

Permutations and Combinations

Question1

Let $S = \{(m, n) : m, n \in \{1, 2, 3, \dots, 50\}\}$. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that $m + n$ is a square of a prime number is q , then $p + q$ is equal to ____.

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Answer: 1333

Solution:

$$5 \mid 6^m + 9^n$$

$$\Rightarrow 5 \mid 1^m + (-1)^n$$

$\Rightarrow m$ and n has to be opposite parity.

$${}^2C_1 \times {}^{25}C_1 \cdot {}^{25}C_1 = 625 \times 2 = 1250$$

For, $m + n = K^2$ for some prime K .

$$m + n \in \{2, 3, \dots, 100\}$$

$$m + n = 4, 9, 25, 49$$

$$m + n = 4 \Rightarrow 3 \text{ pairs}$$

$$m + n = 9 \Rightarrow 8 \text{ pairs}$$

$$m + n = 25 \Rightarrow 24 \text{ pairs}$$

$$m + n = 49 \Rightarrow 48 \text{ pairs}$$

$$\Rightarrow 83 \text{ pairs}$$

$$\Rightarrow 1333$$

Question2

Let ABC be a triangle. Consider four points p_1, p_2, p_3, p_4 on the side AB, five points p_5, p_6, p_7, p_8, p_9 on the side BC, and four points $p_{10}, p_{11}, p_{12}, p_{13}$ on the side AC. None of these points is a vertex of the triangle ABC. Then the total number of pentagons, that can be formed by taking all the vertices from the points $p_1, p_2, \dots, p_{13}$, is _____

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Answer: 660

Solution:

$${}^4C_2 \cdot {}^5C_2 \cdot {}^4C_1 + {}^4C_2 \cdot {}^5C_2 \cdot {}^4C_1 + {}^4C_2 \cdot {}^4C_2 \cdot {}^5C_1 = 240 + 240 + 180 = 660$$

Question3

The number of 4 -letter words, with or without meaning, which can be formed using the letters PQRQRSTUVP, is _____.

JEE Main 2026 (Online) 23rd January Morning Shift

Answer: 1422

Solution:

Letter frequency

P : 3

R, Q : 2

S, T, U, V : 1

4 letter words can be of type

$\Rightarrow ABCD$ or $AABC, AABB, AAAB$

$$\Rightarrow {}^7C_4 \cdot 4! + ({}^3C_1) \cdot {}^6C_2 \cdot \frac{4!}{2!} + {}^3C_2 \cdot \frac{4!}{2!1!} + ({}^1C_1) \cdot {}^6C_1 \cdot \frac{4!}{3!} \\ = 1422$$

Question4

Let S denote the set of 4-digit numbers $abcd$ such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to _____

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Answer: 260

Solution:

for set S : 4-digit numbers $abcd$ with $a > b > c > d$

digits are chosen from $\{0, 1, 2, \dots, 9\}$

since order is strictly fixed, any 4 distinct digits chosen will form exactly one such number. number of ways to choose 4 digits from 10 is ${}^{10}C_4$

$$n(S) = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

for set P : 5-digit numbers with product of digits = 20

possible sets of digits (using digits 1-9):

case 1: $\{5, 4, 1, 1, 1\}$

$$\text{number of arrangements} = \frac{5!}{3!} = 20$$

case 2: $\{5, 2, 2, 1, 1\}$

$$\text{number of arrangements} = \frac{5!}{2!2!} = 30$$

$$n(P) = 20 + 30 = 50$$

calculating final value.

$$n(S) + n(P) = 210 + 50 = 260$$

the value of $n(S) + n(P)$ is 260.

Question5

The number of numbers greater than 5000 , less than 9000 and divisible by 3 , that can be formed using the digits 0, 1, 2, 5, 9, if the repetition of the digits is allowed, is _____

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Answer: 42

Solution:

To find numbers of number greater than 5000 and less than 9000 so number will start with 5 or 9 of the form 5abc and $a, b, c \in \{0, 1, 2, 5, 9\}$ also the number is divisible by 3 .

This means $5 + a + b + c$ is divisible by 3

so the sum $a + b + c$ is of the type $3n + 1$.

unit place c determined by the sum of previous 3 places.

$\text{rem}(a+b) = \text{remainder of } a+b$

rem(a+b)	combination (a,b)	value of c	total = combination (a, b) $\times$ c(choice)
0	(0, 0), (0, 9), (9, 0), (9, 9), (1, 2), (2, 1), (5, 1), (1, 5)	1	$8 \times 1 = 8$
1	(0, 1), (9, 1), (1, 0), (1, 9), (2, 2), (2, 5), (5, 2), (5, 5)	0, 9	$8 \times 2 = 16$
2	(0, 2), (0, 5), (9, 2), (9, 5), (2, 0), (2, 9), (5, 0), (5, 9), (1, 1)	2, 5	$9 \times 2 = 18$

Total = $8 + 16 + 18 = 42$

Question6

Three persons enter in a lift at the ground floor. The lift will go up to 10th floor. The number of ways, in which the three persons can exit

the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to _____.

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Answer: 210

Solution:

Given:

- The lift goes up to the 10th floor.
- The lift does not stop at 1st, 2nd, 3rd floors.
- So, the persons can get down only at the remaining floors.

4, 5, 6, 7, 8, 9, 10 $\Rightarrow$ 7 floors

- Now, all three persons must exit at three different floors (no two can get down at the same floor).
- Also, the three persons are distinct, so who gets down where matters.

First, choose which 3 floors (out of 7) will be used:

$${}^7C_3 = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$$

Next, assign (arrange) the 3 distinct persons to these 3 chosen floors:

Number of arrangements of 3 persons:

$$3! = 6$$

So, total number of ways:

$$\text{Total ways} = {}^7C_3 \times 3! = 35 \times 6 = 210$$

Question 7

Two players A and B play a series of games of badminton. The player, who wins 5 games first, wins the series. Assuming that no game ends in a draw, the number of ways, in which player A wins the series is _____.

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Answer: 126

Solution:

For player A to win the series, A must win the 5th game before B does.

So, when the series ends:

- A has won exactly 5 games.
- B can have won 0, 1, 2, 3, or 4 games.

Now count each case.

Case 1: B wins 0 games

Then the sequence is only

AAAAA

Number of ways = 1

Case 2: B wins 1 game

The series has 6 games, and the last game must be won by A.

So in the first 5 games, A must win 4 games and B must win 1 game.

Number of ways

$$\binom{5}{1} = 5$$

Case 3: B wins 2 games

The series has 7 games, and the last game must be won by A.

So in the first 6 games, A must win 4 games and B must win 2 games.

Number of ways

$$\binom{6}{2} = 15$$

Case 4: B wins 3 games

The series has 8 games, and the last game must be won by A.

So in the first 7 games, A must win 4 games and B must win 3 games.

Number of ways

$$\binom{7}{3} = 35$$

Case 5: B wins 4 games

The series has 9 games, and the last game must be won by A.

So in the first 8 games, A must win 4 games and B must win 4 games.

Number of ways

$$\binom{8}{4} = 70$$

Total number of ways

$$1 + 5 + 15 + 35 + 70 = 126$$

Hence, the number of ways in which player A wins the series is

$$\boxed{126}$$

Question 8

The number of strictly increasing functions f from the set $\{1, 2, 3, 4, 5, 6\}$ to the set $\{1, 2, 3, \dots, 9\}$ such that $f(i) \neq i$ for $1 \leq i \leq 6$, is equal to :

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Options:

A.

21

B.

28

C.

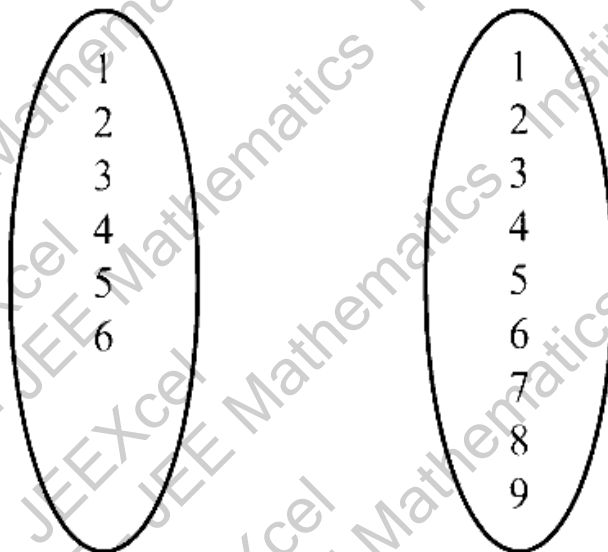
27

D.

22

Answer: B

Solution:



Case (i) If $f(1) = 2$ then ${}^7C_5$ functions

Case (ii) $f(1) = 3$ then ${}^6C_5$

Case (iii) $f(1) = 4$ then ${}^5C_5$

$$\Rightarrow 21 + 6 + 1 = 28$$

Question9

The largest $n \in \mathbb{N}$, for which 7^n divides $101!$, is :

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Options:

A.

18

B.

15

C.

19

D.

16

Answer: D

Solution:

$$\text{Exponent of 7 in } 101! = \left[\frac{101}{7} \right] + \left[\frac{101}{7^2} \right] + \dots \text{ (Where } [\cdot] \text{ is G.I.F.)} = 14 + 12 = 16$$

Question 10

The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is

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Options:

A.

384

B.

403

C.

429

D.

455

Answer: D

Solution:

To distribute 16 oranges to 4 children such that each child gets atleast one orange.

First distribute one orange to each child so that each child get atleast one.

Now, distribute remaining ($n = 12$) oranges to ($r = 4$) children using beggar's method $= {}^{n+r-1}C_{r-1}$

$$= {}^{12+4-1}C_{4-1} = {}^{15}C_3$$

$$= \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 5 \times 7 \times 13 = 455$$

Question 11

The largest value of n , for which 40^n divides $60!$, is

JEE Main 2026 (Online) 24th January Evening Shift

Options:

A.

14

B.

13

C.

11

D.

12

Answer: A

Solution:

To find largest value of n for which $60!$ is divisible by 40^n

This means we need to find exponent of 40 in $60!$

$$40 = 2^3 \times 5$$

Key Concept: formula for exponent of prime p in $k!$ is given as $\left[\frac{k}{p} \right] + \left[\frac{k}{p^2} \right] + \left[\frac{k}{p^3} \right] + \dots$ where $[\]$ is greatest integer function.

so, exponent of 2 in $60!$

$$\left\lfloor \frac{60}{2} \right\rfloor + \left\lfloor \frac{60}{2^2} \right\rfloor + \left\lfloor \frac{60}{2^3} \right\rfloor + \left\lfloor \frac{60}{2^4} \right\rfloor + \left\lfloor \frac{60}{2^5} \right\rfloor + \left\lfloor \frac{60}{2^6} \right\rfloor + \dots$$

$$30 + 15 + 7 + 3 + 1 = 56 - (1)$$

exponent of 5 in 60 !

$$\left\lfloor \frac{60}{5} \right\rfloor + \left\lfloor \frac{60}{5^2} \right\rfloor + \left\lfloor \frac{60}{5^3} \right\rfloor + \dots$$

$$12 + 2 + 0 \dots = 14 - (2)$$

Combining (1) and (2)

60 ! is divisible by $2^{56} \times 5^{14} \times N$, where N is natural number.

60 ! is divisible by $4 \times (2^3)^{18} \times 5^{14} \times N$

60 ! is divisible by $4 \times (2^3)^4 \times (2^3)^{14} \times 5^{14} \times N$

60! is divisible by $2^{14} \times 40^{14} \times N$

So largest value of n is 14

Question12

The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is

JEE Main 2026 (Online) 24th January Evening Shift

Options:

A.

1579

B.

1578

C.

1580

D.

1581

Answer: C

Solution:

Arrange UDAYPUR in alphabetical order A, D, P, R, UU, Y

Number of words start with A. fix A and arrange DPRUUY = $\frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$ ways.

Similarly, number words starts from D, P, R will be same as there is arrangement of 6 letters in which U is repeating twice.

So, total number of words starting with A, D, P & R are = $4 \times 360 = 1440$.

Now, number of words starts with UA

fix UA and arrange D, P, R, U, Y = $5!$ ways = 120 .

the next letter is D after A, so number of words starts with UDAP arranging (R, U, Y) = $3!$ ways = 6

So, number of words start with UDAR and UDAU is same = $3! + 3! = 12$

next is UDAY matches with pattern(UDAYPUR)

number of words start with UDAYP arrang(RU) = $2! = 2$

number of words start with UDAYPR is UDAYPRU = 1

final pattern UDAYPUR = 1

So, rank of word UDAYPUR is =

$1440 + 120 + 6 + 12 + 1 + 1 = 1580$.

Question13

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9 - digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then,

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Options:

A.

$$56x = 9y$$

B.

$$21x = 4y$$

C.

$$45x = 7y$$

D.

$$29x = 5y$$

Answer: B

Solution:

Given, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

1. Calculation for x

For x , we have to form a 9-digit number in which exactly one digit is repeated, and it is repeated exactly twice. So, one digit appears twice, and the remaining 7 digits appear once each.

First, select the digit that will be repeated: from S , this can be done in ${}^9C_1$ ways.

Now, from the remaining 8 digits, choose 7 digits that will appear once: this can be done in ${}^8C_7$ ways.

Now we have 9 digits in total, but one digit is repeated twice. So, the number of different arrangements is $= \frac{9!}{2!}$ ways.

$$x = {}^9C_1 \times {}^8C_7 \times \frac{9!}{2!}$$

2. Calculation for y

For y , we have to form a 9-digit number in which exactly two digits are repeated, and each of these is repeated exactly twice. So, two digits appear twice each (total 4 places), and the remaining 5 digits appear once each.

First, select the 2 digits that will be repeated from S : this can be done in ${}^9C_2$ ways.

Now, from the remaining 7 digits, choose 5 digits that will appear once: this can be done in ${}^7C_5$ ways.

Now we have 9 digits in total, with two digits repeated twice each. So, the number of different arrangements is $= \frac{9!}{2!2!}$ ways.

$$y = {}^9C_2 \times {}^7C_5 \times \frac{9!}{2!2!}$$

To find the relation between x and y , divide x by y :

To find the relation, divide x by y :

$$\frac{x}{y} = \frac{{}^9C_1 \times {}^8C_7 \times \frac{1}{2}}{{}^9C_2 \times {}^7C_5 \times \frac{1}{4}} = \frac{9 \times 8 \times \frac{1}{2}}{36 \times 21 \times \frac{1}{4}}$$

So,

$$\frac{x}{y} = \frac{36}{189} = \frac{4}{21}$$

$$\Rightarrow 21x = 4y$$

Question14

The number of elements in the set

$$S = \left\{ (r, k) : k \in \mathbb{Z} \text{ and } {}^{36}C_{r+1} = \frac{6({}^{35}C_r)}{(k^2-3)} \right\} \text{ is :}$$

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Options:

A.

2

B.

4

C.

8

D.

16

Answer: B

Solution:

We need to find the number of ordered pairs (r, k) satisfying

$${}^{36}C_{r+1} = \frac{6({}^{35}C_r)}{k^2-3},$$

where $k \in \mathbb{Z}$.

Let us simplify the relation using the standard NCERT combination formula.

