5$:

Possible values for n are 6, 7, 8, 9.

Total pairs: 4.

For $m = 6$:

Possible value for n is 7.

Total pairs: 1.

For $m = 7$:

Possible values for n are 8, 9, 10.

Total pairs: 3.

For $m = 8$:

Possible value for n is 9.

Total pairs: 1.

For $m = 9$:

Possible value for n is 10.

Total pairs: 1.

Adding all these up, the total number of elements in set B is:

$$9 + 4 + 5 + 3 + 4 + 1 + 3 + 1 + 1 = 31$$

Question21

Let $R = \{(1, 2), (2, 3), (3, 3)\}$ be a relation defined on the set $\{1, 2, 3, 4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

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Options:

- A. 9
- B. 8
- C. 7
- D. 10

Answer: C

Solution:

$$A = \{1, 2, 3, 4\}$$

For relation to be reflexive

$$R = \{(1, 2), (2, 3), (3, 3)\}$$

Minimum elements added will be

$$(1, 1), (2, 2), (4, 4), (2, 1), (3, 2), (3, 2), (3, 1), (1, 3)$$

$$\therefore \text{Minimum number of elements} = 7$$

Question 22

Let $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x + y| \geq 3\}$ and $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$. If $C = \{(x, y) \in A \cap B : x = 0 \text{ or } y = 0\}$, then $\sum_{(x,y) \in C} |x + y|$ is :

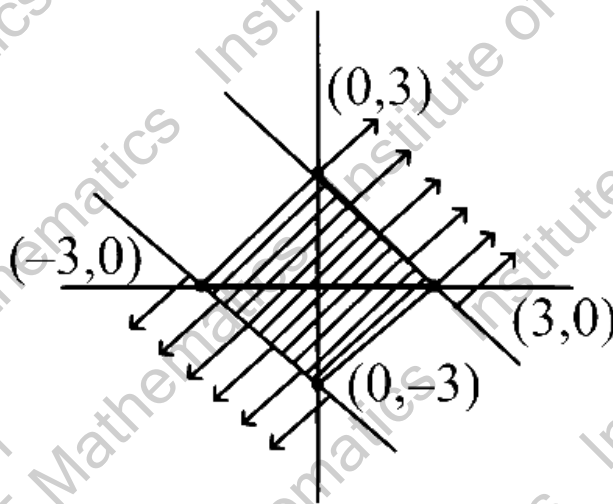
JEE Main 2025 (Online) 23rd January Evening Shift

Options:

- A. 18
- B. 24
- C. 15
- D. 12

Answer: D

Solution:



$$C = \{(3, 0), (-3, 0), (0, 3), (0, -3)\}$$

$$\Sigma |x + y| = 12$$

Question 23

Let $X = \mathbf{R} \times \mathbf{R}$. Define a relation R on X as :

$$(a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$$

Statement I : R is an equivalence relation.

Statement II : For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)R(a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below :

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Options:

- A. Both Statement I and Statement II are true
- B. Statement I is true but Statement II is false
- C. Both Statement I and Statement II are false
- D. Statement I is false but Statement II is true

Answer: B

Solution:

Statement - I :

$$\text{Reflexive : } (a_1, b_1)R(a_1, b_1) \Rightarrow b_1 = b_1 \quad \text{True}$$

$$\text{Symmetric : } \left. \begin{aligned} (a_1, b_1)R(a_2, b_2) &\Rightarrow b_1 = b_2 \\ (a_2, b_2)R(a_1, b_1) &\Rightarrow b_2 = b_1 \end{aligned} \right\} \text{True}$$

Transitive: $(a_1, b_1)R(a_2, b_2) \Rightarrow b_1 = b_2$
 $\& (a_2, b_2)R(a_3, b_3) \Rightarrow b_2 = b_3 \} b_1 = b_3$

$\Rightarrow (a_1, b_1)R(a_3, b_3) \Rightarrow \text{True}$

Hence Relation R is an equivalence relation Statement-I is true.

For statement - II $\Rightarrow y = b$ so False

Question 24

Let $A = \left\{ x \in (0, \pi) - \left\{ \frac{\pi}{2} \right\} : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2 \right\}$ and
 $B = \{ x \geq 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0 \}$. Then $n(A \cup B)$ is equal to :

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Options:

- A. 4
- B. 8
- C. 6
- D. 2

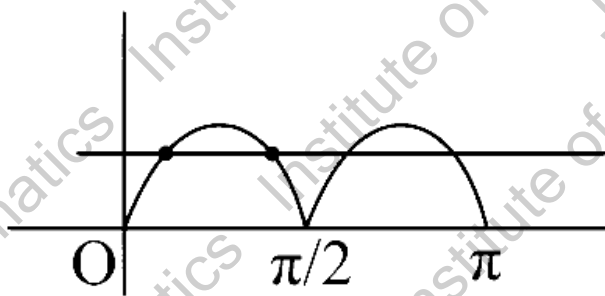
Answer: B

Solution:

$$A : \log_{2/\pi} |\sin x| + \log_{2/\pi} |\cos x| = 2$$

$$\Rightarrow \log_{2/\pi} (|\sin x \cdot \cos x|) = 2$$

$$\Rightarrow |\sin 2x| = \frac{8}{\pi^2}$$



Number of solution 4

$$B : \text{let } \sqrt{x} = t < 2$$

$$\text{Then } \sqrt{x}(\sqrt{x} - 4) + 3(\sqrt{x} - 2) + 6 = 0$$

$$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$$

$$\Rightarrow t^2 - t = 0, t = 0, t = 1$$

$$x = 0, x = 1$$

$$\text{again let } \sqrt{x} = t > 2$$

$$\text{then } t^2 - 4t - 3t + 6 + 6 = 0$$

$$\Rightarrow t^2 - 7t + 12 = 0$$

$$\Rightarrow t = 3, 4$$

$$x = 9, 16$$

Total number of solutions

$$n(A \cup B) = 4 + 4 = 8$$

Question 25

The relation $R = \{(x, y) : x, y \in \mathbb{Z} \text{ and } x + y \text{ is even}\}$ is:

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Options:

- A. reflexive and transitive but not symmetric
- B. reflexive and symmetric but not transitive
- C. an equivalence relation
- D. symmetric and transitive but not reflexive

Answer: C

Solution:

$$R = \{(x, y) : x, y \in \mathbb{Z} \text{ and } x + y \text{ is even}\}$$

reflexive $x + x = 2x$ even

symmetric if $x + y$ is even, then $(y + x)$ is also even

transitive if $x + y$ is even & $y + z$ is even then $x + z$ is also even

So, relation is an equivalence relation.

Question 26

Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by $x R y$ if and only if $\sec^2 x - \tan^2 y = 1$.

