

Mathematical Reasoning**Question1**

The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to
[24-Jan-2023 Shift 1]

Options:

- A. $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
- B. $(\sim Q) \vee P$
- C. $((\sim P) \vee Q) \wedge (\sim Q)$
- D. $(\sim P) \vee Q$

Answer: A

Solution:

Solution:

Let $r = (\sim(P \wedge Q)) \vee ((\sim P) \wedge Q)$; $s = ((\sim P) \wedge (\sim Q))$

Option (A) : $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

is equivalent to (not of only P) $\wedge$ (not of only Q)

= (Both P, Q) and (neither P nor Q)

Question2

Let p and q be two statements.
Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to
[24-Jan-2023 Shift 2]

Options:

- A. $p \vee (p \wedge (\sim q))$
- B. $p \vee ((\sim p) \wedge q)$
- C. $(\sim p) \vee q$
- D. $p \vee (p \wedge q)$

Answer: C

Solution:

Solution:

$\sim(p \wedge (p \Rightarrow \sim q))$

$$\begin{aligned}
 &\equiv \sim p \vee \sim(\sim p \vee \sim q) \\
 &\equiv \sim p \vee (p \wedge q) \\
 &\equiv (\sim p \vee p) \wedge (\sim p \vee q) \\
 &\equiv t \wedge (\sim p \vee q) \\
 &\equiv \sim p \vee q
 \end{aligned}$$

Question3

The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is
[25-Jan-2023 Shift 1]

Options:

- A. equivalent to $(\sim p) \vee (\sim q)$
- B. a tautology
- C. equivalent to $p \vee q$
- D. a contradiction

Answer: B

Solution:

Solution:

$$\begin{aligned}
 &(p \wedge \sim q) \rightarrow (p \rightarrow \sim q) \\
 &\equiv (\sim(p \wedge \sim q)) \vee (\sim p \vee \sim q) \\
 &\equiv (\sim p \vee q) \vee (\sim p \vee \sim q) \\
 &\equiv \sim p \vee t \equiv t
 \end{aligned}$$

Question4

The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is
[25-Jan-2023 Shift 1]

Options:

- A. equivalent to $(\sim p) \vee (\sim q)$
- B. a tautology
- C. equivalent to $p \vee q$
- D. a contradiction

Answer: B

Solution:

Solution:

$$\begin{aligned}
 &(p \wedge \sim q) \rightarrow (p \rightarrow \sim q) \\
 &\equiv (\sim(p \wedge \sim q)) \vee (\sim p \vee \sim q) \\
 &\equiv (\sim p \vee q) \vee (\sim p \vee \sim q) \\
 &\equiv \sim p \vee t \equiv t
 \end{aligned}$$

Question5

Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \rightarrow q) \Delta (p \nabla q)$ is a tautology. Then
[25-Jan-2023 Shift 2]

Options:

- A. $\Delta = \wedge, \nabla = \vee$
- B. $\Delta = \vee, \nabla = \wedge$
- C. $\Delta = \vee, \nabla = \vee$
- D. $\Delta = \wedge, \nabla = \wedge$

Answer: C

Solution:

Solution:

Given $(p \rightarrow q) \Delta (p \nabla q)$

Option 1 $\Delta = \wedge, \nabla = \vee$

Option 2 $\Delta = \vee, \nabla = \wedge$

Option 3 $\Delta = \vee, \nabla = \vee$

Hence, it is tautology.

Option 4 $\Delta = \wedge, \nabla = \wedge$

Question6

If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false ?
[29-Jan-2023 Shift 1]

Options:

- A. $p = T, q = F, r = T$
- B. $p = T, q = T, r = F$
- C. $p = F, q = T, r = F$
- D. $p = T, q = F, r = F$

Answer: C

Solution:

Solution:

Option (3) $(p \vee q) \wedge (\sim q \vee r) \rightarrow (\sim p \vee r)$ will be False.

Question7

The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :
[29-Jan-2023 Shift 2]

Options:

A. $B \Rightarrow (A \Rightarrow B)$

B. $A \Rightarrow (A \Leftrightarrow B)$

C. $A \Rightarrow ((\sim A) \Rightarrow B)$

D. $B \Rightarrow ((\sim A) \Rightarrow B)$

Answer: 0

Solution:

Solution:

A	B	$\sim A$	$\sim A \vee B$	$B \Rightarrow ((\sim A) \vee B)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$A \Rightarrow B$	$\sim A \Rightarrow B$	$B \Rightarrow (A \Rightarrow B)$	$A \Rightarrow ((\sim A) \Rightarrow B)$	$B \Rightarrow ((\sim A) \Rightarrow B)$
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

Question8

Among the statements:

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

[30-Jan-2023 Shift 1]

Options:

A. Only (S1) is a tautology

B. Neither (S1) nor (S2) is a tautology

C. Only (S2) is a tautology

D. Both (S1) and (S2) are tautologies

Answer: B

Solution:

Solution:

$S_1 \equiv ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

$S_2 \equiv (p \vee q) \Rightarrow r \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

$S_2 \rightarrow$ not a tautology

Question9

Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

[30-Jan-2023 Shift 2]

Options:

A. $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$

B. $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$

C. $(P \vee Q) \wedge ((\sim P) \vee R)$

D. $(P \vee \sim Q) \wedge (P \vee \sim R)$

Answer: A

Solution:

Solution:

$P \rightarrow (\sim Q \wedge R)$

$$\sim P \vee (\sim Q \wedge R)$$

$$(\sim P \vee \sim Q) \wedge (\sim P \vee R)$$

Question10

(S1)($p \Rightarrow q \vee (p \wedge (\sim q))$) is a tautology
 (S2)(($\sim p$) $\Rightarrow (\sim q)$) $\wedge ((\sim p) \vee q)$ is a Contradiction. Then
 [31-Jan-2023 Shift 1]

Options:

- A. only (S2) is correct
- B. both (S1) and (S2) are correct
- C. both (S1) and (S2) are wrong
- D. only (S1) is correct

Answer: D

Solution:

p	q	$p \Rightarrow q$	$\sim q$	$p \wedge \sim q$	$(p \Rightarrow q) \vee (p \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$	$\sim p \vee q$	$((\sim p) \Rightarrow (\sim q)) \wedge (\sim p \vee q)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

Question11

The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which
 (($p \wedge q$) $\Rightarrow (r \vee q)$) $\wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is:
 [31-Jan-2023 Shift 2]

Options:

- A. 3
- B. 2
- C. 1
- D. 4

Answer: B

Solution:

$$((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$$

We know, $p \Rightarrow q$ is equivalent to

$$\begin{aligned} & \sim p \vee q \\ & ((\sim(p \wedge q) \vee (r \vee q)) \wedge (\sim(p \wedge r) \vee q)) \\ & \Rightarrow (\sim p \vee \sim q \vee r \vee q) \wedge (\sim p \vee \sim r \vee q) \\ & \Rightarrow (\sim p \vee r \vee q) \wedge (\sim p \vee \sim r \vee q) \\ & \Rightarrow (t) \wedge (\sim p \vee \sim r \vee q) \end{aligned}$$

For this to be tautology, $(\sim p \vee \sim r \vee q)$ must be always true which follows for $r = \sim p$ or $r = q$.

Question12

The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to [1-Feb-2023 Shift 1]

Options:

- A. $(\sim p) \wedge (\sim q)$
- B. $p \wedge (\sim q)$
- C. $(\sim p) \vee (\sim q)$
- D. $(\sim p) \vee q$

Answer: A

Solution:

$$\begin{aligned} & \sim (q \vee ((\sim q) \wedge p)) \\ & = \sim q \wedge \sim((\sim q) \wedge p) \\ & = \sim q \wedge (q \vee \sim p) \\ & = (\sim q \wedge q) \vee (\sim q \wedge \sim p) \\ & = (\sim q \wedge \sim p) \end{aligned}$$

Question13

Which of the following statements is a tautology ? [1-Feb-2023 Shift 2]

Options:

- A. $p \rightarrow (p \wedge (p \rightarrow q))$
- B. $(p \wedge q) \rightarrow (\sim(p) \rightarrow q)$
- C. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
- D. $p \vee (p \wedge q)$

Answer: B

Solution:

Solution:

(i) $p \rightarrow (p \wedge (p \rightarrow q))$
 $(\sim p) \vee (p \wedge (\sim p \vee q))$
 $(\sim p) \vee (p \wedge q)$
 $\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
 $= \sim p \vee q$
(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$
 $\sim (p \wedge q) \vee (p \vee q) = t$
 $\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
 $\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tautology
(iv) $p \vee (p \wedge q) = p$
Not tautology.

