

## Question 1

## Determinants

Let  $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ ,  $B = [B_1, B_2, B_3]$ , where  $B_1, B_2, B_3$  are column matrices, and  $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

If  $\alpha = |B|$  and  $\beta$  is the sum of all the diagonal elements of  $B$ , then  $\alpha^3 + \beta^3$  is equal to

**[27-Jan-2024 Shift 1]**

**Options:**

**Answer: 28**

**Solution:**

$$x_1 = 1, y_1 = -1, z_1 = -1$$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$x_2 = 2, y_2 = 1, z_2 = -2$$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$x_3 = 2, y_3 = 0, z_3 = -1$$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\alpha = |B| = 3$$

$$\beta = 1$$

$$\alpha^3 + \beta^3 = 27 + 1 = 28$$

## Question2

The values of  $\alpha$ , for which  $\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$ , lie in the interval

**[27-Jan-2024 Shift 2]**

**Options:**

A.

$(-2, 1)$

B.

$(-3, 0)$

C.

$\left(-\frac{3}{2}, \frac{3}{2}\right)$

D.

$(0, 3)$

**Answer: B**

**Solution:**

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2\alpha + 3) \left\{ \frac{7\alpha}{6} \right\} - (3\alpha + 1) \left\{ \frac{-7}{6} \right\} = 0$$

$$\Rightarrow (2\alpha + 3) \cdot \frac{7\alpha}{6} + (3\alpha + 1) \cdot \frac{7}{6} = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

Hence option (2) is correct.

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### Question3

Let for any three distinct consecutive terms  $a, b, c$  of an A.P, the lines  $ax + by + c = 0$  be concurrent at the point P and  $Q(\alpha, \beta)$  be a point such that the system of equations

$$x + y + z = 6,$$

$$2x + 5y + az = \beta \text{ and}$$

$$x + 2y + 3z = 4,$$

has infinitely many solutions. Then  $(PQ)^2$  is equal to \_\_\_\_

[29-Jan-2024 Shift 2]

**Answer: 113**

**Solution:**

$\therefore a, b, c$  are in A.P

$$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$$

$\therefore ax + by + c$  passes through fixed point  $(1, -2)$

$$\therefore P = (1, -2)$$

For infinite solution,

$$D = D_1 = D_2 = D_3 = 0$$

$$D : \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow a = 8$$

$$D_1 : \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & a \\ 4 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \beta = 6$$

$$\therefore Q = (8, 6)$$

$$\therefore Q^2 = 113$$

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### Question4

If  $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$  then  $\frac{1}{5}f'(0)$  is equal to

**[30-Jan-2024 Shift 1]**

**Options:**

A.

0

B.

1

C.

2

D.

6

**Answer: B**

**Solution:**

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$f(x) = 45$$

$$f'(x) = 0$$

## Question5

Consider the system of linear equation  $x + y + z = 4\mu$ ,  $x + 2y + 2\lambda z = 10\mu$ ,  $x + 3y + 4\lambda^2 z = \mu^2 + 15$  where  $\lambda, \mu \in \mathbb{R}$ . Which one of the following statements is NOT correct?

**[30-Jan-2024 Shift 1]**

**Options:**

A.

The system has unique solution if  $\lambda \neq 1/2$  and  $\mu \neq 1, 15$

B.

The system is inconsistent if  $\lambda = 1/2$  and  $\mu \neq 1$

C.

The system has infinite number of solutions if  $\lambda = 1/2$  and  $\mu = 15$

D.

The system is consistent if  $\lambda \neq 1/2$

**Answer: B**

**Solution:**

$$x + y + z = 4\mu, x + 2y + 2\lambda z = 10\mu, x + 3y + 4\lambda^2 z = \mu^2 + 15$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution  $\Delta \neq 0, 2\lambda - 1 \neq 0, \left(\lambda \neq \frac{1}{2}\right)$

Let  $\Delta = 0, \lambda = \frac{1}{2}$

$$\Delta_y = 0, \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix} \\ = (\mu - 15)(\mu - 1)$$

For infinite solution  $\lambda = \frac{1}{2}, \mu = 1$  or  $15$

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## Question6

**Consider the system of linear equations**

$$x + y + z = 5, x + 2y + \lambda^2 z = 9$$

$x + 3y + \lambda z = \mu$ , where  $\lambda, \mu \in \mathbb{R}$ . Then, which the following statement is NOT correct?

**[30-Jan-2024 Shift 2]**

**Options:**

A.

System has infinite number of solution if  $\lambda =$  and  $\mu = 13$

B.

System is inconsistent if  $\lambda = 1$  and  $\mu \neq 13$

C.

