



Probability

Question1

From the first 100 natural numbers, two numbers first a and then b are selected randomly without replacement. If the probability that $a - b \geq 10$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to

_____ .

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Answer: 311

Solution:

Let $b = 1$

$\Rightarrow a > 11 \Rightarrow a$ can take 90 values from $\{11, 12, \dots, 100\}$

Let $b = 2 \Rightarrow a > 12 \Rightarrow 8 >$ values and so on till $b = 99 \Rightarrow a$ can only take $a = 100$, only 1 value $\Rightarrow$ favourable cases $= 1 + 2 + 3 + \dots + 90$

$$= \frac{90 \times 91}{2} = \frac{91}{220} \Rightarrow m + n = 311$$

Question2

Let S be a set of 5 elements and $P(S)$ denote the power set of S . Let E be an event of choosing an ordered pair (A, B) from the set $P(S) \times P(S)$ such that $A \cap B = \emptyset$. If the probability of the event E is $\frac{3^p}{2^q}$, where $p, q \in \mathbb{N}$, then $p + q$ is equal to

JEE Main 2026 (Online) 24th January Evening Shift

Answer: 15

Solution:

Let $S = \{a_1, a_2, a_3, a_4, a_5\}$

$n(S) = 5$

$P(S)$ = power set of S

$nP(S) = 2^{n(S)} = 2^5 = 32$

$n(T) = n[P(S) \times P(S)] = 32 \times 32$ (Total outcomes)

E = event that choosing ordered pair (A, B) from $P(S) \times P(S)$ such that $A \cap B = \phi$

for each element $x \in S, n(S) = 5$

there are 4 possibilities such that $A \cap B = \phi$

1. $x \in A$ and $x \notin B$
2. $x \notin A$ and $x \in B$
3. $x \notin A$ and $x \notin B$
4. $x \in A$ and $x \in B$

for $A \cap B = \phi$, exclude the fourth.

so, for each of 5 elements of S there are 3 choices.

The number of favourable outcome $n(E) = 3 \times 3 \times 3 \times 3 \times 3$

$$P(E) = \frac{\text{favourable outcome}}{\text{Total outcome}} = \frac{3^5}{32 \times 32} = \frac{3^5}{2^{10}}$$

It is given that $P(E) = \frac{3^p}{2^q}$

compare $p = 5, q = 10 \implies p + q = 15$ Ans

Question3

Let $a, b, c \in \{1, 2, 3, 4\}$. If the probability, that $ax^2 + 2\sqrt{2}bx + c > 0$ for all $x \in \mathbb{R}$, is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

JEE Main 2026 (Online) 2nd April Morning Shift

Answer: 81

Solution:

For the quadratic expression

$$ax^2 + 2\sqrt{2}bx + c$$

to be positive for all $x \in \mathbb{R}$, we use the NCERT condition for a quadratic $Ax^2 + Bx + C$ to satisfy

$$Ax^2 + Bx + C > 0 \quad \text{for all } x \in \mathbb{R},$$

which is:

$$1. A > 0$$

$$2. \text{Discriminant} < 0$$

Here,

$$A = a, \quad B = 2\sqrt{2}b, \quad C = c$$

Since $a \in \{1, 2, 3, 4\}$, we always have

$$a > 0$$

So we only need

$$D = B^2 - 4AC < 0$$

Now,

$$D = (2\sqrt{2}b)^2 - 4ac = 8b^2 - 4ac$$

Thus,

$$8b^2 - 4ac < 0$$

Dividing by 4:

$$2b^2 - ac < 0$$

So the required condition is

$$ac > 2b^2$$

Now $a, b, c \in \{1, 2, 3, 4\}$, so total number of choices is

$$4 \times 4 \times 4 = 64$$

We count favourable triples (a, b, c) .

Case 1: $b = 1$

Then

$$ac > 2$$

Now $a, c \in \{1, 2, 3, 4\}$. Total pairs (a, c) are 16.

We exclude pairs with $ac \leq 2$:

- $(1, 1)$ gives $ac = 1$
- $(1, 2)$ gives $ac = 2$
- $(2, 1)$ gives $ac = 2$

So favourable pairs:

$$16 - 3 = 13$$

Case 2: $b = 2$

Then

$$ac > 8$$

Possible products greater than 8 are obtained from:

- $(3, 3)$ gives 9
- $(3, 4)$ gives 12
- $(4, 3)$ gives 12

- $(4, 4)$ gives 16

So favourable pairs = 4.

Case 3: $b = 3$

Then

$$ac > 18$$

But maximum possible value of ac is

$$4 \cdot 4 = 16$$

So no favourable pair.

Case 4: $b = 4$

Then

$$ac > 32$$

Again impossible, since maximum $ac = 16$.

So no favourable pair.

Hence total favourable triples are

$$13 + 4 + 0 + 0 = 17$$

Therefore, the required probability is

$$\frac{17}{64}$$

Thus,

$$m = 17, \quad n = 64$$

and

$$m + n = 17 + 64 = 81$$

So the answer is

$$\boxed{81}$$

Question4

A coin is tossed 8 times. If the probability that exactly 4 heads appear in the first six tosses and exactly 3 heads appear in the last five tosses is p , then $96p$ is equal to _____ .

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Answer: 9

Solution:

Let the outcomes of the 8 tosses be denoted by $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$.

The condition “exactly 4 heads in the first six tosses” means the block $(x_1, x_2, x_3, x_4, x_5, x_6)$ contains 4 heads.

The condition “exactly 3 heads in the last five tosses” means the block $(x_4, x_5, x_6, x_7, x_8)$ contains 3 heads.

Notice that these two blocks overlap in the common tosses (x_4, x_5, x_6) .

Let k be the number of heads in the common tosses (x_4, x_5, x_6) .

Then:

- In (x_1, x_2, x_3) we must have $4 - k$ heads (so that the first six have total 4 heads).
- In (x_7, x_8) we must have $3 - k$ heads (so that the last five have total 3 heads).

Now we count the number of valid outcome sequences for each possible value of k .

$$k = 1 \therefore \text{ways to overlap} = {}^3C_1$$

Here, we choose:

- ${}^3C_3$ ways to put $4 - k = 3$ heads in (x_1, x_2, x_3) ,
- ${}^3C_1$ ways to put $k = 1$ head in (x_4, x_5, x_6) ,
- ${}^2C_2$ ways to put $3 - k = 2$ heads in (x_7, x_8) .

$${}^3C_3 \times {}^3C_1 \times {}^2C_2 = 3 \text{ ways}$$

$$k = 2$$

Here, we choose:

- ${}^3C_2$ ways to put $4 - k = 2$ heads in (x_1, x_2, x_3) ,
- ${}^3C_2$ ways to put $k = 2$ heads in (x_4, x_5, x_6) ,
- ${}^2C_1$ ways to put $3 - k = 1$ head in (x_7, x_8) .

$${}^3C_2 \times {}^3C_2 \times {}^2C_1 = 18$$

$$k = 3$$

Here, we choose:

- ${}^3C_1$ ways to put $4 - k = 1$ head in (x_1, x_2, x_3) ,
- 1 way to put $k = 3$ heads in (x_4, x_5, x_6) (all heads),
- 1 way to put $3 - k = 0$ heads in (x_7, x_8) (all tails).

$${}^3C_1 \times 1 = 3$$

$$\text{Total favourable outcomes} = 3 + 18 + 3 = 24.$$

Since each full sequence of 8 tosses has probability $\left(\frac{1}{2}\right)^8$, we get:

$$\text{Total ways } 24 \left(\frac{1}{2}\right)^8$$

$$\text{So } p = 24 \left(\frac{1}{2}\right)^8 = \frac{24}{2^8} = \frac{24}{256} = \frac{3}{32}, \text{ hence } 96p = 96 \cdot \frac{3}{32} = 9.$$

$$96 \times \frac{24}{2^8} = \frac{6 \times 24}{16} = 9$$

Question 5

From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P. is equal to $\frac{a}{b}$, where $a, b \in \mathbf{N}$ and $\gcd(a, b) = 1$, then $a + b$ is equal to _____

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Answer: 944

Solution:

Total number of ways to select any 3 different dates from 31 days is

$$\binom{31}{3}$$

Now we count the number of selections in which the three dates are in an increasing A.P.

Let the three dates be

$$a - d, a, a + d$$

where $d \geq 1$ is an integer.

So we need all triples of the form

$$x, x + r, x + 2r$$

with

$$x \geq 1, \quad x + 2r \leq 31$$

For a fixed common difference r , the first term x can be chosen from

$$1, 2, 3, \dots, 31 - 2r$$

So number of A.P.s with common difference r is

$$31 - 2r$$

Now r can take values

$$1, 2, 3, \dots, 15$$

because

$$x + 2r \leq 31 \Rightarrow 2r \leq 30 \Rightarrow r \leq 15$$

Hence total favourable selections are

$$\sum_{r=1}^{15} (31 - 2r)$$

This is

$$15 \cdot 31 - 2 \sum_{r=1}^{15} r$$

$$= 465 - 2 \cdot \frac{15 \cdot 16}{2}$$

$$= 465 - 240$$

$$= 225$$

Therefore,

$$P = \frac{225}{\binom{31}{3}}$$

Now,

$$\binom{31}{3} = \frac{31 \cdot 30 \cdot 29}{6} = 4495$$

So,

$$P = \frac{225}{4495}$$

Divide numerator and denominator by 5:

$$P = \frac{45}{899}$$

Thus,

$$a = 45, \quad b = 899$$

Therefore,

$$a + b = 45 + 899 = 944$$

So the required answer is

944

Question6

Let the mean and variance of 7 observations 2, 4, 10, x , 12, 14, y , $x > y$, be 8 and 16 respectively. Two numbers are chosen from $\{1, 2, 3, x - 4, y, 5\}$ one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4, is :

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Options:

A.

$$\frac{4}{5}$$

B.

$$\frac{3}{5}$$

C.

$$\frac{2}{5}$$

D.

$$\frac{1}{3}$$

Answer: A

Solution:

$$\text{Mean} = 8$$

$$\frac{2 + 4 + 10 + x + y + 12 + 14}{7} = 8$$

$$\Rightarrow x + y = 14$$

$$\text{variance} = 16$$

$$16 = \frac{2^2 + 4^2 + 10^2 + x^2 + y^2 + 12^2 + 14^2 + y^2}{7} - 8^2$$

$$\Rightarrow x^2 + y^2 = 100$$

$$\therefore x = 8, y = 6$$

New set : $\{1, 2, 3, 4, 5, 6\}$

Required probability = $1 - P(\text{both number are } \geq 4)$

$$= 1 - \frac{{}^3C_2}{{}^6C_2}$$

$$= 1 - \frac{3}{15}$$

$$= \frac{4}{5}$$

Question7

If a random variable x has the probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	$2k$	k	$3k$	$2k^2$	$2k$	$k^2 + k$	$7k^2$

then $P(3 < x \leq 6)$ is equal to

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Options:

A.

0.34

B.

0.64

C.

0.22

D.

0.33

Answer: D

Solution:

$$2K + K + 3K + 2K^2 + 2K + K^2 + K + 7K^2 = 1$$
$$(K + 1)(10K - 1) = 0$$

$$K = -1 \text{ or } K = \frac{1}{10}$$

$$P(3 < x \leq 6) = \frac{2}{100} + \frac{2}{10} + \frac{1}{100} + \frac{1}{10} = 0.33$$

Question 8

Two distinct numbers a and b are selected at random from $1, 2, 3, \dots, 50$. The probability, that their product ab is divisible by 3, is

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Options:

A.

$$\frac{272}{1225}$$

B.

$$\frac{561}{1225}$$

C.

$$\frac{664}{1225}$$

D.

$$\frac{8}{25}$$

Answer: C

Solution:

Total ways to choose two distinct numbers from $1, 2, \dots, 50$ is

$$\binom{50}{2} = 1225.$$

The product ab is **not** divisible by 3 only if **neither** a nor b is divisible by 3.

Multiples of 3 in 1 to 50: $\left\lfloor \frac{50}{3} \right\rfloor = 16$.

So numbers **not** divisible by 3: $50 - 16 = 34$.

Unfavorable pairs:

$$\binom{34}{2} = \frac{34 \cdot 33}{2} = 561.$$

Therefore favorable pairs:

$$1225 - 561 = 664,$$

and the required probability is

$$\frac{664}{1225}.$$

Answer: $\frac{664}{1225}$ (Option C).

Question9

Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is $\frac{p}{q}$, $\gcd(p, q) = 1$, then $p + q$ is equal to

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Options:

A.

24

B.

22

C.

23

D.

21

Answer: C

Solution:

Bag A : 9 white, 8 black (Total = 17)

Bag B : 6 white, 4 black (Total = 10)

One ball is moved from Bag B to Bag A. There are two mutually exclusive events for this transfer:

Case 1 : The ball moved is white

The probability of choosing a white ball from Bag B is:

$$P(W_B) = \frac{6}{10}$$

After adding this white ball to Bag A, Bag A now contains 10 white and 8 black balls (18 total). The probability of then drawing a white ball from Bag A is :

$$P(W_A | W_B) = \frac{10}{18}$$

Case 2: The ball moved is black

The probability of choosing a black ball from Bag B is :

$$P(B_B) = \frac{4}{10}$$

After adding this black ball to Bag A, Bag A now contains 9 white and 9 black balls (18 total). The probability of then drawing a white ball from Bag A is :

$$P(W_A | B_B) = \frac{9}{18}$$

Using total probability theorem

$$P(W_A) = P(W_B) \cdot P(W_A | W_B) + P(B_B) \cdot P(W_A | B_B)$$

$$\Rightarrow P(W_A) = \left(\frac{6}{10} \times \frac{10}{18}\right) + \left(\frac{4}{10} \times \frac{9}{18}\right)$$

$$P(W_A) = \frac{60}{180} + \frac{36}{180} = \frac{96}{180} = \frac{8}{15}$$

It is given that $P(W_A) = \frac{p}{q}$.

$$\gcd(8, 15) = 1$$

$$\text{So } p = 8 \& q = 15$$

$$p + q = 8 + 15 = 23$$

Question10

From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is

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Options:

A.

$$\frac{73}{10^8}$$

B.

$$\frac{67}{10^8}$$

C.

$$\frac{7}{10^7}$$

D.

$$\frac{81}{10^8}$$

Answer: A

Solution:

total number of defective bulbs $n(D) = 10$

total number of non defective bulbs $n(\bar{D}) = 90$

total bulbs $= n(D) + n(\bar{D}) = 100$

probability of picking a defective bulbs: $p = \frac{10}{100} = \frac{1}{10}$

probability of picking a non-defective bulbs $q = \frac{90}{100} = \frac{9}{10}$

number of trials $n = 8$ bulbs are selected

let X be the event of getting atleast 7 defective bulbs

$$P(X) = P(X = 7) + P(X = 8)$$

$$= {}^8C_7 p^7 \cdot q^1 + {}^8C_8 p^8 q^0$$

$$= 8 \cdot \left(\frac{1}{10}\right)^7 \cdot \frac{9}{10} + 1 \cdot \left(\frac{1}{10}\right)^8$$

$$P(X) = \left(\frac{1}{10}\right)^8 [72 + 1] = \frac{73}{10^8}$$

Question 11

A bag contains 10 balls out of which k are red and $(10 - k)$ are black, where $0 \leq k \leq 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is:

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Options:

A.

$$\frac{7}{110}$$

B.

$$\frac{7}{11}$$

C.

$$\frac{7}{55}$$

D.

Answer: D

Solution:

Given Terms

- **Total Balls (n) :** 10
- **Red Balls (k) :** Variable from 0 to 10 .
- **Black Balls :** $(10 - k)$.
- **Sample Drawn (r) :** 3 balls, without replacement.
- **Observed Event (A) :** All 3 balls drawn are black.
- **Target Event :** Find the probability that the bag contained exactly 1 red and 9 black balls (which means $k = 1$).

Let E_k be the event that the bag contains k red balls. Since k can be any integer from 0 to 10 , there are 11 possible compositions. Assuming they are equally likely:

$$P(E_k) = \frac{1}{11}$$

The event A is drawing 3 black balls. The probability of this happening depends on how many black balls $(10 - k)$ were in the bag :

$$P(A | E_k) = \frac{{}^{10-k}C_3}{{}^{10}C_3}$$

Note : For this to be possible, there must be at least 3 black balls, so $10 - k \geq 3$, meaning $k \leq 7$.

Using Bayes' Theorem

We need $P(E_1 | A)$, which is the probability of having 1 red ball given that 3 black balls were drawn :

$$P(E_1 | A) = \frac{P(E_1) \cdot P(A|E_1)}{\sum_{k=0}^7 P(E_k) \cdot P(A|E_k)}$$

Since $P(E_k)$ is constant $(\frac{1}{11})$ for all k , it cancels out :

$$P(E_1 | A) = \frac{{}^{10-1}C_3}{{}^{10}C_3 + {}^9C_3 + \dots + {}^3C_3}$$

denominator ${}^{10}C_3 + {}^9C_3 + \dots + {}^3C_3$ can be written as ${}^3C_3 + {}^4C_3 + {}^5C_3 + \dots + {}^{10}C_3$

using identity ${}^kC_k + {}^{k+1}C_k + {}^{k+2}C_k + \dots + {}^nC_k = {}^{n+1}C_{k+1}$

so, denominator become $= {}^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330$

Numerator : ${}^{10-1}C_3 = {}^9C_3 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$

$$P(E_1 | A) = \frac{84}{330} = \frac{14}{55}$$

Question 12

The probability distribution of a random variable X is given below :

X	$4k$	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	$6k$
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P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$
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If $E(X) = \frac{263}{15}$, then $P(X < 20)$ is equal to :

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Options:

A.

$$\frac{3}{5}$$

B.

$$\frac{14}{15}$$

C.

$$\frac{8}{15}$$

D.

$$\frac{11}{15}$$

Answer: D

Solution:

The expected value of a discrete random variable is given by the formula :

$$E(X) = \sum X_i \cdot P(X_i)$$

Using the data from the table :

$$E(X) = \left(4k \cdot \frac{2}{15}\right) + \left(\frac{30k}{7} \cdot \frac{1}{15}\right) + \left(\frac{32k}{7} \cdot \frac{2}{15}\right) + \left(\frac{34k}{7} \cdot \frac{1}{5}\right) + \left(\frac{36k}{7} \cdot \frac{1}{15}\right) + \left(\frac{40k}{7} \cdot \frac{1}{5}\right) + \left(6k \cdot \frac{1}{15}\right)$$

Take $\frac{k}{15}$ factor common

$$E(X) = \frac{k}{15} \left[8 + \frac{30}{7} + \frac{64}{7} + \frac{102}{7} + \frac{36}{7} + \frac{76}{7} + \frac{120}{7} + 6 \right]$$

$$E(X) = \frac{k}{15} \left[14 + \frac{30 + 64 + 102 + 36 + 76 + 120}{7} \right]$$

$$E(X) = \frac{k}{15} \left[14 + \frac{428}{7} \right]$$

$$E(X) = \frac{k}{15} \left[\frac{98 + 428}{7} \right] = \frac{k}{15} \cdot \frac{526}{7} = \frac{526k}{105}$$

Given that $E(X) = \frac{263}{15}$:

$$\frac{263}{15} = \frac{526k}{105}$$

$$263 = \frac{526k}{7}$$

$$k = \frac{263 \times 7}{526} = \frac{1 \times 7}{2} = \frac{7}{2} = 3.5$$

Identify X values where $X < 20$

Substitute $k = \frac{7}{2}$ into each X :

$$X_1 = 4 \times \frac{7}{2} = 14 (< 20) \quad X_4 = \frac{34}{7} \times \frac{7}{2} = 17 (< 20)$$

$$X_2 = \frac{30}{7} \times \frac{7}{2} = 15 (< 20) \quad X_5 = \frac{36}{7} \times \frac{7}{2} = 18 (< 20)$$

$$X_3 = \frac{32}{7} \times \frac{7}{2} = 16 (< 20) \quad X_6 = \frac{38}{7} \times \frac{7}{2} = 19 (< 20)$$

$$X_7 = \frac{40}{7} \times \frac{7}{2} = 20 (\text{Not } < 20)$$

$$X_8 = 6 \times \frac{7}{2} = 21 (\text{Not } < 20)$$

Calculate $P(X < 20)$:

Sum the probabilities for $X = 14, 15, 16, 17, 18$, and 19 :

$$P(X < 20) = P(X = 14) + P(X = 15) + P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19)$$

$$\Rightarrow P(X < 20) = \frac{2}{15} + \frac{1}{15} + \frac{2}{15} + \frac{3}{15} + \frac{1}{15} + \frac{2}{15}$$

$$\Rightarrow P(X < 20) = \frac{11}{15}$$

Question 13

A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

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Options:

A.