Question 14

**Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to:
[6-Apr-2023 shift 1]**

Options:

- A. $(P \vee R) \Rightarrow Q$
- B. $(P \Rightarrow R) \vee (Q \Rightarrow R)$
- C. $(P \Rightarrow R) \wedge (Q \Rightarrow R)$
- D. $(P \wedge R) \Rightarrow Q$

Answer: A

Solution:

Solution:

$(P \Rightarrow Q) \wedge (R \Rightarrow Q)$
We know that $P \Rightarrow Q \equiv \sim P \vee Q$
 $\Rightarrow (\sim P \vee Q) \wedge (\sim R \vee Q)$
 $\Rightarrow (\sim P \wedge \sim R) \vee Q$
 $\Rightarrow \sim (P \vee R) \vee Q$
 $\Rightarrow (P \vee R) \Rightarrow Q$

Question 15

Among the statements :

(S1) : $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology

(S2) : $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction

[6-Apr-2023 shift 2]

Options:

A. only (S2) is True

B. only (S1) is True

C. neither (S1) and (S2) is True

D. both (S1) and (S2) are True

Answer: C

Solution:

S1

P	Q	$\sim p$	$\sim p \wedge q$	$p \Rightarrow q$	$(p \Rightarrow q) \vee (\sim p \wedge q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	T	T

S2

P	Q	$q \Rightarrow p$	$\sim p$	$(\sim p) \wedge q$	$(q \Rightarrow p) \vee (\sim p \wedge q)$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	F	T	T	F
F	F	T	T	F	T

Ans. Option 3

Question16

Negation of $(p \rightarrow q) \rightarrow (q \rightarrow p)$ is
[8-Apr-2023 shift 1]

Options:

- A. $(\neg q) \wedge p$
- B. $p \vee (\neg q)$
- C. $(\neg p) \vee q$
- D. $q \wedge (\neg p)$

Answer: D

Solution:

Solution:

$$\begin{aligned} & (p \rightarrow q) \rightarrow (q \rightarrow p) \\ & \sim [\sim p \rightarrow q \wedge q \rightarrow p] \\ & \Rightarrow p \rightarrow q \wedge \sim q \rightarrow p \\ & \Rightarrow \sim p \vee q \wedge q \wedge \sim p \\ & \Rightarrow q \wedge \sim p. \end{aligned}$$

Question17

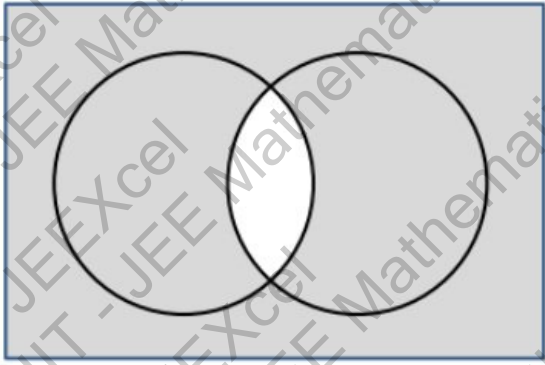
The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to
[8-Apr-2023 shift 2]

Options:

- A. $p \wedge (\sim q)$
- B. $p \wedge (q \wedge (\sim p))$
- C. $p \vee (q \vee (\sim p))$
- D. $p \wedge q$

Answer: D

Solution:



Question18

The negation of the statement :
 $(p \vee q) \wedge (q \vee (\sim r))$ is
[10-Apr-2023 shift 1]

Options:

- A. $((\sim p) \vee r) \wedge (\sim q)$
- B. $((\sim p) \vee (\sim q)) \wedge (\sim r)$
- C. $((\sim p) \vee (\sim q)) \vee (\sim r)$
- D. $(p \vee r) \wedge (\sim q)$

Answer: A

Solution:

$$\begin{aligned} & (p \vee q) \wedge (q \vee (\sim r)) \\ & \sim [(p \vee q) \wedge (q \vee (\sim r))] \\ & = \sim (p \vee q) \wedge (\sim q \wedge r) \\ & = (\sim p \wedge \sim q) \vee (\sim q \wedge r) \\ & = (\sim p \vee r) \wedge (\sim q) \end{aligned}$$

Question19

The statement $\sim [p \vee (\sim (p \wedge q))]$ is equivalent to
[10-Apr-2023 shift 2]

Options:

- A. $(\sim (p \wedge q)) \wedge q$
- B. $\sim (p \vee q)$

C. $\sim(p \wedge q)$

D. $(p \wedge q) \wedge (\sim p)$

Answer: D

Solution:

Solution:

$$\sim[p \vee (\sim(p \wedge q))]$$

$$\sim p \wedge (p \wedge q)$$

Question20

The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _____.

[11-Apr-2023 shift 1]

Answer: 7

Solution:

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$q \vee r$	$(p \vee q) \wedge (p \vee r) \rightarrow q \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

Question21

The converse of $((\sim p) \wedge q) \Rightarrow r$ is

[11-Apr-2023 shift 2]

Options:

A. $(p \vee (\sim q)) \Rightarrow (\sim r)$

B. $((\sim p) \vee q) \Rightarrow r$

C. $(\sim r) \Rightarrow ((\sim p) \wedge q)$

D. $(\sim r) \Rightarrow p \wedge q$

Answer: A

Solution:

Solution:

$$((\sim P) \wedge Q) \Rightarrow R$$

Converse

$$\sim ((\sim P) \wedge Q) \Rightarrow (\sim R)$$

$$(P \vee (\sim Q)) \Rightarrow (\sim R)$$

Question22

Among the two statements

(S1): $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology

[12-Apr-2023 shift 1]

Options:

A. only (S2) is true

B. only (S1) is true

C. both are true

D. both are false

Answer: C

Solution:

$$S_1: (p \rightarrow q) \wedge (p \wedge (\sim q))$$

P	Q	$p \rightarrow q$	$p \wedge (\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow S_1$ is Contradiction

S_2

p	q	$p \wedge q$	$(p \wedge \sim q)$	$(p \wedge \sim q)$	$(\sim p) \wedge (\sim p)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

Question23

The negation of the statement $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ is
[13-Apr-2023 shift 1]

Options:

- A. equivalent to $B \vee \sim C$
- B. a fallacy
- C. equivalent to $\sim C$
- D. equivalent to $\sim A$

Answer: D

Solution:

Solution:

$$\begin{aligned} p : & ((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A \\ & [\sim(A \wedge (B \vee C)) \vee (A \vee B)] \Rightarrow A \\ & [(A \wedge (B \vee C)) \wedge \sim(A \vee B)] \vee A \\ & (f \vee A) = A \\ & \sim p \equiv \sim A \end{aligned}$$

Question24

The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to
[13-Apr-2023 shift 2]

Options:

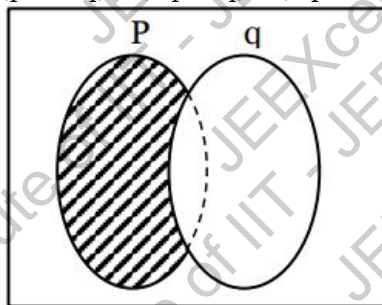
- A. $(\sim p) \vee (\sim q)$
- B. $(\sim p) \wedge (\sim q)$
- C. $p \vee (\sim q)$
- D. $p \vee q$

Answer: A

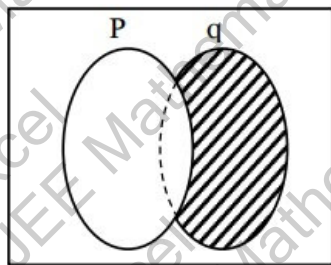
Solution:

