



## Matrices

### Question 1

For some  $\alpha, \beta \in \mathbf{R}$ , let  $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$  be such that  $A^2 - 4A + 2I = B^2 - 3B + I = O$ . Then  $(\det(\operatorname{adj}(A^3 - B^3)))^2$  is equal to \_\_\_\_\_.

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**Answer: 225**

### Solution:

Using characteristic equation,

$$\begin{aligned} \alpha - \lambda & \quad 2 \\ 1 & \quad 2 - \lambda \\ (\lambda - 2)(\lambda - \alpha) - 2 &= 0 \\ \lambda^2 + \lambda(-2 - \alpha) + 2\alpha - 2 &= 0 \\ \Rightarrow A^2 + A(-2 - \alpha) + (2\alpha - 2)I &= 0 \\ \Rightarrow -2 - \alpha = -4 \Rightarrow \alpha &= 2 \end{aligned}$$

Similarly,

$$\begin{aligned} 1 - K & \quad 2 \\ 1 & \quad \beta - K \\ \Rightarrow (K - \beta)(K - 1) - 1 &= 0 \\ K^2 + K(-1 - \beta) + \beta - 1 &= 0 \\ \Rightarrow -1 - \beta = -3 \Rightarrow \beta &= 2 \end{aligned}$$

$$\Rightarrow A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\text{Let } A^3 - B^3 = \begin{bmatrix} 15 & 20 \\ 6 & 7 \end{bmatrix} = C, \det(C) = -15$$

$$|\operatorname{adj} C|^2 = |C|^2 = 225$$

## Question2

Let  $A$  be a  $3 \times 3$  matrix such that  $A + A^T = O$ . If

$$A \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}, A^2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 19 \\ -24 \end{bmatrix} \text{ and}$$

$\det(\text{adj}(2 \text{adj}(A + I))) = (2)^\alpha \cdot (3)^\beta \cdot (11)^\gamma$ ,  $\alpha, \beta, \gamma$  are non-negative integers, then  $\alpha + \beta + \gamma$  is equal to \_\_\_\_\_

**JEE Main 2026 (Online) 22nd January Morning Shift**

**Answer: 18**

**Solution:**

$$A = -A^T$$

$$A = \begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$\alpha = -3, -\beta + \gamma = 2$$

$$\begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 19 \\ \beta = 3 \end{bmatrix}$$

$$|A + I| = \begin{vmatrix} 1 & -3 & 3 \\ 3 & 1 & 5 \\ -3 & -5 & 1 \end{vmatrix} = 44$$

$$\det(\text{adj}(2 \text{adj}(A + I))) = 2^6 \det(\text{adj}(\text{adj}(A + I))) = 2^6 (44)^4 = 2^{14} \times 11^4$$

## Question3

Let  $|A| = 6$ , where  $A$  is a  $3 \times 3$  matrix. If

$|\text{adj}(3 \text{adj}(A^2 \cdot \text{adj}(2A)))| = 2^m \cdot 3^n$ ,  $m, n \in \mathbb{N}$ , then  $m + n$  is equal to

\_\_\_\_\_.

## JEE Main 2026 (Online) 23rd January Morning Shift

**Answer: 62**

**Solution:**

Let

$$X = A^2 \cdot \text{adj}(2A)$$

so the given expression is

$$|\text{adj}(3 \text{adj}(X))|.$$

Since  $A$  is a  $3 \times 3$  matrix and  $|A| = 6$ :

**Step 1: Find  $|A^2|$**

$$|A^2| = |A|^2 = 6^2 = 36$$

**Step 2: Find  $|\text{adj}(2A)|$**

First,

$$|2A| = 2^3 |A| = 8 \cdot 6 = 48$$

For a  $3 \times 3$  matrix  $M$ ,

$$|\text{adj}(M)| = |M|^{3-1} = |M|^2$$

So,

$$|\text{adj}(2A)| = |2A|^2 = 48^2$$

**Step 3: Find  $|X|$**

$$|X| = |A^2| \cdot |\text{adj}(2A)| = 36 \cdot 48^2$$

Now,

$$36 = 2^2 \cdot 3^2, \quad 48 = 2^4 \cdot 3$$

So,

$$48^2 = 2^8 \cdot 3^2$$

Hence,

$$|X| = (2^2 \cdot 3^2)(2^8 \cdot 3^2) = 2^{10} \cdot 3^4$$

**Step 4: Find  $|3 \text{adj}(X)|$**

First,

$$|\text{adj}(X)| = |X|^2 = (2^{10} \cdot 3^4)^2 = 2^{20} \cdot 3^8$$

Since  $3 \text{ adj}(X)$  is a  $3 \times 3$  matrix,

$$|3 \text{ adj}(X)| = 3^3 \cdot |\text{adj}(X)|$$

Therefore,

$$|3 \text{ adj}(X)| = 27 \cdot 2^{20} \cdot 3^8 = 2^{20} \cdot 3^{11}$$

**Step 5: Find the required determinant**

Again using

$$|\text{adj}(M)| = |M|^2$$

we get

$$|\text{adj}(3 \text{ adj}(X))| = (2^{20} \cdot 3^{11})^2 = 2^{40} \cdot 3^{22}$$

So,

$$m = 40, \quad n = 22$$

and

$$m + n = 40 + 22 = 62$$

$$\boxed{62}$$

## Question4

Let  $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$  and  $B$  be a matrix such that

$B(I - A) = I + A$ . Then the sum of the diagonal elements of  $B^T B$  is equal to         

**JEE Main 2026 (Online) 23rd January Evening Shift**

**Answer: 3**

**Solution:**

Observing properties of Matrix  $A$

$$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix} = -A$$

Rearranging the given equation

$$B(I - A) = I + A$$

$$B = (I + A)(I - A)^{-1}$$

Finding  $B^T$

$$B^T = [(I + A)(I - A)^{-1}]^T$$

$$B^T = ((I - A)^{-1})^T (I + A)^T$$

$$B^T = (I - A^T)^{-1} (I + A^T)$$

Since  $A^T = -A$ :

$$B^T = (I + A)^{-1} (I - A)$$

$$B^T B = [(I + A)^{-1} (I - A)] \cdot [(I + A)(I - A)^{-1}]$$

$$B^T B = (I + A)^{-1} (I + A)(I - A)(I - A)^{-1}$$

$$B^T B = I \cdot I = I$$

$$B^T B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the sum of diagonal elements  $= 1 + 1 + 1 = 3$

The sum of the diagonal elements of  $B^T B$  is 3.

## Question5

**The number of  $3 \times 2$  matrices  $A$ , which can be formed using the elements of the set  $\{-2, -1, 0, 1, 2\}$  such that the sum of all the diagonal elements of  $A^T A$  is 5, is**

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**Answer: 312**

**Solution:**

Let matrix  $A$  of order  $3 \times 2$  is

$$A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}, a, b, c, d, e, f \in \{-2, -1, 0, 1, 2\}$$

$$A^T = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

$$A^T A = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} = \begin{bmatrix} a^2 + c^2 + e^2 & ab + cd + ef \\ ab + cd + ef & b^2 + d^2 + f^2 \end{bmatrix}$$

sum of diagonal elements of  $A^T A = 5$

$$a^2 + c^2 + e^2 + b^2 + d^2 + f^2 = 5$$

possible values of squares is  $\{0, 1, 4\}$

**Case 1 :** one element is 4, 1 element is 1 and four elements are 0 .

one element is 4 = 2 choices  $\{-2, 2\}$

one element is 1 = 2 choices  $\{-1, 1\}$

4 element is zero = 1 choice  $\{0\}$

Selecting one element 4 out of 6 choice  $\{a^2, b^2, c^2, d^2, e^2, f^2\}$ , and have 2 choice  $\{-2, 2\}$ , total  $= {}^6C_1 \times 2 = 12$

Selecting one element which is 1 from remaining 5 element  $= {}^5C_1$ , and have two choice  $\{-1, 1\}$ , total  $= {}^5C_1 \times 2 = 10$

Selecting 4 element which is zero from remaining 4 elements  $= {}^4C_4 = 1$  total matrices for Case 1  $= 12 \times 10 \times 1 = 120$

**Case 2 :** five elements are 1 and one element is 0 .

squares  $a^2, b^2, c^2, d^2, e^2, f^2 \in \{1, 1, 1, 1, 1, 0\}$

selection of 5 one from  $a^2, b^2, c^2, d^2, e^2, f^2 = {}^6C_5$

and for 1 there are two choice for each elements  $a, b, c, d, e, f = \{-1, 1\}$ , total  $= 2^5$

for 0 only 1 choices

total matrices of case 2  $= {}^6C_5 \times 2^5 = 192$

total number of such matrices = Case 1 + Case 2

$$= 120 + 192 = 312$$

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## Question6

Let  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$  and  $B$  be two matrices such that  $A^{100} = 100B + I$ .

Then the sum of all the elements of  $B^{100}$  is \_\_\_\_\_

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**Answer: 0**

### Solution:

Decompose Matrix A We can write matrix  $A$  as the sum of the Identity matrix  $I$  and a residual matrix  $C$  :

$$A = I + C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \quad \text{Let } C = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}.$$
$$C^2 = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4-4 & -8+8 \\ 2-2 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Since  $C^2 = 0$ ,  $C$  is a nilpotent matrix. Consequently,  $C^n = 0$  for all  $n \geq 2$ .

**Find  $A^{100}$  using Binomial Expansion :**

$$A^{100} = (I + C)^{100} = {}^{100}C_0 I^{100} + {}^{100}C_1 I^{99} C + {}^{100}C_2 I^{98} C^2 + \dots + {}^{100}C_{100} C^{100}$$

Since  $C^2 = C^3 = \dots = C^{100} = 0$ , the expansion simplifies to :

$$A^{100} = {}^{100}C_0 I + {}^{100}C_1 C$$

$$A^{100} = 1 \cdot I + 100 \cdot C$$

$$A^{100} = I + 100C$$

**Find Matrix B :**

The problem states that  $A^{100} = 100B + I$ . Comparing this to our result:

$$100B + I = 100C + I$$

$$100B = 100C$$

$$B = C$$

**Calculate the Sum of Elements of  $B^{100}$  :**

Since  $B = C$  and we know  $C^2 = 0$ , then:  $B^2 = 0$

Consequently,  $B^{100} = 0$  (the zero matrix).

$$B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The sum of all the elements of  $B^{100}$  is 0 .

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## Question7

Consider the matrices  $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$ . If matrices  $P$  and  $Q$  are such that  $PA = B$  and  $AQ = B$ , then the absolute value of the sum of the diagonal elements of  $2(P + Q)$  is \_\_\_\_\_.

**JEE Main 2026 (Online) 2nd April Evening Shift**

**Answer: 34**

**Solution:**

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore PA = B$$

$$\Rightarrow P = BA^{-1}$$

$$= \frac{1}{2} \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -21 & 12 \\ -7 & 4 \end{bmatrix}$$

$$\therefore AQ = B$$

$$\Rightarrow Q = A^{-1}B$$

$$= \frac{1}{2} \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -2 & -6 \\ -5 & -15 \end{bmatrix}$$

$$\therefore |\text{tr}(2(P + Q))| = |(-21) + (4) + (-2) + (-15)|$$

$$= |-34| = 34$$

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## Question8

Let  $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  satisfy

$$A^2 + \alpha(\text{adj}(\text{adj}(A))) + \beta(\text{adj}(A)(\text{adj}(\text{adj}(A)))) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \text{ for}$$

some  $\alpha, \beta \in \mathbb{R}$ .

Then  $(\alpha - \beta)^2$  is equal to \_\_\_\_\_

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**Answer: 4**

### Solution:

We need to find  $\alpha$  and  $\beta$  from

$$A^2 + \alpha(\text{adj}(\text{adj}(A))) + \beta(\text{adj}(A)\text{adj}(\text{adj}(A))) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Let us solve step by step.

#### 1. Given matrix

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

#### 2. Find $\det(A)$

Since the third row is  $(0, 0, 1)$ , expand along the third row:

$$\det(A) = 1 \cdot \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = (-1)(0) - (1)(1) = -1.$$

So,

$$\det(A) = -1 \neq 0.$$

Hence  $A$  is invertible.

#### 3. Use NCERT property of adjoint of adjoint

For an  $n \times n$  invertible matrix,

$$\text{adj}(\text{adj}(A)) = (\det A)^{n-2}A.$$

Here  $n = 3$ , so

$$\text{adj}(\text{adj}(A)) = (\det A)^1 A = (-1)A = -A.$$

Thus,

$$\text{adj}(\text{adj}(A)) = -A.$$

Also, for any invertible matrix,

$$A \text{adj}(A) = \det(A)I.$$

So,

$$A \text{adj}(A) = -I.$$

Multiplying both sides by  $A^{-1}$  is not necessary; instead use

$$\text{adj}(A) = \det(A)A^{-1} = -A^{-1} \text{ if needed.}$$

Now,

$$\text{adj}(A) \text{adj}(\text{adj}(A)) = \text{adj}(A)(-A) = -(\text{adj}(A)A).$$

And since

$$\text{adj}(A)A = \det(A)I = -I,$$

we get

$$\text{adj}(A) \text{adj}(\text{adj}(A)) = -(-I) = I.$$

So the given equation becomes

$$A^2 + \alpha(-A) + \beta I = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}.$$

That is,

$$A^2 - \alpha A + \beta I = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}.$$

#### 4. Compute $A^2$

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now multiply  $A$  by itself:

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Entrywise calculation:

- First row:

$$(1, 1) : (-1)(-1) + (1)(1) + (-1)(0) = 2$$

$$(1, 2) : (-1)(1) + (1)(0) + (-1)(0) = -1$$

$$(1, 3) : (-1)(-1) + (1)(1) + (-1)(1) = 1$$

• Second row:

$$(2, 1) : (1)(-1) + (0)(1) + (1)(0) = -1$$

$$(2, 2) : (1)(1) + (0)(0) + (1)(0) = 1$$

$$(2, 3) : (1)(-1) + (0)(1) + (1)(1) = 0$$

• Third row:

$$(3, 1) = 0, \quad (3, 2) = 0, \quad (3, 3) = 1$$

Hence

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

### 5. Form $A^2 - \alpha A + \beta I$

First,

$$-\alpha A = \begin{bmatrix} \alpha & -\alpha & \alpha \\ -\alpha & 0 & -\alpha \\ 0 & 0 & -\alpha \end{bmatrix}$$

and

$$\beta I = \begin{bmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta \end{bmatrix}.$$

So,

$$A^2 - \alpha A + \beta I = \begin{bmatrix} 2 + \alpha + \beta & -1 - \alpha & 1 + \alpha \\ -1 - \alpha & 1 + \beta & -\alpha \\ 0 & 0 & 1 - \alpha + \beta \end{bmatrix}.$$

This must equal

$$\begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}.$$

### 6. Compare corresponding entries

From the (1, 2) entry:

$$-1 - \alpha = -2 \Rightarrow \alpha = 1.$$

From the (2, 2) entry:

$$1 + \beta = 0 \Rightarrow \beta = -1.$$

These also satisfy the other entries:

- $(1, 1)$ :

$$2 + \alpha + \beta = 2 + 1 - 1 = 2$$

- $(1, 3)$ :

$$1 + \alpha = 2$$

- $(2, 1)$ :

$$-1 - \alpha = -2$$

- $(2, 3)$ :

$$-\alpha = -1$$

- $(3, 3)$ :

$$1 - \alpha + \beta = 1 - 1 - 1 = -1$$

All correct.

Thus,

$$\alpha = 1, \quad \beta = -1.$$

**7. Find  $(\alpha - \beta)^2$**

$$\alpha - \beta = 1 - (-1) = 2$$

Therefore,

$$(\alpha - \beta)^2 = 2^2 = 4.$$

So the required answer is

**4.**

## Question9

**For the matrices  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$  and  $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$ , if**

**$(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ , then among the following which one is true?**

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**Options:**

A.

$$x = 16, y = 3$$

B.

$$x = 5, y = 7$$

C.

$$x = 11, y = 2$$

D.

$$x = 18, y = 11$$

**Answer: C**

**Solution:**

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 23 & 49 \\ 45 & 21 \end{bmatrix}$$

Characteristic equation of A,

$$(3 - \lambda)(-1 - \lambda) + 4 = 0$$

$$(\lambda - 3)(\lambda + 1) + 4 = \lambda^2 - 2\lambda + 4 = 0$$

$$\Rightarrow A^2 - 2A + I = 0$$

$$A^2 = 2A - I, A^3 = 2A^2 - A = 2(2A - I) - A = 3A - 2I$$

$$A^4 = 4A^2 + I - 4A = 4(2A - I) - 4A + I$$

$$= 4(2A - I) - 3A = 5A - 4I$$

$$(A^5)^3 = (5A - 4I)^3$$

$$= 125A^3 - 3 \times 25A^2(4) + 3(5A)(4^2) - 64I$$

$$= 125(3A - 2I) - 300(2A - I) + 240A - 64I$$

$$= A(375 - 600 + 240) + I(-250 + 300 - 64)$$

$$= 15A - 14I$$

$$\Rightarrow A^{15} + B = 15A - 14I + B$$

$$= \begin{bmatrix} 2 & -11 \\ 1 & -11 \end{bmatrix} (A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2x = 11y$$

## Question 10

If the system of equations

$$3x + y + 4z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has no solution, then the value of  $\alpha$  is equal to:

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Options:

A.

13

B.

4

C.

19

D.

23

**Answer: C**

**Solution:**

$$\Delta = \begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\det(\Delta) = 0$$

$$\alpha = 19$$

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## Question 11

If  $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ , then the determinant of the matrix

$\left( A^{2025} - 3 A^{2024} + A^{2023} \right)$  is

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Options:

A.

12

B.

24

C.

28

D.

16

**Answer: D****Solution:**

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, |A| = 1$$

$$|A^{2025} - 3A^{2024} + A^{2023}| = |A^{2023}| |A^2 - 3A + I| = 16$$


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## Question 12

If  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  is a solution of the system of equations  $AX = B$ , where

$$\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \text{ then } |x + y + z| \text{ is equal to :}$$

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A.

3

B.

2

C.

$$\frac{3}{2}$$

D.

$$1$$

**Answer: B**

**Solution:**

$$X = A^{-1}B = \left( \frac{\text{adj } A}{|A|} \right) B = \pm \frac{1}{10} \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \pm \frac{1}{10} \begin{pmatrix} 20 \\ -10 \\ 10 \end{pmatrix} = \pm \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\therefore |x + y + z| = 2$$

## Question 13

Let  $n$  be the number obtained on rolling a fair die. If the probability that the system

$$x - ny + z = 6$$

$$x + (n - 2)y + (n + 1)z = 8$$

$$(n - 1)y + z = 1$$

has a unique solution is  $\frac{k}{6}$ , then the sum of  $k$  and all possible values of  $n$  is :

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**Options:**

A.

$$22$$

B.

$$20$$

C.

$$24$$

D.

21

**Answer: A**

**Solution:**

$$x - ny + z = 6$$

$$x - (n - 2)y + (n + 1)z = 8$$

$$(n - 1)y + z = 1$$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & (n - 2) & n + 1 \\ 0 & n - 1 & 1 \end{vmatrix} = 0 \Rightarrow n = 1, 2 \text{ or } n = -1 \text{ (rejected)}$$

For unique solution  $n = 3, 4, 5, 6$

Now P (Probability when system of equations has unique solution) =  $\frac{4}{6}$

So  $k = 4$

Now required sum =  $4 + (3 + 4 + 5 + 6) = 22$

## Question 14

**Among the statements :**

**I: If**  $\begin{vmatrix} 1 & \cos \alpha & \cos \beta \\ \cos \alpha & 1 & \cos \gamma \\ \cos \beta & \cos \gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos \alpha & \cos \beta \\ \cos \alpha & 0 & \cos \gamma \\ \cos \beta & \cos \gamma & 0 \end{vmatrix}$ , **then**  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2}$ , **and**

**II: If**  $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = px + q$ , **then**  $p^2 = 196q^2$ ,

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**Options:**

A.

both are true

B.

both are false

C.

only I is true

D.

only II is true

**Answer: B**

**Solution:**

$$\begin{vmatrix} 1 & \cos \alpha & \cos \beta \\ \cos \alpha & 1 & \cos \gamma \\ \cos \beta & \cos \gamma & 1 \end{vmatrix} = 1(1 - \cos^2 \gamma) - \cos \alpha(\cos \alpha - \cos \beta \cos \gamma) + \cos \beta(\cos \alpha \cos \gamma - \cos \beta)$$

$$= 1 - \cos^2 \gamma - \cos^2 \alpha + \cos^2 \beta + 2 \cos \beta + 2 + \cos \alpha \cos \beta \cos \gamma$$

$$\begin{vmatrix} 0 & \cos \alpha & \cos \beta \\ \cos \alpha & 0 & \cos \gamma \\ \cos \beta & \cos \gamma & 0 \end{vmatrix} = -\cos \alpha(-\cos \beta \cos \gamma) + \cos \beta(\cos \alpha \cos \gamma) = 2 \cos \alpha \cos \beta - \cos \gamma$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\therefore q = \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix} = -10 - 2 = -12$$

$$\text{And } -p + q = \begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & -6 \\ 2 & -3 & -3 \end{vmatrix} = -36$$

$$\Rightarrow p = q + 36 = 24$$

$$\Rightarrow p^2 \neq 196q^2$$

Option (4) is correct.

## Question 15

**The system of linear equations**

$$x + y + z = 6$$

$$2x + 5y + az = 36$$

$$x + 2y + 3z = b$$

**has :**

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### Options:

A.

unique solution for  $a = 8$  and  $b = 16$

B.

infinitely many solutions for  $a = 8$  and  $b = 14$

C.

infinitely many solutions for  $a = 8$  and  $b = 16$

D.

unique solution for  $a = 8$  and  $b = 14$

**Answer: B**

### Solution:

For the system to have infinitely many solutions, the determinant of the coefficient matrix ( $\Delta$ ) must be zero.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$1(15 - 2a) - 1(6 - a) + 1(4 - 5) = 0$$

$$15 - 2a - 6 + a - 1 = 0$$

$$8 - a = 0 \Rightarrow a = 8$$

Now, for infinitely many solutions,  $\Delta_x = \Delta_y = \Delta_z = 0$ .

Check  $\Delta_x$  :

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 36 & 5 & 8 \\ b & 2 & 3 \end{vmatrix} = 0$$

$$6(15 - 16) - 1(108 - 8b) + 1(72 - 5b) = 0$$

$$6(-1) - 108 + 8b + 72 - 5b = 0$$

$$-6 - 108 + 72 + 3b = 0$$

$$-42 + 3b = 0 \Rightarrow 3b = 42 \Rightarrow b = 14$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 36 & 8 \\ 1 & 14 & 3 \end{vmatrix} = 1(108 - 112) - 6(6 - 8) + 1(28 - 36) = -4 + 12 - 8 = 0$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & 36 \\ 1 & 2 & 14 \end{vmatrix} = 1(70 - 72) - 1(28 - 36) + 6(4 - 5) = -2 + 8 - 6 = 0$$

If  $a = 8$  and  $b \neq 14$ , the system has no solution.

If  $a = 8$  and  $b = 14$ , the system has infinitely many solutions.

If  $a \neq 8$ , the system has a unique solution regardless of  $b$ .

---

## Question 16

Let  $f(x) = \int \frac{7x^{10} + 9x^8}{(1+x^2+2x^9)^2} dx$ ,  $x > 0$ ,  $\lim_{x \rightarrow 0} f(x) = 0$  and  $f(1) = \frac{1}{4}$ .

If  $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$  and  $B = \text{adj}(\text{adj } A)$  be such that  $|B| = 81$ , then  $\alpha^2$  is equal to

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**Options:**

A.

2

B.

4

C.

3

D.

1

**Answer: B**

**Solution:**

$$\begin{aligned} f(x) &= \int \frac{7x^{10} + 9x^8}{(1+x^2+2x^9)^2} dx \\ &= \int \frac{7x^{10} + 9x^8}{x^{18} \left( \frac{1}{x^9} + \frac{1}{x^7} + 2 \right)^2} = \int \frac{\frac{7}{x^8} + \frac{9}{x^{10}}}{\left( \frac{1}{x^9} + \frac{1}{x^7} + 2 \right)^2} \end{aligned}$$

substitute  $\frac{1}{x^9} + \frac{1}{x^7} + 2 = t$

$$\left( \frac{-9}{x^{10}} - \frac{7}{x^8} \right) dx = dt \Rightarrow \left( \frac{7}{x^8} + \frac{9}{x^{10}} \right) dx = -dt$$

$$f(t) = - \int \frac{dt}{t^2} = \frac{1}{t} + C$$

put value of t

$$f(x) = \frac{1}{\frac{1}{x^9} + \frac{1}{x^7} + 2} + C$$

$$f(x) = \frac{x^9}{2x^9 + x^2 + 1} + C$$

It is given that  $\lim_{x \rightarrow 0} f(x) = 0$

$$\lim_{x \rightarrow 0} \frac{x^9}{2x^9 + x^2 + 1} + C = 0$$

$$0 + C = 0 \Rightarrow C = 0$$

so  $f(x) = \frac{x^9}{2x^9 + x^2 + 1}$

$$f'(x) = \frac{9x^8(2x^9 + x^2 + 1) - x^9(18x^8 + 2x)}{(2x^9 + x^2 + 1)^2}$$

$$f'(1) = \frac{9(1)^8(2(1)^9 + (1)^2 + 1) - (1)^9(18(1)^8 + 2(1))}{(2(1)^9 + (1)^2 + 1)^2}$$

$$= \frac{9 \times 4 - 20}{16} = \frac{16}{16} = 1$$

It is given that  $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$

substitute  $f'(1) = 1 \Rightarrow A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & 1 & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$

$$|A| = 1 \left( 4 \times \frac{1}{4} - \alpha^2 \right) = 1 - \alpha^2$$

It is given that  $B = \text{adj}(\text{adj } A) \& |B| = 81$

$$|B| = |A|^{(n-1)^2}, n = 3 \text{ as } A \text{ is } 3 \times 3 \text{ matrix}$$

$$|B| = |A|^{(3-1)^2} = 81$$

$$|A|^4 = 81 = 3^4$$

$$|A| = \pm 3$$

$$1 - \alpha^2 = \pm 3 \{ |A| = 1 - \alpha^2 \}$$

$$1 - \alpha^2 = -3 \text{ or } 1 - \alpha^2 = 3$$

$$\alpha^2 = 4 \text{ or } \alpha^2 = -2 \text{ not possible}$$

so value of  $\alpha^2$  is 4

---

## Question17

Let  $P = [p_{ij}]$  and  $Q = [q_{ij}]$  be two square matrices of order 3 such that  $q_{ij} = 2^{(i+j-1)} p_{ij}$  and  $\det(Q) = 2^{10}$ . Then the value of  $\det(\text{adj}(\text{adj} P))$  is:

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**Options:**

A.