Then R is :

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Options:

- A. both reflexive and symmetric but not transitive
- B.

both reflexive and transitive but not symmetric

C.

reflexive but neither symmetric nor transitive

D.

an equivalence relation

Answer: D

Solution:

$$\sec^2 x - \tan^2 x = 1 \quad (\text{on replacing } y \text{ with } x)$$

$\Rightarrow$ Reflexive

$$\sec^2 x - \tan^2 y = 1$$

$$\Rightarrow 1 + \tan^2 x + 1 - \sec^2 y = 1$$

$$\Rightarrow \sec^2 y - \tan^2 x = 1$$

$\Rightarrow$ symmetric

$$\sec^2 x - \tan^2 y = 1$$

$$\sec^2 y - \tan^2 z = 1$$

Adding both

$$\Rightarrow \sec^2 x - \tan^2 y + \sec^2 y - \tan^2 z = 1 + 1$$

$$\sec^2 x + 1 - \tan^2 z = 2$$

$$\sec^2 x - \tan^2 z = 1$$

$\Rightarrow$ Transitive

hence equivalence relation

Question 27

Let $S = \mathbb{N} \cup \{0\}$. Define a relation R from S to $\mathbb{R}$ by :

$$R = \left\{ (x, y) : \log_e y = x \log_e \left(\frac{2}{5} \right), x \in S, y \in \mathbb{R} \right\}.$$

Then, the sum of all the elements in the range of R is equal to :

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Options:

A. $\frac{3}{2}$

B. $\frac{10}{9}$

C. $\frac{5}{2}$

D. $\frac{5}{3}$

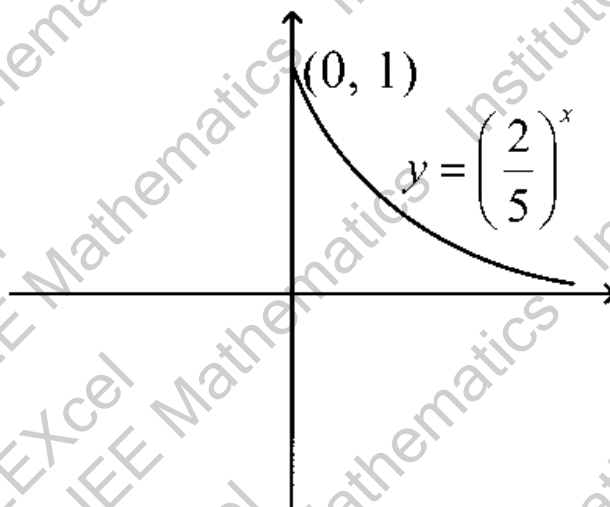
Answer: D

Solution:

$$S = \{0, 1, 2, 3, \dots\}$$

$$\log_e y = \log_e \left(\frac{2}{5} \right)$$

$$\Rightarrow y = \left(\frac{2}{5} \right)^x$$



Required

$$\text{Sum} = 1 + \left(\frac{2}{5} \right)^1 + \left(\frac{2}{5} \right)^2 + \left(\frac{2}{5} \right)^3 + \dots = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

Question 28

Let A be the set of all functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ and R be a relation on A such that $R = \{(f, g) : f(0) = g(1) \text{ and } f(1) = g(0)\}$. Then R is :

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Options:

- A. Symmetric and transitive but not reflexive
- B. Symmetric but neither reflexive nor transitive
- C. Transitive but neither reflexive nor symmetric
- D. Reflexive but neither symmetric nor transitive

Answer: B

Solution:

For R to be reflexive, (f, f) must be in R .

The means $f(0) = f(1)$ and $f(1) = f(0)$ must be true for all f .

But $f(0) \neq f(1)$ always

Therefore, R is not reflexive

If $(f, g) \in R$, then $f(0) = g(1)$ and $f(1) = g(0)$

$$\because f(0) = g(1) \Rightarrow g(1) = f(0)$$

$$\text{and } f(1) = g(0) \Rightarrow g(0) = f(1)$$

R is symmetric

If $(f, g) \in R$ and $(g, h) \in R$, then $f(0) = g(1)$, $f(1) = g(0)$, $g(0) = h(1)$ & $g(1) = h(0)$

Since, $f(0) = g(1)$ and $g(1) = h(0)$, then $f(0)$ is not necessarily equal to $h(0)$.

Therefore, R is not transitive.

$\therefore$ The relation R is symmetric but not reflexive or transitive.

Question 29

Let $A = \{1, 2, 3, \dots, 100\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_2, a_3), (a_3, a_4), \dots, (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k , for which such a sequence exists, is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 6

B. 8

C. 7

D. 5

Answer: D

Solution:

The relation R is defined on the set $A = \{1, 2, 3, \dots, 100\}$ such that $R = \{(a, b) : a = 2b + 1\}$. We need to find the largest integer k for which there exists a sequence of k ordered pairs from R where the second element of each pair is the first element of the next pair.

The sequence in terms of k is:

$$(a_1, a_2), (a_2, a_3), \dots, (a_k, a_{k+1})$$

Here, each a_i satisfies the equation $a_i = 2a_{i+1} + 1$. Consequently, $a_1 = 2a_2 + 1$, making a_1 an odd number.

Let's examine the pattern:

$$a_2 = 2a_3 + 1, \text{ implying } a_1 = 2(2a_3 + 1) + 1 = 4a_3 + 3.$$

$$a_3 = 2a_4 + 1, \text{ leading to } a_1 = 4(2a_4 + 1) + 3 = 8a_4 + 7.$$

Continuing this pattern, we find:

$$a_k = 2a_{k+1} + 1 \implies a_1 = 2^k \cdot a_{k+1} + (2^k - 1)$$

where a_{k+1} needs to be in set A . This implies:

$$a_{k+1} = \frac{a_1 + 1 - 2^k}{2^k}$$

Thus, $2^k \mid (a_1 + 1)$. The task is to find the highest k where 2^k divides any e_i in $\{2, \dots, 101\}$.

The largest power of 2 that divides an element within this range determines k .

After computation, we find that k can be a maximum of 6 because $2^6 = 64$ divides $95 + 1 = 96$, but $2^7 = 128$ does not divide any e_i for $e_i \in A$. Therefore, the maximum k is 6.

The sequence corresponding to this maximum k is:

$(95, 47), (47, 23), (23, 11), (11, 5), (5, 2)$

Question 30

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then $l + m$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

A. 18

B. 20

C. 17

D. 19

Answer: A

Solution:

The relation R is defined for the set $A = \{-3, -2, -1, 0, 1, 2, 3\}$ with the condition $0 \leq x^2 + 2y \leq 4$. Let's determine the pairs (x, y) that satisfy this condition.

For $y = -3$:

Solving $x^2 + 2(-3) \leq 4$, we find $x = \{3, -3\}$.

For $y = -2$:

Solving $x^2 + 2(-2) \leq 4$, we find $x = \{-2, 2\}$.

For $y = -1$:

Solving $x^2 + 2(-1) \leq 4$, we find $x = \{-2, 2\}$.

For $y = 0$:

Solving $x^2 + 0 \leq 4$, we find $x = \{-2, -1, 0, 1, 2\}$.

