



Sequences and Series

Question1

Let $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = \frac{1}{2}a_n + \frac{n^2 - 2n - 1}{n^2(n+1)^2}$. Then $\sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2}\right)$ is equal to _____.

JEE Main 2026 (Online) 21st January Morning Shift

Answer: 2

Solution:

$$a_{n+1} = \frac{1}{2}a_n + \frac{1}{(n+1)^2} - \frac{((n+1)^2 - n^2)}{n^2(n+1)^2}$$

$$a_{n+1} = \frac{a_n}{2} + \frac{2}{(n+1)^2} - \frac{1}{n^2}$$

$$a_{n+1} - \frac{2}{(n+1)^2} = \frac{1}{2} \left(a_n - \frac{2}{n^2} \right)$$

$$\text{Let } b_n = a_n - \frac{2}{n^2}$$

then $\langle b \rangle$ is geometric progression with ratio $= \frac{1}{2}$

$$b_1 = a_1 - \frac{2}{1} = 1 - 2 = (-1)$$

$$\sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2} \right) = \sum_{n=1}^{\infty} b_n = \frac{(-1)}{1 - \left(\frac{1}{2}\right)} = \frac{-1}{1/2} = -2$$

$$\Rightarrow \sum_{n=1}^{\infty} \left(a_n - \frac{2}{n^2} \right) = 2$$

Question2

Suppose a, b, c are in A.P. and $a^2, 2b^2, c^2$ are in G.P. If $a < b < c$ and $a + b + c = 1$, then $9(a^2 + b^2 + c^2)$ is equal to _____.

JEE Main 2026 (Online) 22nd January Evening Shift

Answer: 9

Solution:

$$a = b - d, c = b + d, \Rightarrow b = \frac{1}{3} \Rightarrow 4b^4 = a^2 c^2$$

$$4b^4 = [(b-d)(b+d)]^2$$

$$\frac{4}{81} = \left(\frac{1}{9} - d^2\right)^2 \Rightarrow \frac{4}{81} = \frac{1}{81} - \frac{2d^2}{9} + d^4 \Rightarrow d^4 - \frac{2d^2}{9} - \frac{1}{27} = 0 \Rightarrow 27d^4 - 6d^2 - 1 = 0$$

$$d^2 = 1/3 \Rightarrow d = +\frac{1}{\sqrt{3}} \text{ (as } a > b > c)$$

$$9(a^2 + b^2 + c^2) = 9 \left[\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} + \frac{1}{\sqrt{3}}\right)^2 \right] = 9 \left[\frac{1}{3} + \frac{2}{3} \right] = 3 + 6 = 9$$

Question3

In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R} - (a, b)$, then $a^2 + b^2$ is equal to

_____ .

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Answer: 90

Solution:

Product of first three terms is 27.

Let the terms be $a/r, a, ar$.

$$(a/r)(a)(ar) = 27 \Rightarrow a^3 = 27 \Rightarrow a = 3$$

$$\text{Sum } S = \frac{3}{r} + 3 + 3r = 3 \left(r + \frac{1}{r} + 1 \right).$$

Find the range of the sum

$$\text{The sum } S = 3 \left(\frac{1}{r} + 1 + r \right).$$

Using the Arithmetic Mean-Geometric Mean (AM-GM) inequality for $r + \frac{1}{r}$:

$$\text{If } r > 0, \text{ then } r + \frac{1}{r} \geq 2, \text{ so } S \geq 3(2 + 1) = 9.$$

$$\text{If } r < 0, \text{ then } r + \frac{1}{r} \leq -2, \text{ so } S \leq 3(-2 + 1) = -3.$$

The possible values for the sum are $(-\infty, -3] \cup [9, \infty)$.

This is expressed as $\mathbb{R} - (-3, 9)$.

Thus, $a = -3$ and $b = 9$.

$$\text{The final value is } a^2 + b^2 = (-3)^2 + 9^2 = 9 + 81 = \mathbf{90}.$$

Question4

If $\sum_{r=1}^{25} \left(\frac{r}{r^4+r^2+1} \right) = \frac{p}{q}$, where p and q are positive integers such that $\gcd(p, q) = 1$, then $p + q$ is equal to _____.

JEE Main 2026 (Online) 28th January Evening Shift

Answer: 976

Solution:

Given

$$\sum_{r=1}^{25} \left(\frac{r}{r^4+r^2+1} \right) = \frac{p}{q}$$

The expression $r^4 + r^2 + 1$ can be factored using a completion of squares technique:

$$r^4 + r^2 + 1 = (r^4 + 2r^2 + 1) - r^2 = (r^2 + 1)^2 - r^2$$

Using the difference of squares formula, $a^2 - b^2 = (a - b)(a + b)$:

$$r^4 + r^2 + 1 = (r^2 - r + 1)(r^2 + r + 1)$$

Split into Partial Fractions

Now, let's rewrite the general term T_r :

$$T_r = \frac{r}{(r^2-r+1)(r^2+r+1)}$$

We can express the numerator r in terms of the factors in the denominator:

$$(r^2 + r + 1) - (r^2 - r + 1) = 2r$$

So, $r = \frac{1}{2} [(r^2 + r + 1) - (r^2 - r + 1)]$. Substituting this back:

$$T_r = \frac{1}{2} \left[\frac{(r^2+r+1)-(r^2-r+1)}{(r^2-r+1)(r^2+r+1)} \right]$$

$$T_r = \frac{1}{2} \left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1} \right]$$

Evaluate the Telescoping Sum

$$\text{Let } f(r) = \frac{1}{r^2-r+1}.$$

$$\text{Note that } f(r+1) = \frac{1}{(r+1)^2-(r+1)+1} = \frac{1}{r^2+2r+1-r-1+1} = \frac{1}{r^2+r+1}.$$

$$\text{Our sum becomes: } S = \frac{1}{2} \sum_{r=1}^{25} [f(r) - f(r+1)]$$

Expanding this:

- For $r = 1$: $f(1) - f(2)$
- For $r = 2$: $f(2) - f(3)$
- For $r = 3$: $f(3) - f(4)$
- ...
- For $r = 24$: $f(24) - f(25)$
- For $r = 25$: $f(25) - f(26)$

Result of summation: $\sum_{r=1}^{25} [f(r) - f(r+1)] = f(1) - f(26)$

So,

$$S = \frac{1}{2} [f(1) - f(26)]$$

Calculate the Final Value

$$\bullet f(1) = \frac{1}{1^2 - 1 + 1} = 1$$

$$f(26) = \frac{1}{25^2 + 25 + 1} = \frac{1}{625 + 25 + 1} = \frac{1}{651}$$

$$S = \frac{1}{2} \left[1 - \frac{1}{651} \right] = \frac{1}{2} \left(\frac{651 - 1}{651} \right) = \frac{650}{2 \times 651}$$

$$S = \frac{325}{651}$$

$325 = (5)^2 \times 13$ and $651 = 3 \times 7 \times 31$ they share no common factor $\gcd(325, 651) = 1$

So $p = 325$ and $q = 651$. $p + q = 325 + 651 = 976$

Question5

If $\sum_{k=1}^n a_k = 6n^3$, then $\sum_{k=1}^6 \left(\frac{a_{k+1} - a_k}{36} \right)^2$ is equal to _____.

JEE Main 2026 (Online) 2nd April Morning Shift

Answer: 91

Solution:

$$a_1 + a_2 + \dots + a_n = 6n^3.$$

To get a formula for one term, write the sum up to $n + 1$ terms:

$$a_1 + a_2 + \dots + a_n + a_{n+1} = 6(n+1)^3$$

Now subtract the first equation from this to isolate a_{n+1} :

$$a_{n+1} = 6(n+1)^3 - 6n^3$$

Use the identity $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ with $x = n + 1$ and $y = n$:

$$= 6((n+1) - n)((n+1)^2 + n^2 + n(n+1))$$

Since $(n+1) - n = 1$, simplify the bracket:

$$a_{n+1} = 6(1)(3n^2 + 3n + 1)$$

So, replacing n by $n - 1$ gives a formula for a_n :

$$a_n = 6(3(n-1)^2 + 3(n-1) + 1)$$

Now simplify inside:

$$= 6(3(n^2 - 2n + 1 + n - 1) + 1)$$

Combine terms:

$$= 6(3(n^2 - n) + 1)$$

So we get:

$$a_n = 6(3n^2 - 3n + 1)$$

Now compute $a_{k+1} - a_k$ using the formulas:

$$\sum_{k=1}^6 \left(\frac{6(3k^2+3k+1)-6(3k^2-3k+1)}{36} \right)^2$$

Inside the bracket, cancel common parts:

$$\sum_{k=1}^6 \left(\frac{36k}{36} \right)^2 = \sum_{k=1}^6 k^2 = \frac{6 \times 7 \times 13}{6} = 91$$

Question 6

For the functions $f(\theta) = \alpha \tan^2 \theta + \beta \cot^2 \theta$, and $g(\theta) = \alpha \sin^2 \theta + \beta \cos^2 \theta$, $\alpha > \beta > 0$, let $\min_{0 < \theta < \frac{\pi}{2}} f(\theta) = \max_{0 < \theta < \pi} g(\theta)$. If the first term of a G.P. is $\left(\frac{\alpha}{2\beta} \right)$, its common ratio is $\left(\frac{2\beta}{\alpha} \right)$ and the sum of its first 10 terms is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

JEE Main 2026 (Online) 6th April Morning Shift

Answer: 1279

Solution:

For

$$f(\theta) = \alpha \tan^2 \theta + \beta \cot^2 \theta$$

and

$$g(\theta) = \alpha \sin^2 \theta + \beta \cos^2 \theta, \quad \alpha > \beta > 0$$

we are given that

$$\min_{0 < \theta < \frac{\pi}{2}} f(\theta) = \max_{0 < \theta < \pi} g(\theta).$$

We will first find these two values.

1. Minimum value of $f(\theta)$

Let

$$x = \tan^2 \theta.$$

Since $0 < \theta < \frac{\pi}{2}$, we have $x > 0$.

Then

$$\cot^2 \theta = \frac{1}{\tan^2 \theta} = \frac{1}{x}.$$

So

$$f(\theta) = \alpha x + \frac{\beta}{x}.$$

Now use the standard result for $x > 0$:

$$\alpha x + \frac{\beta}{x} \geq 2\sqrt{\alpha\beta}.$$

Hence,

$$\alpha x + \frac{\beta}{x} \geq 2\sqrt{\alpha\beta}.$$

Therefore,

$$\min f(\theta) = 2\sqrt{\alpha\beta}.$$

2. Maximum value of $g(\theta)$

We have

$$g(\theta) = \alpha \sin^2 \theta + \beta \cos^2 \theta.$$

Using

$$\cos^2 \theta = 1 - \sin^2 \theta,$$

we get

$$g(\theta) = \alpha \sin^2 \theta + \beta(1 - \sin^2 \theta)$$

$$= \beta + (\alpha - \beta) \sin^2 \theta.$$

Since $\alpha > \beta$, the coefficient of $\sin^2 \theta$ is positive. So $g(\theta)$ is maximum when

$$\sin^2 \theta = 1.$$

For $0 < \theta < \pi$, this happens at $\theta = \frac{\pi}{2}$.

Thus,

$$\max g(\theta) = \beta + (\alpha - \beta) = \alpha.$$

3. Use the given condition

Given

$$\min f(\theta) = \max g(\theta),$$

so

$$2\sqrt{\alpha\beta} = \alpha.$$

Now square both sides:

$$4\alpha\beta = \alpha^2.$$

Since $\alpha > 0$, divide by α :

$$4\beta = \alpha.$$

So,

$$\alpha = 4\beta.$$

4. G.P. data

First term:

$$a = \frac{\alpha}{2\beta} = \frac{4\beta}{2\beta} = 2.$$

Common ratio:

$$r = \frac{2\beta}{\alpha} = \frac{2\beta}{4\beta} = \frac{1}{2}.$$

So the G.P. is

$$2, 1, \frac{1}{2}, \frac{1}{4}, \dots$$

5. Sum of first 10 terms

For a G.P.,

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1.$$

Thus,

$$S_{10} = \frac{2\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}.$$

Now simplify:

$$S_{10} = 2 \cdot \frac{1-\frac{1}{1024}}{\frac{1}{2}}$$

$$= 2 \cdot 2 \left(1 - \frac{1}{1024}\right)$$

$$= 4 \cdot \frac{1023}{1024}$$

$$= \frac{4092}{1024}$$

$$= \frac{1023}{256}.$$

So,

$$\frac{m}{n} = \frac{1023}{256}.$$

Hence,

$$m + n = 1023 + 256 = 1279.$$

Therefore, the answer is

$$\boxed{1279}.$$

Question 7

Let $a_1, a_2, a_3, \dots$ be a G.P. of increasing positive terms such that $a_2 \cdot a_3 \cdot a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$. Then $a_3 + a_5 + a_7$ is equal to :

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Options:

A.

$$3256$$

B.

$$3252$$

C.

3248

D.

3244

Answer: B

Solution:

$$a_2 \cdot a_3 \cdot a_4 = 64$$

$$\frac{a}{r} \cdot a \cdot ar = 64$$

$$\Rightarrow a = 4$$

$$a_1 + a_3 + a_5 = \frac{813}{7}$$

$$= \frac{4}{r^2} + 4 + 4r^2 = \frac{813}{7}$$

$$\text{Let } r^2 = t$$

$$28t^2 - 785t + 28 = 0$$

$$(28t - 1)(t - 28) = 0$$

$$\Rightarrow t = 28, \frac{1}{28}$$

$$r^2 = 28 (\because \text{G.P. is increasing})$$

$$a_3 + a_5 + a_7$$

$$= a + ar^2 + ar^4 = r^2 \left(a + \frac{a}{r^2} + r^2 \right)$$

$$= 28 \times \frac{813}{7}$$

$$= 3252$$

Question8

Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \dots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $a_1 + a_2 + \dots + a_{10} = 62$, then a_1 is equal to:

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Options:

A.

$$\sqrt{2} - 1$$

B.

$$2(\sqrt{2} - 1)$$

C.

$$2 - \sqrt{2}$$

D.

$$2(2 - \sqrt{2})$$

Answer: B

Solution:

$$\frac{a_2}{2a_1} = \frac{a_3}{2a_2} = \frac{a_4}{2a_3} = \dots = \frac{a_{10}}{2a_9} = \frac{1}{\sqrt{2}}$$

$\therefore a_1, a_2, a_3, \dots, a_{10}$ are in G.P. With common ratio $\sqrt{2}$

$$\therefore \sum_{i=1}^{10} a_i = \frac{a_1 \left((\sqrt{2})^{10} - 1 \right)}{\sqrt{2} - 1} = 62 \Rightarrow a_1 = 2(\sqrt{2} - 1)$$

Question9

If the sum of the first four terms of an A.P. is 6 and the sum of its first six terms is 4, then the sum of its first twelve terms is

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Options:

A.

-26

B.

-20

C.

-24

D.

-22

Answer: D

Solution:

$$a + (a + d) + (a + 2d) + (a + 3d) = 6$$

$$2a + 3d = 3$$

$$6a + 15d = 4$$

$$d = -5/6$$

$$a = 11/4$$

$$S_{12} = \frac{12}{2} \left[2 \left(\frac{11}{4} \right) + 11 \times (-5/6) \right] = -22$$

Question10

Let $\sum_{k=1}^n a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$, then $\alpha + \beta$ is equal to :

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Options:

A.

3

B.

5

C.

7

D.

12

Answer: B

Solution:

$$\text{Let } S_n = \sum_{k=1}^n a_k = \alpha n^2 + \beta n$$

$$a_1 + a_2 + a_3 + a_4 + \dots + a_n = \alpha n^2 + \beta n$$

Since the sum S_n is a quadratic expression in n with no constant term then the sequence a_k is in A.P (arithmetic progression).

Then n th term of the sequence can be found using the formula

$$a_n = S_n - S_{n-1}$$

$$\begin{aligned} a_n &= \alpha n^2 + \beta n - [\alpha(n-1)^2 + \beta(n-1)] \\ &= \alpha n^2 + \beta n - [\alpha(n^2 - 2n + 1) + \beta n - \beta] \\ &= \alpha n^2 + \beta n - \alpha n^2 + 2\alpha n - \alpha - \beta n + \beta \\ &\Rightarrow a_n = \beta - \alpha + 2\alpha n - (1) \end{aligned}$$

It is given that $a_{10} = 59$, substitute $n = 10$ in equation (1).

$$a_{10} = \beta - \alpha + 2\alpha(10) = 59$$

$$\Rightarrow 19\alpha + \beta = 59 \quad (2)$$

also, $a_6 = 7a_1$ (given)

substitute $n = 6$ and $n = 1$ in equation (1).

$$\beta - \alpha + 2\alpha(6) = 7(\beta - \alpha + 2\alpha(1))$$

$$\Rightarrow 11\alpha + \beta = 7\alpha + 7\beta$$

$$\Rightarrow 4\alpha = 6\beta$$

$$\Rightarrow 2\alpha = 3\beta \Rightarrow \alpha = \frac{3\beta}{2}$$

Substitute $\alpha = \frac{3\beta}{2}$ in equation (2)

$$19\left(\frac{3\beta}{2}\right) + \beta = 59$$

$$\Rightarrow 57\beta + 2\beta = 118$$

$$\Rightarrow 59\beta = 118$$

$$\Rightarrow \beta = \frac{118}{59} = 2$$

$$\text{And } \alpha = \frac{3\beta}{2} = \frac{3 \times 2}{2} = 3$$

So, value of $\alpha + \beta = 3 + 2 = 5$.

Question 11

Consider an A.P.: $a_1, a_2, \dots, a_n$; $a_1 > 0$. If $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{1}{4}a_1$, and

$\sum_{i=1}^n a_i = \frac{525}{2}$, then $\sum_{i=1}^{17} a_i$ is equal to

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Options:

A.

238

B.

136

C.

476

D.

952

Answer: A

Solution:

Given $a_1, a_2, \dots, a_n$ are in A.P; $a_1 > 0$

Let d be the common difference of this A.P.

$$\text{So } d = a_2 - a_1 = -\frac{3}{4} \text{ (given)}$$

$$\text{It is given that } \sum_{i=1}^n a_i = \frac{525}{2} \text{ and } a_n = \frac{1}{4}a_1$$

$$\therefore a_1 + a_2 + a_3 + \dots + a_n = \frac{525}{2}$$

$$\text{Using sum of } n \text{ terms of A.P } S_n = \frac{n}{2}(a_1 + a_n)$$

$$\Rightarrow \frac{n}{2}(a_1 + \frac{1}{4}a_1) = \frac{525}{2}$$

$$\Rightarrow 5a_1n = 525 \times 4$$

$$a_1n = 420 - (1)$$

Also using formula for general term

$$a_n = a_1 + (n-1)d$$

$$\Rightarrow \frac{1}{4}a_1 = a_1 + (n-1)\left(-\frac{3}{4}\right)$$

$$\Rightarrow (n-1)\frac{3}{4} = a_1 - \frac{1}{4}a_1 = \frac{3a_1}{4}$$

$$n = a_1 + 1 - (2)$$

Substitute in equation (1)

$$a_1(a_1 + 1) = 420$$

$$\Rightarrow a_1^2 + a_1 - 420 = 0$$

$$\Rightarrow a_1^2 + 21a_1 - 20a_1 - 420 = 0$$

$$\Rightarrow a_1(a_1 + 21) - 20(a_1 + 21) = 0$$

$$\Rightarrow (a_1 + 21)(a_1 - 20) = 0$$

Since $a_1 > 0$ so $a_1 = 20$

$$\text{To find } \sum_{i=1}^{17} a_i = a_1 + a_2 + \dots + a_{17}$$

$$\text{Sum of 17 terms} = \frac{17}{2}[2a_1 + (17-1)d]$$

$$= \frac{17}{2}\left[2 \times 20 + 16 \times -\frac{3}{4}\right]$$

$$= \frac{17}{2}[40 - 12] = \frac{17}{2} \times 28$$

$$= 17 \times 14 = 238$$

Question 12

Let 729, 81, 9, 1, ... be a sequence and P_n denote the product of the first n terms of this sequence.

If $2 \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{3^\beta}$ and $\gcd(\alpha, \beta) = 1$, then

$\alpha + \beta$ is equal to

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Options:

A.

73

B.

74

C.

75

D.

76

Answer: A

Solution:

729, 81, 9, 1, ... is a sequence in G.P. Can be written as $9^3, 9^2, 9^1, 9^0, \frac{1}{9}, \left(\frac{1}{9}\right)^2, \dots$

P_n is product of first n term

$$P_n = (9^3)(9^2)(9^1) \dots$$

$$P_n = 9^{3+2+1+0+(-1)+(-2)\dots}$$

Clearly power of 9 are in A.P. with first term = 3

common difference = -1

So, sum of first n terms of this A.P. = $\frac{n}{2}[2 \times 3 + (n-1)(-1)]$

$$= \frac{n}{2}[6 - n + 1] = \frac{n}{2}(7 - n)$$

$$\text{So, } P_n = (9)^{\frac{n}{2}(7-n)}$$

It is given that

$$2 \sum_{n=1}^{40} (P_n)^{1/n} = \frac{3^\alpha - 1}{3^\beta}$$

Solving

$$\sum_{n=1}^{40} (P_n)^{1/n} = \sum_{n=1}^{40} (9)^{\frac{n}{2}(7-n) \times \frac{1}{n}} = \sum_{n=1}^{40} 9^{\frac{7-n}{2}}$$

$$\sum_{n=1}^{40} (9^{1/2})^{7-n} = \sum_{n=1}^{40} (3)^{7-n} = 3^7 \sum_{n=1}^{40} \left(\frac{1}{3}\right)^n$$

$$= 3^7 \left[\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 \dots \left(\frac{1}{3}\right)^{40} \right]$$

$$= 3^7 \left[\text{G.P. with first term} = \frac{1}{3} \text{ common ratio} = \frac{1}{3} \right]$$

$$= 3^7 \times \frac{1}{3} \left[\frac{1 - \left(\frac{1}{3}\right)^{40}}{1 - \frac{1}{3}} \right]$$

$$\sum_{n=1}^{40} (P_n)^{1/n} = 3^6 \left(\frac{\frac{3^{40}-1}{3^{40}}}{\frac{2}{3}} \right) = \frac{3^{40} - 1}{2 \cdot 3^{33}}$$

$$2 \sum_{n=1}^{40} (P_n)^{1/n} = \frac{3^{40} - 1}{3^{33}} = \frac{3^\alpha - 1}{3^\beta}$$

Comparing $\alpha = 40, \beta = 33$

g.c.d (40, 33) = 1

So $\alpha + \beta = 40 + 33 = 73$

Question 13

Let a_1, a_2, a_3, a_4 be an A.P. of four terms such that each term of the A.P. and its common difference l are integers. If $a_1 + a_2 + a_3 + a_4 = 48$ and $a_1 a_2 a_3 a_4 + l^4 = 361$, then the largest term of the A.P. is equal to

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Options:

A.

27

B.

24

C.

23

D.

21

Answer: A

Solution:

It is given that a_1, a_2, a_3, a_4 are in A.P with common difference l .

four terms in A.P are $(a - 3d), (a - d), (a + d), (a + 3d)$ common difference $l = 2d \Rightarrow d = \frac{l}{2}$

$$\text{so } a_1 = a - \frac{3l}{2}, a_2 = \left(a - \frac{l}{2}\right)$$

$$a_3 = a + \frac{l}{2}, a_4 = \left(a + \frac{3l}{2}\right)$$

It is given that $a_1 + a_2 + a_3 + a_4 = 48$

$$a - \frac{3l}{2} + a - \frac{l}{2} + a + \frac{l}{2} + a + \frac{3l}{2} = 48$$

$$\Rightarrow 4a = 48 \Rightarrow a = 12$$

$$\text{also, } a_1 a_2 a_3 a_4 + l^4 = 361$$

$$\Rightarrow \left(12 - \frac{3l}{2}\right) \left(12 - \frac{l}{2}\right) \left(12 + \frac{l}{2}\right) \left(12 + \frac{3l}{2}\right) + l^4 = 361$$

$$\Rightarrow \left(144 - \frac{l^2}{4}\right) \left(144 - \frac{9l^2}{4}\right) + l^4 = 361$$

$$\Rightarrow 20736 - 324l^2 - 36l^2 + \frac{9}{16}l^4 + l^4 = 361$$

$$\Rightarrow \frac{25}{16}l^4 - 360l^2 + 20375 = 0$$

multiply both side by $\frac{16}{5}$.

$$5l^4 - 1152l^2 + 65200 = 0$$

using quadratic formula

$$l^2 = \frac{1152 \pm \sqrt{(1152)^2 - 4 \times 5 \times 65200}}{2 \times 5}$$

$$\Rightarrow l^2 = \frac{1152 \pm \sqrt{23104}}{10} = \frac{1152 \pm 152}{10}$$

$$\Rightarrow l^2 = \frac{1152 + 152}{10}, \frac{1152 - 152}{10}$$

$$\Rightarrow l^2 = \frac{1304}{10}, \frac{1000}{10}$$

$$\Rightarrow l^2 = \frac{1304}{10}, 100$$

Since l is integer so $l = \pm 10$

If $l = 10$

$$a_1 = 12 - \frac{3}{2} \times 10 = -3$$

$$a_2 = 12 - \frac{10}{2} = 7$$

$$a_3 = 12 + \frac{10}{2} = 17$$

$$a_4 = 12 + \frac{3}{2} \times 10 = 27$$

If $l = -10$

$$a_1 = 12 + \frac{3}{2} \times 10 = 27$$

$$a_2 = 12 + \frac{10}{2} = 17$$

$$a_3 = 12 - \frac{10}{2} = 7$$

$$a_4 = 12 - \frac{3}{2} \times 10 = -3$$

So, numbers are same but in reverse order therefore, largest term of A.P is 27.

Question 14

The common difference of the A.P.: $a_1, a_2, \dots, a_m$ is 13 more than the common difference of the A.P.: $b_1, b_2, \dots, b_n$. If $b_{31} = -277, b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

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Options:

A.

21

B.

19

C.

24

D.

16

Answer: B

Solution:

Find common difference of second A.P.

It is given that $b_{31} = -277$, $b_{43} = -385$

Let d_b is common difference of second A.P.

$$b_{31} = b_1 + 30d_b \quad \text{--- (1)}$$

$$b_{43} = b_1 + 42d_b \quad \text{--- (2)}$$

Subtract (1) from (2)

$$b_{43} - b_{31} = (42 - 30)d_b$$

$$-385 - (-277) = 12d_b$$

$$d_b = \frac{-108}{12} = -9$$

Find the common difference (d_a) of the first A.P.

The problem states that d_a is 13 more than d_b :

$$d_a = d_b + 13$$

$$d_a = -9 + 13 = 4$$

Calculate a_1

We are given $a_{78} = 327$. Using the formula $a_n = a_1 + (n - 1)d_a$:

$$327 = a_1 + (78 - 1)(4)$$

$$327 = a_1 + (77)(4)$$

$$327 = a_1 + 308$$

$$a_1 = 327 - 308 = 19$$

Question 15

The value of $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k(k+1)}{k!} \right)$ is

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Options:

A.

$$e/2$$

B.

$$\sqrt{e}$$

C.

$$2/e$$

D.

$$1/e$$

Answer: D

Solution:

$$S = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k(k+1)}{k!}$$

First, let's simplify the expression inside the summation. We can cancel out the k in the numerator with the k in $k!$:

$$\frac{k(k+1)}{k!} = \frac{k(k+1)}{k \cdot (k-1)!} = \frac{k+1}{(k-1)!}$$

Now, we rewrite the numerator $(k+1)$ in terms of $(k-1)$ to further simplify the fraction:

$$k+1 = (k-1) + 2$$

Substituting this back into the expression:

$$\frac{k+1}{(k-1)!} = \frac{(k-1)+2}{(k-1)!} = \frac{k-1}{(k-1)!} + \frac{2}{(k-1)!}$$

For the first part, $\frac{k-1}{(k-1)!} = \frac{1}{(k-2)!}$ (valid for $k \geq 2$). So, the general term a_k becomes:

$$a_k = (-1)^{k+1} \left[\frac{1}{(k-2)!} + \frac{2}{(k-1)!} \right]$$

Expand the Series

Let's apply the summation $\sum_{k=1}^{\infty}$ to both parts. Note that for $k=1$, the term $1/(k-2)!$ is effectively 0 because the factorial of a negative integer is infinite in the denominator.

$$S = \sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{(k-2)!} + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{(k-1)!}$$

Using exponential expansion (Taylor series expansion for e^x :)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

for $x = -1$:

$$e^{-1} = \frac{1}{e} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots$$

Evaluate the Summations

$$\sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{(k-2)!}$$

Let $n = k-2$. When $k=2$, $n=0$. When $k \rightarrow \infty$, $n \rightarrow \infty$. The exponent becomes $k+1 = (n+2)+1 = n+3$.

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n!} = (-1)^3 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = -1 \cdot \left(\frac{1}{e}\right) = -\frac{1}{e}$$

$$\sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{(k-1)!}$$

Let $m = k-1$. When $k=1$, $m=0$. When $k \rightarrow \infty$, $m \rightarrow \infty$. The exponent becomes $k+1 = (m+1)+1 = m+2$.

$$= 2 \sum_{m=0}^{\infty} \frac{(-1)^{m+2}}{m!} = 2(-1)^2 \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} = 2 \cdot \left(\frac{1}{e}\right) = \frac{2}{e}$$

Required S

$$S = -\frac{1}{e} + \frac{2}{e} = \frac{1}{e}$$

Question 16

$$\frac{6}{3^{26}} + \frac{10 \cdot 1}{3^{25}} + \frac{10 \cdot 2}{3^{24}} + \frac{10 \cdot 2^2}{3^{23}} + \dots + \frac{10 \cdot 2^{24}}{3} \text{ is equal to :}$$

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Options:

A.

$$2^{26}$$

B.

$$3^{25}$$

C.

$$3^{26}$$

D.

$$2^{25}$$

Answer: A

Solution:

$$\text{Let } S = \frac{6}{3^{26}} + \frac{10 \cdot 1}{3^{25}} + \frac{10 \cdot 2}{3^{24}} + \frac{10 \cdot 2^2}{3^{23}} + \cdots + \frac{10 \cdot 2^{24}}{3}$$

Notice that from the second term onwards, the terms follow a pattern. Let S' be the sum of these terms:

$$S' = \frac{10 \cdot 2^0}{3^{25}} + \frac{10 \cdot 2^1}{3^{24}} + \frac{10 \cdot 2^2}{3^{23}} + \cdots + \frac{10 \cdot 2^{24}}{3^1}$$

We can factor out $\frac{10}{3^{25}}$:

$$S' = \frac{10}{3^{25}} \left(1 + \frac{2^1}{3^{-1}} + \frac{2^2}{3^{-2}} + \cdots + \frac{2^{24}}{3^{-24}} \right)$$

$$S' = \frac{10}{3^{25}} (2^0 \cdot 3^0 + 2^1 \cdot 3^1 + 2^2 \cdot 3^2 + \cdots + 2^{24} \cdot 3^{24})$$

$$S' = \frac{10}{3^{25}} \sum_{k=0}^{24} (2 \cdot 3)^k = \frac{10}{3^{25}} \sum_{k=0}^{24} 6^k$$

Calculate the sum of the GP

The series $\sum_{k=0}^{24} 6^k$ is a geometric progression with first term $a = 1$, common ratio $r = 6$, and $n = 25$ terms.