Solution:

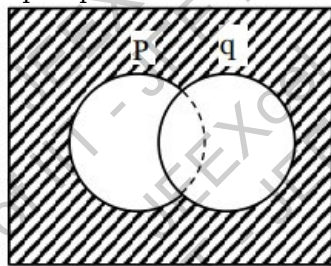
$$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$



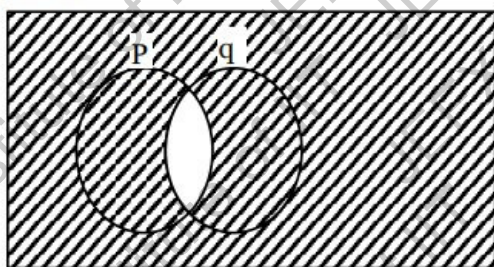
$$p \wedge \sim q \Rightarrow$$



$$\sim p \wedge q =$$



$$\sim p \wedge \sim q =$$



$$(p \wedge \sim q) \vee (\sim p \wedge q)(\sim p \wedge \sim q)$$

$$(\alpha, \beta)$$

$$(\sim p) \vee (\sim q)$$

Plane passing through $(0, -1, 2)$ and $(-1, 2, 1)$

then vector in plane $\langle -1, 3, -1 \rangle$ vector parallel to plane is $\langle 4, 2, -6 \rangle$ normal vector to plane L_2

$$(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$$

$$= \hat{i}(-16) - \hat{j}(10) + \hat{k}(-14)$$

$$\vec{n} = \langle 8, 5, 7 \rangle$$

Equation of plane

$$8(x - 0) + 5(y + 1) + 7(z - 2) = 0$$

$$\Rightarrow 8x + 5y + 7z = 9$$

From given options point $(-2, 5, 0)$ lies on plane.

Question 25

Negation of $p \wedge (q \wedge \sim(p \wedge q))$ is
[15-Apr-2023 shift 1]

Options:

A. $(\sim(p \wedge q)) \wedge q$

B. $\sim(p \vee q)$

C. $p \vee q$

D. $(\sim(p \wedge q)) \vee p$

Answer: D

Solution:

Solution:

$$\begin{aligned}& \sim [p \wedge (q \wedge \sim (p \wedge q))] \\& \sim p \vee (\sim q \vee (p \wedge q)) \\& \sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q)) \\& \sim p \vee (\sim q \vee p) \\& \sim (p \wedge q) \vee p\end{aligned}$$

Question26

The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p \Delta q) \Rightarrow ((p \Delta \sim q) \vee ((\sim p) \Delta q))$ is a tautology, is :
[24-Jun-2022-Shift-1]

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

Solution:

Then $(p \Delta \sim q) \vee (\sim p \Delta q)$ becomes
 $(p \vee \sim q) \vee (\sim p \vee q)$ which is always true, so x becomes a tautology.

Case-II

When Δ is same as $\wedge$

Then $(p \wedge q) \Rightarrow (p \wedge \sim q) \vee (\sim p \wedge q)$

If $p \wedge q$ is T , then $(p \wedge \sim q) \vee (\sim p \wedge q)$ is F

so x cannot be a tautology.

Case-III

When Δ is same as $\Rightarrow$ Then $(p \Rightarrow \sim q) \vee (\sim p \Rightarrow q)$ is same as $(\sim p \vee \sim q) \vee (p \vee q)$, which is always true, so x becomes a tautology.

Case-IV

When Δ is same as $\Leftrightarrow$

Then $(p \Leftrightarrow q) \Rightarrow (p \Leftrightarrow \sim q) \vee (\sim p \Leftrightarrow q)$

$p \Leftrightarrow q$ is true when p and q have same truth values, then $p \Leftrightarrow \sim q$ and $\sim p \Leftrightarrow q$ both are false. Hence x cannot be a tautology.

Question27

Consider the following statements:

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is [24-Jun-2022-Shift-2]

Options:

- A. $B \rightarrow (A \vee C)$
- B. $(\sim B) \wedge (A \wedge C)$
- C. $B \rightarrow ((\sim A) \vee (\sim C))$
- D. $B \rightarrow (A \wedge C)$

Answer: B

Solution:

Solution:

∴ Given statement is

$$(A \wedge C) \rightarrow B$$

Then its negation is

$$\sim \{(A \wedge C) \rightarrow B\}$$

$$\text{or } \sim \{\sim(A \wedge C) \vee B\}$$

$$\therefore (A \wedge C) \wedge (\sim B)$$

$$\text{or } (\sim B) \wedge (A \wedge C)$$

Question28

Consider the following two propositions:

$$\text{P1 : } \sim(p \rightarrow \sim q)$$

$$\text{P2 : } (p \wedge \sim q) \wedge ((\sim p) \vee q)$$

If the proposition $p \rightarrow ((\sim p) \vee q)$ is evaluated as FALSE, then :

[25-Jun-2022-Shift-1]

Options:

- A. P1 is TRUE and P2 is FALSE
- B. P1 is FALSE and P2 is TRUE
- C. Both P1 and P2 are FALSE
- D. Both P1 and P2 are TRUE

Answer: C

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \rightarrow (\sim p \vee q)$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$	$p \wedge \sim q$	p_2
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$p \rightarrow (\sim p \vee q)$ is F when p is true q is false

From table

P1 & P2 both are false

Question29

The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to :
[25-Jun-2022-Shift-2]

Options:

- A. $p \Rightarrow q$
- B. $q \Rightarrow p$
- C. $\sim(p \Rightarrow q)$
- D. $\sim(q \Rightarrow p)$

Answer: C

Solution:

Solution:

Let $S : ((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$
 $\Rightarrow S : \sim((\sim q) \wedge p) \vee ((\sim p) \vee q)$
 $\Rightarrow S : (q \vee (\sim p)) \vee ((\sim p) \vee q)$
 $\Rightarrow S : (\sim p) \vee q$
 $\Rightarrow S : p \Rightarrow q$
 So, negation of S will be $\sim(p \Rightarrow q)$

Question30

Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :
[26-Jun-2022-Shift-1]

Options:

A. $(p \Delta r) \vee q$

B. $(p \Delta r) \wedge q$

C. $(p \wedge r) \Delta q$

D. $(p \nabla r) \wedge q$

Answer: A

Solution:

Solution:

Case-I If $\Delta \equiv \nabla \equiv \wedge$

$$(p \wedge q) \rightarrow ((p \wedge q) \wedge r)$$

it can be false if r is false,

so not a tautology

Case-II If $\Delta \equiv \nabla \equiv \vee$

$$(p \vee q) \rightarrow ((p \vee q) \vee r) \equiv \text{tautology}$$

$$\text{then } (p \vee q) \vee r \equiv (p \Delta r) \vee q$$

Case-III if $\Delta = \vee, \nabla = \wedge$

$$\text{then } (p \wedge q) \rightarrow \{(p \vee q) \wedge r\}$$

Not a tautology

$$(\text{Check } p \rightarrow T, q \rightarrow T, r \rightarrow F)$$

Case-IV if $\Delta = \wedge, \nabla = \vee$

$$(p \wedge q) \rightarrow \{(p \wedge q) \vee r\}$$

Not a tautology

Question31

Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then r is equal to :
[26-Jun-2022-Shift-2]

Options:

- A. p
- B. q
- C. $\sim p$
- D. $\sim q$

Answer: C

Solution:

Clearly r must be equal to $\sim p$

$$\because \sim p \vee \sim p = \sim p$$

$$\text{and } (p \wedge q) \vee \sim p = p$$

$$\therefore \sim p \Rightarrow p = \text{tautology.}$$

Question32

The boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :
[27-Jun-2022-Shift-1]

Options:

- A. $q \rightarrow (p \wedge q)$
- B. $p \rightarrow q$
- C. $p \rightarrow (p \rightarrow q)$
- D. $p \rightarrow (p \vee q)$

Answer: D

Solution:

p	q	$p \wedge q$	$\sim p \wedge q$	$(\sim(p \wedge q)) \vee q$	$p \vee q$	$p \rightarrow q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T	T
T	F	F	T	T	T	F	T
F	F	F	T	T	T	T	T
F	F	F	T	T	F	T	T
				Tautology			Tautology

Question33

Which of the following statement is a tautology?