We know that

$${}^nC_{r+1} = \frac{n!}{(r+1)!(n-r-1)!}$$

and

$${}^{n-1}C_r = \frac{(n-1)!}{r!(n-r-1)!}.$$

So,

$$\frac{{}^{36}C_{r+1}}{{}^{35}C_r} = \frac{\frac{36!}{(r+1)!(35-r)!}}{\frac{35!}{r!(35-r)!}} = \frac{36!}{35!} \cdot \frac{r!}{(r+1)!} = 36 \cdot \frac{1}{r+1} = \frac{36}{r+1}.$$

Hence,

$${}^{36}C_{r+1} = {}^{35}C_r \cdot \frac{36}{r+1}.$$

Given,

$${}^{36}C_{r+1} = \frac{6({}^{35}C_r)}{k^2-3}.$$

Comparing both expressions,

$${}^{35}C_r \cdot \frac{36}{r+1} = \frac{6({}^{35}C_r)}{k^2-3}.$$

Since ${}^{35}C_r \neq 0$, cancel it:

$$\frac{36}{r+1} = \frac{6}{k^2-3}.$$

Cross-multiplying,

$$36(k^2-3) = 6(r+1).$$

Divide by 6:

$$6(k^2-3) = r+1.$$

So,

$$r = 6k^2 - 19.$$

Now r must be valid for the combinations ${}^{35}C_r$ and ${}^{36}C_{r+1}$.

Thus,

$$0 \leq r \leq 35.$$

Substitute $r = 6k^2 - 19$:

$$0 \leq 6k^2 - 19 \leq 35.$$

Add 19 throughout:

$$19 \leq 6k^2 \leq 54.$$

Divide by 6:

$$\frac{19}{6} \leq k^2 \leq 9.$$

Now possible integer square values are

$$k^2 = 4 \quad \text{or} \quad k^2 = 9.$$

So the integer values of k are:

- If $k^2 = 4$, then $k = \pm 2$
- If $k^2 = 9$, then $k = \pm 3$

Now find corresponding r :

For $k^2 = 4$,

$$r = 6(4) - 19 = 24 - 19 = 5.$$

So pairs are

$$(5, 2), (5, -2).$$

For $k^2 = 9$,

$$r = 6(9) - 19 = 54 - 19 = 35.$$

So pairs are

$$(35, 3), (35, -3).$$

Hence total number of elements in the set S is

4.

Therefore, the correct answer is:

4

So, Option B is correct.

Question 15

The number of seven-digit numbers, that can be formed by using the digits 1, 2, 3, 5 and 7 such that each digit is used at least once, is:

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Options:

A.

15400

B.

17800

C.

16800

D.

29400

Answer: C

Solution:

We have to form a 7-digit number using the digits 1, 2, 3, 5, 7 such that **each digit is used at least once**.

Since there are 5 distinct digits and the number has 7 places, after using each digit once, there are 2 extra places.

So, the repetition can happen in the following ways:

Case 1: One digit is repeated 3 times

Then the distribution of digits is:

3, 1, 1, 1, 1

Choose the digit which appears 3 times:

$${}^5C_1 = 5$$

Now arrange the 7 digits:

$$\frac{7!}{3!}$$

So, total numbers in this case:

$$5 \times \frac{7!}{3!} = 5 \times \frac{5040}{6} = 5 \times 840 = 4200$$

Case 2: Two digits are repeated 2 times each

Then the distribution is:

2, 2, 1, 1, 1

Choose the two digits which are repeated:

$${}^5C_2 = 10$$

Now arrange the 7 digits:

$$\frac{7!}{2!2!}$$

So, total numbers in this case:

$$10 \times \frac{7!}{2!2!} = 10 \times \frac{5040}{4} = 10 \times 1260 = 12600$$

Total number of such 7-digit numbers

$$4200 + 12600 = 16800$$

Hence, the required number is

$$\boxed{16800}$$

So, the correct option is **Option C**.

Question 16

Let p_n denote the total number of triangles formed by joining the vertices of an n -side regular polygon.

If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is :

JEE Main 2026 (Online) 2nd April Evening Shift

Options:

A.

7

B.

8

C.

5

D.

6

Answer: C

Solution:

$$P_n = {}^nC_3$$

In an n -sided polygon, any 3 vertices form a triangle, so the total number of triangles is $P_n = {}^nC_3$.

$$P_{n+1} - P_n = 66$$

This means when we increase the number of sides from n to $n + 1$, the number of triangles increases by 66.

$${}^{n+1}C_3 - {}^nC_3 = 66$$

Now we write both combinations using the formula ${}^nC_3 = \frac{n(n-1)(n-2)}{6}$.

$$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 66$$

Take the common factor $\frac{n(n-1)}{6}$ from both terms.

$$\Rightarrow \frac{n(n-1)}{6} [n+1 - n+2] = 66$$

Inside the bracket, simplify: $n+1 - n+2 = 3$.

$$\Rightarrow n(n-1) = 132$$

So we need two consecutive integers whose product is 132. That gives $n = 12$ because $12 \cdot 11 = 132$.

$$n = 12$$

Prime divisors of 12 are 2 & 3

The distinct prime divisors of 12 are 2 and 3.

$$\text{Sum} = 2 + 3 = 5$$

Question 17

The number of ways, of forming a queue of 4 boys and 3 girls such that all the girls are not together, is:

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Options:

A.

5040

B.

3050

C.

3410

D.

4320

Answer: D

Solution:

We have a total of 4 boys and 3 girls.

Total number of children = $4 + 3 = 7$.

The total number of ways to arrange 7 children in a queue is given by the permutation of 7 distinct objects taken all at a time, which is $7!$.

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

Number of arrangements where all the girls are together

To find this, let us treat all 3 girls as a single unit or block, since they must remain together.

Now, we have 4 boys and 1 unit of girls. This gives us a total of $4 + 1 = 5$ entities to arrange in the queue.

These 5 entities can be arranged among themselves in $5!$ ways.

Additionally, the 3 girls within their single unit can rearrange themselves in $3!$ ways.

By the Fundamental Principle of Counting (Multiplication Principle), the total number of ways in which all the girls are together is:

$$5! \times 3!$$

We know that $5! = 120$ and $3! = 6$, so:

$$120 \times 6 = 720$$

Number of arrangements where all the girls are NOT together

To find the required number of ways, we subtract the arrangements where all girls are together from the total possible arrangements.

Required ways = Total arrangements – Arrangements with all girls together

$$\text{Required ways} = 5040 - 720 = 4320$$

Therefore, the number of ways of forming a queue such that all the girls are not together is 4320.

The correct choice is Option D.

Question 18

Let

$$A = \{(a, b, c) : a, b, c \text{ are non-negative integers and } a + b + 2c = 22\}.$$

Then $n(A)$ is equal to :

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Options:

A.

121

B.

124

C.

144

D.

169

Answer: C

Solution:

We need to count the number of ordered triples (a, b, c) of non-negative integers satisfying

$$a + b + 2c = 22$$

Since $a, b, c \geq 0$, let us fix c first.

For a fixed value of c , the equation becomes

$$a + b = 22 - 2c$$

Now, the number of non-negative integer solutions of

$$a + b = n$$

is

$$n + 1$$

So, for fixed c , number of solutions is

$$(22 - 2c) + 1 = 23 - 2c$$

Now find all possible values of c .

Since $2c \leq 22$,

$$c = 0, 1, 2, \dots, 11$$

Hence total number of solutions is

$$\sum_{c=0}^{11} (23 - 2c)$$

This is an arithmetic series:

- first term = 23
- last term = $23 - 2(11) = 1$
- number of terms = 12

So,

$$n(A) = \frac{12}{2}(23 + 1)$$

$$n(A) = 6 \cdot 24 = 144$$

Therefore,

$$n(A) = 144$$

So the correct option is:

144

Hence, Option C.

Question 19

A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is :

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Options:

A.

4100

B.

4140

C.

4230

D.

4290

Answer: A

Solution:

Let us first understand the problem carefully. We are given a box containing:

Total number of blue balls = 5

Total number of yellow balls = 6

Total number of red balls = 4

We need to draw a total of 8 balls such that there are at least two balls of each colour. Let the number of blue, yellow, and red balls drawn be denoted by b , y , and r respectively.

According to the question, we have the following equation:

$$b + y + r = 8$$

Since we need at least 2 balls of each colour, we must satisfy the conditions:

$$b \geq 2, y \geq 2, \text{ and } r \geq 2$$

Since $2 + 2 + 2 = 6$, we only need to decide how to distribute the remaining 2 balls among the three colours. Let us list all the possible combinations for (b, y, r) :

Case 1: 4 blue, 2 yellow, 2 red ($b = 4, y = 2, r = 2$)

Case 2: 2 blue, 4 yellow, 2 red ($b = 2, y = 4, r = 2$)

Case 3: 2 blue, 2 yellow, 4 red ($b = 2, y = 2, r = 4$)

Case 4: 3 blue, 3 yellow, 2 red ($b = 3, y = 3, r = 2$)

Case 5: 3 blue, 2 yellow, 3 red ($b = 3, y = 2, r = 3$)

Case 6: 2 blue, 3 yellow, 3 red ($b = 2, y = 3, r = 3$)

Now, we will find the number of ways for each case using the combinations formula ${}^nC_r = \frac{n!}{r!(n-r)!}$.

1. Number of ways for Case 1:

$${}^5C_4 \times {}^6C_2 \times {}^4C_2 = 5 \times 15 \times 6 = 450$$

2. Number of ways for Case 2:

$${}^5C_2 \times {}^6C_4 \times {}^4C_2 = 10 \times 15 \times 6 = 900$$

3. Number of ways for Case 3:

$${}^5C_2 \times {}^6C_2 \times {}^4C_4 = 10 \times 15 \times 1 = 150$$

4. Number of ways for Case 4:

$${}^5C_3 \times {}^6C_3 \times {}^4C_2 = 10 \times 20 \times 6 = 1200$$

5. Number of ways for Case 5:

$${}^5C_3 \times {}^6C_2 \times {}^4C_3 = 10 \times 15 \times 4 = 600$$

6. Number of ways for Case 6:

$${}^5C_2 \times {}^6C_3 \times {}^4C_3 = 10 \times 20 \times 4 = 800$$

To find the total number of ways to draw the 8 balls satisfying the given conditions, we simply add the number of ways from all the above cases:

$$\text{Total ways} = 450 + 900 + 150 + 1200 + 600 + 800$$

$$\text{Total ways} = 4100$$

Therefore, the total number of ways of drawing 8 balls containing at least two balls of each colour is 4100.

Option A is the correct answer.

Question20

The number of 4-letter words, with or without meaning, each consisting of two vowels and two consonants that can be formed from the letters of the word INCONSEQUENTIAL, without repeating any letter, is:

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A.

2670

B.

2840

C.

2920

D.

3600

Answer: D

Solution:

From the word **INCONSEQUENTIAL**, let us first identify the vowels and consonants available.

The letters are:

$I, N, C, O, N, S, E, Q, U, E, N, T, I, A, L$

We have to form 4-letter words containing:

- exactly 2 vowels
- exactly 2 consonants
- no letter repeated

Since letters of the word may repeat in the original word, we count only **distinct available letters** for selection.

Step 1: Distinct vowels

The vowels present are:

A, E, I, O, U

So, number of distinct vowels = 5.

Step 2: Distinct consonants

The consonants present are:

N, C, S, Q, T, L

So, number of distinct consonants = 6.

Step 3: Choose 2 vowels and 2 consonants

Number of ways to choose 2 vowels from 5:

$${}^5C_2 = 10$$

Number of ways to choose 2 consonants from 6:

$${}^6C_2 = 15$$

So, total selections of letters:

$${}^5C_2 \times {}^6C_2 = 10 \times 15 = 150$$

Step 4: Arrange the 4 chosen letters

All 4 letters are distinct, so number of arrangements:

$$4! = 24$$

Hence, total number of such 4-letter words is:

$$150 \times 24 = 3600$$

Therefore, the correct answer is:

3600

So, **Option D** is correct.

Question21

A building has ground floor and 10 more floors. Nine persons enter in a lift at the ground floor. The lift goes up to the 10th floor. The number of ways, in which any 4 persons exit at a floor and the remaining 5 persons exit at a different floor, if the lift does not stop at the first and the second floors, is equal to :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

2184

B.

3064

C.

7056

D.

11340

Answer: C

Solution:

First, decide which 4 out of the 9 persons will get down together. The remaining 5 persons are then fixed automatically.

Number of ways to form the two groups of sizes 4 and 5 is $\frac{9!}{4!5!} = 126$

Now, the lift does not stop at the 1st and 2nd floors. So people can get down only on floors 3 to 10, i.e., 8 possible floors.

Choose any 2 different floors out of these 8 floors for the two groups to exit: ${}^8C_2 = 28$

After choosing the two floors, we must decide which floor is for the group of 4 and which floor is for the group of 5. This can be done in 2 ways.

So, total number of ways is $126 \times 28 \times 2 = 7056$

Question22

A person has three different bags and four different books. The number of ways, in which he can put these books in the bags so that no bag is empty, is :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

18

B.

36

C.

39

D.

72

Answer: B

Solution:

Step	Entity	Books	Bags	Combination	Picture	Number of ways (n)	Remarks																
1	Number 4	3		Any 2 books to any 1 bag	<table><tr><td></td><td>Bag</td><td>Bag</td><td>Bag</td></tr><tr><td>Books</td><td>Any 2</td><td></td><td></td></tr><tr><td>Books</td><td></td><td>Any 2</td><td></td></tr><tr><td>Books</td><td></td><td></td><td>Any 2</td></tr></table>		Bag	Bag	Bag	Books	Any 2			Books		Any 2		Books			Any 2	$({}^4C_2) \cdot ({}^3C_1)$	selective combinations
						Bag	Bag	Bag															
					Books	Any 2																	
					Books		Any 2																
Books			Any 2																				
2	Number 2	2		Any 1 book to any 1 bag	<table><tr><td>Bag</td><td>Bag</td></tr><tr><td>Any Book</td><td>Other book</td></tr><tr><td>Other book</td><td>Any book</td></tr></table>	Bag	Bag	Any Book	Other book	Other book	Any book	2	$({}^3C_1) \cdot ({}^2C_1) \dots$ Double counting										
					Bag	Bag																	
					Any Book	Other book																	
Other book	Any book																						
3	Number 1	1		1 book to the bag	<table><tr><td>Bag</td></tr><tr><td>Book</td></tr></table>	Bag	Book	1	Default														
Bag																							
Book																							

$$\text{Number of ways } (n) = ({}^4C_2) \cdot ({}^3C_1) \cdot (2) \cdot (1) = 36$$

Question23

The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

JEE Main 2025 (Online) 23rd January Evening Shift

Answer: 17280

Solution:

A : number of ways that all boys sit together = $5! \times 5!$

B : number of ways if no 2 boys

sit together = $4! \times 5!$

$A \cap B = \phi$

Required no. of ways = $5! \times 5! + 4! \times 5! = 17280$

Question24

The number of 3 -digit numbers, that are divisible by 2 and 3 , but not divisible by 4 and 9 , is _____.

