



## Inverse Trigonometric Functions

### Question1

Let the maximum value of  $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$  for  $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$  be  $\frac{m}{n} \pi^2$ , where  $\gcd(m, n) = 1$ . Then  $m + n$  is equal to \_\_\_\_\_.

**JEE Main 2026 (Online) 21st January Evening Shift**

**Answer: 65**

**Solution:**

$$f(x) = (\sin^{-1} x)^2 + (\cos^{-1} x)^2, \quad x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$$

For  $x \in [-1, 1]$ ,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}.$$

Let  $a = \sin^{-1} x$ . Then  $\cos^{-1} x = \frac{\pi}{2} - a$ , and

$$f = a^2 + \left(\frac{\pi}{2} - a\right)^2 = 2a^2 - \pi a + \frac{\pi^2}{4}.$$

This is a quadratic in  $a$  opening upward, so its maximum on the allowed interval occurs at an endpoint of  $a$ .

Now,

$$x = -\frac{\sqrt{3}}{2} \Rightarrow a = -\frac{\pi}{3}, \quad x = \frac{1}{\sqrt{2}} \Rightarrow a = \frac{\pi}{4}.$$

Evaluate  $f$ :

- At  $a = \frac{\pi}{4}$ : this is the vertex, giving the **minimum**.
- At  $a = -\frac{\pi}{3}$ :

$$f = \left(-\frac{\pi}{3}\right)^2 + \left(\frac{5\pi}{6}\right)^2 = \frac{\pi^2}{9} + \frac{25\pi^2}{36} = \frac{29\pi^2}{36}.$$

Hence the maximum value is

$$\frac{m}{n}\pi^2 = \frac{29}{36}\pi^2,$$

with  $\gcd(29, 36) = 1$ .

$$\boxed{m + n = 65}$$

## Question2

If  $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$ , then

the number of solutions of the equation

$$\sin^{-1}(kx - 1) = \sin^{-1}x - \cos^{-1}x \text{ is } \underline{\hspace{2cm}}.$$

**JEE Main 2026 (Online) 28th January Morning Shift**

**Answer: 1**

**Solution:**

Given :

$$k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$$

$$\text{Let } \cos^{-1}\frac{2}{3} = \theta, \text{ so } \sin^{-1}\frac{2}{3} = \frac{\pi}{2} - \theta$$

$$k = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$\Rightarrow k = \frac{1 + \tan\frac{\theta}{2}}{1 - \tan\frac{\theta}{2}} + \frac{1 - \tan\frac{\theta}{2}}{1 + \tan\frac{\theta}{2}}$$

$$\Rightarrow k = \frac{(1 + \tan\frac{\theta}{2})^2 + (1 - \tan\frac{\theta}{2})^2}{(1 - \tan\frac{\theta}{2})(1 + \tan\frac{\theta}{2})}$$

$$\Rightarrow k = \frac{1 + \tan^2\frac{\theta}{2} + 2\tan\frac{\theta}{2} + 1 + \tan^2\frac{\theta}{2} - 2\tan\frac{\theta}{2}}{(1 - \tan\frac{\theta}{2})(1 + \tan\frac{\theta}{2})}$$

$$k = \frac{2(1 + \tan^2\frac{\theta}{2})}{(1 - \tan^2\frac{\theta}{2})}$$

$$\text{Using the identity } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} :$$

$$k = \frac{2}{\cos \theta}$$

$$\text{Since } \theta = \cos^{-1} \frac{2}{3} \Rightarrow \cos \theta = \frac{2}{3}$$

$$k = \frac{2}{2/3} = 3$$

Solving the main equation :

$$\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$$

Put  $k = 3$  :

$$\sin^{-1}(3x - 1) = \sin^{-1} x - \left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$\Rightarrow \sin^{-1}(3x - 1) = 2 \sin^{-1} x - \frac{\pi}{2} \dots\dots(1)$$

$$\text{Domain of } \sin^{-1}(3x - 1) \Rightarrow 3x - 1 \in [-1, 1] \Rightarrow x \in \left[0, \frac{2}{3}\right]$$

$$\text{Domain of } \sin^{-1} x \Rightarrow x \in [-1, 1]$$

$$\text{Intersection domain of Equation 1 is } x \in \left[0, \frac{2}{3}\right]$$

Take sine of both sides of Equation 1:

$$3x - 1 = \sin \left(2 \sin^{-1} x - \frac{\pi}{2}\right)$$

$$\Rightarrow 3x - 1 = -\cos \left(2 \sin^{-1} x\right)$$

Using  $\cos 2A = 1 - 2 \sin^2 A$  :

$$3x - 1 = - \left(1 - 2(\sin(\sin^{-1} x))^2\right)$$

$$\Rightarrow 3x - 1 = 2x^2 - 1$$

$$\Rightarrow 3x = 2x^2 \Rightarrow x(2x - 3) = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{3}{2}$$

$\frac{3}{2}$  is outside the domain.

So,  $x = 0$  is the only solution.

Number of solutions = 1.

### Question3

If  $\frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1} \left( \frac{2^{p-1}}{1+2^{2p-1}} \right) = \alpha$ , then  $\tan \alpha$  is equal to \_\_\_\_.

## JEE Main 2026 (Online) 5th April Morning Shift

**Answer: 2048**

### Solution:

Use the identity

$$\tan^{-1} \left( \frac{x-y}{1+xy} \right) = \tan^{-1} x - \tan^{-1} y$$

when  $xy > -1$ .

Now observe the given term:

$$\frac{2^{p-1}}{1+2^{2p-1}}$$

We want to write it in the form

$$\frac{x-y}{1+xy}$$

Take

$$x = 2^p, \quad y = 2^{p-1}$$

Then

$$x - y = 2^p - 2^{p-1} = 2^{p-1}$$

and

$$xy = 2^p \cdot 2^{p-1} = 2^{2p-1}$$

So,

$$\frac{2^{p-1}}{1+2^{2p-1}} = \frac{2^p - 2^{p-1}}{1+2^p \cdot 2^{p-1}}$$

Hence,

$$\tan^{-1} \left( \frac{2^{p-1}}{1+2^{2p-1}} \right) = \tan^{-1}(2^p) - \tan^{-1}(2^{p-1})$$

Therefore,

$$\sum_{p=1}^{11} \tan^{-1} \left( \frac{2^{p-1}}{1+2^{2p-1}} \right) = \sum_{p=1}^{11} [\tan^{-1}(2^p) - \tan^{-1}(2^{p-1})]$$

This is a telescoping sum:

$$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 4 - \tan^{-1} 2) + \cdots + (\tan^{-1} 2^{11} - \tan^{-1} 2^{10})$$

So all middle terms cancel, and we get

$$\sum_{p=1}^{11} \tan^{-1} \left( \frac{2^{p-1}}{1+2^{2p-1}} \right) = \tan^{-1}(2^{11}) - \tan^{-1}(1)$$

Since

$$\tan^{-1}(1) = \frac{\pi}{4}$$

given

$$\alpha = \frac{\pi}{4} + \sum_{p=1}^{11} \tan^{-1} \left( \frac{2^{p-1}}{1+2^{2p-1}} \right)$$

becomes

$$\alpha = \frac{\pi}{4} + \tan^{-1}(2^{11}) - \frac{\pi}{4} = \tan^{-1}(2^{11})$$

Thus,

$$\tan \alpha = \tan(\tan^{-1}(2^{11})) = 2^{11} = 2048$$

So, the answer is

2048

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## Question4

**If the domain of the function**

**$f(x) = \cos^{-1} \left( \frac{2x-5}{11-3x} \right) + \sin^{-1} (2x^2 - 3x + 1)$  is the interval  $[\alpha, \beta]$ , then  $\alpha + 2\beta$  is equal to :**

**JEE Main 2026 (Online) 21st January Morning Shift**

**Options:**

A.

5

B.

2

C.

3

D.

1

**Answer: C**

**Solution:**

$$\underbrace{-1 \leq \frac{2x-5}{11-3x}}_{(2)} \leq 1 \text{ and } \underbrace{-1 \leq 2x^2 - 3x + 1}_{(4)} \leq 1 \quad \underbrace{\phantom{-1 \leq 2x^2 - 3x + 1}}_{(3)}$$

$$(1) \frac{2x-5}{11-3x} - 1 \leq 0$$

$$\Rightarrow \frac{2x-5-11+3x}{11-3x} \leq 0$$

$$\Rightarrow \frac{5x-16}{11-3x} \leq 0$$

$$\Rightarrow x \in \left(-\infty, \frac{16}{5}\right] \cup \left(\frac{11}{3}, \infty\right) \dots (i)$$

$$(2) \frac{2x-5}{11-3x} \geq -1$$

$$\Rightarrow \frac{2x-5}{11-3x} + 1 \geq 0$$

$$\Rightarrow \frac{6-x}{11-3x} \geq 0$$

$$\Rightarrow \frac{x-6}{3x-11} \geq 0$$

$$\Rightarrow x \in \left(-\infty, \frac{11}{3}\right) \cup [6, \infty) \dots (ii)$$

$$(3) 2x^2 - 3x + 1 \leq 1$$

$$\Rightarrow x(2x-3) \leq 0$$

$$x \in \left[0, \frac{3}{2}\right]$$

$$(4) 2x^2 - 3x + 1 \geq -1$$

$$\Rightarrow 2x^2 - 3x + 2 \geq 0$$

$$x \in R \dots (iv)$$

taking intersection of (i), (ii), (iii), (iv)

$$x \in \left[0, \frac{3}{2}\right]$$

$$\therefore \alpha + 2\beta = 3$$

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**Question5**

The number of solutions of  $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{6}$ , where  $-\frac{1}{2\sqrt{6}} < x < \frac{1}{2\sqrt{6}}$ , is equal to :

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**Options:**

A.

2

B.

0

C.

3

D.

1

**Answer: D**

**Solution:**

$$\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{6}$$

$$\tan^{-1} \left( \frac{10x}{1 - 24x^2} \right) = \frac{\pi}{6}$$

$$24x^2 + 10\sqrt{3}x - 1 = 0$$

$$x = \frac{-5\sqrt{3} \pm 3\sqrt{14}}{24}$$

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## Question 6

If the domain of the function  $f(x) = \sin^{-1} \left( \frac{1}{x^2 - 2x - 2} \right)$ , is  $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$ , then  $\alpha + \beta + \gamma + \delta$  is equal to

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Options:

A.

4

B.

2

C.

5

D.

3

Answer: A

Solution:

Domain of  $\sin^{-1} \theta$  is  $-1 \leq \theta \leq 1$

$f(x) = \sin^{-1} \left( \frac{1}{x^2 - 2x - 2} \right)$  is defined if  $-1 \leq \frac{1}{x^2 - 2x - 2} \leq 1$

s.t  $\frac{1}{x^2 - 2x - 2} \geq -1$  and  $\frac{1}{x^2 - 2x - 2} \leq 1$

so these two inequality and final solution will be intersection of solution of these two inequality.

solve  $\frac{1}{x^2 - 2x - 2} \geq -1$

$$\frac{1}{x^2 - 2x - 2} + 1 \geq 0$$

$$\frac{1 + x^2 - 2x - 2}{x^2 - 2x - 2} \geq 0$$

$$\frac{(x^2 - 2x - 1)}{x^2 - 2x - 2} \geq 0$$

roots of  $x^2 - 2x - 1$  is  $x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

roots of  $x^2 - 2x - 2$  is  $x = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$



$$\text{so, } x \in (-\infty, 1 - \sqrt{3}) \cup [1 - \sqrt{2}, 1 + \sqrt{2}] \cup (1 + \sqrt{3}, \infty) - S_1$$

$$\text{solve } \frac{1}{x^2 - 2x - 2} \leq 1$$

$$\frac{1}{x^2 - 2x - 2} - 1 \leq 0$$

$$\frac{1 - x^2 + 2x + 2}{x^2 - 2x - 2} \leq 0$$

$$\frac{3 - x^2 + 2x}{x^2 - 2x - 2} \leq 0$$

$$\frac{x^2 - 2x - 3}{x^2 - 2x - 2} \geq 0 \text{ multiply by } -1 \text{ sign of inequality change}$$

$$\text{roots of } x^2 - 2x - 3 \text{ is } x = \frac{2 \pm \sqrt{4 + 12}}{2} = 3, -1$$

$$\text{roots of } x^2 - 2x - 2 \text{ is } x = 1 \pm \sqrt{3}$$



$$x \in (-\infty, -1] \cup (1 - \sqrt{3}, 1 + \sqrt{3}) \cup [3, \infty) - S_2$$

taking intersection of  $S_1$  &  $S_2$  for final solution.

$$\text{so domain of } f(x) \text{ is } x \in (-\infty, -1] \cup [1 - \sqrt{2}, 1 + \sqrt{2}] \cup [3, \infty)$$

$$\text{compare this with given domain } x \in (-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$$

$$\text{so, } \alpha = -1, \beta = 1 - \sqrt{2}, \gamma = 1 + \sqrt{2}, \delta = 3$$

$$\alpha + \beta + \gamma + \delta = -1 + 1 - \sqrt{2} + 1 + \sqrt{2} + 3 = 4$$

## Question7

**Considering the principal values of inverse trigonometric functions, the value of the expression**

$$\tan \left( 2 \sin^{-1} \left( \frac{2}{\sqrt{13}} \right) - 2 \cos^{-1} \left( \frac{3}{\sqrt{10}} \right) \right)$$

**is equal to :**

**JEE Main 2026 (Online) 28th January Evening Shift**

**Options:**

A.

$$\frac{33}{56}$$

B.

$$-\frac{33}{56}$$

C.

$$-\frac{16}{63}$$

D.

$$\frac{16}{63}$$

**Answer: A**

**Solution:**

Simplify the individual components

Let  $\alpha = \sin^{-1}\left(\frac{2}{\sqrt{13}}\right)$  and  $\beta = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$ .