[27-Jun-2022-Shift-2]

Options:

- A. $((\sim q) \wedge p) \wedge q$
- B. $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$
- C. $((\sim q) \wedge p) \vee (p \vee (\sim p))$
- D. $(p \wedge q) \wedge (\sim p \wedge q)$

Answer: C

Solution:

Solution:

$\because ((\sim q) \wedge p) \vee (p \vee (\sim p))$
 $= (\sim q \wedge p) \vee t$ (t is tautology)
 $\equiv t$
 $\therefore$ option (C) is correct.

Question34

Let p, q, r be three logical statements. Consider the compound statements $S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and $S_2 : p \rightarrow (q \vee r)$

Then, which of the following is NOT true?

[28-Jun-2022-Shift-1]

Options:

- A. If S_2 is True, then S_1 is True
- B. If S_2 is False, then S_1 is False
- C. If S_2 is False, then S_1 is True
- D. If S_1 is False, then S_2 is False

Answer: C

Solution:

Solution:

$S_1 : (\sim p \vee q) \vee (\sim p \vee r)$
 $\equiv (\sim p \vee q \vee r)$
 $S_2 : \sim p \vee (q \vee r)$
Both are same
So, option (C) is incorrect.

Question35

Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is _____
[28-Jun-2022-Shift-2]

Answer: 0

Solution:

Solution:

According to given data

$$\frac{\sum_{i=1}^7 (x_i - 62)^2}{7} = 20$$

$$\Rightarrow \sum_{i=1}^7 (x_i - 62)^2 = 140$$

So for any x_i , $(x_i - 62)^2 \leq 140$

$$\Rightarrow x_i > 50 \quad \forall i = 1, 2, 3, \dots, 7$$

So no student is going to score less than 50.

Question36

The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p , q , r and s , is equal to _____
[28-Jun-2022-Shift-2]

Answer: 9

Solution:

Solution:

There are total 9 compound propositions, out of which 6 contain $\sim s$. So if we assign s as false, these 6 propositions will be true.

In remaining 3 compound propositions, two contain p and the third contains $\sim r$. So if we assign p and r as true and false respectively, these 3 propositions will also be true.

Hence maximum number of propositions that can be true are 9.

Question37

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta ((p \vee q) \Rightarrow q)$ is a tautology.

Then Δ is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\wedge$

B. $\vee$

C. $\Rightarrow$

D. $\Leftrightarrow$

Answer: C

Solution:

Solution:

$p \vee q \Rightarrow q$
 $\Rightarrow \sim(p \vee q) \vee q$
 $\Rightarrow (\sim p \wedge \sim q) \vee q$
 $\Rightarrow (\sim p \vee q) \wedge (\sim q \vee q)$
 $\Rightarrow (\sim p \vee q) \wedge t = \sim p \vee q$
Now by taking option C
 $(p \wedge q) \Rightarrow \sim p \vee q$
 $\Rightarrow \sim p \vee \sim q \vee \sim p \vee q$
 $\Rightarrow t$

Question38

Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to
[29-Jun-2022-Shift-2]

Options:

A. $p \wedge (\sim q) \wedge r$

B. $(\sim p) \wedge (\sim q) \wedge r$

C. $(\sim p) \wedge q \wedge r$

D. $p \wedge q \wedge (\sim r)$

Answer: C

Solution:

Solution:

Given,
 $(p \vee q) \Rightarrow ((\sim r) \vee p)$
Negation is
 $\sim((p \vee q) \Rightarrow ((\sim r) \vee p))$
 $= (p \vee q) \wedge \sim((\sim r) \vee p)$
 $= (p \vee q) \wedge (r \wedge \sim p)$
 $[(p \wedge \sim p) \vee (q \wedge \sim p)] \wedge r$
 $= q \wedge \sim p \wedge r$

Question39

Which of the following statements is a tautology?

[25-Jul-2022-Shift-1]

Options:

A. $((\sim p) \vee q) \Rightarrow p$

B. $p \Rightarrow ((\sim p) \vee q)$

C. $((\sim p) \vee q) \Rightarrow q$

D. $q \Rightarrow ((\sim p) \vee q)$

Answer: D

Solution:

Solution:

Question40

Consider the following statements:

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as:

[25-Jul-2022-Shift-2]

Options:

A. $((P \wedge (\sim R)) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee R))$

B. $((P \wedge R) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

C. $((P \wedge R) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

D. $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee R))$

Answer: D

Solution:

Solution:

P: Ramu is intelligent

Q: Ramu is rich

R: Ramu is not honest

Given statement, "Ramu is intelligent and honest if and only if Ramu is not rich" = $(P \wedge \sim R) \Leftrightarrow \sim Q$

So, negation of the statement is

$$\sim [(P \wedge \sim R) \Leftrightarrow \sim Q]$$

$$= \sim [\{ \sim (P \wedge \sim R) \vee \sim Q \} \wedge \{ Q \vee (P \wedge \sim R) \}]$$

$$= ((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$$

Question41

The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :
[26-Jul-2022-Shift-1]

Options:

- A. a tautology
- B. a contradiction
- C. equivalent to $(p \Rightarrow q) \wedge q$
- D. equivalent to $(p \Rightarrow q) \wedge p$

Answer: D

Solution:

Solution:

$$\sim(p \Leftrightarrow \sim q) \wedge q \\ = (p \Leftrightarrow q) \wedge q$$

p	q	$p \leftrightarrow q$	$(p \rightarrow q) \wedge q$	$(p \rightarrow q)$	$(p \rightarrow q) \wedge q$	$(p \rightarrow q) \wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

$\therefore (\sim(p \Leftrightarrow \sim q)) \wedge q$ is equivalent to $(p \Rightarrow q) \wedge p$.

Question42

Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is
[26-Jul-2022-Shift-2]

Options:

- A. $(\sim p) \wedge q$
- B. $p \wedge (\sim q)$
- C. $(\sim p) \vee (\sim q)$
- D. $(\sim p) \wedge (\sim q)$

Answer: D

Solution:

$$\begin{aligned}
 & p \Leftrightarrow (q \Rightarrow p) \\
 & \sim(p \Leftrightarrow (q \Rightarrow p)) \\
 & \equiv p \Leftrightarrow \sim(q \Rightarrow p) \\
 & \equiv p \Leftrightarrow (q \wedge \sim p) \\
 & \equiv (p \Rightarrow (q \wedge \sim p)) \wedge ((q \wedge \sim p) \Rightarrow p) \\
 & \equiv (\sim p \vee (q \wedge \sim p)) \wedge ((\sim q \vee p) \vee p) \\
 & \equiv ((\sim p \vee q) \wedge \sim p) \wedge (\sim q \vee p) \\
 & \equiv \sim p \wedge (\sim q \vee p) \\
 & \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge p) \\
 & \equiv (\sim p \wedge \sim q) \vee \text{c} \\
 & \equiv (\sim p \wedge \sim q)
 \end{aligned}$$

Question43

$(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is
[27-Jul-2022-Shift-1]

Options:

- A. p
- B. $\sim p$
- C. q
- D. $\sim q$

Answer: C

Solution:

Solution:

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $p \wedge q \Leftrightarrow p \wedge \sim q \equiv \sim p$
 $\therefore r = q$

Question44

If the truth value of the statement $(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F ?
[27-Jul-2022-Shift-2]

Options:

- A. $P \vee Q \rightarrow \sim R$
- B. $R \vee Q \rightarrow \sim P$

C. $\sim(P \vee Q) \rightarrow \sim R$

D. $\sim(R \vee Q) \rightarrow \sim P$

Answer: D

Solution:

Solution:

$X \Rightarrow Y$ is a false

when X is true and Y is false

So, $P \rightarrow T, Q \rightarrow F, R \rightarrow F$

(A) $P \vee Q \rightarrow \sim R$ is T

(B) $R \vee Q \rightarrow \sim P$ is T

(C) $\sim(P \vee Q) \rightarrow \sim R$ is T

(D) $\sim(R \vee Q) \rightarrow \sim P$ is F

Question45

Let the operations $*$, $\odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is:
[28-Jul-2022-Shift-1]

Options:

A. $(\vee, \wedge)$

B. $(\vee, \vee)$

C. $(\wedge, \wedge)$

D. $(\wedge, \vee)$

Answer: B

Solution:

Solution:

$*, \odot \in \{\wedge, \vee\}$

Now for $(p * q) \odot (p \odot \sim q)$ is tautology

(A) $(\vee, \wedge) : (p \vee q) \wedge (p \wedge \sim q)$ not a tautology

(B) $(\vee, \vee) : (p \vee q) \vee (p \vee \sim q)$

$= P \vee T$ is tautology

(C) $(\wedge, \wedge) : (p \wedge q) \wedge (p \wedge \sim q)$

$= (p \wedge p) \wedge (q \wedge \sim q) = p \wedge F$ not a tautology (Fallacy)

(D) $(\wedge, \vee) : (p \wedge q) \vee (p \vee \sim q)$ not a tautology

Question46

Let

p : Ramesh listens to music.

q : Ramesh is out of his village.

r : It is Sunday.

s : It is Saturday.