For $y = 1$:

Solving $x^2 + 2(1) \leq 4$, we find $x = \{-1, 0, 1\}$.

For $y = 2$:

Solving $x^2 + 2(2) \leq 4$, we find $x = \{0\}$.

The relation R consists of the following pairs:

$R = \{(3, -3), (-3, -3), (-2, -2), (2, -2), (-2, -1), (2, -1), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (-1, 1), (0, 1), (1, 1), (0, 2)\}$

Currently, R has $l = 15$ elements. To make R reflexive, it must include all pairs (x, x) for every $x \in A$. We identify the missing reflexive pairs $(-1, -1)$, $(2, 2)$, and $(3, 3)$, which are required to satisfy reflexivity.

Thus, $m = 3$ more elements are needed. Therefore, the total $l + m = 15 + 3 = 18$.

Question 31

Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

- A. 11
- B. 12
- C. 14
- D. 13

Answer: B

Solution:

To solve the problem, we start by defining the set $A = \{-2, -1, 0, 1, 2, 3\}$ and the relation R on set A , where an element x is related to y (written as xRy) if and only if $y = \max\{x, 1\}$.

This leads us to the following pairs in the relation R :

For $x = -2$, $y = \max\{-2, 1\} = 1$, so $(-2, 1)$ is in R .

For $x = -1$, $y = \max\{-1, 1\} = 1$, so $(-1, 1)$ is in R .

For $x = 0$, $y = \max\{0, 1\} = 1$, so $(0, 1)$ is in R .

For $x = 1$, $y = \max\{1, 1\} = 1$, so $(1, 1)$ is in R .

For $x = 2$, $y = \max\{2, 1\} = 2$, so $(2, 2)$ is in R .

For $x = 3$, $y = \max\{3, 1\} = 3$, so $(3, 3)$ is in R .

Thus, the relation R consists of the pairs: $\{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3)\}$, and there are $l = 6$ elements in R .

Making the Relation Reflexive

A relation is reflexive if every element in the set A relates to itself. Therefore, the missing reflexive pairs are:

$(-2, -2)$

$(-1, -1)$

$(0, 0)$

Adding these three pairs will make the relation reflexive, so $m = 3$.

Making the Relation Symmetric

A relation is symmetric if whenever (x, y) is in R , (y, x) must also be in R . Therefore, the missing symmetric pairs are:

$(1, -2)$

$(1, -1)$

$(1, 0)$

Thus, we need to add these three pairs for symmetry, so $n = 3$.

Finally, we calculate the sum $l + m + n = 6 + 3 + 3 = 12$.

Question 32

Consider the sets

$$A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}, B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\},$$

$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$ and $D = A \cap B$. The total number of one-one functions from the set D to the set C is:

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. 15120

B. 18290

C. 17160

D. 19320

Answer: C

Solution:

$$A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}, B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$$

$$x^2 + 9y^2 - (x^2 + y^2) = 144 - 25$$

Plug in $y^2 = \frac{119}{8}$ into either equation to find x .

$$x^2 = 25 - \frac{119}{8}$$

$$x^2 = \frac{200 - 119}{8}$$

$$x^2 = \frac{81}{8}$$

$$x = \pm \sqrt{\frac{81}{8}}, y = \pm \sqrt{\frac{119}{8}}$$

$$\text{Now, } C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$$

Valid points are $(-2, 0), (-1, -1), (-1, 0), (-1, 1), (0, -2), (0, -1), (0, 0), (0, 1), (0, 2), (1, -1), (1, 0), (1, 1)$

$\therefore$ Total valid points in $C = 13$

$\Rightarrow$ There are 4 distinct real points in set D

$\therefore$ The number of one-one functions from D to C

$$\Rightarrow {}^{13}P_4 \Rightarrow \frac{13!}{(13-4)!} = \frac{13!}{9!} = 17160$$

Question 33

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to:

JEE Main 2025 (Online) 4th April Evening Shift

Options:

- A. 17
- B. 18
- C. 15
- D. 16

Answer: A

Solution:

$$xRy \Leftrightarrow 2x - y \in \{0, 1\}$$

$$\Rightarrow y = 2x \text{ or } y = 2x - 1$$

$$A = \{-3, -2, -1, 0, 1, 2, 3\}$$

$$R = \{(-1, -2), (0, 0), (1, 2), (-1, -3), (0, -1), (1, 1), (2, 3)\}$$

$$\Rightarrow I = 7$$

For R to be reflexive $(0, 0), (1, 1) \in R$

But other (a, a) such that $2a - a \in \{0, 1\}$

$$\Rightarrow a \in \{0, 1\}$$

5 other pairs needs to be added $\Rightarrow m = 5$

$xRy \Rightarrow yRx$ to be symmetric

$$(-1, -2) \Rightarrow (-2, -1)$$

$$(1, 2) \Rightarrow (2, 1)$$

$$(-1, -3) \Rightarrow (-3, -1)$$

$$(0, -1) \Rightarrow (-1, 0)$$

$$(2, 3) \Rightarrow (3, 2) \Rightarrow 5 \text{ needs to be added, } n = 5$$

$$\Rightarrow l + m + n = 17$$

Question 34

Let $A = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : |\alpha - 1| \leq 4 \text{ and } |\beta - 5| \leq 6\}$

and $B = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \leq 144\}$.

Then

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Options:

A.

$$A \subset B$$

B.

$$B \subset A$$

C.

neither $A \subset B$ nor $B \subset A$

D.

$$A \cup B = \{(x, y) : -4 \leq x \leq 4, -1 \leq y \leq 11\}$$

Answer: B

Solution:

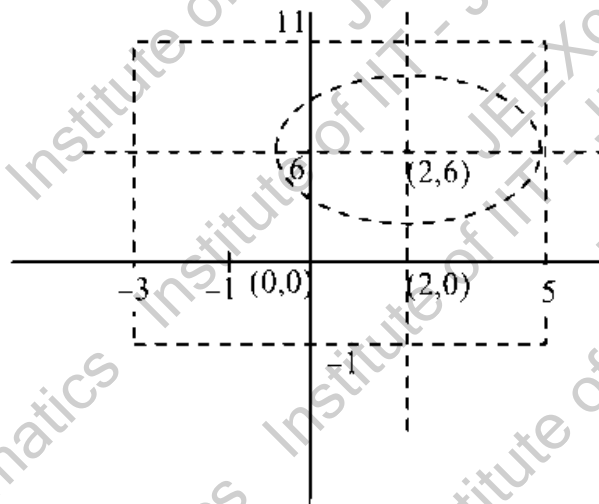
$$A: |x - 1| \leq 4 \text{ and } |y - 5| \leq 6$$

$$\Rightarrow -4 \leq x - 1 \leq 4 \Rightarrow -6 \leq y - 5 \leq 6$$

$$\Rightarrow -3 \leq x \leq 5 \Rightarrow -1 \leq y \leq 11$$

$$B: 16(x - 2)^2 + 9(y - 6)^2 \leq 144$$

$$B: \frac{(x - 2)^2}{9} + \frac{(y - 6)^2}{16} \leq 1$$



From Diagram $B \subset A$

Question 35

Let $A = \{0, 1, 2, 3, 4, 5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$. Then among the statements

(S₁): The number of elements in R is 18, and

(S₂): The relation R is symmetric but neither reflexive nor transitive

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Options:

- A.
both are false
- B.
only (S₁) is true
- C.
only (S₂) is true
- D.
both are true

Answer: C

Solution:

To evaluate the relation R on the set $A = \{0, 1, 2, 3, 4, 5\}$, we first need to understand the conditions for an element (x, y) to be in R . Specifically, $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$.