The sum formula is $S_n = a \frac{r^n - 1}{r - 1}$:

$$\sum_{k=0}^{24} 6^k = \frac{6^{25} - 1}{6 - 1} = \frac{6^{25} - 1}{5}$$

Substituting this back into S' :

$$S' = \frac{10}{3^{25}} \left(\frac{6^{25} - 1}{5} \right) = \frac{2}{3^{25}} (6^{25} - 1) = \frac{2 \cdot 6^{25}}{3^{25}} - \frac{2}{3^{25}}$$

Since $6^{25} = (2 \cdot 3)^{25} = 2^{25} \cdot 3^{25}$:

$$S' = \frac{2 \cdot 2^{25} \cdot 3^{25}}{3^{25}} - \frac{2}{3^{25}} = 2 \cdot 2^{25} - \frac{2}{3^{25}} = 2^{26} - \frac{2}{3^{25}}$$

Find the total sum S :

Now, add the first term $\frac{6}{3^{26}}$ back to S' :

$$S = \frac{6}{3^{26}} + 2^{26} - \frac{2}{3^{25}}$$

$$[\text{Note that, } \frac{6}{3^{26}} = \frac{2 \cdot 3}{3^{26}} = \frac{2}{3^{25}}]$$

$$\Rightarrow S = \frac{2}{3^{25}} + 2^{26} - \frac{2}{3^{25}} = 2^{26}$$

Question 17

Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a > 2$. If α is such that $a, 4, \alpha, b$ are in A.P., then the equation $\alpha x^2 - ax + 2(\alpha - 2b) = 0$ has :

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Options:

A.

one root in $(1, 4)$ and another in $(-2, 0)$

B.

one root in $(0, 2)$ and another in $(-4, -2)$

C.

both roots in the interval $(-2, 0)$

D.

complex roots of magnitude less than 2

Answer: A

Solution:

The arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{5}{16}$.

$$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{5}{16}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{5}{8}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{5}{8}$$

$$\Rightarrow 8(a+b) = 5ab \quad \dots (i)$$

$a, 4, \alpha, b$ are in A.P. Let the common difference be d .

$$\circ \text{ (A). } 4 = a + d \implies d = 4 - a$$

$$\circ \text{ (B). } \alpha = a + 2d = a + 2(4 - a) = 8 - a$$

$$\circ \text{ (C). } b = a + 3d = a + 3(4 - a) = 12 - 2a$$

Solve for a, b , and α

Substitute $b = 12 - 2a$ into equation (1):

$$8(a + 12 - 2a) = 5a(12 - 2a)$$

$$8(12 - a) = 60a - 10a^2$$

$$96 - 8a = 60a - 10a^2$$

$$10a^2 - 68a + 96 = 0$$

$$5a^2 - 34a + 48 = 0$$

Using the quadratic formula: $a = \frac{34 \pm \sqrt{34^2 - 4(5)(48)}}{2(5)} = \frac{34 \pm \sqrt{1156 - 960}}{10} = \frac{34 \pm 14}{10}$

$$a = \frac{48}{10} = 4.8, \quad a = \frac{20}{10} = 2 \text{ (Discarded because the problem states } a > 2 \text{)}$$

With $a = 4.8$: $\alpha = 8 - 4.8 = \mathbf{3.2}$, $b = 12 - 2(4.8) = 12 - 9.6 = \mathbf{2.4}$

Analyze the Quadratic Equation

The given equation is $\alpha x^2 - ax + 2(\alpha - 2b) = 0$. Substitute the values:

$$3.2x^2 - 4.8x + 2(3.2 - 2(2.4)) = 0$$

$$3.2x^2 - 4.8x + 2(3.2 - 4.8) = 0$$

$$3.2x^2 - 4.8x - 3.2 = 0$$

$$2x^2 - 3x - 2 = 0$$

Factorise $(2x + 1)(x - 2) = 0$

The roots are $x = 2$ and $x = -0.5$.

Looking at the options: Root $x = 2$ lies in the interval $(1, 4)$. Root $x = -0.5$ lies in the interval $(-2, 0)$.

Question 18

Let A be the set of first 101 terms of an A.P., whose first term is 1 and the common difference is 5 and let B be the set of first 71 terms of an A.P., whose first term is 9 and the common difference is 7. Then the number of elements in $A \cap B$, which are divisible by 3, is :

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Options:

A.

4

B.

5

C.

6

D.

7

Answer: B

Solution:

Understand Set A

Set A consists of the first 101 terms of an A.P. with the first term $a = 1$ and common difference $d_1 = 5$.

The n -th term of this A.P. is given by the formula:

$$a_n = a + (n - 1)d_1$$

Therefore, the last term (101st term) of Set A is:

$$a_{101} = 1 + (101 - 1) \times 5 = 1 + 100 \times 5 = 501$$

$$\text{So, } A = \{1, 6, 11, 16, 21, \dots, 501\}.$$

Understand Set B

Set B consists of the first 71 terms of an A.P. with the first term $b = 9$ and common difference $d_2 = 7$.

The m -th term of this A.P. is:

$$b_m = b + (m - 1)d_2$$

Therefore, the last term (71st term) of Set B is:

$$b_{71} = 9 + (71 - 1) \times 7 = 9 + 70 \times 7 = 499$$

$$\text{So, } B = \{9, 16, 23, 30, \dots, 499\}.$$

Find the elements in $A \cap B$

The elements common to both sets A and B will themselves form an A.P.

From the initial terms listed above, we can easily see that the first common term is 16.

The common difference of this new A.P. will be the Least Common Multiple (LCM) of the common differences of the two original sets:

$$\text{Common difference of } A \cap B = \text{LCM}(5, 7) = 35$$

The k -th term of the common sequence is given by:

$$C_k = 16 + (k - 1)35$$

Since a common term must exist in both sets, it cannot be greater than the smallest of their last terms. Thus, $C_k \leq \min(501, 499)$.

$$16 + (k - 1)35 \leq 499$$

$$35(k - 1) \leq 483$$

$$k - 1 \leq \frac{483}{35}$$

$$k - 1 \leq 13.8$$

Since k must be a positive integer, the maximum possible value for $(k - 1)$ is 13. Thus, $k \leq 14$.

There are altogether 14 common terms in $A \cap B$.

Find elements in $A \cap B$ divisible by 3

We want to find terms C_k that are divisible by 3, which means:

$$C_k \equiv 0 \pmod{3}$$

$$16 + 35(k - 1) \equiv 0 \pmod{3}$$

Since $16 = 3 \times 5 + 1 \implies 16 \equiv 1 \pmod{3}$ and $35 = 3 \times 11 + 2 \implies 35 \equiv 2 \pmod{3}$, we can simplify:

$$1 + 2(k - 1) \equiv 0 \pmod{3}$$

Checking the first few integer values for k :

- If $k = 1$: $1 + 2(0) = 1$ (Not divisible by 3)
- If $k = 2$: $1 + 2(1) = 3 \equiv 0 \pmod{3}$ (Divisible by 3)
- If $k = 3$: $1 + 2(2) = 5 \equiv 2 \pmod{3}$ (Not divisible by 3)

The condition holds true when $k = 2$. Because we are operating in modulo 3, the values of k that satisfy this condition will form an A.P. with a common difference of 3.

Thus, the valid values of k within the range $1 \leq k \leq 14$ are:

$$k \in \{2, 5, 8, 11, 14\}$$

Counting these, there are exactly 5 valid values of k .

For confirmation, the terms are:

- For $k = 2 \rightarrow 51$
- For $k = 5 \rightarrow 156$
- For $k = 8 \rightarrow 261$
- For $k = 11 \rightarrow 366$
- For $k = 14 \rightarrow 471$

All 5 of these terms are common elements between the sets and are completely divisible by 3.

Correct Answer:

Option B

Question 19

The sum $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ up to 8 terms, is :

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Options:

A.

70

B.

71

C.

72

D.

73

Answer: B

Solution:

$$\begin{aligned} T_r &= \frac{1^3 + 2^3 + 3^3 + \dots + r^3}{1 + 3 + 5 + \dots + (2r-1)} = \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2} \\ &= \frac{r^2 + 2r + 1}{4} \end{aligned}$$

$$\begin{aligned}
 S_n &= \sum_{r=1}^n T_r \\
 S_n &= \frac{1}{4} \sum (r^2 + 2r + 1) \\
 &= \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + 2 \frac{n(n+1)}{2} + n \right] \\
 S_8 &= \frac{1}{4} \left[\frac{8 \times 9 \times 17}{6} + 8 \times 9 + 8 \right] \\
 &= \frac{1}{4} [204 + 72 + 8] = 71
 \end{aligned}$$

Question 20

Let $a_1, a_2, a_3, \dots$ be an A.P. and $g_1 = a_1, g_2, g_3, \dots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to:

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Options:

A.

81

B.

76

C.

62

D.

55

Answer: D

Solution:

Let the common difference of the given A.P. be d and the common ratio of the given G.P. be r .

From the general formulas for the n -th term of an A.P. and a G.P., we know:

$$a_n = a_1 + (n-1)d$$

$$g_n = g_1 r^{n-1}$$

We are given that $g_1 = a_1$ and $a_1 = 1$.

Thus, the first term of both the A.P. and the G.P. is 1. So, $g_1 = 1$.

Next, we are given the first relationship:

$$a_2 + g_2 = 1$$

Using our formulas, we can write the second terms as $a_2 = 1 + d$ and $g_2 = 1 \cdot r = r$. Let us substitute these values into the given equation:

$$(1 + d) + r = 1$$

Subtracting 1 from both sides, we establish a relationship between d and r :

$$d + r = 0 \implies d = -r$$

We are also given a second relationship:

$$a_3 + g_3 = 4$$

Again, applying the n -th term formulas, we have the third terms as $a_3 = 1 + 2d$ and $g_3 = 1 \cdot r^2 = r^2$. Substituting these in the equation gives:

$$(1 + 2d) + r^2 = 4$$

Now, let us substitute $d = -r$ into this equation to form a quadratic equation in terms of r :

$$1 + 2(-r) + r^2 = 4$$

$$r^2 - 2r + 1 - 4 = 0$$

$$r^2 - 2r - 3 = 0$$

Let us factorize this quadratic equation by splitting the middle term:

$$r^2 - 3r + r - 3 = 0$$

$$r(r - 3) + 1(r - 3) = 0$$

$$(r - 3)(r + 1) = 0$$

This gives us two possible values for r : $r = 3$ or $r = -1$.

The problem clearly states that the G.P. is an **increasing** sequence. Since the first term $g_1 = 1$ is positive, an increasing sequence requires the common ratio to be strictly greater than 1. Therefore, we reject $r = -1$ and take the valid value:

$$r = 3$$

Since $d = -r$, we find the common difference of the A.P.:

$$d = -3$$

Now, we are asked to find the value of $a_{10} + g_5$. Let us find a_{10} and g_5 individually.

For the 10th term of the A.P.:

$$a_{10} = a_1 + (10 - 1)d$$

$$a_{10} = a_1 + 9d$$

$$a_{10} = 1 + 9(-3)$$

$$a_{10} = 1 - 27 = -26$$

For the 5th term of the G.P.:

$$g_5 = g_1 r^{5-1}$$

$$g_5 = g_1 r^4$$

$$g_5 = 1 \cdot (3)^4$$

$$g_5 = 81$$

Finally, adding these two values together:

$$a_{10} + g_5 = -26 + 81 = 55$$

Therefore, the value of $a_{10} + g_5$ is 55.

Correct Answer:

Option D: 55

Question21

The first term of an A.P. of 30 non-negative terms is $\frac{10}{3}$. If the sum of this A.P. is the cube of its last term, then its common difference is:

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Options:

A.

$$\frac{5}{87}$$

B.

$$\frac{25}{83}$$

C.

$$\frac{15}{29}$$

D.

$$\frac{5}{29}$$

Answer: A

Solution:

To solve this problem, we will use the standard formulas for an Arithmetic Progression (A.P.) as taught in the NCERT syllabus. Let us break down the information given in the question step by step.

Given:

1. The number of terms in the A.P. is $n = 30$.
2. The first term is $a = \frac{10}{3}$.
3. All the terms in the A.P. are non-negative.
4. The sum of the A.P. is equal to the cube of its last term.

Let the common difference of the A.P. be d and the last term (which is the 30th term) be l .

The formula for the n -th term of an A.P. is:

$$l = a + (n - 1)d$$

$$l = \frac{10}{3} + (30 - 1)d$$

$$l = \frac{10}{3} + 29d \quad \text{--- (Equation 1)}$$

The formula for the sum of n terms of an A.P. when the first and last terms are known is:

$$S_n = \frac{n}{2}(a + l)$$

For $n = 30$, we substitute our known values:

$$S_{30} = \frac{30}{2} \left(\frac{10}{3} + l \right)$$

$$S_{30} = 15 \left(\frac{10}{3} + l \right)$$

$$S_{30} = 50 + 15l$$

According to the problem, the sum of this A.P. is equal to the cube of its last term. So, we can write:

$$S_{30} = l^3$$

Now, substitute the value of S_{30} into this equation:

$$50 + 15l = l^3$$

Rearranging the terms to form a standard cubic equation, we get:

$$l^3 - 15l - 50 = 0$$

To find the roots of this cubic equation, we can use the trial and error method for integer values.

Let us check $l = 5$:

$$(5)^3 - 15(5) - 50 = 125 - 75 - 50 = 0$$

Since $l = 5$ satisfies the equation, $(l - 5)$ is a factor.

Now, we can factorise the cubic polynomial completely:

$$l^3 - 15l^2 + 5l^2 - 25l + 10l - 50 = 0$$

$$l^2(l - 5) + 5l(l - 5) + 10(l - 5) = 0$$

$$(l - 5)(l^2 + 5l + 10) = 0$$

This gives us two parts:

$$1. l - 5 = 0 \implies l = 5$$

$$2. l^2 + 5l + 10 = 0$$

For the quadratic equation $l^2 + 5l + 10 = 0$, the discriminant is:

$$D = (5)^2 - 4(1)(10) = 25 - 40 = -15$$

Since the discriminant is negative ($D < 0$), this part has no real roots. Therefore, the only real solution for the last term is:

$$l = 5$$

Now, substitute $l = 5$ back into Equation 1 to find the common difference d :

$$5 = \frac{10}{3} + 29d$$

Subtract $\frac{10}{3}$ from both sides:

$$29d = 5 - \frac{10}{3}$$

$$29d = \frac{15-10}{3}$$

$$29d = \frac{5}{3}$$

Now, divide both sides by 29:

$$d = \frac{5}{3 \times 29}$$

$$d = \frac{5}{87}$$

Thus, the common difference of the given A.P. is $\frac{5}{87}$. Since the first term and the common difference are positive, all terms are indeed non-negative, satisfying all conditions.

Option A is the correct answer.

Question 22

$\sum_{n=1}^{10} \left(\frac{528}{n(n+1)(n+2)} \right)$ is equal to:

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Options:

A.

65

B.

130

C.

220

D.

440

Answer: B

Solution:

$$S = \sum_{n=1}^{10} \frac{528}{n(n+1)(n+2)}$$

We will simplify the general term using partial fractions.

Step 1: Use the standard identity

$$\frac{1}{n(n+1)(n+2)} = \frac{1}{2} \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right)$$

So,

$$\begin{aligned} \frac{528}{n(n+1)(n+2)} &= 528 \cdot \frac{1}{2} \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right) \\ &= 264 \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right) \end{aligned}$$

Hence,

$$S = 264 \sum_{n=1}^{10} \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right)$$

Step 2: Write a few terms

$$S = 264 \left[\left(\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right) + \left(\frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} \right) + \left(\frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} \right) + \cdots + \left(\frac{1}{10 \cdot 11} - \frac{1}{11 \cdot 12} \right) \right]$$

Now the middle terms cancel out telescopically.

So we get

$$S = 264 \left(\frac{1}{1 \cdot 2} - \frac{1}{11 \cdot 12} \right)$$

$$S = 264 \left(\frac{1}{2} - \frac{1}{132} \right)$$

Take LCM:

$$\frac{1}{2} = \frac{66}{132}$$

So,

$$\frac{1}{2} - \frac{1}{132} = \frac{66-1}{132} = \frac{65}{132}$$

Therefore,

$$S = 264 \cdot \frac{65}{132}$$

$$S = 2 \cdot 65 = 130$$

130

So, the correct option is **Option B**.

Question 23

Let the sum of the first n terms of an A.P. be $3n^2 + 5n$. Then the sum of squares of the first 10 terms of the A.P. is:

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Options:

A.

10220

B.

12860

C.

15220

D.

19780

Answer: C

Solution:

Given,

$$S_n = 3n^2 + 5n$$

where S_n is the sum of the first n terms of the A.P.

To get the n th term, we use

$$a_n = S_n - S_{n-1}$$

So,

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$= 3(n^2 - 2n + 1) + 5n - 5$$

$$= 3n^2 - 6n + 3 + 5n - 5$$

$$= 3n^2 - n - 2$$

Now,

$$a_n = (3n^2 + 5n) - (3n^2 - n - 2)$$

$$a_n = 6n + 2$$

So the terms of the A.P. are:

8, 14, 20, ...

Now we need the sum of squares of the first 10 terms:

$$\sum_{n=1}^{10} (6n + 2)^2$$

Expand:

$$(6n + 2)^2 = 36n^2 + 24n + 4$$

So,

$$\sum_{n=1}^{10} (6n + 2)^2 = 36 \sum_{n=1}^{10} n^2 + 24 \sum_{n=1}^{10} n + 4 \sum_{n=1}^{10} 1$$

Now use the standard sums:

$$\sum_{n=1}^{10} n^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385$$

$$\sum_{n=1}^{10} n = \frac{10 \cdot 11}{2} = 55$$

$$\sum_{n=1}^{10} 1 = 10$$

Substitute:

$$36(385) + 24(55) + 4(10)$$

$$= 13860 + 1320 + 40$$

$$= 15220$$

So the correct answer is

15220

Option C

Question 24

Let $A_1, A_2, A_3, \dots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to :

JEE Main 2026 (Online) 5th April Evening Shift

Options:

A.

129

B.

136

C.

131.50

D.

134

Answer: D

Solution:

Since there are 39 arithmetic means between 59 and 159, the full arithmetic progression has

$$59, A_1, A_2, A_3, \dots, A_{39}, 159$$

So total number of terms is

$$39 + 2 = 41$$

Hence, the number of intervals is

$$41 - 1 = 40$$

Therefore, the common difference is

$$d = \frac{159-59}{40} = \frac{100}{40} = 2.5$$

Now the terms are:

$$A_1 = 59 + 2.5 = 61.5$$

In general,

$$A_n = 59 + n(2.5)$$

Now find the required terms:

$$A_{25} = 59 + 25(2.5) = 59 + 62.5 = 121.5$$

$$A_{28} = 59 + 28(2.5) = 59 + 70 = 129$$

$$A_{31} = 59 + 31(2.5) = 59 + 77.5 = 136.5$$

$$A_{36} = 59 + 36(2.5) = 59 + 90 = 149$$

Now their mean is

$$\frac{A_{25}+A_{28}+A_{31}+A_{36}}{4} = \frac{121.5+129+136.5+149}{4}$$
$$= \frac{536}{4} = 134$$

So the correct answer is:

134

Hence, Option D is correct.

Question 25

If the sum of the first 10 terms of the series

$\frac{1}{1+1^4 \times 4} + \frac{2}{1+2^4 \times 4} + \frac{3}{1+3^4 \times 4} + \frac{4}{1+4^4 \times 4} + \dots$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

JEE Main 2026 (Online) 5th April Evening Shift

Options:

A.

256

B.

264

C.

276

D.

284

Answer: C

Solution:

The given series is

$$\frac{1}{1+4 \cdot 1^4} + \frac{2}{1+4 \cdot 2^4} + \frac{3}{1+4 \cdot 3^4} + \dots$$

So the general term is

$$\frac{r}{1+4r^4}$$

We need the sum of the first 10 terms:

$$S_{10} = \sum_{r=1}^{10} \frac{r}{1+4r^4}$$

Now factor the denominator:

$$1 + 4r^4 = (2r^2 - 2r + 1)(2r^2 + 2r + 1)$$

So,

$$\frac{r}{1+4r^4} = \frac{r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

We now try to split it:

$$\frac{r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)} = \frac{1}{4} \left(\frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1} \right)$$

Check:

$$\frac{1}{4} \left(\frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1} \right)$$

Taking LCM,

$$= \frac{1}{4} \cdot \frac{(2r^2 + 2r + 1) - (2r^2 - 2r + 1)}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

$$= \frac{1}{4} \cdot \frac{4r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

$$= \frac{r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

Correct.

Also note that

$$2r^2 + 2r + 1 = 2(r + 1)^2 - 2(r + 1) + 1$$

So if we define

$$a_r = 2r^2 - 2r + 1$$

then

$$2r^2 + 2r + 1 = a_{r+1}$$

Hence,

$$\frac{r}{1+4r^4} = \frac{1}{4} \left(\frac{1}{a_r} - \frac{1}{a_{r+1}} \right)$$

Therefore,

$$S_{10} = \frac{1}{4} \sum_{r=1}^{10} \left(\frac{1}{a_r} - \frac{1}{a_{r+1}} \right)$$

This is a telescoping series:

$$S_{10} = \frac{1}{4} \left(\frac{1}{a_1} - \frac{1}{a_{11}} \right)$$

Now compute:

$$a_1 = 2(1)^2 - 2(1) + 1 = 1$$

and

$$a_{11} = 2(11)^2 - 2(11) + 1 = 242 - 22 + 1 = 221$$

So,

$$S_{10} = \frac{1}{4} \left(1 - \frac{1}{221} \right)$$

$$= \frac{1}{4} \cdot \frac{220}{221}$$

$$= \frac{55}{221}$$

Thus,

$$m = 55, \quad n = 221$$

So,

$$m + n = 55 + 221 = 276$$

Hence, the correct answer is:

276

So, **Option C** is correct.

Question 26

The sum $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \dots$ upto 10 terms is equal to :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

130

B.

155

C.

$$\frac{315}{2}$$

D.

$$\frac{325}{2}$$

Answer: C

Solution:

$$T_n = \frac{1}{n} (1^2 + 2^2 + \dots + n^2)$$

$$T_n = \frac{n(n+1)(2n+1)}{6n}$$

$$T_n = \frac{1}{6} [2n^2 + 3n + 1]$$

$$S_n = \sum T_n = \frac{1}{6} [2\sum n^2 + 3\sum n + \sum 1]$$

$$\sum T_n = \frac{1}{6} \left[2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n \right]$$

$$S_{10} = \frac{1}{6} \left[\frac{2 \times 10 \times 11 \times 21}{6} + \frac{3}{2} \times 10 \times 11 + 10 \right]$$

$$= \frac{10}{6} \left[77 + \frac{33}{2} + 1 \right] = \frac{315}{2}$$

Question 27

Consider the quadratic equation

$(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0, n \in \mathbf{R}$. Let α be the minimum value of the product of its roots and β be the maximum value of the sum of its roots.

Then the sum of the first six terms of the G.P., whose first term is α and the common ratio is $\frac{\alpha}{\beta}$, is :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

$$\frac{61}{37}$$

B.

$$\frac{121}{81}$$

C.

$$\frac{364}{243}$$

D.

$$\frac{1093}{729}$$

Answer: C

Solution:

For the quadratic equation

$$(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0,$$

let us first compare it with the standard quadratic form

$$ax^2 + bx + c = 0.$$

So,

$$a = n^2 - 2n + 2, \quad b = -3, \quad c = (n^2 - 2n + 2)^2.$$

Now rewrite a in a simpler form:

$$n^2 - 2n + 2 = (n - 1)^2 + 1.$$

Hence,

$$a = (n - 1)^2 + 1.$$

Since $(n - 1)^2 \geq 0$, we get

$$a \geq 1.$$

Now, for a quadratic equation, by Vieta's formulas:

- Sum of roots $= -\frac{b}{a}$
- Product of roots $= \frac{c}{a}$

So here,

$$\text{Sum of roots} = \frac{3}{a}$$

and

$$\text{Product of roots} = \frac{a^2}{a} = a.$$

Therefore,

$$\text{Product of roots} = (n - 1)^2 + 1.$$

So the minimum value of the product is clearly

$$\alpha = 1,$$

attained when $n = 1$.

Now the sum of roots is

$$\frac{3}{(n-1)^2+1}.$$

Since the denominator is minimum when $(n - 1)^2 = 0$, the maximum value of the sum is

$$\beta = \frac{3}{1} = 3.$$

So we have

$$\alpha = 1, \quad \beta = 3.$$

Now the G.P. has

- first term $a = \alpha = 1$
- common ratio $r = \frac{\alpha}{\beta} = \frac{1}{3}$

We need the sum of first six terms:

$$S_6 = a \cdot \frac{1-r^6}{1-r}.$$

Substitute the values:

$$S_6 = 1 \cdot \frac{1-(\frac{1}{3})^6}{1-\frac{1}{3}}.$$

Now,

$$\left(\frac{1}{3}\right)^6 = \frac{1}{729}$$

and

$$1 - \frac{1}{3} = \frac{2}{3}.$$

Hence,

$$S_6 = \frac{1 - \frac{1}{729}}{\frac{2}{3}} = \frac{\frac{728}{729}}{\frac{2}{3}} = \frac{728}{729} \cdot \frac{3}{2}.$$

So,

$$S_6 = \frac{1092}{729} = \frac{364}{243}.$$

Therefore, the required sum is

$$\boxed{\frac{364}{243}}.$$

So the correct option is

Option C.

Question 28

Let $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots$ upto 40 terms. If $(\tan \beta)^{\frac{\alpha}{1020}}$ is a root of the equation $x^2 + x - 2 = 0, \beta \in (0, \frac{\pi}{2})$, then $\sin^2 \beta + 3 \cos^2 \beta$ is equal to :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

2

B.

$\frac{7}{4}$

C.

$\frac{5}{2}$

D.

$\frac{3}{2}$

Answer: A

Solution:

1. A series sum is defined : $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots t_{40}$

2. $(\tan \beta)^{\frac{2}{1020}}$ is a root of the quadratic equation $x^2 + x - 2 = 0$ where $\beta \in (0, \frac{\pi}{2})$

3. To evaluate the expression $\sin^2 \beta + 3 \cos^2 \beta$

$$\therefore \alpha = 3 + 4 + 8 + 9 + 13 + 14 + \dots t_{40}$$

$$= 7 + 17 + 27 + \dots + T_{20}$$

$$= \frac{20}{2} \{ (2) \cdot (7) + (20-1) \cdot (10) \}$$

$$= (10) \cdot (14 + 190) = 2,040 \dots (i)$$

Equation (i) gives the value of $\alpha = 2,040$:

$$(\tan \beta)^{\frac{2}{1020}} = (\tan \beta)^{\frac{2040}{1020}} = \tan^2 \beta \dots (ii)$$

Now let us solve the quadratic equation $x^2 + x - 2 = 0$:

$$x^2 + x - 2 = 0 \Rightarrow x^2 + 2x - x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow \begin{array}{|c|c|} \hline p & q \\ \hline -2 & 1 \\ \hline \end{array}$$

where p and q are the roots

We have derived in equation (ii) $(\tan \beta)^{\frac{2}{1020}} = \tan^2 \beta$ which is also a root of the equation $x^2 + x - 2 = 0$:

From the pair of roots we get $\tan^2 \beta = 1 \Rightarrow \tan \beta = \pm 1$ As $\tan^2 \beta \geq 0$ always

$$\beta \in (0, \frac{\pi}{2}) \Rightarrow \tan \beta = 1 \Rightarrow \beta = \frac{\pi}{4}$$

$$\Rightarrow \sin^2 \beta + 3 \cos^2 \beta = \sin^2 \frac{\pi}{4} + 3 \cos^2 \frac{\pi}{4} = \left(\frac{1}{2}\right) + \left(\frac{3}{2}\right) = 2$$

Question 29

The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to _____.

JEE Main 2025 (Online) 23rd January Evening Shift

Answer: 474

Solution:

$$S_{11} = \frac{11}{2}(2a + 10d) = 88$$

$$a + 5d = 8$$

$$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$$

Roots are

$$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$$

$$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$$

$$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$$

$$p = \frac{177}{2}$$

$$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$$

$$q = 651$$

$$q - 2p$$

$$= 651 - 177$$

$$= 474$$

Question30

The interior angles of a polygon with n sides, are in an A.P. with common difference 6° . If the largest interior angle of the polygon is 219° , then n is equal to _____.

JEE Main 2025 (Online) 28th January Evening Shift

Answer: 20

Solution:

$$\frac{n}{2}(2a + (n-1)6) = (n-2) \cdot 180^\circ$$

$$an + 3n^2 - 3n = (n-2) \cdot 180^\circ \quad \dots (1)$$

Now according to question

$$a + (n-1)6^\circ = 219^\circ$$

$$\Rightarrow a = 225^\circ - 6n^\circ \quad \dots (2)$$

Putting value of a from equation (2) in (1)

We get

$$(225n - 6n^2) + 3n^2 - 3n = 180n - 360$$

$$\Rightarrow 2n^2 - 42n - 360 = 0$$

$$\Rightarrow n^2 - 14n - 120 = 0$$

$$n = 20, -6 \text{ (rejected)}$$

Question31

Let $a_1, a_2, \dots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (a_5 + a_{10} + a_{15} + \dots + a_{2020}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \dots + a_{2024}$ is equal to _____.

JEE Main 2025 (Online) 29th January Evening Shift

Answer: 11132

Solution:

$$a_1 + a_5 + a_{10} + \dots + a_{2020} + a_{2024} = 2233$$

In an A.P. the sum of terms equidistant from ends is equal.

$$a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} \dots$$

$$\Rightarrow 203 \text{ pairs}$$

$$\Rightarrow 203(a_1 + a_{2024}) = 2233$$

Hence,

$$\begin{aligned} S_{2024} &= \frac{2024}{2}(a_1 + a_{2024}) \\ &= 1012 \times 11 \\ &= 11132 \end{aligned}$$

Question32

If the sum of the first 10 terms of the series $\frac{4 \cdot 1}{1+4 \cdot 1^4} + \frac{4 \cdot 2}{1+4 \cdot 2^4} + \frac{4 \cdot 3}{1+4 \cdot 3^4} + \dots$ is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

JEE Main 2025 (Online) 2nd April Evening Shift

Answer: 441

Solution:

$$\begin{aligned} &\frac{4 \cdot 1}{1+4 \cdot 1^4} + \frac{4 \cdot 2}{1+4 \cdot 2^4} + \frac{4 \cdot 3}{1+4 \cdot 3^4} + \dots \\ T_r &= \frac{4r}{1+4r^4} = \frac{4r}{4r^4 + 4r^2 + 1 - 4r^2} \\ &= \frac{4r}{(2r^2 + 1)^2 - (2r)^2} \\ T_r &= \frac{4r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)} \end{aligned}$$

$$T_r = \frac{(2r^2 + 2r + 1) - (2r^2 - 2r + 1)}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

$$T_r = \left(\frac{1}{r^2 + (r-1)^2} - \frac{1}{r^2 + (r+1)^2} \right)$$

$$\sum_{r=1}^{10} T_r = \left(\frac{1}{0^2 + 1^2} - \frac{1}{1^2 + 2^2} + \frac{1}{1^2 + 2^2} - \frac{1}{2^2 + 3^2} + \dots \right)$$

$$\frac{1}{9^2 + 10^2} - \frac{1}{10^2 + 11^2}$$

$$= 1 - \frac{1}{221}$$

$$= \frac{220}{221}$$

$$\therefore m + n = 220 + 221$$

$$= 441$$

Question33

Let $a_1, a_2, a_3, \dots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to:

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

A. 812

B. 784

C. 628

D. 526

Answer: B

Solution:

First, let us denote the first term of the G.P. by $a_1 = A$ and the common ratio (which is > 1 , since the G.P. is increasing) by r . Then the terms are:

$$a_1 = A, \quad a_2 = Ar, \quad a_3 = Ar^2, \quad a_4 = Ar^3, \quad a_5 = Ar^4, \quad a_6 = Ar^5, \dots$$

We are given:

$$a_1 \cdot a_5 = 28, \text{ i.e.}$$

$$A \cdot (Ar^4) = A^2 r^4 = 28. \quad (1)$$

$$a_2 + a_4 = 29, \text{ i.e.}$$

$$Ar + Ar^3 = A(r + r^3) = 29. \quad (2)$$

We want to find $a_6 = Ar^5$.