- For  $\alpha$  : Since  $\sin \alpha = \frac{2}{\sqrt{13}}$ , using Pythagoras' theorem (base =  $\sqrt{(\sqrt{13})^2 - 2^2} = 3$ ), we get:

$$\tan \alpha = \frac{2}{3}$$

- For  $\beta$  : Since  $\cos \beta = \frac{3}{\sqrt{10}}$ , using Pythagoras' theorem (perpendicular =  $\sqrt{(\sqrt{10})^2 - 3^2} = 1$ ), we get:

$$\tan \beta = \frac{1}{3}$$

Rewrite the expression

The expression becomes:  $\tan(2\alpha - 2\beta) = \tan[2(\alpha - \beta)]$ .

First, find  $\tan(\alpha - \beta)$  using the formula  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\tan(\alpha - \beta) = \frac{\frac{2}{3} - \frac{1}{3}}{1 + \left(\frac{2}{3} \cdot \frac{1}{3}\right)} = \frac{\frac{1}{3}}{1 + \frac{2}{9}} = \frac{\frac{1}{3}}{\frac{11}{9}} = \frac{1}{3} \cdot \frac{9}{11} = \frac{3}{11}$$

3. Calculate the final value

Now, use the double angle formula  $\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ , where  $\theta = (\alpha - \beta)$  :

$$\tan[2(\alpha - \beta)] = \frac{2\left(\frac{3}{11}\right)}{1 - \left(\frac{3}{11}\right)^2}$$

$$\tan[2(\alpha - \beta)] = \frac{\frac{6}{11}}{1 - \frac{9}{121}} = \frac{\frac{6}{11}}{\frac{112}{121}}$$

$$\tan[2(\alpha - \beta)] = \frac{6}{11} \cdot \frac{121}{112} = \frac{6 \cdot 11}{112} = \frac{66}{112} = \frac{33}{56}$$


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## Question8

Let  $0 < \alpha < 1$ ,  $\beta = \frac{1}{3\alpha}$  and  $\tan^{-1}(1 - \alpha) + \tan^{-1}(1 - \beta) = \frac{\pi}{4}$ . Then  $6(\alpha + \beta)$  is equal to:

**JEE Main 2026 (Online) 6th April Morning Shift**

**Options:**

- A. 6
- B. 7
- C. 8
- D. 9

**Answer: B**

**Solution:**

Let

$$A = \tan^{-1}(1 - \alpha), \quad B = \tan^{-1}(1 - \beta)$$

and it is given that

$$A + B = \frac{\pi}{4}.$$

Also,

$$0 < \alpha < 1, \quad \beta = \frac{1}{3\alpha}.$$

We have to find  $6(\alpha + \beta)$ .

Using the tangent addition formula,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Since

$$A+B = \frac{\pi}{4},$$

we get

$$\tan(A+B) = 1.$$

Now,

$$\tan A = 1 - \alpha, \quad \tan B = 1 - \beta.$$

So,

$$\frac{(1-\alpha)+(1-\beta)}{1-(1-\alpha)(1-\beta)} = 1.$$

Simplifying the numerator:

$$(1-\alpha) + (1-\beta) = 2 - \alpha - \beta.$$

Now simplify the denominator:

$$1 - (1-\alpha)(1-\beta) = 1 - (1 - \alpha - \beta + \alpha\beta) = \alpha + \beta - \alpha\beta.$$

Hence,

$$\frac{2-\alpha-\beta}{\alpha+\beta-\alpha\beta} = 1.$$

Therefore,

$$2 - \alpha - \beta = \alpha + \beta - \alpha\beta.$$

So,

$$2 = 2\alpha + 2\beta - \alpha\beta.$$

Now use

$$\beta = \frac{1}{3\alpha}.$$

Substituting:

$$2 = 2\alpha + \frac{2}{3\alpha} - \alpha \cdot \frac{1}{3\alpha}.$$

Since

$$\alpha \cdot \frac{1}{3\alpha} = \frac{1}{3},$$

we get

$$2 = 2\alpha + \frac{2}{3\alpha} - \frac{1}{3}.$$

Add  $\frac{1}{3}$  to both sides:

$$\frac{7}{3} = 2\alpha + \frac{2}{3\alpha}.$$

Multiply by  $3\alpha$ :

$$7\alpha = 6\alpha^2 + 2.$$

So,

$$6\alpha^2 - 7\alpha + 2 = 0.$$

Factorising:

$$(3\alpha - 2)(2\alpha - 1) = 0.$$

Thus,

$$\alpha = \frac{2}{3} \text{ or } \alpha = \frac{1}{2}.$$

Now find  $\beta = \frac{1}{3\alpha}$ .

**Case 1:**

If

$$\alpha = \frac{2}{3},$$

then

$$\beta = \frac{1}{2}.$$

So,

$$\alpha + \beta = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}.$$

Hence,

$$6(\alpha + \beta) = 7.$$

**Case 2:**

If

$$\alpha = \frac{1}{2},$$

then

$$\beta = \frac{2}{3}.$$

Again,

$$\alpha + \beta = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}.$$

Hence,

$$6(\alpha + \beta) = 7.$$

Therefore, the required value is

**7**

So, the correct option is **Option B**.

## Question9

If  $\sin \left( \tan^{-1}(x\sqrt{2}) \right) = \cot \left( \sin^{-1} \sqrt{1-x^2} \right)$ ,  $x \in (0, 1)$ , then the value of  $x$  is:

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**Options:**

A.

$$\frac{1}{2}$$

B.

$$\frac{1}{3}$$

C.  $\frac{2}{3}$

D.  $\frac{5}{8}$

**Answer: A**

**Solution:**

Let us solve

$$\sin \left( \tan^{-1}(x\sqrt{2}) \right) = \cot \left( \sin^{-1} \sqrt{1-x^2} \right), \quad x \in (0, 1)$$

step by step.

**Step 1: Simplify the left-hand side**

Let

$$\theta = \tan^{-1}(x\sqrt{2})$$

Then

$$\tan \theta = x\sqrt{2}$$

We know that if  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ , then we can take

- opposite =  $x\sqrt{2}$
- adjacent = 1

So hypotenuse is

$$\sqrt{1 + (x\sqrt{2})^2} = \sqrt{1 + 2x^2}$$

Hence,

$$\sin \theta = \frac{x\sqrt{2}}{\sqrt{1+2x^2}}$$

Therefore,

$$\sin \left( \tan^{-1}(x\sqrt{2}) \right) = \frac{x\sqrt{2}}{\sqrt{1+2x^2}}$$

## Step 2: Simplify the right-hand side

Let

$$\phi = \sin^{-1} \sqrt{1 - x^2}$$

Then

$$\sin \phi = \sqrt{1 - x^2}$$

Since  $x \in (0, 1)$ , we have  $\sqrt{1 - x^2} \in (0, 1)$ , so  $\phi$  is acute.

Now,

$$\cos \phi = \sqrt{1 - \sin^2 \phi} = \sqrt{1 - (1 - x^2)} = \sqrt{x^2} = x$$

because  $x > 0$ .

Therefore,

$$\cot \phi = \frac{\cos \phi}{\sin \phi} = \frac{x}{\sqrt{1-x^2}}$$

So,

$$\cot \left( \sin^{-1} \sqrt{1 - x^2} \right) = \frac{x}{\sqrt{1-x^2}}$$

## Step 3: Equate both sides

Now the given equation becomes

$$\frac{x\sqrt{2}}{\sqrt{1+2x^2}} = \frac{x}{\sqrt{1-x^2}}$$

Since  $x \in (0, 1)$ , we have  $x \neq 0$ , so divide both sides by  $x$ :

$$\frac{\sqrt{2}}{\sqrt{1+2x^2}} = \frac{1}{\sqrt{1-x^2}}$$

#### Step 4: Cross multiply

$$\sqrt{2}\sqrt{1-x^2} = \sqrt{1+2x^2}$$

Now square both sides:

$$2(1-x^2) = 1+2x^2$$

$$2-2x^2 = 1+2x^2$$

$$1 = 4x^2$$

$$x^2 = \frac{1}{4}$$

Since  $x \in (0, 1)$ ,

$$x = \frac{1}{2}$$

#### Final Answer

$$\boxed{\frac{1}{2}}$$

So, the correct option is **Option A**.

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### Question10

Let  $\alpha = 3 \sin^{-1} \left( \frac{6}{11} \right)$  and  $\beta = 3 \cos^{-1} \left( \frac{4}{9} \right)$ , where inverse trigonometric functions take only the principal values.

Given below are two statements :

**Statement I :**  $\cos(\alpha + \beta) > 0$ .

**Statement II :**  $\cos(\alpha) < 0$ .

In the light of the above statements, choose the correct answer from the options given below :

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**Options:**

A.

Both Statement I and Statement II are true

B.

Both Statement I and Statement II are false

C.

Statement I is true but Statement II is false

D.

Statement I is false but Statement II is true

**Answer: A**

**Solution:**

$$\frac{1}{2} < \frac{6}{11} < \frac{1}{\sqrt{2}} \Rightarrow \sin^{-1}\left(\frac{1}{2}\right) < \sin^{-1}\left(\frac{6}{11}\right) < \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \dots (i)$$

$$\Rightarrow 3 \sin^{-1}\left(\frac{1}{2}\right) < 3 \sin^{-1}\left(\frac{6}{11}\right) < 3 \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \frac{\pi}{2} < \alpha < \frac{3\pi}{4} \Rightarrow \cos \alpha < 0 \dots (ii)$$

$$\frac{4}{9} < \frac{1}{2} < \frac{1}{\sqrt{2}} \Rightarrow \cos^{-1}\left(\frac{4}{9}\right) > \frac{\pi}{3} > \frac{\pi}{4}$$

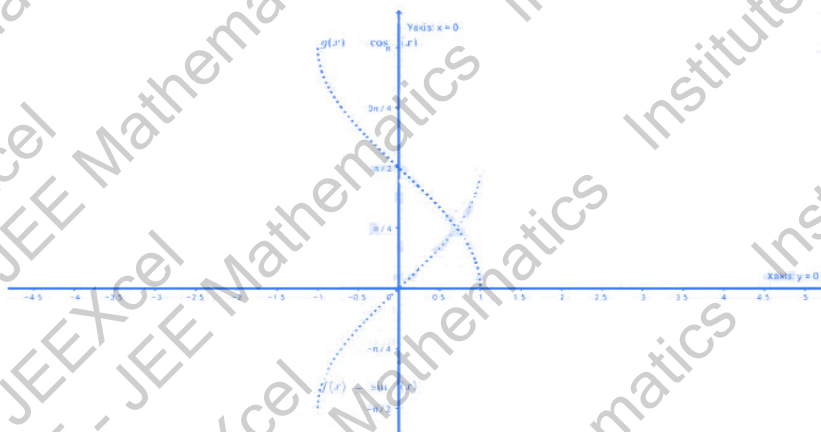
$$\Rightarrow 3 \cos^{-1}\left(\frac{4}{9}\right) > \pi > \frac{3\pi}{4} \Rightarrow \beta > \pi \dots (ii)$$

Inequalities (i) and (ii) shows the mathematical logic for  $\alpha + \beta > \frac{\pi}{2} + \pi$

$$\Rightarrow \cos(\alpha + \beta) > 0 \dots (iii)$$

From equations (i) and (iii) we can conclude :

| Statement | I   | II  |
|-----------|-----|-----|
| True      | Yes | Yes |



## Question11

If for some  $\alpha, \beta; \alpha \leq \beta, \alpha + \beta = 8$  and  $\sec^2(\tan^{-1} \alpha) + \operatorname{cosec}^2(\cot^{-1} \beta) = 36$ , then  $\alpha^2 + \beta$  is \_\_\_\_\_

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**Answer: 14**

**Solution:**

$$\text{If } \left( \tan(\tan^{-1}(\alpha)) + 1(\cot(\cot^{-1} \beta)) \right)^2 = 36$$

$$\alpha^2 + \beta^2 = 34$$

$$\alpha\beta = 15$$

$$\alpha = 3, \beta = 5$$

$$\therefore \alpha^2 + \beta = 9 + 5 = 14$$

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## Question12

Let  $S = \{x : \cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}[2x + 1]\}$ . Then  $\sum_{x \in S} (2x - 1)^2$  is equal to \_\_\_\_\_.