System is consistent if  $\lambda \neq 1$  and  $\mu = 13$

D.

System has unique solution if  $\lambda \neq 1$  and  $\mu \neq 13$

**Answer: D**

**Solution:**

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\lambda = 1, -\frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Infinite solution  $\lambda = 1$  &  $\mu = 13$

For unique sol<sup>n</sup>  $\lambda \neq 1$

For no sol<sup>n</sup>  $\lambda = 1$  &  $\mu \neq 13$

If  $\lambda \neq 1$  and  $\mu \neq 13$

Considering the case when  $\lambda = -\frac{1}{2}$  and  $\mu \neq 13$  this will generate no solution case

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## Question 7

**If the system of linear equations**

$$x - 2y + z = -4$$

$$2x + \alpha y + 3z = 5$$

$$3x - y + \beta z = 3$$

has infinitely many solutions, then  $12\alpha + 13\beta$  is equal to

**[31-Jan-2024 Shift 1]**

**Options:**

A.

60

B.

64

C.

54

D.

58

**Answer: D**

**Solution:**

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$$

$$= \alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha$$

For infinite solutions  $D = 0$ ,  $D_1 = 0$ ,  $D_2 = 0$  and

$$D_3 = 0$$

$$D = 0$$

$$\alpha\beta - 3\alpha + 4\beta = 17 \dots\dots (1)$$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$$

$$13\beta - 9 - 36 - 9 = 0$$

$$13\beta = 54, \beta = \frac{54}{13} \text{ put in (1)}$$

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$$54\alpha - 39\alpha + 216 = 221$$

$$15\alpha = 5 \quad \alpha = \frac{1}{3}$$

$$\text{Now, } 12\alpha + 13\beta = 12 \cdot \frac{1}{3} + 13 \cdot \frac{54}{13}$$

$$= 4 + 54 = 58$$

## Question8

$$\text{If } f(x) = \begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 3x^2 + 2 & 2x & x^3 + 6 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix} \text{ for all } x \in \mathbb{R}, \text{ then } 2f'(0) + f'(0) \text{ is equal to}$$

**[31-Jan-2024 Shift 1]**

**Options:**

A.

48

B.

24

C.

42

D.

18

**Answer: C**

**Solution:**

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 12$$

$$f'(x) = \begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$\therefore f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= 24 - 6 = 18$$

$$\therefore 2f(0) + f'(0) = 42$$

## Question9

Let  $A$  be a  $3 \times 3$  real matrix such that

$$A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, A \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 4 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Then, the system  $(A - 3I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  has

**[31-Jan-2024 Shift 2]**

**Options:**

A.  
unique solution

B.  
exactly two solutions

C.  
no solution

D.

infinitely many solutions

**Answer: A**

**Solution:**

$$\text{Let } A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$\text{Given } A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \dots\dots(1)$$

$$\therefore \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \\ x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore x_1 + z_1 = 2 \dots\dots\dots(2)$$

$$x_2 + z_2 = 0 \dots\dots\dots(3)$$

$$x_3 + z_3 = 0 \dots\dots\dots(4)$$

$$\text{Given } A \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -x_1 + z_1 \\ -x_2 + z_2 \\ -x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow -x_1 + z_1 = 4 \dots\dots\dots(5)$$

$$-x_2 + z_2 = 0 \dots\dots\dots(6)$$

$$-x_3 + z_3 = 4 \dots\dots\dots(7)$$

$$\text{Given } A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore y_1 = 0, y_2 = 2, y_3 = 0$$

$$\therefore \text{ from (2), (3), (4), (5), (6) and (7)}$$

$$x_1 = 3x, x_2 = 0, x_3 = -1$$

$$y_1 = 0, y_2 = 2, y_3 = 0$$

$$z_1 = -1, z_2 = 0, z_3 = 3$$

$$\therefore A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\therefore \text{ Now } (A - 3I) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -z \\ -y \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$[z = -1], [y = -2], [x = -3]$$

## Question 10

If the system of equations

$$2x + 3y - z = 5$$

$$x + \alpha y + 3z = -4$$

$$3x - y + \beta z = 7$$

has infinitely many solutions, then  $13\alpha\beta$  is equal to

[1-Feb-2024 Shift 1]

Options:

A.

1110

B.

1120

C.

1210

D.