81

B.

16

C.

124

D.

32

**Answer: B**

**Solution:**

Given two  $3 \times 3$  matrices  $P = [p_{ij}]$ ,  $Q = [q_{ij}]$  are given

$$q_{ij} = 2^{(i+j-1)} p_{ij}, \det(Q) = 2^{10}.$$

To calculate  $\det(\text{adj}(\text{adj} P))$

$$\text{We know } \det(\text{adj}(\text{adj} P)) = (\det(P))^{(n-1)^2}$$

where  $n$  is order of matrix which is 3

$$\text{so, } \det(\text{adj}(\text{adj} P)) = (\det(P))^4 - (1)$$

Let P matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$\text{and } Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$$

use  $q_{ij} = 2^{i+j-1}p_{ij}$  to find Q matrix

$$\text{e.g. } q_{11} = 2^{1+1-1}p_{11} = 2p_{11}, q_{23} = 2^{2+3-1}p_{23} = 2^4p_{23}$$

$$Q = \begin{bmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{bmatrix}$$

$$Q = \begin{bmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{bmatrix}$$

$$\det(Q) = \begin{vmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{vmatrix} = 2^{10}$$

taking common 2 from  $R_1$ ,  $2^2$  from  $R_2$  &  $2^3$  from  $R_3$

$$\det(Q) = 2 \cdot 2^2 \cdot 2^3 \begin{vmatrix} p_{11} & 2p_{12} & 2^2p_{13} \\ p_{21} & 2p_{22} & 2^2p_{23} \\ p_{31} & 2p_{32} & 2^2p_{33} \end{vmatrix} = 2^{10}$$

taking common 2 from  $C_2$  and  $2^2$  from  $C_3$

$$2 \cdot 2^2 \cdot 2^3 \cdot 2 \cdot 2^2 \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix} = 2^{10}$$

$$\Rightarrow 2^9 \cdot \det(P) = 2^{10}$$

$$\Rightarrow \det(P) = 2$$

$$\text{from (1) } \det(\text{adj}(\text{adj } P)) = (\det(P))^4 = 2^4 = 16$$

## Question 18

Let  $A$ ,  $B$  and  $C$  be three  $2 \times 2$  matrices with real entries such that  $B = (I + A)^{-1}$  and  $A + C = I$ .

If  $BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$  and  $CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$ , then  $x_1 + x_2$  is

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**Options:**

A.

4

B.

2

C.

0

D.

-2

**Answer: C**

**Solution:**

We are given:

$$\bullet B = (I + A)^{-1} \implies B^{-1} = I + A$$

$$\bullet A + C = I \implies A = I - C$$

Substitute  $A$  into the first equation:

$$B^{-1} = I + (I - C)$$

$$B^{-1} = 2I - C \dots\dots\dots (1)$$

$$\text{pre-multiply } B : B \cdot B^{-1} = B(2I - C)I = 2B - BC \implies BC = 2B - I \dots\dots\dots (2)$$

$$\text{post multiply in eqn (1): } B^{-1}B = (2I - C) \cdot BI = 2B - CB \implies CB = 2B - I \dots\dots\dots (3)$$

from (2) & (3),  $BC = CB$

3. Set up the System of Equations Since  $BC = CB$ , we use the given matrix for  $BC$  :

$$CB = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$$

Now, substitute this into the vector equation :

$$\begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

This results in two linear equations :

$$\bullet x_1 - 5x_2 = 12$$

$$\bullet -x_1 + 2x_2 = -6$$

1. Solve for  $x_1$  and  $x_2$

Adding the two equations :

$$(x_1 - x_1) + (-5x_2 + 2x_2) = 12 - 6$$

$$\Rightarrow x_2 = -2$$

Substitute  $x_2 = -2$  into the second equation :

$$-x_1 + 2(-2) = -6$$

$$-x_1 - 4 = -6 \Rightarrow -x_1 = -2 \Rightarrow x_1 = 2$$

$$\text{Final result } x_1 + x_2 = 2 + (-2) = 0$$

---

## Question 19

Let  $A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$ . If  $A^2 - 4A + I = O$  and  $B^2 - 5B - 6I = O$ , then among the two statements:

(S1) :  $[(B - A)(B + A)]^T = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$

and

(S2) :  $\det(\text{adj}(A + B)) = -5$

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**Options:**

A.

only (S1) is correct

B.

only (S2) is correct

C.

both (S1) and (S2) are correct

D.

both (S1) and (S2) are wrong

**Answer: B**

### Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & \alpha \end{bmatrix}$$

$$A^2 - 4A + I = 0$$

trace of matrix

$$\alpha + 1 = 4$$

$$\alpha = 3$$

$$B = \begin{bmatrix} 3 & 3 \\ \beta & 2 \end{bmatrix}$$

$$B^2 - 5B - 6I = 0$$

$$\det.(B) = 6 - 3\beta = -6$$

$$3\beta = 12$$

$$\beta = 4$$

$$B - A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}; B + A = \begin{bmatrix} 4 & 5 \\ 5 & 5 \end{bmatrix}$$

$$(B - A)(B + A) = \begin{bmatrix} 13 & 15 \\ 7 & 10 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}, \text{adj}B = \begin{bmatrix} 2 & -3 \\ -4 & 3 \end{bmatrix}$$

$$\text{adj}(A + B) = \begin{bmatrix} 5 & -5 \\ -5 & 4 \end{bmatrix}$$

---

## Question 20

Let  $\alpha, \beta \in \mathbb{R}$  be such that the system of linear equations

$$x + 2y + z = 5$$

$$2x + y + \alpha z = 5$$

$$8x + 4y + \beta z = 18$$

has no solution. Then  $\frac{\beta}{\alpha}$  is equal to :

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**Options:**

A.

B.

4

C.

8

D.

-8

**Answer: B**

**Solution:**

$$x + 2y + z = 5$$

$$2x + y + \alpha z = 5$$

$$8x + 4y + \beta z = 18$$

For a system of three linear equations to have **no solution**, it is necessary that the coefficient matrix be **singular**, i.e. its determinant is zero:  $\Delta = 0$ .

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 8 & 4 & \beta \end{vmatrix} = 0$$

$$\Delta = 1(\beta - 4\alpha) - 2(2\beta - 8\alpha) + 1(0) = 0$$

$$= \beta - 4\alpha - 4\beta + 16\alpha = 0$$

$$\Rightarrow 4\alpha = \beta$$

Now substitute  $\beta = 4\alpha$  in the third equation. Then the left side of the third equation becomes  $8x + 4y + 4\alpha z$ , which is exactly 4 times the left side of the second equation  $2x + y + \alpha z$ .

But the constants do not match:  $4 \times 5 = 20$ , whereas the third equation has 18. So we get

$4 \times (2x + y + \alpha z) = 20$  but  $8x + 4y + 4\alpha z = 18$ , which is a contradiction. Hence, the system is inconsistent and has **no solution**.

$$\text{Therefore, } \frac{\beta}{\alpha} = \frac{4\alpha}{\alpha} = 4.$$

---

## Question 21

**If the system of equations**

$$x + 5y + 6z = 4$$

$$2x + 3y + 4z = 7$$

$$x + 6y + az = b$$

has infinitely many solutions, then the point  $(a, b)$  lies on the line

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Options:

A.

$$y - x = 3$$

B.

$$x - y = 3$$

C.

$$x + y = 11$$

D.

$$x + y = 12$$

**Answer: B**

**Solution:**

The given system of equations is:

$$x + 5y + 6z = 4$$

$$2x + 3y + 4z = 7$$

$$x + 6y + az = b$$

**Find the determinant of the coefficient matrix ( $\Delta$ )**

The coefficient matrix is formed by the coefficients of  $x$ ,  $y$ , and  $z$ .

$$\Delta = \begin{vmatrix} 1 & 5 & 6 \\ 2 & 3 & 4 \\ 1 & 6 & a \end{vmatrix}$$

Let us expand this determinant along the first row:

$$\Delta = 1(3 \cdot a - 6 \cdot 4) - 5(2 \cdot a - 1 \cdot 4) + 6(2 \cdot 6 - 1 \cdot 3)$$

$$\Delta = 1(3a - 24) - 5(2a - 4) + 6(12 - 3)$$

$$\Delta = 3a - 24 - 10a + 20 + 6(9)$$

$$\Delta = -7a - 4 + 54$$

$$\Delta = -7a + 50$$

For the system to have infinitely many solutions, we equate  $\Delta$  to 0:

$$-7a + 50 = 0 \implies 7a = 50 \implies a = \frac{50}{7}$$

**Find the determinant  $\Delta_z$**

To find  $\Delta_z$ , we replace the  $z$ -column (third column) with the constant terms from the right-hand side of the equations.

$$\Delta_z = \begin{vmatrix} 1 & 5 & 4 \\ 2 & 3 & 7 \\ 1 & 6 & b \end{vmatrix}$$

We again expand along the first row:

$$\Delta_z = 1(3 \cdot b - 6 \cdot 7) - 5(2 \cdot b - 1 \cdot 7) + 4(2 \cdot 6 - 1 \cdot 3)$$

$$\Delta_z = 1(3b - 42) - 5(2b - 7) + 4(12 - 3)$$

$$\Delta_z = 3b - 42 - 10b + 35 + 4(9)$$

$$\Delta_z = -7b - 7 + 36$$

$$\Delta_z = -7b + 29$$

Equating  $\Delta_z$  to zero for infinitely many solutions, we get:

$$-7b + 29 = 0 \implies 7b = 29 \implies b = \frac{29}{7}$$

**Check the options for the point  $(a, b)$**

We have obtained the point coordinates as  $(a, b) = \left(\frac{50}{7}, \frac{29}{7}\right)$ .

Let us evaluate the difference  $a - b$  to find out which linear profile equation matches our outcome:

$$a - b = \frac{50}{7} - \frac{29}{7}$$

$$a - b = \frac{50-29}{7}$$

$$a - b = \frac{21}{7} = 3$$

Writing this relation in terms of  $x$  and  $y$  (where  $x = a$  and  $y = b$ ), it perfectly satisfies the line equation:

$$x - y = 3$$

Thus, the point  $(a, b)$  lies on the line given in Option B.

---

## Question 22

Let  $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ . Then the sum of all elements of the matrix  $\text{adj}(\text{adj}(2(\text{adj } A)^{-1}))$  is equal to:

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Options:

A.

3

B.

4

C.

-4

D.

-3

**Answer: D**

**Solution:**

Given

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 5 \end{bmatrix}$$

We have to find the sum of all elements of

$$\text{adj}(\text{adj}(2(\text{adj } A)^{-1}))$$

$$\begin{aligned} \det A &= 1 \begin{vmatrix} 0 & 1 \\ 3 & 5 \end{vmatrix} - 1 \begin{vmatrix} -2 & 1 \\ 1 & 5 \end{vmatrix} + 2 \begin{vmatrix} -2 & 0 \\ 1 & 3 \end{vmatrix} \\ &= 1(0 \cdot 5 - 1 \cdot 3) - 1((-2) \cdot 5 - 1 \cdot 1) + 2((-2) \cdot 3 - 0 \cdot 1) \end{aligned}$$

$$= -3 - (-11) + 2(-6)$$

$$= -3 + 11 - 12 = -4$$

So,

$$\det A = -4$$

For an  $n \times n$  matrix,

$$\det(\text{adj } A) = (\det A)^{n-1}$$

Since  $A$  is  $3 \times 3$ ,

$$\det(\text{adj } A) = (\det A)^2 = (-4)^2 = 16$$

Now let

$$B = 2(\text{adj } A)^{-1}$$

We need  $\text{adj}(\text{adj } B)$ .

For a  $3 \times 3$  matrix, the property is:

$$\text{adj}(\text{adj } B) = (\det B) B$$

Since

$$B = 2(\text{adj } A)^{-1}$$

and for a  $3 \times 3$  matrix,

$$\det(2M) = 2^3 \det(M) = 8 \det(M)$$

therefore,

$$\det B = 8 \det((\text{adj } A)^{-1})$$

Also,

$$\det((\text{adj } A)^{-1}) = \frac{1}{\det(\text{adj } A)} = \frac{1}{16}$$

Hence,

$$\det B = 8 \cdot \frac{1}{16} = \frac{1}{2}$$

So,

$$\text{adj}(\text{adj } B) = (\det B) B = \frac{1}{2} B$$

But

$$B = 2(\text{adj } A)^{-1}$$

Thus,

$$\text{adj}(\text{adj } B) = \frac{1}{2} \cdot 2(\text{adj } A)^{-1} = (\text{adj } A)^{-1}$$

So the required matrix is simply

$$(\text{adj } A)^{-1}$$

We know,

$$\text{adj } A = (\det A) A^{-1}$$

Hence,

$$(\text{adj } A)^{-1} = \frac{1}{\det A} A$$

Since  $\det A = -4$ ,

$$(\text{adj } A)^{-1} = -\frac{1}{4} A = -\frac{1}{4} A$$

Therefore the required matrix is

$$-\frac{1}{4} A$$

First find the sum of elements of  $A$ :

$$1 + 1 + 2 - 2 + 0 + 1 + 1 + 3 + 5 = 12$$

Therefore sum of elements of  $-\frac{1}{4}A$  is

$$-\frac{1}{4} \cdot 12 = -3$$

**Final Answer**

$$\boxed{-3}$$

So, the correct option is **D**.

---

## Question 23

Let  $S = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \{0, 1, 2, 3, 4\} \text{ and } \right.$

$A^2 - 4A + 3I = 0 \left. \right\}$  be a set of  $2 \times 2$  matrices. Then the number of matrices in  $S$ , for which the sum of the diagonal elements is equal to 4, is :

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**Options:**

A.

20

B.

17

C.

21

D.

19

**Answer: D**

**Solution:**

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad a, b, c, d \in \{0, 1, 2, 3, 4\}$$

and it satisfies

$$A^2 - 4A + 3I = 0.$$

We have to count those matrices for which the sum of diagonal elements is

$$a + d = 4.$$

So we must find the number of matrices satisfying both:

$$1. A^2 - 4A + 3I = 0$$

$$2. \operatorname{tr}(A) = a + d = 4$$

## Step 1: Factor the matrix equation

We write

$$A^2 - 4A + 3I = (A - I)(A - 3I) = 0.$$

So the matrix  $A$  satisfies the polynomial

$$(x - 1)(x - 3) = 0.$$

Hence the only possible eigenvalues are 1 and 3.

Now the trace is given to be 4. Since trace is sum of eigenvalues, the eigenvalues must be 1 and 3.

So every such matrix has distinct eigenvalues 1 and 3, hence it is diagonalizable.

## Step 2: Use trace and determinant

For a  $2 \times 2$  matrix with eigenvalues 1 and 3,

- trace  $= 1 + 3 = 4$
- determinant  $= 1 \cdot 3 = 3$

So we need all matrices with entries in  $\{0, 1, 2, 3, 4\}$  such that

$$a + d = 4, \quad ad - bc = 3.$$

Let  $d = 4 - a$ . Then

$$a(4 - a) - bc = 3$$

that is,

$$bc = a(4 - a) - 3.$$

Now check all possible values of  $a \in \{0, 1, 2, 3, 4\}$ .

## Step 3: Case-wise counting

**Case 1:  $a = 0$** 

Then  $d = 4$ , and

$$bc = 0 \cdot 4 - 3 = -3$$

not possible.

**Case 2:  $a = 1$** 

Then  $d = 3$ , and

$$bc = 1 \cdot 3 - 3 = 0$$

So we need ordered pairs  $(b, c)$  with  $b, c \in \{0, 1, 2, 3, 4\}$  and  $bc = 0$ .

That means at least one of  $b, c$  is 0.

Number of such ordered pairs:

- $b = 0$ : 5 choices for  $c$
- $c = 0$ : 5 choices for  $b$
- but  $(0, 0)$  counted twice

So total

$$5 + 5 - 1 = 9$$

matrices.

**Case 3:  $a = 2$** 

Then  $d = 2$ , and

$$bc = 2 \cdot 2 - 3 = 1$$

So

$$bc = 1.$$

Since  $b, c \in \{0, 1, 2, 3, 4\}$ , this is possible only for

$$(b, c) = (1, 1).$$

So total matrices in this case:

1

**Case 4:  $a = 3$** 

Then  $d = 1$ , and

$$bc = 3 \cdot 1 - 3 = 0$$

Again, number of ordered pairs  $(b, c)$  with  $bc = 0$  is

9.

**Case 5:  $a = 4$** 

Then  $d = 0$ , and

$$bc = 4 \cdot 0 - 3 = -3$$

not possible.

#### Step 4: Total count

Total number of matrices is

$$9 + 1 + 9 = 19.$$

Hence the required number of matrices is

19

So, the correct option is:

Option D

---

### Question 24

Let  $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$  and  $\det(A - \alpha I) = 0$ , where  $\alpha$  is a real

number. If the largest possible value of  $\alpha$  is  $p$ , then the circle  $(x - p)^2 + (y - 2p)^2 = 320$ , intersects the co-ordinate axes at

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Options:

A.

1 point

B.

2 points

C.

3 points

D.

4 points

Answer: C

## Solution:

We need the largest real value of  $\alpha$  such that

$$\det(A - \alpha I) = 0$$

That means  $\alpha$  is an eigenvalue of the matrix

$$A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}.$$

So first we find the eigenvalues of  $A$ .

### Step 1: Form $A - \alpha I$

$$A - \alpha I = \begin{bmatrix} 1 - \alpha & 2 & 7 \\ 4 & -2 - \alpha & 8 \\ 3 & 8 & -7 - \alpha \end{bmatrix}$$

Now,

$$\det(A - \alpha I) = \begin{vmatrix} 1 - \alpha & 2 & 7 \\ 4 & -2 - \alpha & 8 \\ 3 & 8 & -7 - \alpha \end{vmatrix} = 0$$

### Step 2: Expand the determinant

Expanding along the first row:

$$\det(A - \alpha I) = (1 - \alpha) \begin{vmatrix} -2 - \alpha & 8 \\ 8 & -7 - \alpha \end{vmatrix} - 2 \begin{vmatrix} 4 & 8 \\ 3 & -7 - \alpha \end{vmatrix} + 7 \begin{vmatrix} 4 & -2 - \alpha \\ 3 & 8 \end{vmatrix}$$

Now calculate each minor:

$$\begin{vmatrix} -2 - \alpha & 8 \\ 8 & -7 - \alpha \end{vmatrix} = (-2 - \alpha)(-7 - \alpha) - 64 \\ = (\alpha + 2)(\alpha + 7) - 64 = \alpha^2 + 9\alpha + 14 - 64 = \alpha^2 + 9\alpha - 50$$

Next,

$$\begin{vmatrix} 4 & 8 \\ 3 & -7 - \alpha \end{vmatrix} = 4(-7 - \alpha) - 24 = -28 - 4\alpha - 24 = -52 - 4\alpha$$

And,

$$\begin{vmatrix} 4 & -2 - \alpha \\ 3 & 8 \end{vmatrix} = 32 - 3(-2 - \alpha) = 32 + 6 + 3\alpha = 38 + 3\alpha$$

So,

$$\det(A - \alpha I) = (1 - \alpha)(\alpha^2 + 9\alpha - 50) - 2(-52 - 4\alpha) + 7(38 + 3\alpha)$$

Now simplify:

$$(1 - \alpha)(\alpha^2 + 9\alpha - 50) = \alpha^2 + 9\alpha - 50 - \alpha^3 - 9\alpha^2 + 50\alpha$$

$$= -\alpha^3 - 8\alpha^2 + 59\alpha - 50$$

Also,

$$-2(-52 - 4\alpha) = 104 + 8\alpha$$

and

$$7(38 + 3\alpha) = 266 + 21\alpha$$

Therefore,

$$\det(A - \alpha I) = -\alpha^3 - 8\alpha^2 + 59\alpha - 50 + 104 + 8\alpha + 266 + 21\alpha$$

$$= -\alpha^3 - 8\alpha^2 + 88\alpha + 320$$

So the equation becomes

$$-\alpha^3 - 8\alpha^2 + 88\alpha + 320 = 0$$

Multiplying by  $-1$ :

$$\alpha^3 + 8\alpha^2 - 88\alpha - 320 = 0$$

### Step 3: Factorise

Try integer roots. On putting  $\alpha = 8$ :

$$8^3 + 8(8^2) - 88(8) - 320 = 512 + 512 - 704 - 320 = 0$$

So  $(\alpha - 8)$  is a factor.

Divide:

$$\alpha^3 + 8\alpha^2 - 88\alpha - 320 = (\alpha - 8)(\alpha^2 + 16\alpha + 40)$$

Now solve

$$\alpha^2 + 16\alpha + 40 = 0$$

$$\alpha = \frac{-16 \pm \sqrt{256 - 160}}{2} = \frac{-16 \pm \sqrt{96}}{2} = \frac{-16 \pm 4\sqrt{6}}{2} = -8 \pm 2\sqrt{6}$$

Hence the eigenvalues are

$$8, \quad -8 + 2\sqrt{6}, \quad -8 - 2\sqrt{6}$$

So the largest possible value is

$$p = 8$$

### Step 4: Put $p = 8$ in the circle

Given circle:

$$(x - p)^2 + (y - 2p)^2 = 320$$

Substitute  $p = 8$ :

$$(x - 8)^2 + (y - 16)^2 = 320$$

Its centre is  $(8, 16)$  and radius is

$$r = \sqrt{320} = 8\sqrt{5}$$

### Step 5: Find intersections with the coordinate axes

#### On the $x$ -axis

For points on the  $x$ -axis,  $y = 0$ .

$$(x - 8)^2 + (0 - 16)^2 = 320$$

$$(x - 8)^2 + 256 = 320$$

$$(x - 8)^2 = 64$$

$$x - 8 = \pm 8$$

So,

$$x = 16 \quad \text{or} \quad x = 0$$

Thus the circle meets the  $x$ -axis at

$$(16, 0), (0, 0)$$

#### On the $y$ -axis

For points on the  $y$ -axis,  $x = 0$ .

$$(0 - 8)^2 + (y - 16)^2 = 320$$

$$64 + (y - 16)^2 = 320$$

$$(y - 16)^2 = 256$$

$$y - 16 = \pm 16$$

So,

$$y = 32 \quad \text{or} \quad y = 0$$

Thus the circle meets the  $y$ -axis at

$$(0, 32), (0, 0)$$

### Step 6: Count distinct intersection points

The distinct points are

$$(16, 0), (0, 0), (0, 32)$$

So the circle intersects the coordinate axes at

**3 points**

Hence, the correct option is:

**Option C**

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## Question 25

If the system of equations :

$$x + y + z = 5$$

$$x + 2y + 3z = 9$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of  $\lambda + \mu$  is :

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**Options:**

A.

16

B.

18

C.

19

D.

21

**Answer: B**

**Solution:**

$$D = \lambda - 5$$

$$D_1 = \lambda + \mu - 18$$

$$D_2 = 4\lambda - 2\mu + 6$$

$$D_3 = \mu - 13$$

$$\text{Here } x = \frac{D_1}{D}; y = \frac{D_2}{D}; z = \frac{D_3}{D}$$

For the system to have infinitely many solutions, the determinant of the coefficient matrix and all the corresponding determinants must be zero.

$$D = D_1 = D_2 = D_3 = 0$$

First, set  $D = 0$ :

$$\lambda - 5 = 0 \Rightarrow \lambda = 5$$

Next, set  $D_3 = 0$ :

$$\mu - 13 = 0 \Rightarrow \mu = 13$$

Now check  $D_1 = 0$  using  $\lambda = 5$  and  $\mu = 13$ :

$$\lambda + \mu - 18 = 5 + 13 - 18 = 0$$

Also check  $D_2 = 0$ :

$$4\lambda - 2\mu + 6 = 4(5) - 2(13) + 6 = 20 - 26 + 6 = 0$$

So the values are consistent. Therefore,

$$\therefore \lambda + \mu = 5 + 13 = 18$$

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## Question 26

Consider the system of linear equations in  $x, y, z$ :

$$x + 2y + tz = 0$$

$$6x + y + 5tz = 0$$

$$3x + t^2y + f(t)z = 0$$

where  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a differentiable function. If this system has infinitely many solutions for all  $t \in \mathbb{R}$ , then  $f$

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**Options:**

A.

is a constant function

B.

is strictly increasing on  $\mathbb{R}$

C.

is strictly decreasing on  $\mathbb{R}$

D.

has two critical points

**Answer: B**

### Solution:

For a homogeneous system of three linear equations in  $x, y, z$  to have infinitely many solutions, the coefficient matrix must be singular for every  $t \in \mathbb{R}$ .