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**Answer: 5**

**Solution:**

$$\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x + 1)$$

$$2 \cos^{-1} x - \sin^{-1}(2x + 1) = \frac{3\pi}{2}$$

$$2\alpha - \beta = \frac{3\pi}{2} \text{ where } \cos^{-1} x = \alpha, \sin^{-1}(2x + 1) = \beta$$

$$2\alpha = \frac{3\pi}{2} + \beta$$

$$\cos 2\alpha = \sin \beta$$

$$2 \cos^2 \alpha - 1 = \sin \beta$$

$$2x^2 - 1 = 2x + 1$$

$$x^2 - x - 1 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{5}}{2}, \left\{ x = \frac{1 + \sqrt{5}}{2} \text{ rejected} \right\}$$

$$\therefore 4x^2 - 4x = 4$$

$$(2x - 1)^2 = 5$$

## Question13

If  $y = \cos \left( \frac{\pi}{3} + \cos^{-1} \frac{x}{2} \right)$ , then  $(x - y)^2 + 3y^2$  is equal to

**JEE Main 2025 (Online) 2nd April Evening Shift**

**Answer: 3**

**Solution:**

$$\begin{aligned} y &= \cos \left( \frac{\pi}{3} + \cos^{-1} \frac{x}{2} \right) \\ &= \cos \left( \frac{\pi}{3} \right) \cos \left( \cos^{-1} \left( \frac{x}{2} \right) \right) - \sin \left( \frac{\pi}{3} \right) \sin \left( \cos^{-1} \left( \frac{x}{2} \right) \right) \\ &= \frac{1}{2} \cdot \frac{x}{2} - \frac{\sqrt{3}}{2} \cdot \sqrt{1 - \frac{x^2}{4}} \\ &\Rightarrow 4y = x - \sqrt{3} \sqrt{4 - x^2} \\ &\Rightarrow (4y - x)^2 = 3(4 - x^2) \\ &\Rightarrow 16y^2 + x^2 - 8xy = 12 - 3x^2 \\ &x^2 + 4y^2 - 2xy = 3 \\ &(x - y)^2 + 3y^2 = 3 \end{aligned}$$

## Question14

Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of

$16 \left( (\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2 \right)$  is :

**JEE Main 2025 (Online) 22nd January Morning Shift**

**Options:**

A.  $24\pi^2$

B.  $18\pi^2$

C.  $22\pi^2$

D.  $31\pi^2$

**Answer: C**

**Solution:**

Let  $f(x) = 16 \left[ (\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2 \right]$ .

We can express  $f(x)$  as:

$$f(x) = 16 \left[ (\sec^{-1} x + \operatorname{cosec}^{-1} x)^2 - 2 (\sec^{-1} x) \left( \frac{\pi}{2} - \sec^{-1} x \right) \right]$$

This simplifies to:

$$f(x) = 16 \left[ \frac{\pi^2}{4} - \pi \sec^{-1} x + 2 (\sec^{-1} x)^2 \right], \quad \text{where } \sec^{-1} x \in [0, \pi] - \left\{ \frac{\pi}{2} \right\}$$

Further simplification gives:

$$f(x) = 16 \left[ 2 (\sec^{-1} x - \frac{\pi}{4})^2 + \frac{\pi^2}{4} - \frac{\pi^2}{8} \right]$$

For the maximum value when  $\sec^{-1} x = \pi$ :

$$\max = 16 \left[ 2\pi^2 - \pi^2 + \frac{\pi^2}{4} \right] = 20\pi^2$$

For the minimum value when  $\sec^{-1} x = \frac{\pi}{4}$ :

$$\min = 16 \left[ \frac{2 \times \pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4} \right] = 2\pi^2$$

Therefore, the sum of the maximum and minimum values is:

$$\text{Sum} = 22\pi^2$$

---

## Question 15

If  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$ , then  $\cos^{-1} \left( \frac{12}{13} \cos x + \frac{5}{13} \sin x \right)$  is equal to

**JEE Main 2025 (Online) 23rd January Morning Shift**

**Options:**

A.  $x + \tan^{-1} \frac{5}{12}$

B.  $x - \tan^{-1} \frac{4}{3}$

C.  $x + \tan^{-1} \frac{4}{5}$

D.  $x - \tan^{-1} \frac{5}{12}$

**Answer: D**

**Solution:**

$$\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$$

$$\cos^{-1} \left( \frac{12}{13} \cos x + \frac{5}{12} \sin x \right)$$

$$\cos^{-1} (\cos x \cos \alpha + \sin x \sin \alpha)$$

$$\cos^{-1} (\cos(x - \alpha))$$

$$\Rightarrow x - \alpha \text{ because } x - \alpha \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$\Rightarrow x - \tan^{-1} \frac{5}{12}$$

---

## Question 16

If  $\alpha > \beta > \gamma > 0$ , then the expression

$\cot^{-1} \left\{ \beta + \frac{(1+\beta^2)}{(\alpha-\beta)} \right\} + \cot^{-1} \left\{ \gamma + \frac{(1+\gamma^2)}{(\beta-\gamma)} \right\} + \cot^{-1} \left\{ \alpha + \frac{(1+\alpha^2)}{(\gamma-\alpha)} \right\}$  is equal to :

## JEE Main 2025 (Online) 24th January Evening Shift

Options:

- A.  $3\pi$
- B.  $\frac{\pi}{2} - (\alpha + \beta + \gamma)$
- C.  $\pi$
- D. 0

Answer: C

Solution:

$$\begin{aligned} &\Rightarrow \cot^{-1} \left( \frac{\alpha\beta + 1}{\alpha - \beta} \right) + \cot^{-1} \left( \frac{\beta\gamma + 1}{\beta - \gamma} \right) + \cot^{-1} \left( \frac{\alpha\gamma + 1}{\gamma - \alpha} \right) \\ &\Rightarrow \tan^{-1} \left( \frac{\alpha - \beta}{1 + \alpha\beta} \right) + \tan^{-1} \left( \frac{\beta - \gamma}{1 + \beta\gamma} \right) + \pi + \tan^{-1} \left( \frac{\gamma - \alpha}{1 + \gamma\alpha} \right) \\ &\Rightarrow (\tan^{-1} \alpha - \tan^{-1} \beta) + (\tan^{-1} \beta - \tan^{-1} \gamma) + (\pi + \tan^{-1} \gamma - \tan^{-1} \alpha) \\ &\Rightarrow \pi \end{aligned}$$

---

## Question17

$\cos \left( \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{33}{65} \right)$  is equal to:

## JEE Main 2025 (Online) 28th January Morning Shift

Options:

- A.  $\frac{33}{65}$
- B. 1
- C.  $\frac{32}{65}$
- D. 0

**Answer: D**

**Solution:**

$$\begin{aligned}& \cos \left( \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{33}{65} \right) \\& \cos \left( \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{33}{56} \right) \\& \cos \left( \tan^{-1} \left( \frac{\frac{3}{4} + \frac{5}{12}}{1 + \frac{3}{4} \cdot \frac{5}{12}} \right) + \tan^{-1} \frac{33}{56} \right) \\& \cos \left( \tan^{-1} \frac{56}{33} + \cot^{-1} \frac{56}{33} \right) \\& \cos \left( \frac{\pi}{2} \right) = 0\end{aligned}$$

---

## Question 18

Let  $[x]$  denote the greatest integer less than or equal to  $x$ . Then the domain of  $f(x) = \sec^{-1}(2[x] + 1)$  is:

**JEE Main 2025 (Online) 28th January Evening Shift**

**Options:**

A.

$$(-\infty, \infty)$$

B.

$$(-\infty, \infty) - \{0\}$$

C.

$$(-\infty, -1] \cup [0, \infty)$$

D.

$$(-\infty, -1] \cup [1, \infty)$$

**Answer: A**

**Solution:**

$$\begin{aligned}
 2[x] + 1 &\leq -1 \text{ or } 2[x] + 1 \geq 1 \\
 \Rightarrow [x] &\leq -1 \cup [x] \geq 0 \\
 \Rightarrow x &\in (-\infty, 0) \cup x \in [0, \infty) \\
 \Rightarrow x &\in (-\infty, \infty)
 \end{aligned}$$


---

## Question19

Considering the principal values of the inverse trigonometric functions,  $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$ ,  $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$ , is equal to

**JEE Main 2025 (Online) 4th April Morning Shift**

**Options:**

A.  $\frac{-5\pi}{6} - \sin^{-1} x$

B.  $\frac{5\pi}{6} - \sin^{-1} x$

C.  $\frac{\pi}{6} + \sin^{-1} x$

D.  $\frac{\pi}{4} + \sin^{-1} x$

**Answer: C**

**Solution:**

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} < x < \frac{1}{\sqrt{2}}$$

$$\text{Let } x = \cos \theta, \theta \in \left(\frac{\pi}{4}, \frac{2\pi}{3}\right)$$

$$\Rightarrow \sqrt{1-x^2} = \sin \theta \text{ as } \sin \theta > 0$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\cos \theta + \frac{1}{2}\sin \theta\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{3} + \theta\right)\right)$$

$$\frac{\pi}{3} + \theta \in \left(\frac{7\pi}{2}, \pi\right)$$

$$\begin{aligned}
 &= \sin^{-1} \left( \sin \left( \pi - \left( \frac{\pi}{3} + \theta \right) \right) \right) \\
 &= \sin^{-1} \left( \sin \left( \frac{2\pi}{3} - \theta \right) \right) \\
 &= \frac{2\pi}{3} - \theta \\
 &= \frac{2\pi}{3} - \cos^{-1} x \\
 &= \frac{2\pi}{3} - \left( \frac{\pi}{2} - \sin^{-1} x \right) \\
 &= \frac{\pi}{6} + \sin^{-1} x
 \end{aligned}$$


---

## Question 20

**The sum of the infinite series**

$$\cot^{-1} \left( \frac{7}{4} \right) + \cot^{-1} \left( \frac{19}{4} \right) + \cot^{-1} \left( \frac{39}{4} \right) + \cot^{-1} \left( \frac{67}{4} \right) + \dots \text{ is :}$$