1. Solve for r

From (1):

$$A^2 r^4 = 28 \implies A^2 = \frac{28}{r^4}.$$

From (2):

$$A(r + r^3) = 29 \implies A = \frac{29}{r + r^3}.$$

Plug A from (2) into (1). After some algebra (or by a systematic approach), one finds that

$$r^2 = 28 \implies r = \sqrt{28} = 2\sqrt{7}$$

(since $r > 1$, we take the positive root).

2. Solve for A

From (2), using $r = 2\sqrt{7}$:

$$r + r^3 = 2\sqrt{7} + (2\sqrt{7})^3 = 2\sqrt{7} + 56\sqrt{7} = 58\sqrt{7}.$$

Hence,

$$A = \frac{29}{58\sqrt{7}} = \frac{1}{2\sqrt{7}}.$$

3. Find a_6

$$a_6 = A r^5 = \frac{1}{2\sqrt{7}} \times (2\sqrt{7})^5.$$

Compute $(2\sqrt{7})^5$. First, $(2\sqrt{7})^2 = 28$; thus

$$(2\sqrt{7})^5 = (2\sqrt{7})^4 \cdot (2\sqrt{7}) = 784 \times (2\sqrt{7}) = 1568\sqrt{7}.$$

Therefore,

$$a_6 = \frac{1}{2\sqrt{7}} \cdot 1568\sqrt{7} = \frac{1568}{2} \text{ (since } \sqrt{7} \text{ cancels)} = 784.$$

Answer: $a_6 = 784$.

Hence, the correct option is **Option B**.

Question 34

Suppose that the number of terms in an A.P. is $2k$, $k \in \mathbb{N}$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to:

JEE Main 2025 (Online) 22nd January Evening Shift

Options:

A. 8

B. 6

C. 4

D. 5

Answer: D

Solution:

$a_1, a_2, a_3, \dots, a_{2k} \rightarrow \text{A.P.}$

$$\sum_{r=1}^k a_{2r-1} = 40, \sum_{r=1}^k a_{2r} = 55, a_{2k} - a_1 = 27$$

$$\frac{k}{2}[2a_1 + (k-1)2d] = 40, \frac{k}{2}[2a_2 + (k-1)2d] = 55,$$

$$d = \frac{27}{2k-1}$$

$$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$$

$$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9k = 10k - 5$$

$$\therefore k = 5$$

Question 35

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. -120

B. -1200

C. -1080

D. -1020

Answer: C

Solution:

The first term, $a = 3$

Common difference, d

The formula for the sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Given:

$$S_4 = \frac{1}{5}(S_8 - S_4)$$

This implies:

$$5S_4 = S_8 - S_4 \Rightarrow 6S_4 = S_8$$

Substituting the sum formulas:

$$6 \cdot \frac{4}{2}[2 \times 3 + (4-1)d] = \frac{8}{2}[2 \times 3 + (8-1)d]$$

Simplifying:

$$6 \times 2[6 + 3d] = 4[6 + 7d]$$

$$12(6 + 3d) = 4(6 + 7d)$$

$$72 + 36d = 24 + 28d$$

$$36d - 28d = 24 - 72$$

$$8d = -48$$

$$d = -6$$

Now, to find S_{20} :

$$S_{20} = \frac{20}{2}[2 \times 3 + (20 - 1)(-6)]$$

$$S_{20} = 10[6 + 19 \times (-6)]$$

$$S_{20} = 10[6 - 114]$$

$$S_{20} = 10 \times (-108)$$

$$S_{20} = -1080$$

Thus, the sum of the first 20 terms is -1080 .

Question 36

Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots$ upto n terms. If the sum of the first six terms of an A.P. with first term $-p$ and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference between 20^{th} and 15^{th} terms of the A.P. is

JEE Main 2025 (Online) 24th January Morning Shift

Options:

A. 20

B. 45

C. 90

D. 25

Answer: D

Solution:

To find the value of S_{2025} , calculate the partial sum

$$S_n = \sum_{n=1}^{2025} \frac{1}{n(n+1)} = \sum_{n=1}^{2025} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

This series is telescopic, meaning most terms cancel out with each other:

$$= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots + \left(\frac{1}{2025} - \frac{1}{2026} \right)$$

Simplifying the expression, we find:

$$= \frac{1}{1} - \frac{1}{2026} = 1 - \frac{1}{2026} = \frac{2025}{2026}$$

Now, evaluate $\sqrt{2026 \cdot S_{2025}}$:

$$\sqrt{2026 \cdot \frac{2025}{2026}} = \sqrt{2025} = 45$$

For the arithmetic progression with the first term $-p$ and common difference p , the sum of the first six terms is given by:

$$\frac{6}{2}[-2p + (6-1)p] = 3(5p) = 15p$$

Given that:

$$15p = 45$$

We find:

$$p = 3$$

To determine the absolute difference between the 20th and 15th terms of the A.P., compute:

$$|A_{20} - A_{15}| = |(-p + 19p) - (-p + 14p)|$$

Substituting the value of p :

$$= |18p - 13p| = |5p| = 5 \times 5 = 25$$

Thus, the absolute difference is 25.

Question 37

If $7 = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \frac{1}{7^3}(5 + 3\alpha) + \dots \infty$, then the value of α is :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. $\frac{1}{7}$

B. 1

C. $\frac{6}{7}$

D. 6

Answer: D

Solution:

$$\text{Let } S = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \dots$$

$$\frac{1}{7}S = \frac{1}{7}(5) + \frac{1}{7^2}(5 + \alpha) + \dots \infty$$

$$\frac{6}{7}(S) = 5 + \frac{1}{7}\alpha \left(\frac{1}{1 - \frac{1}{7}} \right)$$

$$6 = 5 + \frac{\alpha}{6} \Rightarrow \alpha = 6$$

Question38

In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 525

B. 505

C. 510

D. 515

Answer: D

Solution:

Let a & d are first term and common diff of an AP.

$$S_{40} = \frac{40}{2} [2a + 39d] = 1030 \quad \dots (1)$$

$$S_{12} = \frac{12}{2} [2a + 11d] = 57 \quad \dots (2)$$

by (1) & (2)

$$a = -\frac{7}{2} \quad d = \frac{3}{2}$$

$$\begin{aligned} \therefore S_{30} - S_{10} &= \frac{30}{2} [2a + 29d] - \frac{10}{2} [2a + 9d] \\ &= 20a - 390d \\ &= 515 \end{aligned}$$

Question39

Let T_r be the r^{th} term of an A.P. If for some m , $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$, and

$20 \sum_{r=1}^{25} T_r = 13$, then $5 \sum_{r=m}^{2m} T_r$ is equal to

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Options:

A. 98

B. 126

C. 112

D. 142

Answer: B

Solution:

To solve this problem, we start by analyzing the terms of an arithmetic progression (A.P.) where:

$$T_m = \frac{1}{25}$$

$$T_{25} = \frac{1}{20}$$

$$20 \sum_{r=1}^{25} T_r = 13$$

The formula for the r^{th} term of an A.P. is:

$$T_r = a + (r - 1)d$$

Given:

$$T_m = a + (m - 1)d = \frac{1}{25} \quad (\text{Equation 1})$$

$$T_{25} = a + 24d = \frac{1}{20}$$

The sum of the first 25 terms (S_{25}) is given by:

$$S_{25} = \frac{25}{2} \times (2a + 24d)$$

Substituting into the equation for the sum:

$$20 \times \frac{25}{2} \times (2a + 24d) = 13$$

This simplifies to:

$$a = \frac{1}{500}$$

Substituting $a = \frac{1}{500}$ into $T_{25} = a + 24d = \frac{1}{20}$, we find:

$$\frac{1}{500} + 24d = \frac{1}{20}$$

$$24d = \frac{1}{20} - \frac{1}{500}$$

$$d = \frac{1}{500}$$

Using Equation 1 again:

$$\frac{1}{500} + \frac{m-1}{500} = \frac{1}{25}$$

$$\frac{m}{500} = \frac{1}{25}$$

$$m = 20$$

Now to find $5m \sum_{r=m}^{2m} T_r$:

$$5m = 5 \times 20 = 100$$

$$\sum_{r=20}^{40} T_r = \frac{21}{2} \times (T_{20} + T_{40})$$

Since $m = 20$, the summation covers terms from T_{20} to T_{40} , and:

$$100 \times \sum_{r=20}^{40} T_r = 126$$

Therefore, $5m \sum_{r=m}^{2m} T_r = 126$.

Question40

Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and

$2a_{n+2} = 5a_{n+1} - 3a_n, n = 0, 1, 2, 3, \dots$. Then $\sum_{k=1}^{100} a_k$ is equal to

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Options:

A. $3a_{100} + 100$

B. $3a_{100} - 100$

C. $3a_{99} - 100$

D. $3a_{99} + 100$

Answer: B

Solution:

$$a_0 = 0, a_1 = \frac{1}{2}$$

$$2a_{n+2} = 5a_{n+1} - 3a_n$$

$$2x^2 - 5x + 3 = 0 \Rightarrow x = 1, 3/2$$

$$\therefore a_n = A(1)^n + B\left(\frac{3}{2}\right)^n$$

$$n = 0 \quad 0 = A + B \quad \left. \begin{array}{l} A = -1 \\ B = 1 \end{array} \right\}$$

$$n = 1 \quad \frac{1}{2} = A + \frac{3}{2}B$$

$$\Rightarrow a_n = -1 + \left(\frac{3}{2}\right)^n$$

$$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100} (-1) + \left(\frac{3}{2}\right)^k$$

$$= -100 + \frac{\left(\frac{3}{2}\right) \left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{3}{2} - 1}$$

$$= -100 + 3 \left(\left(\frac{3}{2}\right)^{100} - 1\right)$$

$$= 3 \cdot (a_{100}) - 100$$

Question41

For positive integers n , if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^n \left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is :

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Options:

A.

540

B.

675

C.

1350

D.

135

Answer: B

Solution:

$$\begin{aligned}a_n &= \frac{n^2 + 5n + 6}{4} \\S_n &= S_n = \sum_{k=1}^n \frac{1}{a_k} = \sum_{k=1}^n \frac{4}{k^2 + 5k + 6} \\&= 4 \sum_{k=1}^n \frac{1}{(k+2)(k+3)} \\&= 4 \sum_{k=1}^n \left(\frac{1}{k+2} - \frac{1}{k+3} \right) \\&= 4 \left(\frac{1}{3} - \frac{1}{4} \right) + 4 \left(\frac{1}{4} - \frac{1}{5} \right) + \dots \\&= 4 \left(\frac{1}{n+2} - \frac{1}{n+3} \right) \\&= 4 \left(\frac{1}{3} - \frac{1}{n+3} \right) \\&= \frac{4n}{3(n+3)} \\S_{2025} &= \frac{(507)(4)(2025)}{3(2028)} \\&= 675\end{aligned}$$

Question 42

Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

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Options:

A.

108

B.

90

C.

122

D.

84

Answer: B

Solution:

$$S_3 = 3a + 3d = 54$$

$$\Rightarrow a + d = 18$$

$$S_{20} = 10(2a + 19d)$$

$$\Rightarrow 10(36 + 17d)$$

$$\Rightarrow 1600 < 10(36 + 17d) < 1800$$

$$\Rightarrow 160 < 36 + 17d < 180$$

$$\Rightarrow 124 < 17d < 144$$

$$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$$

Common difference will be natural number

$$\Rightarrow d = 8 \Rightarrow a = 10$$

$$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$$

Question43

Let $a_1, a_2, a_3, \dots$ be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^n a_k = 0$, then n is :

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Options:

A. 18

B. 17

C. 11

D. 10

Answer: C

Solution:

Given:

$$\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$$

This means:

$$a_1 + a_3 + \cdots + a_{23} = -\frac{72}{5}a_1$$

Express each term in A.P.:

The general term of the A.P. is given by $a_k = a + (k-1)d$. Thus, for the odd indices:

$$a_1 + (a + 2d) + (a + 4d) + \cdots + (a + 22d)$$

Simplify the equation:

$$12a + 2d(1 + 2 + \cdots + 11) = -\frac{72}{5}a$$

Use the formula for sum of an arithmetic series (first 11 terms):

$$1 + 2 + \cdots + 11 = \frac{11 \times 12}{2} = 66$$

So, the equation becomes:

$$12a + 2d(66) = -\frac{72}{5}a$$

$$12a + 132d = -\frac{72}{5}a$$

Solve for the relationship between a and d :

$$132d = -\frac{72}{5}a - 12a$$

$$132d = -\frac{132}{5}a$$

So,

$$a = -5d \quad (\text{i})$$

Second condition:

$$\sum_{k=1}^n a_k = 0 \Rightarrow S_n = 0$$

$$\frac{n}{2}[2a + (n-1)d] = 0$$

$$2a = -(n-1)d \quad (\text{ii})$$

Combine equation (i) and (ii):

Substitute $a = -5d$ into equation (ii):

$$2(-5d) = -(n-1)d$$

$$-10d = -(n-1)d$$

Therefore:

$$n-1 = 10$$

$$n = 11$$

Thus, $n = 11$.

Question 44

The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

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Options:

- A. 6
- B. 4
- C. 8
- D. 10

Answer: B

Solution:

Let the number of terms be $2n$

$$\begin{array}{r} T_1 + T_3 + T_5 \dots T_{2n-1} = 24 \\ T_2 + T_4 + T_6 \dots T_{2n} = 30 \\ \hline T_2 - T_1 + (T_4 - T_3) + \dots (T_{2n} - T_{2n-1}) = 6 \end{array}$$

$$nd = 6$$

$$(a + (2n + 1)d) - a = \frac{21}{2}$$

$$\Rightarrow 2nd - d = \frac{21}{2}$$

$$\Rightarrow 12 - \frac{21}{2} = d$$

$$\Rightarrow d = \frac{3}{2}$$

$$\therefore n = 4$$

$$\therefore \text{Total terms} = 8$$

Question 45

The sum $1 + 3 + 11 + 25 + 45 + 71 + \dots$ upto 20 terms, is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

- A. 7240
- B. 8124
- C. 7130
- D. 6982

Answer: A

Solution:

$T_r = 3r^2 - 7r + 5$ using second order difference

$$\begin{aligned}\sum_{r=1}^{20} (3r^2 - 7r + 5) &= 3\Sigma r^2 - 7\Sigma r + 5\Sigma(1) \\ &= \frac{3(n)(n+1)(2n+1)}{6} - \frac{7n(n+1)}{2} - 5n, n = 20 \\ &= 7240\end{aligned}$$

Question 46

Let $a_1, a_2, a_3, \dots$ be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

- A. 128
- B. 129
- C. 131
- D. 130

Answer: B

Solution:

We start with the given sequence $a_1, a_2, a_3, \dots$ of an increasing geometric progression (G.P.). Two key conditions are provided:

$$a_3 \cdot a_5 = 729$$

$$a_2 + a_4 = \frac{111}{4}$$

Calculating using given conditions:

Using the sequence terms:

$$a_3 = a \cdot r^2, \quad a_5 = a \cdot r^4,$$

Then:

$$a_3 \cdot a_5 = (a \cdot r^2)(a \cdot r^4) = a^2 \cdot r^6 = 729 = 27^2$$

It follows:

$$a \cdot r^3 = 27 \Rightarrow a_4 = a \cdot r^3 = 27 \quad (\text{i})$$

Using the second condition:

$$a_2 + a_4 = \frac{111}{4}$$

Substituting $a_4 = 27$:

$$a_2 = \frac{111}{4} - 27$$

$$a_2 = a \cdot r = \frac{3}{4} \quad (\text{ii})$$

Solving for r and a :

From equation (i) and (ii), we derive r^2 :

$$r^2 = \frac{4 \cdot 27}{3} = 4 \times 9 = 36$$

Since the G.P. is increasing, $r = 6$.

Solve for a :

Substituting $r = 6$ into $a \cdot r = \frac{3}{4}$:

$$a \cdot 6 = \frac{3}{4} \Rightarrow a = \frac{3}{24} = \frac{1}{8}$$

Calculating the requested sum:

The task is to find:

$$24(a_1 + a_2 + a_3) = 24(a + ar + ar^2)$$

Substitute the values:

$$= 24 \times \frac{1}{8}(1 + 6 + 6^2) = 3 \times (1 + 6 + 36) = 3 \times 43 = 129$$

Therefore, the value is 129.

Question 47

The sum $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \dots$ upto ∞ terms, is equal to

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. $3e$

B. $2e$

C. $4e$

D. $6e$

Answer: B

Solution:

The series given is:

$$1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \dots$$

In general, the r -th term of the series, denoted as T_r , takes the form:

$$T_r = \frac{r^2}{r!}$$

To simplify T_r , we write it in terms of factorials:

$$T_r = \frac{r}{(r-1)!} = \frac{(r-1)+1}{(r-1)!} = \frac{1}{(r-2)!} + \frac{1}{(r-1)!}$$

Thus, the series can be expressed as:

$$\sum_{r=1}^{\infty} T_r = \sum_{r=1}^{\infty} \left(\frac{1}{(r-2)!} + \frac{1}{(r-1)!} \right)$$

This breaks into two parts:

$$= \sum_{r=1}^{\infty} \frac{1}{(r-2)!} + \sum_{r=1}^{\infty} \frac{1}{(r-1)!}$$

Each part is a well-known series. Specifically:

$\sum_{r=1}^{\infty} \frac{1}{(r-2)!} = e$, starting from the term where $r - 2 = 0$, which aligns with the expansion of e .

$\sum_{r=1}^{\infty} \frac{1}{(r-1)!} = e$, for the terms starting where $r - 1 = 0$.

Consequently, the sum of the entire series is:

$$e + e = 2e$$

Question 48

$1 + 3 + 5^2 + 7 + 9^2 + \dots$ upto 40 terms is equal to

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. 40870

B. 41880

C. 43890

D. 33980

Answer: B

Solution:

$$\begin{aligned} & 1 + 3 + 5^2 + 7 + 9^2 + \dots \text{ upto 40 terms} \\ & (1^2 + 5^2 + 9^2 + \dots) + (3 + 7 + 11 + \dots) \\ & = \left(\sum_{k=1}^{20} (4k-3)^2 \right) + \frac{20}{2} [6 + (20-1)4] \\ & = 16 \sum_{k=1}^{20} k^2 - 24 \sum_{k=1}^{20} k + 9 \times 20 + 10[82] \\ & = 16 \left(\frac{20 \times 21 \times 41}{6} \right) - 24 \left(\frac{20 \times 21}{2} \right) + 1000 \\ & = 45920 - 5040 + 1000 \\ & = 41880 \end{aligned}$$

Question 49

Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. 3814

B. 4003

C. 4027

D. 3761

Answer: D

Solution:

$$1^{\text{st}} \text{ A.P. : } 1, 6, 11, \dots \Rightarrow T_n = S_n - 4$$

$$2^{\text{nd}} \text{ A.P.: } 9, 16, 23, \dots \Rightarrow T_m = 2 + 7m$$

Let's find when they are equal for the first time:

$$5n - 4 = 2 + 7m$$

$$\Rightarrow 5n - 7m = 6$$

$$\Rightarrow n = 4, m = 2$$

$\Rightarrow$ 16 is the first term, common difference will be

$$\text{LCM}(d_1, d_2) = \text{LCM}(5, 7) = 35$$

$\Rightarrow$ Common terms will be 16, 51, 86, ...

The last term of 1st A.P.

$$= T_{2025} = 5 \times 2025 - 4 = 10121$$

$\Rightarrow$ Common term must be less than that

$$\Rightarrow 35n - 19$$

$$\Rightarrow 35n - 19 \leq 10121 \Rightarrow 35n \leq 10140$$

$$\Rightarrow n \leq 289.7$$

$$\Rightarrow n = 289$$

$$\Rightarrow \text{in } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 2025 + 2025 - 289$$

$$= 3761$$

Question 50

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that $D = d + 3, d \geq 0$. If

$$\frac{p+q}{p-q} = \frac{19}{5}, \text{ then } p - q \text{ is equal to}$$

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 540

B. 450

C. 600

D. 630

Answer: A

Solution:

Let the terms in A be $a_1 - d, a_1, a_1 + d$

and in B be $a_2 - D, a_2, a_2 + D$

$$\text{Now } 3a_1 = 36$$

$$\Rightarrow a_1 = 12$$

$$\text{and } 3a_2 = 36$$

$$\Rightarrow a_2 = 12$$

$$\text{Now } (12 - d)(12)(12 + d) = p$$

$$\text{and } (12 - D)(12)(12 + D) = q$$

$$\text{Also } \frac{p+q}{p-q} = \frac{19}{5}$$

$$\Rightarrow 12q = 7p$$

$$\Rightarrow 12(12 - D)(12)(12 + D) = 7(12 - d)(12)(12 + d)$$

$$\Rightarrow 12(9 - d)(12)(15 - d) = 7(12 - d)(12)(12 + d)$$

$$\Rightarrow 12(135 - d^2 - 6d) = 7(144 - d^2)$$

$$\Rightarrow d = 6, D = 9$$

$$p = 6 \times 12 \times 18 = 1296$$

$$q = 756$$

$$p - q = 540$$

Question 51

If the sum of the first 20 terms of the series

$\frac{4 \cdot 1}{4 + 3 \cdot 1^2 + 1^4} + \frac{4 \cdot 2}{4 + 3 \cdot 2^2 + 2^4} + \frac{4 \cdot 3}{4 + 3 \cdot 3^2 + 3^4} + \frac{4 \cdot 4}{4 + 3 \cdot 4^2 + 4^4} + \dots$ is $\frac{m}{n}$, where m and n are coprime, then $m + n$ is equal to :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 423

B. 421

C. 422

D. 420

Answer: B

Solution:

$$\begin{aligned}
 S_n &= \sum_{r=1}^n \frac{4r}{4 + 3r^2 + r^4} \\
 &= 2 \sum_{r=1}^n \frac{2r}{(r^2 + 2)^2 - r^2} = 2 \sum_{r=1}^n \frac{(r^2 + 2 + r) - (r^2 + 2 - r)}{(r^2 + 2 + r)(r^2 + 2 - r)} \\
 &= 2 \sum_{r=1}^n \left(\frac{1}{r^2 + 2 - r} - \frac{1}{r^2 + 2 + r} \right) \\
 S_{20} &= 2 \left[\left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{8} \right) + \dots \right] \\
 &= 2 \left(\frac{1}{2} - \frac{1}{20^2 + 2 + 20} \right) \\
 &= 2 \left(\frac{1}{2} - \frac{1}{422} \right) \\
 &= 2 \left(\frac{422 - 2}{422 \times 2} \right) = \frac{420}{422} = \frac{210}{211} = \frac{m}{n} \\
 m + n &= 421
 \end{aligned}$$

Question 52

Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4 , then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1 x_2 x_3 x_4)$ is:

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Options:

- A. 18
- B. 216
- C. 36
- D. 72

Answer: B

Solution:

Given the sequence x_1, x_2, x_3, x_4 in geometric progression:

$$x_1 = a$$

$$x_2 = ar$$

$$x_3 = ar^2$$

$$x_4 = ar^3$$

When you subtract 2, 7, 9, and 5 from x_1, x_2, x_3, x_4 respectively, the sequence becomes an arithmetic progression. Thus, the new sequence is:

$$a - 2$$

$$ar - 7$$

$$ar^2 - 9$$

$$ar^3 - 5$$

For these to form an arithmetic progression, the common differences must be equal, so:

$$(ar - 7) - (a - 2) = (ar^2 - 9) - (ar - 7)$$

Simplifying gives:

$$a(r - 1) - 5 = ar(r - 1) - 2$$

$$a(r - 1)(r - 1) = -3 \quad (i)$$

$$(ar - 7) - (a - 2) = (ar^3 - 5) - (ar^2 - 9)$$

Simplifying gives:

$$a(r - 1) - 5 = ar^2(r - 1) + 4$$

$$a(r - 1)(r^2 - 1) = -9 \quad (ii)$$

Using the ratio of equations (ii) and (i):

$$\frac{a(r-1)(r^2-1)}{a(r-1)(r-1)} = \frac{-9}{-3}$$

$$r + 1 = 3 \implies r = 2$$

Plugging back into equation (i):

$$a(1)(1) = -3 \implies a = -3$$

So, the sequence x_1, x_2, x_3, x_4 is:

$$x_1 = -3$$

$$x_2 = -6$$

$$x_3 = -12$$

$$x_4 = -24$$

The expression for $\frac{1}{24}(x_1 \cdot x_2 \cdot x_3 \cdot x_4)$ is:

$$\frac{1}{24}((-3) \cdot (-6) \cdot (-12) \cdot (-24)) = 216$$

Question 53

If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is :

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Options:

A.

757

B.

755

C.

750

D.

760

Answer: A

Solution:

To solve this problem, we have to work with two equations derived from the geometric progression (G.P.):

$$ar + ar^3 + ar^5 = 21$$

$$ar^7 + ar^9 + ar^{11} = 15309$$

From these equations, we can extract the terms:

$$\text{Equation (1): } ar(1 + r^2 + r^4) = 21$$

$$\text{Equation (2): } ar^7(1 + r^2 + r^4) = 15309$$

Now, divide equation (2) by equation (1):

$$\frac{ar^7}{ar} = \frac{15309}{21}$$

From this division, we get:

$$r^6 = 729$$

Which implies:

$$r = 3 \quad (\text{since both terms and ratios are positive})$$

Using this value of r , calculate the sum of the first nine terms of the G.P. using the formula for the sum of a G.P.:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Substitute known values:

$$\Rightarrow \frac{a(3^9 - 1)}{3 - 1} = \frac{a(19683 - 1)}{2}$$

$$\text{Given that } \frac{7}{91} \times (19683 - 1)/2 = \frac{7 \times 19682}{91 \times 2}:$$

$$= \frac{7 \times 19682}{91 \times 2} = \frac{9841}{13} = 757$$

Therefore, the sum of the first nine terms of the G.P. is **757**.

Question 54

Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to :

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Options:

A.

65

B.

56

C.

70

D.

64

Answer: D

Solution:

Sum of n terms of an AP (S_n): $S_n = \frac{n}{2}[2a + (n - 1)d]$ where 'a' is the first term and 'd' is the common difference.

nth term of an AP (a_n): $a_n = a + (n - 1)d$

Given Information:

$S_n = 700$ (The sum of the first n terms is 700)

$a_6 = 7$ (The 6th term is 7)

$S_7 = 7$ (The sum of the first 7 terms is 7)

Steps to Solve:

Use S_7 to find a relationship between 'a' and 'd':

$$S_7 = \frac{7}{2}[2a + (7 - 1)d] = 7$$

$$\frac{7}{2}[2a + 6d] = 7$$

$$2a + 6d = 2$$

$$a + 3d = 1 \text{ (Equation 1)}$$

Use a_6 to find another relationship between 'a' and 'd':

$$a_6 = a + (6 - 1)d = 7$$

$$a + 5d = 7 \text{ (Equation 2)}$$

Solve the system of equations (Equation 1 and Equation 2) to find 'a' and 'd':

Subtract Equation 1 from Equation 2:

$$(a + 5d) - (a + 3d) = 7 - 1$$

$$2d = 6$$

$$d = 3$$

Substitute $d = 3$ into Equation 1:

$$a + 3(3) = 1$$

$$a + 9 = 1$$

$$a = -8$$

Find 'n' using $S_n = 700$:

$$S_n = \frac{n}{2}[2a + (n - 1)d] = 700$$

$$\frac{n}{2}[2(-8) + (n - 1)(3)] = 700$$

$$n[-16 + 3n - 3] = 1400$$

$$n[3n - 19] = 1400$$

$$3n^2 - 19n - 1400 = 0$$

Solve the quadratic equation for 'n':

We can use the quadratic formula: $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$n = \frac{19 \pm \sqrt{(-19)^2 - 4(3)(-1400)}}{2(3)}$$

$$n = \frac{19 \pm \sqrt{361 + 16800}}{6}$$

$$n = \frac{19 \pm \sqrt{17161}}{6}$$

$$n = \frac{19 \pm 131}{6}$$

We have two possible values for n:

$$n = \frac{19 + 131}{6} = \frac{150}{6} = 25$$

$$n = \frac{19 - 131}{6} = \frac{-112}{6} = -\frac{56}{3} \text{ (Since 'n' must be a positive integer, we discard this solution)}$$

So, $n = 25$

Find a_n (which is a_{25}):

$$a_{25} = a + (25 - 1)d$$

$$a_{25} = -8 + (24)(3)$$

$$a_{25} = -8 + 72$$

$$a_{25} = 64$$

Therefore, $a_n = 64$. The correct answer is Option D.

Question 55

$$\text{If } \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90},$$

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \infty = \alpha,$$

$$\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots \infty = \beta,$$

then $\frac{\alpha}{\beta}$ is equal to :

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Options:

A.

23

B.

14

C.

18

D.

15

Answer: D

Solution:

$$\text{If } \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90} \quad \dots\dots (i)$$

$$\beta = \frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots,$$

$$= \frac{1}{16} \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \right],$$

$$= \frac{1}{16} \times \frac{\pi^4}{90} \text{ using (i) } \dots\dots\dots (ii)$$

$$\alpha = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \infty$$

$$\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \dots \right)$$

$$- \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots \right)$$

$$\alpha = \frac{\pi^4}{90} - \frac{1}{16} \times \frac{\pi^4}{90} \quad [\text{using (i) and (ii)}]$$

$$\alpha = \frac{16-1}{16 \times 90} \times \pi^4 = \frac{15}{16 \times 90} \pi^4 = \frac{\pi^4}{96}$$

$$\therefore \frac{\alpha}{\beta} = \frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16 \times 90}} = \frac{16 \times 90}{96} = 15$$

Question 56

The number of common terms in the progressions 4, 9, 14, 19,....., up to 25th term and 3, 6, 9, 12, up to 37th term is :

[27-Jan-2024 Shift 1]

Options:

A.

9

B.

5

C.

7

D.

8

Answer: C

Solution:

4, 9, 14, 19, ... , up to 25th term

$$T_{25} = 4 + (25 - 1)5 = 4 + 120 = 124$$

3, 6, 9, 12, ... , up to 37th term

$$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$$

Common difference of Ist series $d_1 = 5$

Common difference of IInd series $d_2 = 3$

First common term = 9, and

their common difference = $15(\text{LCM}^{\text{2}} \text{ of } d_1 \text{ and } d_2)$ then common terms are

9, 24, 39, 54, 69, 84, 99

Question 57

If

$$8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \dots \infty,$$

then the value of p is _____

[27-Jan-2024 Shift 1]

Answer: 9

Solution:

$$8 = \frac{3}{1 - \frac{1}{4}} + \frac{p \cdot \frac{1}{4}}{\left(1 - \frac{1}{4}\right)^2}$$

$$(\text{sum of infinite terms of A.G.P} = \frac{a}{1-r} + \frac{dr}{(1-r)^2})$$

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

Question 58

The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$ is :-

[27-Jan-2024 Shift 2]

Options:

A.

-118

B.

-110

C.

-115

D.

-100

Answer: C

Solution:

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$-129\frac{1}{4}, \dots, 19\frac{1}{4}, 20$$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$$\begin{aligned} & -129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right) \\ & = -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115 \end{aligned}$$

Question 59

If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

[29-Jan-2024 Shift 1]

Options:

A.

7

B.

4

C.

5

D.

6

Answer: D

Solution:

$$\begin{aligned} & a + ar + ar^2 + ar^3 + \dots + a^{63} \\ &= 7(a + ar^2 + ar^4 + \dots + ar^{62}) \\ &\Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2} \\ &r = 6 \end{aligned}$$

Question60

In an A.P., the sixth terms $a_6 = 2$. If the $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P., is equal to

[29-Jan-2024 Shift 1]

Options:

A.

$\frac{3}{2}$

B.

$\frac{8}{5}$

C.

$\frac{2}{3}$

D.