So, we form the coefficient matrix:

$$A = \begin{pmatrix} 1 & 2 & t \\ 6 & 1 & 5t \\ 3 & t^2 & f(t) \end{pmatrix}$$

Since the system has infinitely many solutions for all  $t$ , we must have

$$\det(A) = 0 \quad \text{for all } t \in \mathbb{R}.$$

Now compute the determinant.

$$\det(A) = \begin{vmatrix} 1 & 2 & t \\ 6 & 1 & 5t \\ 3 & t^2 & f(t) \end{vmatrix}$$

Expanding along the first row,

$$\det(A) = 1 \begin{vmatrix} 1 & 5t \\ t^2 & f(t) \end{vmatrix} - 2 \begin{vmatrix} 6 & 5t \\ 3 & f(t) \end{vmatrix} + t \begin{vmatrix} 6 & 1 \\ 3 & t^2 \end{vmatrix}$$

Now evaluate each minor:

$$\begin{vmatrix} 1 & 5t \\ t^2 & f(t) \end{vmatrix} = f(t) - 5t^3$$

$$\begin{vmatrix} 6 & 5t \\ 3 & f(t) \end{vmatrix} = 6f(t) - 15t$$

$$\begin{vmatrix} 6 & 1 \\ 3 & t^2 \end{vmatrix} = 6t^2 - 3$$

So,

$$\det(A) = (f(t) - 5t^3) - 2(6f(t) - 15t) + t(6t^2 - 3)$$

$$= f(t) - 5t^3 - 12f(t) + 30t + 6t^3 - 3t$$

$$= -11f(t) + t^3 + 27t$$

Since this is zero for all  $t$ ,

$$-11f(t) + t^3 + 27t = 0$$

Hence,

$$f(t) = \frac{t^3 + 27t}{11}$$

Now differentiate:

$$f'(t) = \frac{3t^2+27}{11} = \frac{3(t^2+9)}{11}$$

Since  $t^2 + 9 > 0$  for all  $t \in \mathbb{R}$ , we get

$$f'(t) > 0 \quad \text{for all } t \in \mathbb{R}.$$

Therefore,  $f$  is strictly increasing on  $\mathbb{R}$ .

Hence, the correct option is:

Option B

## Question 27

Let  $A$  be a  $3 \times 3$  matrix such that

$$A^T \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}, A^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} \text{ and } A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

If  $\det(A) = 1$ , then  $\det(\text{adj}(A^2 + A))$  is equal to:

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**Options:**

A.

16

B.

25

C.

49

D.

64

**Answer: D**

**Solution:**

Let

$$u = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We are given

$$A^T u = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}, \quad A^T v = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

and

$$Au = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}, \quad Av = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

We have to find

$$\det(\operatorname{adj}(A^2 + A))$$

given that

$$\det(A) = 1$$

**Step 1: Use the relation  $A(u - v)$**

First note that

$$u - v = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

So,

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = A(u - v) = Au - Av$$

Now,

$$Au - Av = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence the first column of  $A$  is

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Also, since

$$A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

the third column of  $A$  is

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Now use

$$A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

which gives

$$\begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

This is consistent.

So let

$$A = \begin{bmatrix} 2 & a & 1 \\ 1 & b & 3 \\ 3 & c & 1 \end{bmatrix}$$

Now use  $\det(A) = 1$ .

## Step 2: Find the second column using determinant

Compute determinant:

$$\det(A) = \begin{vmatrix} 2 & a & 1 \\ 1 & b & 3 \\ 3 & c & 1 \end{vmatrix}$$

Expanding along the first row,

$$\det(A) = 2 \begin{vmatrix} b & 3 \\ c & 1 \end{vmatrix} - a \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & b \\ 3 & c \end{vmatrix}$$

$$= 2(b - 3c) - a(1 - 9) + (c - 3b)$$

$$= 2b - 6c + 8a + c - 3b$$

$$= 8a - b - 5c$$

$$\text{Given } \det(A) = 1,$$

$$8a - b - 5c = 1$$

But we still need more information. For that use the conditions involving  $A^T$ .

## Step 3: Use transpose conditions

Since

$$A^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

this means the third column of  $A^T$  is  $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ , so the third row of  $A$  is

$$[3 \quad 1 \quad 1]$$

Thus,

$$c = 1$$

Now  $A$  becomes

$$A = \begin{bmatrix} 2 & a & 1 \\ 1 & b & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Next, use

$$A^T \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$$

But

$$A^T \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + A^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now  $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  is the first column of  $A^T$ , i.e. the first row of  $A$ , which is

$$\begin{bmatrix} 2 \\ a \\ 1 \end{bmatrix}$$

And

$$A^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} 2 \\ a \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$$

So,

$$\begin{bmatrix} 5 \\ a+1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$$

Hence,

$$a = 1$$

Now use  $\det(A) = 1$ :

$$8a - b - 5c = 1$$

with  $a = 1, c = 1$ ,

$$8 - b - 5 = 1$$

$$3 - b = 1$$

$$b = 2$$

Thus,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

**Step 4: Find  $A^2 + A$**

First compute  $A^2$ :

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Multiplying row by column:

$$A^2 = \begin{bmatrix} 8 & 5 & 6 \\ 13 & 8 & 10 \\ 10 & 6 & 7 \end{bmatrix}$$

So,

$$A^2 + A = \begin{bmatrix} 8 & 5 & 6 \\ 13 & 8 & 10 \\ 10 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 6 & 7 \\ 14 & 10 & 13 \\ 13 & 7 & 8 \end{bmatrix}$$

**Step 5: Use determinant property**

For a  $3 \times 3$  matrix  $M$ ,

$$\det(\text{adj } M) = \det(M)^{3-1} = \det(M)^2$$

So we first find

$$\det(A^2 + A)$$

Now,

$$A^2 + A = A(A + I)$$

Hence,

$$\det(A^2 + A) = \det(A) \det(A + I)$$

Given  $\det(A) = 1$ , so

$$\det(A^2 + A) = \det(A + I)$$

Now

$$A + I = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

Its determinant is

$$\det(A + I) = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 3 \\ 3 & 1 & 2 \end{vmatrix}$$

Expanding along the first row:

$$= 3 \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 3(6 - 3) - (2 - 9) + (1 - 9)$$

$$= 3 \cdot 3 - (-7) - 8$$

$$= 9 + 7 - 8 = 8$$

So,

$$\det(A^2 + A) = 8$$

Therefore,

$$\det(\text{adj}(A^2 + A)) = \det(A^2 + A)^2 = 8^2 = 64$$

**Final Answer**

64

So, the correct option is **D**.

## Question 28

Let **M** be a  $3 \times 3$  matrix such that  $M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

and  $M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ . If  $M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$ , then  $x + y + z$  equals :

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Options:

A.

4

B.

5

C.

7

D.

11

**Answer: B**

**Solution:**

Since the images of the standard basis vectors are given, the columns of matrix  $M$  are exactly these vectors.

So,

$$M = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

Now we are given

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$$

So,

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$$

This gives the system:

$$x - z = 1$$

$$2x + y + z = 7$$

$$3x + 2y + z = 11$$

Now solve step by step.

From the first equation,

$$x = 1 + z$$

Substitute into the second equation:

$$2(1+z) + y + z = 7$$

$$2 + 3z + y = 7$$

$$y + 3z = 5 \Rightarrow y = 5 - 3z$$

Now substitute  $x = 1 + z$  and  $y = 5 - 3z$  into the third equation:

$$3(1+z) + 2(5-3z) + z = 11$$

$$3 + 3z + 10 - 6z + z = 11$$

$$13 - 2z = 11$$

$$-2z = -2$$

$$z = 1$$

Then,

$$x = 1 + z = 2$$

and

$$y = 5 - 3z = 2$$

Therefore,

$$x + y + z = 2 + 2 + 1 = 5$$

So the correct answer is:

**5**

Option B

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## Question29

If  $f : \mathbf{N} \rightarrow \mathbf{Z}$  is defined by

$$f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}, k \in \mathbf{N},$$

and  $\sum_{n=1}^k f(n) = 98$ , then  $k$  is equal to :

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**Options:**

A.

3

B.

4

C.

5

D.

6

**Answer: A**

**Solution:**

$$f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$$

$$\sum_{n=1}^k f(n) = \begin{vmatrix} \frac{k(k+1)}{2} & -1 & -5 \\ -2\frac{k(k+1)(2k+1)}{6} & 3(2k+1) & 2k+1 \\ -3\frac{k^2(k+1)^2}{4} & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix} = 98$$

$$\Rightarrow \frac{k(k+1)(2k+1)}{2} \cdot \frac{7}{3} = 98$$

$$\Rightarrow k(k+1)(2k+1) = 84$$

$$\Rightarrow k = 3$$

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## Question 30

Let  $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$  and  $B = [b_{ij}], 1 \leq i, j \leq 3$ . If  $B = A^{99} - I$ , then the value of  $\frac{b_{31} - b_{21}}{b_{32}}$  is :

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**Options:**

A.

99

B.

199

C.

149

D.

159

**Answer: C**

**Solution:**

$$A = P + I$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = P + I \text{ where } P^3 = 0$$

$$A^{99} = I + 99P + {}^{99}C_2 P^2$$

$$A^{99} - I = 99 \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + {}^{99}C_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 99 \times 3 & 0 & 0 \\ 44550 & 99 \times 3 & 0 \end{bmatrix}$$

$$b_{31} = 44550, b_{21} = 99 \times 3, b_{32} = 99 \times 3$$

$$\therefore \frac{b_{31} - b_{21}}{b_{32}} = \frac{44550 - 297}{297} = 149$$

## Question 31

The sum of all possible values of  $\theta \in [0, 2\pi]$ , for which the system of equations :

$$x \cos 3\theta - 8y - 12z = 0$$

$$x \cos 2\theta + 3y + 3z = 0$$

$$x + y + 3z = 0$$

has a non-trivial solution, is equal to :

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**Options:**

A.

$$\pi$$

B.

$$2\pi$$

C.

$$3\pi$$

D.

$$4\pi$$

**Answer: D**

**Solution:**

$$\begin{vmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 3 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$$

$$\begin{vmatrix} \cos 3\theta & -8 & -4 \\ \cos 2\theta & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$(C_1 \rightarrow C_1 - C_2) \& C_2 \rightarrow C_2 - C_3$$

$$\begin{vmatrix} \cos 3\theta + 8 & -4 & -3 \\ \cos 2\theta - 3 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$2 \cos 3\theta + 16 + 4 \cos 2\theta - 12 = 0$$

$$(4 \cos^3 \theta - 3 \cos \theta) + 2(2 \cos^2 \theta - 1) + 2 = 0$$

$$4 \cos^3 \theta + 4 \cos^2 \theta - 3 \cos \theta = 0$$

$$\cos \theta (4 \cos^2 \theta + 4 \cos \theta - 3) = 0$$

$$\cos \theta (2 \cos \theta + 3)(2 \cos \theta - 1) = 0$$

$$\cos \theta = 0, \frac{1}{2}, -\frac{3}{2} \text{ (rejected)}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\text{Sum} = 4\pi$$

## Question32

Let  $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$ . If  $\det(B) = 66$ , then  $\det(\text{adj}(A))$  equals :

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**Options:**

A.

289

B.

361

C.

441

D.

529

**Answer: C**

**Solution:**

$$A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow |A| = 15\alpha - 10 + 16 = 15\alpha + 6$$

$$\text{Now adj } A = \begin{bmatrix} 15 & 3 & -6 \\ -10 & 5\alpha & 4 \\ 8 & -4\alpha & 3\alpha - 2 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \text{adj}(A)$$

$$B = \begin{bmatrix} 16 & 3 & -6 \\ -10 & 0 & 4 \\ 8 & 0 & \alpha - 2 \end{bmatrix}$$

$$\&|B| = 6(5\alpha + 6) = 66 \text{ (given in question)}$$

$$\Rightarrow \alpha = 1$$

$$\Rightarrow |A| = 21$$

$$A \cdot \text{Adj}(A) = |A|I \Rightarrow |A \cdot \text{Adj}(A)| = ||A|I| = (|A|^3)|I| = |A|^3$$

$$\Rightarrow |A| |\text{Adj}(A)| = |A|^3 \Rightarrow |\text{Adj}(A)| = |A|^2 = (21)^2 = 441$$

## Question33

If the system of linear equations :

$$x + y + z = 6$$

$$x + 2y + 5z = 10$$

$$2x + 3y + \lambda z = \mu$$

has infinitely many solutions, then the value of  $\lambda + \mu$  equals:

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**Options:**

A.

12

B.

16

C.

22

D.

28

**Answer: C****Solution:**

To find the condition for infinitely many solutions, we first calculate the determinant  $D$  of the coefficients of  $x, y, z$ :

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{vmatrix} = \lambda - 6$$

For infinitely many solutions,  $D = 0$ . So,  $\lambda - 6 = 0 \Rightarrow \lambda = 6$ .

Now, let us find  $D_1$ , the determinant obtained by replacing the first column with the constants:

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 5 \\ \mu & 3 & \lambda \end{vmatrix} = 2\lambda + 3\mu - 60$$

Next, the determinant  $D_2$  is obtained by replacing the second column with the constants:

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 5 \\ 2 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 10)$$

Finally,  $D_3$  is obtained by replacing the third column with the constants:

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 2 & 3 & \mu \end{vmatrix} = \mu - 16$$

For the system to have infinitely many solutions, all the determinants must be zero:

$$D = D_1 = D_2 = D_3 = 0$$

Substitute  $\lambda = 6$ :

$$\text{From } D_3 = 0, \mu - 16 = 0 \Rightarrow \mu = 16.$$

Thus,

$$\lambda + \mu = 6 + 16 = 22$$

**Question 34**

Let  $A$  be a square matrix of order 3 such that  $\det(A) = -2$  and  $\det(3 \operatorname{adj}(-6 \operatorname{adj}(3A))) = 2^{m+n} \cdot 3^{mn}$ ,  $m > n$ . Then  $4m + 2n$  is equal to \_\_\_\_\_.

**JEE Main 2025 (Online) 22nd January Morning Shift**

**Answer: 34**

**Solution:**

$$\text{As } A \operatorname{adj} A = |A|I, \det(\lambda A) = \lambda^n \det A$$

$$\det(3 \operatorname{adj}(-6 \operatorname{adj}(3A))) = 3^3 \det(\operatorname{adj}(-6 \operatorname{adj}(3A)))$$

$$= 3^3 (-6 \operatorname{adj}(3A))^2$$

$$= 3^3 (-6)^6 |3A|^4$$

$$= 3^9 2^6 \cdot 3^{12} \cdot (-2)^4$$

$$= 3^{21} \cdot 2^{10}$$

Now comparing with given condition

$$2^{m+n} 3^{mn} = 2^{10} \cdot 3^{21}$$

$$m + n = 10, mn = 21$$

$$\Rightarrow m = 7, n = 3 (m > n)$$

$$\therefore 4m + 2n = 28 + 6 = 34$$

## Question 35

Let  $A$  be a  $3 \times 3$  matrix such that  $X^T A X = O$  for all nonzero  $3 \times 1$

matrices  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ . If  $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$ ,  $A \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$ , and

$\det(\operatorname{adj}(2(A + I))) = 2^\alpha 3^\beta 5^\gamma$ ,  $\alpha, \beta, \gamma \in N$ , then  $\alpha^2 + \beta^2 + \gamma^2$  is

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Answer: 44

**Solution:**

$$X^T A X = 0$$

$$(xyz) \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$(xyz) \begin{pmatrix} a_1 x + a_2 y + a_3 z \\ b_1 x + b_2 y + b_3 z \\ c_1 x + c_2 y + c_3 z \end{pmatrix} = 0$$

$$x(a_1 x + a_2 y + a_3 z) + y(b_1 x + b_2 y + b_3 z) + z(c_1 x + c_2 y + c_3 z) = 0$$

$$a_1 = 0, b_2 = 0, c_3 = 0$$

$$a_2 + b_1 = 0, a_3 + c_1 = 0, b_3 = c_2 = 0$$

$A$  = skew symm matrix

$$A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}; \quad A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$x + y = 1$$

$$-x + z = 4$$

$$y + z = 5$$

$$\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$$2x + y = 0 \quad x = -1$$

$$-x + z = 4 \quad y = 2$$

$$-y - 2z = -8 \quad z = 3$$

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$$

$$2(A + I) = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -2 & -6 & 2 \end{pmatrix}$$

$$2(A + I) = 120 \Rightarrow \det | \text{adj}(2(A + I)) |$$

$$= 120^2 = 2^6 \cdot 3^2 \cdot 5^2$$

$$\alpha = 6, \beta = 2, \gamma = 2$$

---

## Question 36

Let  $M$  denote the set of all real matrices of order  $3 \times 3$  and let  $S = \{-3, -2, -1, 1, 2\}$ . Let

$$S_1 = \{A = [a_{ij}] \in M : A = A^T \text{ and } a_{ij} \in S, \forall i, j\},$$

$$S_2 = \{A = [a_{ij}] \in M : A = -A^T \text{ and } a_{ij} \in S, \forall i, j\},$$

$$S_3 = \{A = [a_{ij}] \in M : a_{11} + a_{22} + a_{33} = 0 \text{ and } a_{ij} \in S, \forall i, j\}.$$

If  $n(S_1 \cup S_2 \cup S_3) = 125\alpha$ , then  $\alpha$  equals \_\_\_\_\_.

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**Answer: 1613**

**Solution:**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{No. of elements in } S_1 : A = A^T \Rightarrow 5^3 \times 5^3$$

$$\text{No. of elements in } S_2 : A = -A^T \Rightarrow 0$$

since no zero in  $S_2$

$$\text{No. of elements in } S_3 \Rightarrow$$

$$\left. \begin{array}{l} a_{11} + a_{22} + a_{33} = 0 \Rightarrow (1, 2, -3) \Rightarrow 31 \\ \text{or} \\ (1, 1, -2) \Rightarrow 3 \\ \text{or} \\ (-1, -1, 2) \Rightarrow 3 \end{array} \right\} \Rightarrow 12 \times 5^6$$

$$n(S_1 \cap S_3) = 12 \times 5^3$$

$$n(S_1 \cup S_2 \cup S_3) = 5^6(1 + 12) - 12 \times 5^3$$

$$\Rightarrow 5^3 \times [13 \times 5^3 - 12] = 125\alpha$$

$$\alpha = 1613$$

---

## Question37

Let  $S = \left\{ m \in \mathbf{Z} : A^{m^2} + A^m = 3I - A^{-6} \right\}$ , where  $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ .

Then  $n(S)$  is equal to \_\_\_\_\_.

**JEE Main 2025 (Online) 29th January Morning Shift**

**Answer: 2**

**Solution:**

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}, A^3 = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}, A^4 = \begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$$

and so on

$$A^6 = \begin{bmatrix} 7 & -6 \\ 6 & -5 \end{bmatrix}$$

$$A^m = \begin{bmatrix} m+1 & -m \\ m & -m+1 \end{bmatrix},$$

$$A^{m^2} = \begin{bmatrix} m^2+1 & -m^2 \\ m^2 & -(m^2-1) \end{bmatrix}$$

$$A^{m^2} + A^m = 3I - A^{-6}$$

$$\begin{bmatrix} m^2+1 & -m^2 \\ m^2 & -(m^2-1) \end{bmatrix} + \begin{bmatrix} m+1 & -m \\ m & -m+1 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -5 & 6 \\ -6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -6 \\ 6 & -4 \end{bmatrix}$$

$$= m^2 + 1 + m + 1 = 8$$

$$= m^2 + m - 6 = 0 \Rightarrow m = -3, 2$$

$$n(s) = 2$$

---

## Question38

Let  $I$  be the identity matrix of order  $3 \times 3$  and for the matrix

$$A = \begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}, |A| = -1. \text{ Let } B \text{ be the inverse of the matrix}$$

$\text{adj}(A \text{adj}(A^2))$ . Then  $|(\lambda B + I)|$  is equal to \_\_\_\_\_

**JEE Main 2025 (Online) 3rd April Evening Shift**

**Answer: 38**

**Solution:**

$$\begin{aligned} B &= [\text{adj}(A \text{adj}(A^2))]^{-1} \\ \text{Adj}(A^2) &= (\text{adj } A)^2 \Rightarrow A \text{adj}(A^2) = A \text{adj}(A) \cdot (\text{adj } A) \\ &= A(|A|A^{-1})^2 = |A|^2(A^{-1}) = A^{-1} \\ \Rightarrow B &= (\text{adj}(A^{-1}))^{-1} = (|(A^{-1})|A)^{-1} = \frac{A^{-1}}{-1} = -A^{-1} \\ \Rightarrow B &= -A^{-1} \end{aligned}$$

$$\begin{aligned} |A| = -1 &= \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1 \Rightarrow \lambda = 3 \\ |3B + I| &= |I - 3A^{-1}| = \frac{|A| |I - 3A^{-1}|}{|A|} = \frac{|A - 3I|}{|A|} \\ &= \frac{|A - 3I|}{-1} = \frac{\begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix}}{-1} = 38 \\ \Rightarrow |3B + I| &= 38 \end{aligned}$$

## Question 39

Let  $A = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$ . If for some  $\theta \in (0, \pi)$ ,  $A^2 = A^T$ , then the sum of the diagonal elements of the matrix  $(A + I)^3 + (A - I)^3 - 6A$  is equal to \_\_\_\_\_.

## JEE Main 2025 (Online) 4th April Morning Shift

**Answer: 6**

**Solution:**

Note that  $A$  is orthogonal:

$$AA^T = A^T A = I \text{ and } A^T = A^{-1}$$

Given  $A^2 = A^T$ , then:

$$A^3 = I$$

$$\text{Tr}(A + I)^3 + (A - I)^3 - 6A = \text{Tr}(2A^3 + 6A - 6A)$$

$$= \text{Tr}(2A^3) = \text{Tr}(2I)$$

$$(\text{Using } (A + I)^3 + (A - I)^3 = 2A^3 + 6A \text{ and } 2A^3 = 2I) =$$

$$6$$

---

## Question 40

The number of singular matrices of order 2, whose elements are from the set  $\{2, 3, 6, 9\}$ , is \_\_\_\_\_.

## JEE Main 2025 (Online) 7th April Morning Shift

**Answer: 36**

**Solution:**

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

for  $A$  to be singular matrix

$$ad = bc$$

Case 1: exactly 1 number is used  $\Rightarrow {}^4C_1$  ways

Case 2 : exactly 2 numbers is used  $\Rightarrow {}^4C_2$  ways

Case 3 : exactly 3 numbers used  $\Rightarrow$  none will be singular.

Case 4: exactly 4 numbers is used

$$\Rightarrow ab = cd \Rightarrow 2 \times 9 = 3 \times 6$$

$$\Rightarrow {}^4C_1 \times 2! = 8 \text{ matrix .}$$

$\therefore$  Total ways  $\Rightarrow 4 + 6 \times 4 + 8 = 36$  matrices.

---

## Question 41

For a  $3 \times 3$  matrix  $M$ , let  $\text{trace}(M)$  denote the sum of all the diagonal elements of  $M$ . Let  $A$  be a  $3 \times 3$  matrix such that  $|A| = \frac{1}{2}$  and  $\text{trace}(A) = 3$ . If  $B = \text{adj}(\text{adj}(2A))$ , then the value of  $|B| + \text{trace}(B)$  equals :

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Options:

A. 56

B. 132

C. 174

D. 280

**Answer: D**

**Solution:**

$$B = \text{adj}(\text{adj}(2A)) = \det(2A) \cdot (2A)$$

Since  $A$  is a  $3 \times 3$  matrix with

$$\det(A) = \frac{1}{2},$$

the determinant of  $2A$  is computed as

$$\det(2A) = 2^3 \det(A) = 8 \cdot \frac{1}{2} = 4.$$

Thus,

$$B = 4 \cdot (2A) = 8A.$$

Now, compute the determinant and the trace of  $B$ :

Determinant of  $B$ :

$$\det(B) = \det(8A) = 8^3 \det(A) = 512 \cdot \frac{1}{2} = 256.$$

Trace of  $B$ :

$$\text{trace}(B) = \text{trace}(8A) = 8 \cdot \text{trace}(A) = 8 \cdot 3 = 24.$$

Finally, adding these results:

$$\det(B) + \text{trace}(B) = 256 + 24 = 280.$$

---

## Question 42

If the system of linear equations:

$$x + y + 2z = 6$$

$$2x + 3y + az = a + 1$$

$$-x - 3y + bz = 2b$$

where  $a, b \in \mathbf{R}$ , has infinitely many solutions, then  $7a + 3b$  is equal to :

**JEE Main 2025 (Online) 22nd January Evening Shift**

**Options:**

A. 12

B. 9

C. 22

D. 16

**Answer: D**

**Solution:**

We begin with the system:

$$x + y + 2z = 6,$$

$$2x + 3y + az = a + 1,$$

$$-x - 3y + bz = 2b.$$

**Step 1.** Solve the first equation for  $x$ :

$$x = 6 - y - 2z.$$

**Step 2.** Substitute  $x = 6 - y - 2z$  into the second equation:

$$2(6 - y - 2z) + 3y + az = a + 1.$$

Expanding and simplifying:

$$12 - 2y - 4z + 3y + az = a + 1 \implies y + (a - 4)z = a - 11.$$

Call this **Equation (I)**.

**Step 3.** Substitute  $x = 6 - y - 2z$  into the third equation:

$$-(6 - y - 2z) - 3y + bz = 2b.$$

Expanding and simplifying:

$$-6 + y + 2z - 3y + bz = 2b \implies -2y + (b + 2)z = 2b + 6.$$

Call this **Equation (II)**.

**Step 4.** For the system to have infinitely many solutions, the two equations in  $y$  and  $z$  must be dependent—that is, one must be a constant multiple of the other. Assume there exists a constant  $k$  such that

$$-2 = k \cdot 1 \implies k = -2.$$

Apply this to the coefficient of  $z$  and the constant term.