Considering this, let's list the pairs:

For $\max\{x, y\} = 3$, the possible pairs are:

$(0, 3), (3, 0), (1, 3), (3, 1), (2, 3), (3, 2), (3, 3)$

For $\max\{x, y\} = 4$, the possible pairs are:

$(0, 4), (4, 0), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)$

Combining these, the set R consists of the following elements:

$R = \{(0, 3), (3, 0), (1, 3), (3, 1), (2, 3), (3, 2), (3, 3), (0, 4), (4, 0), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)\}$

This gives us a total of 16 elements in R , not 18 as initially claimed in statement S_1 .

Next, we analyze the properties of the relation R :

Reflexivity: A relation is reflexive if $(x, x) \in R$ for all $x \in A$. For example, $(0, 0), (1, 1), (2, 2)$ are not in R , so R is not reflexive.

Symmetry: A relation is symmetric if whenever $(a, b) \in R$, then $(b, a) \in R$ as well. For all pairs (x, y) listed, both (x, y) and (y, x) are present. Thus, R is symmetric.

Transitivity: A relation is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$. An example where transitivity fails is $(0, 3)$ and $(3, 1)$ are in R but $(0, 1)$ is not in R . Therefore, R is not transitive.

In conclusion, statement S_2 is correct as R is symmetric but neither reflexive nor transitive.

Question 36

Let $S = \{1, 2, 3, \dots, 10\}$. Suppose M is the set of all the subsets of S , then the relation $R = \{(A, B) : A \cap B \neq \emptyset ; A, B \in M\}$ is :

[27-Jan-2024 Shift 1]

Options:

A.

symmetric and reflexive only

B.

reflexive only

C.

symmetric and transitive only

D.

symmetric only

Answer: D

Solution:

Let $S = \{1, 2, 3, \dots, 10\}$

$R = \{(A, B) : A \cap B \neq \phi, A, B \in M\}$

For Reflexive,

M is subset of ' S '

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

$A \cap B \neq \phi,$

$\Rightarrow B \cap A \neq \phi,$

So it is symmetric.

For transitive,

If $A = \{(1, 2), (2, 3)\}$

$B = \{(2, 3), (3, 4)\}$

$C = \{(3, 4), (5, 6)\}$

$A \cap B \neq \phi$ & $B \cap C \neq \phi$ but $A \cap C = \phi$

So it is not transitive

Question 37

Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) R (c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

[29-Jan-2024 Shift 1]

Options:

- A.
Reflexive and symmetric but not transitive
- B.
Reflexive but neither symmetric nor transitive
- C.
Reflexive, symmetric and transitive
- D.
Reflexive and transitive but not symmetric

Answer: A

Solution:

$$(a, b)R(a, b) \text{ as } ab - ab = 0$$

Therefore reflexive

$$\text{Let } (a, b)R(c, d) \Rightarrow ad - bc \text{ is divisible by } 5$$

$$\Rightarrow bc - ad \text{ is divisible by } 5 \Rightarrow (c, d)R(a, b)$$

Therefore symmetric

Relation not transitive as $(3, 1)R(10, 5)$ and $(10, 5)R(1, 1)$ but $(3, 1)$ is not related to $(1, 1)$

Question 38

If R is the smallest equivalence relation on the set $\{1, 2, 3, 4\}$ such that $\{(1, 2), (1, 3)\} \subset R$, then the number of elements in R is

[29-Jan-2024 Shift 2]

Options:

- A.
10
- B.
12
- C.
8

D.
15

Answer: A

Solution:

Given set $\{1, 2, 3, 4\}$

Minimum order pairs are

$(1, 1), (2, 2), (3, 3), (4, 4), (3, 1), (2, 1), (2, 3), (3, 2), (1, 3), (1, 2)$

Thus no. of elements = 10

Question 39

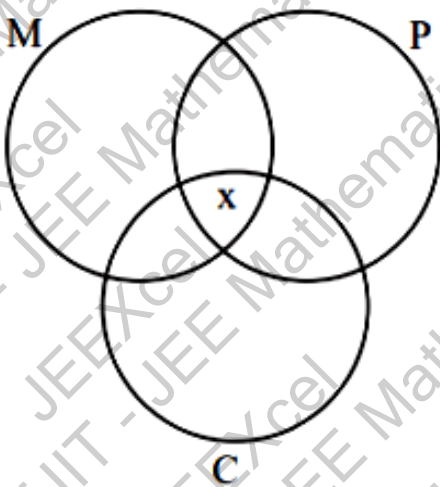
A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _____

[30-Jan-2024 Shift 1]

Options:

Answer: 10

Solution:

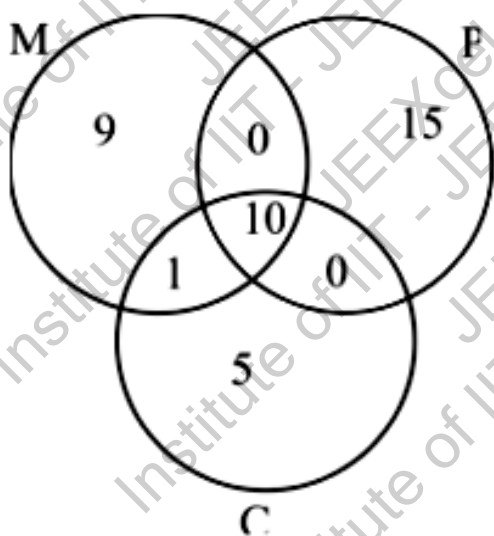


$11 - x \geq 0$ (Maths and Physics)

$x \leq 11$

$x = 11$ does not satisfy the data.

For $x = 10$



Hence maximum number of students passed in all the three subjects is 10 .