JEE Main 2025 (Online) 24th January Morning Shift

Answer: 125

Solution:

No. of 3 digits = $999 - 99 = 900$

No. of 3 digit numbers divisible by 2 & 3 i.e. by 6

$$\frac{900}{6} = 150$$

No. of 3 digit numbers divisible by 4 & 9 i.e. by 36

$$\frac{900}{36} = 25$$

∴ No of 3 digit numbers divisible by 2&3 but not by 4&9

$$150 - 25 = 125$$

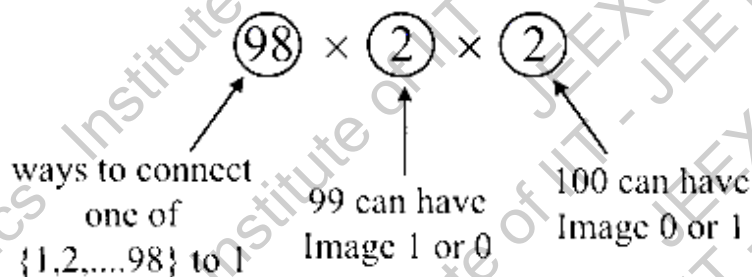
Question 25

Number of functions $f: \{1, 2, \dots, 100\} \rightarrow \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to _____.

JEE Main 2025 (Online) 24th January Evening Shift

Answer: 392

Solution:



392 Ans.

Question 26

The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is _____.

JEE Main 2025 (Online) 28th January Evening Shift

Answer: 64

Solution:

x	y	z
---	---	---

Let $x = 2 \Rightarrow y + z = 13$

$(4, 9), (5, 8), (6, 7), (7, 6), (8, 5), (9, 4) \rightarrow 6$

Let $x = 3 \Rightarrow y + z = 12$

$(3, 9), (4, 8), \dots, (9, 3) \rightarrow 7$

Let $x = 4 \Rightarrow y + z = 11$

$(2, 9), (3, 8), \dots, (9, 1) \rightarrow 9$

Let $x = 5 \Rightarrow y + z = 10$

$(1, 9), (2, 8), \dots, (9, 1) \rightarrow 10$

Let $x = 6 \Rightarrow y + z = 9$

$(0, 9), (1, 8), \dots, (9, 0) \rightarrow 9$

Let $x = 7 \Rightarrow y + z = 8$

$(0, 9), (1, 7), \dots, (8, 0) \rightarrow 9$

Let $x = 8 \Rightarrow y + z = 7$

$(0, 7), (1, 6), \dots, (7, 0) \rightarrow 8$

Let $x = 9 \Rightarrow y + z = 6$

$(0, 6), (1, 5), \dots, (6, 0) \rightarrow 7$

Total = $6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64$

Question 27

The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _____.

JEE Main 2025 (Online) 29th January Morning Shift

Answer: 1405

Solution:

(i) Single letter is used, then no. of words = 5

(ii) Two distinct letters are used, then no. of words

$${}^5C_2 \times \left(\frac{6!}{2!4!} \times 2 + \frac{6!}{3!3!} \right) = 10(30 + 20) = 500$$

(iii) Three distinct letters are used, then no. of words

$${}^5C_3 \times \frac{6!}{2!2!2!} = 900$$

Total no. of words = 1405

Question28

If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^n$; $m, n \in \{1, 2, 3, \dots, 9\}$, then $m + n$ is equal to _____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 14

Solution:

When numbers are uniformly distributed, half of them have even digit sums and half have odd digits sums.

Number of 7-digit numbers with even digit sum =

$$\frac{1}{2} \cdot 9 \cdot 10^6 = 4.5 \cdot 10^6$$

Note that $9 \cdot 5 \cdot 10^5 = 4.5 \cdot 10^6$

$$m + n = 9 + 5 = 14$$

Question29

All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n . Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_{n-1})$, $n > 1$.

If $P(\text{CDBEA}) = \frac{2^\alpha}{2^\beta - 1}$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to : _____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 183

Solution:

Firstly, by this rule, we note:

$$P(W_1) = p$$

$$P(W_2) = 2p$$

$$P(W_3) = 4p$$

...

$$P(W_n) = 2^{n-1}p$$

To find the initial probability p , consider the total probability must sum to 1 across all 120 possible words (since $5! = 120$):

$$\sum_{n=1}^{120} P(W_n) = 1$$

This is a geometric series sum where:

$$p(1 + 2 + 2^2 + \dots + 2^{119}) = 1$$

Since the series sum $1 + 2 + 2^2 + \dots + 2^{119}$ is equal to $2^{120} - 1$, we have:

$$p(2^{120} - 1) = 1 \Rightarrow p = \frac{1}{2^{120} - 1}$$

Thus, the probability for the n -th word is:

$$P(W_n) = \frac{2^{n-1}}{2^{120} - 1} \quad (\text{i})$$

Next, determine the position of "CDBEA". Starting from the first letter:

Words beginning with 'A': $4! = 24$

Words beginning with 'B': $4! = 24$

Words beginning with 'C':

CA*: $3! = 6$

$$\text{CB}^*: 3! = 6$$

$$\text{CDA}^{**}: 2! = 2$$

$$\text{CDBA}^*: 1! = 1$$

Summing these, the position of "CDBEA" is the 64th word.

Substitute into equation (i):

$$P(\text{CDBEA}) = P(W_{64}) = \frac{2^{63}}{2^{120}-1}$$

Given $P(\text{CDBEA}) = \frac{2^\alpha}{2^\beta-1}$, we find:

$$\alpha = 63$$

$$\beta = 120$$

Thus, the sum $\alpha + \beta = 63 + 120 = 183$.

Question30

Let m and n , ($m < n$), be two 2-digit numbers. Then the total numbers of pairs (m, n) , such that $\gcd(m, n) = 6$, is _____.

JEE Main 2025 (Online) 4th April Evening Shift

Answer: 64

Solution:

$$m = 6a, n = 6b$$

$$\text{So } \gcd(m, n) = 6 \Rightarrow \gcd(a, b) = 1$$

$$m = 6a \geq 10 \Rightarrow a \geq \left\lceil \frac{10}{6} \right\rceil = 2$$

$$m = 6a \leq 99 \Rightarrow a \leq \left\lfloor \frac{99}{6} \right\rfloor = 16$$

So $a, b \in \{2, 3, \dots, 16\}$, and we count how many coprime pairs (a, b) with $a < b$, $\gcd(a, b) = 1$

$$\begin{aligned}
 a = 2 &\Rightarrow b = 3, 5, 7, 9, 11, 13, 15 \Rightarrow 7 \\
 a = 3 &\Rightarrow b = 4, 5, 7, 8, 10, 11, 13, 14, 16 \Rightarrow 9 \\
 a = 4 &\Rightarrow b = 5, 7, 9, 11, 13, 15 \Rightarrow 6 \\
 a = 5 &\Rightarrow b = 6, 7, 8, 9, 11, 12, 13, 14, 16 \Rightarrow 9 \\
 a = 6 &\Rightarrow b = 7, 11, 13 \Rightarrow 3 \\
 a = 7 &\Rightarrow b = 8, 9, 10, 11, 12, 13, 15, 16 \Rightarrow 8 \\
 a = 8 &\Rightarrow b = 9, 11, 13, 15 \Rightarrow 4 \\
 a = 9 &\Rightarrow b = 10, 11, 13, 14, 16 \Rightarrow 5 \\
 a = 10 &\Rightarrow b = 11, 13 \Rightarrow 2 \\
 a = 11 &\Rightarrow b = 12, 13, 14, 15, 16 \Rightarrow 5 \\
 a = 12 &\Rightarrow b = 13, 17 \times \rightarrow \text{only 13 is valid} \Rightarrow 1 \\
 a = 13 &\Rightarrow b = 14, 15, 16 \Rightarrow 3 \\
 a = 14 &\Rightarrow b = 15, \Rightarrow 1 \\
 a = 15 &\Rightarrow b = 16 \Rightarrow 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Total} &= 7 + 9 + 6 + 9 + 3 + 8 + 4 + 5 + 2 + 5 + 1 \\
 &+ 3 + 1 + 1 = 64
 \end{aligned}$$

Question31

From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

- A. 6084
- B. 5148
- C. 14950
- D. 4356

Answer: B

Solution:

First, note that we are choosing **5 distinct letters** (in strictly increasing alphabetical order) such that the **middle (third) letter is 'M'**. Symbolically, if we denote the chosen letters as:

$$L_1 < L_2 < L_3 < L_4 < L_5,$$

we want $L_3 = M$. The English alphabet has 26 letters, and M is the 13th.

Step 1: Letters before M

The letters before M are $\{A, B, C, \dots, L\}$.

There are **12** letters here (A through L).

We need to pick **2** of these 12 letters to occupy L_1 and L_2 .

The number of ways to choose 2 letters out of 12 is ${}^{12}C_2$.

Step 2: Letters after M

The letters after M are $\{N, O, P, \dots, Z\}$.

There are **13** letters here (N through Z).

We need to pick **2** of these 13 letters to occupy L_4 and L_5 .

The number of ways to choose 2 letters out of 13 is ${}^{13}C_2$.

Step 3: Multiply the choices

Since these choices are independent (picking the two letters before M and two letters after M), the total number of ways is:

$${}^{12}C_2 \times {}^{13}C_2$$

Calculate each combination:

$${}^{12}C_2 = \frac{12 \times 11}{2} = 66, \quad {}^{13}C_2 = \frac{13 \times 12}{2} = 78.$$

So,

$${}^{12}C_2 \times {}^{13}C_2 = 66 \times 78 = 5148.$$

Answer: 5148 (Option B)

Question32

In a group of 3 girls and 4 boys, there are two boys B_1 and B_2 . The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

Options:

- A. 120
- B. 96
- C. 72
- D. 144

Answer: D

Solution:

Let's break the problem down step by step:

There are two blocks because all girls must stand together and all boys must stand together. The two blocks can be arranged in:

2 ways (i.e., girls first then boys, or boys first then girls).

The girls can be arranged among themselves in:

$$3! = 6 \text{ ways.}$$

For the boys (4 in total), they must be arranged such that the specific boys B_1 and B_2 are not adjacent.

First, calculate the total number of arrangements of 4 boys:

$$4! = 24.$$

Next, count the arrangements where B_1 and B_2 are adjacent. Think of B_1 and B_2 as a single unit. This unit can be arranged in:

$$2! = 2 \text{ ways (since } B_1 \text{ and } B_2 \text{ can swap positions).}$$

Now, with this new unit, we have 3 units in total (the B_1B_2 unit and the other 2 boys), which can be arranged in:

$$3! = 6 \text{ ways.}$$

So, the number of arrangements where B_1 and B_2 are adjacent is:

$$2! \times 3! = 2 \times 6 = 12.$$

Therefore, the number of valid arrangements for the boys where B_1 and B_2 are not adjacent is:

$$24 - 12 = 12.$$

Finally, multiply all the factors together:

$$\text{Total ways} = 2 \times 6 \times 12 = 144.$$

Thus, the number of ways in which the girls and boys can stand in the queue under the given conditions is

144.

This corresponds to Option D.

Question33

The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is :

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. 34000

B. 37000

C. 35000

D. 36000

Answer: D

Solution:

DAUGHTER

Total words = $8!$

Total words in which vowels are together = $6! \times 3!$ words in which all vowels are not together

$$= 8! - 6! \times 3!$$

$$= 6![56 - 6]$$

$$= 720 \times 50$$

$$= 36000$$

Question34

Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 8925

B. 9100

C. 8575

D. 8750

Answer: A

Solution:



C-I (3G&2 B)&(1G&2 B)

C-II (2G&3 B)&(2G&1 B)

C-III (1G&4 B)&(3G&0 B)

Total = C-I + C-II + C-III

$$= {}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1 + {}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1 \cdot {}^5C_2 + {}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$$

$$= 8925$$

Question 35

The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8, is

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. 5720

B. 5719

C. 4608

D. 4607

Answer: D

Solution:

Case I 5____0

Case II 5____1

5 2

5 3

6 0

6 1

6 2

7 0

Case IX 7____1

$9 \times (8 \times 8 \times 8) = 4608$ but 50000 is not included, so total numbers $4608 - 1 = 4607$

Question36

Let ${}^nC_{r-1} = 28$, ${}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let

$A(4 \cos t, 4 \sin t)$, $B(2 \sin t, -2 \cos t)$ and $C(3r - n, r^2 - n - 1)$ be the vertices of a triangle ABC , where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. 18

B. 8

C. 20

D. 6

Answer: C

Solution:

$${}^nC_{r-1} = 28, {}^nC_r = 56$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{28}{56}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{1}{2}$$

$$\frac{r}{(n-r+1)} = \frac{1}{2}$$

$$3r = n + 1 \quad \dots (i)$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{56}{70}$$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \dots (ii)$$

By (i) & (ii)

$$(r = 3), (n = 8)$$

$$A(4 \cos t, 4 \sin t) \quad B(2 \sin t, -2 \cos t) \quad C(3r - n, r^2 - n - 1)$$

$$A(4 \cos t, 4 \sin t) \quad B(2 \sin t, -2 \cos t) \quad C(1, 0)$$

$$(3x - 1)^2 + (3y)^2 = (4 \cos t + 2 \sin t)^2 + (4 \sin t - \cos t)^2$$

$$(3x - 1)^2 + (3y)^2 = 20 \quad \therefore \text{option (1)}$$

Question 37

Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

164

B.

158

C.

161

D.

173

Answer: C

Solution:

(i) number of numbers created using

$$1111133 = \frac{7!}{5!2!} \Rightarrow 21$$

(ii) number of numbers created using

$$1111223 = \frac{7!}{4!2!} \Rightarrow 105$$

(iii) number of numbers created using

$$1112222 = \frac{7!}{4!3!} \Rightarrow 35$$

$$\text{Total} = 161$$

Question38

If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement is :

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

PRNAKU

B.

PRKAUN

C.

PRKANU

D.

PRNAUK

Answer: B

Solution:

A, K, N, P, R, U

$$\boxed{A} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{K} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{N} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{P} \boxed{A} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{K} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{N} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{R} \boxed{A} \dots\dots\dots = \underline{3} = 6$$

$$\boxed{P} \boxed{R} \boxed{K} \boxed{A} \boxed{N} \boxed{U} = 1$$

$$\boxed{P} \boxed{R} \boxed{K} \boxed{A} \boxed{U} \boxed{N} = 1$$

Total = 440

$\Rightarrow 440^{\text{th}}$ word

Question39

The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1 s and exactly three 2 s, is equal to :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 360

B. 2520

C. 1820

D. 45

Answer: B

Solution:

To find the number of sequences of ten terms, each being either 0, 1, or 2, containing exactly five 1s and exactly three 2s, follow these steps:

Determine the Remaining Terms: Since there are 5 ones and 3 twos, you will need 2 zeros to fill the sequence (because $5 + 3 + 2 = 10$).

Calculate the Number of Arrangements: You have a total of 10 positions to fill with these numbers (5 ones, 3 twos, 2 zeros). The formula for calculating permutations of a multiset is:

$$\frac{10!}{5! \times 3! \times 2!}$$

Where:

10! is the factorial of the total number of terms.

5! is the factorial for the number of 1s.

3! is the factorial for the number of 2s.

2! is the factorial for the number of 0s.

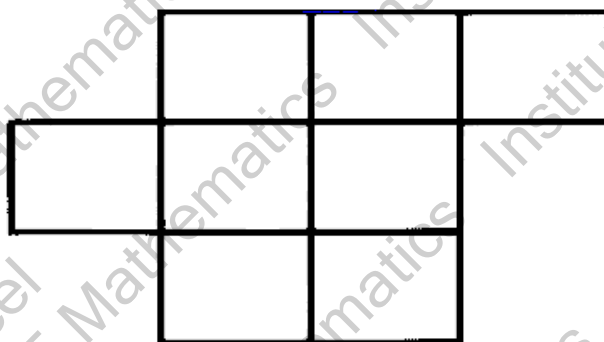
Result: Simplifying the calculation gives:

$$\frac{10!}{5! \times 3! \times 2!} = 2520$$

Thus, there are 2520 possible sequences with the given conditions.