**JEE Main 2025 (Online) 4th April Evening Shift**

**Options:**

A.  $\frac{\pi}{2} + \cot^{-1} \left( \frac{1}{2} \right)$

B.  $\frac{\pi}{2} - \cot^{-1} \left( \frac{1}{2} \right)$

C.  $\frac{\pi}{2} - \tan^{-1} \left( \frac{1}{2} \right)$

D.  $\frac{\pi}{2} + \tan^{-1} \left( \frac{1}{2} \right)$

**Answer: C**

**Solution:**

$$\cot^{-1} \left( \frac{7}{4} \right) + \cot^{-1} \left( \frac{19}{4} \right) + \cot^{-1} \left( \frac{39}{4} \right) + \cot^{-1} \left( \frac{67}{4} \right) + \dots$$

$$T_r = \cot^{-1} \left( \frac{4r^2 + 3}{4} \right)$$

$$T_r = \tan^{-1} \left( \frac{1}{\left( \frac{3}{4} + r^2 \right)} \right)$$

$$T_r = \tan^{-1} \left( \frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + r^2 - 1/4} \right)$$

$$T_r = \tan^{-1} \left( \frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r + \frac{1}{2}\right) \left(r - \frac{1}{2}\right)} \right)$$

$$T_r = \tan^{-1} \left( r + \frac{1}{2} \right) - \tan^{-1} \left( r - \frac{1}{2} \right)$$

$$T_1 = \tan^{-1} \left( \frac{3}{2} \right) - \tan^{-1} \left( \frac{1}{2} \right)$$

$$T_2 = \tan^{-1} \left( \frac{5}{2} \right) - \tan^{-1} \left( \frac{3}{2} \right)$$

$$\vdots \quad \quad \quad \vdots$$

$$T_n = \tan^{-1} \left( \frac{2n+1}{2} \right) - \tan^{-1} \left( \frac{1}{2} \right)$$

$$\Sigma T_r = \tan^{-1} \left( \frac{2n+1}{2} \right) - \tan^{-1} \left( \frac{1}{2} \right)$$

$$\Sigma T_r = \frac{\pi}{2} - \tan^{-1} \left( \frac{1}{2} \right)$$

## Question 21

The value of  $\cot^{-1} \left( \frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)} \right) - \cot^{-1} \left( \frac{\sqrt{1+\tan^2(\frac{1}{2})}+1}{\tan(\frac{1}{2})} \right)$  is equal to

**JEE Main 2025 (Online) 8th April Evening Shift**

**Options:**

A.

$$\pi - \frac{3}{2}$$

B.

$$\pi + \frac{5}{2}$$

C.

$$\pi - \frac{5}{4}$$

D.

$$\pi + \frac{3}{2}$$

**Answer: C**

**Solution:**

$$\begin{aligned} & \cot^{-1} \left( \frac{|\sec 2| - 1}{\tan 2} \right) - \cot^{-1} \left( \frac{|\sec \frac{1}{2}| + 1}{\tan \frac{1}{2}} \right) \\ &= \cot^{-1} \left( \frac{-1 - \cos 2}{\sin 2} \right) - \cot^{-1} \left( \frac{1 + \cos \frac{1}{2}}{\sin \frac{1}{2}} \right) \\ &= \pi - \cot^{-1}(\cot 1) - \cot^{-1} \left( \cot \frac{1}{4} \right) \\ &= \pi - 1 - \frac{1}{4} = \pi - \frac{5}{4} \end{aligned}$$

---

## Question 22

Considering only the principal values of inverse trigonometric functions, the number of positive real values of  $x$  satisfying

$$\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4} \text{ is :}$$

**[27-Jan-2024 Shift 2]**

**Options:**

A.

More than 2

B.

1

C.

2

D.

0

**Answer: B**

### Solution:

$$\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4}; x > 0$$

$$\Rightarrow \tan^{-1}2x = \frac{\pi}{4} - \tan^{-1}x$$

Taking tan both sides

$$\Rightarrow 2x = \frac{1-x}{1+x}$$

$$\Rightarrow 2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+8}}{8} = \frac{-3 \pm \sqrt{17}}{8}$$

$$\text{Only possible } x = \frac{-3 + \sqrt{17}}{8}$$

---

### Question23

Let  $x = m/n$  ( $m, n$  are co-prime natural numbers) be a solution of the equation  $\cos(2\sin^{-1}x) = 1/9$  and let  $\alpha, \beta$  ( $\alpha > \beta$ ) be the roots of the equation  $mx^2 - nx - m + n = 0$ . Then the point  $(\alpha, \beta)$  lies on the line

[29-Jan-2024 Shift 2]

Options:

A.

$$3x + 2y = 2$$

B.

$$5x - 8y = -9$$

C.

$$3x - 2y = -2$$

D.

$$5x + 8y = 9$$

**Answer: D**

**Solution:**

Assume  $\sin^{-1}x = \theta$

$$\cos(2\theta) = \frac{1}{9}$$

$$\sin \theta = \pm \frac{2}{3}$$

as m and n are co-prime natural numbers,

$$x = \frac{2}{3}$$

i.e.  $m = 2, n = 3$

So, the quadratic equation becomes  $2x^2 - 3x + 1 = 0$  whose roots are  $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2}\right)$  lies on  $5x + 8y = 9$

---

## Question24

For  $\alpha, \beta, \gamma \neq 0$ . If  $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$  and  $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$ , then  $\gamma$  equal to

**[31-Jan-2024 Shift 1]**

**Options:**

A.

$$\sqrt{3}/2$$

B.

$$1/\sqrt{2}$$

C.

$$\frac{\sqrt{3}-1}{2\sqrt{2}}$$

D.

$$\sqrt{3}$$

**Answer: A**

**Solution:**

$$\text{Let } \sin^{-1} \alpha = A, \sin^{-1} \beta = B, \sin^{-1} \gamma = C$$

$$A + B + C = \pi$$

$$(\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$$

$$\frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$$

$$\Rightarrow \cos C = \frac{1}{2}$$

$$\sin C = \gamma$$

$$\cos C = \sqrt{1 - \gamma^2} = \frac{1}{2}$$

$$\gamma = \frac{\sqrt{3}}{2}$$

## Question 25

If  $a = \sin^{-1}(\sin(5))$  and  $b = \cos^{-1}(\cos(5))$ , then  $a^2 + b^2$  is equal to

**[31-Jan-2024 Shift 2]**

**Options:**

A.

$$4\pi^2 + 25$$

B.

$$8\pi^2 - 40\pi + 50$$

C.

$$4\pi^2 - 20\pi + 50$$

D.

$$25$$

**Answer: B**

**Solution:**

$$a = \sin^{-1}(\sin 5) = 5 - 2\pi$$

$$\text{and } b = \cos^{-1}(\cos 5) = 2\pi - 5$$

$$\therefore a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$$

$$= 8\pi^2 - 40\pi + 50$$

---

## Question 26

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$  is equal to

**[24-Jan-2023 Shift 1]**

**Options:**

A.  $\frac{\pi}{4}$

B.  $\frac{\pi}{2}$

C.  $\frac{\pi}{3}$

D.  $\frac{\pi}{6}$

**Answer: C**

**Solution:**

$$\begin{aligned} & \tan^{-1} \left( \frac{1 + \sqrt{3}}{3 + \sqrt{3}} \right) + \sec^{-1} \left( \sqrt{\frac{8 + 4\sqrt{3}}{6 + 3\sqrt{3}}} \right) \\ &= \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) + \sec^{-1} \left( \frac{2}{\sqrt{3}} \right) = \frac{\pi}{3} \end{aligned}$$


---

## Question27

If the sum of all the solutions of

$$\tan^{-1} \left( \frac{2x}{1-x^2} \right) + \cot^{-1} \left( \frac{1-x^2}{2x} \right) = \frac{\pi}{3}$$

$-1 < x < 1$ ,  $x \neq 0$ , is  $\alpha - \frac{4}{\sqrt{3}}$ , then  $\alpha$  is equal to \_\_\_\_\_.

[25-Jan-2023 Shift 1]

**Answer: 2**

**Solution:**

Case I :  $x > 0$

$$\tan^{-1} \frac{2x}{1-x^2} + \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$$

$$x = 2 - \sqrt{3}$$

Case II :  $x < 0$

$$\tan^{-1} \frac{2x}{1-x^2} + \tan^{-1} \frac{2x}{1-x^2} + \pi = \frac{\pi}{3}$$

$$x = \frac{-1}{\sqrt{3}} \Rightarrow \alpha = 2$$


---

## Question28

Let  $a_1 = 1, a_2, a_3, a_4, \dots$  be consecutive natural numbers. Then

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$  is equal to

[30-Jan-2023 Shift 2]

Options:

A.  $\frac{\pi}{4} - \cot^{-1}(2022)$

B.  $\cot^{-1}(2022) - \frac{\pi}{4}$

C.  $\tan^{-1}(2022) - \frac{\pi}{4}$

D.  $\frac{\pi}{4} - \tan^{-1}(2022)$

Answer: 0

Solution:

Sol.  $a_2 - a_1 = a_3 - a_2 = \dots = a_{2022} - a_{2021} = 1$ .

$$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021}a_{2022}}\right)$$

$$= [(\tan^{-1}a_2) - \tan^{-1}a_1] + [\tan^{-1}a_3 - \tan^{-1}a_2] + \dots$$

$$+ [\tan^{-1}a_{2022} - \tan^{-1}a_{2021}]$$

$$= \tan^{-1}a_{2022} - \tan^{-1}a_1$$

$$= \tan^{-1}(2022) - \tan^{-1}1 = \tan^{-1}2022 - \frac{\pi}{4} \text{ (option 3)}$$

$$= \left(\frac{\pi}{2} - \cot^{-1}(2022)\right) - \frac{\pi}{4}$$

$$= \frac{\pi}{4} - \cot^{-1}(2022) \text{ (option 1)}$$

## Question29

If  $\sin^{-1}\frac{\alpha}{17} + \cos^{-1}\frac{4}{5} - \tan^{-1}\frac{77}{36} = 0, 0 < \alpha < 13$ , then

$\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$  is equal to

[31-Jan-2023 Shift 1]

**Options:**

- A.  $\pi$
- B. 16
- C. 0
- D.  $16 - 5\pi$

**Answer: A**

**Solution:**

$$\cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4}$$

$$\therefore \sin^{-1} \frac{\alpha}{17} = \tan^{-1} \frac{77}{36} - \tan^{-1} \frac{3}{4} = \tan^{-1} \left( \frac{\frac{77}{36} - \frac{3}{4}}{1 + \frac{77}{36} \cdot \frac{3}{4}} \right)$$

$$\sin^{-1} \frac{\alpha}{17} = \tan^{-1} \frac{8}{15} = \sin^{-1} \frac{8}{17}$$

$$\Rightarrow \frac{\alpha}{17} = \frac{8}{17} \Rightarrow \alpha = 8$$

$$\begin{aligned} \therefore \sin^{-1}(\sin 8) + \cos^{-1}(\cos 8) \\ = 3\pi - 8 + 8 - 2\pi \\ = \pi \end{aligned}$$

---

## Question 30

**If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such GPs is**  
**[31-Jan-2023 Shift 1]**

**Options:**

- A. 7
- B.  $\frac{9}{2}$
- C. 3
- D. 14

**Answer: A**

**Solution:**

$$a, ar, ar^2, ar^3 (a, r > 0)$$

$$a^4 r^6 = 1296$$

$$a^2 r^3 = 36$$

$$a = \frac{6}{r^{3/2}}$$

$$a + ar + ar^2 + ar^3 = 126$$

$$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6} = 21$$

$$(r^{-3/2} + r^{3/2}) + (r^{1/2} + r^{-1/2}) = 21$$

$$r^{1/2} + r^{-1/2} = A$$

$$r^{-3/2} + r^{3/2} + 3A = A^3$$

$$A^3 - 3A + A = 21$$

$$A^3 - 2A = 21$$

$$A = 3$$

$$\sqrt{r} + \frac{1}{\sqrt{r}} = 3$$

$$r_r + 1 = 3\sqrt{r}$$

$$r^2 + 2r + 1 = 9r$$

$$r^2 - 7r + 1 = 0$$

---

## Question 31

Let  $(a, b) \subset (0, 2\pi)$  be the largest interval for which

$\sin^{-1}(\sin \theta) - \cos^{-1}(\sin \theta) > 0, \theta \in (0, 2\pi)$  holds. If

$$\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}$$

$$(x^2 - 6x + 10) = 0$$

and  $\alpha - \beta = b - a$ , then  $\alpha$  is equal to :

[31-Jan-2023 Shift 2]