For the  $z$ -coefficient in Equations (I) and (II):

$$b + 2 = k(a - 4) = -2(a - 4) = -2a + 8.$$

Thus,

$$b = -2a + 6.$$

For the constant term:

$$2b + 6 = k(a - 11) = -2(a - 11) = -2a + 22.$$

Substitute  $b = -2a + 6$  into this equation:

$$2(-2a + 6) + 6 = -2a + 22 \implies -4a + 12 + 6 = -2a + 22.$$

Simplify:

$$-4a + 18 = -2a + 22.$$

Solve for  $a$ :

$$-4a + 18 + 4a = -2a + 22 + 4a \implies 18 = 2a + 22,$$

$$2a = 18 - 22 = -4 \implies a = -2.$$

Substitute  $a = -2$  into  $b = -2a + 6$ :

$$b = -2(-2) + 6 = 4 + 6 = 10.$$

**Step 5.** We now compute

$$7a + 3b = 7(-2) + 3(10) = -14 + 30 = 16.$$

Thus, the value of  $7a + 3b$  is

**16.**

---

## Question43

If  $A, B$ , and  $(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))$  are non-singular matrices of same order, then the inverse of  $A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B$ , is equal to

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**Options:**

A.  $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

B.  $\text{adj}(B^{-1}) + \text{adj}(A^{-1})$

C.  $AB^{-1} + A^{-1}B$

D.  $\frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$

**Answer: D**

**Solution:**

$$\begin{aligned} & \left[ A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1} \cdot B \right]^{-1} \\ & B^{-1} \cdot (\text{adj}(A^{-1}) + \text{adj}(B^{-1})) \cdot A^{-1} \\ & B^{-1} \text{adj}(A^{-1})A^{-1} + B^{-1}(\text{adj}(B^{-1})) \cdot A^{-1} \\ & B^{-1} |A^{-1}|I + |B^{-1}|IA^{-1} \\ & \frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|} \\ & \Rightarrow \frac{\text{adj}B}{|B||A|} + \frac{\text{adj}A}{|A||B|} \\ & = \frac{1}{|A||B|}(\text{adj}B + \text{adj}A) \end{aligned}$$

---

## Question44

If the system of equations

$$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$$

$$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$$

$$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$$

has infinitely many solutions, then  $\lambda^2 + \lambda$  is equal to

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**Options:**

A. 20

B. 10

C. 6

D. 12

**Answer: D**

**Solution:**

$$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$$

$$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$$

$$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$$

For infinitely many solutions

$$D = \begin{vmatrix} \lambda - 1 & \lambda - 4 & \lambda \\ \lambda & \lambda - 1 & \lambda - 4 \\ \lambda + 1 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$$

$$(\lambda - 3)(2\lambda + 1) = 0$$

$$D_x = \begin{vmatrix} 5 & \lambda - 4 & \lambda \\ 7 & \lambda - 1 & \lambda - 4 \\ 9 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$$

$$2(3 - \lambda)(23 - 2\lambda) = 0$$

$$\lambda = 3$$

$$\therefore \lambda^2 + \lambda = 9 + 3 = 12$$

---

**Question 45**

Let  $A = [a_{ij}]$  be a  $3 \times 3$  matrix such that  $A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ,  $A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

and  $A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ , then  $a_{23}$  equals :

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**Options:**

A. 2

B.  $-1$

C. 1

D. 0

**Answer: B**

**Solution:**

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow a_{22} = 0; a_{12} = 0$$

$$a_{32} = 1$$

$$A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} 4a_{11} + a_{12} + 3a_{13} &= 0 \\ 4a_{21} + a_{22} + 3a_{23} &= 1 \Rightarrow 4a_{21} + 3a_{23} = 1 \\ 4a_{31} + a_{32} + 3a_{33} &= 0 \end{aligned}$$

$$A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} 2a_{11} + a_{12} + 2a_{13} &= 1 \\ 2a_{21} + a_{22} + 2a_{23} &= 0 \Rightarrow a_{21} + a_{23} = 0 \\ 2a_{31} + a_{32} + 2a_{33} &= 0 \end{aligned}$$

$$-4a_{23} + 3a_{23} = 1 \Rightarrow a_{23} = -1$$

**Question 46**

**The system of equations**

$$\begin{aligned}x + y + z &= 6, \\x + 2y + 5z &= 9, \\x + 5y + \lambda z &= \mu,\end{aligned}$$

has no solution if

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**Options:**

A.  $\lambda = 17, \mu = 18$

B.  $\lambda = 17, \mu \neq 18$

C.  $\lambda = 15, \mu \neq 17$

D.  $\lambda \neq 17, \mu \neq 18$

**Answer: B**

**Solution:**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$$

$$\lambda = 17$$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$$

$$\mu \neq 18$$

## Question 47

If the system of equations

$$2x - y + z = 4$$

$$5x + \lambda y + 3z = 12$$

$$100x - 47y + \mu z = 212$$

has infinitely many solutions, then  $\mu - 2\lambda$  is equal to

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Options:

A. 56

B. 59

C. 57

D. 55

Answer: C

Solution:

$$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$$

$$2(\lambda\mu + 141) + (5\mu - 300) - 235 - 100\lambda = 0 \dots (1)$$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$$

$$6\lambda = -12 \Rightarrow \lambda = -2$$

Put  $\lambda = -2$  in (1)

$$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$$

$$\mu = 53$$

$$\therefore 57$$

---

## Question 48

If the system of equations

$$x + 2y - 3z = 2$$

$$2x + \lambda y + 5z = 5$$

$$14x + 3y + \mu z = 33$$

has infinitely many solutions, then  $\lambda + \mu$  is equal to :

**Options:**

- A. 13
- B. 10
- C. 12
- D. 11

**Answer: C**

**Solution:**

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0, \lambda\mu + 42\lambda - 4\mu + 107 = 0$$

$$D_1 = 2\lambda\mu + 99\lambda - 10\mu + 255$$

$$D_2 = 13 - \mu$$

$$D_3 = 5\lambda + 5$$

$$D_2 = 0 \Rightarrow \mu = 13 \text{ \& } D_3 = 0 \Rightarrow \lambda = -1$$

check & verify for these values  $D \text{ \& } D_2 = 0$

## Question49

**For some  $a, b$ , let**

$$f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}, x \neq 0, \lim_{x \rightarrow 0} f(x) = \lambda + \mu a + \nu b.$$

**Then  $(\lambda + \mu + \nu)^2$  is equal to :**

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**Options:**

- A. 25
- B. 16
- C. 9
- D. 36

**Answer: B**

**Solution:**

$$\begin{aligned}\lim_{x \rightarrow 0} f(x) &= \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix} \\ &= (a+1)(2(b+1)-b) + 1(ab - a(b+1)) + ba \\ &= (a+1)(b+2) - a + ab \\ &= b+a+2 = \lambda + \mu a + \nu b \\ \lambda = 2, \mu = 1, \nu = 1 &\Rightarrow (\lambda + \mu + \nu)^2 = 16\end{aligned}$$

---

## Question 50

Let  $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$  and  $P = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ ,  $\theta > 0$ . If

$B = PAP^T$ ,  $C = P^T B^{10} P$  and the sum of the diagonal elements of  $C$  is  $\frac{m}{n}$ , where  $\gcd(m, n) = 1$ , then  $m + n$  is :

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**Options:**

A.

127

B.

2049

C.

258

D.

65

**Answer: D**

**Solution:**

$$P = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore P^T P = I$$

$$B = P A P^T$$

Pre multiply by  $P^T$  ( Given)

$$P^T B = P^T P A P^T = A P^T$$

Now post multiply by  $P$

$$P^T B P = A P^T P = A$$

$$\text{So } A^2 = \underbrace{P^T B P}_{I} P^T B P$$

$$A^2 = P^T B^2 P$$

$$\text{Similarly } A^{10} = P^T B^{10} P = C$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix} \text{ (Given)}$$

$$\Rightarrow A^2 = \begin{bmatrix} \frac{1}{2} & \sqrt{2} - 2 \\ 0 & 1 \end{bmatrix}$$

Similarly check  $A^3$  and so on since  $C = A^{10}$

$$\Rightarrow \text{Sum of diagonal elements of } C \text{ is } \left(\frac{1}{\sqrt{2}}\right)^{10} + 1$$

$$= \frac{1}{32} + 1 = \frac{33}{32} = \frac{m}{n}$$

$$\text{gcd}(m, n) = 1 \text{ (Given)}$$

$$\Rightarrow m + n = 65$$

## Question 51

Let  $M$  and  $m$  respectively be the maximum and the minimum values of

$$f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 4x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 4x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 4x \end{vmatrix}, x \in R$$

Then  $M^4 - m^4$  is equal to:

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Options:

A.

1280

B.

1040

C.

1215

D.

1295

**Answer: A**

**Solution:**

$$\begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 4x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 4x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 4x \end{vmatrix}, x \in R$$

$$R_2 \rightarrow R_2 - R_1 \text{ \& } R_3 \rightarrow R_3 - R_1$$

$$f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 4x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expand about  $R_1$ , we get

$$f(x) = 2 + 4 \sin 4x$$

$$\therefore M = \text{max value of } f(x) = 6$$

$$m = \text{min value of } f(x) = -2$$

$$\therefore M^4 - m^4 = 1280$$

---

## Question 52

Let  $A = [a_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$ . If  $A_{ij}$  is the cofactor of  $a_{ij}$ ,  
 $C_{ij} = \sum_{k=1}^2 a_{ik} A_{jk}$ ,  $1 \leq i, j \leq 2$ , and  $C = [C_{ij}]$ , then  $8|C|$  is equal to :

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**Options:**

A.

288

B.

262

C.

222

D.

242

**Answer: D**

**Solution:**

To solve the problem, we need to determine the determinant of matrix  $A$ :

$$A = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$$

The determinant of  $A$ , denoted as  $|A|$ , is calculated as:

$$|A| = (\log_5 128)(\log_4 25) - (\log_4 5)(\log_5 8)$$

Evaluating each component:

$\log_5 128$  can be simplified using change of base formula:

$$\log_5 128 = \frac{\log_{10} 128}{\log_{10} 5}$$

$\log_4 5$  using change of base:

$$\log_4 5 = \frac{\log_{10} 5}{\log_{10} 4}$$

$\log_5 8$ :

$$\log_5 8 = \frac{\log_{10} 8}{\log_{10} 5}$$

$$\log_4 25:$$

$$\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4}$$

Now, substitute these values into  $|A|$ :

$$|A| = \left( \frac{\log_{10} 128}{\log_{10} 5} \right) \left( \frac{\log_{10} 25}{\log_{10} 4} \right) - \left( \frac{\log_{10} 5}{\log_{10} 4} \right) \left( \frac{\log_{10} 8}{\log_{10} 5} \right)$$

Next, we find the cofactors of matrix  $A$ :

$$A_{11} = \log_4 25$$

$$A_{12} = -\log_5 8$$

$$A_{21} = -\log_4 5$$

$$A_{22} = \log_5 128$$

Then, calculate matrix  $C$  whose elements are given by  $C_{ij} = \sum_{k=1}^2 a_{ik} A_{jk}$ :

$$C_{11} = a_{11} A_{11} + a_{12} A_{12} = |A| = \frac{11}{2}$$

$$C_{12} = a_{11} A_{21} + a_{12} A_{22} = 0$$

$$C_{21} = a_{21} A_{11} + a_{22} A_{12} = 0$$

$$C_{22} = a_{21} A_{21} + a_{22} A_{22} = |A| = \frac{11}{2}$$

Thus, matrix  $C$  is:

$$C = \begin{bmatrix} \frac{11}{2} & 0 \\ 0 & \frac{11}{2} \end{bmatrix}$$

To find  $|C|$ , we compute:

$$|C| = \left( \frac{11}{2} \right) \left( \frac{11}{2} \right) = \frac{121}{4}$$

Finally, calculate  $8|C|$ :

$$8|C| = 8 \times \frac{121}{4} = 242$$

## Question 53

Let  $A = [a_{ij}]$  be a matrix of order  $3 \times 3$ , with  $a_{ij} = (\sqrt{2})^{i+j}$ . If the sum of all the elements in the third row of  $A^2$  is  $\alpha + \beta\sqrt{2}$ ,  $\alpha, \beta \in \mathbf{Z}$ , then  $\alpha + \beta$  is equal to:

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Options:

A.

210

B.

280

C.

224

D.

168

**Answer: C**

**Solution:**

$$A = \begin{bmatrix} (\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4 \\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5 \\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$$

$$A^2 = 2^2 \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}$$

$$= 4 \begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{bmatrix}$$

$$\text{Sum of elements of 3}^{\text{rd}} \text{ row} = 4(14 + 14\sqrt{2} + 28)$$

$$= 4(42 + 14\sqrt{2})$$

$$= 168 + 56\sqrt{2}$$

$$\alpha + \beta\sqrt{2}$$

$$\therefore \alpha + \beta = 168 + 56 = 224$$

---

### Question 54

Let  $\alpha, \beta$  ( $\alpha \neq \beta$ ) be the values of  $m$ , for which the equations  $x + y + z = 1$ ,  $x + 2y + 4z = m$  and  $x + 4y + 10z = m^2$  have infinitely many solutions. Then the value of  $\sum_{n=1}^{10} (n^\alpha + n^\beta)$  is equal to :

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**Options:**

A.

3410

B.

560

C.

3080

D.

440

**Answer: D**

**Solution:**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20 - 16) - 1(10 - 4) + 1(4 - 2) \\ = 4 - 6 + 2 = 0$$

For infinite solutions

$$\Delta_x = \Delta_y = \Delta_z = 0$$

$$m^2 - 3m + 2 = 0$$

$$m = 1, 2$$

$$\alpha = 1, \beta = 2$$

$$\therefore \sum_{n=1}^{10} (n^\alpha + n^\beta) = \sum_{n=1}^{10} n^1 + \sum_{n=1}^{10} n^2 \\ = \frac{10(11)}{2} + \frac{10(11)(21)}{6} \\ = 55 + 385 \\ = 440$$

## Question 55

Let  $A = [a_{ij}]$  be a  $2 \times 2$  matrix such that  $a_{ij} \in \{0, 1\}$  for all  $i$  and  $j$ . Let the random variable  $X$  denote the possible values of the determinant of the matrix  $A$ . Then, the variance of  $X$  is:

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**Options:**

A.

$$\frac{5}{8}$$

B.

$$\frac{1}{4}$$

C.

$$\frac{3}{4}$$

D.

$$\frac{3}{8}$$

**Answer: D**

**Solution:**

$$\begin{aligned} |A| &= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \\ &= a_{11}a_{22} - a_{21}a_{12} \\ &= \{-1, 0, 1\} \end{aligned}$$

| x  | $P_i$           | $P_i X_i$          | $P_i X_i^2$                    |
|----|-----------------|--------------------|--------------------------------|
| -1 | $\frac{3}{16}$  | $-\frac{3}{16}$    | $\frac{3}{16}$                 |
| 0  | $\frac{10}{16}$ | 0                  | 0                              |
| 1  | $\frac{3}{16}$  | $\frac{3}{16}$     | $\frac{3}{16}$                 |
|    |                 | $\sum P_i X_i = 0$ | $\sum P_i X_i^2 = \frac{3}{8}$ |

$$\begin{aligned} \therefore \text{var}(x) &= \sum P_i X_i^2 - \left( \sum P_i X_i \right)^2 \\ &= \frac{3}{8} - 0 = \frac{3}{8} \end{aligned}$$

---

## Question 56

If the system of linear equations

$$3x + y + \beta z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has infinitely many solutions, then the value of  $22\beta - 9\alpha$  is :

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**Options:**

A. 31

B. 37

C. 43

D. 49

**Answer: A**

**Solution:**

$$3x + y + \beta z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has infinite solution

$$\Rightarrow \Delta = 0, \Delta_1 = \Delta_2 = \Delta_3$$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Delta_2 = 0 \Rightarrow \begin{vmatrix} 3 & 3 & \beta \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 3(-3+4) - 3(2+1) + \beta(8+3) = 0$$

$$\Rightarrow 3 - 9 + 11\beta = 0$$

$$\Rightarrow \beta = \frac{6}{11}$$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 3(4\alpha+6) - 1(8+3) + 3(4-\alpha) = 0$$

$$12\alpha + 18 - 11 + 12 - 3\alpha = 0$$

$$9\alpha = -19$$

$$\alpha = \frac{-19}{9}$$

$$\therefore 22\beta - 9\alpha = 31$$

## Question 57

Let  $a \in R$  and  $A$  be a matrix of order  $3 \times 3$  such that  $\det(A) = -4$  and

$$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}, \text{ where } I \text{ is the identity matrix of order } 3 \times 3. \text{ If}$$

$\det((a+1)\text{adj}((a-1)A))$  is  $2^m 3^n$ ,  $m, n \in \{0, 1, 2, \dots, 20\}$ , then  $m + n$  is equal to :

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**Options:**

A. 14

B. 17

C. 15

D. 16

**Answer: D**

**Solution:**

To solve for the value of  $m + n$ , we first establish the matrix  $A$  and determine the value of  $a$ . The condition given is:

$$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$$

Since  $I$  is the identity matrix of size  $3 \times 3$ , we have:

$$A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$$

We know  $\det(A) = -4$ . Calculating the determinant:

$$\det(A) = 0(0 \times 1 - 0 \times 1) - a(2 \times 1 - 0 \times a) + 1(2 \times 1 - 0 \times 1) = -2a + 2$$

Setting  $-2a + 2 = -4$ , we solve for  $a$ :

$$-2a + 2 = -4$$

$$-2a = -6$$

$$a = 3$$

With  $a = 3$ , the problem is to find  $\det((a+1) \operatorname{adj}((a-1)A))$ . With  $a = 3$ :

Calculate  $(a+1) \operatorname{adj}((a-1)A)$ :

$$= 4 \operatorname{adj}(3A)$$

Calculate the determinant:

$$\det(4 \operatorname{adj}(3A)) = 4^3 \times \det(\operatorname{adj}(3A))$$

Using the property  $\det(\operatorname{adj}(A)) = \det(A)^{n-1}$  for a  $n \times n$  matrix:

$$\operatorname{adj}(3A) = (3A)^{2-1} = (3 \times \det(A))^2$$

$$= (3^3 \times (-4))^2$$

Therefore, calculate  $\det(3A)^2$ :

$$\det((3A)^2) = (3^3 \times (-4))^2 = 3^6 \times 4^2$$

Substitute back:

$$4^3 \times \det((3A)^2) = 4^3 \times 3^6 \times 4^2 = 2^6 \times 3^6 \times 2^4$$

Simplifying gives:

$$= 2^{10} \times 3^6$$

Finally, we have powers  $m = 10$  and  $n = 6$ . Therefore:

$$m + n = 10 + 6 = 16$$

---

## Question 58

Let  $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$ ,  $\alpha > 0$ , such that  $\det(A) = 0$  and  $\alpha + \beta = 1$ . If  $I$  denotes  $2 \times 2$  identity matrix, then the matrix  $(I + A)^8$  is :

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Options:

A.  $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

B.  $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

C.  $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$

D.  $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$

**Answer: B**

**Solution:**

Let  $|A| = 0 \Rightarrow \alpha\beta - (-6) = 0 \Rightarrow \alpha\beta = -6$  and  $\alpha + \beta = 1 \Rightarrow \alpha, \beta$  are roots of the equation

$$x^2 - x - 6 = 0 \Rightarrow x = 3, -2. \text{ Since } \alpha > 0$$

$$\Rightarrow \alpha = 3, \beta = -2$$

$$\Rightarrow A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow I + A = \begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$$

$$(I + A)^2 = \begin{bmatrix} 10 & -3 \\ 18 & -5 \end{bmatrix} \Rightarrow (I + A)^4 = \begin{bmatrix} 46 & -15 \\ 90 & -29 \end{bmatrix}$$

$$\Rightarrow (I + A)^8 = \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$$

---

## Question 59

Let  $A$  be a  $3 \times 3$  real matrix such that  $A^2(A - 2I) - 4(A - I) = O$ , where  $I$  and  $O$  are the identity and null matrices, respectively. If  $A^5 = \alpha A^2 + \beta A + \gamma I$ , where  $\alpha, \beta$ , and  $\gamma$  are real constants, then  $\alpha + \beta + \gamma$  is equal to :

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**Options:**

A. 76

B. 12

C. 4

D. 20

**Answer: B**

**Solution:**

$$A^2(A - 2I) - 4(A - I) = 0$$

$$A^3 - 2A^2 - 4A + 4I = 0$$

Multiply by  $A$

$$A^4 = 2A^3 + 4A^2 - 4A$$

$$A^4 = 2(2A^2 + 4A - 4I) + 4A^2 - 4A$$

$$A^4 = 8A^2 + 4A - 8I$$

Multiply again by  $A$

$$\Rightarrow A^5 = 8A^3 + 4A^2 - 8A$$

$$\Rightarrow A^5 = 8(2A^2 + 4A - 4I) + 4A^2 - 8A$$

$$\Rightarrow A^5 = 20A^2 + 24A - 32I$$

Comparing with  $A^5 = \alpha A^2 + \beta A + \gamma I$

$$\alpha = 20, \beta = 24, \gamma = -32$$

$$\therefore \alpha + \beta + \gamma = 20 + 24 - 32$$

$$= 44 - 32$$

$$= 12$$

---

**Question 60**

**If the system of equations**

$$2x + \lambda y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \mu z = 9$$

has infinitely many solutions, then  $(\lambda^2 + \mu^2)$  is equal to :

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**Options:**

A. 30

B. 26

C. 22

D. 18

**Answer: B**

**Solution:**

$$2x + \lambda y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \mu z = 9$$

For infinite solutions  $\Rightarrow \Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$$\Rightarrow -4\lambda - 3\lambda\mu + 4\mu + 31 = 0$$

$$\Delta_1 = \begin{vmatrix} 5 & \lambda & 3 \\ 7 & 2 & -1 \\ 9 & 5 & \mu \end{vmatrix} = 0 \Rightarrow -9\lambda - 7\lambda\mu + 10\mu + 76 = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 & 5 \\ 3 & -1 & 7 \\ 4 & \mu & 9 \end{vmatrix} = 0 \Rightarrow \mu + 5 = 0 \Rightarrow \mu = -5$$

$$\Delta_3 = \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0 \Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$$

$\therefore$  For infinite solution  $\mu = -5$  and  $\lambda = -1$

$$\text{Now } \mu^2 + \lambda^2 = 25 + 1$$

$$= 26$$

## Question61

Let  $A$  be a matrix of order  $3 \times 3$  and  $|A| = 5$ . If

$|2 \operatorname{adj}(3A \operatorname{adj}(2A))| = 2^\alpha \cdot 3^\beta \cdot 5^\gamma$ ,  $\alpha, \beta, \gamma \in N$ , then  $\alpha + \beta + \gamma$  is equal to

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**Options:**

A. 26

B. 27

C. 25

D. 28

**Answer: B**

**Solution:**

To find the expression  $|2 \operatorname{adj}(3A \operatorname{adj}(2A))|$ , we break it down as follows:

Recognize that:

$$|2 \operatorname{adj}(3A \operatorname{adj}(2A))| = 2^3 |3A \operatorname{adj}(2A)|^2$$

Apply properties of determinants:

$$= 2^3 (3^3)^2 |A|^2 |\operatorname{adj}(2A)|^2$$

Further simplify using  $|\operatorname{adj}(B)| = |B|^{n-1}$  for a  $3 \times 3$  matrix:

$$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (|2A|^2)^2$$

Simplify  $|2A|$ :

$$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot |A|^4$$

Continue to simplify:

$$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot 5^4$$

Expand and combine powers:

$$= 2^{15} \cdot 3^6 \cdot 5^6$$

Therefore,  $\alpha = 15$ ,  $\beta = 6$ , and  $\gamma = 6$ . So,  $\alpha + \beta + \gamma = 15 + 6 + 6 = 27$ .

---

## Question62

Let the matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  satisfy  $A^n = A^{n-2} + A^2 - I$  for  $n \geq 3$ .

Then the sum of all the elements of  $A^{50}$  is :

**JEE Main 2025 (Online) 4th April Evening Shift**

**Options:**

A. 44

B. 39

C. 52

D. 53

**Answer: D**

**Solution:**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^3 = A + A^2 - I$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^4 = A^2 + A^2 - I = 2A^2 - I$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \text{ and } A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

$$\text{Sum of elements} = 53$$

## Question 63

Let the system of equations :

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$12x + 3y - (4 + \lambda)z = 16 - \mu$$

have infinitely many solutions. Then the radius of the circle centred at  $(\lambda, \mu)$  and touching the line  $4x = 3y$  is :

**JEE Main 2025 (Online) 7th April Morning Shift**

**Options:**

A.  $\frac{7}{5}$

B.  $\frac{21}{5}$

C. 7

D.  $\frac{17}{5}$

**Answer: A**

**Solution:**

$$\begin{aligned}\Delta &= \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(4 + \lambda) \end{vmatrix} \\ &= 2(-12 - 3\lambda + 6) - 3(-28 - 7\lambda + 24) + 5(21 - 36) \\ &= -12 - 6\lambda + 12 + 21\lambda - 75 \\ &= 15\lambda - 75 \\ &\Rightarrow 15\lambda - 75 = 0 \\ &\Rightarrow \lambda = 5 \\ \Delta_1 &= \begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16 - \mu & 3 & -9 \end{vmatrix} \\ &= 9(-27 + 6) - 3(-72 + 32 - 2\mu) + 5(24 - 48 + 3\mu) \\ &= -189 + 120 + 6\mu - 120 + 15\mu \\ &= 21\mu - 189 = 0 \\ &\Rightarrow \mu = 9\end{aligned}$$

$$\therefore r = \left| \frac{4(5) - 3(9)}{\sqrt{(4)^2 + (3)^2}} \right|$$

$$r = \frac{7}{5}$$


---

## Question 64

Let  $A$  be a  $3 \times 3$  matrix such that  $|\text{adj}(\text{adj}(\text{adj } A))| = 81$ .