Question 40

The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :



JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 5880

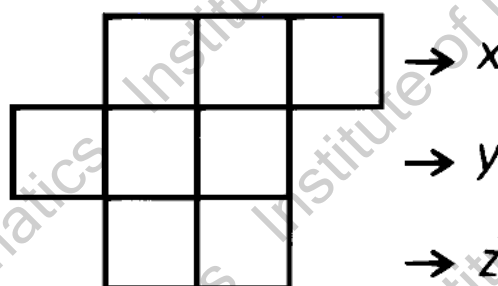
B. 840

C. 960

D. 5760

Answer: D

Solution:



Let x, y, z be the number of box which are filled

$$\Rightarrow 1 \leq x \leq 3, 1 \leq y \leq 3, 1 \leq z \leq 2$$

x	y	z	Number of ways
3	1	1	${}^3C_3 \cdot {}^3C_1 \cdot {}^2C_1 = 6$

x	y	z	Number of ways
2	2	1	${}^3C_2 \cdot {}^3C_2 \cdot {}^2C_1 = 18$
1	3	1	${}^3C_1 \cdot {}^3C_3 \cdot {}^2C_1 = 6$
2	1	2	${}^3C_2 \cdot {}^3C_1 \cdot {}^2C_2 = 9$
1	2	2	${}^3C_1 \cdot {}^3C_2 \cdot {}^2C_2 = 9$

Total ways = (48) to fill boxes

Now to arrange a, b, c, d and e

Number of ways will be $48 \cdot 5! = 5760$

Question 41

Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O. In the first quadrant, $P_1, P_2, \dots, P_{12}$ are 12 points on line L_1 and $Q_1, Q_2, \dots, Q_9$ are 9 points on line L_2 . Then the total number of triangles, that can be formed having vertices at three of the 22 points O, $P_1, P_2, \dots, P_{12}, Q_1, Q_2, \dots, Q_9$, is:

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. 1026

B. 1188

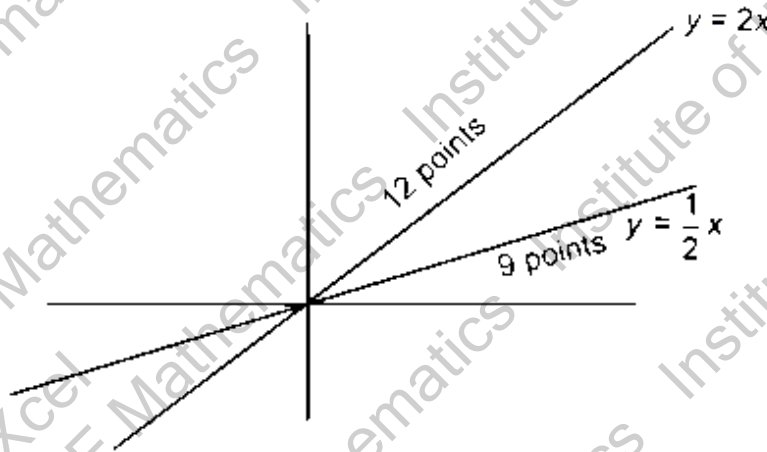
C. 1134

D. 1080

Answer: C

Solution:

Total triangles



$$\begin{aligned}
 & (2 \text{ points as } y = \frac{x}{2}, 1 \text{ point on } y = \frac{x}{2}) \\
 & + 2 (\text{ points and } y = \frac{x}{2}, 1 \text{ point on } y = 2x) \\
 & + (1 \text{ point on } y = 2x, 1) \text{ point on } y = \frac{x}{2} \text{ and origin} \\
 & = {}^9C_2, {}^{12}C_1 + {}^9C_1 {}^{12}C_2 + {}^9C_1 \cdot {}^{12}C_1 \cdot {}^1C_1 \\
 & = 1134
 \end{aligned}$$

Question 42

From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

JEE Main 2025 (Online) 7th April Morning Shift

Options:

- A. 145
- B. 165
- C. 155
- D. 135

Answer: C

Solution:

1 Captain, 1 vice-captain are already present

⇒ We need to select 8 players such that atleast 3 batsman and bowler must be there

Batsman	Bowler	Number of ways
3	5	${}^6C_3 \cdot {}^5C_5 = 20$
4	4	${}^6C_4 \cdot {}^5C_4 = 75$
5	3	${}^6C_5 \cdot {}^5C_3 = 60$

Total = 155 ways

Question43

There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

230

B.

210

C.

200

D.

220

Answer: B

Solution:

To count the number of distinct triangles:

Start with all possible triples of points.

From 12 points, the number of ways to choose any 3 is

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$$

Subtract the “invalid” triples that are collinear.

The only collinear sets of three points arise from the single line containing the 5 collinear points.

Number of ways to pick 3 points from those 5 is

$$\binom{5}{3} = 10.$$

Valid triangles = total triples – collinear triples.

$$220 - 10 = 210.$$

Hence, the total number of triangles that can be formed is **210**.

Correct option: B

Question 44

Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

[27-Jan-2024 Shift 2]

Options:

A.

$$\alpha \in \mathbb{N} \text{ and } \beta \notin \mathbb{N}$$

B.

$$\alpha \notin \mathbb{N} \text{ and } \beta \in \mathbb{N}$$

C.

$$\alpha \in \mathbb{N} \text{ and } \beta \in \mathbb{N}$$

D.

$$\alpha \notin \mathbb{N} \text{ and } \beta \notin \mathbb{N}$$

Answer: C

Solution:

$$\alpha = \frac{(4!)!}{(4!)^{3!}}, \beta = \frac{(5!)!}{(5!)^{4!}}$$

$$\alpha = \frac{(24)!}{(4!)^6}, \beta = \frac{(120)!}{(5!)^{24}}$$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

$$\text{No. of ways of formation of group} = \frac{24!}{(4!)^6 \cdot 6!} \in \mathbb{N}$$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

$$= \frac{(120)!}{(5!)^{24} \cdot 24!} \in \mathbb{N}$$

Question 45

All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

[29-Jan-2024 Shift 1]

Answer: 553

Solution:

Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

Question 46

Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

[29-Jan-2024 Shift 2]

Options:

A.

18

B.

16

C.

12

D.

15

Answer: D

Solution:

3 Shelf empty : $(8, 0, 0, 0) \rightarrow 1$ way

2 shelf empty: $\left. \begin{array}{l} (7, 1, 0, 0) \\ (6, 2, 0, 0) \\ (5, 3, 0, 0) \\ (4, 4, 0, 0) \end{array} \right\} \rightarrow 4$ ways

1 shelf empty : $\left. \begin{array}{l} (6, 1, 1, 0) \quad (3, 3, 2, 0) \\ (5, 2, 1, 0) \quad (4, 2, 2, 0) \\ (4, 3, 1, 0) \end{array} \right\} \rightarrow 5$ ways

0 Shelf empty : $\left. \begin{array}{l} (1, 2, 3, 2) \quad (5, 1, 1, 1) \\ (2, 2, 2, 2) \\ (3, 3, 1, 1) \\ (4, 2, 1, 1) \end{array} \right\} \rightarrow 5$ ways

Total = 15 ways

Question 47

In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections: A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _____

[30-Jan-2024 Shift 2]

Answer: 11376

Solution:

If 4 questions from each section are selected

Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$$\begin{aligned}\therefore \text{Total ways} &= {}^8C_5 \cdot {}^6C_5 \cdot {}^6C_5 + {}^8C_6 \cdot {}^6C_5 \cdot {}^6C_4 \times 2 + {}^8C_5 \cdot {}^6C_6 \cdot {}^6C_4 \times 2 + {}^8C_4 \cdot {}^6C_6 \cdot {}^6C_5 \times 2 + {}^8C_7 \cdot {}^6C_4 \cdot {}^6C_4 \\&= 56 \cdot 6 \cdot 6 + 28 \cdot 6 \cdot 15 \cdot 2 + 56 \cdot 15 \cdot 2 + 70 \cdot 6 \cdot 2 + 8 \cdot 15 \cdot 15 \\&= 2016 + 5040 + 1680 + 840 + 1800 = 11376\end{aligned}$$

Question 48

The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

[31-Jan-2024 Shift 1]

Answer: 3734

Solution:

We have I, T, T, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$$= {}^8C_1 \times \frac{4!}{3!} = 32$$

Number of words with selection (a, a, b, b)

$$= \frac{4!}{2!2!} = 6$$

Number of words with selection (a, a, b, c)

$$= {}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$$

Number of words with selection (a, b, c, d)

$$= {}^9C_4 \times 4! = 3024$$

$$\therefore \text{total} = 3024 + 672 + 6 + 32$$

$$= 3734$$

Question49

The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

[31-Jan-2024 Shift 2]

Options:

A.

406

B.

130

C.

142

D.

136

Answer: D

Solution:

After giving 2 apples to each child 15 apples left now 15 apples can be distributed in ${}^{15+3-1}C_2 = {}^{17}C_2$ ways

$$= \frac{17 \times 16}{2} = 136$$

Question50

If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

[1-Feb-2024 Shift 1]

Options:

A.

47

B.

53

C.

51

D.

43

Answer: C

Solution:

Total ways to partition 5 into 4 parts are :

$$5, 0, 0, 0 \Rightarrow 1 \text{ way}$$

$$4, 1, 0, 0 \Rightarrow \frac{5!}{4!} = 5 \text{ ways}$$

$$3, 2, 0, 0 \Rightarrow \frac{5!}{3!2!} = 10 \text{ ways}$$

$$2, 2, 0, 1 \Rightarrow \frac{5!}{2!2!2!} = 15 \text{ ways}$$

$$2, 1, 1, 1 \Rightarrow \frac{5!}{2!(1!)^3 3!} = 10 \text{ ways}$$

$$3, 1, 1, 0 \Rightarrow \frac{5!}{3!2!} = 10 \text{ ways}$$

$$\text{Total} \Rightarrow 1 + 5 + 10 + 15 + 10 + 10 = 51 \text{ ways}$$

Question51

The lines $L_1, L_2, \dots, L_{20}$ are distinct. For $n = 1, 2, 3, \dots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \dots, L_{20}\}$ is equal to :

[1-Feb-2024 Shift 2]

Answer: 101

Solution:

$L_1, L_3, L_5, \dots, L_{19}$ are Parallel

$L_2, L_4, L_6, \dots, L_{20}$ are Concurrent

$$\text{Total points of intersection} = {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1 = 101$$

Question52

The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is ____

[24-Jan-2023 Shift 1]

Answer: 60

Solution:

Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

Question53

A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

[24-Jan-2023 Shift 1]

Answer: 546

Solution:

For at most two language courses

$$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$$

Question54

The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

[24-Jan-2023 Shift 2]

Options:

- A. 120
- B. 168
- C. 220
- D. 48

Answer: B

Solution:

Four digit numbers greater than 7000 $= 2 \times 4 \times 3 \times 2 = 48$

Five digit number $= 5! = 120$

Total number greater than 7000 $= 120 + 48 = 168$

Question 55

Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of nonempty subsets of S that have the sum of all elements a multiple of 3, is _____.
[25-Jan-2023 Shift 1]

Answer: 43

Solution:

Elements of the type $3k = 3$

Elements of the type $3k + 1 = 1, 7, 9$

Elements of the type $3k + 2 = 2, 5, 11$

Subsets containing one element $S_1 = 1$

Subsets containing two elements

$$S_2 = {}^3C_1 \times {}^3C_1 = 9$$

Subsets containing three elements

$$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$$

Subsets containing four elements

$$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$$

Subsets containing five elements

$$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$$

Subsets containing six elements $S_6 = 1$

Subsets containing seven elements $S_7 = 1$

$$\Rightarrow \text{sum} = 43$$

Question 56

The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is
[25-Jan-2023 Shift 2]

Options:

A. 6

B. 12

C. 120

D. 72

Answer: D

Solution:

Numbers between 5000 & 10000

Using digits 1, 3, 5, 7, 9

$$\text{Total Numbers} = 3 \times 4 \times 3 \times 2 = 72$$

Question 57

Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 orange, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is _____
[25-Jan-2023 Shift 2]

Answer: 6860

Solution:

7 Red apple(RA), 5 white apple(WA), 8 oranges (O)
5 fruits to be selected (Note:- fruits taken different)
Possible selections :- (2O, 1 RA, 2 WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)
 $\Rightarrow {}^8C_2 {}^7C_1 {}^5C_2 + {}^8C_2 {}^7C_2 {}^5C_1 + {}^8C_3 {}^7C_1 {}^5C_1$
 $\Rightarrow 1960 + 2940 + 1960$
 $\Rightarrow 6860$

Question 58

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is _____.
[29-Jan-2023 Shift 1]

Answer: 32

Solution:

1	2				
---	---	--	--	--	--

$$= {}^7C_4 = 35$$

1	3				
---	---	--	--	--	--

$$= {}^6C_4 = 15$$

1	4				
---	---	--	--	--	--

$$= {}^5C_4 = 5$$

1	5				
---	---	--	--	--	--

$$= {}^4C_4 = 1$$

2	3				
---	---	--	--	--	--

$$= {}^6C_4 = 15$$

71 words

245678 $\rightarrow$ 72th word

$$2+4+5+6+7+8=32$$

$$= {}^6C_4 = 15$$

71 words

$$245678 \rightarrow 72^{\text{th}} \text{ word}$$

$$2 + 4 + 5 + 6 + 7 + 8 = 32$$

Question 59

Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____.
[29-Jan-2023 Shift 1]

Answer: 1436

Solution:

No of 5 digit numbers starting with digit 1

$$= 5 \times 5 \times 5 \times 5 = 625$$

No of 5 digit numbers starting with digit 2

$$= 5 \times 5 \times 5 \times 5 = 625$$

No of 5 digit numbers starting with 31

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 32

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 33

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 351

$$= 5 \times 5 = 25$$

No of 5 digit numbers starting with 352

$$= 5 \times 5 = 25$$

No of 5 digit numbers starting with 3531 = 5

No of 5 digit numbers starting with 3532 = 5

Before 35337 will be 4 numbers,

So rank of 35337 will be 1690

So, in descending order serial number will be

$$3125 - 1690 + 1 = 1436$$

Question60

The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48 , is
[29-Jan-2023 Shift 2]

Options:

- A. 472
- B. 432
- C. 507
- D. 400

Answer: B

Solution:

Total 3 digit number = 900

Divisible by 3 = 300 (Using $\frac{900}{3} = 300$)

Divisible by 4 = 225 (Using $\frac{900}{4} = 225$)

Divisible by 3 & 4 = 108, ...

(Using $\frac{900}{12} = 75$)

Number divisible by either 3 or 4
= $300 + 225 - 75 = 450$

We have to remove divisible by 48 ,
144, 192, ..., 18 terms

Required number of numbers = $450 - 18 = 432$

Question61

The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is:
[29-Jan-2023 Shift 2]

Options:

A. 89

B. 84

C. 86

D. 79

Answer: A

Solution:

Lets arrange the letters of OUGHT in alphabetical order.

G, H, O, T, U

Words starting with

G---- $\rightarrow 4!$

H---- $\rightarrow 4!$

O---- $\rightarrow 4!$

TG--- $\rightarrow 3!$

TH--- $\rightarrow 3!$

TO G-- $\rightarrow 2!$

T O H -- $\rightarrow 2!$

T O U G H $\rightarrow 1!$

— Total = 89

Question62

The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is _____.