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as [28-Jul-2022-Shift-2]

Options:

- A. $((\sim q) \wedge (r \vee s)) \Rightarrow p$
- B. $(q \wedge (r \vee s)) \Rightarrow p$
- C. $p \Rightarrow (q \wedge (r \vee s))$
- D. $p \Rightarrow ((\sim q) \wedge (r \vee s))$

Answer: D

Solution:

Solution:

p : Ramesh listens to music

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

$p \rightarrow q$ conveys the same p only if q

Statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday"

$p \Rightarrow ((\sim q) \wedge (r \vee s))$

Question47

The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to : [29-Jul-2022-Shift-1]

Options:

- A. $q \Rightarrow (p \wedge r)$
- B. $p \Rightarrow (p \wedge r)$
- C. $(p \wedge r) \Rightarrow (p \wedge q)$
- D. $(p \wedge q) \Rightarrow r$

Answer: D

Solution:

Solution:

$(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to $(p \wedge q) \Rightarrow r$

Question48

The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to [29-Jul-2022-Shift-2]

Options:

- A. $(p \wedge (\sim r)) \Rightarrow q$
- B. $(\sim q) \Rightarrow ((\sim r) \vee p)$
- C. $p \Rightarrow (q \vee r)$
- D. $(p \wedge (\sim q)) \Rightarrow r$

Answer: B

Solution:

Solution:

(A) $(p \wedge (\sim r)) \Rightarrow q$
 $\equiv \sim(p \wedge \sim r) \vee q$
 $\equiv (\sim p \vee r) \vee q$
 $\equiv \sim p \vee (r \vee q)$
 $\equiv p \rightarrow (q \vee r)$
 $\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$
(C) $p \Rightarrow (q \vee r)$
 $\equiv \sim p \vee (q \vee r)$
 $\equiv (\sim p \vee q) \vee (\sim p \vee r)$
 $\equiv (p \rightarrow q) \vee (p \rightarrow r)$
(D) $(p \wedge \sim q) \Rightarrow r$
 $\equiv p \Rightarrow (q \vee r)$
 $\equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

Question49

**The contrapositive of the statement; "If you will work, you will earn money" is
[2021, 25 Feb. Shift-II]**

Options:

- A. to earn money, you need to work
- B. you will earn money, if you will not work
- C. if you will not earn money, you will not work
- D. if you will earn money, you will work

Answer: C

Solution:

Solution:

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$
p: you will work
 $\Rightarrow$ p: you will not work
q: you will earn money $\Rightarrow$
q: you will not earn money
Then,
 $\sim q \rightarrow \sim p$: if you will not earn money, you will not work.

Question50

Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then, [2021, 26 Feb. Shift-II]

Options:

- A. F_1 and F_2 both are tautologies
- B. F_1 is a tautology but F_2 is not a tautology
- C. F_1 is not tautology but F_2 is a tautology
- D. Both F_1 and F_2 are not tautologies

Answer: C

Solution:

Solution:

$F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$
 $\equiv \{ (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \} \vee \sim A$ (Associative law)
 $\equiv \{ (A \vee \sim A) \wedge (\sim B \vee \sim A) \} \vee [(A \vee B) \wedge C]$ (distributive law)
 $\equiv (\sim B \vee \sim A) \vee [(A \vee B) \wedge \sim C]$
 $\equiv [(\sim A \vee \sim B) \vee (A \vee B)] \wedge [(\sim A \vee \sim B) \vee \sim C]$
 $\equiv T \wedge (\sim A \vee \sim B) \vee \sim C$
 $\equiv (\sim A \vee \sim B) \vee \sim C$
Which is not a tautology.
Now, $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$
 $\equiv (A \vee B) \vee (\sim B \vee \sim A)$
Hence, F_2 is a tautology.

Question51

The statement $A \rightarrow (B \rightarrow A)$ is equivalent to [2021, 25 Feb. Shift-1]

Options:

- A. $A \rightarrow (A \wedge B)$
- B. $A \rightarrow (A \rightarrow B)$
- C. $A \rightarrow (A \leftrightarrow B)$
- D. $A \rightarrow (A \vee B)$

Answer: D

Solution:

Solution:

Given, statement $A \rightarrow (B \rightarrow A)$

$$\begin{aligned}
 &\equiv A \rightarrow (\sim B \vee A) \\
 &\equiv \sim A \vee (\sim B \vee A) \\
 &\equiv (\sim A \vee A) \vee \sim B \\
 &\equiv T \vee \sim B \equiv T \\
 \therefore T \vee B &= T \equiv (\sim A \vee A) \vee B \\
 &\equiv \sim A \vee (A \vee B) \\
 &\equiv A \rightarrow (A \vee B)
 \end{aligned}$$

Question 52

For the statements p and q , consider the following compound statements

A. $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

B. $[(p \vee q) \wedge \sim p] \rightarrow q$

Then, which of the following statement(s) is/are correct?

[2021, 24 Feb. Shift-II]

Options:

A. (A) and (B) both are not tautologies.

B. (A) and (B) both are tautologies.

C. (A) is a tautology but not (B).

D. (B) is a tautology but not (A).

Answer: B

Solution:

Given statements,

(A) $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

(B) $[(p \vee q) \wedge \sim p] \rightarrow q$

For statement (A),

$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$
T	F	T
F	F	T
T	F	T
T	T	T

$\therefore$ Statement (A) is tautology.

For statement (B),

$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	F	T
T	F	T
T	T	T
F	T	T

$\therefore$ Statement (B) is tautology.

$\therefore$ (A) and (B) both are tautologies.

Question53

The negative of the statement $\sim p \wedge (p \vee q)$ is
[2021, 24 Feb. Shift-II]

Options:

- A. $p \vee q$
- B. $p \vee \sim q$
- C. $p \wedge q$
- D. $p \wedge \sim q$

Answer: B

Solution:

Solution:

Given, statement $[\sim p \wedge (p \vee q)]$

Negative of given statement $\sim[\sim p \wedge (p \vee q)]$

$= p \vee \sim[p \vee q]$ [By De Morgan's law]

$= p \vee (\sim p \wedge \sim q)$ [By De Morgan's law]

$= (p \vee \sim p) \wedge (p \vee \sim q)$ [Using Distribution property]

Question54

The statement among the following that is a tautology is
[2021, 24 Feb. Shift-1]

Options:

- A. $A \wedge (A \vee B)$
- B. $A \vee (A \wedge B)$
- C. $[A \wedge (A \rightarrow B)] \rightarrow B$
- D. $B \rightarrow [A \wedge (A \rightarrow B)]$

Answer: C

Solution:

Solution:

Given, $[A \wedge (A \rightarrow B)] \rightarrow B$

$$= A \wedge (\sim A \vee B) \rightarrow B$$

$$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$$

$$= (A \wedge B) \rightarrow B$$

$$= \sim A \vee \sim B \vee B = t$$

Hence, $[A \wedge (A \rightarrow B)] \rightarrow B$ is a tautology.