Question40

The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____

[30-Jan-2024 Shift 2]

Answer: 960

Solution:

Total number of relation both symmetric and reflexive $= 2^{\frac{n^2-n}{2}}$

Total number of symmetric relation $= 2^{\left(\frac{n^2+n}{2}\right)}$

$\Rightarrow$ Then number of symmetric relation which are not reflexive

$$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$$

$$\Rightarrow 2^{10} - 2^6$$

$$\Rightarrow 1024 - 64$$

$$= 960$$

Question41

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (1, 4)\}$ be a relation on A . Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n . Then, the minimum value of n is _____

[31-Jan-2024 Shift 1]

Answer: 16

Solution:

All elements are included

Answer is 16

Question42

Let $A = \{1, 2, 3, \dots, 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n . Then, the minimum value of n is _____

[31-Jan-2024 Shift 2]

Answer: 66

Solution:

$$R = \{(3, 2), (6, 4), (9, 6), (12, 8), \dots, (99, 66)\}$$

$$n(R) = 33$$

$$\therefore 66$$

Question43

Let $A = \{1, 2, 3, \dots, 20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

Then, number of elements in $R_1 - R_2$ is equal to _____

[1-Feb-2024 Shift 1]

Answer: 46

Solution:

$$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + \underbrace{[1 + \dots + 1]}_{10 \text{ times}}$$

$$n(R_1) = 66$$

$$R_1 \cap R_2 = \{(1, 1), (2, 2), \dots, (20, 20)\}$$

$$n(R_1 \cap R_2) = 20$$

$$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$$

$$= n(R_1) - 20$$

$$= 66 - 20$$

$$R_1 - R_2 = 46 \text{ Pair}$$

Question44

The number of elements in the set

$S = \{(x, y, z) : x, y, z \in \mathbb{Z}, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals _____

[1-Feb-2024 Shift 1]

Answer: 169

Solution:

$$\begin{array}{ll} x+2y+3z=42, & x,y,z \geq 0 \\ z=0 & x+2y=42 \Rightarrow 22 \\ z=1 & x+2y=39 \Rightarrow 20 \\ z=2 & x+2y=36 \Rightarrow 19 \\ z=3 & x+2y=30 \Rightarrow 17 \\ z=4 & x+2y=30 \Rightarrow 16 \\ z=5 & x+2y=27 \Rightarrow 14 \\ z=6 & x+2y=24 \Rightarrow 13 \\ z=7 & x+2y=21 \Rightarrow 11 \\ z=8 & x+2y=18 \Rightarrow 10 \\ z=9 & x+2y=15 \Rightarrow 8 \\ z=10 & x+2y=12 \Rightarrow 7 \\ z=11 & x+2y=9 \Rightarrow 5 \\ z=12 & x+2y=6 \Rightarrow 4 \\ z=13 & x+2y=3 \Rightarrow 2 \\ z=14 & x+2y=0 \Rightarrow 1 \end{array}$$

Total : 169

Question45

Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in \mathbb{R}$ and $(a, b) \in R_2 \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Then

[1-Feb-2024 Shift 2]

Options:

A.

Only R_1 is an equivalence relation

B.

Only R_2 is an equivalence relation

C.

R_1 and R_2 both are equivalence relations

D.

Neither R_1 nor R_2 is an equivalence relation

Answer: B

Solution:

$$aR_1b \Leftrightarrow a^2 + b^2 = 1; a, b \in R$$

$$(a, b)R_2(c, d) \Leftrightarrow a + d = b + c; (a, b), (c, d) \in N$$

for R_1 : Not reflexive symmetric not transitive

for R_2 : R_2 is reflexive, symmetric and transitive

Hence only R_2 is equivalence relation.

Question46

The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is ____
[24-Jan-2023 Shift 2]

Answer: 13

Solution:

Given $R = \{(a, b), (b, c), (b, d)\}$

In order to make it equivalence relation as per given set, R must be

$\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)\}$

There already given so 13 more to be added.

Question47

In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β , then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is :

[6-Apr-2023 shift 2]

Options:

A.

$$\frac{\sqrt{129}}{12}$$

B.

$$\frac{\sqrt{117}}{12}$$

C.

$$\frac{\sqrt{119}}{12}$$

D.

$$\frac{3\sqrt{15}}{12}$$

Answer: C

Solution:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(ABB) = 75 + 40 - 100$$

$$n(A \cap B) = 15$$

$$\text{Only E} \rightarrow 60 \quad \alpha = 60$$

$$\text{Only H} \rightarrow 25 \quad \beta = 25$$

$$\text{Both} = 15$$

$$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1$$

$$\frac{25x^2}{(60)^2} + \frac{(25y^2)}{(25)^2} = 1$$

$$e^2 = 1 - \left[\frac{25 \times 25}{(60)^2} \right]$$

$$e^2 = \frac{(60)^2 - (25)^2}{(60)^2}$$

$$e^2 = \frac{(60 - 25)(60 + 25)}{60 \times 60}$$

$$e^2 = \frac{(35)(85)}{60 \times 60} = \frac{119}{144}$$

$$e = \frac{\sqrt{119}}{12}$$

Question48

Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is:

[8-Apr-2023 shift 1]

Options:

A.

752

B.

772

C.

782

D.

792

Answer: D

Solution:

$$n(A \times B) = 10$$

$${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$$

Question49

The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| < 6\}$ is

[10-Apr-2023 shift 1]

Answer: 6

Solution:

$$-6 < n^2 - 10n + 19 < 6$$

$$\Rightarrow n^2 - 10n + 25 > 0 \text{ and } n^2 - 10n + 13 < 0$$

$$(n-5)^2 > 0 \quad 5-3\sqrt{2} < n < 5+3\sqrt{2}$$

$$N \in \mathbb{Z} - \{5\} \quad n = \{2, 3, 4, 5, 6, 7, 8\}$$

... (i) ... (ii)

From (i) $\cap$ (ii)

$$N = \{2, 3, 4, 5, 6, 8, \}$$

Number of values of $n = 6$

Question50

The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is :

[11-Apr-2023 shift 1]

Options:

A.

10

B.

9

C.

8

D.

12

Answer: B

Solution:

$$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$$

$$\Rightarrow 3\cos^2\theta(\cos^2\theta - 1) + 2\sin^2\theta(\sin^4\theta - 1) = 0$$

$$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta(1 + \sin^2\theta)\cos^2\theta - 1$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2 + 2\sin^2\theta - 3) = 0$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta - 1) = 0$$

$$(C1) \sin^2\theta = 0 \rightarrow 3 \text{ solution; } \theta = \{0, \pi, 2\pi\}$$

$$(C2) \cos^2\theta = 0 \rightarrow 2 \text{ solution; } \theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$(C3) \sin^2\theta = \frac{1}{2} \rightarrow 4 \text{ solution; } \theta = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

No. of solution = 9

Question 51

An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

[11-Apr-2023 shift 1]

Options:

A.

15

B.

9

C.

21

D.

10

Answer: C

Solution:

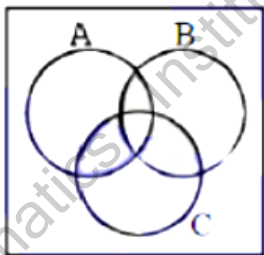
$$|A| = 48$$

$$|B| = 25$$

$$|C| = 18$$

$$|A \cup B \cup C| = 60 \text{ [Total]}$$

$$|A \cap B \cap C| = 5$$



$$|A \cap B \cap C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$$

$$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$$

$$= 36$$

No. of men who received exactly 2 medals

$$\Rightarrow \sum |A \cap B| - 3|A \cap B \cap C|$$

$$= 36 - 15$$

$$= 21$$

Question 52

The number of the relations, on the set $\{1,2,3\}$ containing $(1,2)$ and $(2,3)$, which are reflexive and transitive but not symmetric, is _____.