[29-Jan-2023 Shift 2]

Answer: 3000

Solution:

N should be divisible by 2 but not by 3

$N = (\text{Numbers divisible by 2}) - (\text{Numbers divisible by 6})$

by 6)

$$N = \frac{9000}{2} - \frac{9000}{6} = 4500 - 1500 = 3000$$

Question 63

Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to _____
[30-Jan-2023 Shift 1]

Answer: 21

Solution:

For number to be divisible by 15, last digit should be 5 and sum of digits must be divisible by 3. Possible combinations are

1	2	1	5
---	---	---	---

Numbers = 3

2	2	3	5
---	---	---	---

Numbers = 3

3	3	1	5
---	---	---	---

Numbers = 3

1	1	5	5
---	---	---	---

Numbers = 3

2	3	5	5
---	---	---	---

Numbers = 3

3	3	5	5
---	---	---	---

Total Numbers = 21

Question64

The number of ways of selecting two numbers a and b , $a \in \{2, 4, 6, \dots, 100\}$ and $b \in \{1, 3, 5, \dots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is
[30-Jan-2023 Shift 2]

Options:

A. 186

B. 54

C. 108

D. 268

Answer: C

Solution:

$$a \in \{2, 4, 6, 8, 10, \dots, 100\}$$

$$b \in \{1, 3, 5, 7, 9, \dots, 99\}$$

$$\text{Now, } a + b \in \{25, 71, 117, 163\}$$

$$(i) a + b = 25, \text{ no. of ordered pairs } (a, b) \text{ is } 12$$

$$(ii) a + b = 71, \text{ no. of ordered pairs } (a, b) \text{ is } 35$$

$$(iii) a + b = 117, \text{ no. of ordered pairs } (a, b) \text{ is } 42$$

$$(iv) a + b = 163, \text{ no. of ordered pairs } (a, b) \text{ is } 19 \therefore \text{total} = 108 \text{ pairs}$$

Question 65

The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.
[30-Jan-2023 Shift 2]

Answer: 240

Solution:

Digits are 1, 2, 2, 2, 3, 3, 5

$$\text{If unit digit 5, then total numbers} = \frac{6!}{3!2!}$$

$$\text{If unit digit 3, then total numbers} = \frac{6!}{3!}$$

$$\text{If unit digit 1, then total numbers} = \frac{6!}{3!2!}$$

$$\therefore \text{total numbers} = 60 + 60 + 120 = 240$$

Question 66

Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to _____.
[31-Jan-2023 Shift 1]

Answer: 710

Solution:

$$1000 - 2799$$

Divisible by 3

$$1002 + (n-1)3 = 2799$$

$$n = 600$$

Divisible by 11

$$1 - 2799 \rightarrow \left[\frac{2799}{11} \right] = [254] = 254$$

$$1 - 999 = \left[\frac{999}{11} \right] = 90$$

$$1000 - 2799 = 254 - 90 = 164$$

Divisible by 33

$$1 - 2799 \rightarrow \left[\frac{2799}{33} \right] = 84$$

$$1 - 999 \rightarrow \left[\frac{999}{33} \right] = 30$$

$$1000 - 2799 \rightarrow 54$$

$$\therefore n(3) + n(11) - n(33)$$

$$600 + 164 - 54 = 710$$

Question 67

Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____.
[31-Jan-2023 Shift 1]

Answer: 2997

Solution:

$$42920 = 1$$

$$42922 = 1$$

$$42923 = 1$$

$$= 2997$$

Question68

If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to:
[31-Jan-2023 Shift 2]

Answer: 45

Solution:

$$\begin{aligned}\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} &= \frac{11}{21} \\ \Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} &= \frac{11}{21} \\ \Rightarrow \frac{2n+1}{(n+1)(n+2)} &= \frac{11}{42} \\ \Rightarrow n &= 5 \\ \Rightarrow n^2 + n + 15 &= 25 + 5 + 15 = 45\end{aligned}$$

Question69

The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____.
[1-Feb-2023 Shift 1]

Answer: 50400

Solution:

Vowels : A,A,A,I,I,O
Consonants : S,S,S,S,N,N,T
 $\therefore$ Total number of ways in which vowels come together

$$= \frac{x8}{[4]2} \times \frac{x6}{[x3]2} = 50400$$

Question70

Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1, y \geq 3, z \geq 4$, is equal to _____.
[1-Feb-2023 Shift 2]

Answer: 105

Solution:

$${}^{15}C_2 = \frac{15 \times 14}{2} = 105$$

Question71

The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is _____.
[1-Feb-2023 Shift 2]

Answer: 81

Solution:

Taking single digit $\rightarrow 444444 \quad \frac{6!}{6!} = 1$

Taking two digit $\rightarrow$

(4, 5) 444555 (4, 9) 444999

$$\frac{5!}{3!2!} = 10 \quad \frac{5!}{3!2!} = 10$$

Taking three digit

$$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$$

$$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$$

$$\text{Total} = 81$$

Question72

The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _____.

[6-Apr-2023 shift 1]

Answer: 171

Solution:

20 distinct oranges distributed among 3 children so

that each child gets at least one orange

$$= 3^{20} - {}^3C_1 2^{20} + {}^3C_2 1^{20}$$

Question73

All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers.

Then the serial number of the word PUBLIC is :

[6-Apr-2023 shift 2]

Options:

A. 580

B. 578

C. 576

D. 582

Answer: D

Solution:

$$B\text{-----} = 5! = 120$$

$$C\text{-----} = 5! = 120$$

$$I\text{-----} = 5! = 120$$

$$L\text{-----} = 5! = 120$$

$$PB\text{-----} = 4! = 24$$

$$PC\text{-----} = 4! = 24$$

$$PI\text{-----} = 4! = 24$$

$$PL\text{-----} = 4! = 24$$

$$PUBC\text{-----} = 2! = 2$$

$$PUBI\text{-----} = 2! = 2$$

$$PUBLC\text{-----} = 1$$

$$PUBLIC\text{-----} = \frac{1}{582}$$

$$\text{Rank} = 582$$

Ans. Option 4

Question 74

The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from

the letters of the word **UNIVERSE** without repetition is _____ :
[6-Apr-2023 shift 2]

Answer: 432

Solution:

Case 1 2 vowels different, 2 consonant different

$$\begin{aligned} &({}^3C_2)({}^4C_2)(4!) \\ &= (3)(6)(24) \\ &= 432 \end{aligned}$$

Question 75

The number of arrangements of the letters of the word
"INDEPENDENCE" in which all the vowels always occur together is.
[8-Apr-2023 shift 1]

Options:

- A. 16800
- B. 14800
- C. 18000
- D. 33600

Answer: A

Solution:

$$\begin{aligned} &\text{IEEEE,} \\ &\text{NNN, DD, P, C} \\ &\frac{8!}{3!2!} \times \frac{6!}{4!} = 16800 \end{aligned}$$

Question 76

The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is
[8-Apr-2023 shift 1]

Options:

A. $7(720)^2$

B. 720

C. $7(360)^2$

D. $126(5!)^2$

Answer: D

Solution:

$$\begin{aligned} & 6! \times {}^7C_5 \times 5! \\ & \Rightarrow 720 \times 21 \times 120 \\ & \Rightarrow 2 \times 360 \times 7 \times 3 \times 120 \\ & \Rightarrow 126 \times (5!)^2 \end{aligned}$$

Question 77

If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)k$, is equal to
[8-Apr-2023 shift 2]

Options:

A. 1890

B. 945

C. 2835

D. 5670

Answer: D

Solution:

$M_2A_2T_2$ HEICS

= total words - when C&S are together

$$\frac{11!}{2!2!2!} - \frac{10!}{2!2!} \times 2$$
$$\frac{1 \times 10}{2!2!} \times 9$$
$$= \frac{9 \times 10 \times 9 \times 8 \times 7}{8} \downarrow 6$$

$$= 5670 \downarrow 6$$

$$k = 5670 \text{ (Option 4)}$$

Question 78

The number of permutations of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is

[10-Apr-2023 shift 1]

Answer: 4898

Solution:

Numbers are 1, 2, 3, 4, 5, 6, 7

Numbers having string (154) = (154), 2, 3, 6, 7 = 5 !

Numbers having string (2467) = (2467), 1, 3, 5 = 4 !

Number having string (154) and (2467)

$= (154), (2467) = 2!$
 Now $n(154 \cup 2467) = 5! + 4! - 2!$
 $= 120 + 24 - 2 = 142$
 Again total numbers $= 7! = 5040$
 Now required numbers $= n$ (neither 154 nor 2467)
 $= 5040 - 142$
 $= 4898$

Question 79

Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is _____.
 [10-Apr-2023 shift 1]

Answer: 16

Solution:

Let number of couples $= n$
 $\therefore {}^nC_2 \times {}^{n-2}C_2 \times 2 = 840$
 $\Rightarrow n(n-1)(n-2)(n-3) = 840 \times 2$
 $= 21 \times 40 \times 2$
 $= 7 \times 3 \times 8 \times 5 \times 2$
 $n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$
 $\therefore n = 8$
 Hence, number of persons $= 16$.

Question 80

Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is
 [10-Apr-2023 shift 2]

Options:

A. 1120

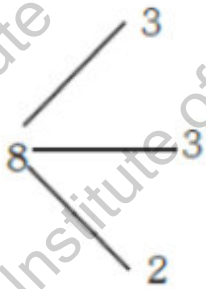
B. 560

C. 3360

D. 1680

Answer: D

Solution:



$$\begin{aligned}\text{Ways} &= \frac{8!}{3!3!2!} \times 3! \\ &= \frac{8!}{3!3!2!} \times 3! \\ &= 56 \times 30 \\ &= 1680\end{aligned}$$

Question 81

The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____.

[10-Apr-2023 shift 2]

Answer: 26664

Solution:

$$\begin{aligned}&2, 1, 2, 3 \\ &-- \frac{1}{2!} = 3 \\ &-- \times 2 \quad 3! = 6\end{aligned}$$

$$--x3 \frac{3!}{2!} = 3$$

$$\text{Sum of digits of unit place} = 3 \times 1 + 6 \times 2 + 3 \times 3 = 24$$

Required sum

$$= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$$

$$= 24 \times 1111$$

$$= 26664$$

Question82

The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is :

[11-Apr-2023 shift 1]

Options:

A. 136

B. 114

C. 80

D. 92

Answer: B

Solution:

$$x + y + z = 15$$

$$\text{Total no. solution} = {}^{15+3-1}C_3 = 136... (1)$$

$$\text{Let } x = y \neq z$$

$$2x + z = 15 \Rightarrow z = 15 - 2t$$

$$\Rightarrow t \in \{0, 1, 2, \dots, 7\} - \{5\}$$

$\therefore 7$ solutions

$\therefore$ there are 21 solutions in which exactly

Two of x, y, z are equal ... (2)

There is one solution in which $x = y = z...$ (3)

$$\text{Required answer} = 136 - 21 - 1 = 114$$

Question83

In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the

students sits on the allotted seat, is _____.
[11-Apr-2023 shift 1]

Answer: 44

Solution:

Derangement of 5 students

$$\begin{aligned}D_5 &= 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right) \\&= 120 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} \right) \\&= 60 - 20 + 5 - 1 \\&= 40 + 4 \\&= 44\end{aligned}$$

Question84

If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is
[11-Apr-2023 shift 2]

Options:

- A. 102
- B. 103
- C. 101
- D. 104

Answer: A

Solution:

5	2	1	3	4
T	H	A	M	S
4	1	0	0	0
4!	3!	2!	1!	0!

$$\Rightarrow 4 \times 4! + 3! \times 1 + 0 + 0 + 0$$

$$\Rightarrow 96 + 6 = 102$$

$$\text{Rank THAMS} = 102 + 1 = 103$$

Question 85

The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to
[12-Apr-2023 shift 1]

Options:

A. 132

B. 120

C. 72

D. 96

Answer: B

Solution:

5	x	x	x	0
7	x	x	x	0
5	x	x	x	5
9	x	x	x	0
9	x	x	x	5

$$\text{So Required numbers} = 5 \times {}^4P_3 = 120$$

Question86

Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

[12-Apr-2023 shift 1]

Answer: 1260

Solution:

$$\begin{array}{l} abc \text{ or } cba \\ abc \\ - - - - \\ cba \end{array}$$
$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

Question87

The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _____.

[13-Apr-2023 shift 1]

Answer: 413

Solution:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12, x_i \in \{1, 2, 3, 4\}$$
$$\text{No. of solutions} = {}^{5+7-1}C_{7-1} = \frac{7!}{6!} - \frac{7!}{5!} = 413$$

Question88

All words, with or without meaning, are made using all the letters of the word **MONDAY**. These words are written as in a dictionary with serial numbers. The serial number of the word **MONDAY** is [13-Apr-2023 shift 2]

Options:

- A. 328
- B. 327
- C. 324
- D. 326

Answer: B

Solution:

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{A} & & & & & \\ \hline \end{array} \overset{5!}{=} 120$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{D} & & & & & \\ \hline \end{array} \overset{5!}{=} 120$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{A} & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{D} & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{N} & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{O} & \text{A} & & & \\ \hline \end{array} \overset{3!}{=} 6$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{O} & \text{D} & & & \\ \hline \end{array} \overset{3!}{=} 6$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{O} & \text{N} & \text{A} & & \\ \hline \end{array} \overset{2!}{=} 2$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{O} & \text{N} & \text{D} & \text{A} & \text{Y} \\ \hline \end{array} = 1$$

$$\begin{aligned} \text{Rank} &= 120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1 \\ &= 327 \end{aligned}$$

Question89

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is _____.
[13-Apr-2023 shift 2]

Answer: 16

Solution:

a	b	2
---	---	---

$(a, b) = (1, 3), (3, 1), (2, 2), (2, 5), (5, 2), (3, 4), (4, 3), (5, 5)$
 $= 8$ numbers

a	b	4
---	---	---

$(a, b) = (1, 1), (1, 4), (4, 1), (2, 3), (3, 2)$
 $(4, 4), (3, 5), (5, 3) = 8$ numbers
total $8 + 8 = 16$

Question90

The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is

[15-Apr-2023 shift 1]

Options:

A. 21

B. 18

C. 20

D. 22

Answer: D

Solution:

(1, 1, 1)(3, 3, 3)(5, 5, 5)(8, 8, 8)

(5, 5, 8)(8, 8, 5)(1, 3, 5)(1, 3, 8)

$$\text{Total number} = 1 + 1 + 1 + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 3! + 3! = 22$$

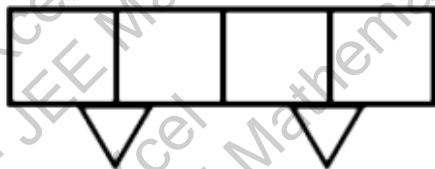
Question 91

A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is

[15-Apr-2023 shift 1]

Answer: 72

Solution:



Sum of first two digits

Sum of last two digits = α

Case-I : $\alpha = 7$

$$2 \times 12 = 24 \text{ ways.}$$

7	0		
0	7	1	6
		2	5
		3	4
		4	3
		5	2
		6	1

Case - II : $\alpha = 8$

1	7	2	6
7	1	6	2
		3	5
		5	3

--	--	--	--

$2 \times 8 \text{ ways}$

$= 16 \text{ ways}$

Case-III : $\alpha = 9$

2	7	3	6
7	2	6	3
		4	5
		5	4

--	--	--	--

$2 \times 8 \text{ ways}$

$= 16 \text{ ways}$

Case IV : $\alpha = 10$

3	7	4	6
7	3	6	4

$2 \times 4 \text{ ways}$

8 ways

Case V : $\alpha = 11$

4	7	5	6
7	4	6	5

$2 \times 4 \text{ ways}$

8 ways

$$\text{Ans. } 24 + 16 + 16 + 8 + 8 = 72$$

Question92

The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is
[25-Jul-2022-Shift-1]

Answer: 1492

Solution:

M	A	N	K	I	N	D
---	---	---	---	---	---	---

$$\left(\frac{4 \times 6!}{2!} \right) + (5! \times 0) + \left(\frac{4! \times 3}{2!} \right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

Question93

The number of 5-digit natural numbers, such that the product of their digits is 36, is _____.
[26-Jul-2022-Shift-1]

Answer: 180

Solution:

Factors of $36 = 2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

(1, 2, 2, 3, 3), (1, 4, 3, 3, 1), (1, 9, 2, 2, 1)

(1, 4, 9, 11), (1, 2, 3, 6, 1), (1, 6, 6, 1, 1)

i.e., total numbers

$$\frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{3!2!}$$

$$= (30 \times 3) + 20 + 60 + 10 = 180$$

Question 94

Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _____.