53

B.

55

C.

107

D.

105

Answer: C

Solution:

$$P = (6 \text{ tail}) + (4 \text{ tail} + 1 \text{ head}) + (2 \text{ tail} + 2 \text{ head}) + (3 \text{ head})$$

$$P = \binom{1}{2}^6 + \frac{5!}{1!4!} \binom{1}{2}^5 + \frac{4!}{2!2!} \binom{1}{2}^4 + \binom{1}{2}^3$$

$$P = \frac{1}{64} + \frac{5}{32} + \frac{3}{8} + \frac{1}{8}$$

$$P = \frac{1+10+24+8}{64} = \frac{43}{64} = \frac{m}{n}$$

$$m+n=107$$

Question 14

A letter is known to have arrived by post either from KANPUR or from ANANTPUR. On the envelope just two consecutive letters AN are visible. The probability, that the letter came from ANANTPUR, is:

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Options:

A.

$$\frac{7}{10}$$

B.

$$\frac{10}{17}$$

C.

$$\frac{12}{19}$$

D.

$$\frac{7}{19}$$

Answer: B

Solution:

Let

- A = event that the letter came from ANANTPUR
- K = event that the letter came from KANPUR
- B = event that on the envelope, the two visible consecutive letters are AN

We have to find:

$$P(A | B)$$

Using Bayes' theorem,

$$P(A | B) = \frac{P(A) P(B|A)}{P(A) P(B|A) + P(K) P(B|K)}$$

Since the letter is known to have come either from ANANTPUR or KANPUR, and no other information is given, we take

$$P(A) = P(K) = \frac{1}{2}$$

So,

$$P(A | B) = \frac{P(B|A)}{P(B|A) + P(B|K)}$$

Now we find these probabilities.

1. Probability of seeing AN in ANANTPUR

Word: ANANTPUR

Its consecutive pairs are:

AN, NA, AN, NT, TP, PU, UR

There are 7 consecutive pairs in all, and AN occurs 2 times.

Therefore,

$$P(B | A) = \frac{2}{7}$$

2. Probability of seeing AN in KANPUR

Word: KANPUR

Its consecutive pairs are:

KA, AN, NP, PU, UR

There are 5 consecutive pairs in all, and AN occurs 1 time.

Therefore,

$$P(B | K) = \frac{1}{5}$$

3. Apply Bayes' theorem

$$P(A | B) = \frac{\frac{2}{7}}{\frac{2}{7} + \frac{1}{5}}$$

Take LCM:

$$\frac{2}{7} + \frac{1}{5} = \frac{10+7}{35} = \frac{17}{35}$$

So,

$$P(A | B) = \frac{\frac{2}{7}}{\frac{17}{35}} = \frac{2}{7} \cdot \frac{35}{17} = \frac{10}{17}$$

Therefore, the required probability is

$$\boxed{\frac{10}{17}}$$

So, the correct option is **B**.

Question 15

The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4, respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7. Then the probability, that the team wins the tournament, is :

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Options:

A.

0.74

B.

0.76

C.

0.72

D.

0.78

Answer: B

Solution:

Let

- A = event that player A is selected as captain
- B = event that player B is selected as captain
- W = event that the team wins the tournament

Given:

$$P(A) = 0.6, \quad P(B) = 0.4$$

Also,

$$P(W|A) = 0.8, \quad P(W|B) = 0.7$$

We have to find $P(W)$.

Using the formula of total probability:

$$P(W) = P(A) P(W|A) + P(B) P(W|B)$$

Substituting the values:

$$P(W) = 0.6 \times 0.8 + 0.4 \times 0.7$$

$$P(W) = 0.48 + 0.28$$

$$P(W) = 0.76$$

So, the required probability is

0.76

Hence, the correct option is **Option B**.

Question 16

A bag contains $(N + 1)$ coins - N fair coins, and one coin with 'Head' on both sides. A coin is selected at random and tossed. If the probability of getting 'Head' is $\frac{9}{16}$, then N is equal to:

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A.

5

B.

7

C.

8

D.

9

Answer: B

Solution:

Let E_1 be the event of selecting a fair coin.

Let E_2 be the event of selecting the two-headed coin.

Let A be the event of getting a 'Head' when the selected coin is tossed.

We are given that there are N fair coins and 1 two-headed coin.

Hence, the total number of coins in the bag is $N + 1$.

Since a coin is selected at random, the probability of choosing a fair coin is:

$$P(E_1) = \frac{N}{N+1}$$

The probability of choosing the two-headed coin is:

$$P(E_2) = \frac{1}{N+1}$$

Now, let us find the conditional probabilities of getting a 'Head' for each type of coin:

If a fair coin is selected, the probability of getting a 'Head' is:

$$P(A|E_1) = \frac{1}{2}$$

If the two-headed coin is selected, the probability of getting a 'Head' is a certainty (since both sides are 'Head'):

$$P(A|E_2) = 1$$

According to the Law of Total Probability, the total probability of getting a 'Head', denoted as $P(A)$, is given by:

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

We are given that the probability of getting a 'Head' is $\frac{9}{16}$. Let us substitute the known values into the equation:

$$\frac{9}{16} = \left(\frac{N}{N+1} \cdot \frac{1}{2}\right) + \left(\frac{1}{N+1} \cdot 1\right)$$

Let us simplify the equation:

$$\frac{9}{16} = \frac{N}{2(N+1)} + \frac{1}{N+1}$$

To add the terms on the right side, let us take the common denominator as $2(N+1)$:

$$\frac{9}{16} = \frac{N+2(1)}{2(N+1)}$$

$$\frac{9}{16} = \frac{N+2}{2N+2}$$

Now, we can solve for N by cross-multiplying:

$$9 \cdot (2N+2) = 16 \cdot (N+2)$$

$$18N+18 = 16N+32$$

Transpose the terms containing N to one side and the constant terms to the other side:

$$18N - 16N = 32 - 18$$

$$2N = 14$$

Divide both sides by 2:

$$N = 7$$

Therefore, the value of N is equal to 7.

Option B is the correct answer.

Question 17

A bag contains 6 blue and 6 green balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each drawn pair consists of one blue and one green ball is :

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Options:

A.

$$\frac{63}{925}$$

B.

$$\frac{17}{231}$$

C.

$$\frac{16}{231}$$

D.

$$\frac{64}{925}$$

Answer: C

Solution:

We want every pair drawn to have **one blue** and **one green**. Since there are 6 blue and 6 green balls, we will draw a total of 6 pairs.

For each pair to be mixed (one of each colour), we must match the 6 blue balls with the 6 green balls in pairs. The number of such “good” pairings is counted by choosing and matching one ball of one colour with one ball of the other colour.

The total number of ways to split 12 balls into 6 pairs (without caring about the order of drawing inside a pair) is counted separately. The required probability is:

$$\text{Probability} = \frac{6 \times 6}{\binom{12}{(2)^6} \times 6} = \frac{16}{231}$$

So, the probability that each drawn pair consists of one blue and one green ball is $\frac{16}{231}$.

Question 18

A candidate has to go to the examination centre to appear in an examination. The candidate uses only one means of transportation for the entire distance out of bus, scooter and car. The probabilities of the candidate going by bus, scooter and car, respectively, are $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{2}{5}$. The probabilities that the candidate reaches late at the examination centre are $\frac{1}{5}$, $\frac{1}{3}$ and $\frac{1}{4}$ if the candidate uses bus, scooter and car, respectively. Given that the candidate reached late at the examination centre, the probability that the candidate travelled by bus is :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

$$\frac{11}{37}$$

B.

$$\frac{12}{37}$$

C.

$$\frac{13}{37}$$

D.

$$\frac{14}{37}$$

Answer: B

Solution:

Conveyance	Bus	Scooter	Car	Arrive late	Bus	Scooter	Car
Probability	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	Probability	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$

We label the events in the chart below :

Conveyance	Bus	Scooter	Car	Arrive late by	Bus	Scooter	Car
Event	B	S	C	Event	L_b	L_s	L_c

We label the event that the candidate arrives late be L

Event	B	S	C	L_b	L_s	L_c
Probability	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$

$$\Rightarrow P(L \cap B) = P(L) \cdot P(B/L) = P(B \cap L)$$

$$\Rightarrow P(B/L) = \frac{P(B \cap L)}{P(L)}$$

$$= \frac{P(B) \cdot P(L/B)}{P(B \cap L) + P(S \cap L) + P(C \cap L)}$$

$$= \frac{P(B) \cdot P(L/B)}{P(B) \cdot P(L/B) + P(S) \cdot P(L/S) + P(C) \cdot P(L/C)}$$

$$P(B/L) = \frac{P(B) \cdot P(L_b)}{P(B) \cdot P(L_b) + P(S) \cdot P(L_s) + P(C) \cdot P(L_c)}$$

$$= \frac{\left(\frac{2}{5}\right) \cdot \left(\frac{1}{5}\right)}{\left(\frac{2}{5}\right) \cdot \left(\frac{1}{5}\right) + \left(\frac{1}{5}\right) \cdot \left(\frac{1}{3}\right) + \left(\frac{2}{5}\right) \cdot \left(\frac{1}{4}\right)}$$

$$= \left(\frac{2}{25}\right) \cdot \left(\frac{300}{24+20+30}\right) = \frac{12}{37}$$

Question 19

Three distinct numbers are selected randomly from the set $\{1, 2, 3, \dots, 40\}$. If the probability, that the selected numbers are in an increasing G.P., is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

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Answer: 2477

Solution:

Common ratio	Last triplet	Total
$r = 2$	10, 20, 40	10
$r = 3$	4, 12, 36	4
$r = 4$	2, 8, 32	2
$r = 5$	1, 5, 25	1
$r = 6$	1, 6, 36	1
Total = 18		

Total choices = ${}^{40}C_3 = 9880$

$$\text{Required probability} = \frac{18}{9880} = \frac{9}{4940} = \frac{m}{n}$$

$$m + n = 4949$$

Question 20

A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to _____.

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Answer: 2

Solution:

$$\begin{aligned} &P\left(\frac{\text{Lost (spade)}}{n \text{ cards are spade}}\right) \\ &= \frac{P\left(\frac{n_s}{L_s}\right)P(L_s)}{P\left(\frac{n_s}{L_s}\right)P(L_s) + P\left(\frac{n_s}{\bar{L}_s}\right)P(\bar{L}_s)} \\ &= \frac{\frac{{}^{12}C_n}{{}^{51}C_n} \times \frac{1}{4}}{\frac{{}^{12}C_n}{{}^{51}C_n} \times \frac{1}{4} + \frac{3}{4} \times \frac{{}^{13}C_n}{{}^{51}C_n}} = \frac{1}{1 + 3 \cdot \frac{{}^{13}C_n}{{}^{12}C_n}} = \frac{13 - n}{52 - n} \\ &\Rightarrow \frac{13 - n}{52 - n} = \frac{11}{50} \\ &\Rightarrow n = 2 \end{aligned}$$

Question 21

Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

- A. 4
- B. 14
- C. 11
- D. 13

Answer: B

Solution:

In a bag containing 4 white and 6 black balls, two balls are selected one by one without replacement. We need to find the probability that the first ball is black, given that the second ball is black. Let's define the events:

A: The first ball selected is black.

B: The second ball selected is black.

The probability $P(A|B)$ is given by the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Let's compute each part separately:

Probability of both balls being black $P(A \cap B)$:

The chance that the first ball is black is $\frac{6}{10}$. Given that one black ball is removed, the probability that the second is also black is $\frac{5}{9}$.

$$P(A \cap B) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

Probability that the second ball is black $P(B)$:

This can occur if either the first ball was white or black:

$$\text{First ball white, second black: } \frac{4}{10} \times \frac{6}{9}$$

$$\text{Both balls black: } \frac{6}{10} \times \frac{5}{9}$$

$$P(B) = \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{5}{9} = \frac{24}{90} + \frac{30}{90} = \frac{54}{90} = \frac{3}{5}$$

Now, using these probabilities:

$$P(A|B) = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{1}{3} \times \frac{9}{5} = \frac{3}{5}$$

Here, $\frac{3}{5}$ is in its simplest form ($\gcd(3, 5) = 1$), so $m = 3$ and $n = 5$.

Therefore, $m + n = 3 + 5 = 8$.

Question 22

A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X , then the value of $64(\mu + \sigma^2)$ is:

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Options:

A. 64

B. 32

C. 51

D. 48

Answer: D

Solution:

Outcome	x_i	p_i
HHH	0	$\frac{1}{8}$
TTT	0	$\frac{1}{8}$
HHT	1	$\frac{1}{8}$
HTH	1	$\frac{1}{8}$
THH	0	$\frac{1}{8}$
TTH	0	$\frac{1}{8}$
THT	1	$\frac{1}{8}$
HTT	1	$\frac{1}{8}$

$$\begin{aligned}\mu &= \sum x_i P_i = \frac{1}{2} \\ \sigma^2 &= \sum x_i^2 P_i - \mu^2 \\ &= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\end{aligned}$$

$$\begin{aligned}64(\mu + \sigma^2) &= 64 \left[\frac{1}{2} + \frac{1}{4} \right] \\ &= 64 \times \frac{3}{4} = 48\end{aligned}$$

Question 23

If A and B are two events such that $P(A \cap B) = 0.1$, and $P(A | B)$ and $P(B | A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A} \cup \bar{B})}{P(\bar{A} \cap \bar{B})}$ is :

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Options:

- A. $\frac{4}{3}$
- B. $\frac{7}{4}$
- C. $\frac{9}{4}$
- D. $\frac{5}{3}$

Answer: C

Solution:

To solve this problem, start by considering the equation given for the probabilities:

$$12x^2 - 7x + 1 = 0$$

The roots of this equation are:

$$x = \frac{1}{3}, \frac{1}{4}$$

Assume $P(A | B) = \frac{1}{3}$ and $P(B | A) = \frac{1}{4}$.

From the definitions of conditional probability, we have:

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{4}$$

Given that $P(A \cap B) = 0.1$, we can use these equations to find $P(B)$ and $P(A)$:

$$\text{From } \frac{0.1}{P(B)} = \frac{1}{3}, \text{ we find } P(B) = 0.3.$$

$$\text{From } \frac{0.1}{P(A)} = \frac{1}{4}, \text{ we find } P(A) = 0.4.$$

Now, calculate $P(A \cup B)$ using the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - 0.1 = 0.6$$

The task is to find the value of:

$$\frac{P(\overline{A \cup B})}{P(A \cap B)}$$

Using De Morgan's laws and the complements:

$$P(\overline{A \cup B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.1 = 0.9$$

$$P(\overline{A \cap B}) = 1 - P(A \cup B) = 1 - 0.6 = 0.4$$

Finally, compute the ratio:

$$\frac{P(\overline{A \cup B})}{P(A \cap B)} = \frac{0.9}{0.4} = \frac{9}{4}$$

Question 24

One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

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Options:

A. $\frac{2}{3}$

B. $\frac{3}{5}$

C. $\frac{4}{9}$

D. $\frac{1}{2}$

Answer: D

Solution:

a = number on dice 1

b = number on dice 2

$$(a, b) = (1, 3), (3, 1), (2, 2), (2, 3), (3, 2), (1, 4), (4, 1)$$

Required probability

$$\begin{aligned} &= \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6} \\ &= \frac{18}{36} = \frac{1}{2} \end{aligned}$$

Question 25

A board has 16 squares as shown in the figure :

Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

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Options:

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{23}{30}$

D. $\frac{7}{10}$

Answer: B

Solution:

Total ways for selecting any two squares = ${}^{16}C_2 = 120$

Total ways for selecting common side squares

= 24

so required probability

$$= 1 - \frac{24}{120}$$

$$= \frac{4}{5}$$

Question 26

A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability, that A wins if A makes the first throw, is

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Options:

A. $\frac{8}{19}$

B. $\frac{9}{19}$

C. $\frac{8}{17}$

D. $\frac{9}{17}$

Answer: B

Solution:

$$P(S_5) = \frac{1}{9}$$

$$P(S_8) = \frac{5}{36}$$

$$\begin{aligned} \text{required prob} &= \frac{1}{9} + \frac{8}{9} \cdot \frac{31}{36} \cdot \frac{1}{9} + \left(\frac{8}{9} \cdot \frac{31}{36}\right)^2 \cdot \frac{1}{9} + \dots \infty \\ &= \frac{\frac{1}{9}}{1 - \frac{62}{81}} = \frac{9}{19} \end{aligned}$$

Question 27

Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability $P(E)$ is :

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Options:

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. $\frac{3}{16}$

D. $\frac{5}{8}$

Answer: A

Solution:

$$\text{C-I } \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \rightarrow 4 \text{ ways}$$

$$\text{C-II } \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \& \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \rightarrow 2 \text{ ways}$$

$$P = \frac{\text{favourable}}{\text{total}} = \frac{6}{16} = \frac{3}{8}$$

Question28

Three defective oranges are accidentally mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges, then the variance of x is

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Options:

A. $26/75$

B. $14/25$

C. $18/25$

D. $28/75$

Answer: D

Solution:

Probability distribution

x_i	p_i
$x = 0$	$\frac{{}^7C_2}{{}^{10}C_2} = \frac{42}{90}$
$x = 1$	$\frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{42}{90}$
$x = 2$	$\frac{{}^3C_2}{{}^{10}C_2} = \frac{6}{90}$

Now,

$$\mu = \sum x_i p_i = \frac{42}{90} + \frac{12}{90} = \frac{54}{90}$$

$$\sigma^2 = \sum p_i x_i^2 - \mu^2 = \frac{42}{90} + \frac{24}{90} - \left(\frac{54}{90}\right)^2$$

$$\Rightarrow \frac{66}{90} - \left(\frac{54}{90}\right)^2$$

$$\sigma^2 \Rightarrow \frac{28}{75} \therefore (1)$$

Question29

Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}$, ($i = \sqrt{-1}$) is non-zero, equals

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Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

Answer: A

Solution:

$$i^{k_1} + i^{k_2} \neq 0 \quad i^{k_1} \rightarrow 4 \text{ option for } i, -1, -i, 1$$

$$\text{Total cases} \Rightarrow 4 \times 4 = 16$$

$$\text{Unfavourable cases} \Rightarrow i^{k_1} + i^{k_2} = 0$$

$$\left\{ \begin{matrix} 1, -1 \\ -1, 1 \\ i, -i \\ -i, i \end{matrix} \right\}$$

$$4 \text{ Cases} \Rightarrow \text{Probability} = \frac{16-4}{16} = \frac{3}{4}$$

Question30

Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The

probability that the selected word will NOT have vowels in alphabetical order is:

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Options:

A.

$$\frac{1}{4}$$

B.

$$\frac{1}{2}$$

C.

$$\frac{1}{3}$$

D.

$$\frac{2}{3}$$

Answer: B

Solution:

A, E, G R D N

$$\text{Probability (P)} = \frac{\text{favourable case}}{\text{Total case}}$$

(when A & E are in order)

Total case = $6!$

Favourable case = ${}^6C_2 \cdot 4!$

$$P = \frac{(15)4!}{(30)4!}$$

$$\text{Probability when not in order} = 1 - \frac{1}{2} = \frac{1}{2}$$

Question 31

Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability that the ball is drawn from Bag B_2 is:

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Options:

A.

$$\frac{2}{5}$$

B.

$$\frac{4}{15}$$

C.

$$\frac{1}{3}$$

D.

$$\frac{2}{3}$$

Answer: B

Solution:

E_1 : Bag B_1 is selected

B_1	B_2	B_3
6 W 4 B	4 W 6 B	5 W 5 B

E_2 : bag B_2 is selected

E_3 : Bag B_3 is selected

A : Drawn ball is white

We have to find $P\left(\frac{E_2}{A}\right)$

$$\begin{aligned} P\left(\frac{E_2}{A}\right) &= \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)} \\ &= \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{6}{10} + \frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10}} \\ &= \frac{4}{15} \end{aligned}$$

Question 32

Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is $\frac{29}{45}$, then n is equal to:

Options:

A.