[12-Apr-2023 shift 1]

Answer: 3

Solution:

$$A = \{1, 2, 3\}$$

For Reflexive $(1, 1), (2, 2), (3, 3) \in R$

For transitive : $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$

Not symmetric : $(2, 1)$ and $(3, 2) \notin R$

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)\}$$

Question 53

The number of elements in the set $\{n \in \mathbb{N} : 10 \leq n \leq 100, \text{ and } 3^n - 3 \text{ is a multiple of } 7\}$ is

[15-Apr-2023 shift 1]

Answer: 15

Solution:

$$n \in [10, 100]$$

$3^n - 3$ is multiple of 7

$$3^n = 7\lambda + 3$$

$$n = 1, 7, 13, 20, \dots, 97$$

Number of possible values of $n = 15$

Question 54

Let $A = \{z \in \mathbb{C} : 1 \leq |z - (1 + i)| \leq 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B :

Options:

A. is an empty set

B. contains exactly two elements

C. contains exactly three elements

D. is an infinite set

Answer: D

Solution:

Let, $z = x + iy$

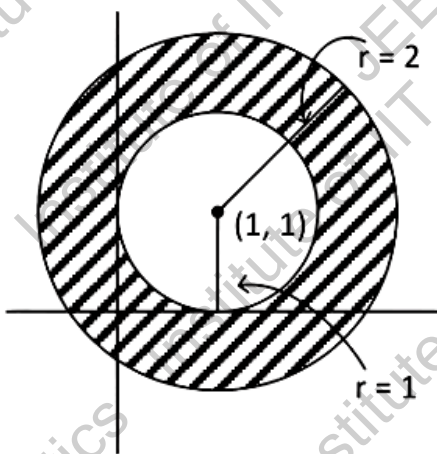
Given, $1 \leq |z - (1 + i)| \leq 2$

$$\Rightarrow 1 \leq |x + iy - 1 - i| \leq 2$$

$$\Rightarrow 1 \leq |(x-1) + i(y-1)| \leq 2$$

$$\Rightarrow 1 \leq \sqrt{(x-1)^2 + (y-1)^2} \leq 2$$

It represents two concentric circles both having center at $(1, 1)$ and radii 1 and 2.



Also given,

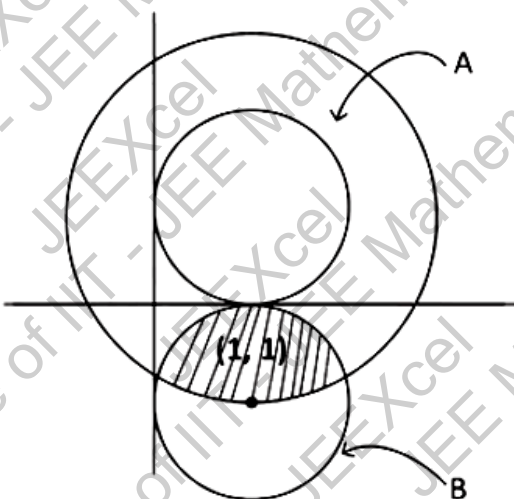
$$|z - (1 - i)| = 1$$

$$\Rightarrow |x + iy - 1 + i| = 1$$

$$\Rightarrow |(x-1) + i(y+1)| = 1$$

$$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = 1$$

This represents a circle with center at $(1, -1)$ and radius = 1.



Question55

Let $A = \{x \in \mathbb{R} : |x + 1| < 2\}$ and $B = \{x \in \mathbb{R} : |x - 1| \geq 2\}$. Then which one of the following statements is NOT true?

[25-Jun-2022-Shift-2]

Options:

A. $A - B = (-1, 1)$

B. $B - A = \mathbb{R} - (-3, 1)$

C. $A \cap B = (-3, -1]$

D. $A \cup B = \mathbb{R} - [1, 3)$

Answer: B

Solution:

$$A = (-3, 1) \text{ and } B = (-\infty, -1] \cup [3, \infty)$$

$$\text{So, } A - B = (-1, 1)$$

$$B - A = (-\infty, -3] \cup [3, \infty) = \mathbb{R} - (-3, 3)$$

$$A \cap B = (-3, -1]$$

$$\text{and } A \cup B = (-\infty, 1) \cup [3, \infty) = \mathbb{R} - [1, 3)$$

So Option B is not True

Question56

Let $A = \{n \in \mathbb{N} : \text{H.C.F.}(n, 45) = 1\}$ and

Let $B = \{2k : k \in \{1, 2, \dots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is _____

[26-Jun-2022-Shift-1]

Answer: 5264

Solution:

Sum of all elements of $A \cap B = 2$ [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$$\begin{aligned}
 &= 2 \left[\frac{100 \times 101}{2} - 3 \left(\frac{33 \times 34}{2} \right) - 5 \left(\frac{20 \times 21}{2} \right) + 15 \left(\frac{6 \times 7}{2} \right) \right] \\
 &= 10100 - 3366 - 2100 + 630 \\
 &= 5264
 \end{aligned}$$

Question 57

Let a set $A = A_1 \cup A_2 \cup \dots \cup A_k$, where $A_i \cap A_j = \emptyset$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i, \text{ if and only if } x \in A_i, 1 \leq i \leq k\}$. Then, R is :

[29-Jun-2022-Shift-1]

Options:

- A. reflexive, symmetric but not transitive.
- B. reflexive, transitive but not symmetric.
- C. reflexive but not symmetric and transitive.
- D. an equivalence relation.

Answer: D

Solution:

$$R = \{(x, y) : y \in A_i, \text{ iff } x \in A_i, 1 \leq i \leq k\}$$

(1) Reflexive

$$(a, a) \Rightarrow a \in A_i \text{ iff } a \in A_i$$

(2) Symmetric

$$(a, b) \Rightarrow a \in A_i \text{ iff } b \in A_i$$

$$(b, a) \in R \text{ as } b \in A_i \text{ iff } a \in A_i$$

(3) Transitive

$$(a, b) \in R \text{ \& } (b, c) \in R$$

$$\Rightarrow a \in A_i \text{ iff } b \in A_i \text{ \& } b \in A_i \text{ iff } c \in A_i$$

$$\Rightarrow a \in A_i \text{ iff } c \in A_i$$

$$\Rightarrow (a, c) \in R.$$

$\Rightarrow$ Relation is equivalence.

Question 58

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and $C = \{T \subseteq A : T \text{ the sum of all the elements of } T \text{ is a prime number}\}$. Then the number of elements in the set $B \cup C$ is _____

[25-Jul-2022-Shift-2]

Answer: 107

Solution:

$$\therefore (B \cup C)' = B' \cap C'$$

B' is a set containing sub sets of A containing element 1 and not containing 2.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of $\{3, 4, 5, 6, 7\}$ whose sum of elements plus 1 is composite.