1220

**Answer: B**

**Solution:**

Using family of planes

$$2x + 3y - z - 5 = k_1(x + ay + 3z + 4) + k_2(3x - y + \beta z - 7)$$

$$2 = k_1 + 3k_2, 3 = k_1 a - k_2, -1 = 3k_1 + \beta k_2, -5 = 4k_1 - 7k_2$$

On solving we get

$$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, a = -70, \beta = \frac{-16}{13}$$

$$13a\beta = 13(-70)\left(\frac{-16}{13}\right)$$

$$= 1120$$

## Question11

Let the system of equations  $x + 2y + 3z = 5$ ,  $2x + 3y + z = 9$ ,  $4x + 3y + \lambda z = \mu$  have infinite number of solutions. Then  $\lambda + 2\mu$  is equal to :

**[1-Feb-2024 Shift 2]**

**Options:**

A.

28

B.

17

C.

22

D.

15

**Answer: B**

**Solution:**

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 9$$

$$4x + 3y + \lambda z = \mu$$

for infinite following  $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for  $\lambda = -13, \mu = 15$  system of equation has infinite solution hence  $\lambda + 2\mu = 17$

## Question 12

If the system of equations

$$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair  $(\lambda, \mu)$  is equal to  
[24-Jan-2023 Shift 2]

Options:

A.  $\left(\frac{72}{5}, \frac{21}{5}\right)$

B.  $\left( \frac{-72}{5}, \frac{-21}{5} \right)$

C.  $\left( \frac{72}{5}, \frac{-21}{5} \right)$

D.  $\left( \frac{-72}{5}, \frac{21}{5} \right)$

**Answer: C**

**Solution:**

**Solution:**

$$x + 2y + 3z = 3 \dots (i)$$

$$4x + 3y - 4z = 4 \dots (ii)$$

$$8x + 4y - \lambda z = 9 + \mu \dots (iii)$$

$$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8 \dots (iv)$$

$$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu \dots (v)$$

$$(iv) \times 2 - (v) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$$

$$\text{For infinite solutions} \Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$$

$$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$$

$$\Rightarrow (\lambda, \mu) \equiv \left( \frac{72}{5}, \frac{-21}{5} \right)$$

## Question 13

Let  $S_1$  and  $S_2$  be respectively the sets of all  $a \in \mathbb{R} - \{0\}$  for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$$

$$(3a + 5)x + (a + 5)y + (a + 2)z = 3$$

has unique solution and infinitely many solutions. Then

[25-Jan-2023 Shift 1]

**Options:**

A.  $n(S_1) = 2$  and  $S_2$  is an infinite set

B.  $S_1$  is an infinite set and  $n(S_2) = 2$

C.  $S_1 = \emptyset$  and  $S_2 = \mathbb{R} - \{0\}$

D.  $S_1 = \mathbb{R} - \{0\}$  and  $S_2 = \emptyset$

**Answer: D**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$= a(15a^2 + 31a + 36) = 0 \Rightarrow a = 0$$

$$\Delta \neq 0 \text{ for all } a \in \mathbb{R} - \{0\}$$

$$\text{Hence } S_1 = \mathbb{R} - \{0\} \quad S_2 = \emptyset$$

## Question14

Consider the following system of questions

$$\alpha x + 2y + z = 1$$

$$2\alpha x + 3y + z = 1$$

$$3x + \alpha y + 2z = \beta$$

For some  $\alpha, \beta \in \mathbb{R}$ . Then which of the following is NOT correct.

[29-Jan-2023 Shift 1]

Options:

- A. It has no solution if  $\alpha = -1$  and  $\beta \neq 2$
- B. It has no solution for  $\alpha = -1$  and for all  $\beta \in \mathbb{R}$
- C. It has no solution for  $\alpha = 3$  and for all  $\beta \neq 2$
- D. It has a solution for all  $\alpha \neq -1$  and  $\beta = 2$

Answer: B

Solution:

Solution:

$$D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1, 3$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 2$$

$$D_y = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$$

$$\beta = 2, \alpha = -1$$

$$\alpha = -1, \beta = 2 \text{ Infinite solution}$$

## Question15

Let the system of linear equations

$$x + y + kz = 2$$

$$2x + 3y - z = 1$$

$$3x + 4y + 2z = k$$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$   
 $(2k + 1)x + (k + 5)y = 10$  has:  
[30-Jan-2023 Shift 1]

**Options:**

- A. infinitely many solutions
- B. unique solution satisfying  $x - y = 1$
- C. no solution
- D. unique solution satisfying  $x + y = 1$