5

B.

6

C.

4

D.

3

Answer: B

Solution:

$$\text{Bag 1} = \{4W, 5B\}$$

$$\text{Bag 2} = \{nW, 3B\}$$

$$P\left(\frac{W}{\text{Bag2}}\right) = \frac{29}{45}$$

$$\Rightarrow P\left(\frac{W}{B_1}\right) \times P\left(\frac{W}{B_2}\right) + P\left(\frac{B}{B_1}\right) \times P\left(\frac{W}{B_2}\right) = \frac{29}{45}$$

$$\frac{4}{9} \times \frac{n+1}{n+4} + \frac{5}{9} \times \frac{n}{n+4} = \frac{29}{45}$$

$$n = 6$$

Question33

Given three indential bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the ball is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p} + \frac{1}{q}\right)$ is:

Options:

A. 6

- B. 9
C. 7
D. 8

Answer: C

Solution:

Probability that a Red ball comes from Bag I (p):

$$p(B_1/R) = \frac{p(B_1) \cdot p(R/B_1)}{p(R)}$$

Substituting the values,

$$p(B_1/R) = \frac{\frac{1}{3} \times \frac{3}{10}}{\frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{6}{10}}$$

Simplify to find p :

$$= \frac{\frac{1}{10}}{\frac{1}{3}(1.2)} = \frac{1}{4}$$

So, $p = \frac{1}{4}$.

Probability that a Green ball comes from Bag III (q):

$$p(B_3/G) = \frac{p(B_3) \cdot p(G/B_3)}{p(G)}$$

Substitute the values,

$$p(B_3/G) = \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10}}$$

Simplify to find q :

$$= \frac{\frac{2}{15}}{\frac{1}{3} \times 1.2} = \frac{1}{3}$$

So, $q = \frac{1}{3}$.

Calculation of $\left(\frac{1}{p} + \frac{1}{q}\right)$:

$$\frac{1}{p} = 4, \quad \frac{1}{q} = 3$$

Therefore,

$$\left(\frac{1}{p} + \frac{1}{q}\right) = 4 + 3 = \boxed{7}$$

Question 34

If the probability that the random variable X takes the value x is given by

$P(X = x) = k(x + 1)3^{-x}, x = 0, 1, 2, 3, \dots$, where k is a constant, then $P(X \geq 3)$ is equal to

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Options:

A. $\frac{1}{9}$

B. $\frac{8}{27}$

C. $\frac{7}{27}$

D. $\frac{4}{9}$

Answer: A

Solution:

To find $P(X \geq 3)$, we first determine the constant k using the total probability for X .

The probability $P(X = x)$ is given by:

$$P(X = x) = k(x + 1) \cdot 3^{-x}, \quad x = 0, 1, 2, 3, \dots$$

The total probability must equal 1:

$$s = \sum_{x=0}^{\infty} k(x + 1) \cdot 3^{-x}$$

Calculating that series:

$$s = \frac{k}{3^0} + \frac{2k}{3} + \frac{3k}{3^2} + \dots$$

Therefore, dividing the series by 3:

$$\frac{s}{3} = \frac{k}{3} + \frac{2k}{3^2} + \dots$$

Subtracting these:

$$s - \frac{s}{3} = k + \frac{k}{3} + \frac{k}{3^2} + \dots$$

The resulting series is a geometric series:

$$\frac{2s}{3} = k \left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots \right)$$

The sum of the infinite geometric series is:

$$\frac{2s}{3} = k \cdot \frac{1}{1 - \frac{1}{3}} = \frac{3k}{2}$$

Equating:

$$s = \frac{9k}{4} = 1$$

Thus, solving for k :

$$k = \frac{4}{9}$$

Next, compute $P(X \geq 3)$:

$$P(X \geq 3) = 1 - (P(X = 0) + P(X = 1) + P(X = 2))$$

Calculating these:

$$P(X = 0) = k = \frac{4}{9}$$

$$P(X = 1) = \frac{2k}{3} = \frac{8}{27}$$

$$P(X = 2) = \frac{3k}{9} = \frac{4}{27}$$

Adding these probabilities:

$$P(X = 0) + P(X = 1) + P(X = 2) = \frac{4}{9} + \frac{8}{27} + \frac{4}{27} = \frac{8}{9}$$

Finally, calculate $P(X \geq 3)$:

$$P(X \geq 3) = 1 - \frac{8}{9} = \frac{1}{9}$$

Question 35

A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. $\frac{11}{15}$

B. $\frac{2}{15}$

C. $\frac{3}{5}$

D. $\frac{28}{75}$

Answer: D

Solution:

X	$P(X)$	$XP(X)$	$(X_i - \mu)^2$	$P_i X(X_i - \mu)^2$
$X = 0$	$\frac{{}^7C_2}{{}^{10}C_2}$	0	$(0 - \frac{3}{5})^2$	$\frac{7}{15} \left(\frac{9}{25} \right)$
$X = 1$	$\frac{{}^7C_1 {}^3C_1}{{}^{10}C_2}$	$\frac{7}{15}$	$(1 - \frac{3}{5})^2$	$\frac{7}{15} \left(\frac{4}{25} \right)$
$x = 2$	$\frac{{}^7C_0 {}^3C_2}{{}^{10}C_2}$	$\frac{2}{15}$	$(2 - \frac{3}{5})^2$	$\frac{2}{15} \left(\frac{49}{25} \right)$

$$\mu = \sum X_i P(X_i) = 0 + \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$$

Variance (X) =

$$\sum P_i (X_i - \mu)^2 = \frac{7}{15} \left(\frac{9}{25} \right) + \frac{7}{15} \left(\frac{4}{25} \right) + \frac{2}{15} \left(\frac{49}{25} \right) = \frac{28}{75}$$

Question36

The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

- A. $\frac{129}{182}$
- B. $\frac{17}{26}$
- C. $\frac{19}{26}$
- D. $\frac{103}{182}$

Answer: A

Solution:

$3E, 1D, 8P$
 $3E, 2D, 7P$
 $4E, 1D, 7P$
 $4E, 2D, 6P$

$$P = \frac{{}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_8 + {}^4C_3 \cdot {}^2C_2 \cdot {}^{10}C_7 + {}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_7 + {}^4C_4 \cdot {}^2C_2 \cdot {}^{10}C_6}{{}^{10}C_{12}}$$
$$= \frac{129}{182}$$

Question37

Let a random variable X take values 0, 1, 2, 3 with $P(X=0)=P(X=1)=p$, $P(X=2)=P(X=3)$ and $E(X^2)=2E(X)$. Then the value of $8p-1$ is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

- A.
- 2

B.

0

C.

3

D.

1

Answer: A

Solution:

$$2p + 2q = \frac{1}{2}$$

$$p + q$$

$$E(x^2) = \sum_{i=0}^3 x_i^2 p(x_i) = 0 \cdot p + 1 \cdot p + 4 \cdot q + 9q$$

$$= p + 13q$$

$$E(x) = \sum_{i=0}^3 x_i p(x_i) = 0 \cdot p + 1 \cdot p + 2q + 3q = p + 5q$$

$$p + 13q = 2(p + 5q)$$

$$p = 3q$$

$$\text{So, } q = \frac{1}{8} \text{ \& } p = \frac{3}{8}$$

$$\text{So, } 8p - 1 = 2 \quad \text{Option (2)}$$

Question38

A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $n^2 - m^2$ is equal to :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

64

B.

80

C.

60

D.

72

Answer: B

Solution:

Define the Events:

Let A be the event that an unbiased coin is drawn.

Let B be the event that the coin with heads on both sides is drawn.

Let H be the event that a head turns up when the coin is tossed.

State the Given Probabilities:

$P(A) = \frac{19}{20}$ (since there are 19 unbiased coins out of 20 total coins)

$P(B) = \frac{1}{20}$ (since there is 1 coin with two heads out of 20 total coins)

$P(H|A) = \frac{1}{2}$ (probability of getting a head given an unbiased coin is drawn)

$P(H|B) = 1$ (probability of getting a head given the coin with two heads is drawn)

Use Bayes' Theorem:

We want to find $P(A|H)$, which is the probability that the drawn coin was unbiased given that a head turned up. Bayes' Theorem states:

$$P(A|H) = \frac{P(H|A) \cdot P(A)}{P(H)}$$

Calculate $P(H)$:

We can find $P(H)$ using the law of total probability:

$$P(H) = P(H|A) \cdot P(A) + P(H|B) \cdot P(B)$$

$$P(H) = \left(\frac{1}{2}\right) \cdot \left(\frac{19}{20}\right) + (1) \cdot \left(\frac{1}{20}\right) = \frac{19}{40} + \frac{1}{20} = \frac{19}{40} + \frac{2}{40} = \frac{21}{40}$$

Apply Bayes' Theorem to find $P(A|H)$:

$$P(A|H) = \frac{P(H|A) \cdot P(A)}{P(H)} = \frac{\left(\frac{1}{2}\right) \cdot \left(\frac{19}{20}\right)}{\frac{21}{40}} = \frac{\frac{19}{40}}{\frac{21}{40}} = \frac{19}{21}$$

Determine m and n :

Since $P(A|H) = \frac{m}{n}$ and $\gcd(m, n) = 1$, we have $m = 19$ and $n = 21$.

Calculate $n^2 - m^2$:

$$n^2 - m^2 = 21^2 - 19^2 = (21 + 19)(21 - 19) = (40)(2) = 80$$

Therefore, $n^2 - m^2 = 80$.

So, the correct answer is:

Option B: 80

Question39

If A and B are two events such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cap \overline{B}) = 0.5$, where $\overline{B}$ denotes the complement of B , then $P\left(B \mid (A \cup \overline{B})\right)$ is equal to

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

$$\frac{1}{3}$$

B.

$$\frac{1}{2}$$

C.

$$\frac{1}{4}$$

D.

$$\frac{1}{6}$$

Answer: C

Solution:

$$P(A) = \frac{7}{10}, P(B) = \frac{4}{10}$$

$$P(A \cup \overline{B}) = \frac{5}{10}$$

$$P\left(\frac{B}{A \cup \overline{B}}\right) = \frac{P(B \cap (A \cup \overline{B}))}{P(A \cup \overline{B})}$$

$$= \frac{P((B \cap \overline{B}) \cup (B \cap A))}{P(A \cup \overline{B})} = \frac{P(A \cap B)}{P(A \cup \overline{B})}$$

$$= \frac{P(A) - P(A \cap \overline{B})}{P(A) + P(\overline{B}) - P(A \cap \overline{B})} = \frac{\frac{7}{10} - \frac{5}{10}}{\frac{7}{10} + (1 - \frac{4}{10}) - \frac{5}{10}}$$

$$= \frac{2}{8} = \frac{1}{4}$$

Question 40

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3)$, $b = P(X \geq 3)$ and $c = P(X \geq 6 \mid X > 3)$. Then $b + c/a$ is equal to _____

[27-Jan-2024 Shift 1]

Answer: 12

Solution:

$$a = P(X = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$b = P(X \geq 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \dots$$

$$= \frac{\frac{25}{216}}{1 - \frac{5}{6}} = \frac{25}{216} \times \frac{6}{1} = \frac{25}{36}$$

$$P(X \geq 6) = \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \dots$$

$$= \frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$\frac{b+c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}} = 12$$

Question 41

An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

[27-Jan-2024 Shift 2]

Options:

A.

$\frac{5}{256}$

B.

5/715

C.

3/715

D.

3/256

Answer: C

Solution:

$$\frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$$

Hence option (3) is correct.

Question42

A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

[29-Jan-2024 Shift 1]

Options:

A.

5/6

B.

1/6

C.

5/11

D.

6/11

Answer: C

Solution:

$$z = \frac{1}{2} - 2i$$

$$|z+1| = \alpha z + \beta(1+i)$$

$$\left| \frac{3}{2} - 2i \right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$$

$$\left| \frac{3}{2} - 2i \right| = \left(\frac{\alpha}{2} + \beta \right) + (\beta - 2\alpha)i$$

$$\beta = 2\alpha \text{ and } \frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4}$$

$$\alpha + \beta = 3$$

Question43

An integer is chosen at random from the integers 1, 2, 3,...,50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

[29-Jan-2024 Shift 2]

Options:

A.

$$8/25$$

B.

$$21/50$$

C.

$$9/50$$

D.

$$14/25$$

Answer: B

Solution:

Given set = $\{1, 2, 3, \dots, 50\}$

$P(A)$ = Probability that number is multiple of 4

$P(B)$ = Probability that number is multiple of 6

$P(C)$ = Probability that number is multiple of 7

Now,

$$P(A) = \frac{12}{50}, P(B) = \frac{8}{50}, P(C) = \frac{7}{50}$$

again

$$P(A \cap B) = \frac{4}{50}, P(B \cap C) = \frac{1}{50}, P(A \cap C) = \frac{1}{50}$$

$$P(A \cap B \cap C) = 0$$

Thus

$$\begin{aligned} P(A \cup B \cup C) &= \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0 \\ &= \frac{21}{50} \end{aligned}$$

Question44

Two integers x and y are chosen with replacement from the set $\{0, 1, 2, 3, \dots, 10\}$. Then the probability that $|x - y| > 5$ is :

[30-Jan-2024 Shift 1]

Options:

A.

$$\frac{30}{121}$$

B.

$$\frac{62}{121}$$

C.

$$\frac{60}{121}$$

D.

$$\frac{31}{121}$$

Answer: A

Solution:

If $x = 0$, $y = 6, 7, 8, 9, 10$

If $x = 1$, $y = 7, 8, 9, 10$

If $x = 2$, $y = 8, 9, 10$

If $x = 3$, $y = 9, 10$

If $x = 4$, $y = 10$

If $x = 5$, $y =$ no possible value

Total possible ways $= (5 + 4 + 3 + 2 + 1) \times 2$ Required probability $= \frac{30}{11 \times 11} = \frac{30}{121}$

Question 45

Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is :

[30-Jan-2024 Shift 2]

Options:

A.

$\frac{1}{3}$

B.

$\frac{1}{9}$

C.

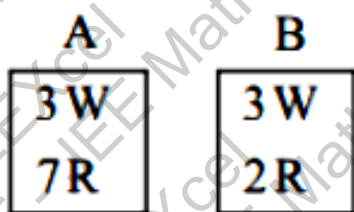
$\frac{1}{3}$

D.

$\frac{3}{10}$

Answer: C

Solution:



E_1 : A is selected

E_2 : B is selected

E : white ball is drawn

$$\begin{aligned}
 P(E_1|E) &= \frac{P(E) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)} = \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{2}{5}} \\
 &= \frac{3}{3+4} = \frac{3}{7}
 \end{aligned}$$

Question 46

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

[31-Jan-2024 Shift 1]

Options:

A.

$\frac{2}{25}$

B.

$\frac{4}{25}$

C.

$\frac{2}{3}$

D.

$\frac{4}{75}$

Answer: D

Solution:

Probability of drawing first red and then white $= \frac{10}{75} \times \frac{30}{75} = \frac{4}{75}$

Question47

Three rotten apples are accidentally mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

[31-Jan-2024 Shift 1]

Options:

A.

$\frac{37}{153}$

B.

$\frac{57}{153}$

C.

$\frac{47}{153}$

D.

$\frac{40}{153}$

Answer: D

Solution:

3 bad apples, 15 good apples.

Let X be no of bad apples

$$\text{Then } P(X=0) = \frac{{}^{15}C_2}{{}^{18}C_2} = \frac{105}{153}$$

$$P(X=1) = \frac{{}^3C_1 \times {}^{15}C_1}{{}^{18}C_2} = \frac{45}{153}$$

$$P(X=2) = \frac{{}^3C_2}{{}^{18}C_2} = \frac{3}{153}$$

$$E(X) = 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 2 \times \frac{3}{153} = \frac{51}{153} \\ = \frac{1}{3}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 \\ = 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 4 \times \frac{3}{153} - \left(\frac{1}{3}\right)^2 \\ = \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$$

Question 48

A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is-

[31-Jan-2024 Shift 2]

Options:

A.

$\frac{2}{9}$

B.

$\frac{1}{9}$

C.

$\frac{2}{27}$

D.

$\frac{1}{27}$

Answer: A

Solution:

Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head = $\frac{2}{3}$

$\therefore$ Probability of getting 2 tails and 1 head

$$= \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \right) \times 3$$

$$= \frac{2}{27} \times 3$$

$$= \frac{2}{9}$$

Question49

A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

[1-Feb-2024 Shift 1]

Options:

A.

$\frac{2}{5}$

B.

$\frac{2}{7}$

C.

$\frac{1}{7}$

D.