If  $S = \left\{ n \in \mathbb{Z} : (|\text{adj}(\text{adj } A)|)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)} \right\}$ , then  $\sum_{n \in S} |A^{(n^2+n)}|$  is equal to :

**JEE Main 2025 (Online) 7th April Morning Shift**

**Options:**

A. 820

B. 866

C. 750

D. 732

**Answer: D**

**Solution:**

$$|\text{adj}(\text{adj}(\text{adj } A))| = 81$$

$$= |A|^{(n-1)^3} = (3)^4 \Rightarrow |A|^8 = 3^4 \Rightarrow |A| = 3^{1/2}$$

$$|\text{adj}(\text{adj } A)|^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)}$$

$$\left[ |A|^{(n-1)^2} \right]^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$$

$$|A|^{2(n-1)^2} = |A|^{3n^2-5n-4}$$

$$\Rightarrow 2(n-1)^2 = 3n^2-5n-4$$

$$n^2-n-6=0$$

$$\Rightarrow n = -2, 3$$

$$\sum_{n \in S} |A^{n^2+n}| = |A^2| + |A^{12}|$$

$$= 3 + 3^6 = 732$$


---

## Question 65

Let the system of equations

$$x + 5y - z = 1$$

$$4x + 3y - 3z = 7$$

$$24x + y + \lambda z = \mu$$

$\lambda, \mu \in \mathbb{R}$ , have infinitely many solutions. Then the number of the solutions of this system,

if  $x, y, z$  are integers and satisfy  $7 \leq x + y + z \leq 77$ , is :

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**Options:**

A.

4

B.

5

C.

3

D.

6

**Answer: C**

**Solution:**

For infinitely many solution

$$\Delta = 0$$

$$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$$

$$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$$

$$\Rightarrow 17\lambda = -289$$

$$\Rightarrow \lambda = -17$$

$$\Delta 1 = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$$

$$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$$

$$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$$

$$\Rightarrow 12\mu = 540$$

$$\mu = 45$$

$$x + 5y - z = 1$$

$$4x + 3y - 3z = 7$$

$$24x + y - 17z = 45$$

$$\text{Let } z = 1$$

$$x + 5y = 1 + \lambda \quad \times 4$$

$$4x + 3y = 7 + 3\lambda$$

$$\frac{4x + 20y = 4 + 4\lambda}{-17y = 3 - \lambda}$$

$$y = \frac{\lambda - 3}{17}, x = 1 + \lambda - \frac{5\lambda - 15}{17}$$

$$= \frac{32 - 12\lambda}{17}$$

$$7 \leq \frac{\lambda - 3}{17} + \frac{32 + 12\lambda}{17} + \lambda \leq 77$$

$$7 \leq \frac{30\lambda + 29}{17} \leq 77$$

$$3 \leq \lambda \leq 42$$

$$\lambda = 3, 20, 37$$

## Question 66

$$\text{Let } A = \begin{bmatrix} 2 & 2 + p & 2 + p + q \\ 4 & 6 + 2p & 8 + 3p + 2q \\ 6 & 12 + 3p & 20 + 6p + 3q \end{bmatrix}.$$

If  $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$ ,  $m, n \in \mathbb{N}$ , then  $m + n$  is equal to

## JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

22

B.

20

C.

24

D.

26

**Answer: C**

**Solution:**

$$|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$$

$$C_3 \rightarrow C_3 - C_2 - C_1 \times \frac{q}{2}$$

$$\text{Then } C_3 \rightarrow C_2 - C_1 \times \left(1 + \frac{p}{2}\right)$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$$

$$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$$

$$|\text{adj}(\text{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$$

$$= (3^3 |A|)^4 = (3^3 \times 2^3)^4 = 2^{12} \times 3^{12}$$

$$\Rightarrow m + n = 24$$

---

## Question 67

Let  $\alpha$  be a solution of  $x^2 + x + 1 = 0$ , and for some  $a$  and  $b$  in

$R, [4 \ a \ b] \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = [0 \ 0 \ 0]$ . If  $\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$ , then  $m + n$  is equal to \_\_\_\_\_

## JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

11

B.

3

C.

8

D.

7

**Answer: A**

**Solution:**

Let  $\alpha$  be a solution of the equation  $x^2 + x + 1 = 0$ . This implies that  $\alpha$  is a cube root of unity, denoted as  $\omega$ , where  $\alpha^2 + \alpha + 1 = 0$ .

We are given matrix equations with parameters  $a$  and  $b$  as follows:

$$[4 \ a \ b] \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = [0 \ 0 \ 0]$$

Solving these equations:

$$\begin{aligned} [4 - a - 2b, 64 - a - 14b, 52 + 2a - 8b] &= [0, 0, 0] \\ \Rightarrow a + 2b &= 4 \\ \Rightarrow a + 14b &= 64 \end{aligned}$$

From these, solve for  $a$  and  $b$ :

$$\begin{aligned} 12b &= 60 \Rightarrow b = 5 \\ a &= -6 \end{aligned}$$

We now substitute  $a$  and  $b$  into the given function:

$$\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$$

We substitute  $\alpha = \omega$  and simplify:

$$\begin{aligned} \frac{4}{\omega} + \frac{m}{1} + \frac{n}{\omega^2} &= 3 \\ \Rightarrow 4\omega^2 + m + n\omega &= 3 \end{aligned}$$

Substitute  $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$  and  $\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ :

$$4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + m + n\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3$$

$$\Rightarrow -2 + m - \frac{n}{2} = 3$$

$$\text{and } \frac{-4\sqrt{3}}{2} + \frac{n\sqrt{3}}{2} = 0$$

From the above, solve for  $n$  and  $m$ :

$$n = 4$$

$$m = 7$$

Thus,  $m + n = 11$ .

## Question68

Consider the matrix  $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

Given below are two statements :

Statement I:  $f(-x)$  is the inverse of the matrix  $f(x)$ .

Statement II:  $f(x)f(y) = f(x+y)$ .

**In the light of the above statements, choose the correct answer from the options given below**

**[27-Jan-2024 Shift 1]**

**Options:**

A.

Statement I is false but Statement II is true

B.

Both Statement I and Statement II are false

C.

Statement I is true but Statement II is false

D.

Both Statement I and Statement II are true

**Answer: D**

**Solution:**

$$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence statement-I is correct

Now, checking statement II

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(x) \cdot f(y) = f(x+y)$$

Hence statement-II is also correct.

---

## Question69

Let A be a  $2 \times 2$  real matrix and I be the identity matrix of order 2. If the roots of the equation  $|A - xI| = 0$  be -1 and 3, then the sum of the diagonal elements of the matrix  $A^2$  is.....

[27-Jan-2024 Shift 2]

**Answer: 10**

**Solution:**

$$|A - xI| = 0$$

Roots are -1 and 3

$$\text{Sum of roots} = \text{tr}(A) = 2$$

$$\text{Product of roots} = |A| = -3$$

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{We have } a + d = 2$$

$$ad - bc = -3$$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

$$\text{We need } a^2 + bc + bc + d^2$$

$$= a^2 + 2bc + d^2$$

$$= (a + d)^2 - 2ad + 2bc$$

$$= 4 - 2(ad - bc)$$

$$= 4 - 2(-3)$$

$$= 4 + 6$$

$$= 10$$

---

**Question 70**

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix} \text{ and } |2A|^3 = 2^{21} \text{ where } \alpha, \beta \in \mathbb{Z}, \text{ Then a value of } \alpha \text{ is}$$

### [29-Jan-2024 Shift 1]

Options:

A.

3

B.

5

C.

17

D.

9

**Answer: B**

**Solution:**

$$|A| = \alpha^2 - \beta^2$$

$$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$$

$$\alpha^2 - \beta^2 = 16$$

$$(\alpha + \beta)(\alpha - \beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$$

---

### Question 71

Let  $A$  be a square matrix such that  $AA^T = I$ . Then  $\frac{1}{2}A[(A + A^T)^2 + (A - A^T)^2]$  is equal to

### [29-Jan-2024 Shift 1]

Options:

A.

$$A^2 + I$$

B.

$$A^3 + I$$

C.

$$A^2 + A^T$$

D.

$$A^3 + A^T$$

**Answer: D**

**Solution:**

$$AA^T = I = A^T A$$

On solving given expression, we get

$$\begin{aligned} \frac{1}{2}A[A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T] \\ = A[A^2 + (A^T)^2] = A^3 + A^T \end{aligned}$$

## Question 72

Let  $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$  and  $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$ . The sum of the prime factors of  $|P^{-1}AP - 2I|$  is equal to

**[29-Jan-2024 Shift 2]**

**Options:**

A.

26

B.

27

C.

66

D.

**Answer: A**

**Solution:**

$$\begin{aligned}
 |P^{-1}AP - 2I| &= |P^{-1}AP - 2P^{-1}P| \\
 &= |P^{-1}(A - 2I)P| \\
 &= |P^{-1}| |A - 2I| |P| \\
 &= |A - 2I| \\
 &= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69
 \end{aligned}$$

So, Prime factor of 69 is 3 & 23

So, sum = 26

## Question 73

Let  $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$  be a non-zero  $3 \times 3$  matrix, where

$$x \sin \theta = y \sin \left( \theta + \frac{2\pi}{3} \right) = z \sin \left( \theta + \frac{4\pi}{3} \right) \neq 0, \theta \in (0, 2\pi).$$

For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

(I) Trace (R) = 0

(II) If trace(adj (adj(R))) = 0, then R has exactly one non-zero entry.

[30-Jan-2024 Shift 2]

**Options:**

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

**Answer: C**

**Solution:**

$$x \sin \theta = y \sin \left( \theta + \frac{2\pi}{3} \right) = z \sin \left( \theta + \frac{4\pi}{3} \right) \neq 0$$

$$\Rightarrow x, y, z \neq 0$$

Also,

$$\sin \theta + \sin \left( \theta + \frac{2\pi}{3} \right) + \sin \left( \theta + \frac{4\pi}{3} \right) = 0 \quad \forall \theta \in \mathbb{R}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$\Rightarrow xy + yz + zx = 0$$

$$(i) \quad \text{Trace}(R) = x + y + z$$

$$\text{If } x + y + z = 0 \text{ and } xy + yz + zx = 0$$

$$\Rightarrow x = y = z = 0$$

Statement (i) is False

$$(ii) \quad \text{Let } P : \text{trace}(\text{Adj}(\text{Adj}(R))) = 0$$

$Q : R$  has exactly one non zero entry

if  $P$  is false then  $P \rightarrow Q$  is always true

Statement (ii) is True

---

**Question 74**

Let  $A$  be a  $3 \times 3$  matrix and  $\det(A) = 2$ . If

$$n = \det(\underbrace{\text{adj}(\text{adj}(\dots \text{adj}(A)))}_{2024 \text{ times}})$$

Then the remainder when  $n$  is divided by 9 is equal to \_\_\_\_\_

**[31-Jan-2024 Shift 2]**

**Options:**

**Answer: 7**

**Solution:**

$$|A| = 2$$

$$\underbrace{\text{adj}(\text{adj}(\dots \text{adj}(A)))}_{2024 \text{ times}} = |A|^{(n-1)2024}$$

$$= |A|^{2024}$$

$$= 2^{2024}$$

$$2^{2024} = (2^2)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$$

$$\Rightarrow 2^{2024} \equiv 4 \pmod{9}$$

$$\Rightarrow 2^{2024} \equiv 9m + 4, m \leftarrow \text{even}$$

$$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} \equiv 16 \pmod{9}$$

$$\equiv 7$$

## Question 75

If  $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ,  $C = ABA^T$  and  $X = A^T C^2 A$ , then  $\det X$  is equal to :

**[1-Feb-2024 Shift 1]**

**Options:**

A.

243

B.

729

C.

27

D.

891

**Answer: B**

**Solution:**

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$$

$$\text{Now } C = ABA^T \Rightarrow \det(C) = (\det(A))^2 \times \det(B)$$

$$|C| = 9$$

$$\text{Now } |X| = |A^T C^2 A|$$

$$= |A^T| |C|^2 |A|$$

$$= |A|^2 |C|^2$$

$$= 9 \times 81$$

$$= 729$$

---

## Question 76

Let  $A = I_2 - 2MM^T$ , where  $M$  is real matrix of order  $2 \times 1$  such that the relation  $M^T M = I_1$  holds. If  $\lambda$  is a real number such that the relation  $AX$

=  $\lambda X$  holds for some non-zero real matrix  $X$  of order  $2 \times 1$ , then the sum of squares of all possible values of  $\lambda$  is equal to :

[1-Feb-2024 Shift 2]

Answer: 2

Solution:

$$A = I_2 - 2MM^T$$

$$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$$

$$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$$

$$= I_2 - 4MM^T + 4MM^T$$

$$= I_2$$

$$AX = \lambda X$$

$$A^2X = \lambda AX$$

$$X = \lambda(\lambda X)$$

$$X = \lambda^2 X$$

$$X(\lambda^2 - 1) = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

Sum of square of all possible values = 2

---

## Question 77

If  $A$  and  $B$  are two non-zero  $n \times n$  matrices such that  $A^2 + B = A^2B$ , then  
[24-Jan-2023 Shift 1]

Options:

A.  $AB = I$

B.  $A^2B = I$

C.  $A^2 = I$  or  $B = I$

D.  $A^2B = BA^2$

**Answer: D**

**Solution:**

$$A^2 + B = A^2B$$

$$(A^2 - I)(B - I) = I$$

$$A^2 + B = A^2B$$

$$A^2(B - I) = B$$

$$A^2 = B(B - I)^{-1}$$

$$A^2 = B(A^2 - I)$$

$$A^2 = BA^2 - B$$

$$A^2 + B = BA^2$$

$$A^2B = BA^2$$

---

## Question 78

The number of square matrices of order 5 with entries from the set  $\{0, 1\}$ , such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is  
[24-Jan-2023 Shift 2]

**Options:**

A. 225

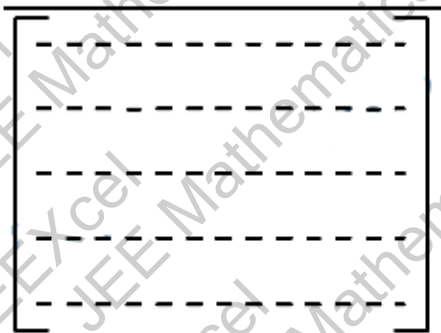
B. 120

C. 150

D. 125

**Answer: B**

**Solution:**



In each row and each column exactly one is to be placed -

$\therefore$  No. of such matrices =  $5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} 5 \text{ ways} \\ 4 \text{ ways} \\ 3 \text{ ways} \\ 2 \text{ ways} \\ 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next row select 1 place for 1's. Automatically some column will get filled with 0's.  $\Rightarrow$

Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways =  $5 \times 4 \times 3 \times 2 \times 1 = 120$

## Question 79

Let A be a  $3 \times 3$  matrix such that  $|\text{adj}(\text{adj}(\text{adj} A))| = 12^4$ . Then

$|A^{-1} \text{adj} A|$  is equal to

[24-Jan-2023 Shift 2]

Options:

A.  $2\sqrt{3}$

B.  $\sqrt{6}$

C. 12

D. 1

Answer: A

Solution:

$$\text{Given } |\text{adj}(\text{adj}(\text{adj} \cdot A))| = 12^4$$

$$\Rightarrow |A|^{(n-1)^3} = 12^4$$

We are asked

$$|A^{-1} \cdot \text{adj} A|$$

$$= |A^{-1}| \cdot |\text{adj} A|$$

$$= \frac{1}{|A|} \cdot |A|^{3-1}$$

$$= |A| = 2\sqrt{3}$$

## Question 80

Let  $x, y, z > 1$  and

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}.$$

Then  $|\text{adj}(\text{adj} A^2)|$  is equal to

[25-Jan-2023 Shift 1]

Options:

A.  $6^4$

B.  $2^8$

C.  $4^8$

D.  $2^4$

**Answer: B**

**Solution:**

$$|A| = \frac{1}{\log x \cdot \log y \cdot \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & 2 \log y & \log z \\ \log x & \log y & 3 \log z \end{vmatrix} = 2$$

$$\Rightarrow |\text{adj}(\text{adj} A^2)| = |A^2|^4 = 2^8$$

## Question81

Let  $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$ , where  $i = \sqrt{-1}$ . If

$M = A^T B A$ , then the inverse of the matrix  $A M^{2023} A^T$  is  
[25-Jan-2023 Shift 2]

Options:

A.  $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C.  $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D.  $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

**Answer: D**

**Solution:**

$$A A^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$$M = A^T B A$$

$$M^2 = M \cdot M = A^T B A A^T B A = A^T B^2 A$$

$$M^3 = M^2 \cdot M = A^T B^2 A A^T B A = A^T B^3 A$$

$$M^{2023} = \dots \dots \dots A^T B^{2023} A$$

$$A M^{2023} A^T = A A^T B^{2023} A A^T = B^{2023}$$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$$\text{Inverse of } (A M^{2023} A^T) \text{ is } \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$$

## Question 82

Let  $A, B, C$  be  $3 \times 3$  matrices such that  $A$  is symmetric and  $B$  and  $C$  are skew-symmetric.

Consider the statements

(S1)  $A^{13} B^{26} - B^{26} A^{13}$  is symmetric

(S2)  $A^{26} C^{13} - C^{13} A^{26}$  is symmetric

Then.

[25-Jan-2023 Shift 2]

Options:

A. Only S2 is true

B. Only S1 is true

C. Both S1 and S2 are false

D. Both S1 and S2 are true

**Answer: A**

**Solution:**

Given,  $A^T = A, B^T = -B, C^T = -C$

Let  $M = A^{13} B^{26} - B^{26} A^{13}$

$$\begin{aligned}
 \text{Then, } M^T &= (A^{13}B^{26} - B^{26}A^{13})^T \\
 &= (A^{13}B^{26})^T - (B^{26}A^{13})^T \\
 &= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26} \\
 &= B^{26}A^{13} - A^{13}B^{26} = -M
 \end{aligned}$$

Hence,  $M$  is skew symmetric

$$\text{Let, } N = A^{26}C^{13} - C^{13}A^{26}$$

$$\begin{aligned}
 \text{then, } N^T &= (A^{26}C^{13})^T - (C^{13}A^{26})^T \\
 &= -(C^T)^{13}(A^T)^{26} + A^{26}C^{13} = N
 \end{aligned}$$

Hence,  $N$  is symmetric.

$\therefore$  Only  $S_2$  is true.

## Question83

Let  $\alpha$  and  $\beta$  be real numbers. Consider a  $3 \times 3$  matrix  $A$  such that  $A^2 = 3A + \alpha I$ . If  $A^4 = 21A + \beta I$ , then  
[29-Jan-2023 Shift 1]

Options:

- A.  $\alpha = 1$
- B.  $\alpha = 4$
- C.  $\beta = 8$
- D.  $\beta = -8$

**Answer: D**

**Solution:**

$$\begin{aligned}
 A^2 &= 3A + \alpha I \\
 A^3 &= 3A^2 + \alpha A \\
 A^3 &= 3(3A + \alpha I) + \alpha A \\
 A^3 &= 9A + \alpha A + 3\alpha I \\
 A^4 &= (9 + \alpha)A^2 + 3\alpha A \\
 &= (9 + \alpha)(3A + \alpha I) + 3\alpha A \\
 &= A(27 + 6\alpha) + \alpha(9 + \alpha) \\
 &\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1 \\
 &\Rightarrow \beta = \alpha(9 + \alpha) = -8
 \end{aligned}$$

## Question84

The set of all values of  $t \in \mathbb{R}$ , for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2 \cos t) & e^{-t}(-2 \sin t - \cos t) \\ e^t & e^{-t}(2 \sin t + \cos t) & e^{-t}(\sin t - 2 \cos t) \\ e^t & e^{-t} \cos t & e^{-t} \sin t \end{bmatrix}$$

is invertible, is

[29-Jan-2023 Shift 2]

Options:

A.  $\left\{ (2k+1) \frac{\pi}{2}, k \in \mathbb{Z} \right\}$

B.  $\left\{ k\pi + \frac{\pi}{4}, k \in \mathbb{Z} \right\}$

C.  $\{k\pi, k \in \mathbb{Z}\}$

D.  $\mathbb{R}$

Answer: D

Solution:

If its invertible, then determinant value  $\neq 0$

So,

$$\begin{vmatrix} e^t & e^{-t}(\sin t - 2 \cos t) & e^{-t}(-2 \sin t - \cos t) \\ e^t & e^{-t}(2 \sin t + \cos t) & e^{-t}(\sin t - 2 \cos t) \\ e^t & e^{-t} \cos t & e^{-t} \sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \sin t - 2 \cos t & -2 \sin t - \cos t \\ 1 & 2 \sin t + \cos t & \sin t - 2 \cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying,  $R_1 \rightarrow R_1 - R_2$  then  $R_2 \rightarrow R_2 - R_3$

We get

$$e^{-t} \begin{vmatrix} 0 & -\sin t - \cos t & -3 \sin t + \cos t \\ 0 & 2 \sin t & -2 \cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$$e^{-t} \times 1(2 \sin t \cos t + 6 \cos^2 t + 6 \sin^2 t - 2 \sin t \cos t) \neq 0$$

$$\Rightarrow e^{-t} \times 6 \neq 0$$

for  $\forall t \in \mathbb{R}$

## Question85

Let A be a symmetric matrix such that  $|A| = 2$  and

$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$ . If the sum of the diagonal elements of A is s, then  $\frac{\beta s}{\alpha^2}$  is equal to \_\_\_\_\_.

[29-Jan-2023 Shift 2]

**Answer: 5**

**Solution:**

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

Now  $ac - b^2 = 2$  and  $2a + b = 1$  and  $2b + c = 2$   
solving all these above equations we get

$$\frac{1-b}{2} \times \left( \frac{2-2b}{1} \right) - b^2 = 2$$

$$\Rightarrow (1-b)^2 - b^2 = 2$$

$$\Rightarrow 1 - 2b = 2$$

$$\Rightarrow b = -\frac{1}{2} \text{ and } a = \frac{3}{4} \text{ and } c = 3$$

$$\text{Hence } \alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$$

$$\text{and } \beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$$

$$\text{also } s = a + c = \frac{15}{4}$$

$$\therefore \frac{\beta s}{\alpha^2} = \frac{3 \times \frac{15}{4}}{4 \times \frac{9}{4}} = 5$$

---

## Question86

Let  $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$ ,  $d = |A| \neq 0$   $|A - d(\text{Adj } A)| = 0$

Then

[30-Jan-2023 Shift 1]

Options:

A.  $(1 + d)^2 = (m + q)^2$

B.  $1 + d^2 = (m + q)^2$

C.  $(1 + d)^2 = m^2 + q^2$

D.  $1 + d^2 = m^2 + q^2$

Answer: A

Solution:

Sol.  $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$ ,  $|A - d(\text{adj } A)| = 0$

$$\Rightarrow |A - d(\text{adj } A)| = \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right|$$

$$= \begin{vmatrix} m - qd & n(1 + d) \\ p(1 + d) & q - md \end{vmatrix} = 0$$

$$\Rightarrow (m - qd)(q - md) - np(1 + d)^2 = 0$$

$$\Rightarrow mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 = 0$$

$$\Rightarrow (mq - np) + d^2(mq - np) - d(m^2 + q^2 + 2np) = 0$$

$$\Rightarrow d + d^3 - d((m + q)^2 - 2d) = 0$$

$$\Rightarrow 1 + d^2 = (m + q)^2 - 2d$$

$$\Rightarrow (1 + d)^2 = (m + q)^2$$

## Question 87

If  $P$  is a  $3 \times 3$  real matrix such that  $P^T = aP + (a - 1)I$ , where  $a > 1$ , then  
[30-Jan-2023 Shift 2]

Options:

A.  $P$  is a singular matrix

B.  $|\text{Adj } P| > 1$

C.  $|\text{Adj } P| = \frac{1}{2}$

D.  $|\text{Adj } P| = 1$

**Answer: D**

**Solution:**

$$\begin{aligned} P^T &= aP + (a-1)I \\ \Rightarrow P &= aP^T + (a-1)I \\ \Rightarrow P^T - P &= a(P - P^T) \\ \Rightarrow P &= P^T, \text{ as } a \neq -1 \\ \text{Now, } P &= aP + (a-1)I \\ \Rightarrow P &= -I \Rightarrow |P| = 1 \\ \Rightarrow |\text{Adj } P| &= 1 \end{aligned}$$


---

## Question 88

Let  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$ . Then the sum of the diagonal elements of the

matrix  $(A + I)^{11}$  is equal to:  
[31-Jan-2023 Shift 1]

**Options:**

A. 6144

B. 4094

C. 4097

D. 2050

**Answer: C**

**Solution:**

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$$\Rightarrow A^3 = A^4 = \dots = A$$

$$(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \dots + {}^{11}C_{10} A + {}^{11}C_{11} I$$

$$= ({}^{11}C_0 + {}^{11}C_1 + \dots + {}^{11}C_{10})A + I$$

$$= (2^{11} - 1)A + I = 2047A + I$$

$$\therefore \text{Sum of diagonal elements} = 2047(1 + 4 - 3) + 3$$

$$= 4094 + 3 = 4097$$

## Question89

Let  $A = [a_{ij}]$ ,  $a_{ij} \in \mathbb{Z} \cap [0, 4]$ ,  $1 \leq i, j \leq 2$ . The number of matrices  $A$  such that the sum of all entries is a prime number  $p \in (2, 13)$  is \_\_\_\_\_.  
[31-Jan-2023 Shift 2]

**Answer: 204**

**Solution:**

As given  $a + b + c + d = 3$  or  $5$  or  $7$  or  $11$

if sum = 3

$$(1+x+x^2+\dots+x^4)^4 \rightarrow x^3$$

$$(1-x^5)^4(1-x)^{-4} \rightarrow x^3$$

$$\therefore {}^{4+3-1}C_3 = {}^6C_3 = 20$$

If sum = 5

$$(1-4x^5)(1-x)^{-4} \rightarrow x^5$$

$$\Rightarrow {}^{4+5-1}C_5 - 4x^{4+0-1}C_0 = {}^8C_5 - 4 = 52$$

If sum = 7

$$(1-4x^5)(1-x)^{-4} \rightarrow x^7$$

$$\Rightarrow {}^{4+5-1}C_4 - {}^{4+0-1}C_0 = {}^8C_5 - 4 = 52$$

If sum = 11

$$(1-4x^5+6x^{10})(1-x)^{-4} \rightarrow x^{11}$$

$$\Rightarrow {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-4}C_6 + 6 \cdot {}^{4+1-1}C_1$$

$$= {}^{14}C_{11} - 4 \cdot {}^9C_6 + 6 \cdot 4 = 364 - 336 + 24 = 52$$

$$\therefore \text{Total matrices} = 20 + 52 + 80 + 52 = 204$$

---

## Question90

Let  $A$  be a  $n \times n$  matrix such that  $|A| = 2$ . If the determinant of the matrix  $\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))$  is  $2^{84}$ , then  $n$  is equal to \_\_\_\_\_.