Question55

The statement among the following that is a tautology is:

24 Feb 2021 Shift 1

Options:

A. $A \vee (A \wedge B)$

B. $A \wedge (A \vee B)$

C. $B \rightarrow [A \wedge (A \rightarrow B)]$

D. $[A \wedge (A \rightarrow B)] \rightarrow B$

Answer: D

Solution:

Solution:

$$(A \wedge (A \rightarrow B)) \rightarrow B$$

$$= (A \wedge (\sim A \vee B)) \rightarrow B$$

$$= ((A \wedge \sim A) \vee (A \wedge B)) \rightarrow B$$

$$= (A \wedge B) \rightarrow B$$

$$= \sim (A \wedge B) \vee B$$

$$= (\sim A \vee \sim B) \vee B$$

$$= T$$

Question56

If p and q are two statements, then which of the following compound statement is a tautology?

[2021, 18 March Shift-II]

Options:

A. $(p \Rightarrow q) \wedge \sim q \Rightarrow 0$

B. $(p \Rightarrow q) \wedge \sim q \Rightarrow \sim p$

C. $(p \Rightarrow q) \wedge \sim q \Rightarrow p$

D. $(p \Rightarrow q) \wedge \sim q \Rightarrow (p \wedge q)$

Answer: B

Solution:

Solution:

To check options one by one,

$$(a) [(p \rightarrow q) \wedge \sim q] \rightarrow q$$

$$\equiv (\sim p \wedge \sim q) \rightarrow q$$

$$\equiv (p \vee q) \vee q \equiv p \vee q$$

$$(b) [(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

$$\equiv (p \vee q) \vee \sim p$$

$$\equiv t$$

$$(c) [(p \rightarrow q) \wedge \sim q] \rightarrow p$$

$$\equiv [(\sim p \vee q) \wedge \sim q] \rightarrow p$$

$$\equiv (\sim p \wedge \sim q) \rightarrow p$$

$$\equiv (p \vee q) \vee p \equiv (p \vee q)$$

$$(d) [(p \rightarrow q) \wedge \sim q] \rightarrow p \wedge q$$

$$\equiv (p \vee q) \vee (p \wedge q)$$

$$\equiv [(p \vee q) \vee p] \wedge [(p \vee q) \vee q]$$

$$\equiv (p \vee q) \wedge (p \vee q) \equiv (p \vee q)$$

Question 57

If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q^* (\sim p))$ is a tautology, then the Boolean expression $(p^* (\sim q))$ is equivalent to
[2021, 17 March Shift-1]

Options:

A. $q \Rightarrow p$

B. $q \Rightarrow p$

C. $p \Rightarrow \sim q$

D. $p \Rightarrow q$

Answer: A

Solution:

Solution:

The Boolean expression

$(p \Rightarrow q) \Leftrightarrow (q^* (\sim p))$ is a tautology.

Making the truth table for this

p	q	$p \rightarrow q$	$q^* \sim p$	$\sim q$	$\sim q \wedge p$	$\sim (\sim q \wedge p)$
T	T	T	T	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	F	T

$$\therefore \sim (\sim q \wedge p) = q \vee \sim p = \sim p \vee q$$

$$\therefore * \text{ is equivalent to } \vee.$$

$$\begin{aligned}
 p^* \sim q &= p \vee \sim q \\
 &= \sim q \vee p \\
 &= q \Rightarrow p
 \end{aligned}$$

Question 58

If the Boolean expression $(p \wedge q)^* (p \otimes q)$ is a tautology, then $*$ and $\otimes$ are respectively, given by
[2021, 17 March Shift-II]

Options:

- A. $\rightarrow, \rightarrow$
- B. $\wedge \vee$
- C. $\vee, \rightarrow$
- D. $\wedge \rightarrow$

Answer: A

Solution:

Solution:

p	q	$p \wedge q$	$p \otimes q$	$(p \wedge q)^* (p \otimes q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Clearly $*$ and $\otimes$ are $\rightarrow, \rightarrow$ respectively for tautology.

Question 59

Which of the following Boolean expression is a tautology?
[2021, 16 March Shift-1]

Options:

- A. $(p \wedge q) \vee (p \vee q)$
- B. $(p \wedge q) \vee (p \rightarrow q)$
- C. $(p \wedge q) \wedge (p \rightarrow q)$
- D. $(p \wedge q) \rightarrow (p \rightarrow q)$

Answer: D

Solution:

Solution:

Let p and q are two statements.

Let's make the truth table and see $(p \wedge q)$ implies $(p \vee q)$ or not.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T	T
F	T	F	T	T	T
T	F	F	T	F	T
F	F	F	F	T	T

Since, last column of the truth table for $(p \wedge q) \rightarrow (p \rightarrow q)$ contains T only.

So, $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology.

Question60

Which of the following is the negation of the statement "for all $M > 0$, there exist $x \in S$ such that $x \geq M$ "?

[2021, 27 July Shift-II]

Options:

- A. there exists $M > 0$, such that $x < M$ for all $x \in S$
- B. there exists $M > 0$, there exists $x \in S$ such that $x \geq M$
- C. there exists $M > 0$, there exists $x \in S$ such that $x < M$
- D. there exists $M > 0$, such that $x \geq M$ for all $x \in S$

Answer: A

Solution:

Solution:

Let P : for all $M > 0$, there exists $x \in S$ such that $x \geq M$

$\sim P$: there exists $M > 0$, for all $x \in S$ such that $x < M$

($\because$ negation of 'there exists' is 'for all')

Question61

Consider the statement "The match will be played only if the weather is good and ground is not wet".

Select the correct negation from the following

[2021,25 July Shift-II]

Options:

- A. The match will not be played and weather is not good and ground is wet.
- B. If the match will not be played, then either weather is not good or ground is wet.
- C. The match will be played and weather is not good or ground is wet.
- D. The match will not be played or weather is good and ground is not wet.

Answer: C

Solution:

Solution:

Let p: The match will be played

q :weather is good

r : ground is not wet

$\sim[p \rightarrow (q \wedge r)] = p \wedge \sim(q \wedge r) = p \wedge (\sim q \vee \sim r)$

The match will be played and weather is not good or ground is wet.

Question62

Consider the following three statements,

(A) If $3 + 3 = 7$, then $4 + 3 = 8$

(B) If $5 + 3 = 8$, then earth is flat

(C) If both (A) and (B) are true, then $5 + 6 = 17$

Then, which of the following statements is correct

[2021, 20 July Shift II]

Options:

- A. (A) is false, but (B) and (C) are true
- B. (A) and (C) are true while (B) is false
- C. (A) is true while (B) and (C) are false
- D. (A) and (B) are false while (C) is true

Answer: B

Solution:

Solution:

To solve this, let's construct a truth table for $p \rightarrow q$

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

For (A), statements pandqare as follows,

$p : 3 + 3 = 7$
 $q : 4 + 3 = 8$
 Since, p is false, then $p \rightarrow q$ is true.
 Hence, (A) is true.
 For (B), statements p and q are as follows,
 $p : 5 + 3 = 8$
 $q : \text{Earth is flat}$
 Here, p is true and q is false.
 $\therefore p \rightarrow q$ is false.
 Hence, (b) is false.
 For (c), statements p and q are as follows,
 $p : \text{Both (A) and (B) are true.}$
 $q : 5 + 6 = 17$
 Here ' p ' is false because (B) is not true, then
 $p \rightarrow q$ is true.
 Hence, (c) is true.