$\frac{5}{8}$

Answer: B

Solution:

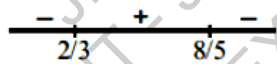
$$a_6 = 2 \Rightarrow a + 5d = 2$$

$$a_1 a_4 a_5 = a(a + 3d)(a + 4d)$$

$$= (2 - 5d)(2 - 2d)(2 - d)$$

$$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$$

$$f'(d) = -2(5d - 8)(3d - 2)$$



$$d = \frac{8}{5}$$

Question61

If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P, then $a:b:c$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$9 : 6 : 4$$

B.

$$16 : 4 : 1$$

C.

$$25 : 10 : 4$$

D.

$$6 : 3 : 2$$

Answer: A

Solution:

$\log_e a, \log_e b, \log_e c$ are in A.P.

$$\therefore b^2 = ac \dots\dots(i)$$

Also

$\log_e\left(\frac{a}{2b}\right), \log_e\left(\frac{2b}{3c}\right), \log_e\left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\frac{b}{c} = \frac{3}{2}$$

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$$\frac{a}{b} = \frac{3}{2}$$

$$a : b : c = 9 : 6 : 4$$

Question62

If each term of a geometric progression $a_1, a_2, a_3, \dots$ with $a_1 = 1/8$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \dots + a_n$, then $S_{20} - S_{18}$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$2^{15}$$

B.

$$-2^{18}$$

C.

$$2^{18}$$

D.

$$-2^{15}$$

Answer: D

Solution:

Let r^{th} term of the GP be ar^{n-1} . Given,

$$2a_r = a_{r+1} + a_{r+2}$$

$$2ar^{n-1} = ar^n + ar^{n+1}$$

$$\frac{2}{r} = 1 + r$$

$$r^2 + r - 2 = 0$$

Hence, we get, $r = -2$ (as $r \neq 1$)

So, $S_{20} - S_{18} = (\text{Sum upto 20 terms}) - (\text{Sum upto 18 terms}) = T_{19} + T_{20}$

$$T_{19} + T_{20} = ar^{18}(1 + r)$$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

Question63

Let S_n denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is :

[30-Jan-2024 Shift 1]

Options:

A.

395

B.

390

C.

405

D.

410

Answer: A

Solution:

$$S_{20} = \frac{20}{2}[2a + 19d] = 790$$

$$2a + 19d = 79 \dots\dots\dots(1)$$

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

$$2a + 9d = 29 \dots\dots\dots(2)$$

From (1) and (2) $a = -8, d = 5$

$$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$$

$$= 405 - 10$$

$$= 395$$

Question 64

Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \dots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to

[30-Jan-2024 Shift 1]

Answer: 353

Solution:

$$\alpha = 1^2 + 4^2 + 8^2 \dots$$

$$t_n = a^2 + bn + c$$

$$1 = a + b + c$$

$$4 = 4a + 2b + c$$

$$8 = 9a + 3b + c$$

On solving we get, $a = \frac{1}{2}, b = \frac{3}{2}, c = -1$

$$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$$

$$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2, \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

Question65

Let a and b be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to pth term of another GP, whose first term is a and fifth term is b. Then p is equal to

[30-Jan-2024 Shift 2]

Options:

A.

20

B.

25

C.

21

D.

24

Answer: C

Solution:

$$1^{\text{st}} \text{ GP} \Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$$

$$t_{11} = ar^{10} = a(r^2)^5 = a \cdot \left(\frac{b}{a}\right)^5$$

$$2^{\text{nd}} \text{ G.P.} \Rightarrow T_1 = a, T_5 = ar^4 = b$$

$$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$$

$$T_p = ar^{p-1} = a \left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$t_{11} = T_p \Rightarrow a \left(\frac{b}{a}\right)^5 = a \left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$$

Question66

Let S_n be the sum to n -terms of an arithmetic progression 3,7,11,.....

If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^n S_k \right) < 42$, then n equals _____

[30-Jan-2024 Shift 2]

Answer: 9

Solution:

$$S_n = 3 + 7 + 11 + \dots \dots n \text{ terms}$$

$$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$$

$$= 2n^2 + n$$

$$\sum_{k=1}^n S_k = 2 \sum_{k=1}^n K^2 + \sum_{k=1}^n K$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n(n+1) \left[\frac{2n+1}{3} + \frac{1}{2} \right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$

$$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^n S_k < 42$$

$$40 < 4n + 5 < 42$$

$$35 < 4n < 37$$

$$n = 9$$

Question67

The sum of the series $\frac{1}{1-3 \cdot 1^2+1^4} + \frac{2}{1-3 \cdot 2^2+2^4} + \frac{3}{1-3 \cdot 3^2+3^4} + \dots$ up to 10 terms is

[31-Jan-2024 Shift 1]

Options:

A.

$$45/109$$

B.

$$- \frac{45}{109}$$

C.

55/109

D.

$$-\frac{55}{109}$$

Answer: D

Solution:

General term of the sequence,

$$T_r = \frac{r}{1 - 3r^2 + r^4}$$

$$T_r = \frac{r}{r^4 - 2r^2 + 1 - r^2}$$

$$T_r = \frac{r}{(r^2 - 1)^2 - r^2}$$

$$T_r = \frac{r}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$T_r = \frac{\frac{1}{2}[(r^2 + r - 1) - (r^2 - r - 1)]}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$= \frac{1}{2} \left[\frac{1}{r^2 - r - 1} - \frac{1}{r^2 + r - 1} \right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2} \left[\frac{1}{-1} - \frac{1}{109} \right] = \frac{-55}{109}$$

Question68

Let 2nd, 8th and 44th, terms of a non-constant A.P. be respectively the 1st, 2nd and 3rd terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to-

[31-Jan-2024 Shift 2]

Options:

A.

980

B.

960

C.

990

D.

970

Answer: D

Solution:

$1 + d, 1 + 7d, 1 + 43d$ are in GP

$$(1 + 7d)^2 = (1 + d)(1 + 43d)$$

$$1 + 49d^2 + 14d = 1 + 44d + 43d^2$$

$$6d^2 - 30d = 0$$

$$d = 5$$

$$S_{20} = \frac{20}{2} [2 \times 1 + (20 - 1) \times 5]$$

$$= 10[2 + 95]$$

$$= 970$$

Question69

Let 3, a, b, c be in A.P. and 3, a-1, b+1, c+9 be in G.P. Then, the arithmetic mean of a, b and c is :

[1-Feb-2024 Shift 1]

Options:

A.

-4

B.

-1

C.

13

D.

11

Answer: D

Solution:

$$3, a, b, c \rightarrow \text{A.P.} \Rightarrow 3, 3+d, 3+2d, 3+3d$$

$$3, a-1, b+1, c+9 \rightarrow \text{G.P.} \Rightarrow 3, 2+d, 4+2d, 12+3d$$

$$a = 3+d \quad (2+d)^2 = 3(4+2d)$$

$$b = 3+2d \quad d = 4, -2$$

$$c = 3+3d$$

$$\text{If } d = 4 \quad \text{G.P.} \Rightarrow 3, 6, 12, 24$$

$$a = 7$$

$$b = 11$$

$$c = 15$$

$$\frac{a+b+c}{3} = 11$$

Question 70

Let 3, 7, 11, 15, ..., 403 and 2, 5, 8, 11, ..., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _____

[1-Feb-2024 Shift 1]

Answer: 6699

Solution:

$$3, 7, 11, 15, \dots, 403$$

$$2, 5, 8, 11, \dots, 404$$

$$\text{LCM}(4, 3) = 12$$

$$11, 23, 35, \dots \text{ let } (403)$$

$$403 = 11 + (n-1) \times 12$$

$$\frac{392}{12} = n-1$$

$$33.66 = n$$

$$n = 33$$

$$\text{Sum } \frac{33}{2}(22 + 32 \times 12)$$

$$= 6699$$

Question71

Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to:

[1-Feb-2024 Shift 2]

Options:

A.

800

B.

890

C.

790

D.

690

Answer: C

Solution:

$$S_{10} = 390$$

$$\frac{10}{2}[2a + (10 - 1)d] = 390$$

$$\Rightarrow 2a + 9d = 78$$

$$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a + 9d}{a + 4d} = \frac{15}{7} \Rightarrow 8a = 3d$$

From (1) & (2) $a = 3$ & $d = 8$

$$\begin{aligned} S_{15} - S_5 &= \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8) \\ &= \frac{15 \times 118 - 5 \times 38}{2} = 790 \end{aligned}$$

Question72

If three successive terms of a G.P. with common ratio $r (r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r , then $3[r] + [-r]$ is equal to :

[1-Feb-2024 Shift 2]

Answer: 1

Solution:

$$a, ar, ar^2 \rightarrow \text{G.P.}$$

Sum of any two sides $>$ third side

$$a + ar > ar^2, a + ar^2 > ar, ar + ar^2 > a$$

$$r^2 - r - 1 < 0$$

$$r \in \left(\frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2} \right) \dots\dots\dots (1)$$

$$r^2 - r + 1 > 0$$

always true

$$r^2 + r - 1 > 0$$

$$r \in \left(-\infty, -\frac{1 - \sqrt{5}}{2} \right) \cup \left(\frac{-1 + \sqrt{5}}{2}, \infty \right)$$

Taking intersection of (1), (2)

$$r \in \left(\frac{-1 + \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2} \right)$$

As $r > 1$

$$r \in \left(1, \frac{1 + \sqrt{5}}{2} \right)$$

$$[r] = 1, [-r] = -2$$

$$3[r] + [-r] = 1$$

Question 73

For three positive integers p, q, r , $x^{pq}y^{q^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to
[24-Jan-2023 Shift 1]

Options:

A. 2

B. 6

C. 12

D. -6

Answer: A

Solution:

$$pq^2 = \log_x \lambda$$

$$qr = \log_y \lambda$$

$$p^2r = \log_z \lambda$$

$$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \dots (1)$$

$$\log_x z = \frac{pq^2}{p^2r} = \frac{q^2}{pr} \dots (2)$$

$$\log_z y = \frac{p^2r}{qr} = \frac{p^2}{q} \dots (3)$$

$$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr} \text{ in A.P.}$$

$$\frac{3r}{pq} - 3 = \frac{1}{2}$$

$$r = \frac{7}{6}pq \dots (4)$$

$$r = pq + 1$$

$$pq = 6 \dots (5)$$

$$r = 7 \dots (6)$$

$$\frac{3p^2}{q} = 4$$

After solving $p = 2$ and $q = 3$

Question 74

The 4th term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is
[24-Jan-2023 Shift 1]

Answer: 12

Solution:

$$T_4 = 500 \text{ where } a = \text{first term,}$$

$$r = \text{common ratio} = \frac{1}{m}, m \in \mathbb{N}$$

$$ar^3 = 500$$

$$\frac{a}{m^3} = 500$$

$$S_n - S_{n-1} = ar^{n-1}$$

$$S_6 > S_5 + 1 \text{ and } S_7 < S_6 + \frac{1}{2}$$

$$S_6 - S_5 > 1 \quad \frac{a}{m^6} < \frac{1}{2}$$

$$ar^5 > 1 \quad m^3 > 10^3$$

$$\frac{500}{m^2} > 1 \quad m > 10$$

$$m^2 < 500 \dots (1)$$

From (1) and (2)
 $m = 11, 12, 13, \dots, 22$

So number of possible values of m is 12

Question 75

If $\frac{1^3 + 2^3 + 3^3 + \dots \text{upto } n \text{ terms}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots \text{upto } n \text{ terms}} = \frac{9}{5}$, then the value of n is
[24-Jan-2023 Shift 2]

Answer: 5

Solution:

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \dots + n^3 &= \left(\frac{n(n+1)}{2} \right)^2 \\ 1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots + n^n + \text{terms} &= \\ \sum_{r=1}^n r(2r+1) &= \sum_{r=1}^n (2r^2 + r) \\ &= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{6} (2(2n+1) + 3) \\ &= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3} \\ &= \frac{n^2(n+1)^2}{4} \\ &\Rightarrow \frac{\frac{5n(n+1)}{2} \times \frac{(4n+5)}{3}}{2} = \frac{9}{5} \\ &\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3} \\ &\Rightarrow 15n(n+1) = 18(4n+5) \\ &\Rightarrow 15n^2 + 15n = 72n + 90 \\ &\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0 \\ &\Rightarrow (n-5)(5n+6) = 0 \\ &\Rightarrow n = \frac{-6}{5} \text{ or } 5 \\ &\Rightarrow n = 5. \end{aligned}$$

Question 76

Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the 7th, 9th, 17th

terms of A_1, A_2, A_3 , respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and

common difference is $\frac{d}{12}$, is equal to _____.

[25-Jan-2023 Shift 1]

Answer: 495

Solution:

$$\begin{vmatrix} A+6d & 7 & 1 \\ 2(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$$
$$\Rightarrow A = -7 \text{ and } d = 6$$
$$\therefore c - a - b = 20$$
$$S_{20} = 495$$

Question 77

For the two positive numbers a , b , if a , b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then, $16a + 12b$ is equal to _____.

[25-Jan-2023 Shift 2]

Answer: 3

Solution:

$$a, b, \frac{1}{18} \rightarrow \text{GP}$$
$$\frac{a}{18} = b^2$$
$$\frac{1}{a}, 10, \frac{1}{b} \rightarrow \text{AP}$$
$$\frac{1}{a} + \frac{1}{b} = 20$$
$$\Rightarrow a + b = 20ab, \text{ from eq. (i) ; we get}$$
$$\Rightarrow 18b^2 + b = 360b^3$$
$$\Rightarrow 360b^2 - 18b - 1 = 0 \quad \{\because b \neq 0\}$$
$$\Rightarrow b = \frac{18 \pm \sqrt{324 + 1440}}{720}$$
$$\Rightarrow b = \frac{18 + \sqrt{1764}}{720} \quad \{\because b > 0\}$$
$$\Rightarrow b = \frac{1}{12}$$
$$\Rightarrow 18 \times \frac{1}{144} = \frac{1}{8}$$
$$\text{Now, } 16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 3$$

Question 78

Let $a_1, a_2, a_3, \dots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1 a_9 + a_2 a_4 a_6 + a_5 + a_7$ is equal to _____.
[29-Jan-2023 Shift 1]

Answer: 60

Solution:

$$\begin{aligned}a_4 \cdot a_6 &= 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3 \\&\&a_5 + a_7 = 24 \Rightarrow a_5 + a_5 r^2 = 24 \Rightarrow (1 + r^2) = 8 \Rightarrow r = \sqrt{7} \\&\Rightarrow a = \frac{3}{49} \\&\Rightarrow a_1 a_9 + a_2 a_4 a_6 + a_5 + a_7 = 9 + 27 + 3 + 21 = 60\end{aligned}$$

Question 79

Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratio r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to ____
[29-Jan-2023 Shift 2]

Answer: 9

Solution:

Given that

$$c_k = a_k + b_k \text{ and } a_1 = b_1 = 4$$

$$\text{also } a_2 = 4r_1, a_3 = 4r_1^2$$

$$b_2 = 4r_2, b_3 = 4r_2^2$$

$$\text{Now } c_2 = a_2 + b_2 = 5 \text{ and } c_3 = a_3 + b_3 = \frac{13}{4}$$

$$\Rightarrow r_1 + r_2 = \frac{5}{4} \text{ and } r_1^2 + r_2^2 = \frac{13}{16}$$

$$\text{Hence } r_1 r_2 = \frac{3}{8} \text{ which gives } r_1 = \frac{1}{2} \text{ \& } r_2 = \frac{3}{4}$$

$$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$$

$$= \frac{4}{1-r_1} + \frac{4}{1-r_2} - \left(\frac{48}{32} + \frac{27}{2} \right)$$

$$= 24 - 15 = 9$$

Question80

Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and

$T = \sum_{n=1}^8 \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 461

Solution:

$$\text{As, } S = \frac{b_1}{2} + \frac{b_2}{2^2} + \dots + \frac{b_9}{2^9} + \frac{b_{10}}{2^{10}}$$

$$\Rightarrow \frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \dots + \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}}$$

subtracting

$$\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \dots + \frac{a_9}{2^{10}} \right) - \frac{b_{10}}{2^{11}}$$

$$\Rightarrow S = b_1 - \frac{b_{10}}{2^{10}} + \left(\frac{a_1}{2} + \frac{a_2}{2^2} + \dots + \frac{a_9}{2^9} \right)$$

$$\Rightarrow \frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \dots + \frac{a_9}{2^{10}} \right)$$

subtracting

$$\Rightarrow \frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}} \right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \dots + \frac{8}{2^9} \right)$$

$$\Rightarrow \frac{S}{2} = \frac{a_1 + b_1}{2} - \frac{(b_{10} + 2a_9)}{2^{11}} + \frac{T}{4}$$

$$\Rightarrow 2S = 2(a_1 + b_1) - \frac{(b_{10} + 2a_9)}{2^9} + T$$

$$\Rightarrow 2^7(2S - T) = 2^8(a_1 + b_1) - \frac{(b_{10} + 2a_9)}{4}$$

$$\text{Also, } b_n - b_{n-1} = a_{n-1}$$

$$\therefore b_{10} - b_1 = a_1 + a_2 + \dots + a_9$$

$$= 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$$

$$\Rightarrow b_{10} = 130 \text{ (As } b_1 = 1)$$

$$\therefore 2^7(2S - T) = 2^8(1 + 1) - (130 + 2 \times 37)$$

$$2^9 - \frac{204}{4} = 461$$

Question81

If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \dots + a_{25}$ is equal to :

[30-Jan-2023 Shift 1]

Options:

A. $\frac{51}{144}$

B. $\frac{49}{138}$

C. $\frac{50}{141}$

D. $\frac{52}{147}$

Answer: C

Solution:

Option (3)

If $a_n = \frac{-2}{4n^2 - 16n + 15}$ then $a_1 + a_2 + \dots + a_{25}$

$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2 - 16n + 15}$$

$$= \sum \frac{-2}{4n^2 - 6n - 10n + 15}$$

$$= \sum \frac{-2}{2n(2n-3) - 5(2n-3)}$$

$$= \sum \frac{-2}{(2n-3)(2n-5)}$$

$$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$$

$$= \frac{1}{47} - \frac{1}{(-3)}$$

$$= \frac{50}{141}$$

Question 82

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n}$. Then $a^2 - b + c$

is equal to _____.

[30-Jan-2023 Shift 1]

Answer: 26

Solution:

$$\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{(n-3)!} + \sum_{n=0}^{\infty} \frac{3}{(n-2)!}$$

$$\begin{aligned}
 & + \sum_{n=0}^{\infty} \frac{1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!} \\
 & = e + 3e + e + \frac{1}{2} \left(e - \frac{1}{e} \right) - \frac{1}{2} \left(e + \frac{1}{e} \right) \\
 & = 5e - \frac{1}{e} \\
 \therefore a^2 - b + c & = 26
 \end{aligned}$$

Question83

Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444 , then abc is equal to
[30-Jan-2023 Shift 2]

Options:

A. 343

B. 216

C. $\frac{343}{8}$

D. $\frac{125}{8}$

Answer: B

Solution:

As a^3, b^3, c^3 be in A.P. $\rightarrow a^3 + c^3 = 2b^3 \dots (1)$

$\log_a b, \log_c a, \log_b c$ are in G.P.

$$\therefore \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} = \left(\frac{\log a}{\log c} \right)^2$$

$$\therefore (\log a)^3 = (\log c)^3 \Rightarrow a = c \dots (2)$$

From (1) and (2)

$$a = b = c$$

$$T_1 = \frac{a+4b+c}{3} = 2a; d = \frac{a-8b+c}{10} = \frac{-6a}{10} = \frac{-3}{5}a$$

$$\therefore S_{20} = \frac{20}{2} \left[4a + 19 \left(-\frac{3}{5}a \right) \right]$$

$$= 10 \left[\frac{20a - 57a}{5} \right]$$

$$= -74a$$

$$\therefore -74a = -444 \Rightarrow a = 6$$

$$\therefore abc = 6^3 = 216$$

Question84

The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then
[30-Jan-2023 Shift 2]

Options:

A. d, e, f are in A.P.

B. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

C. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

D. d, e, f are in G.P.

Answer: C

Solution:

$$ax^2 + 2bx + c = 0$$

$$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 (\because b^2 = ac)$$

$$\Rightarrow (x\sqrt{a} + \sqrt{c})^2 = 0$$

$$x^2 + \frac{\sqrt{c}}{\sqrt{a}} \dots\dots$$

$$\text{Now, } dx^2 + 2ex + f = 0$$

$$\Rightarrow d \left(\frac{c}{a} \right) + 2e \left[-\frac{\sqrt{c}}{\sqrt{a}} \right] + f = 0$$

$$\Rightarrow \frac{dc}{a} + f = 2e \sqrt{\frac{c}{a}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e \sqrt{\frac{1}{ac}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \text{ [as } b = \sqrt{ac} \text{]}$$

$$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

Question 85

The 8th common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \dots$$

$$S_2 = 1 + 6 + 11 + 16 + 21 + \dots$$

is _____.

[30-Jan-2023 Shift 2]

Answer: 151

Solution:

$$T_8 = 11 + (8 - 1) \times 20$$

$$= 11 + 140 = 151$$

Question86

Let $y = f(x)$ represent a parabola with focus $(-\frac{1}{2}, 0)$ and directrix $y = -\frac{1}{2}$.

Then

$$S = \left\{ x \in \mathbb{R} : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2} \right\} :$$

[31-Jan-2023 Shift 1]

Options:

- A. contains exactly two elements
- B. contains exactly one element
- C. is an infinite set
- D. is an empty set

Answer: A

Solution:

$$\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$$

$$y = (x^2 + x)$$

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$$

$$0 \leq x^2 + x + 1 \leq 1$$

$$x^2 + x \leq 0 \dots (1)$$

$$\text{Also } x^2 + x \geq 0 \dots (2)$$

$$\therefore x^2 + x = 0 \Rightarrow x = 0, -1$$

S contains 2 element.

Question87

Let $a_1, a_2, \dots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12 \left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right)$$

is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 8

Solution:

$$2a_7 = a_5 \text{ (given)}$$

$$2(a_1 + 6d) = a_1 + 4d$$

$$a_1 + 8d = 0 \dots (1)$$

$$a_1 + 10d = 18 \dots (2)$$

By (1) and (2) we get $a_1 = -72, d = 9$

$$a_{18} = a_1 + 17d = -72 + 153 = 81$$

$$a_{10} = a_1 + 9d = 9$$

$$12 \left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \dots + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d} \right)$$

$$12 \left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d} \right) = \frac{12(9 - 3)}{9} = \frac{12 \times 6}{6} = 8$$

Question 88

Let $a_1, a_2, a_3, \dots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :
[31-Jan-2023 Shift 2]

Options:

A. 24

B. $\frac{33}{4}$

C. $\frac{381}{4}$

D. 9

Answer: A

Solution:

$$a + 6d = 3 \dots (1)$$

$$Z = a(a + 3d)$$

$$= (3 - 6d)(3 - 3d)$$

$$= 18d^2 - 27d + 9$$

Differentiating with respect to d

$$\Rightarrow 36d - 27 = 0$$

$$\Rightarrow d = \frac{3}{4}, \text{ from (1) } a = \frac{-3}{2}, (Z = \text{minimum})$$

$$\text{Now, } S_n = \frac{n}{2} \left(-3 + (n-1) \frac{3}{4} \right) = 0$$

$$\Rightarrow n = 5$$

Now,

$$n! - 4a_{n(n+2)} = 120 - 4(a_{35})$$

$$= 120 - 4(a + (35-1)d)$$

$$= 120 - 4 \left(\frac{-3}{2} + 34 \cdot \left(\frac{3}{4} \right) \right)$$

$$= 120 - 4 \left(\frac{-6 + 102}{4} \right)$$

$$= 120 - 96 = 24$$

Question 89

The sum

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \dots + 15.29^2$ is _____.

[31-Jan-2023 Shift 2]

Answer: 6952

Solution:

Separating odd placed and even placed terms we get

$$S = (1.1^2 + 3.5^2 + \dots + 15.29^2) - (2.3^2 + 4.7^2 + \dots + 14.27^2)$$

$$S = \sum_{n=1}^8 (2n-1)(4n-3)^2 - \sum_{n=1}^7 (2n)(4n-1)^2$$

Applying summation formula we get
= 29856 - 22904 = 6952

Question 90

The sum to 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ is:-

[1-Feb-2023 Shift 1]

Options:

A. $\frac{59}{111}$

B. $\frac{55}{111}$

C. $\frac{56}{111}$

D. $\frac{58}{111}$

Answer: B

Solution:

$$T_r = \frac{(r^2+r+1) - (r^2-r+1)}{2(r^4+r^2+1)}$$

$$\Rightarrow T_r = \frac{1}{2} \left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1} \right]$$

$$T_1 = \frac{1}{2} \left[\frac{1}{1} - \frac{1}{3} \right]$$

$$T_2 = \frac{1}{2} \left[\frac{1}{3} - \frac{1}{7} \right]$$

$$T_3 = \frac{1}{2} \left[\frac{1}{7} - \frac{1}{13} \right]$$

$$T_{10} = \frac{1}{2} \left[\frac{1}{91} - \frac{1}{111} \right]$$

$$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2} \left[1 - \frac{1}{111} \right] = \frac{55}{111}$$

Question91

Let $a_1 = 8, a_2, a_3, \dots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is _____.

[1-Feb-2023 Shift 1]

Answer: 754

Solution:

$$a_1 + a_2 + a_3 + a_4 = 50$$

$$\Rightarrow 32 + 6d = 50$$

$$\Rightarrow d = 3$$

$$\text{and, } a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$$

$$\Rightarrow 32 + (4n - 10) \cdot 3 = 170$$

$$\Rightarrow n = 14$$

$$a_7 = 26, a_8 = 29$$

$$\Rightarrow a_7 \cdot a_8 = 754$$

Question92

The number of 3 -digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is _____.

[1-Feb-2023 Shift 1]

Answer: 514

Solution:

$$\begin{aligned} \text{Divisible by 2} &\rightarrow 450 \\ \text{Divisible by 3} &\rightarrow 300 \\ \text{Divisible by 7} &\rightarrow 128 \\ \text{Divisible by 2\&7} &\rightarrow 64 \\ \text{Divisible by 3\&7} &\rightarrow 43 \\ \text{Divisible by 2\&3} &\rightarrow 150 \\ \text{Divisible by 2, 3\&7} &\rightarrow 21 \\ \therefore \text{Total numbers} &= 450 + 300 - 150 - 64 - 43 + 21 = 514 \end{aligned}$$

Question93

Which of the following statements is a tautology ?
[1-Feb-2023 Shift 2]

Options:

A. $p \rightarrow (p \wedge (p \rightarrow q))$

B. $(p \wedge q) \rightarrow (\sim(p) \rightarrow q)$

C. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

D. $p \vee (p \wedge q)$

Answer: B

Solution:

(i) $p \rightarrow (p \wedge (p \rightarrow q))$

$(\sim p) \vee (p \wedge (\sim p \vee q))$

$(\sim p) \vee (p \wedge q)$

$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$

$= \sim p \vee q$

(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$

$\sim(p \wedge q) \vee (\sim p \vee q) = t$

$\{a, b, d\} \vee \{a, b, c\} = V$

Tautology

(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

$\sim(p \wedge (\sim p \vee q)) \vee \sim q = \sim(p \wedge q) \vee \sim q = \sim p \vee \sim q$

Not tautology

(iv) $p \vee (p \wedge q) = p$

Not tautology.

Question94

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ..., 399

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ..., 197, is equal to _____.

[1-Feb-2023 Shift 2]

Answer: 321

Solution:

3, 7, 11, 15, ..., 399 $d_1 = 4$

2, 5, 8, 11, ..., 359 $d_2 = 3$

2, 7, 12, 17, ..., 197 $d_3 = 5$

$\text{LCM}(d_1, d_2, d_3) = 60$

Common terms are 47, 107, 167
Sum = 321

Question95

The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \dots$ is :
[6-Apr-2023 shift 1]

Options:

- A. 3450
- B. 3420
- C. 3520
- D. 3250

Answer: C

Solution:

$$\begin{aligned} s_n &= 5 + 11 + 19 + 29 + 41 + \dots + T_n \\ S_n &= 5 + 11 + 19 + 29 + \dots + T_{n-1} + T_n \\ \hline 0 &= 5 + \underbrace{\{6 + 8 + 10 + 12 + \dots\}}_{(n-1) \text{ terms}} - T_n \end{aligned}$$

$$T_n = 5 + \frac{(n-1)}{2} [2 \cdot 6 + (n-2) \cdot 2]$$

$$T_n = 5 + (n-1)(n+4) = 5 + n^2 + 3n - 4 = n^2 + 3n + 1$$

$$\text{Now } S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20} n^2 + 3n + 1$$

$$S_{20} = \frac{20 \cdot 21 \cdot 41}{6} + \frac{3 \cdot 20 \cdot 21}{2} + 20$$

$$S_{20} = 2870 + 630 + 20$$

$$S_{20} = 3520$$

Question96

Let $a_1, a_2, a_3, \dots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then :

$$\lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right) \text{ is}$$

[6-Apr-2023 shift 1]

Options:

A. $\frac{1}{\sqrt{d}}$

B. 1

C. $\sqrt{d}$

D. 0

Answer: B

Solution:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + \dots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{a_{n-1} - a_n} \right) \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_1} - \sqrt{a_2} + \sqrt{a_2} - \sqrt{a_3} + \dots + \sqrt{a_{n-1}} - \sqrt{a_n}}{-d} \right) \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left(\frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{d}} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{d}} \left(\sqrt{\frac{a_1}{n} + d} - \frac{d}{n} - \frac{\sqrt{a_1}}{n} \right) \\ &= 1 \end{aligned}$$

Question 97

If $\gcd(m, n) = 1$ and

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$$

then $m^2 - n^2$ is equal to :

[6-Apr-2023 shift 2]

Options:

A. 180

B. 220

C. 200

D. 240

Answer: D

Solution:

$$\begin{aligned} & (1-2)(1+2) + (3-4)(3+4) + \dots + (2021-2022)(2021+2022) + (2023)^2 = (1012)m^2n \\ & \Rightarrow (-1)[1+2+3+4+\dots+2022] + (2023)^2 = (1012)m^2n \\ & \Rightarrow (-1) \frac{(2022)(2023)}{2} + (2023)^2 = (1012)m^2n \\ & \Rightarrow (2023)[2023 - 1011] = (1012)m^2n \\ & \Rightarrow (2023)(1012) = (1012)m^2n \end{aligned}$$

$$\begin{aligned}\Rightarrow m^2 n &= 2023 \\ \Rightarrow m^2 n &= (17)^2 \times 7 \\ m &= 17, n = 7 \\ m^2 - n^2 &= (17)^2 - 7^2 = 289 - 49 = 240 \\ \text{Ans. Option 4}\end{aligned}$$

Question98

If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \dots + 20(21)^{19} = k(20)^{19}$, then k is equal to

[6-Apr-2023 shift 2]

Answer: 400

Solution:

$$\begin{aligned}S &= (20)^{19} + 2(21)(20)^{18} + \dots + 20(21)^{19} \\ \frac{21}{20}S &= 21(20)^{18} + 2(21)^2(20)^{17} + \dots + (21)^{20} \\ \text{Subtract} \\ \left(1 - \frac{21}{20}\right)S &= (20)^{19} + (21)(20)^{18} + (21)^2(20)^{17} + \dots + (21)^{19} - (21)^{20} \\ \left(-\frac{1}{20}\right)S &= (20)^{19} \left[\frac{1 - \left(\frac{21}{20}\right)^{20}}{1 - \frac{21}{20}} \right] - (21)^{20} \\ \left(-\frac{1}{20}\right)S &= (21)^{20} - (20)^{20} - (21)^{20} \\ S &= (20)^{21} = K(20)^{19} \quad (\text{given}) \\ K &= (20)^2 = 400\end{aligned}$$

Question99

Let $S_K = \frac{1+2+\dots+K}{K}$ and $\sum_{j=1}^n S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D $\in \mathbb{N}$ and A has least value. Then
[8-Apr-2023 shift 1]

Options:

- A. A + B is divisible by D
- B. A + B = 5(D - C)
- C. A + C + D is not divisible by B
- D. A + B + D is divisible by 5

Answer: A

Solution:

$$S_k = \frac{k+1}{2}$$

$$S_k^2 = \frac{k^2 + 1 + 2k}{4}$$

$$\therefore \sum_{j=1}^n S_j^2 = \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + n + n(n+1) \right]$$

$$= \frac{n}{4} \left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1 \right]$$

$$= \frac{n}{4} \left[\frac{2n^2 + 3n + 1}{6} + n + 2 \right]$$

$$= \frac{n}{4} \left[\frac{2n^2 + 9n + 13}{6} \right] = \frac{n}{24} [2n^2 + 9n + 13]$$

$$A = 24, B = 2, C = 9, D = 13$$

Question 100

Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \dots$ and $S_n = \sum_{k=1}^n a_k$.