Number of such 5 elements subset = 1

Number of such 4 elements subset = 3 (except selecting 3 or 7)

Number of such 3 elements subset = 6 (except selecting $\{3, 4, 5\}$, $\{3, 6, 7\}$, $\{4, 5, 7\}$ or $\{5, 6, 7\}$)

Number of such 2 elements subset = 7 (except selecting $\{3, 7\}$, $\{4, 6\}$, $\{5, 7\}$)

Number of such 1 elements subset = 3 (except selecting $\{4\}$ or $\{6\}$)

Number of such 0 elements subset = 1

$$n(B \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$$

Question 59

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \emptyset\}$ is _____.
[26-Jul-2022-Shift-2]

Answer: 112

Solution:

$$A = \{1, 2, 3, 4, 5, 6, 7\} \text{ and } B = \{3, 6, 7, 9\}$$

$$\text{Total subset of } A = 2^7 = 128$$

$C \cap B = \emptyset$ when set C contains the element 1, 2, 4, 5

$$\therefore S = \{C \subseteq A; C \cap B \neq \emptyset\}$$

$$= \text{Total} - (C \cap B = \emptyset)$$

$$= 128 - 2^4 = 112$$

Question 60

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $aR_1b \Leftrightarrow ab \geq 0$ and $aR_2b \Leftrightarrow a \geq b$ Then,
[27-Jul-2022-Shift-1]

Options:

- A. R_1 is an equivalence relation but not R_2
- B. R_2 is an equivalence relation but not R_1
- C. both R_1 and R_2 are equivalence relations
- D. neither R_1 nor R_2 is an equivalence relation

Answer: D

Solution:

$$R_1 = \{xy \geq 0, x, y \in \mathbb{R}\}$$

For reflexive $x \times x \geq 0$ which is true.

For symmetric

$$\text{If } xy \geq 0 \Rightarrow yx \geq 0$$

$$\text{If } x = 2, y = 0 \text{ and } z = -2$$

Then $x \cdot y \geq 0$ & $y \cdot z \geq 0$ but $x \cdot z \geq 0$ is not true

$\Rightarrow$ not transitive relation.

$\Rightarrow R_1$ is not equivalence

R_2 if $a \geq b$ it does not implies $b \geq a$

$\Rightarrow R_2$ is not equivalence relation

Question 61

For $\alpha \in \mathbb{N}$, consider a relation R on $\mathbb{N}$ given by $R = \{(x, y) : 3x + \alpha y \text{ is a multiple of } 7\}$.

The relation R is an equivalence relation if and only if :

[28-Jul-2022-Shift-1]

Options:

- A. $\alpha = 14$
- B. α is a multiple of 4
- C. 4 is the remainder when α is divided by 10
- D. 4 is the remainder when α is divided by 7

Answer: D

Solution:

$R = \{(x, y) : 3x + \alpha y \text{ is multiple of } 7\}$, now R to be an equivalence relation

(1) R should be reflexive : $(a, a) \in R \forall a \in \mathbb{N}$

$$\therefore 3a + \alpha a = 7k$$

$$\therefore (3 + \alpha)a = 7k$$

$$\therefore 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3$$

$$= 7k_1 + 4$$

(2) R should be symmetric : $aRb \Leftrightarrow bRa$

$$aRb : 3a + (7k_1 - 3)b = 7m$$

$$\Rightarrow 3(a-b) + 7kb = 7m$$

$$\Rightarrow 3(b-a) + 7ka = 7m$$

$$\text{So, } aRb \Rightarrow bRa$$

$$\therefore R \text{ will be symmetric for } a = 7k_1 - 3$$

(3) Transitive : Let $(a, b) \in R, (b, c) \in R$

$$\Rightarrow 3a + (7k_1 - 3)b = 7k_1 \text{ and}$$

$$3b + (7k_2 - 3)c = 7k_2$$

$$\text{Adding } 3a + 7kb + (7k_2 - 3)c = 7(k_1 + k_2)$$

$$3a + (7k_2 - 3)c = 7m$$

$$\therefore (a, c) \in R$$

$\therefore R$ is transitive

$$\therefore \alpha = 7k - 3 = 7k + 4$$

Question62

Let R be a relation from the set $\{1, 2, 3, \dots, 60\}$ to itself such that $R = \{(a, b) : b = pq, \text{ where } p, q \geq 3 \text{ are prime numbers}\}$. Then, the number of elements in R is:
[29-Jul-2022-Shift-1]

Options:

A. 600

B. 660

C. 540

D. 720

Answer: B

Solution:

b can take its values as 9, 15, 21, 33, 39, 51, 57, 25, 35, 55, 49

b can take these 11 values and a can take any of 60 values

So, number of elements in $R = 60 \times 11 = 660$

Question63

Let $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$ $B = \{9k + 2 : k \in \mathbb{N}\}$

and $C = \{9k + 1 : k \in \mathbb{N}\}$ for some $1 (0 < 1 < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400 , then 1 is equal to
[2021, 24 Feb. Shift-1]

Answer: 5

Solution:

Given, $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$

$B = \{9k+2 : k \in \mathbb{N}\}$

$C = \{9k+1 : k \in \mathbb{N}\}$

$\therefore$ 3 digit number of the form $3k+2$ are $\{101, 109, \dots, 992\}$

$$\Rightarrow \text{Sum} = \frac{100}{2}[101+992] = \frac{100 \times 1093}{2}$$

Similarly, 3-digit number of the form $9k+5$ is

$$\frac{100}{2}[104+995] = \frac{100 \times 1099}{2}$$

$$[\because \text{numbers are } 104, 113, \dots, 995] \text{ Their sum} = \frac{100 \times 1093}{2} + \frac{100 \times 1099}{2}$$

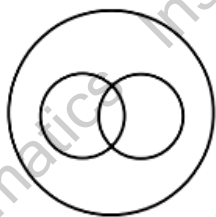
$$= 100 \times 1096 = 400 \times 274$$

Hence, we can say the value of $I = 5$ as the second series of numbers obtained by set C is of the form $9k+5$.

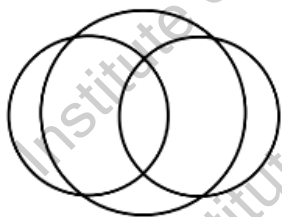
$\therefore$ Required value of $I = 5$

Question 64

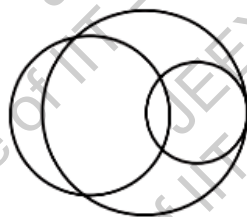
In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagram can justify the above statement?