Question63

The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to
 [2021, 27 July Shift-1]

Options:

- A. $P \vee Q$
- B. $P \wedge \sim Q$
- C. $\sim(P \Rightarrow Q)$
- D. $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

Answer: D

Solution:

P	Q	$P \vee Q$	$\sim P$	$(P \vee Q) \wedge P$	$(P \vee Q) \wedge \sim P \rightarrow Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

P	Q	$\sim Q$	$P \wedge \sim Q$	$P \rightarrow Q$	$\sim(P \rightarrow Q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	F

$\sim(P \rightarrow Q)$	$P \wedge \sim Q$	$\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$
F	F	T
T	T	T
F	F	T
F	F	T

Question64

The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to
[2021, 25 July Shift-1]

Options:

- A. $\sim q$
- B. q
- C. p
- D. $\sim p$

Answer: D

Solution:

$$\begin{aligned}
 & (p \rightarrow q) \wedge (q \rightarrow \sim p) \\
 & (\sim p \vee q) \wedge (\sim q \vee \sim p) \quad [\because p \rightarrow q \equiv \sim p \vee q] \\
 & \Rightarrow (\sim p \vee q) \wedge (\sim p \vee \sim q) \quad [\text{commutative property}] \\
 & \Rightarrow \sim p \vee (q \wedge \sim q) \quad (\text{distributive property}) \\
 & \Rightarrow \sim p
 \end{aligned}$$

Question65

Which of the following Boolean expression is not a tautology?
[2021, 22 July Shift-II]

Options:

- A. $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$
- B. $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$
- C. $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
- D. $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Answer: D

Solution:

- (i) $(p \rightarrow q) \vee (\sim p \rightarrow p)$
 $= (\sim p \vee q) \vee (q \vee p)$
 $= (\sim p \vee q) \vee q = t \vee q = t$
- (ii) $(q \rightarrow p) \vee (\sim q \rightarrow p)$
 $= (\sim q \vee p) \vee (q \vee p)$
 $= (\sim q \vee q) \vee p = t \vee p = \text{True}$
- (iii) $(p \rightarrow \sim q) \vee (\sim q \rightarrow p)$
 $= (\sim p \vee \sim q) \vee (q \vee p)$
 $= (\sim p \vee q) \vee (\sim q \vee q) = T \vee T = T$
- (iv) $(\sim q \rightarrow q) \vee (\sim q \rightarrow p)$
 $= p \vee q \vee q \vee p = p \vee p \vee q \vee q$
 $= p \vee q$
Not a tautology.

Question66

The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to [2021, 20 July Shift-1]

Options:

- A. $q \Rightarrow p$
- B. $p \Rightarrow q$
- C. $q \Rightarrow p$
- D. $p \Rightarrow \sim q$

Answer: B

Solution:

Solution:
 $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$q \vee \sim p$	$p \wedge \sim q \Rightarrow q \vee \sim p$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	F	T	T

p	q	$q \rightarrow p$	$p \rightarrow q$	$\sim q \rightarrow p$	$p \rightarrow \sim q$
T	T	T	T	T	F
T	F	T	F	T	F
F	T	F	T	T	T
F	F	T	T	F	T

So, option (b) is correct.

Question 67

Let $*$, square, $\wedge$, $\vee$ be such that the Boolean expression $(p * q) \rightarrow (p \wedge q)$ is a tautology. Then [2021, 31 Aug. Shift-I]

Options:

A. $\ast = v_1 = v$

B. $\ast = \Lambda_1 = \Lambda$

C. $\ast = \Lambda' = v$

D. $\ast = v_1 = \Lambda$

Answer: C

Solution:

Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

$\therefore (p \wedge \sim q) \rightarrow (p \vee q)$ is tautology.

Question 68

Negation of the statement $(p \vee q) \Rightarrow (q \vee r)$ is [2021, 31 Aug. Shift-II]

Options:

A. $p \wedge \sim q \wedge \sim r$

B. $\sim p \wedge q \wedge \sim r$

C. $\sim p \wedge \wedge r$

D. $p \wedge q \wedge r$

Answer: A

Solution:

Solution:

Negative of $(p \vee r) \Rightarrow (q \vee r)$

$$\equiv \sim ((p \vee r) \Rightarrow (q \vee r))$$

$$\equiv (p \vee r) \wedge (\sim (q \vee r))$$

$$\equiv (p \vee r) \wedge (\sim q \wedge \sim r)$$

$$\equiv (p \vee r) \wedge \sim r) \wedge \sim q$$

$$\equiv ((p \wedge \sim r) \vee (r \wedge \sim r)) \wedge \sim q$$

$$\equiv (p \wedge \sim r) \wedge f) \wedge \sim q$$

$$\equiv (p \wedge \sim r) \wedge (\sim q)$$

$$\equiv p \wedge \sim q \wedge \sim r$$

Question69

The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is
[2021, 27 Aug. Shift-1]

Options:

- A. a tautology
- B. equivalent to $p \rightarrow \sim r$
- C. a fallacy
- D. equivalent to $\rightarrow \rightarrow r$

Answer: A

Solution:

Solution:

Taking True = 1, False = 0

(1)	(2)	(3)	(4)	(5)	(6)	(7)
p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(1) \wedge (4) \wedge (5)$	$(6) \rightarrow (3)$
0	0	0	1	1	0	1
0	0	1	1	1	0	1
0	1	0	1	0	0	1
0	1	1	1	1	0	1
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

Column (7) gives true (1) in each case.

Hence, $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is a tautology.

Question70

The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to [2021,27 Aug. Shift-11]

Options:

- A. $(p \wedge q) \Rightarrow (r \wedge q)$
- B. $(q \wedge r) \Rightarrow (p \wedge q)$
- C. $(p \wedge q) \Rightarrow (tr \vee q)$
- D. $(p \wedge r) \Rightarrow (p \wedge q)$

Answer: A

Solution:

Given, $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$
 $\Leftrightarrow \sim (p \wedge q) \vee ((r \wedge q) \wedge p)$
 $\Leftrightarrow \sim (p \wedge q) \vee ((r \wedge q) \wedge (p \wedge q))$
 $\Leftrightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge [\sim (p \wedge q) \vee (r \wedge q)]$
 $\Leftrightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge q)]$
 $\Leftrightarrow \sim (p \wedge q) \vee (r \wedge q)$
 $\Leftrightarrow (p \wedge q) \Rightarrow (r \wedge q)$

Question71

If the truth value of the Boolean expression $((p \vee q) \wedge (q \rightarrow r) \wedge (\sim r) \rightarrow (p \wedge q))$ is false, then the truth values of the statements, p, q and r respectively can be [2021, 26 Aug. Shift-1]

Options:

- A. T F T
- B. F F T
- C. T F F
- D. F T F

Answer: C

Solution:

	p	q	r	$p \vee q$	$q \rightarrow r$	$\sim$	X	$p \wedge q$	Y
(A)	T	F	T	T	T	F	F	F	T
(B)	F	F	T	F	T	F	F	F	T
(C)	T	F	F	T	T	T	T	F	F
(D)	F	T	F	T	F	T	F	F	T

$$X = (p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)$$

$$Y = X \rightarrow (p \wedge q)$$

Question 72

Consider the two statements :

(S₁) : $(p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology (S₂) : $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy Then,

[2021, 26 Aug. Shift-11]

Options:

- A. only (S₁) is true
- B. both (S₁) and (S₂) are false
- C. both (S₁) and (S₂) are true
- D. only (S₂) is true

Answer: C

Solution:

Solution:

$$S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$$

$$S_1 = q \vee (\sim p \vee p) = q \vee (\text{true})$$

$$S_1 = (\text{Always true})$$

$$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q)$$

$$= (p \wedge \sim q) \wedge (p \wedge \sim q) = \text{fallacy}$$

Question73

Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?
[1 Sep 2021 Shift 2]

Options:

- A. $\sim (q \rightarrow p)$
- B. $\sim p \rightarrow \sim q$
- C. $\sim (p \rightarrow \sim q)$
- D. $\sim (p \rightarrow q)$

Answer: D

Solution:

Solution:

$$\begin{aligned} &\sim(p \rightarrow q) \\ \Rightarrow &\sim(\sim p \vee q) \\ \Rightarrow &p \wedge \sim q \end{aligned}$$

Question74

Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$ " is:
[Jan. 7, 2020 (II)]

Options:

- A. If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
- B. If $A \subseteq C$, then $B \subset A$ or $D \subset B$
- C. If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \not\subseteq D$
- D. If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

Answer: D

Solution:

$$\begin{aligned} \text{Let } P &= A \subseteq B, Q = B \subseteq D, R = A \subseteq C \\ \text{Contrapositive of } (P \wedge Q) \rightarrow R &\text{ is } \neg R \rightarrow \neg (P \wedge Q) \\ &= \neg R \rightarrow \neg P \vee \neg Q \end{aligned}$$

Question75

Negation of the statement:

$\sqrt{5}$ is an integer of 5 is irrational is:

[Jan. 9, 2020 (I)]

Options:

- A. $\sqrt{5}$ is not an integer or 5 is not irrational
- B. $\sqrt{5}$ is not an integer and 5 is not irrational
- C. $\sqrt{5}$ is irrational or 5 is an integer.
- D. $\sqrt{5}$ is an integer and 5 is irrational

Answer: B

Solution:

Solution:

Let p and q the statements such that p = $\sqrt{5}$ is an integer q = 5 is an irrational number.