Then $S_{30} - a_{40}$ is equal to

[8-Apr-2023 shift 2]

Options:

A. 11260

B. 11280

C. 11290

D. 11310

Answer: C

Solution:

$$S_n = 5 + 8 + 14 + 23 + 35 + 50 + \dots + a_n$$

$$S_n = 5 + 8 + 14 + 23 + 35 + \dots + a_n$$

$$O = 5 + 3 + 6 + 9 + 12 + 15 + \dots + a_n$$

$$a_n = 5 + (3 + 6 + 9 + \dots (n-1) \text{ terms})$$

$$a_n = \frac{3n^2 - 3n + 10}{2}$$

$$a_{40} = \frac{3(40)^2 - 3(40) + 10}{2} = 2345$$

$$S_{30} = \frac{3 \sum_{n=1}^{30} n^2 - 3 \sum_{n=1}^{30} n + 10 \sum_{n=1}^{30} 1}{2}$$

$$= \frac{3 \times 30 \times 31 \times 61}{6} - \frac{3 \times 30 \times 31}{2} + 10 \times 30$$

$$S_{30} = 13635$$

$$S_{30} - a_{40} = 13635 - 2345$$

$$= 11290 \text{ (Option (C))}$$

Question101

Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _____.

[8-Apr-2023 shift 2]

Answer: 150

Solution:

$$\begin{aligned}\frac{2}{y} &= \frac{1}{x} + \frac{1}{z} \\ 2y^2 &= xz \\ \frac{2}{y} &= \frac{x+z}{xz} = \frac{x+z}{2y^2} \\ x+z &= 4y \\ xy + yz + zx &= \frac{3}{\sqrt{2}}xyz \\ y(x+z) + zx &= \frac{3}{\sqrt{2}}xz \cdot y \\ 4y^2 + 2y^2 &= \frac{3}{\sqrt{2}}y \cdot 2y^2 \\ 6y^2 &= 3\sqrt{2}y^3 \\ y &= \sqrt{2} \\ x + y + z &= 5y = 5\sqrt{2} \\ 3(x + y + z)^2 &= 3 \times 50 = 150\end{aligned}$$

Question102

Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to :

[10-Apr-2023 shift 1]

Options:

- A. 210
- B. 220
- C. 231
- D. 241

Answer: C

Solution:

Let a, ar, ar^2 be three terms of GP

$$\text{Given : } a^2 + (ar)^2 + (ar^2)^2 = 33033$$

$$a^2(1 + r^2 + r^4) = 11^2 \cdot 3 \cdot 7 \cdot 13$$

$$\Rightarrow a = 11 \text{ and } 1 + r^2 + r^4 = 3 \cdot 7 \cdot 13$$

$$\Rightarrow r^2(1 + r^2) = 273 - 1$$

$$\Rightarrow r^2(r^2 + 1) = 272 = 16 \times 17$$

$$\Rightarrow r^2 = 16$$

$$\therefore r = 4 \text{ [}\because r > 0\text{]}$$

$$\text{Sum of three terms} = a + ar + ar^2 = a(1 + r + r^2)$$

$$= 11(1 + 4 + 16)$$

$$= 11 \times 21 = 231$$

Question 103

If $f(x) = \frac{(\tan 1^\circ)x + \log_e(123)}{x \log_e(1234) - (\tan 1^\circ)}$, $x > 0$, then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is:

[10-Apr-2023 shift 1]

Options:

A. 2

B. 4

C. 8

D. 0

Answer: B

Solution:

$$f(x) = \frac{(\tan 1^\circ)x + \log 123}{x \log 1234 - \tan 1^\circ}$$

$$\text{Let } A = \tan 1^\circ, B = \log 123, C = \log 1234$$

$$f(x) = \frac{Ax + B}{xC - A}$$

$$f(f(x)) = \frac{A\left(\frac{Ax+B}{xC-A}\right) + B}{C\left(\frac{Ax+B}{xC-A}\right) - A}$$

$$= \frac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$$

$$= \frac{x(A^2 + BC)}{(A^2 + BC)} = x$$

$$f(f(x)) = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$$

$$AM \geq GM$$

$$x + \frac{4}{x} \geq 4$$

Question104

The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to _____.
[10-Apr-2023 shift 1]

Answer: 9525

Solution:

A.P: 3, 8, 13, ..., 373

$$T_n = a + (n-1)d$$

$$373 = 3 + (n-1)5$$

$$\Rightarrow n = \frac{370}{5}$$

$$\Rightarrow n = 75$$

$$\text{Now Sum} = \frac{n}{2}[a + 1]$$

$$= \frac{75}{2}[3 + 373] = 14100$$

Now numbers divisible by 3 are,

$$3, 18, 33, \dots, 363$$

$$363 = 3 + (k-1)5$$

$$\Rightarrow k-1 = \frac{360}{5} = 72 \Rightarrow k = 73$$

$$\text{Now, sum} = \frac{73}{2}(3 + 363) = 4575$$

$$\therefore \text{req. sum} = 14100 - 4575 = 9525$$

Question105

If $S_n = 4 + 11 + 21 + 34 + 50 + \dots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 220

B. 227

C. 226

D. 223

Answer: D

Solution:

$$S_n = 4 + 11 + 21 + 34 + 50 + \dots + n \text{ terms}$$

Difference are in A.P.

$$\text{Let } T_n = an^2 + bn + c$$

$$T_1 = a + b + c = 4$$

$$T_2 = 4a + 2b + c = 11$$

$$T_3 = 9a + 3b + c = 21$$

By solving these 3 equations

$$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$$

$$\text{So } T_n = \frac{3}{2}n^2 + \frac{5}{2}n$$

$$S_n = \sum T_n$$

$$= \frac{3}{2} \sum n^2 + \frac{5}{2} \sum n$$

$$= \frac{3}{2} \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \frac{(n)(n+1)}{2}$$

$$= \frac{n(n+1)}{4} [2n+1+5]$$

$$S_n = \frac{n(n+1)}{4} (2n+6) = \frac{n(n+1)(n+3)}{2}$$

$$\frac{1}{60} \left(\frac{29 \times 30 \times 32}{2} - \frac{9 \times 10 \times 12}{2} \right) = 223$$

Question106

Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _____.

[10-Apr-2023 shift 2]

Answer: 16

Solution:

$$\left(\frac{a-2d}{4}, \frac{(a-d)}{2}, a, 2(a+d), 4(a+2d) \right)$$

$$a = 2$$

$$\left(\frac{1}{4} + \frac{1}{2} + 1 + 6 \right) \times 2 + (-1 + 2 + 8)d = \frac{49}{2}$$

$$2 \left(\frac{3}{4} + 7 \right) + 9d = \frac{49}{2}$$

$$9d = \frac{49}{2} - \frac{62}{4} = \frac{98-62}{4} = 9$$

$$d = 1$$

$$\Rightarrow a_4 = 4(a+2d)$$

$$= 16$$

Question107

Let $x_1, x_2, \dots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i)$, $1 \leq i \leq 100$, then the mean of $y_1, y_2, \dots, y_{100}$ is :

[11-Apr-2023 shift 1]

Options:

A. 10051.50

B. 10100

C. 10101.50

D. 10049.50

Answer: D

Solution:

$$\begin{aligned}\text{Mean} &= 200 \\ \frac{100}{2}(2 \times 2 + 99d) &= 200 \\ \Rightarrow \frac{100}{2} &= 200 \\ \Rightarrow 4 + 99d &= 400 \\ \Rightarrow d &= 4 \\ y_i &= i(x_i - 1) \\ &= i(2 + (i-1)4 - i) = 3i^2 - 2i \\ \text{Mean} &= \frac{\sum y_i}{100} \\ &= \frac{1}{100} \sum_{i=1}^{100} 3i^2 - 2i \\ &= \frac{1}{100} \left\{ \frac{3 \times 100 \times 101 \times 201}{6} - \frac{2 \times 100 \times 101}{2} \right\} \\ &= 101 \left\{ \frac{201}{2} - 1 \right\} = 101 \times 99.5 \\ &= 10049.50\end{aligned}$$

Question 108

Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to _____.

[11-Apr-2023 shift 1]

Answer: 2175

Solution:

$$\begin{aligned}S &= 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{1}{5^{108}} \\ \frac{S}{5} &= \frac{109}{5} + \frac{108}{5^2} + \dots + \frac{2}{5^{108}} + \frac{1}{5^{109}} \\ \frac{4S}{5} &= 109 - \frac{1}{5} - \frac{1}{5^2} + \dots - \frac{1}{5^{108}} - \frac{1}{5^{109}}\end{aligned}$$

$$= 109 - \left(\frac{1}{5} \frac{\left(1 - \frac{1}{5^{109}}\right)}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4} \left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$S = \frac{5}{4} \left(109 - \frac{1}{4} + \frac{1}{4 \cdot 5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

Question 109

Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750β , then the value of β is
[11-Apr-2023 shift 2]

Options:

A. 55

B. 108

C. 90

D. 110

Answer: C

Solution:

Given $a + b + c + d = 11$

$(a, b, c, d > 0)$

$(a^5 b^3 c^2 d) \text{ max.} = ?$

Let assume Numbers -

$\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}, \frac{d}{1}$

We know A.M. $\geq$ G.M.

$$\frac{\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

$$\frac{11}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

$$a^5 \cdot b^3 \cdot c^2 \cdot d \leq 5^5 \cdot 3^3 \cdot 2^2$$

$$\max(a^5 b^3 c^2 d) = 5^5 \cdot 3^3 \cdot 2^2 = 337500$$

$$= 90 \times 3750 = \beta \times 3750$$

$$\beta = 90$$

Option (C) 90 correct

Question 110

For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots$ is 10, then the value of k is _____
 [11-Apr-2023 shift 2]

Answer: 2

Solution:

$$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \text{ upto } \infty$$

$$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \text{ upto } \infty$$

$$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \dots \text{ upto } \infty$$

$$S = 9 \left(1 - \frac{1}{k} \right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \dots \text{ upto } \infty$$

$$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \dots \text{ upto } \infty$$

$$\left(1 - \frac{1}{k} \right) S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \dots \text{ upto } \infty$$

$$9 \left(1 - \frac{1}{k} \right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k} \right)}$$

$$9(k-1)^3 = 4k(k-1) + 1$$

$$k = 2$$

Question 111

Let $\langle a_n \rangle$ be a sequence such that $a_1 + a_2 + \dots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$. If

$28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \dots p_m$, where $p_1, p_2, \dots, p_m$ are the first m prime numbers, then m

is equal to

[12-Apr-2023 shift 1]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: C

Solution:

$$\begin{aligned}
 a_n &= S_n - S_{n-1} = \frac{n^2 + 3n}{(n+1)(1+2)} - \frac{(n-1)(n+2)}{n(n+1)} \\
 \Rightarrow a_n &= \frac{4}{n(n+1)(1+2)} \\
 \Rightarrow 28 \sum_{k=1}^{10} \frac{1}{a_k} &= 28 \sum_{k=1}^{10} \frac{k(k+1)(k+2)}{4} \\
 &= \frac{7}{4} \sum_{k=1}^{10} (k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2)) \\
 &= \frac{7}{4} \cdot 10 \cdot 11 \cdot 12 \cdot 13 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13
 \end{aligned}$$

So $m = 6$

Question112

Let $s_1, s_2, s_3, \dots, s_{10}$ respectively be the sum to 12 terms of 10 A.P. s_m whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to
[13-Apr-2023 shift 1]

Options:

- A. 7260
- B. 7380
- C. 7220
- D. 7360

Answer: A

Solution:

$$\begin{aligned}
 S_k &= 6(2k + (11)(2k - 1)) \\
 S_k &= 6(2k + 22k - 11) \\
 S_k &= 144k - 66 \\
 \sum_{k=1}^{10} S_k &= 144 \sum_{k=1}^{10} k - 66 \times 10 \\
 &= 144 \times \frac{10 \times 11}{2} - 660 \\
 &= 7920 - 660 \\
 &= 7260
 \end{aligned}$$

Question113

The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - \dots$ is equal to

_____.
[13-Apr-2023 shift 1]

Answer: 1310

Solution:

$$\begin{aligned}& (2^2 - 3^2 + 4^2 - 5^2 + 20 \text{ terms}) + (2^2 + 4^2 + \dots + 10 \text{ terms}) \\&= (2 + 3 + 4 + 5 + \dots + 11) + 4[1 + 2^2 + \dots + 10^2] \\&= \left[\frac{21 \times 22}{2} - 1 \right] + 4 \times \frac{10 \times 11 \times 21}{6} \\&= 1 - 231 + 14 \times 11 \times 10 \\&= 1540 + 1 - 231 \\&= 1310\end{aligned}$$

Question 114

Let $a_1, a_2, a_3, \dots$ be a G. P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then

$6(a_2 + a_4)(a_4 + a_6)$ is equal to

[13-Apr-2023 shift 2]

Options:

- A. 2
- B. 3
- C. $3\sqrt{3}$
- D. $2\sqrt{2}$

Answer: B

Solution:

$$\begin{aligned}a_3 \cdot a_5 &= \frac{1}{9} \\ \Rightarrow ar^2 \cdot ar^4 &= \frac{1}{9} \\ \Rightarrow (ar^3)^2 &= \frac{1}{9} \\ \Rightarrow ar^3 &= \frac{1}{3} \dots (i) \\ a_6 + a_8 &= 2 \\ \Rightarrow ar^5 + ar^7 &= 2 \\ \Rightarrow ar^3(r^2 + r^4) &= 2 \\ \Rightarrow \frac{1}{3}r^2(1 + r^2) &= 2 \\ \Rightarrow r^2(1 + r^2) &= 2 \times 3 \\ \Rightarrow r^2 &= 2 \Rightarrow r = \sqrt{2} \\ a &= \frac{1}{3} \times \frac{1}{r^3} \\ &= \frac{1}{3} \times \frac{1}{2\sqrt{2}} = \frac{1}{6\sqrt{2}} \\ 6(a_2 + a_4)(a_4 + a_6) \\ \Rightarrow 6(ar + ar^3)(ar^3 + ar^5)\end{aligned}$$

$$\Rightarrow 6 \left(\frac{ar^3}{r^2} + \frac{1}{3} \right) \left(\frac{1}{3} + \frac{1}{3}r^2 \right) = 3$$

Question 115

Let $[a]$ denote the greatest integer $\leq a$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}]$ is equal to _____.
[13-Apr-2023 shift 2]

Answer: 825

Solution:

$$\begin{aligned} S &= [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}] \\ [\sqrt{1}] &\rightarrow [\sqrt{3}] = 1 \times 3 \\ [\sqrt{4}] &\rightarrow [\sqrt{8}] = 2 \times 5 \\ [\sqrt{9}] &\rightarrow [\sqrt{15}] = 3 \times 7 \\ &\vdots \\ [\sqrt{100}] &\rightarrow [\sqrt{120}] = 10 \times 21 \\ S &= 1 \times 3 + 2 \times 5 + 3 \times 7 + \dots + 10 \times 21 \\ &= \sum_{r=1}^{10} r(2r+1) \\ &= 2 \sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r \\ &= \frac{2 \times 10 \times 11 \times 21}{6} + \frac{10 \times 11}{2} \\ &= 770 + 55 \\ &= 825 \end{aligned}$$

Question 116

Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in \mathbb{R}$. If $2f'(2) - f'(2) = 119(2)^n + 1$ then n is equal to _____
[13-Apr-2023 shift 2]

Answer: 10

Solution:

$$\begin{aligned} f(x) &= \sum_{k=1}^{10} kx^k \\ \Rightarrow f(x) &= x + 2x^2 + 3x^3 + \dots + 9x^9 + 10x^{10} \quad \text{(i)} \\ xf(x) &= x^2 + 2x^3 + \dots + 9x^{10} + 10x^{11} \dots \quad \text{(ii)} \\ \text{"(i) - (ii)" } & \end{aligned}$$

$$f(x)(1-x) = x + x^2 + x^3 + \dots + x^{10} - 10x^{11}$$

$$f(x)(1-x) = \frac{x(1-x^{10})}{1-x} - 10x^{11}$$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(2) = 2 + g(2)^{11}$$

$$(1-x)^2 f(x) = x(1-x^{10}) - 10x^{11}(1-x)$$

diff. w.r.t. x

$$(1-x)^2 f'(2) + f(2)2(1-x)(-1)$$

$$= x(-10x^9) + (1-x^{10}) - 10x^{11}(-1) - (1-x)(110)x^{10}$$

put $x = 2$

$$f'(2) + f(2)(2) = -10(2)^{10} + 1 - 2^{10} + 10(2)^{11} - 110(2)^{10} + 110(2)^{11}$$

$$= (-121)2^{10} + (120)2^{11} + 1$$

$$= 2^{10}(240 - 121) + 1$$

$$= 119(2)^{10} + 1$$

$$n = 10$$

Question 117

Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to [15-Apr-2023 shift 1]

Options:

A. $2(A_1 + A_2)G_1 G_3$

B. $(A_1 + A_2)^2 G_1 G_3$

C. $2(A_1 + A_2)G_1^2 G_3^2$

D. $(A_1 + A_2)G_1^2 G_3^2$

Answer: B

Solution:

a, A_1, A_2, b are in A.P.

$$d = \frac{b-a}{3}; A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$A_2 = \frac{a+2b}{3}$$

$$A_1 + A_2 = a + b$$

a, G_1, G_2, G_3, b are in G.P.

$$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$$G_1 = (a^3 b)^{\frac{1}{4}}$$

$$G_2 = (a^2 b^2)^{\frac{1}{4}}$$

$$G_3 = (ab^3)^{\frac{1}{4}}$$

$$\begin{aligned}
 G_1^4 + 2^4 + G_3^4 + G_1^2 G_3^2 &= \\
 a^3 b + a^2 b^2 + ab^3 + (a^3 b) \frac{1}{2} \cdot (ab^3) \frac{1}{2} &= \\
 = a^3 b + a^2 b^2 + ab^3 + a^2 \cdot b^2 &= \\
 = ab(a^2 + 2ab + b^2) &= \\
 = ab(a+b)^2 &= \\
 = G_1 \cdot G_3 \cdot (A_1 + A_2)^2 &=
 \end{aligned}$$

Question118

If the sum of the series $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{2^2 \cdot 3} + \frac{1}{2^2 \cdot 3^2} - \frac{1}{2 \cdot 3^2} + \frac{1}{3^4}\right) + \dots$ is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to _____
 [15-Apr-2023 shift 1]

Answer: 7

Solution:

$$\begin{aligned}
 P\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) + \dots &= P\left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{3^4}\right) + \dots \\
 \frac{5P}{6} &= \frac{\frac{1}{4}}{1 - \frac{1}{2}} - \frac{\frac{1}{9}}{1 + \frac{1}{3}} \\
 \frac{5P}{6} &= \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \\
 \therefore P &= \frac{1}{2} = \frac{\alpha}{\beta} \quad \therefore \alpha = 1, \beta = 2 \\
 \alpha + 3\beta &= 7
 \end{aligned}$$

Question119

If $\{a_i\}_{i=1}^n$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^n a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :
 [24-Jun-2022-Shift-1]

Options:

- A. 48
- B. 96
- C. 92
- D. 104

Answer: B

Solution:

$$\sum_{i=1}^n a_i = 192$$

$$\Rightarrow a_1 + a_2 + a_3 + \dots + a_n = 192$$

$$\Rightarrow \frac{n}{2}[a_1 + a_n] = 192$$

$$\Rightarrow a_1 + a_n = \frac{384}{n} \dots \dots (1)$$

$$\text{Now, } \sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$$

$$\Rightarrow a_2 + a_4 + a_6 + \dots + a_n = 120$$

Here total $\frac{n}{2}$ terms present.

$$\therefore \frac{\frac{n}{2}}{2}[a_2 + a_n] = 120$$

$$\Rightarrow \frac{n}{4}[a_1 + 1 + a_n] = 120$$

$$\Rightarrow a_1 + a_n + 1 = \frac{480}{n} \dots \dots (2)$$

Subtracting (1) from (2), we get

$$1 = \frac{480}{n} - \frac{384}{n}$$

$$\Rightarrow 1 = \frac{96}{n}$$

$$\Rightarrow n = 96$$

Question 120

If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is :
[25-Jun-2022-Shift-1]

Options:

A. 1

B. 2

C. 3

D. 5

Answer: D

Solution:

$$\begin{aligned} \frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots + \frac{1}{2^{10} \cdot 3} &= \frac{K}{2^{10} \cdot 3^{10}} \\ \Rightarrow \frac{1}{2 \cdot 3^{10}} \left[\frac{\left(\frac{3}{2}\right)^{10} - 1}{\frac{3}{2} - 1} \right] &= \frac{K}{2^{10} \cdot 3^{10}} \\ = \frac{3^{10} - 2^{10}}{2^{10} \cdot 3^{10}} &= \frac{K}{2^{10} \cdot 3^{10}} \Rightarrow K = 3^{10} - 2^{10} \end{aligned}$$

$$\begin{aligned} \text{Now } K &= (1+2)^{10} - 2^{10} \\ &= {}^{10}C_0 + {}^{10}C_1 2 + {}^{10}C_2 2^2 + \dots + {}^{10}C_{10} 2^{10} - 2^{10} \\ &= {}^{10}C_0 + {}^{10}C_1 2 + 6\lambda + {}^{10}C_9 \cdot 2^9 \\ &= 1 + 20 + 5120 + 6\lambda \\ &= 5136 + 6\lambda + 5 \\ &= 6\mu + 5 \end{aligned}$$

$$\lambda, \mu \in \mathbb{N}$$

$$\therefore \text{remainder} = 5$$

Question121

The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \dots$ is equal to
[25-Jun-2022-Shift-1]

Answer: 98

Solution:

$$\begin{aligned} S &= \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \dots \\ &= \sum_{r=1}^{100} \left(\frac{3^r - 2^r}{3^r} \right) \\ &= 100 - \frac{2}{3} \left(1 - \left(\frac{2}{3} \right)^{100} \right) \\ &= 98 + 2 \left(\frac{2}{3} \right)^{100} \\ \therefore [S] &= 98 \end{aligned}$$

Question122

The sum $1 + 2 \cdot 3 + 3 \cdot 3^2 + \dots + 10 \cdot 3^9$ is equal to
[25-Jun-2022-Shift-2]

Options:

A. $\frac{2 \cdot 3^{12} + 10}{4}$

B. $\frac{19 \cdot 3^{10} + 1}{4}$

$$C. 5 \cdot 3^{10} - 2$$

$$D. \frac{9 \cdot 3^{10} + 1}{2}$$

Answer: B

Solution:

$$\text{Let } S = 1.3^0 + 2.3^1 + 3.3^2 + \dots + 10.3^9$$

$$3S = 1.3^1 + 2.3^2 + \dots + 10.3^{10}$$

$$-2S = (1.3^0 + 1.3^1 + 1.3^2 + \dots + 1.3^9) - 10.3^{10}$$

$$\Rightarrow S = \frac{1}{2} \left[10.3^{10} - \frac{3^{10} - 1}{-3 - 1} \right]$$

$$\Rightarrow S = \frac{19.3^{10} + 1}{4}$$

Question 123

Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$. Then $A + B$ is equal to
[26-Jun-2022-Shift-1]

Answer: 1100

Solution:

$$A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$$

$$B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$$

$$A = \sum_{j=1}^{10} \min(i, 1) + \min(j, 2) + \dots + \min(i, 10)$$

$$= (1 + 1 + 1 + \dots + 1) + (2 + 2 + 2 + \dots + 2) + (3 + 3 + 3 + \dots + 3) + \dots (1) \text{ 1 times}$$

$$B = \sum_{j=1}^{10} \max(i, 1) + \max(j, 2) + \dots + \max(i, 10)$$

$$= (10 + 10 + \dots + 10) + (9 + 9 + \dots + 9) + \dots + 11 \text{ times}$$

$$A + B = 20(1 + 2 + 3 + \dots + 10)$$

$$= 20 \times \frac{10 \times 11}{2} = 10 \times 110 = 1100$$

Question124

If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then $\frac{A}{B}$ is equal to :

[26-Jun-2022-Shift-2]

Options:

A. $\frac{11}{9}$

B. 1

C. $-\frac{11}{9}$

D. $-\frac{11}{3}$

Answer: C

Solution:

$$A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n} \text{ and } B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$$

$$A = \frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \dots$$

$$B = \frac{-1}{2} + \frac{1}{4^2} - \frac{1}{2^3} + \frac{1}{4^4} + \dots$$

$$A = \frac{\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}, B = \frac{-\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}$$

$$A = \frac{11}{15}, B = \frac{-9}{15}$$

$$\therefore \frac{A}{B} = \frac{-11}{9}$$

Question125

If $a_1(>0)$, a_2 , a_3 , a_4 , a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____
[26-Jun-2022-Shift-2]

Answer: 40

Solution:

Let G.P. be $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$,

$$\because 3a_2 + a_3 = 2a_4$$

$$\Rightarrow 3ar + ar^2 = 2ar^3$$

$$\Rightarrow 2ar^2 - r - 3 = 0$$

$$\therefore r = -1 \text{ or } \frac{3}{2}$$

$$\because a_1 = a > 0 \text{ then } r \neq -1$$

$$\text{Now, } a_2 + a_4 = 2a_3 + 1$$

$$ar + ar^3 = 2ar^2 + 1$$

$$a\left(\frac{3}{2} + \frac{27}{8} - \frac{9}{2}\right) = 1$$

$$\therefore a = \frac{8}{3}$$

$$\therefore a_2 + a_4 + 2a_5 = a(r + r^3 + 2r^4)$$

$$= \frac{8}{3}\left(\frac{3}{2} + \frac{27}{8} + \frac{81}{8}\right) = 40$$

Question 126

$x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a , b , c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :

[27-Jun-2022-Shift-1]

Options:

A. x, y, z are in A.P.

B. x, y, z are in G.P.

C. $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

D. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Answer: C

Solution:

$$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}; y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}; z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$$

Now,

$a, b, c \rightarrow$ AP

$1-a, 1-b, 1-c \rightarrow$ AP

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c} \rightarrow$ HP

$x, y, z \rightarrow$ HP

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \rightarrow$ AP

Question 127

If the sum of the first ten terms of the series

$$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$$

is $\frac{m}{n}$, where m and n are co-prime numbers, then $m + n$ is equal to

[27-Jun-2022-Shift-1]

Answer: 276

Solution:

$$T_r = \frac{r}{(2r^2)^2 + 1}$$
$$= \frac{r}{(2r^2 + 1)^2 - (2r)^2}$$

$$\begin{aligned}
 &= \frac{1}{4} \frac{4r}{(2r^2 + 2r + 1)(2r^2 - 2r + 1)} \\
 S_{10} &= \frac{1}{4} \sum_{r=1}^{10} \left(\frac{1}{(2r^2 - 2r + 1)} - \frac{1}{(2r^2 + 2r + 1)} \right) \\
 &= \frac{1}{4} \left[1 - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \dots + \frac{1}{181} - \frac{1}{221} \right] \\
 \Rightarrow S_{10} &= \frac{1}{4} \cdot \frac{220}{221} = \frac{55}{221} = \frac{m}{n} \\
 \therefore m + n &= 276
 \end{aligned}$$

Question 128

Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \dots$. Then $4S$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. $\left(\frac{7}{3}\right)^2$

B. $\frac{7^3}{3^2}$

C. $\left(\frac{7}{3}\right)^3$

D. $\frac{7^2}{3^3}$

Answer: C

Solution:

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \dots \quad \text{(i)}$$

$$\frac{1}{7}S = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \dots \quad \text{(ii)}$$

(i) - (ii)

$$\frac{6}{7}S = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \dots \quad \text{(iii)}$$

$$\frac{6}{7^2}S = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \dots \quad \text{(iv)}$$

(iii) - (iv)

$$\left(\frac{6}{7}\right)^2 S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \dots$$

$$= 2 \left[\frac{1}{1 - \frac{1}{7}} \right] = 2 \left(\frac{7}{6} \right)$$

$$\therefore 4S = 8 \left(\frac{7}{6} \right)^3 = \left(\frac{7}{3} \right)^3$$

Question 129

If $a_1, a_2, a_3, \dots$ and $b_1, b_2, b_3, \dots$ are A.P., and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$, then $a_4 b_4$ is equal to -
[27-Jun-2022-Shift-2]

Options:

A. $\frac{35}{27}$

B. 1

C. $\frac{27}{28}$

D. $\frac{28}{27}$

Answer: D

Solution:

$a_1, a_2, a_3, \dots$ are in A.P. (Let common difference is d_1)

$b_1, b_2, b_3, \dots$ are in A.P. (Let common difference is d_2)

and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$

$$\therefore a_1 b_1 = 1$$

$$\therefore b_1 = \frac{1}{2}$$

$$a_{10} b_{10} = 1$$

$$\therefore b_{10} = \frac{1}{3}$$

$$\text{Now, } a_{10} = a_1 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$$

$$b_{10} = b_1 + 9d_2 \Rightarrow d_2 = \frac{1}{9} \left[\frac{1}{3} - \frac{1}{2} \right] = -\frac{1}{54}$$

$$\text{Now, } a_4 = 2 + \frac{3}{9} = \frac{7}{3}$$

$$b_4 = \frac{1}{2} - \frac{3}{54} = \frac{4}{9}$$

$$\therefore a_4 b_4 = \frac{28}{27}$$

Question130

Let $A_1, A_2, A_3, \dots$ be an increasing geometric progression of positive real numbers. If $A_1 A_3 A_5 A_7 = \frac{1}{1256}$ and $A_2 + A_4 = \frac{7}{36}$, then the value of $A_6 + A_8 + A_{10}$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. 33

B. 37

C. 43

D. 47

Answer: C

Solution:

$$A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$$

$$(A_4)^4 = \frac{1}{1296}$$

$$A_4 = \frac{1}{6}$$

$$A_2 + A_4 = \frac{7}{36}$$

$$A_2 = \frac{1}{36}$$

$$A_6 = 1$$

$$A_8 = 6$$

$$A_{10} = 36$$

$$A_6 + A_8 + A_{10} = 43$$

Question131

If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is :
[28-Jun-2022-Shift-2]

Options:

- A. 21
B. 22
C. 23
D. 24

Answer: C

Solution:

$a, A_1, A_2, \dots, A_n, 100$

Let d be the common difference of above A.P. then

$$\frac{a+d}{100-d} = \frac{1}{7}$$

$$\Rightarrow 7a + 8d = 100 \dots (i)$$

$$\text{and } a + n = 33$$

$$\text{and } 100 = a + (n+1)d$$

$$\Rightarrow 100 = a + (34-a) \frac{(100-7a)}{8}$$

$$\Rightarrow 800 = 8a + 7a^2 - 338a + 3400$$

$$\Rightarrow 7a^2 - 330a + 2600 = 0$$

$$\Rightarrow a = 10, \frac{260}{7}, \text{ but } a \neq \frac{260}{7}$$

$$\therefore n = 23$$

Question 132

Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$$\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right) \text{ is equal to } \underline{\hspace{2cm}}$$

[28-Jun-2022-Shift-2]

Answer: 41651

Solution:

$$S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{n+2} = (n^2+1) - \frac{2}{n+2}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$$

$$= \frac{1}{26} + \sum_{n=1}^{50} \left\{ (n^2 - n) + 2 \left(\frac{1}{n+1} - \frac{1}{n+2} \right) \right\}$$

$$= \frac{1}{26} + \frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} + 2 \left(\frac{1}{2} - \frac{1}{52} \right)$$

$$= 1 + 25 \times 17(101 - 3)$$

$$= 41651$$

Question133

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

[29-Jun-2022-Shift-1]

Options:

A. $\frac{6}{343}$

B. $\frac{7}{216}$

C. $\frac{8}{343}$

D. $\frac{49}{216}$

Answer: B

Solution:

$$a_{n+2} = 2a_{n+1} - a_n + 1 \text{ and } a_0 = a_1 = 0$$

$$a_2 = 2a_1 - a_0 + 1 = 1$$

$$a_3 = 2a_2 - a_1 + 1 = 3$$

$$a_4 = 2a_3 - a_2 + 1 = 6$$

$$a_5 = 2a_4 - a_3 + 1 = 10$$

$$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \dots$$

$$s = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \dots$$

$$\frac{1}{7}s = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \dots$$

$$\frac{6s}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \dots$$

$$\frac{6s}{49} = \frac{1}{7^3} + \frac{2}{7^4} + \dots$$

$$\frac{36s}{49} = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \dots$$

$$\frac{36s}{49} = \frac{\frac{1}{7^2}}{1 - \frac{1}{7}}$$

$$\frac{36s}{49} = \frac{7}{49 \times 6}$$

$$s = \frac{7}{216}$$

Question134

The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \dots$ is equal to:

[29-Jun-2022-Shift-2]

Options:

A. $\frac{425}{216}$

B. $\frac{429}{216}$

C. $\frac{288}{125}$

D. $\frac{280}{125}$

Answer: C

Solution:

Question 135

Let 3, 6, 9, 12, upto 78 terms and 5, 9, 13, 17, upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to _____

[29-Jun-2022-Shift-2]

Answer: 2223

Solution:

1st AP :

3, 6, 9, 12, ... upto 78 terms

$$t_{78} = 3 + (78 - 1)3$$

$$= 3 + 77 \times 3$$

$$= 234$$

2nd AP:

5, 9, 13, 17, ... upto 59 terms

$$t_{59} = 5 + (59 - 1)4$$

$$= 5 + 58 \times 4$$

$$= 237$$

Common term's AP :

First term = 9

Common difference of first AP = 3

And common difference of second AP = 4

$\therefore$ Common difference of common terms

$$AP = \text{LCM}(3, 4) = 12$$

$\therefore$ New AP = 9, 21, 33, ...

$$t_n = 9 + (n - 1)12 \leq 234$$

$$\Rightarrow n \leq \frac{237}{12}$$

$$\Rightarrow n = 19$$

$$\begin{aligned}\therefore S_{19} &= \frac{19}{2}[2.9 + (19-1)12] \\ &= 19(9 + 108) \\ &= 2223\end{aligned}$$

Question136

Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to
[25-Jul-2022-Shift-1]

Answer: 27560

Solution:

Given,

$$a_n = a_{n-1} + 2$$

$$\Rightarrow a_n - a_{n-1} = 2$$

$\therefore$ In this series between any two consecutive terms difference is 2. So this is an A.P. with common difference 2.