**Options:**

A.  $\frac{\pi}{48}$

B.  $\frac{\pi}{16}$

C.  $\frac{\pi}{8}$

D.  $\frac{\pi}{12}$

**Answer: D**

**Solution:**

$$\sin^{-1} \sin \theta - \left( \frac{\pi}{2} - \sin^{-1} \sin \theta \right) > 0$$

$$\Rightarrow \sin^{-1} \sin \theta > \frac{\pi}{4}$$

$$\Rightarrow \sin \theta > \frac{1}{\sqrt{2}}$$

$$\text{So, } \theta \in \left( \frac{\pi}{4}, \frac{3\pi}{4} \right)$$

$$\theta \in \left( \frac{\pi}{4}, \frac{3\pi}{4} \right) = (a, b)$$

$$\Rightarrow \alpha - a = \frac{\pi}{2} = \alpha - \beta$$

$$\Rightarrow \beta = \alpha - \frac{\pi}{2}$$

$$x + \beta x + \sin^2[(x-3)^2 + 1] + \cos^{-1}[(x-3)^2 + 1] = 0$$

$$x = 3, 9\alpha + 3\beta + \frac{\pi}{2} + 0 = 0$$

$$\Rightarrow 9\alpha + 3\left(\alpha - \frac{\pi}{2}\right) + \frac{\pi}{2} = 0$$

$$\Rightarrow 12\alpha - \pi = 0$$

$$\alpha = \frac{\pi}{12}$$

## Question 32

Let S be the set of all solutions of the equation

$$\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi, \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right].$$

Then  $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$  is equal to

[1-Feb-2023 Shift 1]

**Options:**

A. 0

B.  $\frac{-2\pi}{3}$

C.  $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D.  $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

**Answer: B**

**Solution:**

$$\cos^{-1}(2x) = \pi + 2\cos^{-1}\left(\sqrt{1-x^2}\right)$$

$$\text{LHS} = [0, \pi]$$

For equation to be meaningful  $\cos^{-1}2x = \pi$  and  $\cos^{-1}\left(\sqrt{1-x^2}\right) = 0 \Rightarrow x = \frac{-1}{2}$  and  $x = 0$

which is not possible

$$\therefore x \in \emptyset$$

$$\text{Now } \Sigma(x) = 0$$

$\therefore$  Sum over empty set is always 0

### Question33

Let  $S = \{x \in \mathbb{R} : 0 < x < 1, \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\}$ . If

$n(S)$  denotes the number of elements in  $S$  then :

[1-Feb-2023 Shift 2]

**Options:**

A.  $n(S) = 2$  and only one element in  $S$  is less than  $\frac{1}{2}$ .

B.  $n(S) = 1$  and the element in  $S$  is more than  $\frac{1}{2}$ .

C.  $n(S) = 1$  and the element in  $S$  is less than  $\frac{1}{2}$ .

D.  $n(S) = 0$

**Answer: C**

**Solution:**

$$0 < x < 1$$

$$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\tan^{-1}x = \theta \in \left(0, \frac{\pi}{4}\right) \therefore x = \tan \theta$$

$$2\tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) = \cos^{-1}(\cos 2\theta)$$

$$2\left(\frac{\pi}{4} - \theta\right) = 2\theta \therefore 4\theta = \frac{\pi}{2} \therefore \theta = \frac{\pi}{8}$$

$$x = \tan \frac{\pi}{8} \therefore x = \sqrt{2} - 1 \approx 0.414$$

## Question 34

If  $S = \left\{ x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4} \right\}$ , then is equal to \_\_\_\_\_.

[13-Apr-2023 shift 1]

**Answer: 4**

**Solution:**

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$$

$$\therefore \frac{t}{\sqrt{t^2+1}} \in (-1, 1)$$

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right) = \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) + \frac{\pi}{4}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \left(\frac{1}{\sqrt{2}}\right) \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)\right) + \frac{1}{\sqrt{2}}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{x^2+1}} + \frac{x}{\sqrt{x^2+1}}\right)$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{2}}\left(\frac{1+x}{\sqrt{x^2+1}}\right)$$

After solving this equation, we get

$$x = -1 \text{ or } x = 0$$

$$S = \{-1, 0\}$$

$$\sum_{x \in R} \left( \sin\left((x^2+x+5)\frac{\pi}{2}\right) - \cos((x^2+x+5)\pi) \right)$$

$$= \left[ \sin\left(\frac{5\pi}{2}\right) - \cos(5\pi) \right] + \left[ \sin\left(\frac{5\pi}{2}\right) - \cos(5\pi) \right] = 4$$

## Question 35

For  $x \in (-1, 1]$ , the number of solutions of the equation

$\sin^{-1} x = 2 \tan^{-1} x$  is equal to \_\_\_\_\_.

[13-Apr-2023 shift 2]

**Answer: 2**

### Solution:

$$\sin^{-1}x = 2\tan^{-1}x$$

$$\sin^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow x = \frac{2x}{1+x^2}$$

$$\Rightarrow x\left(\frac{2}{1+x^2} - 1\right) = 0$$

$$\Rightarrow x = 0, 1, -1 \text{ but } -1 \text{ is not included.}$$

Answer 2 solutions

---

### Question36

Let  $x * y = x^2 + y^3$  and  $(x * 1) * 1 = x * (1 * 1)$ . Then a value of

$$2\sin^{-1}\left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2}\right) \text{ is}$$

[24-Jun-2022-Shift-2]

Options:

A.  $\frac{\pi}{4}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{2}$

D.  $\frac{\pi}{6}$

Answer: B

### Solution:

Given,

$$x * y = x^2 + y^3$$

$$\therefore x * 1 = x^2 + 1^3 = x^2 + 1$$

$$\text{Now, } (x * 1) * 1 = (x^2 + 1) * 1$$

$$\Rightarrow (x * 1) * 1 = (x^2 + 1)^2 + 1^3$$

$$\Rightarrow (x * 1) * 1 = x^4 + 1 + 2x^2 + 1$$

$$\text{Also, } x * (1 * 1)$$

$$= x * (1^2 + 1^3)$$

$$= x * 2$$

$$= x^2 + 2^3$$

$$= x^2 + 8$$

Given that,

$$(x * 1) * 1 = x * (1 * 1)$$

$$\therefore x^4 + 1 + 2x^2 + 1 = x^2 + 8$$

$$\Rightarrow x^4 + x^2 - 6 = 0$$

$$\Rightarrow x^4 + 3x^2 - 2x^2 - 6 = 0$$

$$\Rightarrow x^2(x^2 + 3) - 2(x^2 + 3) = 0$$

$$\Rightarrow (x^2 + 3)(x^2 - 2) = 0$$

$$\Rightarrow x^2 = 2, -3$$

[ $x^2 = -3$ . not possible as square of anything should be always possible]

$$\therefore x^2 = 2$$

$\therefore$  Now,

$$\begin{aligned}
 & 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) \\
 &= 2\sin^{-1}\left(\frac{2^2+2-2}{2^2+2+2}\right) \\
 &= 2\sin^{-1}\left(\frac{4}{8}\right) \\
 &= 2\sin^{-1}\left(\frac{1}{2}\right) \\
 &= 2 \times \frac{\pi}{6} \\
 &= \frac{\pi}{3}
 \end{aligned}$$


---

### Question37

The value of  $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$  is equal to :

[25-Jun-2022-Shift-2]

Options:

A.  $-\frac{\pi}{4}$

B.  $-\frac{\pi}{8}$

C.  $-\frac{5\pi}{12}$

D.  $-\frac{4\pi}{9}$

**Answer: B**

**Solution:**

$$\begin{aligned}
 & \tan^{-1} \left( \frac{\cos\left(\frac{15\pi}{4}\right) - 1}{\sin\frac{\pi}{4}} \right) \\
 &= \tan^{-1} \left( \frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}}} \right) \\
 &= \tan^{-1}(1 - \sqrt{2}) = -\tan^{-1}(\sqrt{2} - 1) \\
 &= -\frac{\pi}{8}
 \end{aligned}$$


---

### Question38

If the inverse trigonometric functions take principal values then

$\cos^{-1} \left( \frac{3}{10} \cos \left( \tan^{-1} \left( \frac{4}{3} \right) \right) + \frac{2}{5} \sin \left( \tan^{-1} \left( \frac{4}{3} \right) \right) \right)$  is equal to:

[26-Jun-2022-Shift-2]

Options:

A. 0

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{3}$

D.  $\frac{\pi}{6}$

**Answer: C**

**Solution:**

$$\begin{aligned}
 & \cos^{-1} \left( \frac{3}{10} \cos \left( \tan^{-1} \left( \frac{4}{3} \right) \right) + \frac{2}{5} \sin \left( \tan^{-1} \left( \frac{4}{3} \right) \right) \right) \\
 &= \cos^{-1} \left( \frac{3}{10} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5} \right) \\
 &= \cos^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3}
 \end{aligned}$$

---

### Question39

$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) + \cos^{-1}\left(\cos \frac{7\pi}{6}\right) + \tan^{-1}\left(\tan \frac{3\pi}{4}\right)$  is equal to:

[27-Jun-2022-Shift-1]

Options:

A.  $\frac{11\pi}{12}$

B.  $\frac{17\pi}{12}$

C.  $\frac{31\pi}{12}$

D.  $-\frac{3\pi}{4}$

Answer: A

Solution:

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) + \cos^{-1}\left(\cos \frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos \frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) + \cos^{-1}\cos \frac{7\pi}{6} + \tan^{-1}\tan \frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

---

### Question40

The value of  $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$  is

[27-Jun-2022-Shift-2]

**Options:**

A.  $\frac{26}{25}$

B.  $\frac{25}{26}$

C.  $\frac{50}{51}$

D.  $\frac{52}{51}$

**Answer: A**

**Solution:**

$$\begin{aligned} & \cot \left( \sum_{n=1}^{50} \tan^{-1} \left( \frac{1}{1+n+n^2} \right) \right) \\ &= \cot \left( \sum_{n=1}^{50} \tan^{-1} \left( \frac{(n+1)-n}{1+(n+1)n} \right) \right) \\ &= \cot \left( \sum_{n=1}^{50} (\tan^{-1}(n+1) - \tan^{-1}n) \right) \\ &= \cot(\tan^{-1}51 - \tan^{-1}1) \\ &= \cot \left( \tan^{-1} \left( \frac{51-1}{1+51} \right) \right) \\ &= \cot \left( \cot^{-1} \left( \frac{52}{50} \right) \right) \\ &= \frac{26}{25} \end{aligned}$$

---

## Question41

$50 \tan \left( 3 \tan^{-1} \left( \frac{1}{2} \right) + 2 \cos^{-1} \left( \frac{1}{\sqrt{5}} \right) \right) + 4\sqrt{2} \tan \left( \frac{1}{2} \tan^{-1}(2\sqrt{2}) \right)$  is equal

to  
[29-Jun-2022-Shift-1]

Answer: 29

**Solution:**

$$\begin{aligned} & 50 \tan \left( 3 \tan^{-1} \frac{1}{2} + 2 \cos^{-1} \frac{1}{\sqrt{5}} \right) \\ & + 4\sqrt{2} \tan \left( \frac{1}{2} \tan^{-1} 2\sqrt{2} \right) \\ & = 50 \tan \left( \tan^{-1} \frac{1}{2} + 2 \left( \tan^{-1} \frac{1}{2} + \tan^{-1} 2 \right) \right) \\ & + 4\sqrt{2} \tan \left( \frac{1}{2} \tan^{-1} 2\sqrt{2} \right) \\ & = 50 \tan \left( \tan^{-1} \frac{1}{2} + 2 \cdot \frac{\pi}{2} \right) + 4\sqrt{2} \times \frac{1}{\sqrt{2}} \\ & = 50 \left( \tan \tan^{-1} \frac{1}{2} \right) + 4 \\ & = 25 + 4 = 29 \end{aligned}$$

---

## Question 42

The set of all values of  $k$  for which

$(\tan^{-1} x)^3 + (\cot^{-1} x)^3 - k\pi^3$ ,  $x \in \mathbb{R}_u$  is the interval :

[24-Jun-2022-Shift-1]

**Options:**

A.  $\left[ \frac{1}{32}, \frac{7}{8} \right)$

B.  $\left( \frac{1}{24}, \frac{13}{16} \right)$

C.  $\left[ \frac{1}{48}, \frac{13}{16} \right]$

D.  $\left[ \frac{1}{32}, \frac{9}{8} \right)$

**Answer: A**

**Solution:**

$$(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3$$

$$\text{Let } f(t) = t^3 + \left( \frac{\pi}{2} - t \right)^3$$

$$\text{Where } t = \tan^{-1} x, x \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$= t^3 + \left( \frac{\pi}{2} \right)^3 - \frac{3\pi^2 t}{4} + \frac{3\pi^2}{2} t^2 - t^3$$

$$f(t) = \frac{3\pi^2}{2} t^2 - \frac{3\pi^2}{4} t + \frac{\pi^3}{8}$$

This is a quadratic equation of  $t$ .