[31-Jan-2023 Shift 2]

**Answer: 5**

**Solution:**

$$\begin{aligned} & |\text{Adj}(2 \text{Adj}(2A^{-1}))| \\ &= |2 \text{Adj}(\text{Adj}(2A^{-1}))|^{n-1} \\ &= 2^{n(n-1)} |\text{Adj}(2A^{-1})|^{n-1} \\ &= 2^{n(n-1)} |(2A^{-1})|^{(n-1)(n-1)} \\ &= 2^{n(n-1)} 2^{n(n-1)(n-1)} |A^{-1}|^{(n-1)(n-1)} \\ &= 2^{n(n-1)+n(n-1)(n-1)} \frac{1}{|A|^{(n-1)^2}} \\ &= \frac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}} \\ &= 2^{n(n-1)+n(n-1)^2-(n-1)^2} \\ &= 2^{(n-1)(n^2-n+1)} \end{aligned}$$

$$\text{Now, } 2^{(n-1)(n^2-n+1)}$$

$$2^{(n-1)(n^2-n+1)} = 2^{84}$$

$$\text{So, } n = 5$$

---

## Question91

$$\text{If } A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}, \text{ then:}$$

[1-Feb-2023 Shift 2]

**Options:**

A.  $A^{30} - A^{25} = 2I$

$$\text{B. } A^{30} + A^{25} + A = I$$

$$\text{C. } A^{30} + A^{25} - A = I$$

$$\text{D. } A^{30} = A^{25}$$

**Answer: C**

**Solution:**

$$A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ Here } \alpha = \frac{\pi}{3}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{25} = A$$

$$A^{25} - A = 0$$

## Question 92

Let  $A = [a_{ij}]_{2 \times 2}$ , where  $a_{ij} \neq 0$  for all  $i, j$  and  $A^2 = I$ . Let  $a$  be the sum of all diagonal elements of  $A$  and  $b = |A|$ . Then  $3a^2 + 4b^2$  is equal to:  
[6-Apr-2023 shift 1]

**Options:**

$$\text{A. } 14$$

$$\text{B. } 4$$

$$\text{C. } 3$$

$$\text{D. } 7$$

**Answer: B**

**Solution:**

$$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$$

$$\text{Let } A = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$$

$$A^2 = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = I$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta\delta \\ \alpha\gamma + \gamma\delta & \gamma\beta + \delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \alpha^2 + \beta\gamma = 1$$

$$(\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$$

$$(\alpha + \delta)\gamma = 0$$

$$\beta\gamma + \delta^2 = 0$$

$$\text{Now } 3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$$

## Question 93

Let  $P$  be a square matrix such that  $P^2 = I - P$ . For  $\alpha, \beta, \gamma, \delta \in \mathbb{N}$ , if  $P^\alpha + P^\beta = \gamma I - 29P$  and  $P^\alpha - P^\beta = \delta I - 13P$ , then  $\alpha + \beta + \gamma - \delta$  is equal to :  
[6-Apr-2023 shift 2]

**Options:**

A. 40

B. 22

C. 24

D. 18

**Answer: C**

**Solution:**

$$P^2 = I - P$$

$$P^\alpha + P^\beta = \gamma I - 29P$$

$$P^\alpha - P^\beta = \delta I - 13P$$

$$P^4 = (I - P)^2 = I + P^2 - 2P$$

$$P^4 = I + I - P - 2P = 2I - 3P$$

$$P^8 = (P^4)^2 = (2I - 3P)^2 = 4I + 9P^2 - 12P$$

$$= 4I + 9(I - P) - 12P$$

$$P^8 = 13I - 21P \dots (1)$$

$$P^6 = P^4 \cdot P^2 = (2I - 3P)(I - P)$$

$$= 2I - 5P + 3P^2$$

$$= 2I - 5P + 3(I - P)$$

$$= 5I - 8P \dots (2)$$

$$(1) + (2)$$

$$P^8 + P^6 = 18I - 29P$$

$$(1) - (2)$$

$$P^8 - P^6 = 8I - 13P$$

$$\text{From (A)} \quad \alpha = 8, \quad \beta = 6$$

$$\gamma = 18$$

$$\delta = 8$$

$$\alpha + \beta + \gamma - \delta = 32 - 8 = 24$$

## Question 94

Let  $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ . If  $|\text{adj}(\text{adj}(\text{adj } 2A))| = (16)^n$ , then  $n$  is equal to

[8-Apr-2023 shift 1]

Options:

A. 8

B. 9

C. 12

D. 10

Answer: D

Solution:

$$|A| = 2[3] - 1[2] = 4$$

$$\therefore |\text{adj}(\text{adj}(\text{adj } 2A))|$$

$$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$$

$$\Rightarrow (2^3 |A|)^8 = 16^n$$

$$\Rightarrow (2^3 \times 2^2)^8 = 16^n$$

$$= 2^{40} = 16^n$$

$$= 16^{10} = 16^n \Rightarrow n = 10$$

---

## Question 95

Let  $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ ,  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  and  $Q = PAP^T$ . If

$P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then  $2a + b - 3c - 4d$  equal to

[8-Apr-2023 shift 1]

Options:

A. 2004

B. 2007

C. 2005

D. 2006

Answer: C

Solution:

$$\begin{aligned} Q &= PAP^T \\ P^T \cdot Q^{2007} \cdot P &= P^T \cdot Q \cdot Q \dots Q \cdot P \\ &= P^T (PAP^T)(P \cdot AP^T) \dots (P^T)P \\ &\Rightarrow (P^T P)A(P^T P)A \dots A(P^T P) \end{aligned}$$

$$P^T \cdot P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore P^T \cdot Q^{2007} \cdot P = A^{2007}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = 1, b = 2007, c = 0, d = 1$$

$$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$$

---

## Question96

If  $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$ ,  $A^{-1} = \alpha A + \beta I$  and  $\alpha + \beta = -2$ , then  $4\alpha^2 + \beta^2 + \lambda^2$  is

equal to :

[8-Apr-2023 shift 2]

Options:

A. 14

B. 12

C. 19

D. 10

Answer: A

Solution:

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$$

$$\Rightarrow (10 - 5\lambda)A^{-1} = -A + 11I$$

$$\therefore \alpha = \frac{-1}{10 - 5\lambda} \quad \text{and} \quad \beta = \frac{+11}{10 - 5\lambda}$$

$$\alpha + \beta = -2 \Rightarrow \frac{10}{10 - 5\lambda} = -2 \Rightarrow 10 - 5\lambda = -5 \Rightarrow \lambda = 3$$

$$\therefore \alpha = \frac{1}{5} \quad \& \quad \beta = \frac{-11}{5}$$

$$\therefore 4\alpha^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 3^2 = 14 \quad \text{Ans.}$$

---

## Question97

If A is a  $3 \times 3$  matrix and  $|A| = 2$ , then  $|3 \operatorname{adj}(|3A|A^2)|$  is equal to :

[10-Apr-2023 shift 1]

Options:

A.  $3^{12} \cdot 6^{10}$

B.  $3^{11} \cdot 6^{10}$

C.  $3^{12} \cdot 6^{11}$

D.  $3^{10} \cdot 6^{11}$

**Answer: B**

**Solution:**

Given  $|A| = 2$

Now,  $|3 \operatorname{adj}(|3A|A^2)|$

$|3A| = 3^3 \cdot |A|$

$= 3^3 \cdot (2)$

$\operatorname{Adj}(|3A|A^2) = \operatorname{adj}\{(3^3 \cdot 2)A^2\}$

$= (2 \cdot 3^3)^2 (\operatorname{adj} A)^2$

$= 2^2 \cdot 3^6 \cdot (\operatorname{adj} A)^2$

$|3 \operatorname{adj}(|3A|A^2)| = |2^2 \cdot 3 \cdot 3^6 (\operatorname{adj} A)^2|$

$= (2^2 \cdot 3^7)^3 |\operatorname{adj} A|^2$

$= 2^6 \cdot 3^{21} (|A|^2)^2$

$= 2^6 \cdot 3^{21} (2^2)^2$

$= 2^{10} \cdot 3^{21}$

$= 2^{10} \cdot 3^{10} \cdot 3^{11}$

$|3 \operatorname{adj}(|3A|A^2)| = 6^{10} \cdot 3^{11}$

## Question98

If  $A = \frac{1}{5!6!7!} \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$ , then  $|\operatorname{adj}(\operatorname{adj}(2A))|$  is equal to

[10-Apr-2023 shift 2]

**Options:**

A.  $2^{16}$

B.  $2^8$

C.  $2^{12}$

D.  $2^{20}$

**Answer: A**

**Solution:**

$$|A| = \frac{1}{5!6!7!} \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$$R_3 \rightarrow R_3 \rightarrow R_2$$

$$R_2 \rightarrow R_2 \rightarrow R_1$$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

$$\begin{aligned} |\text{adj}(\text{adj}(2A))| &= |2A|^{(n-1)^2} \\ &= |2A|^4 \\ &= (2^3 |A|)^4 \\ &= 2^{12} |A|^4 \Rightarrow 2^{16} \end{aligned}$$

## Question99

Let  $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$ , where  $a, c \in \mathbb{R}$ . If  $A^3 = A$  and the positive value of

$a$  belongs to the interval  $(n - 1, n]$ , where  $n \in \mathbb{N}$ , then  $n$  is equal to

[11-Apr-2023 shift 1]

**Answer: 2**

**Solution:**

Solution:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = A$$

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$$

Given  $A^3 = A$   
 $2ac+3=0 \dots (1)$  and  $a+2+3c=1$   
 $a+1+3c=0$   
 $a+1-\frac{9}{2a}=0$   
 $2a^2+2a-9=0$   
 $f(1)<0, f(2)>0$   
 $a \in (1, 2]$   
 $n=2$

## Question100

Let  $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$ . If  $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ , then the sum of all

the elements of the matrix  $\sum_{n=1}^{50} B^n$  is equal to  
 [12-Apr-2023 shift 1]

Options:

- A. 50
- B. 75
- C. 125
- D. 100

Answer: D

### Solution:

$$\text{Let } C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B = CAD$$

$$B^n = \underbrace{(CAD)(CAD)(CAD) \dots (CAD)}_{n\text{-times}}$$

$$\Rightarrow B^n = CA^nD \dots (1)$$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

$$\text{Similarly } A^n = A^3 = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$$

$$B^n \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{n}{51} + 2 \\ -1 & -\frac{n}{51} - 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n}{51} + 1 & \frac{n}{51} \\ -\frac{n}{51} & 1 - \frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25 - 50 & 25 \\ -25 & -25 - 50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

$$\text{Sum of the elements} = 100$$

---

## Question 101

The number of symmetric matrices of order 3, with all the entries from the set  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , is  
[13-Apr-2023 shift 1]

Options:

A.  $10^9$

B.  $10^6$

C.  $9^{10}$

D.  $6^{10}$

**Answer: B**

**Solution:**

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \{0, 1, 2, \dots, 9\}, \text{ Number of matrices} = 10^6$$


---

## Question102

Let  $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$ ,  $\alpha > 2$  be the adjoint of a matrix A and  $|A| = 2$ .

then  $\begin{bmatrix} \alpha & -2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$  is equal to

[13-Apr-2023 shift 1]

**Options:**

A. 16

B. 32

C. 0

D. -16

**Answer: D**

**Solution:**

$$\text{Given, } B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$$

$$|B| = 4$$

$$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$$

$$-\alpha^2 + 6\alpha - 8 = 0$$

$$\alpha = 2, 4$$

Given  $\alpha > 2$

So,  $\alpha = 2$  is rejected

$$\begin{bmatrix} 4 & -8 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]_{1 \times 1}$$

## Question 103

Let for  $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ ,  $|A| = 2$ . If  $|2 \operatorname{adj}(2 \operatorname{adj}(2A))| = 32^n$ , then

$3n + \alpha$  is equal to

[13-Apr-2023 shift 2]

Options:

A. 10

B. 9

C. 12

D. 11

**Answer: D**

**Solution:**

$$|A| = 2$$

$$\operatorname{adj}(kA) = k^{m-1} \operatorname{adj} A \quad \{m = \text{order of matrix}\}$$

$$\operatorname{adj}(2A) = 2^2 \operatorname{adj} A = 4 \operatorname{adj}(A)$$

$$\operatorname{adj}(2 \operatorname{adj}(2A)) = \operatorname{adj}(8 \operatorname{adj} A)$$

$$= 8^2 \operatorname{adj} \operatorname{adj}(A)$$

$$|2 \operatorname{adj} 2 \operatorname{adj}(2A)| = |2^7 \operatorname{adj} \operatorname{adj}(A)|$$

$$= (2^7)^3 |A|^{2^2}$$

$$\begin{aligned}
 &= 2^{21} |A|^4 \\
 &= 2^{21} \cdot 2^4 \\
 &\Rightarrow 2^{25} = (32)^n \\
 &\Rightarrow 2^{25} = 2^{5n} \\
 &\Rightarrow n = 5 \\
 &|A| = 2 \\
 &(6-1) - 2(2\alpha-1) + 3(\alpha-3) = 2 \\
 &\Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9 \\
 &\Rightarrow \alpha = -4 \\
 &3n + \alpha = 11
 \end{aligned}$$


---

## Question 104

Let the determinant of a square matrix  $A$  of order  $m$  be  $m - n$ , where  $m$  and  $n$  satisfy  $4m + n = 22$  and  $17m + 4n = 93$ . If  $\det(n \operatorname{adj}(\operatorname{adj}(mA))) = 3^a 5^b 6^c$ , then  $a + b + c$  is equal to [15-Apr-2023 shift 1]

Options:

- A. 101
- B. 84
- C. 109
- D. 96

**Answer: D**

**Solution:**

$$\begin{aligned}
 |A| &= m - n \\
 4m + n &= 22 \\
 17m + 4n &= 93 \\
 m = 5, n &= 2 \\
 |A| &= 3 \\
 |2 \operatorname{adj}(\operatorname{adj}(5A))| &= 2^5 |5A|^{16} \\
 &= 2^5 \cdot 5^{80} |A|^{16} \\
 &= 2^5 \cdot 5^{80} \cdot 3^{16} \\
 &= 3^{11} \cdot 5^{80} \cdot 6^5 \\
 a + b + c &= 96
 \end{aligned}$$


---

## Question 105

Let  $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} ; a, b \in \{1, 2, 3, \dots, 100\} \right\}$  and let

$T_n = \{A \in S : A^{n(n+1)} = I\}$ . Then the number of elements in  $\bigcap_{n=1}^{100} T_n$  is  
[24-Jun-2022-Shift-2]

**Answer: 100**

**Solution:**

$$A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -a+ab \\ 0 & b^2 \end{bmatrix}$$

$$\therefore T_n = \{A \in S : A^{n(n+1)} = I\}$$

$\therefore b$  must be equal to 1

$\therefore$  In this case  $A^2$  will become identity matrix and  $a$  can take any value from 1 to 100

$\therefore$  Total number of common element will be 100.

---

## Question 106

Let  $A$  be a  $3 \times 3$  real matrix such that

$$A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ and}$$

$$A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

If  $X = (x_1, x_2, x_3)^T$  and  $I$  is an identity matrix of order 3, then the

$$\text{system } (A - 2I)X = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

has :

[25-Jun-2022-Shift-1]

Options:

- A. no solution
- B. infinitely many solutions
- C. unique solution
- D. exactly two solutions

Answer: B

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} a+b &= 1 \\ d+e &= 1 \\ g+h &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} a+c &= -1 \\ d+f &= 1 \\ g+i &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{aligned} rc &= 1 \\ f &= 1 \\ i &= 2 \end{aligned}$$

Solving will get

$$a = -2, b = 3, c = 1, d = -1, e = 2, f = 1, g = -1, h = 1, i = 2$$

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A = 2I = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A - 2I)x = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow -4x_1 + 3x_2 + x_3 = 4 \dots (i)$$

$$-x_1 + x_3 = 1$$

$$-x_1 + x_2 = 1 \dots (ii)$$

$$\text{So } 3(iii) + (ii) = (i)$$

∴ Infinite solution

## Question 107

Let  $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ . If  $M$  and  $N$  are two matrices given by  $M = \sum_{k=1}^{10} A^{2k}$

and  $N = \sum_{k=1}^{10} A^{2k-1}$  then  $MN^2$  is :

[25-Jun-2022-Shift-1]

Options:

- A. a non-identity symmetric matrix
- B. a skew-symmetric matrix
- C. neither symmetric nor skew-symmetric matrix
- D. an identity matrix

Answer: A

Solution:

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$$M = A^2 + A^4 + A^6 + \dots + A^{20}$$

$$= -4I + 16I - 64I + \dots \text{ upto 10 terms}$$

$$= -I[4 - 16 + 64 \dots + \text{ upto 10 terms }]$$

$$= -I \cdot 4 \left[ \frac{(-4)^{10} - 1}{-4 - 1} \right] = \frac{4}{5}(2^{20} - 1)I$$

$$N = A^1 + A^3 + A^5 + \dots + A^{19}$$

$$= A - 4A + 16A + \dots \text{ upto 10 terms}$$

$$= A \left( \frac{(-4)^{10} - 1}{-4 - 1} \right) = - \left( \frac{2^{20} - 1}{5} \right) A$$

$$N^2 = \frac{(2^{20} - 1)^2}{2^5} A^2 = \frac{-4}{24} (2^{20} - 1)^2 I$$

$$MN^2 = \frac{-16}{125} (2^{20} - 1)^3 I = KI \quad (K \neq \pm 1)$$

$$(MN^2)^T = (KI)^T = KI$$

∴ A is correct

## Question 108

Let A be a  $3 \times 3$  matrix having entries from the set  $\{-1, 0, 1\}$ . The number of all such matrices A having sum of all the entries equal to 5, is  
[25-Jun-2022-Shift-1]

**Answer: 414**

**Solution:**

Case-I:

$1 \rightarrow 7$  times

and  $-1 \rightarrow 2$  times

$$\text{number of possible matrix} = \frac{9!}{7!2!} = 36$$

Case-II:

$1 \rightarrow 6$  times,

$-1 \rightarrow 1$  times

and  $0 \rightarrow 2$  times

$$\text{number of possible matrix} = \frac{9!}{6!2!} = 252$$

Case-III:

$1 \rightarrow 5$  times

and  $0 \rightarrow 4$  times

$$\text{number of possible matrix} = \frac{9!}{5!4!} = 126$$

Hence total number of all such matrix  $A = 414$

---

## Question 109

Let  $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$ . Then the number of elements in

the set  $\{(n, m) : n, m \in \{1, 2, \dots, 10\} \text{ and } nA^n + mB^m = I\}$  is \_\_\_\_  
[25-Jun-2022-Shift-2]

**Answer: 1**

**Solution:**

$$A^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = A$$

$$\Rightarrow A^K = A, K \in \mathbb{I}$$

$$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$$

So,  $B^K = B, K \in \mathbb{I}$

$$nA^n + mB^m = nA + mB$$

$$= \begin{bmatrix} 2n-2n & \\ n-n & \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So,  $2n - m = 1, -n + m = 0, 2m - n = 1$

So,  $(m, n) = (1, 1)$

## Question 110

Let  $A$  be a  $3 \times 3$  invertible matrix. If  $|\text{adj}(24A)| = |\text{adj}(3 \text{adj}(2A))|$ , then  $|A|^2$  is equal to :  
[26-Jun-2022-Shift-1]

Options:

A.  $6^6$

B.  $2^{12}$

C.  $2^6$

D. 1

Answer: C

Solution:

We know,  $|\text{adj } A| = |A|^{n-1}$

Now,  $|\text{adj } 24A| = |\text{adj } 3(\text{adj } 2A)|$

$$\Rightarrow |24A|^{3-1} = |3\text{adj } 2A|^{3-1}$$

$$\Rightarrow |24A|^2 = |3\text{adj } 2A|^2$$

Also, we know,  $|KA| = K^n |A|$

$$\Rightarrow ((24)^2)^2 |A|^2 = ((3)^2)^2 |\text{adj } 2A|^2$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (|2A|^{3-1})^2$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot |2A|^4$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (2^3)^4 \cdot |A|^4$$

$$\Rightarrow 3^6 \cdot 8^6 \cdot |A|^2 = 3^6 \cdot 8^4 \cdot |A|^4$$

$$\Rightarrow 8^2 = |A|^2$$

$$\Rightarrow |A|^2 = 64 = 2^6$$

---

## Question 111

Let  $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ ,  $Y = \alpha I + \beta X + \gamma X^2$  and

$Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2$ ,  $\alpha, \beta, \gamma \in \mathbb{R}$ .

If  $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ , then  $(\alpha - \beta + \gamma)^2$  is equal to \_\_\_\_\_

[26-Jun-2022-Shift-2]

Answer: 100

**Solution:**

$$\therefore X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Y = \alpha I + \beta X + \gamma X^2 = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$$\therefore Y \cdot Y^{-1} = I$$

$$\therefore \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{\alpha} & \frac{-\beta}{\alpha^2} & \frac{\gamma}{\alpha^3} \\ 0 & \frac{1}{\alpha} & \frac{-\beta}{\alpha^2} \\ 0 & 0 & \frac{1}{\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} & \frac{\alpha - 2\beta + \gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \alpha = 5, \beta = 10, \gamma = 15$$

$$\therefore (\alpha - \beta + \gamma)^2 = 100$$

---

## Question 112

The positive value of the determinant of the matrix A, whose

$$\text{Adj}(\text{Adj}(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}, \text{ is } \underline{\hspace{2cm}}$$

**[27-Jun-2022-Shift-1]**

**Answer: 14**

**Solution:**

$$|\text{adj}(\text{adj}(A))| = |A|^{2^2} = |A|^4$$

$$\therefore |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (14)^3 (3 - 2(-5) - 1(-1))$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

## Question 113

Let  $A$  and  $B$  be two  $3 \times 3$  matrices such that  $AB = I$  and  $|A| = \frac{1}{8}$ . Then  $|\text{adj}(B \text{adj}(2A))|$  is equal to

**[27-Jun-2022-Shift-2]**

**Options:**

A. 16

B. 32

C. 64

D. 128

**Answer: C**

**Solution:**

A and B are two matrices of order  $3 \times 3$ .  
and  $AB = I$ ,

$$|A| = \frac{1}{8}$$

Now,  $|A| |B| = 1$

$$|B| = 8$$

$$\therefore |\text{adj}(B(\text{adj}(2A)))| = |B(\text{adj}(2A))|^2$$

$$= |B|^2 |\text{adj}(2A)|^2$$

$$= 2^6 |2A|^{2 \times 2}$$

$$= 2^6 \cdot 2^{12} \cdot \frac{1}{2^{12}} = 64$$

---

## Question 114

Let A be a matrix of order  $2 \times 2$ , whose entries are from the set  $\{0, 1, 2, 3, 4, 5\}$ . If the sum of all the entries of A is a prime number p,  $2 < p < 8$ , then the number of such matrices A is \_\_\_\_\_

**[27-Jun-2022-Shift-2]**

**Answer: 180**

**Solution:**

∴ Sum of all entries of matrix A must be prime p such that  $2 < p < 8$  then sum of entries may be 3, 5 or 7.

If sum is 3 then possible entries are (0, 0, 0, 3), (0, 0, 1, 2) or (0, 1, 1, 1).

∴ Total number of matrices =  $4 + 4 + 12 = 20$

If sum is 5 then possible entries are

(0, 0, 0, 5), (0, 0, 1, 4), (0, 0, 2, 3), (0, 1, 1, 3), (0, 1, 2, 2) and (1, 1, 1, 2).

∴ Total number of matrices =  $4 + 12 + 12 + 12 + 12 + 4 = 56$

If sum is 7 then possible entries are

(0, 0, 2, 5), (0, 0, 3, 4), (0, 1, 1, 5), (0, 3, 3, 1), (0, 2, 2, 3), (1, 1, 1, 4), (1, 2, 2, 2), (1, 1, 2, 3) and (0, 1, 2, 4).

Total number of matrices with sum 7 = 104

∴ Total number of required matrices

=  $20 + 56 + 104 = 180$

---

## Question 115

Let A be a matrix of order  $3 \times 3$  and  $\det(A) = 2$ . Then  $\det(\det(A) \operatorname{adj}(5 \operatorname{adj}(A^3)))$  is equal to  
[28-Jun-2022-Shift-1]

Options:

A.  $512 \times 10^6$

B.  $256 \times 10^6$

C.  $1024 \times 10^6$

D.  $256 \times 10^{11}$

Answer: A

Solution:

$$|A| = 2$$

$$|A| = \operatorname{adj}(5 \operatorname{adj} A^3) \mid$$

$$= \mid 25 \mid A \mid \operatorname{adj}(\operatorname{adj} A^3) \mid$$

$$= 25^3 \mid A \mid^3 \cdot \mid \operatorname{adj} A^3 \mid^2$$

$$= 25^3 \cdot 2^3 \cdot \mid A^3 \mid^4$$

$$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$$

---

## Question 116

Let  $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$  where  $i = \sqrt{-1}$ . Then, the number of elements in the set  $\{n \in \{1, 2, \dots, 100\} : A^n = A\}$  is \_\_\_\_\_  
[28-Jun-2022-Shift-2]

**Answer: 25**

**Solution:**

$$\therefore A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$$

So  $A^5 = A$ ,  $A^9 = A$  and so on.