Also given $a_1 = 1$

$\therefore$ Series is $= 1, 3, 5, 7, \dots$

$$\therefore a_n = 1 + (n-1)2 = 2n-1$$

$$\text{Also } b_n = a_n + b_{n-1}$$

When $n = 2$ then

$$b_2 - b_1 = a_2 = 3$$

$$\Rightarrow b_2 - 1 = 3 \quad [\text{Given } b_1 = 1]$$

$$\Rightarrow b_2 = 4$$

When $n = 3$ then

$$b_3 - b_2 = a_3$$

$$\Rightarrow b_3 - 4 = 5$$

$$\Rightarrow b_3 = 9$$

$\therefore$ Series is $= 1, 4, 9, \dots$

$$= 1^2, 2^2, 3^2, \dots, n^2$$

$$\therefore b_n = n^2$$

$$\text{Now, } \sum_{n=1}^{15} (a_n \cdot b_n)$$

$$= \sum_{n=1}^{15} [(2n-1)n^2]$$

$$= \sum_{n=1}^{15} 2n^3 - \sum_{n=1}^{15} n^2$$

$$= 2(1^3 + 2^3 + \dots + 15^3) - (1^2 + 2^2 + \dots + 15^2)$$

$$= 2 \times \left(\frac{15 \times 16}{2} \right)^2 - \left(\frac{15(16) \times 31}{6} \right)$$

$$= 27560$$

Question137

The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to
[25-Jul-2022-Shift-2]

Options:

A. $\frac{7}{87}$

B. $\frac{7}{29}$

C. $\frac{14}{87}$

D. $\frac{21}{29}$

Answer: B

Solution:

$$\begin{aligned} \sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} &= \frac{3}{4} \sum_{n=1}^{21} \frac{1}{4n-1} - \frac{1}{4n+3} \\ &= \frac{3}{4} \left[\left(\frac{1}{3} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{11} \right) + \dots + \left(\frac{1}{83} - \frac{1}{87} \right) \right] \\ &= \frac{3}{4} \left[\frac{1}{3} - \frac{1}{87} \right] = \frac{3}{4} \frac{84}{3 \cdot 87} = \frac{7}{29} \end{aligned}$$

Question 138

Consider two G.P.s. $2, 2^2, 2^3, \dots$ and $4, 4^2, 4^3, \dots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^n k(n-k)$ is equal to :
[26-Jul-2022-Shift-1]

Options:

A. 560

B. 1540

C. 1330

D. 2600

Answer: C

Solution:

Given G.P's $2, 2^2, 2^3, \dots$ 60 terms
 $4, 4^2, \dots$ n terms

Now, G.M. = $2^{\frac{225}{8}}$

$(2 \cdot 2^2 \dots 4 \cdot 4^2 \dots) \frac{1}{60+n} = 2^{\frac{225}{8}}$

$$\left(\frac{n^2 + n + 1830}{6 + n} \right) = 2 \frac{225}{8}$$

$$\Rightarrow \frac{n^2 + n + 1830}{6 + n} = \frac{225}{8}$$

$$\Rightarrow 8n^2 - 217n + 1140 = 0$$

$$n = \frac{57}{8}, 20, \text{ so } n = 20$$

$$\therefore \sum_{k=1}^{20} k(20-k) = 20 \times \frac{20 \times 21}{2} - \frac{20 \times 21 \times 41}{6}$$

$$= \frac{20 \times 21}{2} \left[20 - \frac{41}{3} \right] = 1330$$

Question 139

The series of positive multiples of 3 is divided into sets :

{3}, {6, 9, 12}, {15, 18, 21, 24, 27}, ... Then the sum of the elements in the 11th set is equal to _____.
[26-Jul-2022-Shift-1]

Answer: 6993

Solution:

$$\{3 \times 1\}, \{3 \times 2, 3 \times 3, 3 \times 4\}, \{3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8, 3 \times 9\}, \dots$$

1-term 3 terms 5 terms

$\therefore$ 11th set will have $1 + (10)2 = 21$ term

Also upto 10th set total $3 \times k$ type terms will be $1 + 3 + 5 + \dots + 19 = 100$ term

$\therefore$ Set 11 = $\{3 \times 101, 3 \times 102, \dots, 3 \times 121\}$

$\therefore$ Sum of elements = $3 \times (101 + 102 + \dots + 121)$

$$= \frac{3 \times 222 \times 21}{2} = 6993$$

Question 140

If $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to _____.

[26-Jul-2022-Shift-2]

Answer: 166

Solution:

$$\begin{aligned}
 & \sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} \\
 &= \frac{1}{2} \left[\sum_{k=1}^{10} \left(\frac{1}{k^2 - k + 1} - \frac{1}{k^2 + k + 1} \right) \right] \\
 &= \frac{1}{2} \left[1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{13} + \dots + \frac{1}{91} - \frac{1}{111} \right] \\
 &= \frac{1}{2} \left[1 - \frac{1}{111} \right] = \frac{110}{2 \cdot 111} = \frac{55}{111} = \frac{m}{n} \\
 \therefore m + n &= 55 + 111 = 166
 \end{aligned}$$

Question 141

Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is _____.

[26-Jul-2022-Shift-2]

Answer: 53

Solution:

$$d_1 = \frac{199 - 100}{2} \notin \mathbb{I}$$

$$d_2 = \frac{199 - 100}{3} = 33$$

$$d_3 = \frac{199 - 100}{4} \notin \mathbb{I}$$

$$d_n = \frac{199 - 100}{i + 1} \in \mathbb{I}$$

$$d_i = 33 + 11, 9$$

$$\begin{aligned} \text{Sum of CD's} &= 33 + 11 + 9 \\ &= 53 \end{aligned}$$

Question 142

Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to _____.

[27-Jul-2022-Shift-1]

Options:

A. 290

B. 380

C. 460

D. 510

Answer: B

Solution:

$\therefore a_1, a_2, \dots, a_n$ be an A.P of natural numbers and

$$\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}[2a_1 + 4d]}{\frac{9}{2}[2a_1 + 8d]} = \frac{5}{17}$$

$$\Rightarrow 34a_1 + 68d = 18a_1 + 72d$$

$$\Rightarrow 16a_1 = 4d$$

$$\therefore d = 4a_1$$

And $110 < a_{15} < 120$

$$\therefore 110 < a_1 + 14d < 120 \Rightarrow 110 < 57a_1 < 120$$

$$\therefore a_1 = 2 (\because a_1 \in \mathbb{N})$$

$$d = 8$$

$$\therefore S_{10} = 5[4 + 9 \times 8] = 380$$

Question143

Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____.
[27-Jul-2022-Shift-1]

Answer: 10620

Solution:

$$\therefore |f(n)| \leq 800$$

$$\Rightarrow -800 \leq 2n^2 - n - 1 \leq 800$$

$$\Rightarrow 2n^2 - n - 801 \leq 0$$

$$\therefore n \in \left[\frac{-\sqrt{6409+1}}{4}, \frac{\sqrt{6409+1}}{4} \right] \text{ and } n \in \mathbb{Z}$$

$$\therefore n = -19, -18, -17, \dots, 19, 20.$$

$$\therefore \sum (2x^2 - x - 1) = 2 \sum x^2 - \sum x - \sum 1.$$

$$= 2 \cdot 2 \cdot (1^2 + 2^2 + \dots + 19^2) + 2 \cdot 20^2 - 20 - 40$$

$$= 10620$$

Question144

Let the sum of an infinite G.P., whose first term is a and the common ratio is r , be 5 . Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10ar$, n^{th} term is a_n and the common difference is $10ar^2$, is

equal to :
[27-Jul-2022-Shift-2]

Options:

A. $21a_{11}$

B. $22a_{11}$

C. $15a_{16}$

D. $14a_{16}$

Answer: A

Solution:

Let first term of G.P. be a and common ratio is r . Then, $\frac{a}{1-r} = 5 \dots (i)$

$$a \frac{(r^5 - 1)}{(r - 1)} = \frac{98}{25} \Rightarrow 1 - r^5 = \frac{98}{125}$$

$$\therefore r^5 = \frac{27}{125}, r = \left(\frac{3}{5}\right)^{\frac{3}{5}}$$

$$\therefore \text{Then, } S_{21} = \frac{21}{2} [2 \times 10a + 20 \times 10ar^2]$$

$$= 21[10a + 10 \cdot 10ar^2]$$

$$= 21a_{11}$$

Question 145

$$\frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15} + \dots + \frac{30^3 - 29^3 + 28^3 - 27^3 + \dots + 2^3 - 1^3}{15 \times 63} \text{ is equal to}$$

[27-Jul-2022-Shift-2]

Answer: 120

Solution:

$$\begin{aligned} T_n &= \frac{\sum_{k=1}^n [(2k)^3 - (2k-1)^3]}{n(4n+3)} \\ &= \frac{\sum_{k=1}^n [4k^2 + (2k-1)^2 + 2k(2k-1)]}{n(4n+3)} \\ &= \frac{\sum_{k=1}^n (12k^2 - 6k + 1)}{n(4n+3)} \\ &= \frac{2n(2n^2 + 3n + 1) - 3n^2 - 3n + n}{n(4n+3)} \end{aligned}$$

$$= \frac{n^2(4n+3)}{n(4n+3)} = n$$

$$\therefore T_n = n$$

$$S_n = \sum_{n=1}^{15} T_n = \frac{15 \times 16}{2} = 120$$

Question 146

Consider the sequence $a_1, a_2, a_3, \dots$ such that $a_1 - 1, a_2 - 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for

$n = 1, 2, 3, \dots$. If $\left(\frac{a_1 + \frac{1}{a_2}}{a_3} \right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4} \right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5} \right) \dots \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}} \right) = 2^\alpha ({}^{61}C_{31})$, then

α is equal to :

[28-Jul-2022-Shift-1]

Options:

A. -30

B. -31

C. -60

D. -61

Answer: C

Solution:

$$a_{n+2} = \frac{2}{a_{n+1}} + a_n$$

$$\Rightarrow a_n a_{n+1} + 1 = a_{n+1} a_{n+2} - 1$$

$$\Rightarrow a_{n+2} a_{n+1} - a_n \cdot a_{n+1} = 2$$

For

$$n = 1 \quad a_3 a_2 - a_1 a_2 = 2$$

$$n = 2 \quad a_4 a_3 - a_2 a_3 = 2$$

$$n = 3 \quad a_5 a_4 - a_3 a_4 = 2$$

...

...

...

$$n = n \quad \frac{a_{n+2} a_{n+1} - a_n a_{n+1}}{a_{n+2} a_{n+1}} = \frac{2}{2n+1} \cdot \frac{a_n a_{n+1}}{a_1 a_2}$$

Now,

$$\frac{(a_1 a_2 + 1)}{a_2 a_3} \cdot \frac{(a_2 a_3 + 1)}{a_3 a_4} \cdot \frac{(a_3 a_4 + 1)}{a_4 a_5} \dots \frac{(a_{30} a_{31} + 1)}{a_{31} a_{32}}$$

$$= \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \dots \frac{61}{62}$$

$$= 2^{-60} ({}^{61}C_{31})$$

Question 147

For $p, q \in \mathbb{R}_s$ consider the real valued function $f(x) = (x - p)^2 - q, x \in \mathbb{R}$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is _____.
[28-Jul-2022-Shift-1]

Answer: 50

Solution:

$$\because a_1, a_2, a_3, a_4$$

$$\therefore a_2 = p - 3d, a_2 = p - d, a_3 = p + d \text{ and } a_4 = p + 3d$$

Where $d > 0$

$$\because |f(a_i)| = 500$$

$$\Rightarrow |9d^2 - q| = 500$$

$$\text{and } |d^2 - q| = 500$$

$$\text{either } 9d^2 - q = d^2 - q$$

$$\Rightarrow d = 0 \text{ not acceptable}$$

$$\therefore 9d^2 - q = q - d^2$$

$$\therefore 5d^2 - q = 0$$

$$\text{Roots of } f(x) = 0 \text{ are } p + \sqrt{q} \text{ and } p - \sqrt{q}$$

$$\therefore \text{absolute difference between roots} = |2\sqrt{q}| = 50$$

Question 148

Let $x_1, x_2, x_3, \dots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is _____.
[28-Jul-2022-Shift-1]

Answer: 142

Solution:

$x_1, x_2, x_3, \dots, x_{20}$ are in G.P.

$$x_1 = 3, r = \frac{1}{2}$$

$$\bar{x} = \frac{\sum x_i^2 - 2\sum x_i i + i^2}{20}$$

$$= \frac{1}{20} \left[12 \left(1 - \frac{1}{2^{40}} \right) - 6 \left(4 - \frac{11}{2^{18}} \right) + 70 \times 41 \right]$$

$$\begin{cases} S = 1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + \dots \\ \frac{S}{2} = \frac{1}{2} + \frac{2}{2^2} + \dots \\ \frac{S}{2} = 2 \left(1 - \frac{1}{2^{20}} \right) - \frac{20}{2^{20}} = 4 - \frac{11}{2^{18}} \end{cases}$$

$$\therefore [x] = \left[\frac{2858}{20} - \left(\frac{12}{240} - \frac{66}{2^{18}} \right) \cdot \frac{1}{20} \right]$$

$$= 142$$

Question 149

$\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \dots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then $m \cdot n$ is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 12

Solution:

$$\begin{aligned} \frac{1}{3^{12}} + 5 \left(\frac{2^0}{3^{12}} + \frac{2^1}{3^{11}} + \frac{2^2}{3^{10}} + \dots + \frac{2^{11}}{3} \right) &= 2^n \cdot m \\ \Rightarrow \frac{1}{3^{12}} + 5 \left(\frac{1}{3^{12}} \frac{((6)^2 - 1)}{(6 - 1)} \right) &= 2^n \cdot m \\ \Rightarrow \frac{1}{3^{12}} + \frac{5}{5} \left(\frac{1}{3^{12}} \cdot 2^{12} \cdot 3^{12} - \frac{1}{3^{12}} \right) &= 2^n \cdot m \\ \Rightarrow \frac{1}{3^{12}} + 2^{12} - \frac{1}{3^{12}} &= 2^n \cdot m \\ \Rightarrow 2^n \cdot m &= 2^{12} \\ \Rightarrow m = 1 \text{ and } n = 12 \\ m \cdot n &= 12 \end{aligned}$$

Question 150

If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \dots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is _____.

[29-Jul-2022-Shift-1]

Options:

- A. 198
- B. 202
- C. 212
- D. 218

Answer: C

Solution:

$$\begin{aligned}\frac{1}{20} \left(\frac{1}{20-a} - \frac{1}{40-a} + \frac{1}{40-a} - \frac{1}{60-a} + \dots + \frac{1}{180-a} - \frac{1}{200-a} \right) &= \frac{1}{256} \\ \Rightarrow \frac{1}{20} \left(\frac{1}{20-a} - \frac{1}{200-a} \right) &= \frac{1}{256} \\ \Rightarrow \frac{1}{20} \left(\frac{180}{(20-a)(200-a)} \right) &= \frac{1}{256} \\ \Rightarrow (20-a)(200-a) &= 9.256 \\ \text{OR } a^2 - 220a + 1696 &= 0 \\ \Rightarrow a &= 212, 8\end{aligned}$$

Question151

Let $a_1, a_2, a_3, \dots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____.
[29-Jul-2022-Shift-1]

Answer: 16

Solution:

Given

$$\begin{aligned}S &= \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \dots \infty \\ \frac{1}{2}S &= \frac{a_1}{2^2} + \frac{a_2}{2^3} + \dots \infty \\ \hline \frac{S}{2} &= \frac{a_1}{2} + \frac{(a_2 + a_1)}{2^2} + \frac{(a_3 + a_2)}{2^3} + \dots \infty \\ \Rightarrow \frac{S}{2} &= \frac{a_1}{2} + \frac{d}{2} \\ \Rightarrow a_1 + d = a_2 &= 4 \Rightarrow 4a_2 = 16\end{aligned}$$

Question152

If $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots + \frac{1}{100 \times 101 \times 102} = \frac{k}{101}$, then $34k$ is equal to _____.
[29-Jul-2022-Shift-1]

Answer: 286

Solution:

$$\begin{aligned}
 S &= \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots + \frac{1}{100 \times 101 \times 102} \\
 &= \frac{1}{(3-1) \cdot 1} \left[\frac{1}{2 \times 3} - \frac{1}{101 \times 102} \right] \\
 &= \frac{1}{2} \left(\frac{1}{6} - \frac{1}{101 \times 102} \right) \\
 &= \frac{143}{102 \times 101} = \frac{k}{101} \\
 \therefore 34k &= 286
 \end{aligned}$$

Question 153

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0$.

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

[29-Jul-2022-Shift-2]

Options:

A. 483

B. 528

C. 575

D. 624

Answer: B

Solution:

$$\begin{aligned}
 a_0 &= 0, a_1 = 0 \\
 a_{n+2} &= 3a_{n+1} - 2a_n + 1 : n \geq 0 \\
 a_{n+2} - a_{n+1} &= 2(a_{n+1} - a_n) + 1 \\
 n=0 \quad a_2 - a_1 &= 2(a_1 - a_0) + 1 \\
 n=1 \quad a_3 - a_2 &= 2(a_2 - a_1) + 1 \\
 n=2 \quad a_4 - a_3 &= 2(a_3 - a_2) + 1 \\
 n=n \quad a_{n+2} - a_{n+1} &= 2(a_{n+1} - a_n) + 1 \\
 (a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) &= 0 \\
 a_{n+2} &= 2a_{n+1} + (n+1) \\
 n \rightarrow n-2 \\
 a_n - 2a_{n-1} &= n-1 \\
 \text{Now } a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24} \\
 &= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528
 \end{aligned}$$

Question 154

$\sum_{r=1}^{20} (r^2 + 1)(r!)$ is equal to

[29-Jul-2022-Shift-2]

Options:

- A. $22! - 21!$
 B. $22! - 2(21!)$
 C. $21! - 2(20!)$
 D. $21! - 20!$

Answer: B

Solution:

Given,

$$\sum_{r=1}^{20} (r^2 + 1)(r!)$$

$$\text{Let, } f(r) = (r^2 + 1)(r!)$$

$$= (r^2)(r!) + r!$$

$$= r(r!) + r!$$

$$= r[(r+1-1)r!] + r!$$

$$= r[(r+1)r! - r!] + r!$$

$$= r[(r+1)! - (r!)] + r!$$

$$= r(r+1)! - r(r!)+r! = (r+2-2)(r+1)! - r(r!)+r!$$

$$= (r+2)(r+1)! - 2(r+1)! - [(r+1-1)(r!)] + r!$$

$$= (r+2)! - 2(r+1)! - (r+1)! + r! + r!$$

$$= (r+2)! - 3(r+1)! + 2r!$$

$$= [(r+2)! - (r+1)!] - 2[(r+1)! - r!]$$

$$\therefore \sum_{r=1}^{20} f(r)$$

$$= \sum_{r=1}^{20} [(r+2)! - (r+1)!] - 2 \sum_{r=1}^{20} [(r+1)! - r!]$$

$$= [(22! + 21! + 20! + \dots + 4! + 3!) - (21! + 20! + 19! + \dots + 3! + 2!)] - 2[(21! + 20! + \dots + 3! + 2!) - (20! + 19! + \dots + 1!)]$$

$$= [(22!) - (2!)] - 2[(21!) - (1!)]$$

$$= 22! - 2! - 2 \cdot (21!) + 2 \cdot 1!$$

$$= 22! - 2 \cdot (21!)$$

Question 155

If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots \infty) \log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$,

then the value of $\frac{2 \sin x}{\sin x + \sqrt{3} \cos x} \left(0 < x < \frac{\pi}{2} \right)$ is

[24-Feb-2021 Shift 1]

Options:

A. $2\sqrt{3}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. $\frac{1}{2}$

Answer: D

Solution:

$$e^{(\cos^2 x + \cos^4 x + \dots \infty) \ln 2} = 2^{\cos^2 x + \cos^4 x + \dots \infty} = 2^{\cot^2 x}$$

$$\text{Now } t^2 - 9t + 9 = 0 \Rightarrow t = 1, 8$$

$$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$$

$$\Rightarrow \frac{2 \sin x}{\sin x + \sqrt{3} \cos x} = \frac{2}{1 + \sqrt{3} \cot x} = \frac{2}{4} = \frac{1}{2}$$

Question 156

Let $A = \{x : x \text{ is 3-digit number}\}$ $B = \{x : x = 9k + 2, k \in I\}$ and $C = \{x : x = 9k + \ell, k \in I, \ell \in I, 0 < \ell < 9\}$ for some $\ell (0 < \ell < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400 , then ℓ is equal to [24-Feb-2021 Shift 1]

Answer: 5

Solution:

B and C will contain three digit numbers of the form $9k + 2$ and $9k + \ell$ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.

Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$ where $S(k)$ denotes sum of elements of set k .

Also $B = \{101, 110, \dots, 992\}$

$$\therefore S(B) = \frac{100}{2}(101 + 992) = 54650$$

Case-I : If $\ell = 2$

then $B \cap C = B$

$$\therefore S(B \cup C) = S(B)$$

which is not possible as given sum is

$$274 \times 400 = 109600$$

Case-II : If $\ell \neq 2$

then $B \cap C = \emptyset$

$$\therefore S(B \cup C) = S(B) + S(C) = 400 \times 274$$

$$\Rightarrow 54650 + \sum_{k=11}^{110} 9k + \ell = 109600$$

$$\Rightarrow 9 \sum_{k=11}^{110} k + \sum_{k=11}^{110} \ell = 54950$$

$$\Rightarrow 9 \left(\frac{100}{2}(11 + 110) \right) + \ell(100) = 54950$$

$$\Rightarrow 54450 + 100\ell = 54950$$

$$\Rightarrow \ell = 5$$

Question 157

The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1

and the third term is a_1 then $2a$ is

[2021, 24 Feb. Shift-II]

Answer: 9

Solution:

Let four numbers in GP be a, ar, ar^2, ar^3 .

According to the question,

$$a + ar + ar^2 + ar^3 = \frac{65}{12} \dots\dots\dots (i)$$

$$\text{and } \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\Rightarrow \frac{1}{a} \left(\frac{1+r+r^2+r^3}{r^3} \right) = \frac{65}{18} \dots\dots\dots (ii)$$

Dividing Eq. (i) by (ii), we get

$$\frac{a(1+r+r^2+r^3)}{\frac{1}{a} \frac{(1+r+r^2+r^3)}{r^3}} = \frac{65/12}{65/18}$$

$$\Rightarrow a^2 r^3 = \frac{18}{12}$$

$$\Rightarrow a^2 r^3 = \frac{3}{2}$$

Also, product of first three terms = 1

$$a \times ar \times ar^2 = 1$$

$$\Rightarrow a^3 r^3 = 1$$

$$\Rightarrow a^3 \times \frac{3}{2a^2} = 1 \quad \left[\because r^3 = \frac{3/2}{a^2} \right]$$

$$\Rightarrow a = \frac{2}{3}$$

$$\text{and } r^3 = \frac{3/2}{(2/3)^2} = \left(\frac{3}{2} \right)^3 \Rightarrow r = \frac{3}{2}$$

According to the question, third term

$$\alpha = ar^2 = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} = \frac{3}{2} \therefore 2\alpha = 2 \times \frac{3}{2} = 3 \text{ third term}$$

Question 158

The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to

[2021, 26 Feb. Shift-II]

Options:

A. $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

B. $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

C. $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

D. $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Answer: B

Solution:

$$\begin{aligned} \text{Let } \sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!} &= S \\ &= \sum_{n=1}^{\infty} \frac{4n^2 + 24n + 40}{4(2n+1)!} \\ &= \sum_{n=1}^{\infty} \frac{(2n+1)^2 + (2n+1) \cdot 10 + 29}{4(2n+1)!} \\ &= \frac{1}{4} \left[\sum_{n=1}^{\infty} \frac{(2n+1)^2}{(2n+1)(2n)!} + \sum_{n=1}^{\infty} \frac{(2n+1) \cdot 10}{(2n+1)(2n)!} + \sum_{n=1}^{\infty} \frac{29}{(2n+1)!} \right] \\ &= \frac{1}{4} \left[\sum_{n=1}^{\infty} \frac{(2n+1)}{(2n)!} + \sum_{n=1}^{\infty} \frac{10}{(2n)!} + \sum_{n=1}^{\infty} \frac{29}{(2n+1)!} \right] \dots\dots\dots (1) \end{aligned}$$

$$\begin{aligned} \text{Now,} \\ &= \sum_{n=1}^{\infty} \frac{(2n+1)}{(2n)!} = \sum_{n=1}^{\infty} \frac{2n}{(2n)!} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \\ &= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \\ \text{Now,} \\ &= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots = \frac{e - \frac{1}{e}}{2} \dots\dots\dots (ii) \end{aligned}$$

$$\text{and } \sum_{n=1}^{\infty} \frac{1}{(2n)!} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots = \frac{e + \frac{1}{e} - 2}{2} \dots\dots\dots (iii)$$

$$\text{and } \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} = \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots = \frac{e - \frac{1}{e} - 2}{2} \dots\dots\dots (iv)$$

Using Eqs. (ii), (iii), (iv) in (i),

$$\begin{aligned} S &= \frac{1}{4} \left[\frac{e - \frac{1}{e}}{2} + 11 \& \left(\frac{e + \frac{1}{e} - 2}{2} \right) \right. \\ &\quad \left. + 29 \& \left(\frac{e - \frac{1}{e} - 2}{2} \right) \right] \\ &= \frac{1}{4} \left[\frac{e}{2} - \frac{1}{2e} + \frac{11e}{2} + \frac{11}{2e} + \frac{29e}{2} - \frac{29}{2e} - 4 \right] \\ &= \frac{41e}{8} - \frac{19}{8e} - 10 \end{aligned}$$

Question159

The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$ is equal to
[2021, 26 Feb. Shift-1]

Options:

A. $\frac{13}{4}$

B. $\frac{9}{4}$

C. $\frac{15}{4}$

D. $\frac{11}{4}$

Answer: A

Solution:

Given, $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \dots$

Let, $S_1 = \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \dots$

Multiply $1/3$ in series Eq. (i),

$$\frac{S_1}{3} = \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \dots$$

Subtract Eq. (ii) from Eq. (i), we get

$$S_1 - \frac{S_1}{3} = \frac{2}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots$$

$$\Rightarrow \frac{2S_1}{3} = \frac{2}{3} + \left[\frac{5}{3^2} + \frac{5}{3^3} + \dots \right]$$

$$= \frac{2}{3} + \left[\frac{5/3^2}{1-1/3} \right] \left[\because \frac{5}{3^2} + \frac{5}{3^3} + \dots \text{ is a geometric series with } r = 1/3, \text{ sum upto infinity of this series is } \frac{a}{1-r}, \text{ where } a = \text{first term} \right]$$

$$= \frac{2}{3} + \left[\frac{5}{6} \right] = \frac{9}{6} = \frac{3}{2}$$

$$\Rightarrow S_1 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

$$\therefore S = 1 + S_1$$

$$= 1 + \frac{9}{4} = \frac{13}{4}$$

Question 160

If the arithmetic mean and geometric mean of the p th and q th terms of the sequence $-16, 8, -4, 2, \dots$ satisfy the equation

$$4x^2 - 9x + 5 = 0, \text{ then } p + q \text{ is equal to}$$

[2021, 26 Feb. Shift-II]

Answer: 10

Solution:

If AM and GM satisfy the equation $4x^2 - 9x + 5 = 0$, then AM and GM are nothing but roots of this quadratic equation,

$$4x^2 - 9x + 5 = 0$$

$$\Rightarrow 4x^2 - 4x - 5x + 5 = 0$$

$$\Rightarrow 4x(x-1) - 5(x-1) = 0$$

$$\Rightarrow (x-1)(4x-5) = 0$$

$$\Rightarrow x = 1, \frac{5}{4}$$

$$\text{Then, AM} = \frac{5}{4} \text{ and GM} = 1 \text{ } [\because \text{AM} \geq \text{GM}]$$

Again, the given series is

$$-16, 8, -4, 2, \dots$$

which is a geometric progression series with common ratio $\frac{-1}{2}$, then

$$p\text{th term} = -16 \left(\frac{-1}{2} \right)^{p-1} = t_p$$

$$q\text{th term} = -16 \left(\frac{-1}{2} \right)^{q-1} = t_q$$

$$\text{Arithmetic mean} = \frac{5}{4}$$

$$\Rightarrow \frac{t_p + t_q}{2} = \frac{5}{4}$$

$$\text{Geometric mean} = 1$$

$$\Rightarrow \sqrt{t_p t_q} = 1$$

$$\therefore \sqrt{\frac{t_p}{t_q}} = 1$$

$$\Rightarrow (-16) \left(\frac{-1}{2} \right)^{p-1} (-16) \left(\frac{-1}{2} \right)^{q-1} = 1$$

$$\Rightarrow (-16)^2 \left(\frac{-1}{2} \right)^{p+q-2} = 1$$

$$\Rightarrow (-2^4)^2 \left(\frac{-1}{2} \right)^{p+q-2} = 1$$

$$\Rightarrow (-2)^8 \frac{(+1)^{p+q-2}}{(-2)^{p+q-2}} = 1$$

$$\Rightarrow (-2)^8 (+1)^{p+q-2} = (-2)^{p+q-2}$$

$$\Rightarrow (-2)^8 = (-2)^{p+q-2}$$

$$\Rightarrow p+q-2 = 8$$

$$\Rightarrow p+q = 10$$

Question 161

In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th terms is equal to
[2021, 26 Feb. Shift-1]

Options:

A. 30

B. 26

C. 35

D. 32

Answer: C

Solution:

Let the first term of geometric series be 'a' and common ratio be 'r'.