Here, coefficient of  $t^2$  term is  $\frac{3\pi}{2}$  which is  $> 0$ .

$\therefore$  It is a upward parabola.

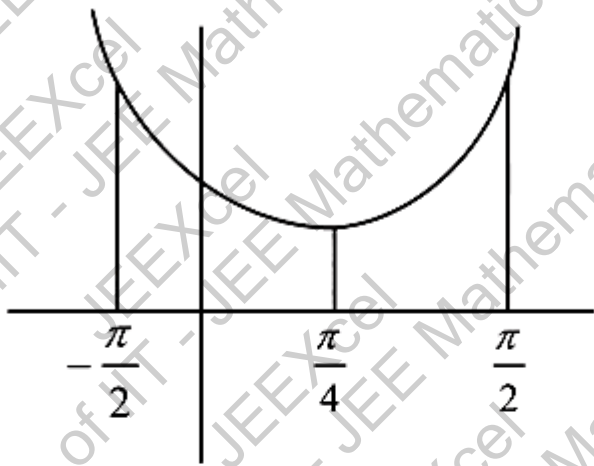
$$\text{Now, } f'(t) = 3\pi t - \frac{3\pi^2}{4}$$

$$f''(t) = 3\pi > 0$$

$$\therefore 3\pi t - \frac{3\pi^2}{4} = 0$$

$$\Rightarrow t = \frac{\pi}{4} \text{ (minima)}$$

$\therefore$  vertex of graph at  $\frac{\pi}{4}$



∴ Minimum value at  $\frac{\pi}{4}$  and maximum value at  $-\frac{\pi}{2}$ .

$$\therefore f\left(\frac{\pi}{4}\right) = \frac{\pi^3}{64} + \left(\frac{\pi}{2} - \frac{\pi}{4}\right)^3 = \frac{\pi^3}{32}$$

$$\begin{aligned} f\left(-\frac{\pi}{2}\right) &= -\frac{\pi^3}{8} + \pi^3 \\ &= \frac{7\pi^3}{8} \end{aligned}$$

$$\therefore k\pi^3 \in \left[ \frac{\pi^3}{32}, \frac{7\pi^3}{8} \right)$$

$$\Rightarrow k \in \left[ \frac{1}{32}, \frac{7}{8} \right)$$

## Question43

Let  $x = \sin(2\tan^{-1}\alpha)$  and  $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$ . If

$S = \{a \in \mathbb{R} : y^2 = 1 - x\}$ , then  $\sum_{\alpha \in S} 16\alpha^3$  is equal to

[25-Jul-2022-Shift-2]

**Answer: 130**

**Solution:**

$$\because x = \sin(2\tan^{-1}\alpha) = \frac{2\alpha}{1+\alpha^2} \dots\dots (i)$$

$$\text{and } y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}}$$

$$\text{Now, } y^2 = 1 - x$$

$$\frac{1}{5} = 1 - \frac{2\alpha}{1+\alpha^2}$$

$$\Rightarrow 1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha$$

$$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$$

$$\therefore \alpha = 2, \frac{1}{2}$$

$$\therefore \sum_{\alpha \in S} 16\alpha^3 = 16 \times 2^3 + 16 \times \frac{1}{2^3} = 130$$

## Question 44

$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$  is equal to :

[26-Jul-2022-Shift-1]

Options:

A. 1

B. 2

C.  $\frac{1}{4}$

D.  $\frac{5}{4}$

**Answer: B**

**Solution:**

$$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$$

$$= \tan\left(2\tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \cdot \frac{1}{8}}\right) + \sec^{-1}\frac{\sqrt{5}}{2}\right)$$

$$= \tan\left[2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right]$$

$$\begin{aligned}
 &= \tan \left[ \tan^{-1} \frac{\frac{2}{3}}{1 - \frac{1}{9}} + \tan^{-1} \frac{1}{2} \right] \\
 &= \tan \left[ \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{2} \right] \\
 &= \tan \left[ \tan^{-1} \frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{8}} \right] = \tan \left[ \tan^{-1} \frac{\frac{5}{4}}{\frac{5}{8}} \right] \\
 &= \tan[\tan^{-1} 2] = 2
 \end{aligned}$$


---

## Question 45

If  $0 < x < \frac{1}{\sqrt{2}}$  and  $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$ , then the value of  $\sin \left( \frac{2\pi\alpha}{\alpha + \beta} \right)$  is

[26-Jul-2022-Shift-2]

Options:

- A.  $4\sqrt{(1-x^2)(1-2x^2)}$
- B.  $4x\sqrt{(1-x^2)(1-2x^2)}$
- C.  $2x\sqrt{(1-x^2)(1-4x^2)}$
- D.  $4\sqrt{(1-x^2)(1-4x^2)}$

**Answer: B**

**Solution:**

$$\text{Let } \frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k \Rightarrow \sin^{-1} x + \cos^{-1} x = k(\alpha + \beta) \Rightarrow \alpha + \beta = \frac{\pi}{2k}$$

$$\text{Now, } \frac{2\pi\alpha}{\alpha + \beta} = \frac{2\pi\alpha}{\frac{\pi}{2k}} = 4k\alpha = 4\sin^{-1} x$$

$$\text{Here } \sin \left( \frac{2\pi\alpha}{\alpha + \beta} \right) = \sin(4\sin^{-1} x)$$

$$\text{Let } \sin^{-1} x = \theta$$

$$\because x \in \left( 0, \frac{1}{\sqrt{2}} \right) \Rightarrow \theta \in \left( 0, \frac{\pi}{4} \right)$$

$$\Rightarrow x = \sin \theta$$

$$\Rightarrow \cos \theta = \sqrt{1-x^2}$$

$$\Rightarrow \sin 2\theta = 2x \cdot \sqrt{1-x^2}$$

$$\Rightarrow \cos 2\theta = \sqrt{1-4x^2(1-x^2)} = \sqrt{(2x^2-1)^2} = 1-2x^2$$

$$\because (\cos 2\theta > 0, \text{ as } 2\theta \in (0, \frac{\pi}{2}))$$

$$\Rightarrow \sin 4\theta = 2 \cdot 2x \sqrt{1-x^2}(1-2x^2)$$

$$= 4x \sqrt{1-x^2}(1-2x^2)$$

## Question 46

For  $k \in \mathbb{R}$ , let the solutions of the equation

$$\cos(\sin^{-1}(x \cot(\tan^{-1}(\cos(\sin^{-1}x)))) = k, 0 < |x| < \frac{1}{\sqrt{2}} \text{ be } \alpha \text{ and } \beta, \text{ where}$$

the inverse trigonometric functions take only principal values. If the

solutions of the equation  $x^2 - bx - 5 = 0$  are  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$  and  $\frac{\alpha}{\beta}$ , then  $\frac{b}{k^2}$  is

equal to \_\_\_\_\_.

[27-Jul-2022-Shift-1]

**Answer: 12**

**Solution:**

$$\cos(\sin^{-1}(x \cot(\tan^{-1}(\cos(\sin^{-1}x)))) = k.$$

$$\Rightarrow \cos(\sin^{-1}(x \cot(\tan^{-1} \sqrt{1-x^2}))) = k$$

$$\Rightarrow \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = k$$

$$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k$$

$$\Rightarrow \frac{1-2x^2}{1-x^2} = k^2$$

$$\Rightarrow 1-2x^2 = k^2 - k^2x^2$$

$$\therefore x^2 - \left(\frac{k^2-1}{k^2-2}\right) = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2 \left( \frac{k^2 - 2}{k^2 - 1} \right) \dots\dots (1)$$

$$\text{and } \frac{\alpha}{\beta} = -1 \dots\dots (2)$$

$$\therefore 2 \left( \frac{k^2 - 2}{k^2 - 1} \right) (-1) = -5$$

$$\Rightarrow k^2 = \frac{1}{3}$$

$$\text{and } b = S \cdot R = 2 \left( \frac{k^2 - 2}{k^2 - 1} \right) - 1 = 4$$

$$\therefore \frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

## Question 47

Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation

$\cos^{-1}(x) - 2\sin^{-1}(x) - \cos^{-1}(2x)$  is equal to :  
[28-Jul-2022-Shift-1]

**Options:**

A. 0

B. 1

C.  $\frac{1}{2}$

D.  $-\frac{1}{2}$

**Answer: A**

**Solution:**

$$\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x$$

$$\text{For Domain : } x \in \left[ -\frac{1}{2}, \frac{1}{2} \right]$$

$$\cos^{-1}x - 2 \left( \frac{\pi}{2} - \cos^{-1}x \right) = \cos^{-1}(2x)$$

$$\Rightarrow \cos^{-1}x + 2\cos^{-1}x = \pi + \cos^{-1}2x$$

$$\Rightarrow \cos(3\cos^{-1}x) = -\cos(\cos^{-1}2x)$$

$$\Rightarrow 4x^3 = x$$

$$\Rightarrow x = 3, \pm \frac{1}{2}$$

## Question 48

If  $\frac{\sin^{-1}(x)}{a} = \frac{\cos^{-1}(x)}{b} = \frac{\tan^{-1}(y)}{c}$ ,  $0 < x < 1$ , then the value of  $\cos\left(\frac{\pi c}{a+b}\right)$  is

[2021, 26 Feb. Shift-1]

Options:

A.  $\frac{1-y^2}{y\sqrt{y}}$

B.  $1-y^2$

C.  $\frac{1-y^2}{1+y^2}$

D.  $\frac{1-y^2}{2y}$

Answer: C

Solution:

$$\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c} \dots (i)$$

Take first two terms of Eq. (i)

$$\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b}$$

$$\Rightarrow \frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\sin^{-1}x + \cos^{-1}x}{a+b}$$

$$\Rightarrow \frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\pi/2}{a+b}$$

$$\left[ \because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \right]$$

$$\Rightarrow \frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\pi/2}{a+b} = \frac{\tan^{-1}y}{c}$$

Using last two terms,

$$\frac{\tan^{-1}y}{c} = \frac{\pi/2}{a+b}$$

$$\Rightarrow \tan^{-1}y = \frac{\pi c}{2(a+b)}$$

$$\Rightarrow 2\tan^{-1}y = \frac{\pi c}{(a+b)}$$

$$\Rightarrow \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = \frac{\pi c}{a+b}$$

$$\left[ \because 2\tan^{-1}y = \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) \right]$$

$$\Rightarrow \frac{1-y^2}{1+y^2} = \cos\left(\frac{\pi c}{a+b}\right)$$

$$\therefore \cos\left(\frac{\pi c}{a+b}\right) = \frac{1-y^2}{1+y^2}$$

## Question 49

**$\operatorname{cosec}\left[2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$  is equal to**

**[2021, 25 Feb. Shift-II]**

**Options:**

A.  $\frac{56}{33}$

B.  $\frac{65}{33}$

C.  $\frac{65}{56}$

D.  $\frac{75}{56}$

**Answer: C**

**Solution:**

$$\operatorname{cosec}[2\cot^{-1}(5) + \cos^{-1}(4/5)]$$

$$= \operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right]$$

$$\left[ \because \tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right) \right]$$

$$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1 - \left(\frac{1}{5}\right)^2}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right]$$

$$\left[ \therefore 2 \tan^{-1} \theta = \tan^{-1} \left( \frac{2\theta}{1-\theta^2} \right) \right]$$

$$= \operatorname{cosec} \left( \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{4}{5} \right)$$

$$\text{Let } \tan^{-1} \left( \frac{5}{12} \right) = x, \text{ then } \tan x = \frac{5}{12} \text{ gives}$$

$$\sin x = \frac{5}{13}, \cos x = \frac{12}{13}$$

$$\text{Let } \cos^{-1} \left( \frac{4}{5} \right) = y, \text{ then } \cos y = \frac{4}{5} \text{ gives,}$$

$$\sin y = \frac{3}{5}$$

$$\text{Now, } \operatorname{cosec}(x+y) = \frac{1}{\sin(x+y)}$$

$$= \frac{1}{\sin x \cos y + \cos x \sin y}$$

$$= \frac{1}{\left( \frac{5}{13} \right) \left( \frac{4}{5} \right) + \left( \frac{12}{13} \right) \left( \frac{3}{5} \right)} = \frac{65}{56}$$