$\frac{1}{5}$

Answer: B

Solution:

$$P(4W4B/2W2B) =$$

$$\frac{P(4W4B) \times P(2W2B/4W4B)}{P(2W6B) \times P(2W2B/2W6B) + P(3W5B) \times P(2W2B/3W5B) + \dots + P(6W2B) \times P(2W2B/6W2B)}$$

$$= \frac{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{5} \times \frac{{}^2C_2 \times {}^6C_2}{{}^8C_4} + \frac{1}{5} \times \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} + \dots + \frac{1}{5} \times \frac{{}^6C_2 \times {}^2C_2}{{}^8C_4}}$$

$$= \frac{2}{7}$$

Question50

Let Ajay will not appear in JEE exam with probability $p = 2/7$, while both Ajay and Vijay will appear in the exam with probability $q = 1/5$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

[1-Feb-2024 Shift 2]

Options:

A.

$9/35$

B.

$18/35$

C.

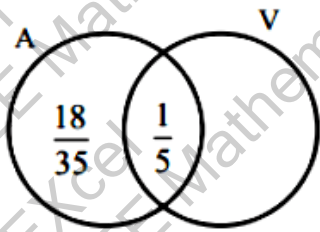
$24/35$

D.

$3/35$

Answer: B

Solution:



$$P(\bar{A}) = \frac{2}{7} = p$$

$$P(A \cap V) = \frac{1}{5} = q$$

$$P(A) = \frac{5}{7}$$

$$\text{Ans. } P(A \cap \bar{V}) = \frac{18}{35}$$

Question51

Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$$x + y + z = 1$$

$$2x + N y + 2z = 2$$

$$3x + 3y + N z = 3$$

has unique solution is $\frac{1}{k}$, then the sum of value of k and all possible values of N is [24-Jan-2023 Shift 1]

Options:

A. 18

B. 19

C. 20

D. 21

Answer: C

Solution:

$$x + y + z = 1$$

$$2x + N y + 2z = 2$$

$$3x + 3y + N z = 3$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$$= (N - 2)(N - 3)$$

For unique solution $\Delta \neq 0$

So $N \neq 2, 3$

$P(\text{system has unique solution}) = -$

So $k = 4$

Therefore sum $= 4 + 1 + 4 + 5 + 6 = 20$

Question 52

Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements :

(S1) : If $P(A) = 0$, then $A = \phi$

(S2) : If $P(A) = 1$, then $A = \Omega$ Then

[24-Jan-2023 Shift 1]

Options:

A. only (S1) is true

B. only (S2) is true

C. both (S1) and (S2) are true

D. both (S1) and (S2) are false

Answer: D

Solution:

Ω = sample space

A = be an event

$A = \left\{ \frac{1}{2} \right\}, \Omega = [0, 1]$

If $P(A) = 0$ $A \neq \phi$

If $P(A) = 1 \Rightarrow A \neq \Omega$

Then both statement are false

Question 53

Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \left\{ x \in \mathbb{Z} : x(66 - x) \geq \frac{5}{9}M \right\}$ and the event

$A = \{ x \in S : x \text{ is a multiple of } 3 \}$. Then $P(A)$ is equal to

[25-Jan-2023 Shift 1]

Options:

A. $\frac{15}{44}$

B. $\frac{1}{3}$

C. $\frac{1}{5}$

D. $\frac{7}{22}$

Answer: B

Solution:

$$M = 33 \times 33$$

$$x(66 - x) \geq \frac{5}{9} \times 33 \times 33$$

$$11 \leq x \leq 55$$

$$A : \{12, 15, 18, \dots, 54\}$$

$$P(A) = \frac{15}{45} = \frac{1}{3}$$

Question54

Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y , such that $x + y$ is divisible by 5, is _____.
[25-Jan-2023 Shift 1]

Answer: 120

Solution:

$$x + y = 5\lambda$$

Cases:

$$\text{Total} = 120$$

Question55

Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N - 2, \sqrt{3}N, N + 2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is
[25-Jan-2023 Shift 2]

Options:

A. 2

B. 4

C. 16

D. 8

Answer: B

Solution:

$$n(s) = 36$$

Given : $N - 2, \sqrt{3N}, N + 2$ are in G.P.

$$3N = (N - 2)(N + 2)$$

$$3N = N^2 - 4$$

$$\Rightarrow N^2 - 3N - 4 = 0$$

$$(N - 4)(N + 1) = 0 \Rightarrow N = 4 \text{ or } N = -1 \text{ rejected}$$

$$(\text{Sum} = 4) = \{(1, 3), (3, 1), (2, 2)\}$$

$$n(A) = 3$$

$$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow k = 4$$

Question 56

25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is

[25-Jan-2023 Shift 2]

Answer: 9

Solution:

E_1 : Smokers

$$P(E_1) = \frac{1}{4}$$

E_2 : non-smokers

$$P(E_2) = \frac{3}{4}$$

E : diagnosed with lung cancer

$$P(E/E_1) = \frac{27}{28}$$

$$P(E/E_2) = \frac{1}{28}$$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)}$$

$$= \frac{\frac{1}{4} \times \frac{27}{28}}{\frac{1}{4} \times \frac{27}{28} + \frac{3}{4} \times \frac{1}{28}} = \frac{27^9}{30_{10}} = \frac{9}{10}$$

$K = 9$

Question 57

Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is [29-Jan-2023 Shift 1]

Options:

A. $\frac{5}{24}$

B. $\frac{2}{15}$

C. $\frac{1}{6}$

D. $\frac{5}{36}$

Answer: D

Solution:

$$\text{Required probability} = 1 - \frac{D_{(15)} + {}^{15}C_1 \cdot D_{(14)} + {}^{15}C_2 D_{(13)}}{15!}$$

$$\text{Taking } D_{(15)} \text{ as } \frac{15!}{e}$$

$$D_{(14)} \text{ as } \frac{14!}{e}$$

$$D_{(13)} \text{ as } \frac{13!}{e}$$

$$\text{We get, } 1 - \left(\frac{\frac{15!}{e} + 15 \cdot \frac{14!}{e} + \frac{15 \times 14}{2} \times \frac{13!}{e}}{15!} \right)$$

$$= 1 - \left(\frac{1}{e} + \frac{1}{e} + \frac{1}{2e} \right) = 1 - \frac{5}{2e} \approx .08$$

Question 58

Let $S = \{w_1, w_2, \dots\}$ be the sample space associated to a random experiment. Let

$$P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2.$$

Let $A = \{2k + 3\ell; k, \ell \in \mathbb{N}\}$ and $B = \{w_n; n \in A\}$. Then $P(B)$ is equal to
[29-Jan-2023 Shift 2]

Options:

A. $\frac{3}{32}$

B. $\frac{3}{64}$

C. $\frac{1}{16}$

D. $\frac{1}{32}$

Answer: B

Solution:

$$\text{Let } P(w_1) = \lambda \text{ then } P(w_2) = \frac{\lambda}{2} \dots P(w_n) = \frac{\lambda}{2^{n-1}}$$

$$\text{As } \sum_{k=1}^{\infty} P(w_k) = 1 \Rightarrow \frac{\lambda}{1 - \frac{1}{2}} = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\text{So, } P(w_n) = \frac{1}{2^n}$$

$$A = \{2k + 3\ell; k, \ell \in \mathbb{N}\} = \{5, 7, 8, 9, 10, \dots\}$$

$$B = \{w_n; n \in A\}$$

$$B = \{w_5, w_7, w_8, w_9, w_{10}, w_{11}, \dots\}$$

$$A = \mathbb{N} - \{1, 2, 3, 4, 6\}$$

$$\therefore P(B) = 1 - [P(w_1) + P(w_2) + P(w_3) + P(w_4) + P(w_6)]$$

$$= 1 - \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} \right]$$

$$= 1 - \frac{32 + 16 + 8 + 4 + 1}{64} = \frac{3}{64}$$

Question 59

If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :
[30-Jan-2023 Shift 1]

Options:

A. $\frac{881}{2592}$

B. $\frac{521}{2592}$

C. $\frac{440}{2592}$

D. $\frac{27}{288}$

Answer: B

Solution:

Either all outcomes are positive or any two are negative.

$$\text{Now, } p = P(\text{positive}) = \frac{3}{6} = \frac{1}{2}$$

$$q = p(\text{negative}) = \frac{2}{6} = \frac{1}{3}$$

Required probability

$$= {}^5C_5 \left(\frac{1}{2}\right)^5 + {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{1}{2}\right)^1$$

$$= \frac{521}{2592}$$

$\therefore$ Option (2) is correct.

Question60

A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to _____.

[30-Jan-2023 Shift 2]

Answer: 14

Solution:

$$p = \frac{{}^6C_1}{6 \times 6} = \frac{1}{6}$$

$$q = \frac{{}^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$$

$$\therefore p : q = 9 : 5 \Rightarrow m + n = 14$$

Question61

A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

[31-Jan-2023 Shift 1]

Options:

A. $\frac{5}{7}$

B. $\frac{2}{7}$

C. $\frac{3}{7}$

D. $\frac{5}{6}$

Answer: A

Solution:

$$\frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2} = \frac{10 + 15}{1 + 3 + 6 + 10 + 15}$$

$$= \frac{25}{35} = \frac{5}{7}$$

Question62

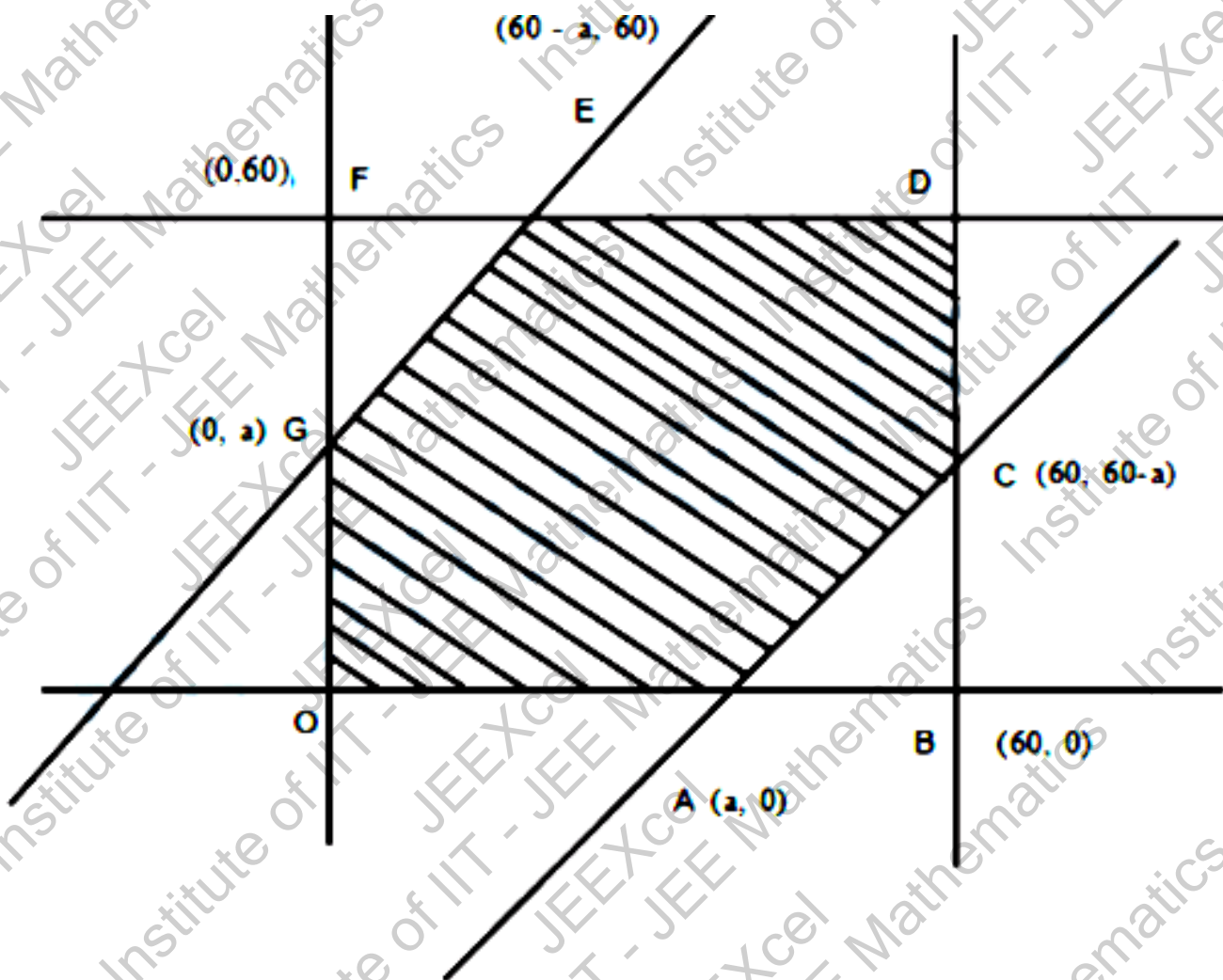
Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space $[0, 60]$ is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to _____.
[31-Jan-2023 Shift 2]

Answer: 10

Solution:

$$|x - y| < a \Rightarrow -a < x - y < a$$

$$\Rightarrow x - y < a \text{ and } x - y > -a$$



$$\begin{aligned}
 P(A) &= \frac{\text{ar}(OACDEG)}{\text{ar}(OBDF)} \\
 &= \frac{\text{ar}(OBDF) - \text{ar}(ABC) - \text{ar}(EFG)}{\text{ar}(OBDF)} \\
 \Rightarrow \frac{11}{36} &= \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600} \\
 \Rightarrow 1100 &= 3600 - (60-a)^2 \\
 \Rightarrow (60-a)^2 &= 2500 \Rightarrow 60-a = 50 \\
 \Rightarrow a &= 10
 \end{aligned}$$

Question 63

In a binomial distribution $B(n, p)$, the sum and product of the mean & variance are 5 and 6 respectively, then find $6(n + p - q)$ is equal to :-
[1-Feb-2023 Shift 1]

Options:

- A. 51
- B. 52
- C. 53
- D. 50

Answer: B

Solution:

$$np + npq = 5, np \cdot npq = 6$$

$$np(1+q) = 5, n^2 p^2 q = 6$$

$$n^2 p^2 (1+q)^2 = 25, n^2 p^2 q = 6$$

$$\frac{6}{q}(1+q)^2 = 25$$

$$6q^2 + 12q + 6 = 25q$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$(3q-2)(2q-3) = 0$$

$$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3} \text{ is accepted}$$

$$p = \frac{1}{3} \Rightarrow n \cdot \frac{1}{3} + n \cdot \frac{1}{3} \cdot \frac{2}{3} = 5$$

$$\frac{3n+2n}{9} = 5$$

$$n = 9$$

$$\text{So } 6(n+p-q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$$

Question64

Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then
[1-Feb-2023 Shift 2]

Options:

- A. the number of favourable cases of the event $(A \cup B) \cap C$ is 6
- B. A and B are mutually exclusive
- C. The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively
- D. B and C are independent

Answer: A

Solution:

$$A : \text{no. on } 1^{\text{st}} \text{ die} < \text{no. on } 2^{\text{nd}} \text{ die}$$

$$A : \text{no. on } 1^{\text{st}} \text{ die} = \text{even \& no. of } 2^{\text{nd}} \text{ die} = \text{odd}$$

$$C : \text{no. on } 1^{\text{st}} \text{ die} = \text{odd \& no. on } 2^{\text{nd}} \text{ die} = \text{even}$$

$$n(A) = 5 + 4 + 3 + 2 + 1 = 15$$

$$\begin{aligned}n(B) &= 9 \\n(C) &= 9 \\n((A \cup B) \cap C) &= (A \cap C) \cup (B \cap C) \\&= (3 + 2 + 1) + 0 = 6.\end{aligned}$$

Question65

A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to :

[6-Apr-2023 shift 1]

Options:

- A. 164
- B. 123
- C. 82
- D. 75

Answer: B

Solution:

$$\begin{aligned}n(\text{total } 5) &= \{1, 4\}, \{2, 3\}, \{3, 2\}, \{4, 1\} = 4 \\P(\text{success}) &= \frac{4}{36} = \frac{1}{9} \\P(\text{at least 4 success}) &= P(4 \text{ success}) + P(5 \text{ success}) \\&= {}^5C_4 \cdot \left(\frac{1}{9}\right)^4 \cdot \frac{8}{9} + {}^5C_5 \left(\frac{1}{9}\right)^5 = \frac{41}{9^5} = \frac{41}{3^{10}} = \frac{123}{3^{11}} = \frac{k}{3^{11}} \\K &= 123\end{aligned}$$

Question66

Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to :

[6-Apr-2023 shift 2]

Options:

- A. 1
- B. 2
- C. 4
- D. 3

Answer: C

Solution:

$$\begin{aligned}F_{av.} &= \frac{{}^6C_3(3!)}{6 \times 6 \times 6} \\&= \frac{(20)(6)}{6 \cdot 6 \cdot 6} = \frac{20}{36} = \frac{5}{9} = \frac{p}{q} \\p &= 5 \\q &= 9 \Rightarrow q - p = 4 \\Ans. Option 3\end{aligned}$$

Question67

In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, then the probability that it is manufactured by the machine C is.