Then, nth term of given series is given as

$$T_n = ar^{n-1}$$

Now, given that sum of second and sixth term is $\frac{25}{2}$.

$$\text{i.e. } T_2 + T_6 = \frac{25}{2}$$

$$\Rightarrow ar + ar^5 = \frac{25}{2}$$

$$\Rightarrow ar(1 + r^4) = \frac{25}{2} \dots\dots\dots (i)$$

Also, given that product of third and fifth term is 25.

$$\text{Irl i.e. } (T_3)(T_5) = 25$$

$$\Rightarrow (ar^2)(ar^4) = 25$$

$$\Rightarrow a^2 r^6 = 25 \quad \dots\dots (ii)$$

$$\text{Squaring Eq. (i), we get } a^2 r^2 (1+r^4)^2 = \left(\frac{25}{2}\right)^2 \quad \dots\dots (iii)$$

Divide Eq. (ii) by (iii),

$$\Rightarrow \frac{a^2 r^2 (1+r^4)^2}{a^2 r^6} = \frac{(25)^2}{4(25)}$$

$$\Rightarrow \frac{(1+r^4)^2}{r^4} = \frac{25}{4}$$

$$\Rightarrow 4(1+r^4)^2 = 25r^4$$

$$\Rightarrow 4(1+r^8+2r^4) = 25r^4$$

$$\Rightarrow 4r^8 - 17r^4 + 4 = 0$$

$$\Rightarrow 4r^8 - 16r^4 - r^4 + 4 = 0$$

$$\Rightarrow 4r^4(r^4 - 4) - 1(r^4 - 4) = 0$$

$$\Rightarrow (r^4 - 4)(4r^4 - 1) = 0$$

$$\text{Gives, } r^4 = 4 \text{ or } r^4 = 1/4$$

We have to find sum of 4th, 6th and 8th term, i.e.

$$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$$

$$= ar(r^2 + r^4 + r^6)$$

$$= ar^3(1 + r^2 + r^4) \quad \dots\dots (iv)$$

Using Eq. (ii),

$$(ar^3)^2 = 25$$

$$\Rightarrow ar^3 = 5$$

Also, we take $r^4 = 4$ because given series is increasing and $r^2 \neq 2$.

$$\therefore T_4 + T_6 + T_8 = 5(1 + 2 + 4)$$

$$= 5(7) = 35$$

Question 162

The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to
[2021, 25 Feb. Shift-II]

Options:

A. $a + 1$

B. $a + \frac{1}{a}$

C. $2\sqrt{a}$

D. $2a$

Answer: C

Solution:

We already know, Arithmetic mean $\geq$ Geometric mean.

Let us take AM and GM of two terms a^{a^x} and a^{1-a^x} ,

$$\Rightarrow \text{AM} = \frac{a^{a^x} + a^{1-a^x}}{2}$$

$$\text{and GM} = \sqrt{a^{a^x} \cdot a^{1-a^x}}$$

$$\therefore \text{AM} \geq \text{GM} \Rightarrow \frac{a^{a^x} + a^{1-a^x}}{2} \geq \sqrt{a^{a^x} \cdot a^{1-a^x}}$$

$$\Rightarrow a^{a^x} + a^{1-a^x} \geq 2\sqrt{a}$$

$\therefore$ Minimum value of $f(x) = a^{a^x} + a^{1-a^x}$ is $2\sqrt{a}$.

Question 163

If $0 < \theta, \varphi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\varphi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$, then

[2021, 25 Feb. Shift-1]

Options:

A. $xy - z = (x + y)z$

B. $xy + yz + zx = z$

C. $xyz = 4$

D. $xy + z = (x + y)z$

Answer: D

Solution:

Given, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$y = \sum_{n=0}^{\infty} \sin^{2n}\varphi$

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$

$\Rightarrow x = 1 + \cos^2\theta + \cos^4\theta + \dots \infty$

$\therefore x = \frac{1}{1 - \cos^2\theta} = \operatorname{cosec}^2\theta$

$\Rightarrow y = 1 + \sin^2\varphi + \sin^4\varphi + \dots \infty$

$\therefore y = \frac{1}{1 - \sin^2\varphi} = \operatorname{cosec}^2\varphi \dots \dots \dots (i)$

$\Rightarrow z = 1 + \sin^2\varphi \cdot \cos^4\theta + \dots \infty$

$\therefore z = \frac{1}{1 - \sin^2\varphi} = \sec^2\varphi \dots \dots \dots (ii)$

$\Rightarrow z = 1 + \cos^2\theta \cdot \sin^2\varphi + \cos^4\theta \sin^4\varphi + \dots \infty$

$\therefore z = \frac{1}{1 - \cos^2\theta \sin^2\varphi} \dots (iii)$

From Eqs. (i), (ii) and (iii), we get

$$z = \frac{1}{1 - \left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right)} \left[\begin{array}{l} \because \cos^2\theta = 1 - \frac{1}{x} \\ \because \sin^2\varphi = 1 - \frac{1}{y} \end{array} \right]$$

$z = \frac{xy}{xy - (x-1)(y-1)}$

$z = \frac{xy}{xy - xy + x + y - 1}$

$\Rightarrow xz + yz - z = xy$

$\Rightarrow xy + z = (x + y)z$

Question164

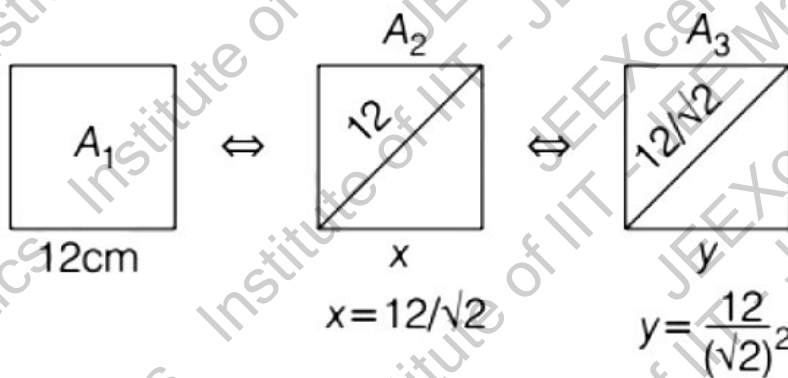
Let $A_1, A_2, A_3, \dots$ be squares, such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1} . If the length of A_1 is 12cm, then the smallest value of n for which area of A_n is less than one, is

[2021, 25 Feb. Shift-I]

Answer: 9

Solution:

According to the question, length of side of A_1 square is 12 cm.



$\therefore$ Side lengths are in GP.

$$\therefore T_n = \frac{12}{(\sqrt{2})^{n-1}}$$

(Side of n th square i.e. A_n)

$$\therefore \text{Area} = (\text{Side})^2 = \left(\frac{12}{(\sqrt{2})^{n-1}} \right)^2 = \frac{144}{2^{n-1}}$$

According to the question, the area of A_n square < 1

$$\Rightarrow 2^{n-1} > 144$$

Here, the smallest possible value of n is 9.

Question165

Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to.....

[2021, 16 March Shift-1]

Answer: 3

Solution:

Given, set $\{11, 8, 21, 16, 26, 32, 4\}$

By observation, we can say that

$$AP = \{11, 16, 21, 26, \dots\}$$

$$GP = \{4, 8, 16, 32, \dots\}$$

$$5m + 6 = 4 \cdot 2^{n-1}$$

$$5m + 6 = 2^{n+1}$$

So, $(2^{n+1} - 6)$ should be a multiple of 5. The unit digit of 2^k is 2, 4, 6, 8. So, when 6 is subtracted from 2^{n+1} , the possible unit digits will be 6, 8, 0, 2. Only 0 is divisible by 5. Hence, 2^{n+1} unit digit has to be 6.

$$2^{n+1} = 2^4, 2^8, 2^{12}, 2^{16} \dots$$

As, 2^{16} will not be a 4 digit number, so, common terms = $\{16, 256, 4096\}$

$\therefore$ Number of common terms = 3

Question 166

Let $\frac{1}{16}$, a and b be in G. P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in (a) P., where a, b > 0. Then, $72(a + b)$ is equal to

[2021, 16 March Shift-II]

Answer: 14

Solution:

Given, GP = $\frac{1}{16}$, a, b

$$\Rightarrow a^2 = \frac{b}{16}$$

and given, AP = $1/a, 1/b, 6$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + 6$$

$$\Rightarrow \frac{2}{16a^2} = \frac{1}{a} + 6$$

$$\Rightarrow \frac{1}{8a^2} = \frac{1+6a}{a}$$

$$\Rightarrow 1 = 8a(1+6a)$$

$$\Rightarrow 48a^2 + 8a - 1 = 0$$

$$\Rightarrow (4a+1)(12a-1) = 0$$

$$\Rightarrow a = -1/4 \text{ or } 1/12$$

As per the question, $a > 0$

$$\therefore a = 1/12$$

$$b = 16a^2 = 16 \cdot \frac{1}{144} = \frac{1}{9}$$

$$\therefore 72(a+b) = 72 \left(\frac{1}{12} + \frac{1}{9} \right)$$

$$= 6 + 8$$

$$= 14$$

Question167

If α, β are natural numbers, such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$, then the slope of the line passing through (α, β) and origin is
[2021,18 March Shift-1]

Options:

- A. 540
- B. 550
- C. 530
- D. 510

Answer: B

Solution:

Given,

$$100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$$

$$\Rightarrow 100^\alpha - 199\beta = \sum_{x=0}^{99} (100-x)(100+x)$$

$$= \sum_{x=0}^{99} (100^2 - x^2)$$

$$= \sum_{x=0}^{99} (100)^2 - \sum_{x=0}^{99} (x)^2$$

$$= (100)^3 - \frac{99 \times 100 \times 199}{6}$$

$$\Rightarrow (100)^\alpha - (199)\beta = (100)^3 - (199)(1650)$$

On comparing, we get $\alpha = 3, \beta = 1650$

Then, the slope of the line passing through (α, β) and origin is

$$= \frac{\beta - 0}{\alpha - 0} = \frac{\beta}{\alpha} = \frac{1650}{3} = 550$$

Question168

$$\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1}$$

is equal to

[2021, 18 March Shift-1]

Options:

- A. $\frac{101}{404}$
- B. $\frac{25}{101}$

C. $\frac{101}{408}$

D. $\frac{99}{400}$

Answer: B

Solution:

$$\begin{aligned} & \frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \dots + \frac{1}{(201)^2-1} \\ &= \sum_{r=1}^{100} \frac{1}{(2r+1)^2-1} \\ &= \sum_{r=1}^{100} \frac{1}{4r^2+4r+1-1} \\ &= \sum_{r=1}^{100} \frac{1}{2r(2r+2)} = \sum_{r=1}^{100} \frac{1}{4r(r+1)} \\ &= \frac{1}{4} \sum_{r=1}^{100} \left(\frac{1}{r} - \frac{1}{r+1} \right) \\ &= \frac{1}{4} \left[\left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{100} - \frac{1}{101} \right) \right] \\ &= \frac{1}{4} \left[1 - \frac{1}{101} \right] = \frac{1}{4} \times \frac{100}{101} \\ &= \frac{25}{101} \end{aligned}$$

Question 169

Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to
[2021, 18 March Shift-11]

Options:

A. 1000

B. 7000

C. 5000

D. 3000

Answer: D

Solution:

$$\begin{aligned} & \text{Given, } S_1 = S_{2n} \text{ and } S_2 = S_{4n} \\ & \text{and } S_2 - S_1 = 1000 \Rightarrow S_{4n} - S_{2n} = 1000 \\ & \Rightarrow \frac{4n}{2} [2a + (4n-1)d] - \frac{2n}{2} [2a + (2n-1)d] = 1000 \\ & \Rightarrow 2n[2a + (4n-1)d] - n[2a + (2n-1)d] = 1000 \\ & \Rightarrow 2an + n(8n-2-2n+1)d = 1000 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow 2an + n(6n - 1)d = 1000 \\
 &\Rightarrow n[2a + (6n - 1)d] = 1000 \\
 &\Rightarrow S_{6n} = \frac{6n}{2}[2a + (6n - 1)d] \\
 &= \frac{6}{2} \times (1000) \\
 &= 6 \times 500 = 3000
 \end{aligned}$$

Question 170

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to
[2021, 27 July Shift-1]

Answer: 3

Solution:

$$\begin{aligned}
 &\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right) \rightarrow \text{AP} \\
 &\Rightarrow 2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right) \\
 &\Rightarrow \log_3(2^x - 5)^2 = \log_3\left[2 \cdot \left(2^x - \frac{7}{2}\right)\right] \\
 &\Rightarrow (2^x - 5)^2 = 2 \cdot 2^x - 7 \\
 &\Rightarrow (2^x)^2 + 25 - 10 \cdot 2^x - 2 \cdot 2^x + 7 = 0 \\
 &\Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 = 0 \\
 &\Rightarrow (2^x - 4)(2^x - 8) = 0 \\
 &\Rightarrow 2^x = 4 \text{ or } 8 \Rightarrow x = 2 \text{ or } 3 \\
 &\text{If } x = 2, \text{ then } \log_3(2^x - 5) = \log_3(2^2 - 5) \\
 &\text{Here, argument is negative, so, } x \neq 2. \\
 &\text{Hence, } x = 3
 \end{aligned}$$

Question 171

If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to
[2021, 27 July Shift-II]

Options:

- A. 4
- B. 3
- C. 0

D. 1

Answer: C

Solution:

If $\tan\left(\frac{\pi}{9}\right)$, x , $\tan\left(\frac{7\pi}{18}\right)$ are in AP.

$$\text{So, } x = \frac{1}{2} \left[\tan\frac{\pi}{9} + \tan\left(\frac{7\pi}{18}\right) \right]$$

($\because$ if a, b, c are in AP, so, $b = \frac{a+c}{2}$)

And $\tan\left(\frac{\pi}{9}\right)$, y , $\tan\left(\frac{5\pi}{18}\right)$ are in AP.

$$\text{Now, } x - 2y = \frac{1}{2} \left[\tan\frac{\pi}{9} + \tan\frac{7\pi}{18} \right] - \left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18} \right)$$

$$\Rightarrow |x - 2y| = \left| \frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18} \right| \left\{ \begin{array}{l} \because \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9} \\ \text{and } \tan\frac{7\pi}{18} = \cot\frac{\pi}{9} \end{array} \right\}$$

$$= \left| \cot\frac{2\pi}{9} - \cot\frac{2\pi}{9} \right| = 0$$

$$\left[\because \cot 2A = \frac{2\cot^2 A - 1}{2\cot A} \right]$$

Question 172

Let S_n be the sum of the first n terms of an arithmetic progression, If $S_{3n} = 3S_{2n}$,

then the value of $\frac{S_{4n}}{S_{2n}}$ is

[2021, 25 July Shift-1]

Options:

A. 6

B. 4

C. 2

D. 8

Answer: A

Solution:

$$\text{Let } S_n = An^2 + Bn = n(An + B) \quad S_{3n} = 3S_{2n}$$

$$\Rightarrow 3n[A(3n) + B] = 3 \cdot 2n \cdot [A(2n) + B]$$

$$\Rightarrow 3An + B = 4An + 2B$$

$$\Rightarrow An + B = 0$$

$$\therefore \frac{S_{4n}}{S_{2n}} = \frac{4n[A(4n) + B]}{2n[A(2n) + B]}$$

$$= 2 \left(\frac{4An - An}{2An - An} \right)$$

$$= 2 \times 3 = 6$$

Question173

Let S_n denote the sum of first n terms of an arithmetic progression. If

$S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to

[2021,22 July Shift-II]

Options:

A. 1862

B. 1842

C. 1852

D. 1872

Answer: A

Solution:

$$S_n = An^2 + Bn$$

$$S_{10} = 100A + 10B = 530$$

$$S_5 = 25A + 5B = 140 \text{ Solving both equations, we get } B = 3 \text{ and}$$

$$A = 5$$

$$\therefore S_n = 5n^2 + 3n$$

$$\therefore S_{20} - S_6 = 5(20^2 - 6^2) + 3(20 - 6)$$

$$= 5 \cdot 26 \cdot 14 + 3 \cdot 14$$

$$= 14(130 + 3) = 14 \times 133 = 1862$$

Question174

The sum of all the elements in the set $\{n \in \{1, 2, \dots, 100\} : \text{HCF of } n \text{ and } 2040 \text{ is } 1\}$ is equal to

[2021, 22 July Shift-II]

Answer: 1251

Solution:

$$n \in \{1, 2, 3, \dots, 100\}$$

$$2040 = 2^3 \times 3 \times 5 \times 17$$

If HCF of n and 2040 is 1, n should not be a multiple of 2, 3, 5, 17.

$$n \in \{1, 7, 11, 13, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97\}$$

$$\sum_n = |1251|$$

Question 175

If sum of the first 21 terms of the series $\log_{9^{1/2}}x + \log_{9^{1/3}}x + \log_{9^{1/4}}x + \dots$ where $x > 0$ is 504, then x is equal to
[2021, 20 July Shift-II]

Options:

- A. 243
- B. 9
- C. 7
- D. 81

Answer: D

Solution:

$$\text{Let } S = \log_{9^{1/2}}x + \log_{9^{1/3}}x + \dots$$

$$\text{Using property, } \log_{ab}x = \frac{1}{b} \log_a x$$

$$S = 2\log_9x + 3\log_9x + \dots + 22\log_9x \\ = \log_9x(2 + 3 + 4 + \dots + 22)$$

$$= \log_9x \left[\frac{21}{2}(4 + 20) \right] = \log_9x(21 \times 12)$$

$$\therefore S = 252\log_9x$$

$$\text{Given, } S = 504, \text{ then} \\ 252\log_9x = 504$$

$$\Rightarrow \log_9x = 2$$

$$\Rightarrow x = (9)^2 = 81$$

Question 176

For $k \in \mathbb{N}$, let

$$\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$$

where $\alpha > 0$. Then the value of $100 \left(\frac{A_{14} + A_{15}}{A_{13}} \right)^2$

[2021, 20 July Shift-II]

Answer: 9

Solution:

Given,

$$\frac{1}{\alpha(\alpha+1)\dots+(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}, \alpha > 0$$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)\dots(\alpha+20)} = \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1} + \dots + \frac{A_{20}}{\alpha+20}$$

$$A_{14} = \frac{1}{(-14)(-13)\dots(-1)(1)\dots(6)} = \frac{1}{14!6!}$$

$$A_{15} = \frac{1}{(-15)(-14)\dots(-1)(1)\dots(5)} = \frac{-1}{15!5!}$$

$$A_{13} = \frac{1}{(-13)\dots(-1)(1)\dots(7)} = \frac{-1}{13!7!}$$

$$\frac{A_{14}}{A_{13}} = \frac{-13!7!}{14!6!} = \frac{-7}{14} = \frac{-1}{2}$$

$$\Rightarrow \frac{A_{15}}{A_{13}} = \frac{13!7!}{15!5!} = \frac{42}{15 \times 14} = \frac{1}{5}$$

$$\therefore 100 \left(\frac{A_{14}}{A_{13}} + \frac{A_{15}}{A_{13}} \right)^2 = 100 \left(-\frac{1}{2} + \frac{1}{5} \right)^2$$

$$= 100 \left(\frac{-3}{10} \right)^2 = 9$$

Question 177

If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \text{ upto } \infty \right)^{\log_{(0.25)} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{ upto } \infty \right)}$$

is 1, then 1^2 is equal to
[2021, 25 July Shift-I]

Answer: 3

Solution:

$$\text{Let } 1 = \alpha^\beta$$

$$\alpha = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \infty \dots \dots (i)$$

$$\frac{\alpha}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \dots \dots \dots (ii)$$

Subtracting Eq. (ii) from Eq. (i),

$$\frac{2\alpha}{3} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$= \frac{4}{3} \left(\frac{1}{1 - \frac{1}{3}} \right) = 2$$

$$\therefore \alpha = 3$$

$$\beta = \log_{0.25} \left(\frac{1}{3} + \frac{1}{3^2} + \dots \right)$$

$$= \log_{0.25} \left(\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right) = \log \frac{1}{4} \frac{1}{2} = \frac{1}{2}$$

$$\therefore L = 3^{1/2}$$

$$\Rightarrow L^2 = 3$$

Question 178

Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such

that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to
[2021, 20 July Shift-II]

Answer: 7

Solution:

$$\text{Let } \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = x \text{ i.e. } \sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$$

$$\text{Given, } a_{n+2} = 2a_{n+1} + a_n$$

Divide the whole by 8^n ,

$$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$$

$$\Rightarrow 8^2 \cdot \frac{a_{n+2}}{8^{n+2}} = 8 \cdot 2 \frac{a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n} \Rightarrow$$

$$64 \left(\frac{a_{n+2}}{8^{n+2}} \right) = 16 \left(\frac{a_{n+1}}{8^{n+1}} \right) + \frac{a_n}{8^n}$$

Now, take the summation,

$$64 \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = 16 \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty} \frac{a_n}{8^n} \quad \dots\dots (i)$$

$$\therefore \sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$$

$$\text{i.e. } \frac{a_1}{8} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + \frac{a_4}{8^4} + \dots = x$$

$$\Rightarrow \frac{a_3}{8^3} + \frac{a_4}{8^4} + \dots = x - \frac{a_1}{8} - \frac{a_2}{8^2}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = x - \frac{a_1}{8} - \frac{a_2}{8^2} \quad \dots\dots (ii)$$

$$\text{Again, } \frac{a_2}{8^2} + \frac{a_3}{8^3} + \dots = x - \frac{a_1}{8}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} = x - \frac{a_1}{8} \quad \dots\dots (iii)$$

From Eqs. (i), (ii) and (iii),

$$64 \left(x - \frac{a_1}{8} - \frac{a_2}{64} \right) = 16 \left(x - \frac{a_1}{8} \right) + x$$

$$\text{Use } a_1 = 1 = a_2$$

$$64 \left(x - \frac{1}{8} - \frac{1}{64} \right) = 16 \left(x - \frac{1}{8} \right) + x$$

$$\begin{aligned} \Rightarrow 64x - 9 &= 2(8x - 1) + x \\ \Rightarrow 64x - 16x - x &= 9 - 2 \Rightarrow 47x = 7 \\ \therefore 47 \sum \frac{a_n}{2^{3n}} &= 7 \end{aligned}$$

Question 179

Let $a_1, a_2, a_3, \dots$ be an (a)P.

If $\frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to

[2021, 31 Aug. Shift-II]

Options:

- A. $\frac{19}{21}$
- B. $\frac{100}{121}$
- C. $\frac{21}{19}$
- D. $\frac{121}{100}$

Answer: C

Solution:

$$\begin{aligned} \frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} &= \frac{100}{p^2} \\ \Rightarrow \frac{S_{10}}{S_p} &= \frac{100}{p^2} \Rightarrow S_p = \frac{S_{10} \cdot p^2}{100} \\ \frac{a_{11}}{a_{10}} &= \frac{S_{11} - S_{10}}{S_{10} - S_9} = \frac{S_{10} \cdot \frac{121}{100} - S_{10}}{S_{10} - S_{10} \cdot \frac{81}{100}} \\ &= \frac{\frac{121}{100} - 1}{1 - \frac{81}{100}} = \frac{21}{19} \end{aligned}$$

Question 180

The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is
[2021, 31 Aug. Shift-II]

Answer: 5143

Solution:

Total 4-digit number

$$= 9 \times 10 \times 10 \times 10 = 9000$$

4 -digit number divisible by 7

1001, 1008, ..., 9996

Number of 4-digit number divisible by 7

$$= \frac{9996 - 1001}{7} + 1 = 1286$$

4 -digit number divisible by 3

1002, 1005, ..., 9999

Number of 4-digit number divisible by 3

$$= \frac{9999 - 1002}{3} + 1 = 3000$$

4 digit number divisible by 21

1008, 1031, ..., 9996

Number of 4 - digit number divisible by 21

$$= \frac{9996 - 1008}{21} + 1 = 429$$

$\therefore$ Number of 4-digit numbers neither divisible by 7 nor 3

$$= 9000 - 1286 - 3000 + 429 = 5143$$

Question181

Three numbers are in an increasing geometric progression with common ratio r . If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d . If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to

[2021, 31 Aug. Shift-I]

Options:

A. $7 - 7\sqrt{3}$

B. $7 + \sqrt{3}$

C. $7 - \sqrt{3}$

D. $7 + 3\sqrt{3}$

Answer: B

Solution:

Let three numbers be $\frac{a}{r}$, a , ar .

According to the question, $\frac{a}{r}$, $2a$, $ar \rightarrow$ AP

$$4a = ar + \frac{a}{r} \Rightarrow r + \frac{1}{r} = 4$$

$$\Rightarrow r^2 - 4r + 1 = 0 \Rightarrow r = 2 \pm \sqrt{3}$$

$$T_4 \text{ of GP} = 3r^2$$

$$3r^2 = ar^2$$

$$a = 3$$

$$r = 2 + \sqrt{3}$$

$$d = 2a - \frac{a}{r} = 3\sqrt{3}$$

$$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$$

$$= 7 + 4\sqrt{3} - 3\sqrt{3} = 7 + \sqrt{3}$$

Question 182

If $0 < x < 1$, then

$\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to

[2021, 27 Aug. Shift-1]

Options:

A. $x \left(\frac{1+x}{1-x} \right) + \log_e(1-x)$

B. $x \left(\frac{1-x}{1+x} \right) + \log_e(1-x)$

C. $\frac{1-x}{1+x} + \log_e(1-x)$

D. $\frac{1+x}{1-x} + \log_e(1-x)$

Answer: A

Solution:

We have,

$$\begin{aligned} & \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots \\ &= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \dots \\ &= 2(x^2 + x^3 + x^4 + \dots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right) \\ &= 2 \cdot \frac{x^2}{1-x} - [-\log_e(1-x) - x] \end{aligned}$$

[using sum of infinite GP = $\frac{a}{1-r}$ and logarithmic series]

$$\begin{aligned} &= \frac{2x^2}{1-x} + x + \log_e(1-x) \\ &= \frac{2x^2 + x - x^2}{1-x} + \log_e(1-x) \\ &= \frac{x^2 + x}{1-x} + \log_e(1-x) \\ &= x \left(\frac{1+x}{1-x} \right) + \log_e(1-x) \end{aligned}$$

Question 183

If $x, y \in \mathbb{R}, x > 0$

$$y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \dots$$

upto ∞ terms and

$\frac{2+4+6+\dots+2y}{3+6+9+\dots+3y} = \frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to

[2021, 27 Aug. Shift-1]

Options:

A. $(10^6, 6)$

B. $(10^4, 6)$

C. $(10^2, 3)$

D. $(10^6, 9)$

Answer: D

Solution:

Given, $\frac{2+4+6+\dots+2y}{3+6+9+\dots+3y} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2(1+2+3+\dots+y)}{3(1+2+3+\dots+y)} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2}{3} = \frac{4}{\log_{10}x} \Rightarrow \log_{10}x = 6$

$\Rightarrow x = 10^6$

Now, $y = \log_{10}x + \log_{10}x^{\frac{1}{3}} + \log_{10}x^{\frac{1}{9}} + \dots$ upto ∞ terms.

$= \log_{10} \left(x \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{9}} \dots \infty \text{ terms} \right)$

$= \log_{10} x^{1 + \frac{1}{3} + \frac{1}{9} + \dots \infty \text{ terms}}$

$= \log_{10} x^{\frac{1}{1 - \frac{1}{3}}} = \log_{10} x^{3/2}$

$= \log_{10} (10^6)^{3/2} [\because x = 10^6]$

$\Rightarrow y = 6 \times \frac{3}{2} = 9$

$\therefore x = 10^6, y = 9$

$(x, y) = (10^6, 9)$

Question 184

If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots$ then the value of e^{1+y} at $x = \frac{1}{2}$ is

[2021, 27 Aug. Shift-II]

Options:

A. $\frac{1}{2}e^2$

B. $2e$

C. $\frac{1}{2}\sqrt{e}$

D. $2e^2$

Answer: A

Solution:

$$y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots$$

$$\Rightarrow y = \left(1 - \frac{1}{2}\right)x^2 + \left(1 - \frac{1}{3}\right)x^3 + \left(1 - \frac{1}{4}\right)x^4 + \dots$$

$$= (x^2 + x^3 + x^4 + \dots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$$

$$= \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$$

$$\therefore y = \frac{x}{1-x} + \ln(1-x)$$

Put $x = \frac{1}{2}$, we get

$$y = 1 - \ln 2$$

Then, $e^{1+y} = e^{1+1-\ln 2} = e^{2-\ln 2}$

$$= e^2 \cdot e^{-\ln 2}$$

$$= \frac{1}{2}e^2$$

Question 185

If the sum of an infinite GP $a, ar, ar^2, ar^3, \dots$ is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, \dots$ is
[2021, 26 Aug. Shift-1]

Options:

A. $5/2$

B. $1/2$

C. $25/2$

D. $9/2$

Answer: B

Solution:

We have, sum of infinite GP $a, ar, ar^2, \dots$ is

$$S_{\infty} = \frac{a}{1-r} = 15 \quad \dots \dots \dots (i)$$

and sum of infinite GP $a^2, a^2r^2, a^2r^4, \dots$ is

$$cS_{\infty}' = \frac{a^2}{1-r^2} = 150$$

$$\Rightarrow \left(\frac{a}{1-r}\right) \left(\frac{a}{1+r}\right) = 150 \quad \dots \dots \dots (ii)$$

Divide Eq. (ii) by Eq. (i)

$$\frac{a}{1+r} = 10 \dots\dots\dots (iii)$$

Divide Eq. (iii) by Eq. (i)

$$\frac{1-r}{1+r} = \frac{10}{15} = \frac{2}{3}$$

$$\Rightarrow 3 - 3r = 2 + 2r$$

$$\Rightarrow 1 = 5r \Rightarrow r = \frac{1}{5}$$

Now, putting $r = \frac{1}{5}$ in Eq. (iii), we get

$$\frac{a}{1 + \frac{1}{5}} = 10$$

$$\Rightarrow \frac{5a}{6} = 10 \Rightarrow a = 12$$

Now, sum of $ar^2, ar^4, ar^6, \dots, \infty$

$$S_{\infty}'' = \frac{ar^2}{1-r^2} = \frac{12 \cdot \left(\frac{1}{25}\right)}{\frac{24}{25}} = \frac{1}{2}$$

Question 186

Let $a_1, a_2, \dots, a_{10}$ be an AP with common difference -3 and $b_1, b_2, \dots, b_{10}$ be a GP with common ratio 2 .