(P)



(Q)



(R)

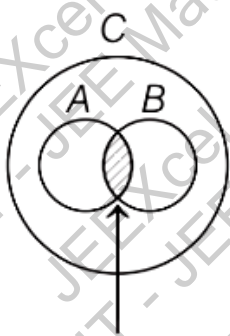
[2021, 17 March Shift-1]

Options:

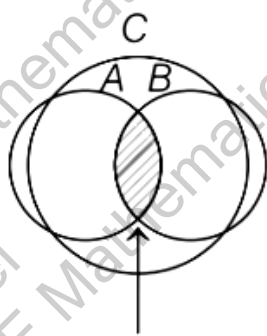
- A. P and Q
- B. P and R
- C. None of these
- D. 0 and R

Answer: C

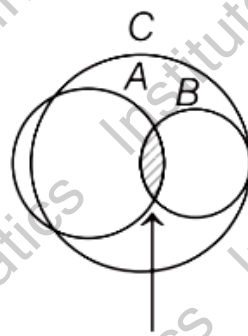
Solution:



The shaded region of this Venn diagram represents the students who play all three types of games.



The shaded region of this Venn diagram represent the students who play all three type of games.



The shaded region of this Venn diagram represent the students who play all three type of games.

Question65

Let $A = \{n \in \mathbb{N} \mid n^2 \leq n + 10,000\}$,
 $B = \{3k + 1 \mid k \in \mathbb{N}\}$ and $C = \{2k \mid k \in \mathbb{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to
[2021, 27 July Shift-II]

Answer: 832

Solution:

$$\text{Let } A = \{n \in \mathbb{N} \mid n^2 \leq n + 10000\}$$

$$n^2 \leq n + 10000$$

$$n^2 - n \leq 10000$$

$$\Rightarrow n(n-1) \leq 100 \times 100$$

$$\Rightarrow A = \{1, 2, 3, \dots, 100\}$$

$$\text{Now, } B = \{3k + 1 \mid k \in \mathbb{N}\}$$

$$B = \{4, 7, 10, 13, \dots\}$$

$$\text{and } C = \{2k \mid k \in \mathbb{N}\}$$

$$C = \{2, 4, 6, 8, \dots\}$$

$$\text{So, } B - C = \{7, 13, 19, \dots, 97, \dots\}$$

$$\text{So, } A \cap (B - C) = \{7, 13, 19, \dots, 97\}$$

This form an AP with common difference (d=6)

$$\Rightarrow 97 = 7 + (n-1)6$$

$$n = \frac{97-7}{6} + 1 = 16 \quad [\because a_n = a + (n-1)d]$$

$$\text{Hence, sum} = \frac{16}{2}[7 + 97]$$

$$= 832 \quad \left\{ \because S_n = \frac{n}{2}(a + 1) \right\}$$

Question66

If $A = \{x \in \mathbb{R} : |x - 2| > 1\}$,

$B = \{x \in \mathbb{R} : \sqrt{x^2 - 3} > 1\}$ and

$C = \{x \in \mathbb{R} : |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$

is

[2021, 27 Aug. Shift-I]

Answer: 256

Solution:

$$A = \{x \in \mathbb{R} : |x - 2| > 1\}$$

$$\Rightarrow A = (-\infty, 1) \cup (3, \infty)$$

$$B = \{x \in \mathbb{R} : \sqrt{x^2 - 3} > 1\}$$

$$\Rightarrow B = (-\infty, -2) \cup (2, \infty)$$

$$C = \{x \in \mathbb{R} : |x - 4| \geq 2\}$$

$$\Rightarrow C = (-\infty, 2] \cup [6, \infty)$$

$$\Rightarrow A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$$

$$\Rightarrow (A \cap B \cap C)^c = [-2, 6)$$

$$\therefore (A \cap B \cap C)^c \cap Z = \{-2, -1, 0, 1, 2, 3, 4, 5\}$$

$$\text{Number of subsets of } (A \cap B \cap C)^c \cap Z$$

$$= 2^8 = 256$$

Question67

Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K % of them are suffering from both ailments, then K can not belong to the set

[2021, 26 Aug. Shift-1]

Options:

A. $\{80, 83, 86, 89\}$

B. $\{84, 86, 88, 90\}$

C. $\{79, 81, 83, 85\}$

D. $\{84, 87, 90, 93\}$

Answer: C

Solution:

Let A = Patient suffering from heart ailment and B = Set of patient suffering from lungs infection

Given, $n(A) = 89\%$ and $n(B) = 98\%$

$$n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow 100 \geq 89 + 98 - n(A \cap B)$$

$$\Rightarrow 87 \leq n(A \cap B)$$

$$\text{Also, } n(A \cap B) = \min\{n(A), n(B)\}$$

$$\Rightarrow n(A \cap B) \leq 89$$

$$\therefore 87 \leq n(A \cap B) \leq 89$$

So, $n(A \cap B) \notin \{79, 81, 83, 85\}$.

Question68

Let $X = \{n \in \mathbb{N} : 1 \leq n \leq 50\}$. If $A = \{n \in X : n \text{ is a multiple of } 2\}$ and $B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is

[Jan. 7, 2020 (II)]

Answer: 29

Solution:

From the given conditions, $n(A) = 25$, $n(B) = 7$ and $n(A \cap B) = 3$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 25 + 7 - 3 = 29$$

Question69

Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is

[Sep. 06, 2020 (I)]

Answer: 28

Solution:

$$2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$$

$$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

$$\therefore m = 7, n = 4 \Rightarrow mn = 28$$

Question70

A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

[Sep. 05, 2020 (I)]

Options:

- A. 63
- B. 36
- C. 54
- D. 38

Answer: B

Solution:

$$\begin{aligned}\text{Given, } n(C) &= 73, n(T) = 65, n(C \cap T) = x \\ \therefore 65 &\geq n(C \cap T) \geq 65 + 73 - 100 \\ \Rightarrow 65 &\geq x \geq 38 \Rightarrow x \neq 36\end{aligned}$$

Question71

A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If $x\%$ of the people read both the newspapers, then a possible value of x can be:

[Sep. 04, 2020 (I)]

Options:

- A. 29
- B. 37
- C. 65
- D. 55

Answer: D

Solution:

$$\begin{aligned}\text{Let } n(U) &= 100, \text{ then } n(A) = 63, n(B) = 76, n(A \cap B) = x \\ \text{Now, } n(A \cup B) &= n(A) + n(B) - n(A \cap B) \leq 100 \\ &= 63 + 76 - x \leq 100 \\ \Rightarrow x &\geq 139 - 100 \Rightarrow x \geq 39 \\ \therefore n(A \cap B) &\leq n(A)\end{aligned}$$

$$\Rightarrow x \leq 63$$
$$\therefore 39 \leq x \leq 63$$

Question 72

Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i 's and exactly 6 of sets Y_i 's, then n is equal to
[Sep. 04, 2020 (II)]

Options:

A. 15

B. 50

C. 45

D. 30

Answer: D

Solution:

$$\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$$

$$\because n(X_i) = 10, n(Y_i) = 5$$

$$\text{So, } \bigcup_{i=1}^{50} X_i = 500, \bigcup_{i=1}^n Y_i = 5n$$

$$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$$