[8-Apr-2023 shift 1]

Options:

A. $\frac{5}{14}$

B. $\frac{3}{7}$

C. $\frac{9}{28}$

D. $\frac{2}{7}$

Answer: A

Solution:

$$\begin{aligned}P(A) &= \frac{2}{10}, P(B) = \frac{3}{10}, P(C) = \frac{5}{10} \\P(\text{Defective} / A) &= \frac{3}{100}, P(\text{Defective} / B) = \frac{4}{100}, P(\text{Defective} / C) = \frac{2}{100} \\P(E) &= \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{2}{10} \times \frac{3}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{5}{10} \times \frac{2}{100}} = \frac{10}{6 + 12 + 10} \\&= \frac{10}{28} \\&= \frac{5}{14}\end{aligned}$$

Question68

If the probability that the random variable X takes values x is given by $P(X = x) = k(x + 1)3^{-x}$, $x = 0, 1, 2, 3, \dots$, where k is a constant, then $P(X \geq 2)$ is equal to

[8-Apr-2023 shift 2]

Options:

A. $\frac{7}{27}$

B. $\frac{11}{18}$

C. $\frac{7}{18}$

D. $\frac{20}{27}$

Answer: A

Solution:

$$\sum_{x=0}^{\infty} P(X = x) = 1$$
$$k(1 + 2 \cdot 3^{-1} + 3 \cdot 3^{-2} + 4 \cdot 3^{-3} + \dots \infty) = 1$$

$$\text{Let } s = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \dots \infty$$

$$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \dots \infty$$

$$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \infty$$

$$\frac{2s}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$$

$$s = \frac{9}{4}$$

$$\text{so } k = \frac{4}{9}$$

$$P(X \geq 2) = 1 - P(x = 0) - P(x = 1)$$

$$= 1 - \frac{4}{9} \left(1 + \frac{2}{3} \right)$$

$$= \frac{7}{27} \text{ (Option 1)}$$

Question69

Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then $4m - 3n$ equal to

∴
[10-Apr-2023 shift 1]

Options:

- A. 12
- B. 8
- C. 10
- D. 6

Answer: B

Solution:

$2^N < N!$ is satisfied for $N \geq 4$

Required probability $P(N \geq 4) = 1 - P(N < 4)$

$N = 1$ (Not possible)

$N = 2(1, 1)$

$$\Rightarrow P(N = 2) = \frac{1}{36}$$

$N = 3(1, 2), (2, 1)$

$$\Rightarrow P(N = 3) = \frac{2}{36}$$

$$P(N < 4) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

$$\therefore P(N \geq 4) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12} = \frac{m}{n}$$

$$\Rightarrow m = 11, n = 12$$

$$\therefore 4m - 3n = 4(11) - 3(12) = 8$$

Question70

Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

[10-Apr-2023 shift 2]

Options:

- A. 60
- B. 30
- C. 90
- D. 15

Answer: A

Solution:

$P(\text{odd number 7 times}) = P(\text{odd number 9 times})$

$${}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

$${}^nC_7 = {}^nC_9$$

$$\Rightarrow n = 16$$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$$

Question 71

Let $S = \{M = [a_{ij}], a_{ij} \in \{0, 1, 2\}, 1 \leq i, j \leq 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then $P(A)$ is equal to :
[11-Apr-2023 shift 1]

Options:

A. $\frac{16}{27}$

B. $\frac{50}{81}$

C. $\frac{47}{81}$

D. $\frac{49}{81}$

Answer: B

Solution:

$$M \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ where } a, b, c, d, \in \{0, 1, 2\}$$

$$n(s) = 3^4 = 81$$

we first bound $p(\overline{A})$

$$|m| = 0 \Rightarrow ad = bc$$

$$ad = bc = 0 \Rightarrow \text{no. of } (a, b, c, d) = (3^2 - 2^2)^2 = 25$$

$$ad = bc = 1 \Rightarrow \text{no. of } (a, b, c, d) = 1^2 = 1$$

$$ad = bc = 2 \Rightarrow \text{no. of } (a, b, c, d) = 2^2 = 4$$

$$ad = bc = 4 \Rightarrow \text{no. of } (a, b, c, d) = 1^2 = 1$$

$$\therefore P(\overline{A}) = \frac{31}{81} \Rightarrow P(A) = \frac{50}{81}$$

Question 72

Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to _____.

[11-Apr-2023 shift 2]

Answer: 27

Solution:

$$64x^2 + 5Nx + 1 = 0$$

$$D = 25N^2 - 256 < 0$$

$$\Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$$

$$\therefore N = 1, 2, 3$$

$$\therefore \text{Probability} = \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64}$$

$$\therefore q - p = 27$$

Question 73

Two dice A and B are rolled. Let numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisor of p is equal to

[12-Apr-2023 shift 1]

Options:

A. 36

B. 31

C. 48

D. 72

Answer: C

Solution:

$\alpha - \beta$	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) (1, 1)	6/36
-1	-----	5/36
-2	-----	4/36
-3	-----	3/36
-4	(2, 6) (1, 5)	2/36
-5	(1, 6)	1/36

$$\sum (x^2) = \sum x^2 P(x) = 2 \left[\frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36} \right]$$

$$= \frac{105}{18} = \frac{35}{6}$$

$$\mu = \sum (x) = 0 \text{ as data is symmetric}$$

$$\sigma^2 = \sum (x^2) = \sum x^2 P(x) = \frac{35}{6} P = 35 = 5 \times 7$$

$$\text{Sum of divisors} = (5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$$

Question 74

A fair n ($n > 1$) faces die is rolled repeatedly until a number less than n appears.

If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to _____.

[12-Apr-2023 shift 1]

Answer: 10

Solution:

$$\text{Mean} = 1 \cdot \frac{n-1}{n} + 2 \cdot \frac{1}{n} \left(\frac{n-1}{n} \right) + 3 \left(\frac{1}{n} \right)^2 \left(\frac{n-1}{n} \right) \dots$$

$$\frac{n}{9} = \left(\frac{n-1}{n} \right) \left(1 + 2 \left(\frac{1}{n} \right) + 3 \left(\frac{1}{n} \right)^2 + \dots \right)$$

$$\frac{n}{9} = \left(\frac{n-1}{n} \right) \left(1 - \frac{1}{n} \right)^{-2} = \left(\frac{n-1}{n} \right) \cdot \frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

Question 75

A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is-
[13-Apr-2023 shift 1]

Options:

A. $\frac{21}{16}$

B. $\frac{15}{16}$

C. $\frac{81}{64}$

D. $\frac{37}{16}$

Answer: A

Solution:

$$P(H) = \frac{3}{4}$$

$$P(T) = \frac{1}{4}$$

$$\text{Mean } \bar{X} = \frac{3}{4} + \frac{3}{8} + 3 \left(\frac{1}{64} + \frac{3}{64} \right)$$

$$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$= 3 \left(\frac{7}{16} \right) = \frac{21}{16}$$

Question 76

The random variable X follows binomial distribution $B(n, p)$, for which the difference of the mean and the variance is 1. If $2P(X = 2) = 3P(X = 1)$, then $n^2 P(X > 1)$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 16

B. 11

C. 12

D. 15

Answer: B

Solution:

$$2P(x=2) = 3P(x=1)$$

$$2 \times {}^n C_2 P^2 (1-P)^{n-2} = 3 {}^n C_1 P^1 (1-P)^{n-1}$$

$$\Rightarrow 2 \frac{n(n-1)}{2} \cdot P = 3n(1-P)$$

$$\Rightarrow (n-1)P = 3(1-P) \dots (i)$$

$$nP - nPq = 1$$

$$\Rightarrow nP - nP(1-p) = 1$$

$$\Rightarrow nP = 1 \Rightarrow n = \frac{1}{p^2}$$

$$\Rightarrow \text{put in equ (i)}$$

$$\left(\frac{1}{p^2} - 1 \right) P = 3(1-p)$$

$$\frac{1}{p} - P = 3 - 3P$$

$$\Rightarrow 1 - P^2 = 3P - 3p^2$$

$$\Rightarrow 2P^2 - 3P + 1 = 0$$

$$\Rightarrow 2P^2 - 2P - P + 1 = 0$$

$$\Rightarrow 2P(P-1) - 1(P-1) = 0$$

$$\Rightarrow P = \frac{1}{2}, P = 1 \{\text{Rejected}\}$$

$$n = \frac{1}{(1/2)^2} = 4$$

$$n^2 P(x > 1) = n^2 p(1 - P(x=0) - P(x=1))$$

$$\Rightarrow n^2 P(1 - (1-P)^n - nP(1-P)^{n-1})$$

$$\Rightarrow (4)^2 \left(1 - \left(\frac{1}{2} \right)^4 - 4 \left(\frac{2}{2} \right)^4 \right)$$

$$\Rightarrow 16 - 1 - 4 = 11$$

Question 77

A bag contains 6 white and 4 black balls. A die is rolled once and the number of ball equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is [15-Apr-2023 shift 1]

Options:

A. $\frac{1}{4}$

B. $\frac{9}{50}$

C. $\frac{11}{50}$

D. $\frac{1}{5}$

Answer: D

Solution:

$$\frac{1}{6} \times \left[\frac{{}^6C_1}{{}^{10}C_1} + \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_3}{{}^{10}C_3} + \frac{{}^6C_4}{{}^{10}C_4} + \frac{{}^6C_5}{{}^{10}C_5} + \frac{{}^6C_6}{{}^{10}C_6} \right]$$

$$= \frac{1}{6} \left(\frac{126 + 70 + 35 + 15 + 5 + 1}{210} \right) = \frac{42}{210} = \frac{1}{5}$$

Question 78

A random variable X has the following probability distribution:

X	0	1	2	3	4
P(X)	k	2k	4k	6k	8k

The value of $P(1 < X < 4 \mid X < 2)$ is equal to:
[24-Jun-2022-Shift-2]

Options:

A. $\frac{4}{7}$

B. $\frac{2}{3}$

C. $\frac{3}{7}$

D. $\frac{4}{5}$

Answer: A

Solution:

∵ x is a random variable

$$\therefore k + 2k + 4k + 6k + 8k = 1$$

$$\therefore k = \frac{1}{21}$$

$$\text{Now, } P(1 < x < 4 \mid x \leq 2) = \frac{4k}{7k} = \frac{4}{7}$$

Question79

In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then

k is equal to _____
[24-Jun-2022-Shift-2]

Answer: 479

Solution:

Student guesses only two wrong. So there are three possibilities.

- (i) Student guesses both wrong from 1st section
- (ii) Student guesses both wrong from 2nd section
- (iii) Student guesses two wrong one from each section

Required probabilities

$$\begin{aligned} &= {}^4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2 \left(\frac{1}{6}\right)^6 + {}^6C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^4 + {}^4C_1 \cdot {}^6C_1 \left(\frac{3}{4}\right) \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^3 \\ &= \frac{1}{4^{10}} [6 \times 9 + 15 \times 9^4 + 24 \times 9^2] \\ &= \frac{27}{4^{10}} [2 + 27 \times 15 + 72] \\ &= \frac{27 \times 479}{4^{10}} \end{aligned}$$

Question80

Let E_1 and E_2 be two events such that the conditional probabilities

$P(E_1 | E_2) = \frac{1}{2}$, $P(E_2 | E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then :

[25-Jun-2022-Shift-1]

Options:

A. $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$$\text{B. } P(E_1 \cap E_2') = P(E_1') \cdot P(E_2)$$

$$\text{C. } P(E_1' \cap E_2') = P(E_1) \cdot P(E_2)$$

$$\text{D. } P(E_1' \cap E_2) = P(E_1) \cdot P(E_2)$$

Answer: C

Solution:

$$P\left(\frac{E_1}{E_2}\right) = \frac{1}{2} \Rightarrow \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{2}$$

$$P\left(\frac{E_2}{E_1}\right) = \frac{3}{4} \Rightarrow \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{3}{4}$$

$$P(E_1 \cap E_2) = \frac{1}{8}$$

$$P(E_2) = \frac{1}{4}, P(E_1) = \frac{1}{6}$$

$$\text{(A) } P(E_1 \cap E_2) = \frac{1}{8} \text{ and } P(E_1) \cdot P(E_2) = \frac{1}{24}$$

$$\Rightarrow P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2)$$

$$\text{(B) } P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$$

$$= 1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{8} \right] = \frac{17}{24}$$

$$P(E_1') = \frac{3}{4} \Rightarrow P(E_1')P(E_2) = \frac{3}{24}$$

$$\Rightarrow P(E_1' \cap E_2') \neq P(E_1') \cdot P(E_2)$$

$$\text{(C) } P(E_1 \cap E_2') = P(E_1) - P(E_1 \cap E_2)$$

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$$

$$P(E_1) \cdot P(E_2) = \frac{1}{24}$$

$$\Rightarrow P(E_1 \cap E_2') = P(E_1) \cdot P(E_2)$$

$$\text{(D) } P(E_1' \cap E_2) = P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

$$P(E_1)P(E_2) = \frac{1}{24}$$

$$\Rightarrow P(E_1' \cap E_2) \neq P(E_1) \cdot P(E_2)$$

Question81

A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is :
[25-Jun-2022-Shift-2]

Options:

A. $\frac{7}{2^{11}}$

B. $\frac{7}{2^{12}}$

C. $\frac{3}{2^{10}}$

D. $\frac{13}{2^{12}}$

Answer: D

Solution:

There are only two ways to get sum 48, which are (32, 8, 8) and (16, 16, 16)

So, required probability

$$\begin{aligned} &= 3 \left(\frac{2}{32} \cdot \frac{1}{8} \cdot \frac{1}{8} \right) + \left(\frac{1}{16} \cdot \frac{1}{16} \cdot \frac{1}{16} \right) \\ &= \frac{3}{2^{10}} + \frac{1}{2^{12}} \\ &= \frac{13}{2^{12}} \end{aligned}$$

Question82

Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$).

The probability that $x_2 = 7$ and $x_4 = 11$ is :

[27-Jun-2022-Shift-1]

Options:

A. $\frac{1}{136}$

B. $\frac{1}{72}$

C. $\frac{1}{68}$

D. $\frac{1}{34}$

Answer: C

Solution:

No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3, ... n(s) = ${}^{18}C_5$

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

Question83

Let X be a random variable having binomial distribution $B(7, p)$. If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is :
[27-Jun-2022-Shift-1]

Options:

A. $\frac{105}{16}$

B. $\frac{7}{16}$

C. $\frac{77}{36}$

D. $\frac{49}{16}$

Answer: C

Solution:

$$B(7, p) \quad n = 7 \quad p = p$$

given

$$P(X = 3) = 5P(X = 4)$$

$${}^7C_3 \times p^3(1-p)^4 = 5 \cdot {}^7C_4 p^4(1-p)^3$$

$$\frac{{}^7C_3}{5 \times {}^7C_4} = \frac{p}{1-p}$$

$$1-p = 5p$$

$$6p = 1$$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

$$\begin{aligned} \text{Sum} &= \frac{7}{6} + \frac{35}{36} \\ &= \frac{42 + 35}{36} \\ &= \frac{77}{36} \end{aligned}$$

Question84

If a point A(x, y) lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is
[27-Jun-2022-Shift-2]

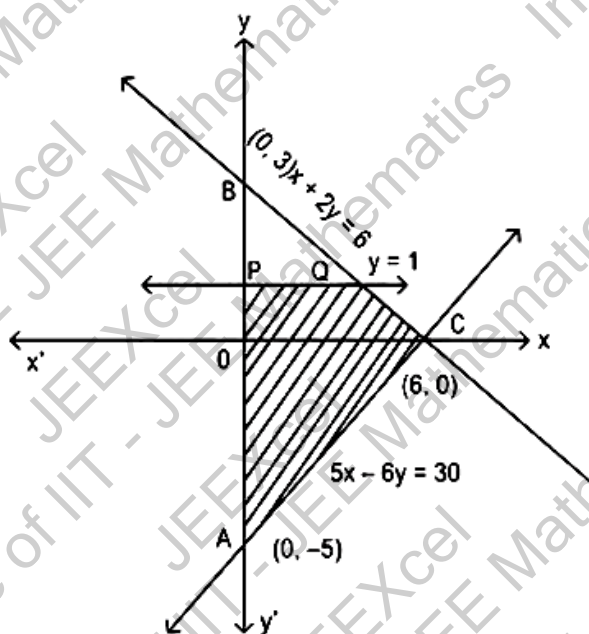
Options:

- A. $\frac{1}{6}$
- B. $\frac{5}{6}$
- C. $\frac{2}{3}$
- D. $\frac{6}{7}$

Answer: B

Solution:

The required probability



$$\begin{aligned}
 &= \frac{\text{Area of Region PQCAP}}{\text{Area of Region ABCA}} \\
 &= \frac{\frac{1}{2} \times 8 \times 6 - \frac{1}{2} \times 2 \times 4}{\frac{1}{2} \times 8 \times 6} \\
 &= \frac{5}{6}
 \end{aligned}$$

Question85

Let $S = \{E_1, E_2, \dots, E_8\}$ be a sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every $n = 1, 2, \dots$

8. Then the number of elements in the set $\{A \subseteq S : P(A) \geq \frac{4}{5}\}$ is _____
[27-Jun-2022-Shift-2]

Answer: 19

Solution:

$$\text{Here } P(E_n) = \frac{n}{36} \text{ for } n = 1, 2, 3, \dots, 8$$

$$\text{Here } P(A) = \frac{\text{Any possible sum of } (1, 2, 3, \dots, 8) (= a \text{ say})}{36}$$

$$\therefore \frac{a}{36} \geq \frac{4}{5}$$

$$\therefore a \geq 29$$

If one of the number from $\{1, 2, \dots, 8\}$ is left then total $a \geq 29$ by 3 ways.

Similarly by leaving terms more 2 or 3 we get 16 more combinations.

$\therefore$ Total number of different set A possible is $16 + 3 = 19$

Question86

The probability, that in a randomly selected 3-digit number at least two digits are odd, is
[28-Jun-2022-Shift-1]

Options:

A. $\frac{19}{36}$

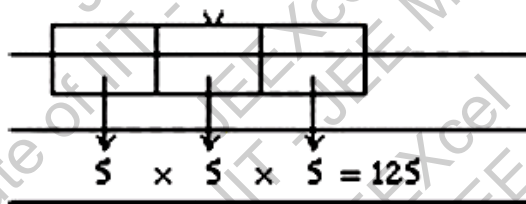
B. $\frac{15}{36}$

C. $\frac{13}{36}$

D. $\frac{23}{36}$

Answer: A

Solution:



Atleast two digits are odd=Exactly 2 digits odd+exactly 3 digits odd

For exactly three digits are odd $= 5 \times 5 \times 5 = 125$ For exactly two digits odd :

If 0 is used then no of 3 digit numbers with 2 odd digits $= 2 \times 5 \times 5 = 50$

If 0 is not used then no of 3 digit numbers with 2 odd digits

$$= {}^3C_1 \times 4 \times 5 \times 5 = 300$$

$$\text{Required Probability} = \frac{125 + 50 + 300}{900} = \frac{475}{900} = \frac{19}{36}$$

Question87

Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to

[24-Jun-2022-Shift-1]

Options:

A. 13

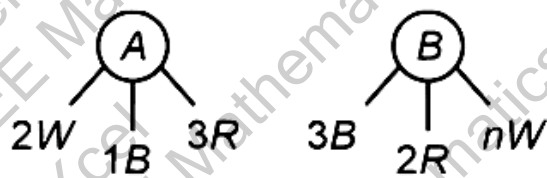
B. 6

C. 4

D. 3

Answer: C

Solution:



$$P(1R \text{ and } 1B) = P(A) \cdot P\left(\frac{1R1B}{A}\right) + P(B) \cdot P\left(\frac{1R1B}{B}\right)$$

$$= \frac{1}{2} \cdot \frac{{}^3C_1 \cdot {}^1C_1}{{}^6C_2} + \frac{1}{2} \cdot \frac{{}^2C_1 \cdot {}^3C_1}{{}^{n+5}C_2}$$

$$P\left(\frac{1R1B}{A}\right) = \frac{\frac{1}{2} \cdot \frac{3}{15}}{\frac{1}{2} \cdot \frac{3}{15} + \frac{1}{2} \cdot \frac{6 \cdot 2}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{11}{10} = \frac{6}{10} + \frac{36}{(n+5)(n+4)}$$

$$\Rightarrow \frac{5}{10 \times 36} = \frac{1}{(n+5)(n+4)}$$

$$\Rightarrow n^2 + 9n - 52 = 0$$

$$\Rightarrow n = 4 \text{ is only possible value}$$

Question88

If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X = 0) = P(X = 1)$, then the value of $\frac{P(X = 15)}{P(X = 18)} - \frac{P(X = 16)}{P(X = 17)}$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. 1320

B. 1088

C. $\frac{120}{1331}$

D. $\frac{1088}{1089}$

Answer: A

Solution:

$$3P(X=0) = P(X=1)$$

$$3 \cdot {}^nC_0 P^0 (1-P)^n = {}^nC_1 P^1 (1-P)^{n-1}$$

$$\frac{3}{n} = \frac{P}{1-P} \Rightarrow \frac{1}{11} = \frac{P}{1-P}$$

$$\Rightarrow 1-P = 11P$$

$$\Rightarrow P = \frac{1}{12}$$

$$\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$$

$$\Rightarrow \frac{{}^{33}C_{15} P^{15} (1-P)^{18}}{{}^{33}C_{18} P^{18} (1-P)^{15}} - \frac{{}^{33}C_{16} P^{16} (1-P)^{17}}{{}^{33}C_{17} P^{17} (1-P)^{16}}$$

$$\Rightarrow \left(\frac{1-P}{P} \right)^3 - \left(\frac{1-P}{P} \right)$$

$$\Rightarrow 11^3 - 11 = 1320$$

Question89

If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is:
[25-Jul-2022-Shift-1]

Options:

A. $\frac{33}{2^{32}}$

B. $\frac{33}{2^{29}}$

C. $\frac{33}{2^{28}}$

D. $\frac{33}{2^{27}}$

Answer: C

Solution:

If n is number of trials, p is probability of success and q is probability of unsuccess then,

Mean = np and variance = npq.