Let $c_k = a_k + b_k, k = 1, 2, \dots, 10$. If $c_2 = 12$ and

$c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to

[2021, 26 Aug. Shift-II]

Answer: 2021

Solution:

$a_1, a_2, a_3, \dots, a_{10}$ are in AP common

difference $= -3$

$b_1, b_2, b_3, \dots, b_{10}$ are in GP common ratio $= 2$

Since, $c_k = a_k + b_k, k = 1, 2, 3, \dots, 10$

$$\therefore c_2 = a_2 + b_2 = 12$$

$$c_3 = a_3 + b_3 = 13$$

$$\text{Now, } c_3 - c_2 = 1$$

$$\Rightarrow (a_3 - a_2) + (b_3 - b_2) = 1$$

$$\Rightarrow -3 + (2b_2 - b_2) = 1$$

$$\Rightarrow b_2 = 4$$

$$\therefore a_2 = 8$$

So, AP is $11, 8, 5, \dots$

and GP is $2, 4, 8, \dots$

$$\text{Now, } \sum_{k=1}^{10} c_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \left(\frac{10}{2}\right)[22 + 9(-3)] + 2\left(\frac{2^{10}-1}{2-1}\right)$$

$$= 5(22 - 27) + 2(1023)$$

$$= 2046 - 25 = 2021$$

Question 187

The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots$ is
[2021, 31 Aug. Shift-1]

Options:

- A. 1
- B. 120/121
- C. 99/100
- D. 143/144

Answer: B

Solution:

$$\begin{aligned} & \frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots \\ &= \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \dots \\ &= \left(\frac{1}{1^2} - \frac{1}{2^2} \right) + \left(\frac{1}{2^2} - \frac{1}{3^2} \right) + \left(\frac{1}{3^2} - \frac{1}{4^2} \right) + \dots + \left(\frac{1}{10^2} - \frac{1}{11^2} \right) \\ &= \frac{1}{1^2} - \frac{1}{11^2} = 1 - \frac{1}{121} = \frac{120}{121} \end{aligned}$$

Question 188

If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$, then 160 S is equal to
[2021, 31 Aug. Shift-II]

Answer: 305

Solution:

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots + \infty \quad \dots\dots (i)$$

$$\frac{S}{5} = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \frac{19}{5^5} + \dots + \infty \quad \dots\dots (ii)$$

Subtracting Eq. (ii) from Eq. (i),

$$c \frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \dots + \infty$$

$$\frac{4S}{5} - \frac{7}{5} = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \dots + \infty = K \quad \dots\dots (iii)$$

$$\frac{K}{5} = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + \frac{8}{5^6} + \dots + \infty \quad \dots\dots (iv)$$

Subtracting Eq. (iv) from Eq (iii),

$$\frac{4K}{5} = \frac{2}{5^2} + \frac{2}{5^3} + \frac{2}{5^4} + \frac{2}{5^5} + \dots \infty$$

$$\frac{4K}{5} = \frac{2}{25} \left(\frac{1}{1 - 1/5} \right) = \frac{1}{10} \Rightarrow K = \frac{1}{8}$$

From Eq. (iii),

$$\frac{4S}{5} - \frac{7}{5} = \frac{1}{8} \Rightarrow S = \frac{61}{32}$$

$$\text{Now, } 106S = 160 \times \frac{61}{32} = 305$$

Question 189

Let $a_1, a_2, \dots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to
[2021, 01 Sep. Shift-II]

Options:

A. 57

B. 72

C. 48

D. 36

Answer: B

Solution:

Let d be the common difference of an AP

$$a_1, a_2, \dots, a_{21} \text{ and } \sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$$

$$\Rightarrow \sum_{n=1}^{20} \frac{1}{a_n(a_n + d)} = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d} \sum_{n=1}^{20} \left(\frac{1}{a_n} - \frac{1}{a_n + d} \right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d} \left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \dots + \frac{1}{a_{20}} - \frac{1}{a_{21}} \right] = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d} \left(\frac{1}{a_1} - \frac{1}{a_{21}} \right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d} \left(\frac{a_{21} - a_1}{a_1 a_{21}} \right) = \frac{4}{9} \Rightarrow a_1 a_{21} = 45$$

$$\Rightarrow a_1(a_1 + 20d) = 45 \dots \dots (i)$$

$$\Rightarrow \frac{21}{2}(2a_1 + 20d) = 189$$

Also sum of first 21 terms = 189

$$\Rightarrow \frac{21}{2}(2a_1 + 20d) = 189$$

$$\Rightarrow a_1 + 10d = 9 \dots \dots (ii)$$

By Eqs. (i) and (ii), we get $a_1 = 3, d = 3/5$

$$\text{or } a_1 = 15, d = -\frac{3}{5}$$

$$\text{So, } a_6 a_{16} = (a_1 + 5d)(a_1 + 15d) = 72$$

Question 190

Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \dots + (n-1) \cdot 1$, $n \geq 4$.

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to
[2021, 01 Sep. Shift-II]

Options:

A. $\frac{e-1}{3}$

B. $\frac{e-2}{6}$

C. $\frac{e}{3}$

D. $\frac{e}{6}$

Answer: A

Solution:

$$S_n = 1 \cdot (n-1) + 2(n-2) + 3(n-3) + \dots + (n-1) \cdot 1, n \geq 4$$

$$= \sum_{r=1}^{n-1} r(n-r) = \frac{n(n^2-1)}{6}$$

$$= \frac{n(n-1)(n+1)}{6}$$

$$\frac{2S_n}{n!} = \frac{(n+1)}{3(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$$

$$= \sum_{n=4}^{\infty} \frac{n-2}{3(n-2)!} = \frac{1}{3} \sum_{n=4}^{\infty} \frac{1}{(n-3)!}$$

$$= \frac{1}{3} \left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right) = \frac{e-1}{3}$$

Question 191

The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is _____.

[NA Jan. 9, 2020 (II)]

Answer: 14

Solution:

First common term of both the series is 23 and common difference is $7 \times 4 = 28$

$\therefore$ Last term $\leq 407 \Rightarrow 23 + (n-1) \times 28 \leq 407$

$$\Rightarrow (n-1) \times 28 \leq 384$$

$$\Rightarrow n \leq \frac{384}{28} + 1$$

$$\Rightarrow n \leq 14.71$$

Hence, $n = 14$

Question 192

If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is:
[Jan. 8, 2020 (II)]

Options:

A. 50

B. $50\frac{1}{4}$

C. 100

D. $100\frac{1}{2}$

Answer: D

Solution:

$$T_{10} = \frac{1}{20} = a + 9d \dots (i)$$

$$T_{20} = \frac{1}{10} = a + 19d \dots (ii)$$

Solving equations (i) and (ii), we get

$$a = \frac{1}{200}, d = \frac{1}{200}$$

$$\Rightarrow S_{200} = \frac{200}{2} \left[\frac{2}{200} + \frac{199}{200} \right] = \frac{201}{2} = 100\frac{1}{2}$$

Question 193

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that for all $x \in \mathbb{R}$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is:
[Jan. 8, 2020 (I)]

Options:

A. 2

B. 3

C. 0

D. 4

Answer: B

Solution:

If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P., then

$$f(x) = \left(\frac{2^{1-x} + 2^{1+x} + 3^x + 3^{-x}}{2} \right)$$

$$2f(x) = 2 \left(2^x + \frac{1}{2^x} \right) + \left(3^x + \frac{1}{3^x} \right)$$

Using AM $\geq$ GM

$$f(x) \geq 3$$

Question 194

Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

[Jan. 7, 2020 (I)]

Options:

A. 27

B. 7

C. $\frac{21}{2}$

D. 16

Answer: D

Solution:

Let 5 terms of A.P. be

$$a - 2d, a - d, a, a + d, a + 2d$$

$$\text{Sum} = 25 \Rightarrow 5a = 25 \Rightarrow a = 5$$

$$\text{Product} = 2520$$

$$(5 - 2d)(5 - d)5(5 + d)(5 + 2d) = 2520$$

$$\Rightarrow (25 - 4d^2)(25 - d^2) = 504$$

$$\Rightarrow 625 - 100d^2 - 25d^2 + 4d^4 = 504$$

$$\Rightarrow 4d^4 - 125d^2 + 625 - 504 = 0$$

$$\Rightarrow 4d^4 - 125d^2 + 121 = 0$$

$$\Rightarrow 4d^4 - 121d^2 - 4d^2 + 121 = 0$$

$$\Rightarrow (d^2 - 1)(4d^2 - 121) = 0$$

$$\Rightarrow d = \pm 1, d = \pm \frac{11}{2}$$

$$d = \pm 1 \text{ and } d = -\frac{11}{2}, \text{ does not give } -\frac{1}{2} \text{ as a term}$$

$$\therefore d = \frac{11}{2}$$

$$\therefore \text{Largest term} = 5 + 2d = 5 + 11 = 16$$

Question 195

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \dots$ to ∞ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. $2^{\frac{1}{2}}$

B. $2^{\frac{1}{4}}$

C. 1

D. 2

Answer: A

Solution:

$$2^{\frac{1}{4}} + \frac{2}{16} + \frac{3}{48} + \dots \infty$$
$$= 2^{\frac{1}{4}} + \frac{1}{8} + \frac{1}{16} + \dots \infty = \sqrt{2}$$

Question 196

Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to :

[Jan. 9, 2020 (II)]

Options:

A. 300

B. 225

C. 175

D. 150

Answer: D

Solution:

Let G.P. be $a, ar, ar^2, \dots$

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \dots + a_{201} = 200$$

$$\Rightarrow \frac{ar^2(r^{200} - 1)}{r^2 - 1} = 200$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots + a_{200} = 100$$

$$\Rightarrow \frac{ar(r^{200} - 1)}{r^2 - 1} = 100$$

From equations (i) and (ii), $r = 2$ and

$$a_2 + a_3 + \dots + a_{200} + a_{201} = 300$$

$$\Rightarrow r(a_1 + \dots + a_{200}) = 300$$

$$\Rightarrow \sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

Question 197

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n} \theta$, for $0 < \theta < \frac{\pi}{4}$, then :

[Jan. 9, 2020 (II)]

Options:

A. $x(1+y) = 1$

B. $y(1-x) = 1$

C. $y(1+x) = 1$

D. $x(1-y) = 1$

Answer: B

Solution:

$$y = 1 + \cos^2 \theta + \cos^4 \theta + \dots$$

$$\Rightarrow y = \frac{1}{1 - \cos^2 \theta} \Rightarrow \frac{1}{y} = \sin^2 \theta$$

$$x = 1 - \tan^2 \theta + \tan^4 \theta + \dots$$

$$x = \frac{1}{1 - (-\tan^2 \theta)} = \frac{1}{\sec^2 \theta} \Rightarrow x = \cos^2 \theta$$

$$y = \frac{1}{\sin^2 \theta} \Rightarrow y = \frac{1}{1-x}$$

$$\therefore y(1-x) = 1$$

Question 198

The greatest positive integer k , for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \dots + 49^2 + 49 + 1$, is:

[Jan. 7, 2020 (I)]

Options:

A. 32

B. 63

C. 60

D. 65

Answer: B

Solution:

$$\frac{(49)^{126} - 1}{48} = \frac{((49)^{63} + 1)(49^{63} - 1)}{48} \left[\because S_n = \frac{a(r^n - 1)}{r - 1} \right]$$
$$\therefore K = 63$$

Question 199

Let $a_1, a_2, a_3, \dots$ be a G. P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to:
[Jan. 7, 2020 (II)]

Options:

A. -513

B. -171

C. 171

D. $\frac{511}{3}$

Answer: B

Solution:

$$\text{Since, } a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4 \dots (i)$$

$$a_3 + a_4 = 16 \Rightarrow a_1 r^2 + a_1 r^3 = 16 \dots (ii)$$

$$\text{From eqn. (i), } a_1 = \frac{4}{1+r} \text{ and substituting the value of } a_1 \text{ in eqn (ii),}$$

$$\left(\frac{4}{1+r} \right) r^2 + \left(\frac{4}{1+r} \right) r^3 = 16$$

$$\Rightarrow 4r^2(1+r) = 16(1+r)$$

$$\Rightarrow r^2 = 4 \therefore r = \pm 2$$

$$r = 2, a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$$

$$r = -2, a_1(1-2) = 4 \Rightarrow a_1 = -4$$

$$\sum_{i=1}^9 a_i = \frac{a_1(r^9 - 1)}{r - 1} = \frac{(-4)((-2)^9 - 1)}{-2 - 1}$$

$$= \frac{4}{3}(-513) = 4\lambda \Rightarrow \lambda = -171$$

Question 200

The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$ is:
[Jan. 7, 2020 (II)]

Options:

- A. 210
- B. 330
- C. 120
- D. 420

Answer: B

Solution:

The given series is in G.P. then $S_n = \frac{a(1-r^n)}{1-r}$

$$\frac{(1+x)^{10} \left[1 - \left(\frac{x}{1+x} \right)^{11} \right]}{\left(1 - \frac{x}{1+x} \right)}$$

$$\Rightarrow \frac{(1+x)^{10} [(1+x)^{11} - x^{11}]}{(1+x)^{11} \times \frac{1}{(1+x)}} = (1+x)^{11} - x^{11}$$

$$\therefore \text{Coefficient of } x^7 \text{ is } {}^{11}C_7 = {}^{11}C_{11-7} = {}^{11}C_4 = 330$$

Question 201

The sum, $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$ is equal to _____.
[Jan. 8, 2020 (II)]

Answer: 504

Solution:

$$\begin{aligned} & \left[\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4} \right] \frac{1}{4} \left[\sum_{n=1}^7 (2n^3 + 3n^2 + n) \right] \\ &= \frac{1}{4} \left[2 \left(\frac{7 \cdot 8}{2} \right)^2 + 3 \left(\frac{7 \cdot 8 \cdot 15}{6} \right) + \frac{7 \cdot 8}{2} \right] \\ &\Rightarrow \frac{1}{4} [2 \times 49 \times 16 + 28 \times 15 + 28] \\ &= \frac{1}{4} [1568 + 420 + 28] = 504 \end{aligned}$$

Question 202

The sum $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$ is _____.
[Jan. 8, 2020 (I)]

Answer: 1540

Solution:

Given series can be written as

$$\begin{aligned}\sum_{k=1}^{20} \frac{k(k+1)}{2} &= \frac{1}{2} \sum_{k=1}^{20} (k^2 + k) \\&= \frac{1}{2} \left[\frac{20(21)(41)}{6} + \frac{20(21)}{2} \right] \\&= \frac{1}{2} \left[\frac{420 \times 41}{6} + \frac{20 \times 21}{2} \right] = \frac{1}{2} [2870 + 210] = 1540\end{aligned}$$

Question 203

If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots$ is (102)m, then m is equal to:
[Jan. 7, 2020 (II)]

Options:

- A. 20
- B. 25
- C. 5
- D. 10

Answer: A

Solution:

$$S = 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots \text{40 terms}$$

$$S = 7 + 17 + 27 + 37 + 47 + \dots \text{20 terms}$$

$$S_{40} = \frac{20}{2} [2 \times 7 + (19)10] = 10[14 + 190]$$

$$= 10[2040] = (102)(20)$$

$$\Rightarrow m = 20$$

Question 204

If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, \dots$ is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \dots, a_{23}$ is ka_1 , where k is equal to :
[Sep. 02, 2020 (II)]

Options:

- A. $-\frac{121}{10}$

B. $\frac{121}{10}$

C. $\frac{72}{5}$

D. $-\frac{72}{5}$

Answer: D

Solution:

Let common difference be d

$$\because S_{11} = 0 \therefore \frac{11}{2} \{2a_1 + 10 \cdot d\} = 0$$

$$\Rightarrow a_1 + 5d = 0 \Rightarrow d = -\frac{a_1}{5} \dots (i)$$

$$\text{Now, } S = a_1 + a_3 + a_5 + \dots + a_{23}$$

$$= a_1 + (a_1 + 2d) + (a_1 + 4d) + \dots + (a_1 + 22d)$$

$$= 12a_1 + 2d \frac{11 \times 12}{2}$$

$$= 12 \left[a_1 + 11 \cdot \left(-\frac{a_1}{5} \right) \right] \quad (\text{From (i)})$$

$$= 12 \times \left(-\frac{6}{5} \right) a_1 = -\frac{72}{5} a_1$$

Question 205

If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

[Sep. 03, 2020 (I)]

Options:

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{7}$

Answer: A

Solution:

$$\text{Given } a = 3 \text{ and } S_{25} = S_{40} - S_{25}$$

$$\Rightarrow 2S_{25} = S_{40}$$

$$\Rightarrow 2 \times \frac{25}{2} [6 + 24d] = \frac{40}{2} [6 + 39d]$$

$$\Rightarrow 25[6 + 24d] = 20[6 + 39d]$$

$$\Rightarrow 5(2 + 8d) = 4(2 + 13d)$$

$$\Rightarrow 10 + 40d = 8 + 52d$$

$$\Rightarrow d = \frac{1}{6}$$

Question 206

In the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ upto n^{th} term is 488 and then n^{th} term is negative, then :
[Sep. 03, 2020 (II)]

Options:

A. $n = 60$

B. n^{th} term is -4

C. $n = 41$

D. n^{th} term is $-4\frac{2}{5}$

Answer: B

Solution:

$$S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$$

$$\therefore S_n = 488$$

$$488 = \frac{n}{2} \left[2 \left(\frac{100}{5} \right) + (n-1) \left(-\frac{2}{5} \right) \right] \quad 488 = \frac{n}{2} (101 - n) \Rightarrow n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

$$\text{For } n = 40 \Rightarrow T_n > 0$$

$$\text{For } n = 61 \Rightarrow T_n < 0$$

$$n^{\text{th}} \text{ term} = T_{61} = \frac{100}{5} + (61-1) \left(-\frac{2}{5} \right) = -4$$

Question 207

Let $a_1, a_2, \dots, a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \dots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :
[Sep. 04, 2020 (II)]

Options:

A. (2490, 249)

B. (2480, 249)

C. (2480, 248)

D. (2490,248)

Answer: D

Solution:

Given that $a_1 = 1$ and $a_n = 300$ and $d \in \mathbb{Z}$

$$\therefore 300 = 1 + (n-1)d$$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)}$$

$\therefore d$ is an integer

$$\therefore n-1 = 13 \text{ or } 23$$

$$\Rightarrow n = 14 \text{ or } 24 \quad (\because 15 \leq n \leq 50)$$

$$\Rightarrow n = 24 \text{ and } d = 13$$

$$a_{20} = 1 + 19 \times 13 = 248$$

$$s_{20} = \frac{20}{2}(2 + 19 \times 13) = 2490$$

Question 208

If $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4 - 2 \sin 2\alpha}$ are the first three terms of an A.P. for some α , then the sixth term of this A.P. is:
[Sep. 05, 2020 (I)]

Options:

A. 66

B. 81

C. 65

D. 78

Answer: A

Solution:

Given that $3^{2 \sin 2\alpha - 1}$, 14, $3^{4 - 2 \sin 2\alpha}$ are in A.P.

$$\text{So, } 3^{2 \sin 2\alpha - 1} + 3^{4 - 2 \sin 2\alpha} = 28$$

$$\Rightarrow \frac{3^{2 \sin 2\alpha}}{3} + \frac{81}{3^{2 \sin 2\alpha}} = 28$$

$$\text{Let } 3^{2 \sin 2\alpha} = x$$

$$\Rightarrow \frac{x}{3} + \frac{81}{x} = 28$$

$$\Rightarrow x^2 - 84x + 243 = 0 \Rightarrow x = 81, x = 3$$

When $x = 81 \Rightarrow \sin 2\alpha = 2$ (Not possible)

$$\text{When } x = 3 \Rightarrow \alpha = \frac{\pi}{12}$$

$$\therefore a = 3^0 = 1, d = 14 - 1 = 13$$

$$a_6 = a + 5d = 1 + 65 = 66$$

Question 209

If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to :
[Sep. 05, 2020 (II)]

Options:

A. 7^2

B. $7^{\frac{1}{2}}$

C. e^2

D. $7^{\frac{46}{21}}$

Answer: A

Solution:

$$S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \dots \text{ 20 terms}$$

$$\therefore S = 460$$

$$\Rightarrow \log_7 (x^2 \cdot x^3 \cdot x^4 \cdot \dots \cdot x^{21}) = 460$$

$$\Rightarrow \log_7 x^{(2+3+4+\dots+21)} = 460$$

$$\Rightarrow (2+3+4+\dots+21)\log_7 x = 460$$

$$\Rightarrow \frac{20}{2}(2+21)\log_7 x = 460$$

$$\Rightarrow \log_7 x = \frac{460}{230} = 2 \Rightarrow x = 7^2 = 49$$

Question 210

If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :
[Sep. 06, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. $\frac{4}{9}$

Answer: D

Solution:

Let $f(1) = k$, then $f(2) = f(1+1) = k^2$

$f(3) = f(2+1) = k^3$

$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow k + k^2 + k^3 + \dots = 2$

$\Rightarrow \frac{k}{1-k} = 2 \Rightarrow k = \frac{2}{3}$

Now, $\frac{f(4)}{f(2)} = \frac{k^4}{k^2} = k^2 = \frac{4}{9}$.

Question 211

Let a, b, c, d and p be any non zero distinct real numbers such that

$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :

[Sep. 06, 2020 (I)]

Options:

A. a, c, p are in A.P.

B. a, c, p are in G.P.

C. a, b, c, d are in G.P.

D. a, b, c, d are in A.P.

Answer: C

Solution:

Rearrange given equation, we get

$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2)$

$+ (c^2p^2 - 2cdp + d^2) = 0$

$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$\therefore ap - b = bp - c = cp - d = 0$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \therefore a, b, c, d$ are in G.P.

Question 212

The common difference of the A.P. $b_1, b_2, \dots, b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \dots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:

[Sep. 06, 2020 (II)]

Options:

A. 81

B. -127

C. -81

D. 127

Answer: C

Solution:

Let common difference of series

$a_1, a_2, a_3, \dots, a_n$ be d

$$\therefore a_{40} = a_1 + 39d = -159 \dots (i)$$

$$\text{and } a_{100} = a_1 + 99d = -399 \dots (ii)$$

From equations (i) and (ii),

$$d = -4 \text{ and } a_1 = -3$$

Since, the common difference of $b_1, b_2, \dots, b_n$ is 2 more than common difference of $a_1, a_2, \dots, a_n$

$\therefore$ Common difference of $b_1, b_2, b_3, \dots$ is (-2)

$$\therefore b_{100} = a_{70}$$

$$\Rightarrow b_1 + 99(-2) = (-3) + 69(-4)$$

$$\Rightarrow b_1 = 198 - 279 \Rightarrow b_1 = -81$$

Question 213

Suppose that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(a) = 3$. If $\sum_{i=1}^n f(i) = 363$, then n is equal to _____.

[NA Sep. 06, 2020 (II)]

Answer: 5

Solution:

$$\therefore f(x+y) = f(x) \cdot f(y) \quad \forall x \in \mathbb{R} \text{ and } f(1) = 3$$

$$\Rightarrow f(x) = 3^x \Rightarrow f(i) = 3^i$$

$$\Rightarrow \sum_{i=1}^n f(i) = 363$$

$$\Rightarrow 3 + 3^2 + 3^3 + \dots + 3^n = 363$$

$$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 363 \quad \left[\because S_n = \frac{a(r^n - 1)}{(r - 1)} \right]$$

$$\Rightarrow 3^n - 1 = \frac{363 \times 2}{3} = 242$$

$$\Rightarrow 3^n = 243 = 3^5 \Rightarrow n = 5$$

Question 214

If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \times 3^9 + 3^{10} = S - 2^{11}$ then S is equal to:

[Sep. 05, 2020 (I)]

Options:

A. $3^{11} - 2^{12}$

B. 3^{11}

C. $\frac{3^{11}}{2} + 2^{10}$

D. $2 \cdot 3^{11}$

Answer: B

Solution:

Given sequence are in G.P. and common ratio 3

$$\therefore \frac{2^{10} \left(\left(\frac{3}{2} \right)^{11} - 1 \right)}{\left(\frac{3}{2} - 1 \right)} = S - 2^{11}$$

$$\Rightarrow 2^{10} \frac{\left(\frac{3^{11} - 2^{11}}{2^{11}} \right)}{\frac{1}{2}} = S - 2^{11}$$

$$\Rightarrow 3^{11} - 2^{11} = S - 2^{11} \Rightarrow S = 3^{11}$$

Question 215

If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:

[Sep. 05, 2020 (II)]

Options:

A. $\frac{1}{26}(3^{49} - 1)$

B. $\frac{1}{26}(3^{50} - 1)$

C. $\frac{2}{13}(3^{50} - 1)$

D. $\frac{1}{13}(3^{50} - 1)$

Answer: B

Solution:

Let the first term be 'a' and common ratio be 'r'.

$$\therefore ar(1 + r + r^2) = 3 \dots (i)$$

$$\text{and } ar^5(1 + r + r^2) = 243 \dots (ii)$$

From (i) and (ii),

$$r^4 = 81 \Rightarrow r = 3 \text{ and } a = \frac{1}{13}$$

$$\therefore S_{50} = \frac{a(r^{50} - 1)}{r - 1} = \frac{3^{50} - 1}{26} \left[\because S_n = \frac{a(r^n - 1)}{(r - 1)} \right]$$

Question 216

Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :
[Sep. 04, 2020 (I)]

Options:

A. 3: 1

B. 9: 7

C. 5: 3

D. 33: 31

Answer: B

Solution:

Let $\alpha, \beta, \gamma, \delta$ be in G.P., then $\alpha\delta = \beta\gamma$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left| \frac{\alpha - \beta}{\alpha + \beta} \right| = \left| \frac{\gamma - \delta}{\gamma + \delta} \right|$$

$$\Rightarrow \frac{\sqrt{9 - 4p}}{3} = \frac{\sqrt{36 - 4q}}{6}$$

$$\Rightarrow 36 - 16p = 36 - 4q \Rightarrow q = 4p$$

$$\therefore \frac{2q + p}{2q - p} = \frac{8p + p}{8p - p} = \frac{9p}{7p} = \frac{9}{7}$$

Question 217

The value of $(0.16) \log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{ to } \infty \right)$ is equal to _____.

[NA Sep. 03, 2020 (I)]

Answer: 4

Solution:

$$\begin{aligned}
 & (0.16)^{\log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \infty \right)} \\
 & 0.16^{\log_{2.5} \left(\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right)} \left[\because S_{\infty} = \frac{a}{1-r} \right] \\
 & = 0.16^{\log_{2.5} \left(\frac{1}{2} \right)} \\
 & = (2.5)^{-2 \log_{2.5} \left(\frac{1}{2} \right)} = \left(\frac{1}{2} \right)^{-2} = 4
 \end{aligned}$$

Question 218

The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

[Sep. 02, 2020 (I)]

Options:

A. $(-\infty, -9] \cup [3, \infty)$

B. $[-3, \infty)$

C. $(-\infty, -3] \cup [9, \infty)$

D. $(-\infty, 9]$

Answer: C

Solution:

Let terms of G.P. be $\frac{a}{r}, a, ar$

$$\therefore a \left(\frac{1}{r} + 1 + r \right) = S \dots (i)$$

$$\text{and } a^3 = 27$$

$$\Rightarrow a = 3 \dots (ii)$$

Put $a = 3$ in eqn. (i), we get

$$S = 3 + 3 \left(r + \frac{1}{r} \right)$$

If $f(x) = x + \frac{1}{x}$, then $f(x) \in (-\infty, -2] \cup [2, \infty)$

$$\Rightarrow 3f(x) \in (-\infty, -6] \cup [6, \infty)$$

$$\Rightarrow 3 + 3f(x) \in (-\infty, -3] \cup [9, \infty)$$

Then, it concludes that

$$S \in (-\infty, -3] \cup [9, \infty)$$

Question 219

If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots \text{ is :}$$

[Sep. 02, 2020 (I)]

Options:

A. $\frac{x+y-xy}{(1+x)(1+y)}$

B. $\frac{x+y+xy}{(1+x)(1+y)}$

C. $\frac{x+y-xy}{(1-x)(1-y)}$

D. $\frac{x+y+xy}{(1-x)(1-y)}$

Answer: C

Solution:

$$\begin{aligned} S &= (x+y) + (x^2+y^2+xy) + (x^3+x^2y+xy^2+y^3) + \dots \infty \\ &= \frac{1}{x-y} [(x^2-y^2) + (x^3-y^3) + (x^4-y^4) + \dots \infty] \\ &= \frac{1}{x-y} \left[\frac{x^2}{1-x} - \frac{y^2}{1-y} \right] = \frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)} \\ \left[\because S_{\infty} = \frac{a}{1-r} \right] \\ &= \frac{x+y-xy}{(1-x)(1-y)} \end{aligned}$$

Question 220

Let S be the sum of the first 9 terms of the series:

$\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\} + \{x^4 + (k+6)a\} + \dots$ where $a \neq 0$ and

$x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to:

[Sep. 02, 2020 (II)]

Options:

A. -5

B. 1

C. -3

D. 3

Answer: C

Solution:

$$\begin{aligned} S &= (x + x^2 + x^3 + \dots 9 \text{ terms}) \\ &\quad + a[k + (k+2) + (k+4) + \dots 9 \text{ terms}] \Rightarrow S = \frac{x(x^9-1)}{x-1} + \frac{9}{2}[2ak + 8 \times (2a)] \\ \Rightarrow S &= \frac{x^{10}-x}{x-1} + \frac{9a(k+8)}{1} = \frac{x^{10}-x+45a(x-1)}{x-1} \quad (\text{Given}) \\ \Rightarrow \frac{x^{10}-x+9a(k+8)(x-1)}{x-1} &= \frac{x^{10}-x+45a(x-1)}{x-1} \\ \Rightarrow 9a(k+8) &= 45a \Rightarrow k+8=5 \Rightarrow k=-3 \end{aligned}$$

Question221

If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to

[Sep. 03, 2020 (II)]

Answer: 39

Solution:

Let m arithmetic mean be $A_1, A_2, \dots, A_m$ and G_1, G_2, G_3 be geometric mean.

The A.P. formed by arithmetic mean is,

3, $A_1, A_2, A_3, \dots, A_m, 243$

$$\therefore d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$$

The G.P. formed by geometric mean

3, $G_1, G_2, G_3, 243$

$$r = \left(\frac{243}{3} \right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$$

$$\therefore A_4 = G_2$$

$$\Rightarrow 3 + 4 \left(\frac{240}{m + 1} \right) = 3(3)^2$$

$$\Rightarrow 3 + \frac{960}{m + 1} = 27 \Rightarrow m + 1 = 40 \Rightarrow m = 39.$$

Question222

If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2) 19 = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :

[Sep. 04, 2020 (I)]

Options:

A. (10,97)

B. (11,103)

C. (10,103)

D. (11,97)

Answer: B

Solution:

The given series is

$$1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots (1 - 20^2 \cdot 19)$$

$$S = 1 + \sum_{r=1}^{10} [1 - (2r)^2(2r-1)]$$

$$= 1 + \sum_{r=1}^{10} (1 - 8r^3 + 4r^2) = 1 + 10 - \sum_{r=1}^{10} (8r^3 - 4r^2)$$

$$= 11 - 8 \left(\frac{10 \times 11}{2} \right)^2 + 4 \times \left(\frac{10 \times 11 \times 21}{6} \right)$$

$$= 11 - 2 \times (110)^2 + 4 \times 55 \times 7$$

$$= 11 - 220(110 - 7)$$

$$= 11 - 220 \times 103 = \alpha - 220\beta$$

$$\Rightarrow \alpha = 11, \beta = 103$$

$$\therefore (\alpha, \beta) = (11, 103)$$

Question 223

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(a) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in \mathbb{N}$, then the value of n , for which $g(n) = 20$, is :
[Sep. 02, 2020 (II)]

Options:

A. 5

B. 20

C. 4

D. 9

Answer: A

Solution:

Given: $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$, $f(1) = 2$

$$\Rightarrow f(2) = f(1) + f(1) = 2 + 2 = 4$$

$$f(3) = f(1) + f(2) = 2 + 4 = 6$$

$$f(n-1) = 2(n-1)$$

$$\text{Now, } g(n) = \sum_{k=1}^{n-1} f(k)$$

$$= f(1) + f(2) + f(3) + \dots + f(n-1)$$

$$= 2 + 4 + 6 + \dots + 2(n-1)$$

$$= 2[1 + 2 + 3 + \dots + (n-1)]$$

$$= 2 \times \frac{(n-1)(n)}{2} = n^2 - n$$

$$\therefore g(n) = 20 \text{ (given)}$$

$$\text{So, } n^2 - n = 20$$

$$\Rightarrow n^2 - n - 20 = 0$$

$$\Rightarrow (n-5)(n+4) = 0$$

$$\Rightarrow n = 5 \text{ or } n = -4 \text{ (not possible)}$$