## Question 50

A possible value of  $\tan \left( \frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right)$  is

[2021, 24 Feb. Shift-11]

Options:

A.  $\frac{1}{\sqrt{7}}$

B.  $2\sqrt{2} - 1$

C.  $\sqrt{7} - 1$

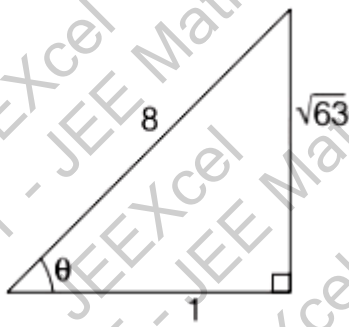
D.  $\frac{1}{2\sqrt{2}}$

**Answer: A**

**Solution:**

Given,  $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let  $\sin^{-1}\frac{\sqrt{63}}{8} = \theta$



$$\Rightarrow \sin \theta = \frac{\sqrt{63}}{8}$$

$$\Rightarrow \cos \theta = \frac{1}{8}$$

Also,  $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$

$$\frac{1 + \frac{1}{8}}{2} = \sqrt{\frac{\frac{9}{8}}{2}} = \frac{3}{4}$$

$$\therefore \tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right)$$

$$= \sqrt{\frac{1 - \cos \frac{\theta}{2}}{1 + \cos \frac{\theta}{2}}}$$

$$= \sqrt{\frac{1 - \frac{3}{4}}{1 + \frac{3}{4}}} = \frac{1}{\sqrt{7}}$$

## Question51

If  $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \dots$  upto 100 terms,  
then  $\alpha$  is  
[2021, 17 March Shift-I]

Options:

A. 1.01

B. 1.00

C. 1.02

D. 1.03

**Answer: A**

**Solution:**

$$\cot^{-1} \alpha = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18$$

+  $\cot^{-1} 32 \dots$  upto 100 terms Let  $T_n$  be the  $n$ th term of  $\cot^{-1} \alpha$ .

$$T_n = \cot^{-1}(2n^2) = \tan^{-1} \left( \frac{1}{2n^2} \right)$$

$$= \tan^{-1} \left[ \frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)} \right]$$

$$\because \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left( \frac{x-y}{1+xy} \right)$$

$$\therefore \tan^{-1} \left\{ \frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)} \right\}$$

$$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$T_1 = \tan^{-1} 3 - \tan^{-1} 1$$

$$T_2 = \tan^{-1} 5 - \tan^{-1} 3$$

$$T_3 = \tan^{-1} 7 - \tan^{-1} 5$$

$\vdots$

$$T_{99} = \tan^{-1} 199 - \tan^{-1} 197$$

$$T_{100} = \tan^{-1} 201 - \tan^{-1} 199$$

$$\Sigma T_r = \tan^{-1} 201 - \tan^{-1} 1$$

$$= \tan^{-1} \left( \frac{201-1}{1+201 \cdot 1} \right) = \tan^{-1} \left( \frac{200}{202} \right)$$

$$\Rightarrow \cot^{-1}(\alpha) = \tan^{-1} \left( \frac{200}{202} \right)$$

$$\Rightarrow \cot^{-1}(\alpha) = \cot^{-1} \left( \frac{202}{200} \right)$$

$$\therefore \alpha = \frac{202}{200} = \frac{101}{100} = 1.01$$

---

## Question 52

The sum of possible values of  $x$  for

$$\tan^{-1}(x+1) + \cot^{-1} \left( \frac{1}{x-1} \right) = \tan^{-1} \left( \frac{8}{31} \right) \text{ is}$$

[2021, 17 March Shift-1]

**Options:**

A.  $\frac{-32}{4}$

B.  $-\frac{31}{4}$

C.  $-\frac{30}{4}$

D.  $-\frac{33}{4}$

**Answer: A**

**Solution:**

$$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}(8/31)$$

$$\Rightarrow \tan^{-1}\left[\frac{(x+1) + (x-1)}{1 - (x+1)(x-1)}\right] = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \tan^{-1}\left[\frac{2x}{1 - (x^2 - 1)}\right] = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \frac{2x}{2 - x^2} = \frac{8}{31}$$

$$\Rightarrow 62x = 16 - 8x^2$$

$$\Rightarrow 8x^2 + 62x - 16 = 0$$

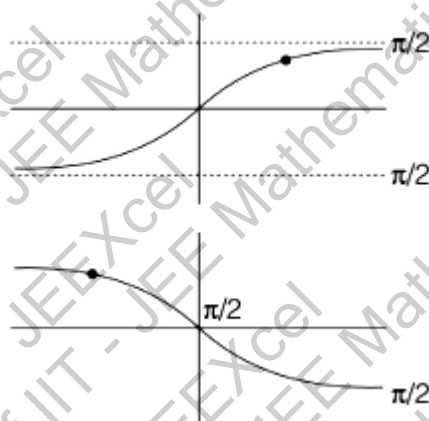
$$\Rightarrow 2(x+8)(4x-1) = 0$$

$$\Rightarrow x = -8, 1/4$$

But at  $x = 1/4$

$$\text{HSS} \Rightarrow \tan^{-1}\left(1 + \frac{1}{4}\right) + \cot^{-1}\left(\frac{1}{\frac{1}{4} - 1}\right)$$

$$= \tan^{-1}\frac{5}{4} + \cot^{-1}\left(\frac{-4}{3}\right)$$



$$= \tan^{-1}\frac{5}{4} + \cot^{-1}\left(\frac{-4}{3}\right) > \pi/2 > \pi/2$$

$$\therefore \text{LHS} > \pi/2$$

$$\text{RHS} = \tan^{-1}\left(\frac{8}{31}\right) < \pi/2$$

As,  $\text{LHS} > \pi/2$  and  $\text{RHS} < \pi/2$ .

So,  $x = -8$  is the only solution.

---

## Question 53

The number of solutions of the equation

$\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$ , for  $x \in [-1, 1]$ , and  $[x]$  denotes the greatest integer less than or equal to  $x$ , is  
[2021, 17 March Shift-II]

Options:

- A. 2
- B. 0
- C. 4
- D. infinite

**Answer: B**

**Solution:**

$$\text{Given, } \sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$$

$$\sin^{-1}\left[\left(x^2 - \frac{2}{3} + 1\right)\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$$

$$\Rightarrow \sin^{-1}\left[\left(x^2 - \frac{2}{3} + 1\right)\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] \dots (i)$$

$$(\because [x+n] = [x] + n, n \in \mathbb{I})$$

$$\therefore \left[x^2 - \frac{2}{3}\right] \text{ gives always integral values.}$$

$$\therefore \left[x^2 - \frac{2}{3}\right] = 0, -1 \text{ are possible values for } x \in [-1, 1]$$

$$[\because -1 \leq x \leq 1]$$

$$\therefore 0 \leq x^2 \leq 1$$

$$\Rightarrow -\frac{2}{3} \leq x^2 - \frac{2}{3} \leq 1 - \frac{2}{3}$$

$$\Rightarrow -\frac{2}{3} \leq x^2 - \frac{2}{3} \leq \frac{1}{3}$$

$$\Rightarrow -0.66 \leq x^2 - \frac{2}{3} \leq 0.33$$

$$\therefore \left[ x^2 - \frac{2}{3} \right] = -1, 0 \text{ are possibilities. } ]$$

$$\text{Case I If } \left[ x^2 - \frac{2}{3} \right] = 0$$

Then, Eq. (i) becomes,

$$\sin^{-1}(1) + \cos^{-1}(0) = x^2$$

$$\Rightarrow x^2 = \pi$$

$$\Rightarrow x = \pm \sqrt{\pi}$$

But at this value of

$$x^2, \left[ x^2 - \frac{2}{3} \right] \neq 0$$

$$\text{Case II If } \left[ x^2 - \frac{2}{3} \right] = -1$$

Then, Eq. (i) becomes,

$$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$$

$$\Rightarrow x^2 = \pi$$

$$\Rightarrow x = \pm \sqrt{\pi}$$

$$\text{But at this value of } x^2, \left[ x^2 - \frac{2}{3} \right] \neq -1$$

$$\therefore x^2 = \pi$$

(Rejected also)

Hence, there is no solution for Eq. (i).

$$\therefore \text{Total number of solution (s)} = 0$$

## Question 54

$$\text{Let } S_k = \sum_{r=1}^k \tan^{-1} \left( \frac{6^r}{2^{2r+1} + 3^{2r+1}} \right).$$

Then,  $\lim_{k \rightarrow \infty} S_k$  is equal to

[2021, 16 March Shift-1]

Options:

A.  $\tan^{-1} \left( \frac{3}{2} \right)$

B.  $\frac{\pi}{2}$

C.  $\cot^{-1} \left( \frac{3}{2} \right)$

D.  $\tan^{-1}(3)$

**Answer: C**

**Solution:**

$$\begin{aligned} S_k &= \sum_{r=1}^k \tan^{-1} \left( \frac{6^r}{2^{2r+1} + 3^{2r+1}} \right) \\ &= \sum \tan^{-1} \frac{2^r 3^{r+1} - 3^r 2^{r+1}}{2^{2r+1} \left[ 1 + \left( \frac{3}{2} \right)^{2r+1} \right]} \\ &= \sum \tan^{-1} \frac{2^{2r+1} \left[ \left( \frac{3}{2} \right)^{r+1} - \left( \frac{3}{2} \right)^r \right]}{2^{2r+1} \left[ 1 + \left( \frac{3}{2} \right)^{2r+1} \right]} \\ &= \sum_{r=1}^k \tan^{-1} \frac{\left( \frac{3}{2} \right)^{r+1} - \left( \frac{3}{2} \right)^r}{1 + \left( \frac{3}{2} \right)^{2r+1}} \\ &= \sum_{r=1}^k \tan^{-1} \left( \frac{3}{2} \right)^{r+1} - \tan^{-1} \left( \frac{3}{2} \right)^r \\ &= \tan^{-1} \left( \frac{3}{2} \right)^2 - \tan^{-1} \left( \frac{3}{2} \right) \\ &\quad \tan^{-1} \left( \frac{3}{2} \right)^3 - \tan^{-1} \left( \frac{3}{2} \right)^2 \\ &\quad \vdots \\ &\quad \tan^{-1} \left( \frac{3}{2} \right)^{k+1} - \tan^{-1} \left( \frac{3}{2} \right)^k \\ S_k &= \tan^{-1} \left( \frac{3}{2} \right)^{k+1} - \tan^{-1} \left( \frac{3}{2} \right) \end{aligned}$$

When  $k \rightarrow \infty$ ,  $\tan^{-1} \left( \frac{3}{2} \right)^{k+1} \rightarrow \pi/2$

$$\lim_{k \rightarrow \infty} S_k = \frac{\pi}{2} - \tan^{-1} \left( \frac{3}{2} \right) = \cot^{-1} \left( \frac{3}{2} \right)$$

## Question 55

**Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy**

$$\sin^{-1} \left( \frac{3x}{2} \right) + \sin^{-1} \left( \frac{4x}{5} \right) = \sin^{-1} x \text{ is equal to}$$

**[2021, 16 March Shift-II]**

**Options:**

A. 2

B. 1

C. 3

D. 0

**Answer: C**

**Solution:**

$$\text{Given, } \sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$$

$$\Rightarrow \sin^{-1}\left(\frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}}\right) = \sin^{-1}x$$

$$\Rightarrow 3x \sqrt{25 - 16x^2} + 4x \sqrt{25 - 9x^2} = 25x$$

$$\Rightarrow \sqrt{225x^2 - 114x^4} + \sqrt{400x^2 - 144x^4} = 25x$$

$$\Rightarrow 25x \quad \text{(i)}$$

$$(400x^2 - 144x^4) - (225x^2 - 144x^4) = 175x^2 \dots \text{(ii)}$$

On dividing Eq. (ii) by Eq. (i),

$$\frac{\sqrt{400x^2 - 144x^4} - \sqrt{225x^2 - 144x^4}}{25x} = 7x \dots \text{(iii)}$$

Now, adding Eqs. (i) and (iii),

$$2\sqrt{400x^2 - 144x^4} = 32x$$

$$\Rightarrow 400x^2 - 144x^4 = 256x^2$$

$$\Rightarrow 144x^2 - 144x^4 = 0$$

$$\Rightarrow 144x^2(1 - x^2) = 0$$

$$x = 0, -1, 1$$

Hence, 3 real values for x satisfies the equation.