Then, negation of the given statement

$\sqrt{5}$ is not an integer and 5 is not an irrational Number

$\sim(p \vee q) = \sim p \wedge \sim q$

Question76

If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively:

[Jan. 9, 2020 (II)]

Options:

- A. F, F
- B. T, F
- C. T, T
- D. F, T

Answer: C

Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Question 77

Which one of the following is a tautology?
[Jan. 8, 2020 (I)]

Options:

A. $(p \wedge (p \rightarrow q)) \rightarrow q$

B. $q \rightarrow (p \wedge (p \rightarrow q))$

C. $p \wedge (p \vee q)$

D. $p \vee (p \wedge q)$

Answer: A

Solution:

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	$q \rightarrow p \wedge (p \rightarrow q)$	$p \vee q$	$p \vee (p \wedge q)$	$p \wedge q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	F	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

Question 78

Which of the following statements is a tautology?

[Jan. 8, 2020 (II)]

Options:

A. $p \vee (\sim q) \rightarrow p \wedge q$

B. $\sim(p \wedge \sim q) \rightarrow p \vee q$

C. $\sim(p \vee \sim q) \rightarrow p \wedge q$

D. $\sim(p \vee \sim q) \rightarrow p \vee q$

Answer: D

Solution:

Solution:

$$(\sim p \wedge q) \rightarrow (p \vee q)$$

$$\Rightarrow \sim \{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$$

$$\Rightarrow \sim \{\sim p \wedge f\}$$

Question 79

The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to:

[Jan. 7, 2020 (I)]

Options:

A. p

B. q

C. $\sim p$

D. $\sim q$

Answer: C

Solution:

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (p \Rightarrow \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

Question80

The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :
[Sep. 06, 2020 (I)]

Options:

- A. $p \wedge \sim q$
- B. $\sim p \wedge \sim q$
- C. $\sim p \vee \sim q$
- D. $\sim p \vee q$

Answer: B

Solution:

Solution:

Negation of given statement = $\sim (p \vee (\sim p \wedge q))$
 $= \sim p \wedge \sim (\sim p \wedge q) = \sim p \wedge (p \vee \sim q)$
 $= (\sim p \wedge q) \vee (\sim p \wedge \sim q)$
 $= F \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

Question81

The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to:
[Sep. 05, 2020 (I)]

Options:

- A. $(x \wedge y) \vee (\sim x \wedge \sim y)$
- B. $(x \wedge y) \wedge (\sim x \vee \sim y)$
- C. $(x \wedge \sim y) \vee (\sim x \wedge y)$

D. $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

Answer: A

Solution:

Solution:

$$\begin{aligned} p : x \leftrightarrow \sim y &= (x \rightarrow \sim y) \wedge (\sim y \rightarrow x) \\ &= (\sim x \vee \sim y) \wedge (y \vee x) \\ &= \sim(x \wedge y) \wedge (x \vee y) \quad (\because \sim(x \wedge y) = \sim x \vee \sim y) \end{aligned}$$

Negation of p is

$$\sim p = (x \wedge y) \vee \sim(x \vee y) = (x \wedge y) \vee (\sim x \wedge \sim y)$$

Question 82

Given the following two statements:

(S₁) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S₂) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :

[Sep. 04, 2020 (I)]

Options:

A. both (S₁) and (S₂) are correct

B. only (S₁) is correct

C. only (S₂) is correct

D. both (S₁) and (S₂) are not correct

Answer: D

Solution:

Solution:

The truth table of both the statements is

p	q	$\sim p$	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	(S ₁)	$\sim p \leftrightarrow q$	(S ₂)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

$\therefore S_1$ is not tautology and S_2 is not fallacy.
Hence, both the statements (S_1) and (S_2) are not correct.

Question83

The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to :
[Sep. 03, 2020 (I)]

Options:

- A. q
- B. $(\sim p) \vee q$
- C. $(\sim p) \wedge q$
- D. $(\sim p) \vee (\sim q)$

Answer: B

Solution:

Solution:

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$p \rightarrow \sim (p \wedge \sim q)$	$\sim p \vee q$
T	T	F	F	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

$\therefore p \rightarrow \sim (p \wedge \sim q)$ is equivalent to $\sim p \vee q$

Question84

Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively:
[Sep. 03, 2020 (II)]

Options:

- A. T, F, T
- B. T, T, T
- C. F, T, F
- D. T, T, F

Answer: D

Solution:

Solution:

$(p \wedge q) \rightarrow (\sim q \vee r)$
 $= \sim(p \wedge q) \vee (\sim q \vee r)$
 $= (\sim p \vee \sim q) \vee (\sim q \vee r)$
 $= (\sim p \vee \sim q \vee r)$
 $\because (\sim p \vee \sim q \vee r)$ is false, then $\sim p$, $\sim q$ and r all these must be false.
 $\Rightarrow p$ is true, q is true and r is false.

Question85

Consider the statement: "For an integer n , if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is: [Sep. 06, 2020 (II)]

Options:

- A. For an integer n , if n is even, then $n^3 - 1$ is odd.
- B. For an integer n , if $n^3 - 1$ is not even, then n is not odd.
- C. For an integer n , if n is even, then $n^3 - 1$ is even.
- D. For an integer n , if n is odd, then $n^3 - 1$ is even.

Answer: A

Solution:

Solution:

Contrapositive statement will be "For an integer n , if n is not odd then $n^3 - 1$ is not even".

Or
"For an integer n , if n is even then $n^3 - 1$ is odd".

Question86

The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is : [Sep. 05, 2020 (II)]

Options:

- A. equivalent to $(p \wedge q) \vee (\sim q)$
- B. a contradiction
- C. equivalent to $(p \vee q) \wedge (\sim p)$
- D. a tautology

Answer: D

Solution:

Solution:

The truth table of $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	T

Hence, the statement is tautology.

Question87

Contrapositive of the statement :

'If a function f is differentiable at a, then it is also continuous at a', is :
[Sep. 04, 2020 (II)]

Options:

- A. If a function f is continuous at a, then it is not differentiable at a.
- B. If a function f is not continuous at a, then it is not differentiable at a.
- C. If a function f is not continuous at a, then it is differentiable at a
- D. If a function f is continuous at a, then it is differentiable at a

Answer: B

Solution:**Solution:**

Contrapositive statement will be "If a function is not continuous at 'a', then it is not differentiable at 'a'".

Question88

The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :
[Sep. 02, 2020 (I)]

Options:

- A. If I do not reach the station in time, then I will catch the train.

B. If I do not reach the station in time, then I will not catch the train.

C. If I will catch the train, then I reach the station in time.

D. If I will not catch the train, then I do not reach the station in time.

Answer: D

Solution:

Solution:

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

i.e. contrapositive of 'if p then q' is 'if not q then not p'.

Question89

The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to

[Jan. 12, 2019 (I)]

Options:

A. $p \wedge q$

B. $p \wedge (\sim q)$

C. $(\sim p) \wedge (\sim q)$

D. $p \vee (\sim q)$

Answer: C

Solution:

Solution:

Consider the Boolean expression

$$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$$

$$= (p \vee \sim q) \wedge (\sim p \wedge \sim q)$$

$$= ((p \vee \sim q) \wedge \sim p) \wedge ((p \vee \sim q) \wedge \sim q)$$

$$= ((p \wedge \sim p) \vee (\sim q \wedge \sim p)) \wedge \sim q$$

$$= (\sim p \wedge \sim q) \wedge \sim q = (\sim p \wedge \sim q)$$