Clearly  $n = 1, 5, 9, \dots, 97$

Number of values of  $n = 25$

## Question 117

The probability that a randomly chosen  $2 \times 2$  matrix with all the entries from the set of first 10 primes, is singular, is equal to :  
[29-Jun-2022-Shift-1]

**Options:**

A.  $\frac{133}{10^4}$

B.  $\frac{18}{10^3}$

C.  $\frac{19}{10^3}$

D.  $\frac{271}{10^4}$

**Answer: C**

### Solution:

Let matrix A is singular then  $|A| = 0$

Number of singular matrix = All entries are same + only two prime number are used in matrix

$$= 10 + 10 \times 9 \times 2$$

$$= 190$$

$$\text{Required probability} = \frac{190}{10^4} = \frac{19}{10^3}$$

---

## Question 118

Let  $A = [a_{ij}]$  be a square matrix of order 3 such that  $a_{ij} = 2^{j-i}$ , for all

$i, j = 1, 2, 3$ . Then, the matrix  $A^2 + A^3 + \dots + A^{10}$  is equal to :

[29-Jun-2022-Shift-1]

Options:

A.  $\left( \frac{3^{10}-3}{2} \right) A$

B.  $\left( \frac{3^{10}-1}{2} \right) A$

C.  $\left( \frac{3^{10}+1}{2} \right) A$

D.  $\left( \frac{3^{10}+3}{2} \right) A$

Answer: A

### Solution:

Given,  $a_{ij} = 2^{j-i}$

$$\text{Now, } A = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 2+2+2 & 4+4+4 \\ \frac{1}{2} + \frac{1}{2} + \frac{1}{2} & 1+1+1 & 2+2+2 \\ \frac{1}{4} + \frac{1}{4} + \frac{1}{4} & \frac{1}{2} + \frac{1}{2} + \frac{1}{2} & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$= 3A$$

$$\text{Similarly, } A^3 = 3^2 A$$

$$A^4 = 3^3 A$$

$$\therefore A^2 + A^3 + \dots + A^{10}$$

$$= 3A + 3^2 A + 3^3 A + \dots + 3^9 A$$

$$= A(3 + 3^2 + 3^3 + \dots + 3^9)$$

$$= A \left( \frac{3(3^9 - 1)}{3 - 1} \right) = \frac{3(3^9 - 1)}{2} A = \left( \frac{3^{10} - 3}{2} \right) A$$

## Question 119

$$\text{Let } A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}. \text{ If}$$

$$B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - \dots - {}^5C_5(\text{adj } A)^5, \text{ then the sum of all}$$

elements of the matrix B is  
[29-Jun-2022-Shift-2]

Options:

A. -5

B. -6

C. -7

D. -8

Answer: C

Solution:

$$\text{Given } A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \text{ and}$$

$$B = I - 5C_1(\text{adj } A) + 5C_2(\text{adj } A)^2 - 5C_3(\text{adj } A)^3 + 5C_4(\text{adj } A)^4 - 5C_5(\text{adj } A)^5$$
$$= (I - (\text{adj } A))^5$$

$$\text{Cofactor of } A = \begin{bmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot 0 \\ (-1)^{2+1} \cdot (-1) & (-1)^{2+2} \cdot 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\text{Transpose of cofactor of } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Now,  $I - \text{adj } A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

Now let,

$$P = I - \text{adj } A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$P^5 = P^4 \cdot P = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

Now sum of elements  $= -1 - 5 - 1 + 0 = -7$

---

## Question 120

Let  $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$ , where  $\alpha$  is a non-zero real number and  $N = \sum_{k=1}^{49} M^{2k}$ .

If  $(I - M^2)N = -2I$ , then the positive integral value of  $\alpha$  is \_\_\_\_\_  
[29-Jun-2022-Shift-2]

**Answer: 1**

**Solution:**

$$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}; M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$$

$$N = M^2 + M^4 + \dots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \dots]I$$

$$= -\alpha^2 \frac{(1 - (-\alpha^2)^{49})}{1 + \alpha^2} \cdot I$$

$$I - M^2 = (1 + \alpha^2)I$$

$$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2I$$

$$\alpha = 1$$


---

## Question 121

Let  $S = \{ \sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd} \}$ .

Let  $a \in S$  and  $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$ .

If  $\sum_{a \in S} \det(\text{adj } A) = 100\lambda$ , then  $\lambda$  is equal to : ion:

[24-Jun-2022-Shift-1]

Options:

A. 218

B. 221

C. 663

D. 1717

Answer: B

Solution:

Given,  $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}_{3 \times 3}$   $S = \{ \sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd} \}$

$$\therefore S = \{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \dots, \sqrt{49}\}$$

We know,

$$|\text{adj } A| = |A|^{n-1}$$

Here,  $n =$  order of matrix.

Here,  $n = 3$

$$\therefore |\text{adj } A| = |A|^{3-1} = |A|^2$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{vmatrix}$$

$$= 1(1 - 0) - 0 + a(0 - (-a))$$

$$= a^2 + 1$$

$$\therefore |\text{adj } A| = |A|^2 = (a^2 + 1)^2$$

$$\text{Now, } \sum_{a \in S} \det(\text{adj } A)$$

$$= \sum_{a \in S} (a^2 + 1)^2$$

$$= (1^2 + 1)^2 + ((\sqrt{3})^2 + 1)^2 + ((\sqrt{5})^2 + 1)^2 + \dots + ((\sqrt{49})^2 + 1)^2$$

$$= (1^2 + 1)^2 + (3 + 1)^2 + (5 + 1)^2 + \dots + (49 + 1)^2$$

$$= 2^2 + 4^2 + 6^2 + \dots + 50^2$$

$$= 2^2(1^2 + 2^2 + 3^2 + \dots + 25^2)$$

$$= 4 \cdot \frac{25 \cdot 26 \cdot 51}{6} = 100.221$$

$$\therefore 100K = 100.221$$

$$\Rightarrow K = 221$$

## Question 122

Let  $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$  and  $B = A - I$ . If  $\omega = \frac{\sqrt{3}i - 1}{2}$ , then the number of

elements in the set  $\{n \in \{1, 2, \dots, 100\} : A^n + (\omega B)^n = A + B\}$  is equal to  
[25-Jul-2022-Shift-1]

**Answer: 17**

**Solution:**

$$\text{Here } A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

We get  $A^2 = A$  and similarly for

$$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

We get  $B^2 = -B \Rightarrow B^3 = B$

$$\therefore A^n + (\omega B)^n = A + (\omega B)^n \quad \text{for } n \in \mathbb{N}$$

For  $\omega^n$  to be unity  $n$  shall be multiple of 3 and for  $B^n$  to be  $B$ ,  $n$  shall be 3, 5, 7, ... 99

$$\therefore n = \{3, 9, 15, \dots, 99\}$$

Number of elements = 17

---

## Question 123

$$\text{Let } A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{C}. \text{ If for some } n \in \mathbb{N}, A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$$

then  $n + a + b$  is equal to  
[25-Jul-2022-Shift-2]

**Answer: 24**

**Solution:**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B^3 = 0$$

$$\therefore A^n = (I + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \dots$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)ab}{2} \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get  $na = 48$ ,  $nb = 96$  and

$$na + \frac{n(n-1)ab}{2} = 2160$$

$$\Rightarrow a = 4, n = 12 \text{ and } b = 8$$

$$n + a + b = 24$$

## Question 124

Let  $A$  be a  $2 \times 2$  matrix with  $\det(A) = -1$  and  $\det((A + I)(\text{Adj}(A) + I)) = 4$ . Then the sum of the diagonal elements of  $A$  can be :

[26-Jul-2022-Shift-1]

Options:

A.  $-1$

B.  $2$

C.  $1$

D.  $-\sqrt{2}$

**Answer: B**

**Solution:**

$$|(A+I)(\text{adj}A+I)| = 4$$

$$\Rightarrow |A \text{adj}A + A + \text{adj}A + I| = 4$$

$$\Rightarrow |(A)I + A + \text{adj}A + I| = 4$$

$$|A| = -1 \Rightarrow |A + \text{adj}A| = 4$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{adj}A = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix} = 4$$

$$\Rightarrow a+d = \pm 2$$

## Question 125

Let  $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$ , then the value of  $A'BA$

is:

[26-Jul-2022-Shift-2]

Options:

A. 1224

B. 1042

C. 540

D. 539

**Answer: D**

**Solution:**

$$\begin{aligned}
 A'BA &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} A \\
 &= \begin{bmatrix} 9^2 + 12^2 - 15^2 & -10^2 + 13^2 + 16^2 & 11^2 - 14^2 + 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= [9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2] \\
 &= [(9^2 - 10^2) + (11^2 + 12^2) + (13^2 - 14^2) + (16^2 - 15^2) + 17^2] \\
 &= [-19 + 265 + (-27) + 31 + 289] \\
 &= [585 - 46] = [539]
 \end{aligned}$$


---

## Question 126

The number of matrices  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , where

$a, b, c, d \in \{-1, 0, 1, 2, 3, \dots, 10\}$ , such that  $A = A^{-1}$ , is \_\_\_\_\_.

[26-Jul-2022-Shift-2]

**Answer: 50**

**Solution:**

$$\therefore A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^2 = \begin{bmatrix} a^2 + bc & b(a+d) \\ c(a+d) & bc + d^2 \end{bmatrix}$$

For  $A^{-1}$  must exist  $ad - bc \neq 0$ ..... (i)

and  $A = A^{-1} \Rightarrow A^2 = I$

$$\therefore a^2 + bc = d^2 + bc = 1$$

$$\text{and } b(a+d) = c(a+d) = 0$$

Case I: When  $a = d = 0$ , then possible values of  $(b, c)$  are  $(1, 1)$ ,  $(-1, 1)$  and  $(1, -1)$  and  $(-1, -1)$ .

Total four matrices are possible.

Case II: When  $a = -d$  then  $(a, d)$  be  $(1, -1)$  or  $(-1, 1)$ .

Then total possible values of  $(b, c)$  are  $(12 + 11) \times 2 = 46$ .

$\therefore$  Total possible matrices =  $46 + 4 = 50$ .

---

## Question 127

Let  $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$ . Let  $\alpha, \beta \in \mathbb{R}$  be such that  $\alpha A^2 + \beta A = 2I$ . Then  $\alpha + \beta$  is equal to  
[27-Jul-2022-Shift-1]

Options:

- A. -10
- B. -6
- C. 6
- D. 10

Answer: D

Solution:

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha + \beta & -8\alpha + 2\beta \\ 8\alpha - 2\beta & 21\alpha - 5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

On Comparing

$$8\alpha = 2\beta, -3\alpha + \beta = 2, 21\alpha - 5\beta = 2$$

$$\Rightarrow \alpha = 2, \beta = 8$$

$$\text{So, } \alpha + \beta = 10$$

## Question 128

Let  $S$  be the set containing all  $3 \times 3$  matrices with entries from  $\{-1, 0, 1\}$ . The total number of matrices  $A \in S$  such that the sum of all the diagonal elements of  $A^T A$  is 6 is \_\_\_\_\_.  
[27-Jul-2022-Shift-1]

Answer: 5376

### Solution:

Sum of all diagonal elements is equal to sum of square of each element of the matrix.

$$\text{i.e., } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \text{ then } t_r(A \cdot A^T)$$

$$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$$

$$\because a_i, b_i, c_i \in \{-1, 0, 1\} \text{ for } i = 1, 2, 3$$

$\therefore$  Exactly three of them are zero and rest are 1 or -1.

Total number of possible matrices  ${}^9C_3 \times 2^6$

$$= \frac{9 \times 8 \times 7}{6} \times 64$$

$$= 5376$$

---

## Question 129

$$\text{Let } A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}.$$

If  $A^2 + \gamma A + 18I = O$ , then  $\det(A)$  is equal to \_\_\_\_\_.

[27-Jul-2022-Shift-2]

Options:

A. -18

B. 18

C. -50

D. 50

**Answer: B**

### Solution:

Characteristic equation of A is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4 - \lambda & -2 \\ \alpha & \beta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (4 + \beta)\lambda + (4\beta + 2\alpha) = 0$$

$$\text{So, } A^2 - (4 + \beta)A + (4\beta + 2\alpha)I = 0$$

$$|A| = 4\beta + 2\alpha = 18$$


---

## Question 130

Consider a matrix  $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{bmatrix}$ , where  $\alpha, \beta, \gamma$  are three

distinct natural numbers. If  $\frac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj } A))))}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = 2^{32} \times 3^{16}$ , then the number of such 3 - tuples  $(\alpha, \beta, \gamma)$  is \_\_\_\_\_.

[27-Jul-2022-Shift-2]

**Answer: 42**

**Solution:**

$$\det(A) = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{vmatrix}$$

$$R_3 \rightarrow R_3 + R_1$$

$$\Rightarrow (\alpha + \beta + \gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore \det(A) = (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$$

$$\text{Also, } \det(\text{adj}(\text{adj}(\text{adj}(\text{adj } A))))$$

$$= (\det(A))^{2^4} = (\det(A))^{16}$$

$$\therefore \frac{(\alpha + \beta + \gamma)^{16} (\alpha - \beta)^{16} (\beta - \gamma)^{16} (\gamma - \alpha)^{16}}{(\alpha - \beta)^{16} (\beta - \gamma)^{16} (\gamma - \alpha)^{16}} = (4.13)^{16}$$

$$\Rightarrow \alpha + \beta + \gamma = 12$$

$$\Rightarrow (\alpha, \beta, \gamma) \text{ distinct natural triplets}$$

$$= {}^{11}C_2 - 1 - {}^3C_2(4) = 55 - 1 - 12 = 42$$


---

## Question 131

Let the matrix  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$  and the matrix  $B_0 = A^{49} + 2A^{98}$ . If

$B_n = \text{Adj}(B_{n-1})$  for all  $n > 1$ , then  $\det(B_4)$  is equal to :

[28-Jul-2022-Shift-1]

Options:

A.  $3^{28}$

B.  $3^{30}$

C.  $3^{32}$

D.  $3^{36}$

Answer: C

Solution:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\text{Now } B_0 = A^{49} + 2A^{98} = (A^3)^{16} \cdot A + 2(A^3)^{32} \cdot A^2$$

$$B_0 = A + 2A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$|B_0| = 9$$

$$\text{Since, } B_n = \text{Adj } |B_{n-1}| \Rightarrow |B_n| = |B_{n-1}|^2$$

$$\text{Hence } |B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16} = |3^2|^{16} = 3^{32}$$

Question 132

Let  $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$  and  $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$ ,  $\alpha, \beta \in \mathbb{R}$ . Let  $\alpha_1$  be the value of  $\alpha$

which satisfies  $(A+B)^2 - A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$  and  $\alpha_2$  be the value of  $\alpha$  which

satisfies  $(A+B)^2 - B^2$ . Then  $|\alpha_1 - \alpha_2|$  is equal to \_\_\_\_\_.

[28-Jul-2022-Shift-1]

**Answer: 2**

**Solution:**

$$(A+B)^2 = A^2 + B^2 + AB + BA$$

$$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \dots\dots (1)$$

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta-1 & 1 \\ \alpha+2\beta & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta+2 & \alpha-\beta \\ 1 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix}$$

By (1) we get

$$\begin{bmatrix} \beta^2+2\beta & \alpha+1 \\ \alpha+3\beta+1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \alpha = 1, \beta = 0 \Rightarrow \alpha_1 = 1$$

Similarly if  $A^2 + AB + BA = 0$  then

$$\left( A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 2\beta & \alpha-\beta+1-1-\alpha \\ \alpha+2\beta+1+2+2\alpha & \alpha^2-2+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \beta = 0 \text{ and } \alpha = -1 \Rightarrow \alpha_2 = -1$$

$$\therefore |\alpha_1 - \alpha_2| = |2| = 2$$

## Question133

Let A and B be any two  $3 \times 3$  symmetric and skew symmetric matrices respectively. Then which of the following is NOT true?

[28-Jul-2022-Shift-2]

Options:

- A.  $A^4 - B^4$  is a symmetric matrix
- B.  $AB - BA$  is a symmetric matrix
- C.  $B^5 - A^5$  is a skew-symmetric matrix
- D.  $AB + BA$  is a skew-symmetric matrix

Answer: C

Solution:

$$\begin{aligned} \text{(A) } M &= A^4 - B^4 \\ M^T &= (A^4 - B^4)^T = (A^T)^4 - (B^T)^4 \\ &= A^4 - (-B)^4 = A^4 - B^4 = M \\ \text{(B) } M &= AB - BA \\ M^T &= (AB - BA)^T = (AB)^T - (BA)^T \\ &= B^T A^T - A^T B^T \\ &= -BA - A(-B) \\ &= AB - BA = M \\ \text{(C) } M &= B^5 - A^5 \\ M^T &= (B^T)^5 - (A^T)^5 = -(B^5 + A^5) \neq -M \\ \text{(D) } M &= AB + BA \\ M^T &= (AB)^T + (BA)^T \\ &= B^T A^T + A^T B^T = -BA - AB = -M \end{aligned}$$

---

## Question134

Let A and B be two  $3 \times 3$  non-zero real matrices such that AB is a zero matrix. Then

[29-Jul-2022-Shift-1]

Options:

- A. the system of linear equations  $AX = 0$  has a unique solution

B. the system of linear equations  $AX = 0$  has infinitely many solutions

C. B is an invertible matrix

D.  $\text{adj}(A)$  is an invertible matrix

**Answer: B**

**Solution:**

$AB$  is zero matrix

$\Rightarrow |A| = |B| = 0$

So neither A nor B is invertible

If  $|A| = 0$

$\Rightarrow |\text{adj } A| = 0$  so  $\text{adj } A$

$AX = 0$  is homogeneous system and  $|A| = 0$

So, it is having infinitely many solutions

---

## Question 135

The number of matrices of order  $3 \times 3$ , whose entries are either 0 or 1 and the sum of all the entries is a prime number, is \_\_\_\_\_.

[29-Jul-2022-Shift-1]

**Answer: 282**

**Solution:**

In a  $3 \times 3$  order matrix there are 9 entries.

These nine entries are zero or one.

The sum of positive prime entries are 2, 3, 5 or 7.

Total possible matrices =  $\frac{9!}{2!7!} + \frac{9!}{3!6!} + \frac{9!}{5!4!} + \frac{9!}{7!2!} = 36 + 84 + 126 + 36 = 282$

---

## Question 136

Which of the following matrices can NOT be obtained from the matrix

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \text{ by a single elementary row operation?}$$

[29-Jul-2022-Shift-2]

Options:

A.  $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C.  $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

D.  $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

**Answer: C**

**Solution:**

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

(1)  $R_1 \rightarrow R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$  possible

(2)  $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$  possible

(3) Option is not possible

(4)  $R_2 \rightarrow R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$  possible

---

**Question 137**

Let  $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$ . For  $k \in \mathbb{N}$ , if  $X^T A^k X = 33$ ,

then  $k$  is equal to \_\_\_\_\_.

[29-Jul-2022-Shift-2]

**Answer: 10**

**Solution:**

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$

$$X^T A^k X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\text{As } A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{for } K \rightarrow \text{Even } A^K = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X^T A^K X = 33 \text{ (This is not correct)}$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3K+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K+3]$$

$$\therefore 3K+3 = 33 \therefore K = 10$$

But it should be dropped as 33 is not matrix

If K is odd

$$X^T A^K X = 33$$

$$X^T A A^{K-1} X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} -1 & 3 & 8 \end{bmatrix} \begin{bmatrix} 3k-2 \\ 1 \\ 1 \end{bmatrix} = [33]$$

$$[-3k+13] = [33]$$

$$k = 20/3 \text{ (not possible)}$$

## Question 138

Let  $M$  be any  $3 \times 3$  matrix with entries from the set  $\{0, 1, 2\}$ . The maximum number of such matrices, for which the sum of diagonal elements of  $M^T M$  is seven, is

24 Feb 2021 Shift 1

**Answer: 540**

**Solution:**

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$$

Case-I : Seven (1's) and two (0's)

$$\frac{9!}{7!2!} = 36$$

Case-II: One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$$\therefore \text{Total} = 540$$

## Question 139

If  $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$  and  $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ , then

$13(a^2 + b^2)$  is .....  
[25 Feb 2021 Shift 1]

**Answer: 13**

**Solution:**

$$A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix} \text{ and } (I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow |(I_2 + A)(I_2 - A)^{-1}| = a^2 + b^2$$

$$\Rightarrow a^2 + b^2 = \frac{|I_2 + A|}{|I_2 - A|} \dots (i)$$

$$\text{Now, } I_2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

Similarly,  $I_2 - A = \begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

Here,  $|I_2 + A| = |I_2 - A| = \left(1 + \tan^2\left(\frac{\theta}{2}\right)\right)$

$\Rightarrow \frac{|I_2 + A|}{|I_2 - A|} = 1 \dots (ii)$

From Eqs. (i) and (ii),

$a^2 + b^2 = 1$

Now,  $13(a^2 + b^2) = 13 \times 1 = 13$

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## Question 140

If for the matrix,  $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$ ,

$AA^T = I_2$ , then the value of  $\alpha^4 + \beta^4$  is

[2021, 25 Feb. Shift-II]

**Options:**

A. 4

B. 1

C. 2

D. 3

**Answer: B**

**Solution:**

Given,  $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$

Given,  $AA^T = I_2$  i.e.

$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 + \alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equating these matrices,

$$c1 = \alpha^2 + 1 \text{ gives, } \alpha = 0$$

$$\alpha(1 - \beta) = 0$$

$$\alpha^2 + \beta^2 = 1$$

Put  $\alpha = 0$  in  $\alpha^2 + \beta^2 = 1$ , we get  $0 + \beta^2 = 1$ , gives  $\beta = \pm 1$

where we take  $\beta^4 = 1$

$$\therefore \alpha^4 + \beta^4 = 0^4 + 1 = 1$$

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## Question 141

Let  $A$  be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of  $A^2$  is 1, then the possible number of such matrices is  
[2021, 26 Feb. Shift-I]

Options:

A. 4

B. 1

C. 6

D. 12

Answer: A

Solution:

Let  $A$  be the matrix as follows,  $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ , since  $A$  is symmetric matrix.

$$\text{Now, } A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{bmatrix}$$

Given that, diagonal entries of  $A^2$  is 1 . i.e.  $a^2 + b^2 + b^2 + c^2 = 1$

$$\text{or } a^2 + 2b^2 + c^2 = 1$$

Case 1  $a = 0$

$$\text{Then, } 2b^2 + c^2 = 1$$

$$(a) \ c = 0 \text{ gives, } b^2 = \frac{1}{2} \text{ or } b = \pm \frac{1}{\sqrt{2}} \therefore a = 0, b = 1/\sqrt{2}, c = 0 \text{ (2 matrices) } a = 0, b = -1/\sqrt{2}, c = 0$$

(b)  $b = 0$ , gives  $c^2 = 1$  or  $c = \pm 1 \therefore a = 0, b = 0, c = 1$   
and  $a = 0, b = 0, c = -1$  (2 matrices)

Case  $2b = 0$ , then  $a^2 + c^2 = 1$

(a)  $a = 0$ , then  $c = \pm 1$   $a = 0, b = 0, c = 1$  and  $a = 0, b = 0, c = -1$

This is repeated case.

(b)  $c = 0$ , then  $a = \pm 1$   $a = 1, b = 0, c = 0$  and  $a = -1, b = 0, c = 0$  Again 2 matrices.

Thus, only acceptable matrices are as follows

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Then possible number of such matrices are 4.

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## Question 142

If the matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$  satisfies the equation

$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  for some real numbers  $\alpha$  and  $\beta$ , then

$\beta - \alpha$  is equal to

[2021, 26 Feb. Shift-II]

**Answer: 4**

**Solution:**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And so on,

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c So, "  $A^{20} + \alpha A^{19} + \beta A$

$$= \begin{bmatrix} 1 + \alpha + \beta & 0 & 0 \\ 0 & 2^{20} + \alpha 2^{19} + 2\beta & 0 \\ 3\alpha + 2\beta & 0 & 1 - \alpha - \beta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get

$$1 - \alpha - \beta = 1$$

$$\Rightarrow \alpha + \beta = 0$$

$$\text{and } 2^{20} + \alpha 2^{19} + 2\beta = 4$$

$$\Rightarrow 2^{20} + \alpha(2^{19} - 2) = 4 \quad [\text{use, } \alpha + \beta = 0]$$

$$\Rightarrow \alpha = \frac{4 - 2^{20}}{2^{19} - 2} = -2$$

$$\Rightarrow \beta = 2$$

$$\therefore \beta - \alpha = 2 - (-2) = 2 + 2 = 4$$

## Question 143

Let  $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$  and  $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ .

If  $\text{Tr}(A)$  denotes the sum of all diagonal elements of the matrix  $A$ , then  $\text{Tr}(A) - \text{Tr}(B)$  has value equal to  
[2021, 18 March Shift-1]

Options:

A. 1

B. 2

C. 0

D. 3

**Answer: B**

**Solution:**

Given,  $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \dots (i)$

and  $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \dots (ii)$

Multiply with 2 in Eq. (ii), we get

$4A - 2B = \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix} \dots (iii)$

Adding Eqs. (i) and (iii),

$5A = \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

$$\therefore \text{Tr}(A) = 1 - 1 + 1 = 1$$

From Eq. (i),

$$B = \frac{1}{2} \left\{ \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\therefore \text{Tr}(B) = 0 - 1 + 0 = -1$$

$$\text{Hence, } \text{Tr}(A) - \text{Tr}(B) = 1 - (-1) = 2$$

## Question 144

Let  $I$  be an identity matrix of order  $2 \times 2$  and  $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ . Then the

value of  $n \in \mathbb{N}$  for which  $P^n = 5I - 8P$  is equal to

[2021, 18 March Shift-II]

**Answer: 6**

**Solution:**

Method (1)

$$\text{Given, } P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$\text{Now, } P^n = 5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix}$$

$$\Rightarrow P^n = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} \dots\dots\dots (i)$$

$$\text{Now, } P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$\text{Again } P^3 = P \cdot P^2 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \text{ Now,}$$

$$P^6 = P^3 \cdot P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$$

$$\Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n \quad [\text{from Eq. (i)}]$$

$$\therefore n=6$$

Method (2)

$$\text{Given } P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

Characteristics equation is  $|P - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(-3-\lambda) + 5 = 0$$

$$\Rightarrow \lambda^2 + \lambda - 1 = 0$$

By Cayley-Hamilton Theorem,

$$P^2 + P - I = 0$$

$$\Rightarrow P^2 = I - P$$

$$\Rightarrow P^3 + P^2 - P = 0$$

$$\Rightarrow P^3 = P - P^2 = P - (I - P) \quad [\text{from Eq. (i)}]$$

$$\Rightarrow P^3 = 2P - I$$

$$\Rightarrow P^3 = 2P - I$$

$$\text{Now, } P \cdot P^3 = P(2P - I)$$

$$\Rightarrow P^4 = 2P^2 - P = 2(I - P) - P \quad [\text{from Eq. (i)}]$$

$$\Rightarrow P^4 = -3P + 2I$$

$$\text{Again, } P \cdot (P^4) = P(-3P + 2I)$$

$$\Rightarrow P^5 = -3P^2 + 2P$$

$$= -3(I - P) + 2P \quad [\text{from Eq. (i)}]$$

$$= 5P - 3I$$

$$\text{and } P(P^5) = P(5P - 3I)$$

$$11 \Rightarrow P^6 = 5P^2 - 3P = 5(I - P) - 3P$$

$$\Rightarrow P^6 = 5I - 8P = P^n \quad (\text{given})$$

$$\therefore n=6$$

## Question 145

Let  $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ ,  $i = \sqrt{-1}$ . Then, the system of linear equations

$$A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \text{ has}$$

[2021, 16 March Shift-I]

**Options:**

- A. a unique solution.
- B. infinitely many solutions.
- C. no solution.
- D. exactly two solutions.