Here

np + npq = 24 ... (i)

np . npq = 128 ... (ii)

and q = 1 - p ... (iii)

from eq. (i), (ii) and (iii) : $p = q = \frac{1}{2}$ and $n = 32$.

$\therefore$ Required probability = $p(X=1) + p(X=2)$

$$\begin{aligned}
 &= {}^{32}C_1 \cdot \left(\frac{1}{2}\right)^{32} + {}^{32}C_2 \cdot \left(\frac{1}{2}\right)^{32} \\
 &= \left(32 + \frac{32 \times 31}{2}\right) \cdot \frac{1}{2^{32}} \\
 &= \frac{33}{2^{28}}
 \end{aligned}$$

Question90

If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then

$P(A | B') + P(B | A')$ is equal to

[25-Jul-2022-Shift-2]

Options:

A. $\frac{3}{4}$

B. $\frac{5}{8}$

C. $\frac{5}{4}$

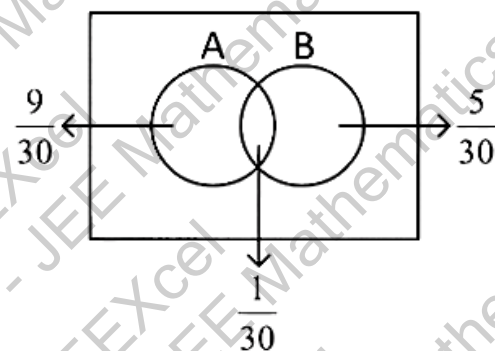
D. $\frac{7}{8}$

Answer: B

Solution:

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{5} \text{ and } P(A \cup B) = \frac{1}{2}$$

$$\therefore P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}$$



$$\text{Now, } P(A | B') + P(B | A') = \frac{P(A \cap B')}{P(B')} + \frac{P(B \cap A')}{P(A')}$$

$$= \frac{\frac{9}{30}}{\frac{4}{5}} + \frac{\frac{5}{30}}{\frac{2}{3}} = \frac{5}{8}$$

Question91

The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If

$P(X = 1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to :

[26-Jul-2022-Shift-1]

Options:

A. $\frac{5}{9}$

B. $\frac{64}{81}$

C. $\frac{16}{27}$

D. $\frac{145}{243}$

Answer: C

Solution:

Given, mean = $np = \alpha$.

and variance = $npq = \frac{\alpha}{3}$

$$\Rightarrow q = \frac{1}{3} \text{ and } p = \frac{2}{3}$$

$$P(X = 1) = n \cdot p^1 \cdot q^{n-1} = \frac{4}{243}$$

$$\Rightarrow n \cdot \frac{2}{3} \cdot \left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$$

$$\Rightarrow n = 6$$

$$P(X = 4 \text{ or } 5) = {}^6C_4 \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^2 + {}^6C_5 \cdot \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3}$$
$$= \frac{16}{27}$$

Question92

Let E_1, E_2, E_3 be three mutually exclusive events such that

$P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2 , then $(p_1 + p_2)$ is equal to

[26-Jul-2022-Shift-1]

Options:

A. $\frac{2}{3}$

B. $\frac{5}{3}$

C. $\frac{5}{4}$

D. 1

Answer: B

Solution:

$$0 \leq \frac{2+3P}{6} \leq 1 \Rightarrow P \in \left[-\frac{2}{3}, \frac{4}{3}\right]$$

$$0 \leq \frac{2-P}{8} \leq 1 \Rightarrow P \in [-6, 2]$$

$$0 \leq \frac{1-P}{2} \leq 1 \Rightarrow P \in [-1, 1]$$

$$0 < P(E_1) + P(E_2) + P(E_3) \leq 1$$

$$0 < \frac{13}{12} - \frac{P}{8} \leq 1$$

$$P \in \left[\frac{2}{3}, \frac{26}{3}\right]$$

Taking intersection of all

$$P \in \left[\frac{2}{3}, 1\right)$$

$$P_1 + P_2 = \frac{5}{3}$$

Question93

Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$.

Then, $54P(X \leq 2)$ is equal to
[26-Jul-2022-Shift-2]

Options:

A. $\frac{73}{27}$

B. $\frac{146}{27}$

C. $\frac{146}{81}$

D. $\frac{126}{81}$

Answer: B

Solution:

$$\text{Mean} = 4 = \mu = np$$

$$\text{Variance} = \sigma^2 = np(1-P) = \frac{4}{3}$$

$$4(1-P) = \frac{4}{3}$$

$$P = \frac{2}{3}$$

$$n \times \frac{2}{3} = 4$$

$$n = 6$$

$$P(X = k) = {}^nC_k P^k (1-P)^{n-k}$$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6C_0 P^0 (1-P)^6 + {}^6C_1 P^1 (1-P)^5 + {}^6C_2 P^2 (1-P)^4$$

$$= {}^6C_0 \left(\frac{1}{3}\right)^6 + {}^6C_1 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^5 + {}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4$$

$$= \left(\frac{1}{3}\right)^6 [1 + 12 + 60] = \frac{73}{3^6}$$

$$54P(X \leq 2) = \frac{73}{3^6} \times 54 = \frac{146}{27}$$

Question94

Let X have a binomial distribution $B(n, p)$ such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$,

then k is equal to :

[27-Jul-2022-Shift-2]

Options:

A. 528

B. 529

C. 629

D. 630

Answer: B

Solution:

$$\text{Mean} = np = 16$$

$$\text{Variance} = npq = 8$$

$$\Rightarrow q = p = \frac{1}{2} \text{ and } n = 32$$

$$P(X > n - 3) = p(X = n - 2) + p(X = n - 1) + p(X = n)$$

$$= ({}^{32}C_2 + {}^{32}C_1 + {}^{32}C_0) \cdot \frac{1}{2^n}$$

$$= \frac{529}{2^n}$$

Question95

A six faced die is biased such that

$$3 \times P(\text{a prime number}) = 6 \times P(\text{a composite number}) = 2 \times P.$$

Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :

[27-Jul-2022-Shift-2]

Options:

A. $\frac{3}{11}$

B. $\frac{5}{11}$

C. $\frac{7}{11}$

D. $\frac{8}{11}$

Answer: D

Solution:

$$\text{Let } P(\text{a prime number}) = \alpha$$

$$P(\text{a composite number}) = \beta$$

$$\text{and } P(1) = \gamma$$

$$\therefore 3\alpha = 6\beta = 2\gamma = k \text{ (say)}$$

$$\text{and } 3\alpha + 2\beta + \gamma = 1$$

$$\Rightarrow k + \frac{k}{3} + \frac{k}{2} = 1 \Rightarrow k = \frac{6}{11}$$

$$\text{Mean} = np \text{ where } n = 2$$

$$\text{and } p = \text{probability of getting perfect square}$$

$$= P(1) + P(4) = \frac{k}{2} + \frac{k}{6} = \frac{4}{11}$$

$$\text{So, mean} = 2 \cdot \left(\frac{4}{11} \right) = \frac{8}{11}$$

Question96

Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is :

[28-Jul-2022-Shift-1]

Options:

A. $\frac{3}{4}$

B. $\frac{11}{16}$

C. $\frac{23}{32}$

D. $\frac{13}{16}$

Answer: A

Solution:

$$P(\text{Female}) = \frac{60}{100} = \frac{3}{5}$$

$$P(\text{Male}) = \frac{2}{5}$$

$$P(\text{Female/Qualified}) = \frac{40}{60} = \frac{2}{3}$$

$$P(\text{Male/qualified}) = \frac{20}{60} = \frac{1}{3}$$

Question 97

Let A and B be two events such that $P(B | A) = \frac{2}{5}$, $P(A | B) = \frac{1}{7}$ and

$P(A \cap B) = \frac{1}{9}$. Consider

(S1) $P(A' \cup B) = \frac{5}{6}$,

(S2) $P(A' \cap B') = \frac{1}{18}$

Then

[28-Jul-2022-Shift-2]

Options:

A. Both (S1) and (S2) are true

B. Both (S1) and (S2) are false

C. Only (S1) is true

D. Only (S2) is true

Answer: A

Solution:

$$P(A/B) = \frac{1}{7} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{7}$$

$$\Rightarrow P(B) = \frac{7}{9}$$

$$P(B/A) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{2}{5}$$

$$P(A) = \frac{5}{2} \cdot \frac{1}{9} = \frac{5}{18}$$

$$S2 : P(A' \cap B') = \frac{1}{18}$$

$$S1 : \text{and } P(A' \cup B) = \frac{1}{9} + \frac{6}{9} + \frac{1}{18} = \frac{5}{6}.$$

Question98

A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X , then $100\sigma^2$ is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 56

Solution:

X = Number of white ball drawn

$$P(X = 0) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{1}{6}$$

$$P(X = 1) = \frac{{}^6C_2 \times {}^4C_1}{{}^{10}C_3} = \frac{1}{2}$$

$$P(X = 2) = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{3}{10}$$

$$\text{and } P(X = 3) = \frac{{}^6C_0 \times {}^4C_3}{{}^{10}C_3} = \frac{1}{30}$$

$$\text{Variance} = \sigma^2 = \sum P_i X_i^2 - \left(\sum P_i X_i \right)^2$$

$$\sigma^2 = \frac{1}{2} + \frac{12}{10} + \frac{3}{10} - \left(\frac{1}{2} + \frac{6}{10} + \frac{1}{10} \right)^2$$

$$= \frac{56}{100}$$

$$100\sigma^2 = 56$$

Question99

Let $S = \{1, 2, 3, \dots, 2022\}$. Then the probability, that a randomly chosen number n from the set S such that $\text{HCF}(n, 2022) = 1$, is :

[29-Jul-2022-Shift-1]

Options:

A. $\frac{128}{1011}$

B. $\frac{166}{1011}$

C. $\frac{127}{337}$

D. $\frac{112}{337}$

Answer: D

Solution:

$$S = \{1, 2, 3, \dots, 2022\}$$

$$\text{HCF}(n, 2022) = 1$$

$\Rightarrow n$ and 2022 have no common factor

$$\text{Total elements} = 2022$$

$$2022 = 2 \times 3 \times 337$$

M : numbers divisible by 2.

$$\{2, 4, 6, \dots, 2022\} \quad n(M) = 1011$$

N : numbers divisible by 3.

$$\{3, 6, 9, \dots, 2022\} \quad n(N) = 674$$

L : numbers divisible by 6.

$$\{6, 12, 18, \dots, 2022\} \quad n(L) = 337$$

$$n(M \cup N) = n(M) + n(N) - n(L)$$

$$= 1011 + 674 - 337$$

$$= 1348$$

O = Number divisible by 337 but not in $M \cup N$

$$\{337, 674\}$$

Number divisible by 2, 3 or 337

$$= 1348 + 2 = 1350$$

$$\text{Required probability} = \frac{2022 - 1350}{2022}$$

$$= \frac{672}{2022}$$

$$= \frac{112}{337}$$

Question 100

Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

[29-Jul-2022-Shift-2]

Options:

A. $\frac{4}{9}$

B. $\frac{5}{18}$

C. $\frac{1}{6}$

D. $\frac{3}{10}$

Answer: B

Solution:

A : Drown ball from boy II is black

B : Red ball transferred

$$\begin{aligned} P\left(\frac{B}{A}\right) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{\frac{3}{9} \times \frac{5}{10}}{\frac{3}{9} \times \frac{5}{10} + \frac{4}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{5}{10}} \\ &= \frac{15}{15 + 24 + 15} = \frac{15}{54} = \frac{5}{18} \end{aligned}$$

Question101

The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is _____.
[29-Jul-2022-Shift-2]

Answer: 96

Solution:

Let two roots of a quadratic equation are mean = np and variance = npq.

Given np + npq = 82.5

and np(npq) = 1350

∴ Quadratic equation is

$$x^2 - 82.5x + 1350 = 0$$

$$\Rightarrow x^2 - 22.5x - 60x + 1350 = 0$$

$$\Rightarrow x - (x - 22.5) - 60(x - 22.5) = 0$$

Mean = 60 and Variance = 22.5

$$np = 60, npq = 22.5$$

$$\Rightarrow q = \frac{9}{24} = \frac{3}{8}, p = \frac{5}{8}$$

$$\therefore n \cdot \frac{5}{8} = 60 \Rightarrow n = 96$$

Question 102

A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is
[2021, 26 Feb. Shift-II]

Options:

A. $\frac{6}{7}$

B. $\frac{1}{7}$

C. $\frac{3}{7}$

D. $\frac{4}{7}$

Answer: C

Solution:

Given, digits 3, 3, 4, 4, 4, 5, 5

Total 7 digit number from given digits will be

$$\frac{7!}{3!2!2!} = \frac{7 \times 6 \times 5 \times 4}{2 \times 2} = 210$$

Now, if the 7 digit number is divisible by 2, then last digit must be 4.

Now, 7 digit number divisible by 2 will be

$$= \frac{6!}{2!2!2!} = \frac{6 \times 5 \times 4 \times 3}{2 \times 2} = 90$$

$$\text{Required probability} = \frac{90}{210} = \frac{9}{21} = \frac{3}{7}$$

Question 103

In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10%, respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is
[2021, 25 Feb. Shift-II]

Options:

A. $\frac{7}{45}$

B. $\frac{8}{45}$

C. $\frac{28}{45}$

D. $\frac{14}{45}$

Answer: C

Solution:

Let 'A' be the event of smokers and non-vegetarian.

Let 'B' be the event of smokers and vegetarian.

Let 'C' be the event of non-smokers and vegetarian.

Let 'E' be the event of chest disorders.

According to question,

$$P(A) = \frac{160}{400} = \frac{2}{5}, P(B) = \frac{100}{400} = \frac{1}{4}$$

$$P(C) = \frac{140}{400} = \frac{7}{20}, P\left(\frac{E}{A}\right) = 35\% = \frac{35}{100},$$

$$P\left(\frac{E}{B}\right) = \frac{20}{100}, P\left(\frac{E}{C}\right) = \frac{10}{100}$$

Thus,

$$P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right) + P(C)P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{2}{5} \cdot \frac{35}{100}}{\frac{2}{5} \cdot \frac{35}{100} + \frac{1}{4} \cdot \frac{20}{100} + \frac{7}{20} \cdot \frac{10}{100}}$$

$$= \frac{14}{100}$$

$$\frac{14}{100} + \frac{5}{100} + \frac{7}{100} = \frac{28}{100} = \frac{7}{25}$$

Question 104

Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α , only B_2 occurs is β and only B_3 occurs is γ . Let P be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$ (All the probabilities are assumed

to lie in the interval $(0, 1)$). Then, $\frac{P(B_1)}{P(B_3)}$ is equal to

[2021, 24 Feb. Shift-I]

Answer: 6

Solution:

Given, B_1, B_2 and B_3 are three independent events.

Let x, y, z be the probability of B_1, B_2, B_3 respectively.

$$P(\text{only } B_1 \text{ occur}) = \alpha$$

$$P(B_1) \cdot P(\bar{B}_2) \cdot P(\bar{B}_3) = \alpha$$

$$\Rightarrow x \cdot (1 - y) \cdot (1 - z) = \alpha$$

$$\Rightarrow P(\text{only } B_2 \text{ occur}) = \beta$$

$$P(\bar{B}_1) \cdot P(B_2) \cdot P(\bar{B}_3) = \beta$$

$$\Rightarrow (1 - x) \cdot y \cdot (1 - z) = \beta$$

$$P(\text{none occur}) = P$$

$$P(\bar{B}_1) \cdot P(\bar{B}_2) \cdot P(\bar{B}_3) = P$$

$$\Rightarrow (1 - x) \cdot (1 - y) \cdot (1 - z) = P$$

Now, we have given relations

$$(\alpha - 2\beta)P = \alpha\beta$$

$$\Rightarrow [x(1 - y)(1 - z) - 2y(1 - x)(1 - z)]$$

$$(1 - x)(1 - y)(1 - z)$$

$$= x \cdot (1 - y)(1 - z) - y(1 - x)(1 - z)$$

$$[\text{putting the value of } \alpha, \beta, P]$$

$$\Rightarrow (1 - z)[x(1 - y) - 2y(1 - x)] = x \cdot y \cdot (1 - z)$$

$$\Rightarrow x - xy - 2y + 2xy = xy$$

$$\Rightarrow x = 2y \dots (i)$$

Similarly, on solving the second relation,

$$(\beta - 3\gamma)P = 2\beta\gamma \text{ by putting } \beta, \gamma \text{ and } P,$$

$$\text{We get, } y = 3z \dots (ii)$$

From Eqs. (i) and (ii), we get

$$x = 2 \times 3z \Rightarrow x = 6z$$

$$\Rightarrow \frac{x}{z} = 6$$

$$\text{Now, } \frac{P(B_1)}{P(B_3)} = \frac{x}{z} = 6$$

$\therefore$ Required ratio is 6.