[26-Jul-2022-Shift-2]

Answer: 30

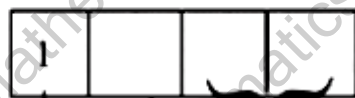
Solution:

Here 1st digit is 1 or 2 only

Case-I

If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64



$$1 \times 3 \times 6 = 18 \text{ ways}$$

Case - II If first digit is 2 then last two digit can be 16, 36, 56, 64



$$1 \times 3 \times 4 = 12 \text{ ways}$$

Total ways = 12 + 18 = 30 ways

Question95

Let S be the sample space of all five digit numbers. If p is the probability that a randomly selected number from S , is a multiple of 7 but not divisible by 5, then $9p$ is equal to
[27-Jul-2022-Shift-1]

Options:

- A. 1.0146
- B. 1.2085
- C. 1.0285
- D. 1.1521

Answer: C

Solution:

Among the 5 digit numbers,
First number divisible by 7 is 10003 and last is 99995 .
 $\Rightarrow$ Number of numbers divisible by 7 .

$$= \frac{99995 - 10003}{7} + 1$$
$$= 12857$$

First number divisible by 35 is 10010 and last is 99995.
 $\Rightarrow$ Number of numbers divisible by 35

$$= \frac{99995 - 10010}{35} + 1$$
$$= 2572$$

Hence number of number divisible by 7 but not by 5

$$= 12857 - 2572$$
$$= 10285$$

$$9p = \frac{10285}{90000} \times 9$$
$$= 1.0285$$

Question96

Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then α is equal to _____.
[28-Jul-2022-Shift-1]

Answer: 7073

Solution:

If password is 6 character long, then

Total number of ways having atleast one number $= 10^6 - 5^6$

Similarly, if 7 character long $= 10^7 - 5^7$

and if 8-character long $= 10^8 - 5^8$

Number of password $= (10^6 + 10^7 + 10^8) - (5^6 + 5^7 + 5^8)$

$= 5^6(2^6 + 5 \cdot 2^7 + 25 \cdot 2^8 - 1 - 5 - 25)$

$= 5^6(64 + 640 + 6400 - 31)$

$= 7073 \times 5^6$

$\therefore \alpha = 7073$

Question97

A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then $b + 3g$ is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 17

Solution:

$${}^bC_3 \cdot {}^gC_2 = 168$$

$$\Rightarrow \frac{b(b-1)(b-2)}{6} \cdot \frac{g(g-1)}{2} = 168$$

$$\Rightarrow b(b-1)(b-2) \cdot g(g-1) = 2^5 \cdot 3^2 \cdot 7$$

$$\Rightarrow b(b-1)(b-2) \cdot g(g-1) = 6 \cdot 7 \cdot 8 \cdot 3 \cdot 2$$

$$\therefore b = 8 \text{ and } g = 3$$

$$\therefore b + 3g = 17$$

Question 98

Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \dots, 1000\}$. If

$A = \{a_1 + a_2 + \dots + a_k : k \in \mathbb{N}, a_1, a_2, a_3, \dots, a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to _____.

[29-Jul-2022-Shift-1]

Answer: 11

Solution:

Here $S = \{4, 6, 9\}$

And $T = \{9, 10, 11, \dots, 1000\}$

We have to find all numbers in the form of $4x + 6y + 9z$, where $x, y, z \in \{0, 1, 2, \dots\}$.

If a and b are coprime number then the least number from which all the number more than or equal to it can be express as $ax + by$ where $x, y \in \{0, 1, 2, \dots\}$ is $(a-1) \cdot (b-1)$.

Then for $6y + 9z = 3(2y + 3z)$

All the number from $(2-1) \cdot (3-1) = 2$ and above can be express as $2x + 3z$ (say t).

Now $4x + 6y + 9z = 4x + 3(t+2)$

$= 4x + 3t + 6$

again by same rule $4x + 3t$, all the number from $(4-1)(3-1) = 6$ and above can be express from $4x + 3t$

Then $4x + 6y + 9z$ express all the numbers from 12 and above.

again 9 and 10 can be express in form $4x + 6y + 9z$.

Then set $A = \{9, 10, 12, 13, \dots, 1000\}$.

Then $T - A = \{11\}$

Only one element 11 is there.

Sum of elements of $T - A = 11$

Question99

The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _____.

[29-Jul-2022-Shift-2]

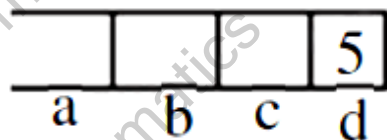
Answer: 6

Solution:

4 digit numbers

For divisibility by 55, no. should be div. by 5 and 11 both

Also, for divisibility by 11



$$a + c = b + 5$$

$$\text{for } b = 1 \quad a = 2, \quad c = 4$$

$$a = 4, \quad c = 2$$

$$\text{for } b = 2 \quad a = 3, \quad c = 4$$

$$a = 4, \quad c = 3$$

$$\text{for } b = 3 \quad a = 6, \quad c = 2$$

$$a = 2, \quad c = 6$$

$\therefore$ 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible



(Not possible)

Question100

In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is ____
[24-Jun-2022-Shift-1]

Answer: 40

Solution:

Let student marks x correct answers and y incorrect. So

$$3x - 2y = 5 \text{ and } x + y \leq 5 \text{ where } x, y \in W$$

Only possible solution is $(x, y) = (3, 2)$

Students can mark correct answers by only one choice but for an incorrect answer, there are two choices. So total number of ways of scoring 5 marks $= {}^5C_3(1)^3 \cdot (2)^2 = 40$

Question101

The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is ____
[24-Jun-2022-Shift-2]

Answer: 576

Solution:

Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 $\rightarrow$ Difference of sum at even backslash & odd place is divisible by 11.

Let number of the form abcdefg

$$\therefore (a + c + e + g) - (b + d + f) = 11x$$

$$a + b + c + d + e + f = 31$$

$$\therefore \text{either } a + c + e + g = 21 \text{ or } 10$$

$$\therefore b + d + f = 10 \text{ or } 21$$

Case-1

$$a + c + e + g = 21$$

$$b + d + f = 10$$

$$(b, d, f) \in \{(1, 2, 7), (2, 3, 5), (1, 4, 5)\}$$

$$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$$

Case-2

$$a + c + e + g = 10$$

$$b + d + f = 21$$

$$(a, b, e, g) \in \{1, 2, 3, 4\}$$

$$(b, d, f) \in \{(5, 7, 9)\}$$

$$\therefore \text{Total number in case 2} = 3! \times 4! = 144$$

$$\therefore \text{Total numbers} = 144 + 432 = 576$$

Question102

The sum of all the elements of the set $\{\alpha \in \{1, 2, \dots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is ____
[24-Jun-2022-Shift-2]

Answer: 1633

Solution:

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100 .

$\therefore$ Complete sum

$$= 96 + (24 \times 8 + 96) + (48 \times 8 + 96) + (72 \times 8 + 96) + 97$$

$$= 1633$$

Question103

The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is ____
[25-Jun-2022-Shift-1]

Answer: 63

Solution:

For odd numbers, unit place shall be 1, 3, 5, 7 or 9.

$xyxy$, $xyxy$, $xyxy$, $xyxz$, $xyx9$ are the type of numbers.

If $x \neq y$ then

$$x + y = 6, 13, 20 \dots$$

$$\text{Total number} = 6 + 6 + 0 + \dots = 12$$

If $x = y$ 3 then

$$x + y = 4, 11, 18, \dots$$

$$\text{Total number} = 4 + 8 + 1 + 0 \dots = 13$$

Similarly for $x \neq y$ 5, we have

$$x + y = 2, 9, 16, \dots$$

$$\text{Total number} = 2 + 9 + 3 = 14$$

for $x = y$ 7 we have

$$x + y = 0, 7, 14, \dots$$

$$\text{Total number} = 0 + 7 + 5 = 12 \text{ ways}$$

And for $x \neq y$ 9 we have

$$x + y = 5, 12, 19 \dots$$

$$\text{Total number} = 5 + 7 + 0 \dots = 12 \text{ ways}$$

$\therefore$ Total odd numbers whose sum of digits is a multiple of 7 is 63.

Question104

The total number of three-digit numbers, with one digit repeated exactly two times, is _____
[25-Jun-2022-Shift-2]

Answer: 243

Solution:

C - 1 : All digits are non-zero

$${}^9C_2 \cdot 2 \cdot \frac{3!}{2} = 216$$

C - 2: One digit is 0

$$0, 0, x \Rightarrow {}^9C_1 \cdot 1 = 9$$

$$0, x, x \Rightarrow {}^9C_1 \cdot 2 = 18$$

$$\text{Total} = 216 + 27 = 243$$

Question105

There are ten boys $B_1, B_2, \dots, B_{10}$ and five girls $G_1, G_2, \dots, G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _____
[26-Jun-2022-Shift-1]

Answer: 1120

Solution:

Number of ways when B_1 and B_2 are not together

= Total number of ways of selecting 3 boys - B_1 and B_2 are together

$$= {}^{10}C_3 - {}^8C_1$$

$$= \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} - 8$$

$$= 112$$

Number of ways to select 3 girls = ${}^5C_3 = 10$

$\therefore$ Total number of ways = $112 \times 10 = 1120$

Question 106

The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _____
[26-Jun-2022-Shift-2]

Answer: 150

Solution:

$$\forall x \in [100, 999], x \in N$$

$$\text{Then } \frac{x}{2} \in [50, 499], \frac{x}{2} \in N$$

Number whose G.C.D. with 18 is 1 in this range have the required condition. There are 6 such number from 18×3 to 18×4 . Similarly from 18×4 to 18×5 , ..., 26×18 to 27×18

$$\therefore \text{Total numbers} = 24 \times 6 + 6 = 150$$

The extra numbers are 53, 487, 491, 493, 497 and 499.

Question 107

The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is
[27-Jun-2022-Shift-1]

Answer: 56

Solution:



$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, \quad x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2$$

$$x_3 = t_3 + 2$$

$$x_4 = t_4 + 2$$

$$x_5 = t_5 + 2$$

$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

$$\text{No. of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

Question108

The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is
[28-Jun-2022-Shift-1]

Options:

A. 36

B. 48

C. 60

D. 72

Answer: D

Solution:

To make a no. divisible by 3 we can use the digits 1, 2, 5, 6, 7 or 1, 2, 3, 5, 7.

Using 1, 2, 5, 6, 7, number of even numbers is

$$= 4 \times 3 \times 2 \times 1 \times 2 = 48$$

Using 1, 2, 3, 5, 7, number of even numbers is

$$= 4 \times 3 \times 2 \times 1 \times 1 = 24$$

Required answer is 72.

Question 109

Let $A = \{1, a_1, a_2, \dots, a_{18}, 77\}$ be a set of integers with

$1 < a_1 < a_2 < \dots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$

contain exactly 39 elements. Then, the value of $a_1 + a_2 + \dots + a_{18}$ is equal to _____

[28-Jun-2022-Shift-1]

Answer: 702

Solution:

$a_1, a_2, a_3, \dots, a_{18}, 77$

are in AP i.e. 1, 5, 9, 13, ..., 77.

Hence $a_1 + a_2 + a_3 + \dots + a_{18} = 5 + 9 + 13 + \dots$ 18 terms $= 702$

Question 110

The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives at least 4 and at most 7 candies, C_3 receives at least 2 and at most 6 candies, is equal to:

[28-Jun-2022-Shift-2]

Options:

A. 205

B. 615

C. 510

D. 430

Answer: D

Solution:

By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2 and C_3, C_4

$$\begin{aligned} &= \text{Coefficient of } x^{30} \text{ in } (x^4 + x^5 + \dots + x^7)(x^2 + x^3 + \dots + x^6)(1 + x + x^2 + \dots)^2 \\ &= \text{Coefficient of } x^{24} \text{ in } \frac{(1-x^4)}{1-x} \frac{(1-x^5)}{1-x} \frac{(1-x^{31})^2}{(1-x)^2} \\ &= \text{Coefficient of } x^{24} \text{ in } (1-x^4-x^5+x^9)(1-x)^{-4} \\ &= {}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430 \end{aligned}$$

Question111

Let $b_1 b_2 b_3 b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \dots, 100\}$ for $1 \leq i \leq 4$ and $b_i \neq b_j$ for $i \neq j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1 b_2 b_3 b_4$ is equal to _____
[29-Jun-2022-Shift-1]

Answer: 18915

Solution:

$$b_i \in \{1, 2, 3, \dots, 100\}$$

Let A = set when $b_1 b_2 b_3$ are consecutive

$$n(A) = \frac{97 + 97 + \dots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2 b_3 b_4$ are consecutive

$$n(A) = 97 \times 98$$

$$n(A \cap B) = \frac{97 + 97 + \dots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2 b_3 b_4$ are consecutive

$$n(B) = 97 \times 98$$

$$n(A \cap B) = 97$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{Number of permutation} = 18915$$

Question112

The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to _____
[29-Jun-2022-Shift-2]

Answer: 1086

Solution:

Let the number is abcd, where a,b,c are divisible by d.

No. of such numbers

$$d = 1, 9 \times 10 \times 10 = 900$$

$$d = 2, 4 \times 5 \times 5 = 100$$

$$d = 3, 3 \times 4 \times 4 = 48$$

$$d = 4, 2 \times 3 \times 3 = 18$$

$$d = 5, 1 \times 2 \times 2 = 4$$

$$d = 6, 7, 8, 9 \quad 4 \times 4 = 16$$

$$\text{Total} = 1086$$

Question113

The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____
[2021, 26 Feb. Shift-II]

Answer: 1000

Solution:

Let x be four digit number, then $\gcd(x, 18) = 3$

This implies x is divisible by 3 but not divisible by 9.

The 4-digit numbers which is an odd multiple of 3 are 1005, 1011, 1017, ..., 9999

These are 1499 in counting i.e. total number of 4-digit numbers which is odd multiple of 3 are 1499.

Now, The 4-digit numbers which is an odd multiple of 9 are, 1017, 1035, ...999 These, are total 499.