## Question56

**The number of real roots of the equation**

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4} \text{ is}$$

**[2021, 20 July Shift-1]**

**Options:**

A. 1

B. 4

C. 3

D. 0

**Answer: D**

**Solution:**

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$$

Domain,  $x(x+1) \geq 0$

$$0 \leq x^2 + x + 1 \leq 1$$

So, only when  $x^2 + x = 0$ , equation will be define(d)

$$x = 0, -1$$

$$\text{At } x = 0, \tan^{-1}0 + \sin^{-1}1 = \frac{\pi}{2}$$

$$x = -1, \tan^{-1}0 + \sin^{-1}1 = \frac{\pi}{2}$$

$\therefore$  No solution.

---

## Question 57

The value of  $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$  is equal to

[2021, 20 July Shift-II]

**Options:**

A.  $\frac{-181}{69}$

B.  $\frac{220}{21}$

C.  $\frac{-291}{76}$

D.  $\frac{151}{63}$

**Answer: B**

**Solution:**

Let

$$A = \tan \left( 2 \tan^{-1} \left( \frac{3}{5} \right) + \sin^{-1} \left( \frac{5}{13} \right) \right) \dots (i)$$

$$\text{Now, using } 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$$

$$2 \tan^{-1} \left( \frac{3}{5} \right) = \tan^{-1} \left( \frac{2 \left( \frac{3}{5} \right)}{1 - \left( \frac{3}{5} \right)^2} \right)$$

$$= \tan^{-1} \left( \frac{\frac{6}{5}}{1 - \frac{9}{25}} \right)$$

$$= \tan^{-1} \left( \frac{30}{16} \right) = \tan^{-1} \left( \frac{15}{8} \right)$$

$$\text{Let } \sin^{-1} \left( \frac{5}{13} \right) = \theta, \text{ then } \sin \theta = \frac{5}{13}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} = \frac{\frac{5}{13}}{\sqrt{1 - \left( \frac{5}{13} \right)^2}} = \frac{\frac{5}{13}}{\frac{12}{13}}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{5}{12} \right) = \sin^{-1} \left( \frac{5}{13} \right)$$

From Eq. (i),

$$A = \tan \left( \tan^{-1} \left( \frac{15}{8} \right) + \tan^{-1} \left( \frac{5}{12} \right) \right)$$

$$= \tan \left[ \tan^{-1} \left( \frac{\frac{15}{8} + \frac{5}{12}}{1 - \left( \frac{15}{8} \right) \left( \frac{5}{12} \right)} \right) \right]$$

$$= \tan \left[ \tan^{-1} \left( \frac{\frac{55}{24}}{\frac{21}{96}} \right) \right]$$

$$= \tan \left[ \tan^{-1} \frac{55 \times 4}{21} \right]$$

$$= \tan \left( \tan^{-1} \frac{220}{21} \right) = \frac{220}{21}$$

---

## Question 58

If  $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = a$ ,  $0 < x < 1$ ,  $a \neq 0$ , then the value of  $2x^2 - 1$  is  
[2021, 27 Aug. Shift-1]

Options:

A.  $\cos\left(\frac{4a}{\pi}\right)$

B.  $\sin\left(\frac{2a}{\pi}\right)$

C.  $\cos\left(\frac{2a}{\pi}\right)$

D.  $\sin\left(\frac{4a}{\pi}\right)$

**Answer: B**

**Solution:**

$$\text{Given, } (\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$$

$$\Rightarrow (\sin^{-1}x + \cos^{-1}x)(\sin^{-1}x - \cos^{-1}x) = a$$

$$\Rightarrow \frac{\pi}{2}(\sin^{-1}x - \cos^{-1}x) = a$$

$$\Rightarrow \frac{\pi}{2} - 2\cos^{-1}x = \frac{2a}{\pi}$$

$$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$$

$$\Rightarrow 2x^2 - 1 = \sin\left(\frac{2a}{\pi}\right)$$

## Question 59

Let  $M$  and  $m$  respectively be the maximum and minimum values of the function  $f(x) = \tan^{-1}(\sin x + \cos x)$  in  $\left[0, \frac{\pi}{2}\right]$ , then the value of  $\tan(M - m)$  is  
[2021, 27 Aug. Shift-II]

**Options:**

A.  $2 + \sqrt{3}$

B.  $2 - \sqrt{3}$

C.  $3 + 2\sqrt{2}$

D.  $3 - 2\sqrt{2}$

**Answer: D**

**Solution:**

We have,  $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\because x \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow 1 \leq \sin x + \cos x \leq \sqrt{2}$$

$$\left[ \because -\sqrt{A^2 + B^2} \leq A \sin x + B \cos x \leq \sqrt{A^2 + B^2} \right]$$

$$\Rightarrow \tan^{-1}(1) \leq \tan^{-1}(\sin x + \cos x) \leq \tan^{-1}(\sqrt{2})$$

$$\therefore m = \tan^{-1}(1) \text{ and } M = \tan^{-1}(\sqrt{2})$$

$$\therefore M - m = \tan^{-1}\sqrt{2} - \tan^{-1}(\sqrt{1})$$

$$= \tan^{-1}\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right)$$

$$= \tan^{-1}(3 - 2\sqrt{2})$$

## Question 60

If  $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$

•  $x \in \left(\frac{\pi}{2}, \pi\right)$ , then  $\frac{dy}{dx}$  at  $x = \frac{5\pi}{6}$  is

**[2021, 27 Aug. Shift-II]**

**Options:**

A.  $-\frac{1}{2}$

B.  $-1$

C.  $\frac{1}{2}$

D. 0

**Answer: A**

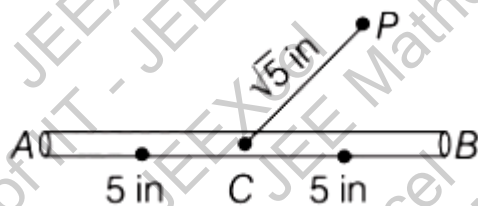
**Solution:**

$$\begin{aligned}y(x) &= \cot^{-1} \left( \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) \\x &\in \left( \frac{\pi}{2}, \pi \right) \\&= \cot^{-1} \frac{\left| \cos \frac{x}{2} + \sin \frac{x}{2} \right| + \left| \cos \frac{x}{2} - \sin \frac{x}{2} \right|}{\left| \cos \frac{x}{2} + \sin \frac{x}{2} \right| - \left| \cos \frac{x}{2} - \sin \frac{x}{2} \right|} \\&= \cot^{-1} \left( \frac{\cos \frac{x}{2} + \sin \frac{x}{2} + \sin \frac{x}{2} - \cos \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2} - \sin \frac{x}{2} + \cos \frac{x}{2}} \right) \\&\left[ \because \frac{x}{2} \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right) \right] \\&= \cot^{-1} \left( \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \right) = \cot^{-1} \left( \tan \frac{x}{2} \right) \\&= \cot^{-1} \left( \cot \left( \frac{\pi}{2} - \frac{x}{2} \right) \right) \\&\Rightarrow y(x) = \frac{\pi}{2} - \frac{x}{2} \\&\therefore y'(x) = \frac{-1}{2}\end{aligned}$$

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## Question 61

A 10 inches long pencil AB with mid-point C and a small eraser P are placed on the horizontal top of a table such that  $PC = \sqrt{5}$  inches and  $\angle PCB = \tan^{-1}(2)$ . The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is



[2021, 26 Aug. Shift-II]

**Options:**

A.  $\tan^{-1}\left(\frac{3}{4}\right)$

B.  $\tan^{-1}(1)$

C.  $\tan^{-1}\left(\frac{4}{3}\right)$

D.  $\tan^{-1}\left(\frac{1}{2}\right)$

**Answer: A**

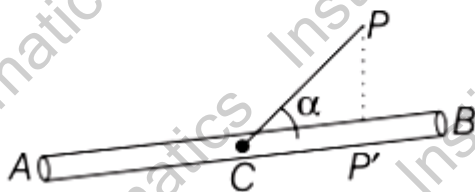
**Solution:**

Let the angle before rotation be  $\theta$ , then

$$\theta = \angle PCB = \tan^{-1}(2)$$



Now, after rotation let angle become  $\alpha$ .



In  $\triangle PCM$

$$PC = \sqrt{5}$$

$$PM = \sqrt{5} \sin \theta = \sqrt{5} \left( \frac{2}{\sqrt{5}} \right) = 2$$

After rotation perpendicular distance becomes  $PP' = 1$

$$PC \sin \alpha = 1$$

$$\sqrt{5} \sin \alpha = 1$$

$$\Rightarrow \sqrt{5} \sin \alpha = 1$$

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \tan \alpha = \frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}} = \frac{1}{2}$$

$\therefore$  Rotated angle

$$= \theta - \alpha = \tan^{-1}(2) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \tan^{-1}\left(\frac{2 - 1/2}{1 + 2 \times 1/2}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

## Question62

$\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$  is equal to (The inverse trigonometric functions take the principal values)  
[2021, 01 Sep. Shift-2]

Options:

A.  $3\pi - 11$

B.  $4\pi - 9$

C.  $4\pi - 11$

D.  $3\pi + 1$

**Answer: C**

**Solution:**

$$\begin{aligned} & \cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) \\ & - \tan^{-1}(\tan(12)) \\ & = 2\pi - 5 + (-2\pi + 6) - (12 - 4\pi) \\ & = 4\pi - 11 \end{aligned}$$

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## Question63

$2\pi - \left( \sin^{-1}\frac{14}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} \right)$  is equal to:

[Sep. 03, 2020 (I)]

Options:

A.  $\frac{\pi}{2}$

B.  $\frac{5\pi}{4}$

C.  $\frac{3\pi}{2}$

D.  $\frac{7\pi}{4}$

**Answer: C**

**Solution:**

$$\begin{aligned}
 & 2\pi - \left( \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} \right) \\
 &= 2\pi - \left( \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{16}{63} \right) \left[ \because \sin^{-1} \frac{4}{5} = \tan^{-1} \frac{4}{3} \right] \\
 &= 2\pi - \left\{ \tan^{-1} \left( \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} \right) + \tan^{-1} \frac{16}{63} \right\} \\
 &= 2\pi - \left( \tan^{-1} \frac{63}{16} + \tan^{-1} \frac{16}{63} \right) \\
 &= 2\pi - \left( \tan^{-1} \frac{63}{16} + \cot^{-1} \frac{63}{16} \right) \\
 &= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}.
 \end{aligned}$$


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## Question 64

If S is the sum of the first 10 terms of the

series  $\tan^{-1} \left( \frac{1}{3} \right) + \tan^{-1} \left( \frac{1}{7} \right) + \tan^{-1} \left( \frac{1}{13} \right) + \tan^{-1} \left( \frac{1}{21} \right) + \dots$  then  $\tan(S)$

is equal to:

[Sep. 05, 2020 (I)]

**Options:**

A.  $\frac{5}{6}$

B.  $\frac{5}{11}$

C.  $-\frac{6}{5}$

D.  $\frac{10}{11}$

**Answer: A**

### Solution:

$$S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \dots \text{ upto 10 terms}$$

$$= \tan^{-1}(2 - 11 + 2 \cdot 1) + \tan^{-1}\left(\frac{3 - 2}{1 + 3 \cdot 2}\right) + \tan^{-1}\left(\frac{4 - 3}{1 + 3 \cdot 4}\right) + \dots + \tan^{-1}\left(\frac{11 - 10}{1 + 11 \cdot 10}\right)$$

$$= (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + \dots + (\tan^{-1}11 - \tan^{-1}10)$$

$$= \tan^{-1}11 - \tan^{-1}1 = \tan^{-1}\left(\frac{11 - 1}{1 + 11 \cdot 1}\right) = \tan^{-1}\left(\frac{5}{6}\right)$$

$$\therefore \tan(S) = \frac{5}{6}$$