**Answer: C**

**Solution:**

$$\text{Let } A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$\text{Similarly, } A^8 = A^4 \cdot A^4$$

$$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 64+64 & -64-64 \\ -64-64 & 64+64 \end{bmatrix}$$

$$= \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \quad \text{Now, } A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$128x - 128y = 8$$

$$\Rightarrow x - y = \frac{1}{16} \dots\dots\dots (i)$$

$$\text{and } -128x + 128y = 64$$

$$\Rightarrow x - y = -\frac{1}{2} \dots\dots\dots (ii)$$

From Eqs. (i) and (ii), we get these two lines are parallel. So, there will be no solution.

## Question 146

The total number of  $3 \times 3$  matrices  $A$  having entries from the set  $(0, 1, 2, 3)$ , such that the sum of all the diagonal entries of  $AA^T$  is 9, is equal to

[2021, 16 March Shift-I]

**Answer: 766**

**Solution:**

Set  $S: \{0, 1, 2, 3\}$

$$\text{Let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{bmatrix}$$

$$\text{where } a = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$b = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$c = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$$

Now, as per question

$$ca \cdot a + b \cdot b + c \cdot c = 9$$

$$\Rightarrow (a_1^2 + a_2^2 + a_3^2) + (b_1^2 + b_2^2 + b_3^2)$$

$$+ (c_1^2 + c_2^2 + c_3^2) = 9$$

$$[a_i, b_i, c_i \in S]$$

$$9 = (1+1+1+1+1+1+1+1+1)$$

$$\text{or } 9 = (1+4+4+0+0+0+0+0+0)$$

$$\text{or } 9 = (9+0+0+0+0+0+0+0+0)$$

$$\text{or } 9 = (4+1+1+1+1+1+1+0+0)$$

Total permutations in case 1 = 1

$$\text{Total permutations in case 2} = \frac{9!}{6!2!} = 252$$

$$\text{In case 3} = \frac{9!}{8!} = 9$$

$$\text{In case 4} = \frac{9!}{5!3!} = 504$$

$$\text{Total permutations} = 1 + 252 + 9 + 504 = 766$$


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## Question 147

Let  $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$  and  $a = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  be two  $2 \times 1$  matrices with real entries

such that  $A = X B$ , where  $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$  and  $k \in \mathbb{R}$ . If

$a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$  and  $(k^2 + 1)b_2^2 \neq -2b_1b_2$ , then the value of  $k$  is  
[2021, 16 March Shift-II]

**Answer: 1**

**Solution:**

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$A = X B$$

$$X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$$

$$X B = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$X B = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + k b_2 \end{bmatrix}$$

$$\text{As, } A = X B$$

$$\text{So, } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + k b_2 \end{bmatrix}$$

$$\sqrt{3}a_1 = b_1 - b_2 \dots\dots\dots (i)$$

$$\sqrt{3}a_2 = b_1 + kb_2 \quad \dots\dots\dots (ii)$$

$$\text{And as given, } a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\text{Adding, Eq. (i)}^2 + \text{Eq. (ii)}^2,$$

$$3a_1^2 + 3a_2^2 = (b_1 - b_2)^2 + (b_1 + kb_2)^2$$

$$\Rightarrow 2(b_1^2 + b_2^2)$$

$$= 2b_1^2 + b_2^2(k^2 + 1) + 2b_1b_2(k - 1)$$

$$\Rightarrow b_2^2(k^2 + 1 - 2) + 2b_1b_2(k - 1) = 0$$

$$\Rightarrow (k - 1)[b_2^2(k + 1) + 2b_1b_2] = 0$$

$$\text{So, } k = 1$$

## Question 148

$$\text{Let } P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

where,  $\omega = \frac{-1 + i\sqrt{3}}{2}$ , and  $I_3$  be the identity matrix of order 3 . If the determinant of the matrix  $(P^{-1}AP - I_3)^2$  is  $\alpha\omega^2$ , then the value of  $\alpha$  is equal to.....

[16 Mar 2021 Shift 1]

**Answer: 36**

**Solution:**

$$\text{Given, } P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

$$|(P^{-1}AP - I_3)|^2 = \alpha\omega^2$$

$$\Rightarrow |(P^{-1}AP - I_3)(P^{-1}AP - I_3)| = \alpha\omega^2$$

$$\Rightarrow P^{-1}APP^{-1}AP - P^{-1}API_3 - I_3P^{-1}AP + I_3 \cdot I_3 = \alpha\omega^2$$

$$\Rightarrow P^{-1}A^2P - P^{-1}AP - P^{-1}AP + I_3 = \alpha\omega^2$$

$$[\because PP^{-1} = I \text{ and } IA = A]$$

$$\Rightarrow |P^{-1}A^2P - 2P^{-1}AP + PP^{-1}| = \alpha\omega^2$$

$$\Rightarrow |P^{-1}(A^2 - 2A + I_3)P| = \alpha\omega^2$$

$$\Rightarrow |P^{-1}| |A - I_3|^2 |P| = \alpha\omega^2$$

$$\Rightarrow |P^{-1}P| |A - I_3|^2 = \alpha\omega^2$$

$$\Rightarrow |A - I_3|^2 = \alpha\omega^2$$

Consider,

$$A - I_3 = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - I_3| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$$

On applying  $C_2 \rightarrow C_2 - C_3$ , we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

On applying  $C_2 \rightarrow C_2 - C_3$ , we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

$$= -\omega[(-\omega - 2) - (-7 + \omega^2)]$$

$$= -\omega(-\omega - 2 + 7 - \omega^2)$$

$$= -\omega(1 - 2 + 7)$$

$$= -6\omega$$

$$|A - I_3| = -6\omega$$

$$|A - I_3|^2 = 36\omega^2 = \alpha\omega^2$$

$$\therefore \alpha = 36$$

## Question 149

If  $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ , then the value of  $\det(A^4) + \det[A^{10} - \text{Adj}(2A)^{10}]$  is

equal to .....

[17 Mar 2021 Shift 1]

**Answer: 16**

**Solution:**

$$\text{If } A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

$$\det(A^4) + \det[A^{10} - [\text{Adj}(2A)]^{10}]$$

$$A \cdot A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$= \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 15 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 2^n & 2^n - (-1)^n \\ 0 & (-1)^n \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \left[ \begin{array}{l} \because x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ \text{adj}(x) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \end{array} \right]$$

$$[\text{adj}(2A)]^2 = 4 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^n = (-2)^n \begin{bmatrix} 1 & (-1)^n [2^n - (-1)^n] \\ 0 & (-1)^n 2^n \end{bmatrix}$$

$$[\text{adj}(2A)]^{10} = 2^{10} \begin{pmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{pmatrix}$$

$$\text{Now, } A^{10} - [\text{adj}(2A)]^{10}$$

$$= \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2^{10} & -(2^{20} - 2^{10}) \\ 0 & 2^{20} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \cdot 2^{10} - 2^{20} - 1 \\ 0 & 1 - 2^{20} \end{bmatrix}$$

$$\det |A^{10} - \text{adj}(2A)^{10}| = 0$$

$$\therefore \det(A^4) + \det[A^{10} - \text{adj}(2A)^{10}] = (16)^4 + 0 = 16$$

## Question 150

If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  and  $M = A + A^2 + A^3 + \dots + A^{20}$ , then the sum of all

the elements of the matrix  $M$  is equal to

[2021, 27 July Shift-II]

**Answer: 2020**

**Solution:**

$$\text{We have, } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } M = A + A^2 + A^3 + \dots + A^{20}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} +$$

$$\dots + \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & \sum n & \sum(\sum r) \\ 0 & 20 & \sum n \\ 0 & 0 & 20 \end{bmatrix}$$

$$\text{Now, } \sum_{r=1}^{20} n = 1 + 2 + \dots + 20 = \frac{20 \times 21}{2} = 210$$

$$\sum_{r=1}^{20} (\sum r) = \sum_{r=1}^{20} \frac{r(r+1)}{2} = \frac{1}{2} \sum_{r=1}^{20} (r^2 + r)$$

$$= \frac{1}{2} \left[ \frac{20 \times 21 \times 41}{6} + \frac{20 \times 21}{2} \right]$$

$$= \frac{1}{2} [2870 + 210] = 1540$$

$$\text{Hence, } M = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\text{Sum of all elements} = 2020.$$

## Question 151

$$S = \left\{ n \in \mathbb{N} \mid \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\}$$

, where  $i = \sqrt{-1}$ . Then the number of 2-digit numbers in the set S is  
[2021, 25 July Shift-1]

**Answer: 11**

**Solution:**

$$\text{Let } X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n$$

$$\Rightarrow AX = IX$$

$$A = I$$

$$A^2 = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\Rightarrow A = i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^8 = i^4 I = I$$

$$\therefore n = 8$$

$\Rightarrow n$  is a multiple of 8.

16, 24, ..... 96

$$\text{Number of elements} = \frac{96 - 16}{8} + 1 = 11$$

## Question 152

If  $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$ , then  $P^{50}$  is

[2021, 25 July Shift-II]

**Options:**

A.  $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

$$\text{C. } \begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$$

$$\text{D. } \begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$$

**Answer: A**

**Solution:**

$$\text{Given, } P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\vdots$

$$\Rightarrow P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$$

$$\text{Hence, } P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

## Question 153

Let  $A = [a_{ij}]$  be a real matrix of order  $3 \times 3$ , such that  $a_{i1} + a_{i2} + a_{i3} = 1$ , for  $i = 1, 2, 3$ . Then, the sum of all the entries of the matrix  $A^3$  is equal to [2021, 22 July Shift-II]

**Options:**

A. 2

B. 1

C. 3

D. 9

**Answer: C**

**Solution:**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$$

Sum of elements of each row is 1 .

Let X be  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  .

Then,  $AX = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$AX = \begin{bmatrix} a_{11} + a_{12} + a_{13} \\ a_{21} + a_{22} + a_{23} \\ a_{31} + a_{32} + a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX = X$$

Replace X with AX .

$$A \cdot AX = AX \Rightarrow A^2X = X$$

Again, replace X with AX ,

$$A^2(AX) = AX$$

$$A^3X = X$$

Let  $A^3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Then,

$$A^3X = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^3 X = \begin{bmatrix} a_{11}' + a_{12}' + a_{13}' \\ a_{21}' + a_{22}' + a_{23}' \\ a_{31}' + a_{32}' + a_{33}' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{So, } a_{11}' + a_{12}' + a_{13}' = 1$$

$$a_{21}' + a_{22}' + a_{23}' = 1$$

$$a_{31}' + a_{32}' + a_{33}' = 1$$

$$\therefore \text{Sum} = 3$$

## Question 154

Let  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . Then, the number of  $3 \times 3$  matrices  $B$  with entries

from the set  $\{1, 2, 3, 4, 5\}$  and satisfying  $AB = BA$  is  
[2021, 22 July Shift-II]

**Answer: 3125**

**Solution:**

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} B = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_1 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$AB = BA$$

$$\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} a_2 & a_1 & a_3 \\ b_2 & b_1 & b_3 \\ c_2 & c_1 & c_3 \end{pmatrix}$$

$$\begin{cases} b_1 = a_2 \\ b_2 = a_1 \\ b_3 = a_3 \end{cases} \begin{cases} a_1 = b_2 \\ a_2 = b_1 \\ a_3 = b_3 \end{cases} \begin{cases} C_1 = C_2 \\ C_2 = C_1 \\ C_3 = C_3 \end{cases}$$

$$B = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Number of distinct elements in B is 5  $\{a_1, a_2, a_3, c_1, c_3\}$  and according to question,  $a_{ij} \in \{1, 2, 3, 4, 5\}$ .

So, number of matrices  $= 5 \times 5 \times 5 \times 5 \times 5 = 3125$

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## Question 155

Let  $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$  and  $B = 7A^{20} - 20A^7 + 2I$ , where  $I$  is an identity

matrix of order  $3 \times 3$ . If  $B = [b_{ij}]$ , then  $b_{13}$  is equal to ...

[2021, 20 July Shift-1]

**Answer: 910**

**Solution:**

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = 7A^{20} - 20A^7 + 2I$$

$$\therefore A^2 = A \cdot A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^2 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^4 = A^3 A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^4 = \begin{pmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^n = \begin{pmatrix} 1 & -n & \frac{n^2 - n}{2} \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore b_{13} = 7 \left( \frac{20 \times 19}{2} \right) - 20 \left( \frac{7 \times 6}{2} \right) + 2(0)$$

$$\Rightarrow b_{13} = 1330 - 420 = 910$$

## Question 156

If  $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$ ,  $i = \sqrt{-1}$ , and  $Q = A^T B A$ , then the

inverse of the matrix  $AQ^{2021}A^T$  is equal to  
[26 Aug 2021 Shift 1]

Options:

A.  $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

B.  $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

C.  $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

D.  $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

**Answer: B**

**Solution:**

$$AA^T = \begin{vmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix}$$

$$AA^T \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

Now,  $Q^2 = A^T B A A^T B A$

$\Rightarrow Q^2 = A^T B^2 A$

Again,  $Q^3 = (A^T B A)(A^T B^2 A) = A^T B^3 A$

Similarly,

$Q^{2021} = A^T B^{2021} A$

$AQ^{2021}A^T = A(A^T B^{2021} A)A^T$

$= (AA^T)B^{2021}(AA^T) = B^{2021}$

$B = \begin{vmatrix} 1 & 0 \\ i & 1 \end{vmatrix}$

$B^2 = \begin{vmatrix} 1 & 0 \\ 2i & 1 \end{vmatrix}$ , similarly  $B^{2021} = \begin{vmatrix} 1 & 0 \\ 2021i & 1 \end{vmatrix}$

$(B^{2021})^{-1} = \frac{\text{adj}(B^{2021})}{|B^{2021}|} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

## Question 157

Let A be a  $3 \times 3$  real matrix. If  $\det(2 \text{ Adj}(2 \text{ Adj}(\text{Adj}(2A)))) = 2^{41}$ , then the value of  $\det(A^2)$  equal  
[26 Aug 2021 Shift 2]

**Answer: 4**

**Solution:**

We have, A is a square matrix of  $3 \times 3$ . Now,

$$\begin{aligned} & 2 \operatorname{Adj}(2 \operatorname{Adj}(\operatorname{Adj}(2A))) \\ &= 2 \operatorname{Adj}(2 \operatorname{Adj}(2^{3-1} \operatorname{adj} A)) \quad [\because \operatorname{adj}(KA) = K^{n-1} \operatorname{adj} A] \\ &= 2 \operatorname{Adj}(2 \operatorname{Adj}(4 \operatorname{adj} A)) \\ &= 2 \operatorname{Adj}(2 \times 4^{3-1} \operatorname{Adj} \operatorname{Adj} A) \\ &= 2 \operatorname{Adj}(32 \operatorname{Adj} \operatorname{Adj} A) \\ &= 2 \times 32^{3-1} \operatorname{Adj} \operatorname{Adj} \operatorname{Adj} A \\ &= 2^{11} \operatorname{Adj} \operatorname{Adj} \operatorname{Adj} A \\ &= 2^{11} \operatorname{Adj}(|A|^{3-2} A) \\ &= 2^{11} \operatorname{Adj}(|A| A) \\ &= 2^{11} \times |A|^{3-1} \operatorname{Adj} A \\ &= 2^{11} \times |A|^2 \operatorname{Adj} A \end{aligned}$$

Now,  $|2 \operatorname{Adj}(2 \operatorname{Adj}(\operatorname{Adj}(2A)))| = 2^{41}$

$$\Rightarrow |2^{11} \times |A|^2 \operatorname{Adj} A| = 2^{41}$$

$$\Rightarrow (2^{11})^3 (|A|^2)^3 |\operatorname{Adj} A| = 2^{41}$$

$$\Rightarrow 2^{33} |A|^6 |A|^{3-1} = 2^{41}$$

$$\Rightarrow |A|^6 \times |A|^2 = 2^8$$

$$\Rightarrow |A|^8 = 2^8$$

$$\Rightarrow |A| = \pm 2$$

Now,  $|A|^2 = |\pm 2|^2 = 4$

---

## Question 158

Let  $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ . If  $A^{-1} = \alpha I + \beta A$ ,  $\alpha, \beta \in \mathbb{R}$ , I is a  $2 \times 2$  identity

matrix, then  $4(\alpha - \beta)$  is equal to :

[27 Jul 2021 Shift 1]

**Options:**

A. 5

B.  $\frac{8}{3}$

C. 2

D. 4

**Answer: D**

**Solution:**

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, |A| = 6$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6} \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left. \begin{aligned} \alpha + \beta &= \frac{2}{3} \\ \beta &= -\frac{1}{6} \end{aligned} \right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$4(\alpha - \beta) = 4(1) = 4$$

---

## Question 159

Let A and B be two  $3 \times 3$  real matrices such that  $(A^2 - B^2)$  is invertible matrix. If  $A^5 = B^5$  and  $A^3B^2 = A^2B^3$ , then the value of the determinant of the matrix  $A^3 + B^3$  is equal to :  
[27 Jul 2021 Shift 2]

**Options:**

A. 2

B. 4

C. 1

D. 0

**Answer: D**

**Solution:**

$$C = A^2 - B^2; |C| \neq 0$$

$$A^5 = B^5 \text{ and } A^3B^2 = A^2B^3$$

$$\text{Now, } A^5 - A^3B^2 = B^5 - A^2B^3$$

$$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$$

$$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$$

Post multiplying inverse of  $A^2 - B^2$ :

$$A^3 + B^3 = 0$$

---

## Question 160

Let  $A = \{a_{ij}\}$  be a  $3 \times 3$  matrix, where  $a_{ij} = \begin{cases} (-1)^{j-1} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$  then

$\det(3 \operatorname{adj}(2A^{-1}))$  is equal to \_\_\_\_\_.  
[20 Jul 2021 Shift 2]

**Answer: 108**

**Solution:**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| = 4$$

$$\begin{aligned} |3 \operatorname{adj}(2A^{-1})| &= |3 \cdot 2^2 \operatorname{adj}(A^{-1})| \\ &= 12^3 |\operatorname{adj}(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108 \end{aligned}$$

---

## Question 161

If the matrix  $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$  satisfies  $A(A^3 + 3I) = 2I$ , then the value of  $K$  is  
[27 Aug 2021 Shift 1]

**Options:**

A.  $\frac{1}{2}$

B.  $-\frac{1}{2}$

C.  $-1$

D.  $1$

**Answer: A**

**Solution:**

Given matrix,

$$A = \begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$$

Characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 2 \\ K & -1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(1 + \lambda) - 2K = 0$$

$$\Rightarrow \lambda^2 + \lambda - 2K = 0$$

$\therefore$  Every square matrix satisfied its own characteristic equation.

$$\therefore A^2 + A - 2KI = 0$$

$$\Rightarrow A^2 = 2KI - A$$

$$\Rightarrow A^4 = 4K^2I + A^2 - 2(2KI)(A)$$

$$\Rightarrow A^4 = 4K^2I + 2KI - A - 4KA$$

$$\Rightarrow A^4 = (4K^2 + 2K)I - (1 + 4K)A \dots (i)$$

$$\text{Now, } A(A^3 + 3I) = 2I$$

$$\Rightarrow A^4 = 2I - 3A \dots (ii)$$

Comparing Eqs. (i) and (ii), we get

$$1 + 4K = 3$$

$$\Rightarrow K = \frac{1}{2}$$

---

## Question 162

Let  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ . Then  $A^{2025} - A^{2020}$  is equal to

[2021, 26 Aug. Shift-II]

**Options:**

A.  $A^6 - A$

B.  $A^5$

C.  $A^5 - A$

D.  $A^6$

**Answer: A**

**Solution:**

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Now,

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

.....

.....

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n-2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^{2025} - A^{2020} = \begin{pmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A^{2025} - A^{2020} = A^6 - A$$

## Question 163

Let  $\alpha$  be a root of the equation  $x^2 + x + 1 = 0$  and the matrix

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}, \text{ then the matrix } A^{31} \text{ is equal to:}$$

[Jan. 7, 2020 (I)]

Options:

A. A

B.  $I_3$

C.  $A^2$

D.  $A^3$

**Answer: D**

**Solution:**

Solution of  $x^2 + x + 1 = 0$  is  $\omega, \omega^2$

So,  $\alpha = \omega$  and

$$\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega$$

$$A^2 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^4 = I$$

$$\Rightarrow A^{30} = A^{28} \times A^2 = A^3$$

## Question 164

The number of all  $3 \times 3$  matrices A, with entries from the set  $\{-1, 0, 1\}$  such that the sum of the diagonal elements of  $AA^T$  is 3, is \_\_\_\_\_.

[NA Jan. 8, 2020 (I)]

**Answer: 672**

### Solution:

Let  $A = [a_{ij}]_{3 \times 3}$

It is given that sum of diagonal elements of  $AA^T$  is 3 i.e.,  $\text{tr}(AA^T) = 3$

$$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \dots + a_{33}^2 = 3$$

Possible cases are

$$\left. \begin{array}{ll} 0,0,0,0,0,1,1,1 & \rightarrow 1 \\ 0,0,0,0,0,0,-1,-1,-1 & \rightarrow 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow 3 \end{array} \right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$

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### Question 165

If  $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$  and  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , then  $10A^{-1}$  is equal to:

[Jan. 8, 2020 (II)]

Options:

A.  $A - 4I$

B.  $6I - A$

C.  $A - 6I$

D.  $4I - A$

Answer: C

### Solution:

Characteristics equation of matrix 'A' is  $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 \\ 9 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 6\lambda - 10 = 0$$

$$\therefore A^2 - 6A - 10I = 0$$

$$\Rightarrow A^{-1}(A^2) - 6A^{-1} - 10I A^{-1} = 0$$

$$\Rightarrow 10A^{-1} = A - 6I$$

---

### Question 166

If  $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$ ,  $\left(\theta = \frac{\pi}{24}\right)$  and  $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , where  $i = \sqrt{-1}$ , then

which one of the following is not true?

[Sep. 04, 2020 (I)]

Options:

A.  $0 \leq a^2 + b^2 \leq 1$

B.  $a^2 - d^2 = 0$

C.  $a^2 - c^2 = 1$

D.  $a^2 - b^2 = \frac{1}{2}$

Answer: D

Solution:

$$\because A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbb{N}$$

$$\therefore A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore A^5 = \begin{bmatrix} \cos 5\theta & i \sin 5\theta \\ i \sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore a = \cos 5\theta, b = i \sin 5\theta = c, d = \cos 5\theta$$

$$\therefore a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$$

$$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$$

$$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$$

$$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos \frac{10\pi}{24}$$

$$\text{and } 0 < \cos \frac{5\pi}{12} < 1 \Rightarrow 0 \leq a^2 + b^2 \leq 1$$

$$\therefore a^2 - b^2 = \frac{1}{2} \text{ is wrong.}$$

Question 167

Let  $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ ,  $x \in \mathbb{R}$  and  $A^4 = [a_{ij}]$ . If  $a_{11} = 109$ , then  $a_{22}$  is equal to

[NA Sep. 03, 2020 (I)]

**Answer: 10**

**Solution:**

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

Given that  $(x^2+1)^2+x^2 = 109$

$$x^4+3x^2-108=0$$

$$\Rightarrow (x^2+12)(x^2-9)=0$$

$$\therefore x^2=9$$

$$a_{22} = x^2+1 = 9+1 = 10$$

## Question 168

Let  $a, b, c \in \mathbb{R}$  be all non-zero and satisfy  $a^3 + b^3 + c^3 = 2$ . If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} \text{ satisfies } A^T A = I, \text{ then a value of } abc \text{ can be:}$$

[Sep. 02, 2020 (II)]

**Options:**

A.  $-\frac{1}{3}$

B.  $\frac{1}{3}$

C. 3

D.  $\frac{2}{3}$

**Answer: B**

**Solution:**

Given :  $A^T A = I$

$$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \sum a^2 & \sum ab & \sum ab \\ \sum ab & \sum a^2 & \sum ab \\ \sum ab & \sum ab & \sum a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So,  $\sum a^2 = 1$  and  $\sum ab = 0$

Now,  $a^3 + b^3 + c^3 - 3abc$

$$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= (a+b+c)(1-0)$$

$$= \sqrt{(a+b+c)^2} = \sqrt{\sum a^2 + 2\sum ab} = \pm 1$$

$$\Rightarrow 2 - 3abc = 1 \Rightarrow abc = \frac{1}{3}$$

$$\text{or } 2 - 3abc = -1 \Rightarrow abc = 1.$$

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