Then, required 4-digit numbers

$$= 1499 - 499 = 1000$$

Question 114

The total number of two digit numbers n , such that $3^n + 7^n$ is a multiple of 10, is
[2021, 25 Feb. Shift-II]

Answer: 45

Solution:

We may write, $7^n = (10 - 3)^n$ or $7^n = 10K + (-3)^n$ (using expansion)

$$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$$

$$= \begin{cases} 10k & n = \text{odd} \\ 10k + 2 \cdot 3^n & n = \text{even} \end{cases}$$

Let $n = \text{even} = 2t, t \in \mathbb{N}$

$$\text{Then, } 3^n = 3^{2t} = 9^t = (10 - 1)^t$$

$$= 10p + (-1)^t$$

$$= 10p \pm 1$$

If $n = \text{even}$, then $7^n + 3^n$ will never be multiple of 10.

This implies $n = 0$ odd

$$n = 11, 13, 15, \dots, 99 \quad (\text{since, } n \text{ is two digit})$$

$$\Rightarrow 10 < n < 100$$

Total possible 'n' are 45.

Question115

The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is
[2021, 25 Feb. Shift-I]

Answer: 32

Solution:

Given, digits = {1, 2, 3, 4, 5}

Numbers divisible by 3 (sum of digits divisible by 3).

--	--	--

Case I When sum is 12 $\rightarrow$ 3, 4, 5 $\rightarrow 3! = 6$

Case II When sum is 9 $\rightarrow$ 2, 3, 4 $\rightarrow 3! = 6$

Case III When sum is 9 $\rightarrow$ 1, 3, 5 $\rightarrow 3! = 6$

Case IV When sum is 6 $\rightarrow$ 1, 2, 3 $\rightarrow 3! = 6$

So, total numbers divisible by 3 = $6 \times 4 = 24$

Numbers divisible by 5 (ending with 5)

So, total numbers divisible by 5 = 12

Numbers divisible by 15, are 145, 415, 345, 435

i.e. total 4 numbers are divisible by both 3 and 5.

		5
--	--	---

$$\begin{array}{c} 4 \times 3 \\ \uparrow \\ 1 \end{array} = 4 \times 3 = 12$$

i.e. divisible by 15.

Hence, the required numbers which are divisible by 3 or 5 = $24 + 12 - 4 = 32$

Question116

A natural number has prime factorisation given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$.

Then, the number of odd divisors of n , including 1, is
[2021, 26 Feb. Shift-II]

Options:

A. 11

B. 6

C. 6x

D. 12

Answer: D

Solution:

Given, $n = 2^x 3^y 5^z$

and $y + z = 5$

$$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \text{ or } \frac{y+z}{yz} = \frac{5}{6}$$

This implies,

$$y + z = 5 \text{ and } yz = 6$$

$$\text{Put } y = \frac{6}{z} \text{ in } y + z = 5$$

$$\Rightarrow \frac{6}{z} + z = 5 \text{ or } z^2 - 5z + 6 = 0$$

$$\Rightarrow z^2 - 3z - 2z + 6 = 0$$

$$(z - 3)(z - 2) = 0$$

$$\Rightarrow z = 3 \text{ or } 2$$

Using $y = \frac{6}{z}$, we get $y = 2$ or 3

For calculating the odd divisor x must be 0 i.e. $x = 10$

$$\therefore n = 2^0 3^3 5^2 \text{ or } n = 2^0 3^2 5^3$$

Total number of odd divisor of n is equal to

$$= (3 + 1)(2 + 1)$$

$$= (4)(3) = 12$$

Question 117

The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is [2021, 26 Feb. Shift-I]

Options:

A. 42

B. 82

C. 77

D. 35

Answer: C

Solution:

To form a seven digit number with sum of digits 10, all the digits can't be 1, 2 or 3. Hence, seven digit number must have the following cases,

Case 1. Using 1, 1, 1, 1, 1, 2, 3

Possible seven digit numbers will be

$$= \frac{7!}{5!} = 7 \times 6 = 42$$

Case 2. Using 2, 2, 2, 1, 1, 1, 1

Possible numbers will be

$$= \frac{7!}{3!4!} = \frac{7 \times 6 \times 5}{3 \times 2} = 35$$

No more cases will be formed

Hence, total number of seven digit numbers possible

$$= 42 + 35 = 77$$

Question 118

The total number of positive integral solutions (x, y, z), such that $xyz = 24$ is

[2021, 25 Feb. Shift-1]

Options:

A. 36

B. 24

C. 45

D. 30

Answer: D

Solution:

Given, $xyz = 24$

$$\Rightarrow xyz = 2^3 \cdot 3^1$$

$$\text{Let } x = 2^{a_1} \cdot 3^{b_1},$$

$$y = 2^{a_2} \cdot 3^{b_2},$$

$$z = 2^{a_3} \cdot 3^{b_3}$$

where, $a_1, a_2, a_3 \in \{0, 1, 2, 3\}$

$b_1, b_2, b_3 \in \{0, 1\}$

Case I $a_1 + a_2 + a_3 = 3$

$\therefore$ Non-negative solution

$$= {}^{3+3-1}C_{3-1} = {}^5C_2 = 10$$

Case II $b_1 + b_2 + b_3 = 1$

$\therefore$ Non-negative solution

$$= {}^{1+3-1}C_{3-1} = {}^3C_2 = 3$$

$\therefore$ Total solutions $= 10 \times 3 = 30$

Question 119

The students $S_1, S_2, \dots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then, the total number of possibilities of forming such groups is

[2021, 24 Feb. Shift-II]

Answer: 31650

Solution:

Given, total students $= 10$

number of groups $= 3$ (i.e. A, B and C)

Each group has atleast one student but group C has atmost 3 students.

$\therefore$ There are 3 cases depending on number of students in group C.

Case I C has 1 student, then $\left. \begin{matrix} A \\ B \end{matrix} \right\} \leftarrow 9 \text{ students}$

$$\therefore \text{Number of ways} = {}^{10}C_1 \times [2^9 - 2]$$

Case II C has 2 students, then $\left. \begin{matrix} A \\ B \end{matrix} \right\} \leftarrow 8 \text{ Students.}$

$$\therefore \text{Number of ways} = {}^{10}C_2 \times [2^8 - 2]$$

Case III C has 3 students, then $\left. \begin{matrix} A \\ B \end{matrix} \right\} \leftarrow 7 \text{ Students.}$

$$\therefore \text{Number of ways} = {}^{10}C_3 \times [2^7 - 2]$$

$\therefore$ Required number of possibilities

$$= {}^{10}C_1(2^9 - 2) + {}^{10}C_2(2^8 - 2) + {}^{10}C_3(2^7 - 2)$$

$$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3]$$

$$= 20 - 90 - 240$$

$$= 128[40 + 90 + 120] - 350$$

$$= (128 \times 250) - 350 = 31650$$

Question 120

A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :
[24-Feb-2021 Shift 1]

Options:

A. 1625

B. 575

C. 560

D. 1050

Answer: A

Solution:

Indians	Foreigners	Number of Ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

Question 121

The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is
[2021, 18 March Shift-I]

Options:

- A. 26664
- B. 122664
- C. 122234
- D. 22264

Answer: A

Solution:

Given, digits are = 1, 2, 2, 3

$\therefore$ Total distinct numbers = $4! / 2! = 12$

1 at unit place $\Rightarrow$ Number of such

numbers = $\frac{3!}{2!} = 3$

2 at unit place $\Rightarrow$ Number of such numbers = $3! = 6$

3 at unit place $\Rightarrow$ Number of such numbers = $\frac{3!}{2!} = 3$

$\therefore$ Sum of digits at unit place is

$3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

Hence, sum of all 4 digit such numbers

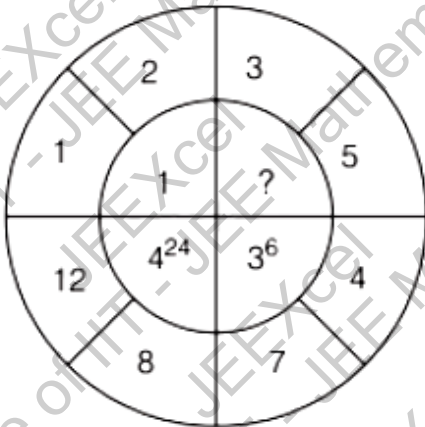
$= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= 1111 \times 24$

$= 26664$

Question 122

The missing value in the following figure is



[2021, 18 March Shift-I]

Answer: 4

Solution:

As, we observe the pattern Inside number

Inside number = (difference)^{(difference)!}

= (Greater number - Smaller number)^{(Greater number - Smaller number)!}

i.e. $1 = (2 - 1)^{(2-1)!}$, $4^{24} = (12 - 8)^{(12-8)!}$,

$3^6 = (7 - 4)^{(7-4)!}$

$\therefore ? = (5 - 3)^{(5-3)!}$

$\therefore$ Required number $= 2^{2!} = 2^{2 \times 1} = 4$

Question 123

If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to

[2021, 18 March Shift-II]

Answer: 160

Solution:

$$\begin{aligned}
 & \sum_{r=1}^{10} r![(r+1)(r+2)(r+3) - 9(r+1) + 8] \\
 &= \sum_{r=1}^{10} \{ (r+3)! - (r+1)! \} - 8 \{ (r+1)! - r! \} \\
 &= (13! + 12! - 2! - 3!) - 8(11! - 1) \\
 &= (12 \cdot 13 + 12 - 8) \cdot 11! - 8 + 8 \\
 &= (160) (11!) \\
 \therefore \alpha &= 160
 \end{aligned}$$

Question 124

The number of times the digit 3 will be written when listing the integers from 1 to 1000 is
[2021, 18 March Shift-1]

Answer: 300

Solution:

Let the number be xyz, $0 \leq x, y, z \leq 9$

Case I '3' appears only one time $\Rightarrow {}^3C_1 \times 9 \times 9 = 243$

Case II '3' appears two times $\Rightarrow {}^3C_2 \times 2 \times 9 = 54$

Case III '3' appears three times

$\Rightarrow {}^3C_3 \times 3 = 3$

$\therefore \text{Total} = 243 + 54 + 3 = 300$

Question 125

Team A consists of 7 boys and n girls and Team B has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams, when a boy plays against a boy and a girl plays against a girl, then n is equal to
[2021, 17 March Shift-I]

Options:

- A. 5
- B. 2
- C. 4
- D. 6

Answer: C

Solution:

	Boys	Girls
Team A	7	n
Team B	4	6

Number of matches between Team A and Team B when a boy play against a boy $({}^7C_1 \times {}^4C_1) = 28$

Similarly, number of matches between Team A and Team B when a girl play against a girl

$$({}^nC_1 \times {}^6C_1) = 6n$$

According to question,

$$28 + 6n = 52$$

$$6n = 24$$

$$n = 4$$

Question126

If the sides AB, BC and CA of a triangle $\triangle ABC$ have 3,5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to
[2021, 17 March Shift-II]

Options:

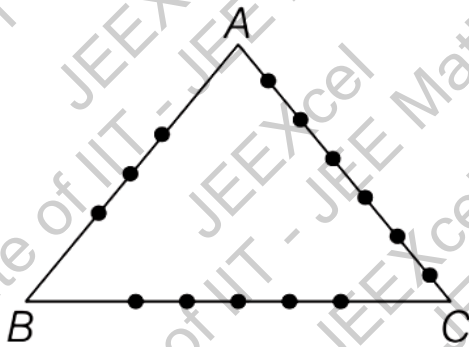
- A. 364
- B. 240
- C. 333
- D. 360

Answer: C

Solution:

Method (I) (Proper Method)

Whenever we construct a triangle, we must require three non-collinear points.



$\therefore$ Total number of triangles using the points 3, 5 and 6 which are on the sides AB, BC and CA

= Either taking (one point from AB, BC and CA)

or (one point from AB and two points from BC)

or (one point from BC and two points from AB)

or (one point from AB and two points from AC)

or (one point from AC and two points from AB)

or (one point from BC and two points from AC)

or (one point from AC and two points from BC)

$\Rightarrow$ Total number of triangles

$$r = ({}^3C_1 \times {}^5C_1 \times {}^6C_1) + ({}^3C_1 \times {}^5C_2)$$

$$+ ({}^3C_1 \times {}^6C_2) + ({}^6C_1 \times {}^3C_2) + ({}^5C_2)$$

$$+ ({}^6C_2)$$

$$= 90 + 30 + 15 + 45 + 18 + 75 + 60$$

$$= 333 \left[\text{using } {}^nC_r = \frac{n!}{r!(n-r)!} \right]$$

$$\text{and } n! = 1 \times 2 \times 3 \times \dots \times n$$

Method (II) (Direct Method)

Total number of points = $3 + 5 + 6 = 14$

Then, when we construct a triangle, we must select 3 points out of 14 but these points never be collinear.

$$\therefore \text{Total number of triangles formed} = {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3 = 333$$

Question 127

Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA, respectively. Let α be the number

of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then, $(\beta - \alpha)$ is equal to
[2021, 16 March Shift-II]

Options:

A. 795

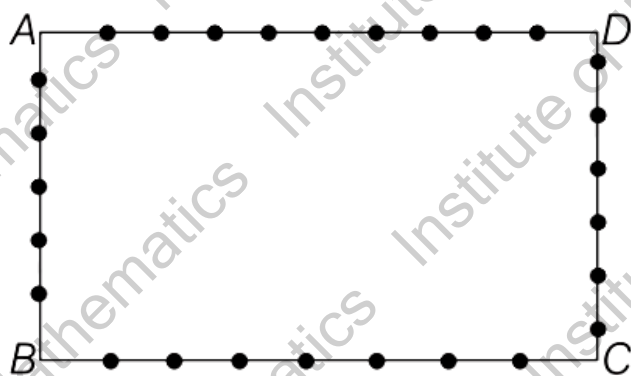
B. 1173

C. 1890

D. 717

Answer: D

Solution:



Number of triangles that can be formed from the points on 3 of the sides.

$${}^5C_1 {}^7C_1 {}^6C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^6C_1 {}^7C_1 {}^9C_1$$

$$= 210 + 315 + 270 + 378$$

$$\Rightarrow \alpha = 1173$$

Number of quadrilaterals that can be formed by taking one point from each of the four vertex

$${}^5C_1 {}^7C_1 {}^6C_1 {}^9C_1 = 5 \times 6 \times 7 \times 9 = 1890$$

$$\Rightarrow \beta = 1890$$

$$\therefore \beta - \alpha = 1890 - 1173$$

$$= 717$$

Question 128

Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is [2021, 20 July Shift-1]

Options:

A. $1/66$

B. $1/11$

C. $1/9$

D. $2/11$

Answer: B

Solution:

$$\begin{array}{ccccccc} \text{E} & \text{X} & \text{A} & \text{M} & \text{I} & \text{N} & \text{A} & \text{T} & \text{I} & \text{O} & \text{N} \\ \left(\begin{array}{c} \downarrow \\ 3 \end{array} \right) & & & & \left(\begin{array}{c} \downarrow \\ 7 \end{array} \right) & & & & & & \end{array}$$

$$\text{Let } x = \text{When M is at fourth place} = \frac{10!}{2!2!2!}$$

$$\text{Let } y = \text{Total number of words} = \frac{11!}{2!2!2!}$$

$$\text{Probability} = \frac{x}{y} = \frac{1}{11}$$

Question 129

There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include atleast 4 bowlers, 5 batsman and 1 wicketkeeper, is [2021, 20 July Shift-I]

Answer: 777

Solution:

Total number of players = 15

Bowlers = 6, Batsman = 7, Wicket keepers = 2

Bowlers	Batsman	Wicket Keepers	Total
4 + 1	5	1	${}^6C_5 \times {}^7C_5 \times {}^2C_1 = 252$
4	5 + 1	1	${}^6C_4 \times {}^7C_6 \times {}^2C_1 = 210$
4	5	1 + 1	${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 315$

Total = 252 + 210 + 315 = 777

Question130

If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10000 is equal to
[2021, 22 July Shift-II]

Answer: 96

Solution:

0, 2, 4, 6, 8



4 options $\times$ 4 options $\times$ 3 $\times$ 2 $\times$ 1

$\therefore$ Total = $4 \times 4 \times 3 \times 2 \times 1 = 96$

Question131

If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to

[2021, 25 July Shift-II]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Given, ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow \frac{n!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow n-r = 1 \dots (i)$$

and

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{{}^nC_r = {}^nC_{r-1}}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$$

$$\Rightarrow n-r+1 = r$$

From Eq. (i),

$$1+1 = r \Rightarrow r = 2$$

Question 132

There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is $100k$, then k is equal to

[2021, 25 July Shift-I]

Answer: 238

Solution:

10(5)	11(6)	12(8)
2+1	2	2+3
2	2+1	2+3
2	2	2+4
	5	5

$$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 8400$$

$$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5 = 11200$$

$$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6 = 4200$$

$$\text{Total} = 8400 + 11200 + 4200 = 23800$$

According to the question, $100K = 23800$

$$K = 238$$

Question133

Let $n \in \mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to x .