Question 105

When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability

that all three hit the target, is

[2021, 25 Feb. Shift-1]

Options:

A. $\frac{1}{27}$

B. $\frac{3}{4}$

C. $\frac{1}{8}$

D. $\frac{3}{8}$

Answer: C

Solution:

Probability of missile to get intercepted $= \frac{1}{3}$

$\therefore$ Probability of missile to not get intercepted $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability of missile to hit the target $= \frac{3}{4}$

$\therefore$ Probability of three missiles to hit the target

$= \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{8}$

Question 106

An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

24 Feb 2021 Shift 1

Options:

A. $\frac{1}{32}$

B. $\frac{5}{16}$

C. $\frac{3}{16}$

D. $\frac{1}{2}$

Answer: D

Solution:

$${}^nC_2 \left(\frac{1}{2}\right)^n = {}^nC_3 \left(\frac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3$$

$$\Rightarrow n = 5$$

Probability of getting an odd number for odd number of times is

$${}^5C_1 \left(\frac{1}{2}\right)^5 + {}^5C_2 \left(\frac{1}{2}\right)^5 + {}^5C_3 \left(\frac{1}{2}\right)^5$$

$$= \frac{1}{2^5} (5 + 10 + 1) = \frac{1}{2}$$

Question 107

Let B_i ($i = 1, 2, 3$) be three independent events in a sample space. The probability that only B_1 occur is α . Only B_2 occurs is β and only B_3 occurs is γ . Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval $(0, 1)$). Then $\frac{P(B_1)}{P(B_3)}$ is equal to

24 Feb 2021 Shift 1

Answer: 6

Solution:

Let $P(B_1) = p_1$, $P(B_2) = p_2$, $P(B_3) = p_3$

given that $p_1(1 - p_2)(1 - p_3) = \alpha$

$p_2(1 - p_1)(1 - p_3) = \beta$ (i)

$p_3(1 - p_1)(1 - p_2) = \gamma$

and $(1 - p_1)(1 - p_2)(1 - p_3) = p$ (ii)

$$\Rightarrow \frac{p_1}{1 - p_1} = \frac{\alpha}{p}, \frac{p_2}{1 - p_2} = \frac{\beta}{p}, \frac{p_3}{1 - p_3} = \frac{\gamma}{p}$$

$$\text{Also } \beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$$

$$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$$

$$\Rightarrow \alpha p - 6p\gamma - 5\alpha\gamma$$

$$\Rightarrow \frac{p_1}{1 - p_1} - \frac{6p_3}{1 - p_3} = \frac{5p_1 p_3}{(1 - p_1)(1 - p_3)}$$

$$\Rightarrow p_1 - 6p_3 = 0$$

$$\Rightarrow \frac{p_1}{p_3} = 6$$

Question 108

Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then, the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is
[2021, 25 Feb. Shift-11]

Options:

- A. $\frac{1}{5}$
- B. $\frac{2}{9}$
- C. $\frac{97}{297}$
- D. $\frac{122}{297}$

Answer: C

Solution:

Here, we have four digit natural numbers, then total cases will exclude those number which contain zero at thousands place.

Hence, total cases will be

$$= ({}^4C_1 \times 9 \times 9 \times 9) - ({}^3C_1 \times 9 \times 9) = 2673$$

Again, only those numbers will have remainder 2 when divided by 5 either they have 2 at its unit place or 7 at its unit place.

When unit digit is 2, then total number of four digit numbers will be

$$= ({}^3C_1 \times 9 \times 9) - ({}^2C_1 \times 9) = 225$$

When unit digit is 7, then total number of four digit numbers will be $= 8 \times 9 \times 9 = 648$

Now, total favourable cases

$$= 225 + 648 = 873$$

Required probability

$$= \frac{\text{Total favourable cases}}{\text{Total number of cases}} \\ = \frac{873}{2673} = \frac{97}{297}$$

Question109

The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is
[2021, 25 Feb. Shift-1]

Options:

- A. $\frac{1}{72}$
- B. $\frac{5}{216}$

C. $\frac{1}{36}$

D. $\frac{1}{54}$

Answer: B

Solution:

Given, $ax^2 + bx + c = 0$

According to the question,

$a, b, c \in \{1, 2, 3, 4, 5, 6\}$

$\therefore n(s) = 6 \times 6 \times 6 = 216$

$\therefore D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

$\Rightarrow ac = \frac{b^2}{4}$

If $b = 2, ac = 1 \Rightarrow (a = 1, c = 1)$

If $b = 4, ac = 4 \Rightarrow (a = 1, c = 4)$

or $(a = 4, c = 1)$

or $(a = 2, c = 2)$

If $b = 6, ac = 9 \Rightarrow (a = 3, c = 3)$

$\therefore$ Required probability $= 5/216$

Question 110

The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is
[2021, 24 Feb. Shift-II]

Options:

A. $\frac{65}{2^7}$

B. $\frac{65}{2^8}$

C. $\frac{135}{2^9}$

D. $\frac{35}{2^7}$

Answer: C

Solution:

Given, set $= \{1, 2, 3, 4, 5\}$

Let the two subsets be A and B.

Then, $n(A \cap B) = 2$ (as given in question)

∴ Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5} = \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

Question111

Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is

[2021, 17 March Shift-1]

Options:

A. $\frac{4}{9}$

B. $\frac{17}{36}$

C. $\frac{5}{12}$

D. $\frac{1}{2}$

Answer: B

Solution:

Six faces have numbers {1, 2, 3, 5, 7, 11}.

Sum of the numbers on top faces of both the sides is less than or equal to 8

$$= (1, 1), (1, 2), (1, 3), (1, 5), (1, 7) \rightarrow 5$$

$$(2, 1), (2, 2), (2, 3), (2, 5) \rightarrow 4$$

$$(3, 1), (3, 2), (3, 3), (3, 5) \rightarrow 4$$

$$(5, 1), (5, 2), (5, 3) \rightarrow 3$$

$$(7, 1) \rightarrow 1$$

$$n(S) = 5 + 4 + 4 + 3 + 1 = 17$$

$$n(T) = 6 \times 6 = 36$$

$$\therefore \text{Probability} = \frac{17}{36}$$

Question112

Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then, the probability that

'10' is followed by '01' is equal to

[2021, 17 March Shift-II]

Options:

A. $\frac{1}{18}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: D

Solution:

Probability of 0 at even place =

$$P(0 \text{ at even place}) = \frac{1}{2} \text{ (given)}$$

$$\text{and } P(0 \text{ at odd places}) = \frac{1}{3} \text{ (given)}$$

$$\therefore P(1 \text{ at even place}) = \frac{1}{2}$$

$$\text{and } P(1 \text{ at odd place}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore P(10 \text{ is followed by } 01) = \text{Concepts}$

$$\left(\frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} \right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} \times \frac{2}{3} \right) \\ = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$$

Question 113

Let A denote the event that a 6 -digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3 . Then, probability of event A is equal to [2021, 16 March Shift-II]

Options:

A. $\frac{9}{56}$

B. $\frac{4}{9}$

C. $\frac{3}{7}$

D. $\frac{11}{27}$

Answer: B

Solution:

Given, digits = {0, 1, 2, 3, 4, 5, 6}

Number of ways in which 6 digit number can be formed using these 7 digits = $6 \cdot 6!$

Number of ways (0 can't be filled here)

If the required number is divisible by 3, then the sum of the digits must be divisible by 3.

Sum of all 7 digits

$$= 0 + 1 + 2 + 3 + 4 + 5 + 6$$

$$= \frac{6 \cdot 7}{2} = 21$$

Now, this further implies we have to remove anyone digit from the given digits. We can remove only the multiple of 3 as the sum of all 7 digits is already 21. So, removing any other digit will lead to a number of 6 digit which will not be a multiple of 3.

Possible digits $\Rightarrow \{1, 2, 3, 4, 5, 6\}$

$\{0, 1, 2, 4, 5, 6\}$

$\{0, 1, 2, 3, 4, 5\}$

Case I $\{1, 2, 3, 4, 5, 6\}$

Number of 6 digit numbers = $6!$

Case II $\{0, 1, 2, 4, 5, 6\}$

Number of 6 digit numbers = $5 \cdot 5!$

Case III $\{0, 1, 2, 3, 4, 5\}$

Number of 6 digit numbers = $5 \cdot 5!$

$$\therefore \text{Probability} = \left(\frac{6! + 5 \cdot 5! + 5 \cdot 5!}{6 \cdot 6!} \right)$$

$$= \left(\frac{1}{6} + \frac{5}{36} + \frac{5}{36} \right) = \frac{16}{36} = \frac{4}{9}$$

Question 114

A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is [2021, 16 March Shift-1]

Options:

A. $\frac{3}{4}$

B. $\frac{52}{867}$

C. $\frac{39}{50}$

D. $\frac{22}{425}$

Answer: C

Solution:

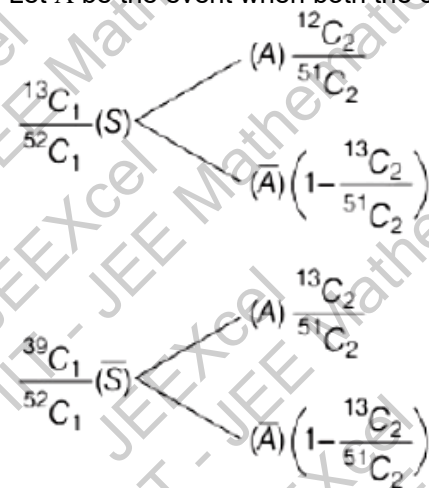
Let S and NS be the respective events of choosing a spade card and a card which is not a spade.

$$P(S) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{1}{4}$$

$$P(NS) = \frac{{}^{39}C_1}{{}^{52}C_1} = \frac{3}{4}$$

Let A be the event when both the chosen cards are spade.

Let $\bar{A}$ be the event when both the chosen cards are not spade.



We have to find the probability of missing card not being spade when both the chosen cards are spade $= P(\bar{S} | A)$ By using Bayes theorem,

$$\begin{aligned}
 P(\bar{S} | A) &= \frac{P(\bar{S} \cap A)}{P(A)} \\
 &= \frac{P(A | \bar{S})P(\bar{S})}{P(A | S)P(S) + P(A | \bar{S})P(\bar{S})} \\
 &= \frac{\left(\frac{13C_2}{51C_2}\right) \left(\frac{39C_1}{52C_1}\right)}{\left(\frac{13C_1}{52C_1}\right) \left(\frac{12C_2}{51C_2}\right) + \left(\frac{13C_2}{51C_2}\right) \left(\frac{39C_1}{52C_1}\right)} \\
 &= \frac{\left(\frac{13 \cdot 12}{51 \cdot 50}\right) \left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right) \left(\frac{12 \cdot 11}{51 \cdot 50}\right) + \left(\frac{3}{4}\right) \left(\frac{13 \cdot 12}{51 \cdot 50}\right)} \\
 &= \frac{13 \times 12 \times 3}{(12 \times 11) + (3 \times 12 \times 13)} \\
 &= \frac{39}{50}
 \end{aligned}$$

Question 115

Let there be three independent events E_1 , E_2 and E_3 . The probability that only E_1 occurs is α , only E_2 occurs is β and only E_3 occurs is γ . Let p denote the probability of none of events occur that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0, 1)$. Then, probability of occurrence of E_1 probability of occurrence of E_3 is equal to

[2021, 17 March Shift-1]

Answer: 6

Solution:

Let $P(E_1) = x$, $P(E_2) = y$, $P(E_3) = z$

Then, $(1-x)(1-y)(1-z) = P \dots (A)$

Probability of only $E_1 = P(E_1 \cap E_2^c \cap E_3^c)$

$$\alpha = x(1-y)(1-z) \dots (B)$$

$$\text{Similarly, } \beta = (1-x)y(1-z) \dots (C)$$

$$\gamma = (1-x)(1-y)z \dots (D)$$

On dividing Eq. (A) $\div$ (B),

$$\frac{P}{\alpha} = \frac{1-x}{x}$$

$$\Rightarrow xP = \alpha - \alpha x \Rightarrow x(P + \alpha) = \alpha$$

$$\Rightarrow x = \frac{\alpha}{\alpha + P}$$

$$\text{Similarly, } y = \frac{\beta}{\beta + P}$$

$$z = \frac{\gamma}{\gamma + P}$$

$$\frac{P(E_1)}{P(E_3)} = \frac{x}{z} = \left(\frac{\alpha}{\alpha + P} \right) \times \left(\frac{\gamma + P}{\gamma} \right)$$

$$\frac{x}{z} = \left(\frac{\alpha}{\gamma} \right) \left(\frac{\gamma + P}{\alpha + P} \right) = \left(\frac{1 + P/\gamma}{1 + P/\alpha} \right)$$

$$\text{Also given, } P = \frac{\alpha\beta}{\alpha - 2\beta} = \frac{2\beta\gamma}{\beta - 3\gamma}$$

$$\alpha P - 2\beta P = \alpha\beta$$

$$\Rightarrow \frac{\alpha P}{\alpha + 2\beta} = \beta$$

$$\beta P - 3\gamma P = 2\beta\gamma$$

$$\beta(P - 2\gamma) = 3\gamma P$$

$$\Rightarrow \beta = \frac{3\gamma P}{P - 2\gamma}$$

From Eqs. (i) and (ii),

$$\Rightarrow \frac{\alpha P}{\alpha + 2\beta} = \frac{3\gamma P}{P - 2\gamma}$$

$$\Rightarrow \frac{1}{1 + 2\left(\frac{\beta}{\alpha}\right)} = \frac{3}{\left(\frac{P}{\gamma}\right) - 2}$$

$$\Rightarrow (P/\gamma) - 2 = 3 + 6(P/\alpha)$$

$$\frac{P}{\gamma} = 5 + 6\left(\frac{P}{\alpha}\right)$$

$$\frac{x}{z} = \frac{1 + P/\gamma}{1 + P/\alpha} = \frac{1 + 5 + 6(P/\alpha)}{1 + P/\alpha}$$

$$= 6\left(\frac{1 + P/\alpha}{1 + P/\alpha}\right) = 6$$

Question 116

Let 9 distinct balls be distributed among 4 boxes, B_1 , B_2 , B_3 and B_4 . If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$, then k lies in the set [2021, 25 July Shift-1]

Options:

A. $\{x \in \mathbb{R} : |x - 3| < 1\}$

B. $\{x \in \mathbb{R} : |x - 2| \leq 1\}$

C. $\{x \in \mathbb{R} : |x - 13| < 1\}$

D. $\{x \in \mathbb{R} : |x - 5| \leq 1\}$

Answer: A

Solution:

B_3 contains exactly 3 balls among 9 balls.

So, number of ways $= {}^9C_3$

And the rest of the 6 balls can be placed in 3^6 ways.

If E is the event and S is the sample,

$$n(E) = {}^9C_3 \times 3^6$$

$$\text{And } n(s) = 4^9$$

$$\text{Probability} = \frac{{}^9C_3 \times 3^6}{4^9}$$

According to the question,

$$\frac{{}^9C_3 \times 3^6}{4^9} = k \frac{3^9}{4^9}$$

$$k = \frac{{}^9C_3}{3^3} = \frac{9 \times 8 \times 7}{1 \times 2 \times 3 \times 27} = \frac{28}{9}$$

Which satisfies $|x - 3| < 1$.

Question 117

The probability that a randomly selected 2-digit number belongs to the set $\{n \in \mathbb{N} : (2^n - 2) \text{ is a multiple of } 3\}$ is equal to
[2021, 27 July Shift-1]

Options:

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: C

Solution:

Sample set $S = \{10, 11, 12, \dots, 99\}$

$$n(S) = 99 - 10 + 1 = 90$$

n	2^n	$2^n - 2$	Divisible by 3
1	2	0	Yes
2	4	2	No
3	8	6	Yes
4	16	14	No
5	32	30	Yes
6	64	62	No
7	128	126	Yes
8	256	254	No
9	512	510	Yes
10	1024	1022	No
11	2048	2046	Yes
12	4096	4094	No

As we can see, for all the odd values of n , $2^n - 2$ is divisible by 3.
So, the event set will be all the two digit odd numbers.

$$E = \{11, 13, 15, \dots, 99\}$$

$$n(E) = \left(\frac{99-11}{2} + 1 \right) = 45$$

$$\therefore \text{Probability} = \frac{n(E)}{n(S)} = \frac{45}{90} = \frac{1}{2}$$

Question 118

The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in \mathbb{R}$, is
[2021, 20 July Shift-1]

Options:

A. $\frac{7}{36}$

B. $\frac{2}{9}$

C. $\frac{1}{6}$

D. $\frac{1}{4}$

Answer: B

Solution:

$$f(x) = x^2 + 2(a+4)x - 5a + 64$$

$$f(x) > 0, \text{ When } D < 0 \text{ as coefficient of } x^2 > 0$$

$$4(a+4)^2 - 4(64 - 5a) < 0$$

$$\Rightarrow a^2 + 16 + 8a - 64 + 5a < 0$$

$$\Rightarrow a^2 + 13a - 48 < 0$$

$$\Rightarrow (a+16)(a-3) < 0$$

$$\Rightarrow n$$

According to the question, $a \in [-5, 30]$

$$n(S) = 30 + 5 + 1 = 36$$

$$n(E) = 2 + 5 + 1 = 8$$

$$\Rightarrow 16, 3 \Rightarrow$$

$$\Rightarrow \text{Probability} = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$$

Question 119

Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non singular, is
[2021, 22 July Shift-II]

Options:

A. $\frac{45}{162}$

B. $\frac{23}{81}$

C. $\frac{22}{81}$

D. $\frac{43}{162}$

Answer: D

Solution:

$$\text{Let } A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, |A| = ad - bc$$

$$n(s) = 6 \times 6 \times 6 \times 6 = 6^4$$

For non-singular matrix $|A| \neq 0$

$$\Rightarrow ad \neq bc$$

Now, a, b, c and d should be different.

$$\text{If } ad = bc$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\text{If } a = 1, \frac{1}{2} = \frac{3}{6} \text{ and } \frac{1}{3} = \frac{2}{6}$$

$$\{a, b, c, d\} = \{1, 2, 3, 6\} \text{ or } \{1, 3, 2, 6\}$$

$$\text{If } a = 2$$

$$\frac{2}{1} = \frac{6}{3}, \frac{2}{3} = \frac{4}{6}, \frac{2}{4} = \frac{3}{6} = \frac{1}{2}, \frac{2}{6} = \frac{1}{3}$$

$$\text{So, } \{a, b, c, d\} = \{2, 1, 6, 3\}, \{2, 3, 4, 6\}, \{2, 4, 3, 6\}, \{2, 4, 1, 2\}, \{2, 6, 1, 3\}$$

$$\text{If } a = 3, \frac{3}{1} = \frac{6}{2}, \frac{3}{2} = \frac{6}{4}$$

$$\{a, b, c, d\} = \{3, 1, 6, 2\}, \{3, 2, 6, 4\}, \{4, 6, 2, 3\}$$

$$\text{If } a = 4, \frac{4}{2} = \frac{6}{3}, \frac{4}{6} = \frac{2}{3}, \{a, b, c, d\} = \{4, 2, 6, 3\}$$

If $a = 5 \Rightarrow$ Not possible

$$\text{If } a = 6, \frac{6}{2} = \frac{3}{1}, \frac{6}{3} = \frac{2}{1}, \frac{6}{4} = \frac{3}{2}$$

$$\{a, b, c, d\} = \{6, 2, 3, 1\}, \{6, 3, 2, 1\}, \{6, 4, 3, 2\}$$

$$\text{So, } n(E) = 2 + 5 + 2 + 2 + 0 + 3 = 16$$

$$\text{So, } n(E) = {}^6C_4 - 16$$

$$\text{Probability} = \frac{{}^6C_4 - 16}{6^4} = \frac{43}{162}$$

Question 120

Let A, B and C be three events such that the probability that exactly one of A and B occurs is $(1 - k)$, the probability that exactly one of B and C occurs is $(1 - 2k)$, the probability that exactly one of C and A occurs is $(1 - k)$ and the probability of all A, B and C occur simultaneously is k^2 , where $0 < k < 1$. Then the probability that at least one of A, B and C occur is [2021, 20 July Shift-II]

Options:

A. greater than $1/8$ but less than $1/4$

B. greater than $1/2$

C. greater than $1/4$ but less than $2/2$

D. exactly equal to $\frac{1}{2}$

Answer: B

Solution:

There are three events A, B, C.