If the sum of $(n+1)$ terms ${}^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3, \dots$ is equal to

$2^{100} \cdot 101$, then $2 \left\lfloor \frac{n-1}{2} \right\rfloor$ is equal to

[2021, 25 July Shift-II]

Answer: 98

Solution:

We have,

$$1^n C_0 + 3^n C_1 + 5^n C_2 + \dots + (2n+1)^n C_n$$

$$T_r = (2r+1)^n C_r$$

$$\text{Now, sum}(S) = \sum T_r$$

$$S = \sum (2r+1)^n C_r$$

$$= 2 \sum r^n C_r + \sum^n C_r$$

$$= 2(n2^{n-1}) + 2^n = n \cdot 2^n + 2^n$$

$$\therefore S = 2^n(n+1)$$

$$\text{Given that, } S = 2^{100} \cdot 101$$

$$\Rightarrow 2^n(n+1) = 2^{100} \cdot 101$$

$$\Rightarrow n = 100$$

$$\text{Now, } 2 \left[\frac{n-1}{2} \right] = 2 \left[\frac{100-1}{2} \right] = 2 \left[\frac{99}{2} \right]$$

$$= 2[49.5] = 2 \times 49 = 98$$

$$(\because [x] \text{ is greatest integer function})$$

Question 134

Let n be a non-negative integer. Then the number of divisors of the form " $4n+1$ " of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to
[2021, 27 July shift-II]

Answer: 924

Solution:

$$\text{Let } N = (10)^{10} \cdot (11)^{11} \cdot (13)^{13}$$

$$N = 2^{10} \cdot 5^{10} \cdot 11^{11} \cdot 13^{13}$$

Now, power of 2 must be zero. Power of 5 can be anything. Power of 13 can be anything. But power of 11 should be even. So, required number of divisor is $= 1 \times 11 \times 14 \times 6 = 924$

Question 135

The sum of all three-digit numbers less than or equal to 500, that are formed without using the digit 1 and they all are multiple of 11, is
[2021, 26 Aug. Shift-II]

Answer: 7744

Solution:

Multiples of 11 such that they are of 3 -digit and less than 500.

121, 132, ..., 495

$$n = \frac{495 - 121}{11} + 1 = 35$$

$$S = \frac{35}{2}(121 + 495) = 10780$$

Again, multiples of 11 which are 3-digits, less than 500 and having 1 at hundred's place are

121, 132, ..., 198

$$n_1 = \left(\frac{198 - 121}{11} \right) + 1 = 8$$

$$S_1 = \frac{8}{2}(121 + 198) = 1276$$

The multiply of 11 which are of 3 -digits, less than 500 and having 1 at ten's place are 319, 418

$$\therefore S_2 = 319 + 418 = 737$$

The multiple of 11 which are 3-digits, less than 500 and having 1 at unit place are 231, 341, 451

$$\therefore S_3 = 231 + 341 + 451 = 1023$$

$$\begin{aligned} \therefore \text{Required sum} &= S - S_1 - S_2 - S_3 \\ &= 7744 \end{aligned}$$

Question 136

The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7, if the repetition of digits is not allowed, is [2021, 26 Aug. Shift-I]

Answer: 52

Solution:

Case I When 0 is at unit place

$$\frac{4}{5} \times \frac{4}{4} \times \frac{1}{1} = 20$$

Case II When 4 or 6 are at unit place

$$\frac{1}{(4)} \times \frac{6}{(4)} \times \frac{6}{(2)} = 32$$

[0 cannot be come at hundredth place]

∴ Total number of required

$$= 20 + 32 = 52$$

Question137

A number is called a palindrome if it reads the same backward as well as forward For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is
[2021, 27 Aug. Shift-I]

Answer: 100

Solution:

Form of six digit palindrome number $xyzyx$

This will be divisible by 55 Hence, $x = 5$ and $5yzy5$ will be divisible by 11.

$\Rightarrow (5 + z + y) - (y + z + 5)$ is divisible by 11 which is true for all values of y and $z \Rightarrow y$ and z can be chosen in 10×10 ways Number of such number = 100

Question138

Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then, the number of elements in the set $T = \{A \subset \text{eq}S : A \neq \emptyset \text{ and the sum of all the elements of } A \text{ is not a multiple of } 3 \}$ is
[2021, 27 Aug. Shift-II]

Answer: 80

Solution:

$S = \{1, 2, 3, 4, 5, 6, 9\}$

3 n Type numbers 3, 6, 9

$3n - 1$ Type numbers 2, 5

$3n - 2$ Type numbers 1, 4

Let N_p = Number of Subset of S

containing p element which are not divisible by 3 .

For $P = 1$

$${}^2C_1 + {}^2C_1 = 4$$

For $P = 2$

$${}^3C_1 {}^2C_1 + {}^3C_1 {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$$

For $P = 3$

$${}^3C_1 ({}^2C_2 + {}^2C_2) + {}^3C_2 ({}^2C_1 + {}^2C_1) + {}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2 = 22$$

For $P = 4$

$${}^3C_1 [{}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2] + {}^3C_2 ({}^2C_2 + {}^2C_2) + {}^3C_3 ({}^2C_1 + {}^2C_1) = 22$$

For $P = 5$

$${}^3C_2 ({}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2) + {}^3C_3 ({}^2C_2 + {}^2C_2) = 14$$

For $P = 6$

$${}^3C_3 ({}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2) = 4$$

Total Subsets

$$= 4 + 14 + 22 + 22 + 14 + 4 = 80$$

Question139

The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is
[2021, 31 Aug. Shift-1]

Answer: 576

Solution:

VOWELS (2 Vowel +4 consonant)

All consonants must not be together

Total possibility of formation of 6 letter word = 6 !

The number of arrangement when all the consonant comes together = 3! × 4 !

Number of arrangement when all the consonants never come together
= Total - All consonant together = $6! - 3!4! = 576$

Question140

The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r$
= $(k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is:

[Jan. 7, 2020 (II)]

Options:

- A. 3
- B. 2
- C. 6
- D. 4

Answer: D

Solution:

$$\frac{36}{r+1} \times {}^{35}C_r (k^2 - 3) = {}^{35}C_r \cdot 6$$

$$\Rightarrow k^2 - 3 = \frac{r+1}{6}$$

$$\Rightarrow k^2 = 3 + \frac{r+1}{6}$$

r can be 5, 35 for $k \in$

$$r = 5, k = \pm 2$$

$$r = 35, k = \pm 3$$

Hence, number of ordered pairs = 4

Question141

An urn contains 5 red marbles, 4 black marbles and 3 white marbles.
Then the number of ways in which 4 marbles can be drawn so that at
the most three of them are red is _____.

[NA Jan. 8, 2020 (I)]

Answer: 490

Solution:

0 Red, 1 Red, 2 Red, 3 Red

Number of ways of selecting atmost three red balls

$$= {}^7C_4 + {}^5C_1 \cdot {}^7C_3 + {}^5C_2 \cdot {}^7C_2 + {}^5C_3 \cdot {}^7C_1$$

$$= 35 + 175 + 210 + 70 = 490$$

Question 142

If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively,

then:

[Jan. 8, 2020 (I)]

Options:

A. $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$

B. $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

C. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

D. $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$

Answer: C

Solution:

We know ${}^nC_r$ is greatest at middle term.

$$\text{So, } a = ({}^{19}C_p)_{\max} = {}^{19}C_{10} = {}^{19}C_9$$

$$b = ({}^{20}C_q)_{\max} = {}^{20}C_{10}$$

$$c = ({}^{21}C_r)_{\max} = {}^{21}C_{10} = {}^{21}C_{11}$$

$$\text{Now, } \frac{a}{19}C_9 = \frac{b}{20}C_{10} = \frac{c}{21}C_{11}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{42/11} \therefore \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

Question143

The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is

[NA Jan. 8, 2020 (II)]

Answer: 2454

Solution:

EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I : If all are different, then

$${}^8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

Case II : If two are same and two are different, then ${}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!} = 3 \cdot 21 \cdot 12 = 756$

Case III : If two are same and other two are same, then

$${}^3C_2 \cdot \frac{4!}{2!2!} = 3 \cdot 6 = 18$$

$$\therefore \text{Total cases} = 1680 + 756 + 18 = 2454$$

Question144

${}_rL_t \equiv {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \dots + (101) \cdot C_{25} = 2^{25} \cdot k$, then k is equal to

[NA Jan. 9, 2020 (II)]

Answer: 51

Solution:

$$\begin{aligned}\sum_{r=0}^{25} (4r+1) {}^{25}C_r &= 4 \sum_{r=0}^{25} r \cdot {}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r \\ &= 4 \sum_{r=1}^{25} r \times \frac{25}{r} {}^{24}C_{r-1} + 2^{25} = 100 \sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25} \\ &= 100 \cdot 2^{24} + 2^{25} = 2^{25}(50+1) = 51 \cdot 2^{25}\end{aligned}$$

Hence, by comparison $k = 51$

Question 145

If the number of five digit numbers with distinct digits and 2 at the 10th place is 336k, then k is equal to:

[Jan. 9, 2020 (I)]

Options:

- A. 4
- B. 6
- C. 7
- D. 8

Answer: D

Solution:

Number of five digit numbers with 2 at 10th place

$$= 8 \times 8 \times 7 \times 6 = 2688$$

∵ It is given that, number of five digit number with 2 at 10th place = 336k

$$\therefore 336k = 2688 \Rightarrow k = 8$$

Question 146

Total number of 6 -digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is:

[Jan. 7, 2020 (I)]

Options:

- A. $\frac{1}{2}(6!)$

B. 6!

C. 5^6

D. $\frac{5}{2}(6!)$

Answer: D

Solution:

Five digits numbers be 1,3,5,7,9

For selection of one digit, we have ${}^5C_1$

choice. And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence, total such numbers = $\frac{5 \cdot 6!}{2!} = \frac{5}{2} \cdot 6!$

Question 147

Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

[Sep. 06, 2020 (I)]

Options:

A. $2!3!4!$

B. $(3!)^3 \cdot (4!)$

C. $(3!)^2 \cdot (4!)$

D. $3!(4!)^3$

Answer: B

Solution:

Number of arrangement

$= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

Question 148

The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51th term)
 $+(1! - 2! + 3! - \dots$ up to 51th term) is equal to :
[Sep. 03, 2020 (I)]

Options:

A. $1 - 51(51)!$

B. $1 + (51)!$

C. $1 + (52)!$

D. 1

Answer: C

Solution:

We know, $(r+1) \cdot {}^rP_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + \dots$ 51 terms) +

$(1! - 2! + 3! - \dots$ upto 51 terms)

$= [2! - 3! + 4! - \dots + 52!] + [1! - 2! + 3! - \dots + 51!]$

$= 52! + 1! = 52! + 1$

Question 149

If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _____.
[NA Sep. 02, 2020 (I)]

Answer: 309

Solution:

M - 3

O - 4

T - 6

H - 2

E - 1

R - 5

$$\Rightarrow 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$$

$$= 240 + 48 + 18 + 2 + 1 = 309$$

Question 150

The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

[NA Sep. 06, 2020 (II)]

Answer: 120

Solution:

For vowels not together

Number of ways to arrange L, T, T, R = $\frac{4!}{2!}$

Then put both E in 5 gaps formed in ${}^5C_2$ ways.

$$\therefore \text{No. of ways} = \frac{4!}{2!} \cdot {}^5C_2 = 120$$

Question 151

The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _____.

[NA Sep. 05, 2020 (I)]

Answer: 240

Solution:

$S \rightarrow 2, L \rightarrow 2, A, B, Y, U$

$\therefore$ Required number of ways $= {}^2C_1 \times {}^5C_2 \times \frac{4!}{2!} = 240$

Question 152

There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

[Sep. 05, 2020 (II)]

Options:

A. 3000

B. 1500

C. 2255

D. 2250

Answer: D

Solution:

Since, each section has 5 questions.

$\therefore$ Total number of selection of 5 questions

$$= 3 \times {}^5C_1 \times {}^5C_1 \times {}^5C_3 + 3 \times {}^5C_1 \times {}^5C_2 \times {}^5C_2$$

$$= 3 \times 5 \times 5 \times 10 + 3 \times 5 \times 10 \times 10$$

$$= 750 + 1500 = 2250$$

Question 153

A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four

of the answers are correct, is
[NA Sep. 04, 2020 (II)]

Answer: 135

Solution:

Select any 4 correct questions in ${}^6C_4$ ways.

Number of ways of answering wrong question = 3

$\therefore$ Required number of ways = ${}^6C_4(1)^4 \times 3^2 = 135$

Question154

The total number of 3 -digit numbers, whose sum of digits is 10, is
[NA Sep. 03, 2020 (II)]

Answer: 54

Solution:

Let xyz be the three digit number

$x + y + z = 10, x \leq 1, y \geq 0, z \geq 0$

$x - 1 = t \Rightarrow x = 1 + t \quad x - 1 \geq 0, t \geq 0$

$t + y + z = 10 - 1 = 9 \quad 0 \leq t, z, z \leq 9$

$\therefore$ Total number of non-negative integral solution = ${}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11 \cdot 10}{2} = 55$

But for $t = 9, x = 10$, so required number of integers
= $55 - 1 = 54$

Question155

Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected

by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is:

[Sep. 02, 2020 (II)]

Options:

A. 201

B. 200

C. 101

D. 199

Answer: A

Solution:

Number of two consecutive stations (Blue lines) = n

Number of two non-consecutive stations (Red lines) = ${}^nC_2 - n$

Now, according to the question, ${}^nC_2 - n = 99n$

$$\Rightarrow \frac{n(n-1)}{2} - 100n = 0 \Rightarrow n(n-1-200) = 0$$

$$\Rightarrow n-1-200 = 0 \Rightarrow n = 201$$