Probability that exactly one of A and B occur is

$$P(A \cap B) + P(A \cap \bar{B})$$

$$= 1 - k \quad \dots (i)$$

Probability that exactly one of B and C occur is

$$P(B \cap C) + P(B \cap \bar{C}) = 1 - 2k \quad \dots (ii)$$

Probability that exactly one of C and A occur is $P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - k \quad \dots (iii)$

Probability of all A, B, C occur simultaneously, is

$$P(A \cap B \cap C) = k^2$$

From Eqs. (i), (ii) and (iii),

$$P(A) + P(B) - 2P(A \cap B) = 1 - k \quad \dots (v)$$

$$P(B) + P(C) - 2P(B \cap C) = 1 - 2k \quad \dots (vi)$$

$$P(A) + P(C) - 2P(A \cap C) = 1 - k \quad \dots (vii)$$

Adding Eqs. (v), (vi) and (vii),

$$2[P(A) + P(B) + P(C) - P(A \cap B)$$

$$- P(B \cap C) - P(C \cap A)]$$

$$= (1 - k) + (1 - 2k) + (1 - k) = 3 - 4k$$

$$\therefore P(A) + P(B) + P(C) - P(A \cap B)$$

$$\begin{aligned}
 -P(B \cap C) - P(A \cap C) &= \frac{3-4k}{2} \\
 \Rightarrow P(A \cup B \cup C) &= \frac{3-4k}{2} + P(A \cap B \cap C) \\
 &= \frac{3-4k}{2} + k^2 = \frac{2k^2 - 4k + 3}{2} \\
 \Rightarrow P(A \cup B \cup C) &= \frac{2(k-1)^2 + 1}{2} \\
 &= (k-1)^2 + \frac{1}{2} > \frac{1}{2} \\
 \therefore P(A \cup B \cup C) &> \frac{1}{2}
 \end{aligned}$$

Question 121

Let $S = \{1, 2, 3, 4, 5, 6\}$. Then, the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is [2021, 31 Aug. Shift-II]

Options:

- A. $\frac{1}{10}$
- B. $\frac{1}{15}$
- C. $\frac{1}{5}$
- D. $\frac{1}{30}$

Answer: A

Solution:

$$\because g : S \rightarrow S$$

$$\text{where } S = \{1, 2, 3, 4, 5, 6\}$$

$$\text{Total number of onto function} = 6!$$

$$\text{Given condition } g(3) = 2g(1)$$

$$\text{If } g(1) = 1, \text{ then } g(3) = 2$$

$$g(1) = 2, \text{ then } g(3) = 4$$

$$\text{and } g(1) = 3, \text{ then } g(3) = 6$$

There are only 3 possible cases

$$\text{Number onto functions} = 3 \times 4!$$

$$\text{Required probability} = \frac{3 \times 4!}{6!} = \frac{1}{10}$$

Question 122

When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$.

All other faces occur with probability $1/6$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice is $13/96$, then the value of x is
[2021, 27 Aug. Shift-1]

Options:

A. $1/16$

B. $1/8$

C. $1/9$

D. $\frac{1}{12}$

Answer: B

Solution:

$$\text{Let } P_1 = \frac{1}{6} - x$$

$$P_6 = \frac{1}{6} + x \quad [\text{sum of opposite faces is } 7]$$

$$P_2 = P_3 = P_4 = P_5 = \frac{1}{6} \quad [P_{\text{sum}} = 7]$$

$$= 2(P_6 \times P_1 + P_5 \times P_2 + P_4 \times P_3)$$

$$\Rightarrow \frac{13}{96} = 2 \left(\left(\frac{1}{6} + x \right) \left(\frac{1}{6} - x \right) + \left(\frac{1}{6} \right)^2 + \left(\frac{1}{6} \right)^2 \right)$$

$$\Rightarrow \frac{13}{96} = 2 \left[\frac{1}{36} - x^2 + \frac{1}{36} + \frac{1}{36} \right]$$

$$\Rightarrow \frac{1}{6} - 2x^2 = \frac{13}{96}$$

$$\Rightarrow x^2 = \frac{1}{64} \Rightarrow x = \frac{1}{8}$$

Question 123

An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is P , then $98P$ is equal to
[2021, 31 Aug. Shift-1]

Answer: 28

Solution:

$P(A)$ = Probability of first unit to function

$P(B)$ = Probability of second unit to function

$$P(A) = 0.9, P(\bar{A}) = 0.1$$

$$P(B) = 0.8, P(\bar{B}) = 0.2$$

$AB \rightarrow$ Operate

$\bar{A}\bar{B} \rightarrow$ Not operate

$\bar{A}B \rightarrow$ Not operate

$A\bar{B} \rightarrow$ Not operate

According to question, first failed and second functioning, given that instrument has failed, So according to Bayes theorem

$$\begin{aligned}\therefore P &= \frac{P(\bar{A}) \cdot P(B)}{P(\bar{A})P(B) + P(A)P(\bar{B}) + P(A)P(B)} \\ &= \frac{(0.1 \times 0.8)}{(0.9 \times 0.2) + (0.1 \times 0.8) + (0.1 \times 0.2)} = \frac{8}{28}\end{aligned}$$

$$98P = \frac{8}{28} \times 98 = 28$$

Question 124

Let A and B be independent events such that $P(A) = p$ and $P(B) = 2p$. The largest value of p, for which $P(\text{exactly one of A, B occurs}) = \frac{5}{9}$, is

[2021, 26 Aug. Shift-1]

Options:

A. $\frac{1}{3}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{5}{12}$

Answer: D

Solution:

$$P(\text{Exactly one out of A or B occurs}) = \frac{5}{9}$$

$$p(A \cap \bar{B}) + p(\bar{A} \cap B) = \frac{5}{9}$$

$$p(A)p(\bar{B}) + p(\bar{A})p(B) = \frac{5}{9}$$

$$\therefore p(1-2p) + (1-p)2p = \frac{5}{9}$$

$$\Rightarrow 9p(1-2p) + 18p(1-p) = 5$$

$$\Rightarrow 9p - 18p^2 + 18p - 18p^2 = 5$$

$$\Rightarrow 36p^2 - 27p + 5 = 0$$

$$\Rightarrow (3p - 1)(12p - 5) = 0$$

$$p = \frac{1}{3} \text{ or } p = \frac{5}{12}$$

$$p_{\max} = \frac{5}{12}$$

Question 125

A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is [2021, 26 Aug. Shift-II]

Options:

A. $\frac{125}{216}$

B. $\frac{11}{36}$

C. $\frac{5}{6}$

D. $\frac{25}{36}$

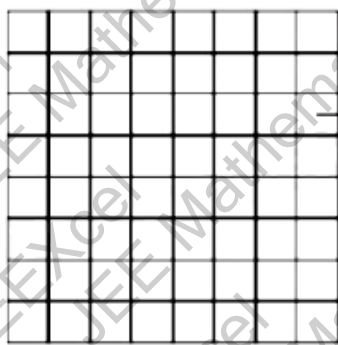
Answer: D

Solution:

$$\begin{aligned} P(X \geq 5 \mid X > 2) &= \frac{P(X \geq 5 \cap X > 2)}{P(X > 2)} = \frac{P(X \geq 5)}{P(X > 2)} \\ &= \frac{\left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \dots \infty}{\left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) + \dots \infty} \\ &= \frac{\left(\frac{5}{6}\right)^4 \cdot \left(\frac{1}{6}\right)}{\left(\frac{5}{6}\right)^2 \cdot \left(\frac{1}{6}\right)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36} \end{aligned}$$

Question 126

Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is



[2021, 01 Sep. Shift-II]

Options:

A. $\frac{2}{7}$

B. $\frac{1}{18}$

C. $\frac{1}{7}$

D. $\frac{1}{9}$

Answer: B

Solution:

$$\text{Total ways} = {}^{64}C_2 = 32 \times 63$$

$$\text{Favourable ways} = 2 \times 7 \times 8$$

$$\text{Required probability} = \frac{2 \times 7 \times 8}{32 \times 63} = \frac{1}{18}$$

Question127

In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is:

[Jan. 9,2020 (I)]

Options:

A. $\frac{9}{16}$

B. $\frac{11}{16}$

C. $\frac{13}{16}$

D. $\frac{15}{16}$

Answer: B

Solution:

$P(\text{second A – card appears before the third B– card}) = P(AA) + P(ABA) + P(BAA) + P(ABBA) + P(BBAA) + P(BABA)$

$$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$$

Question128

Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE?

[Jan. 8, 2020 (I)]

Options:

A. $P(A/B) = \frac{2}{3}$

B. $P(A/B') = \frac{1}{3}$

C. $P(A'/B') = \frac{1}{3}$

D. $P(A/(A \cup B)) = \frac{1}{4}$

Answer: B

Solution:

A and B are independent events.

$$\text{So, } P\left(\frac{A}{B'}\right) = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3} - \frac{1}{3} \cdot \frac{1}{6}}{\frac{1}{6}} = \frac{1}{3}$$

Question129

An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. Then the expected value of X, is:

[Jan. 7, 2020 (I)]

Options:

A. $\frac{3}{16}$

B. $\frac{1}{8}$

C. $-\frac{3}{16}$

D. $-\frac{1}{8}$

Answer: B

Solution:

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = No. of times head occur consecutively

Now expectation

$$= \sum xP(k) = (-1) \times \frac{1}{32} + (-1) \times \frac{12}{32} + (-1) \times \frac{11}{32} + 3 \times \frac{5}{32} + 4 \times \frac{2}{32} + 5 \times \frac{1}{32} = \frac{1}{8}$$

Question130

In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will

be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to:

[Jan. 7,2020 (II)]

Options:

A. $\frac{17}{8}$

B. $\frac{17}{4}$

C. $\frac{17}{2}$

D. 4

Answer: A

Solution:

Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$$\begin{aligned} &= {}^5C_0 \left(\frac{3}{4}\right)^5 + {}^5C_1 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^4 + {}^5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 \\ &= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27 \times 17}{64 \times 8} \\ &= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8} \\ \therefore k &= \frac{17}{8} \end{aligned}$$

Question131

Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is:
[Jan. 8, 2020 (II)]

Options:

- A. 0.02
- B. 0.20
- C. 0.01
- D. 0.10

Answer: D

Solution:

$$P(\text{ exactly one }) = \frac{2}{5}$$

$$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$$

$$P(A \text{ or } B) = P(A \cup B) = \frac{1}{2}$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{1}{2}$$

$$\therefore P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10} = 0.10$$

Question132

If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :
[Jan. 9, 2020 (II)]

Options:

A. $\frac{965}{2^{11}}$

B. $\frac{965}{2^{10}}$

C. $\frac{945}{2^{10}}$

D. (Bonus)

Answer: D

Solution:

Total number of ways placing 10 different balls in 4 distinct boxes $= 4^{10}$
 Since, two of the 4 distinct boxes contains exactly 2 and 3 balls.
 Then, there are three cases to place exactly 2 and 3 balls in 2 of the 4 boxes.
 Case-1: When boxes contains balls in order 2,3,0,5

Then, number of ways of placing the balls $= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4!$

Case-2: When boxes contains ball in order 2, 3, 1, 4.

Then, number of ways of placing the balls $= \frac{10!}{2! \times 3! \times 1! \times 4!} \times 4!$

Case-3: When boxes contains ball in order 2, 3, 2, 3

Then, number of ways of placing the balls $= \frac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$

Therefore, number of ways of placing the balls that contains exactly 2 and 3 balls. $= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4! +$

$\frac{10!}{2! \times 3! \times 1! \times 4!} \times 4! + \frac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$

$= 2^5 \times 17 \times 945$

Hence, the required probability

$= \frac{2^5 \times 17 \times 945}{4^{10}} = \frac{17 \times 945}{2^{15}}$

Question133

A random variable X has the following probability distribution:

X	:	1	2	3	4	5
P(X)	:	K^2	$2K$	K	$2K$	$5K^2$

Then, $P(X > 2)$ is equal to:

[Jan. 9, 2020 (II)]

Options:

A. $\frac{7}{12}$

B. $\frac{1}{36}$

C. $\frac{1}{6}$

D. $\frac{23}{36}$

Answer: D

Solution:

$$\sum P(K) = 1 \Rightarrow 6K^2 + 5K = 1$$

$$6K^2 + 5K - 1 = 0$$

$$6K^2 + 6K - K - 1 = 0$$

$$\Rightarrow (6K - 1)(K + 1) = 0$$

$$\Rightarrow K = \frac{1}{6} \quad (K = -1 \text{ rejected})$$

$$P(X > 2) = K + 2K + 5K^2 \\ = \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6 + 12 + 5}{36} = \frac{23}{36}$$

Question 134

In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins.

The probability of A winning the game is :

[Sep. 04, 2020 (II)]

Options:

A. $\frac{5}{31}$

B. $\frac{31}{61}$

C. $\frac{5}{6}$

D. $\frac{30}{61}$

Answer: D

Solution:

Probability of sum getting 6, $P(A) = \frac{5}{36}$

Probability of sum getting 7, $P(B) = \frac{6}{36} = \frac{1}{6}$

$$P(A \text{ wins}) = P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A}) \cdot P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \dots$$

$$\Rightarrow \frac{5}{36} + \left(\frac{31}{36}\right) \left(\frac{30}{36}\right) \left(\frac{5}{36}\right) + \dots \infty$$

$$\Rightarrow \frac{5}{36} \left(1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + \dots \infty\right)$$

$$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}} = \frac{30}{61} \left(\because S_{\infty} = \frac{a}{1-r}\right)$$

Question 135

A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :
[Sep. 03, 2020 (I)]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{8}$

D. $\frac{1}{9}$

Answer: D

Solution:

E_1 [the event for getting score a multiple of 4] = (1, 3), (3, 1), (2, 2), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4) & (6, 6)

E_2 [4 has appeared atleast once] = (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5) & (4, 6)

$E_1 \cap E_2 = (4, 4)$

$$P\left(\frac{E_2}{E_1}\right) = \frac{1}{9}$$

Question 136

The probability that a randomly chosen 5 -digit number is made from exactly two digits is:
[Sep. 03, 2020 (II)]

Options:

A. $\frac{135}{10^4}$

B. $\frac{121}{10^4}$

C. $\frac{150}{10^4}$

D. $\frac{134}{10^4}$

Answer: A

Solution:

Total outcomes = $9(10^4)$

Favourable outcomes = ${}^9C_2(2^5 - 2) + {}^9C_1(2^4 - 1) = 36(30) + 9(15)$

$$\text{Probability} = \frac{36 \times 30 + 9 \times 15}{9 \times 10^4} = \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

Question 137

Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :
[Sep. 02, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{8}{17}$

C. $\frac{4}{17}$

D. $\frac{2}{5}$

Answer: B

Solution:

Let B_1 and B_2 be the boxes and N be the number of non-prime number.

$$\therefore P(B_1) = P(B_2) = \frac{1}{2}$$

and $P(\text{non-prime number})$

$$= P(B_1) \times P\left(\frac{N}{B_1}\right) + P(B_2) \times P\left(\frac{N}{B_2}\right)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$$

So,

$$P\left(\frac{B_1}{N}\right) = \frac{P(B_1) \times P\left(\frac{N}{B_1}\right)}{P(B_1) \times P\left(\frac{N}{B_1}\right) + P(B_2) \times P\left(\frac{N}{B_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{15}{40}} = \frac{8}{17}$$

Question138

Let E^C denote the complement of an event E . Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$

and $P(E_1 \cap E_2 \cap E_3) = 0$

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to:

[Sep. 02, 2020 (II)]

Options:

A. $P(E_2^C) + P(E_3)$

B. $P(E_3^C) - P(E_2^C)$

C. $P(E_3) - P(E_2^C)$

D. $P(E_3^C) - P(E_2)$

Answer: D

Solution:

$$P\left(\frac{E_2^C \cap E_3^C}{E_1}\right) = \frac{P[E_1 \cap E_2^C \cap E_3^C]}{P(E_1)}$$

$$= \frac{P(E_1) - P[E_1 \cap (E_2 \cup E_3)]}{P(E_1)} = [\because P(A \cap B^C) = P(A) - P(A \cap B)]$$

$$= \frac{P(E_1) - P[(E_1 \cap E_2) \cup (E_1 \cap E_3)]}{P(E_1)}$$

$$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$$

$$= 1 - P(E_2) - P(E_3) [\because P(A \cap B) = P(A) \cdot P(B)]$$

$$= P(E_2^C) - P(E_3) \text{ or } P(E_3^C) - P(E_2)$$

Question139

Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is _____.

[NA Sep. 05, 2020 (I)]

Answer: 11

Solution:

Probability of getting at least two 3's or 5's in one trial

$$= {}^4C_2 \left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^2 + {}^4C_3 \left(\frac{2}{6}\right)^3 \left(\frac{4}{6}\right) + {}^4C_4 \left(\frac{2}{6}\right)^4$$
$$= \frac{33}{3^4} = \frac{11}{27}$$

$$E(x) = np = 27 \left(\frac{11}{27}\right) = 11$$

Question 140

In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____.

[NA Sep. 05, 2020 (II)]

Answer: 11

Solution:

Let 'n' bombs are required, then

$$1 - {}^nC_1 \cdot \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} - {}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n \geq \frac{99}{100}$$

$$\Rightarrow \frac{1}{100} \geq \frac{n+1}{2^n} \Rightarrow 2^n \geq 100(n+1) \Rightarrow n \geq 11$$

Question 141

The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater

than $\frac{1}{4}$, is _____.

[NA Sep. 04, 2020 (I)]

Answer: 3

Solution:

$$p = \frac{1}{10}, q = \frac{9}{10}$$

$$P(\text{not hitting target in } n \text{ trials}) = \left(\frac{9}{10}\right)^n$$

$$P(\text{at least one hit}) = 1 - \left(\frac{9}{10}\right)^n$$

$$\therefore 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4} \Rightarrow (0.9)^n < 0.75$$

$$\therefore n_{\text{minimum}} = 3$$

Question 142

Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:

[Sep. 06, 2020 (I)]

Options:

A. $\frac{15}{101}$

B. $\frac{5}{101}$

C. $\frac{5}{33}$

D. $\frac{10}{99}$

Answer: C

Solution:

For an A.P. $2b = a + c$ (even), so both a and c even numbers or odd numbers from given numbers and b number will be fixed automatically.

$$\text{Required probability} = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$$

Question 143

The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval:
[Sep. 06, 2020 (II)]

Options:

- A. $[0.35, 0.36]$
- B. $[0.25, 0.35]$
- C. $[0.20, 0.25]$
- D. $[0.36, 0.40]$

Answer: B

Solution:

$$\because P(A \cap B) = P(A) + P(B) - P(A \cup B) \\ = 1 - 0.8 = 0.2$$

Now,

$$\because P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\Rightarrow \alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$$

$$\Rightarrow \beta = 1.2 - \alpha$$

$$\because \alpha \in [0.85, 0.95] \text{ then } \beta \in [0.25, 0.35]$$