**Answer: D**

**Solution:**

**Solution:**

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 1(10) - 1(7) + k(-1) - 0$$

$$\Rightarrow k = 3$$

For  $k = 3$ , 2<sup>nd</sup> system is

$$4x + 5y = 7 \dots (1)$$

$$\text{and } 7x + 8y = 10 \dots (2)$$

Clearly, they have a unique solution

$$(2) - (1) \Rightarrow 3x + 3y = 3$$

$$\Rightarrow x + y = 1$$

## Question 16

For  $\alpha, \beta \in \mathbb{R}$ , suppose the system of linear equations

$$x - y + z = 5$$

$$2x + 2y + \alpha z = 8$$

$$3x - y + 4z = \beta$$

has infinitely many solutions. Then  $\alpha$  and  $\beta$  are the roots of

[30-Jan-2023 Shift 2]

**Options:**

A.  $x^2 - 10x + 16 = 0$

B.  $x^2 + 18x + 56 = 0$

C.  $x^2 - 18x + 56 = 0$

D.  $x^2 + 14x + 24 = 0$

**Answer: C**

**Solution:**

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0; 8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$$

$$8 + \alpha + 6 - 3\alpha - 6 = 0$$

$$\alpha = 4$$


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## Question 17

For the system of linear equations

$$x + y + z = 6$$

$$\alpha x + \beta y + 7z = 3$$

$$x + 2y + 3z = 14$$

which of the following is NOT true?

[31-Jan-2023 Shift 1]

Options:

A. If  $\alpha = \beta = 7$ , then the system has no solution

B. If  $\alpha = \beta$  and  $\alpha \neq 7$  then the system has a unique solution.

C. There is a unique point  $(\alpha, \beta)$  on the line  $x + 2y + 18 = 0$  for which the system has infinitely many solutions

D. For every point  $(\alpha, \beta) \neq (7, 7)$  on the line  $x - 2y + 7 = 0$ , the system has infinitely many solutions.

**Answer: D**

**Solution:**

By equation 1 and 3

$$y + 2z = 8$$

$$y = 8 - 2z$$

$$\text{And } x = -2 + z$$

Now putting in equation 2

$$\alpha(z - 2) + \beta(-2z + 8) + 7z = 3$$

$$\Rightarrow (\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$$

So equations have unique solution if

$$\alpha - 2\beta + 7 \neq 0$$

And equations have no solution if

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 \neq 0$$

And equations have infinite solution if

$$\alpha - 2\beta + 7 = 0 \text{ and } 2\alpha - 8\beta + 3 = 0$$


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## Question 18

Let S denote the set of all real values of  $\lambda$  such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent, then  $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$  is equal to

**[1-Feb-2023 Shift 1]**

**Options:**

- A. 2
- B. 12
- C. 4
- D. 6

**Answer: D**

**Solution:**

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$$

$$(\lambda + 2)(\lambda^2 - 2\lambda + 1) = 0.$$

$$(\lambda + 2)(\lambda - 1)^2 = 0 \Rightarrow \lambda = -2, \lambda = 1$$

at  $\lambda = 1$  system has infinite solution, for inconsistent  $\lambda = -2$

$$\text{so } \sum (|-2|^2 + |-2|) = 6$$

## Question 19

**For the system of linear equations  $ax + y + z = 1$ ,  $x + ay + z = 1$ ,  $x + y + az = \beta$ , which one of the following statements is NOT correct?**

**[1-Feb-2023 Shift 2]**

**Options:**

- A. It has infinitely many solutions if  $\alpha = 2$  and  $\beta = -1$
- B. It has no solution if  $\alpha = -2$  and  $\beta = 1$
- C.  $x + y + z = \frac{3}{4}$  if  $\alpha = 2$  and  $\beta = 1$
- D. It has infinitely many solutions if  $\alpha = 1$  and  $\beta = 1$

**Answer: A**

**Solution:**

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$$

$$\alpha^3 - 3\alpha + 2 = 0$$

$$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$$

$$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$$

$$\alpha = 1, \alpha = -2, 1$$

$$\text{For } \alpha = 1, \beta = 1$$

$$x + y + z = 1$$

$$x + y + z = b \} \text{ infinite solution}$$

$$\text{For } \alpha = 2, \beta = 1$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2 - 1 = 1 \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 1 = 1 \Rightarrow z = \frac{1}{4}$$

$$\text{For } \alpha = 2 \Rightarrow \text{unique solution}$$

## Question 20

If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then  $2a + 3b$  is equal to :

[6-Apr-2023 shift 1]

Options:

A. 28

B. 20

C. 25

D. 23

**Answer: D**

**Solution:**

**Solution:**

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

For  $\infty$  solution

$$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence  $2a + 3b = 23$

## Question 21

**For the system of equations**

$$x + y + z = 6$$

$$x + 2y + \alpha z = 10$$

$x + 3y + 5z = \beta$ , which one of the following is NOT true :

[6-Apr-2023 shift 2]

**Options:**

- A. System has a unique solution for  $\alpha = 3, \beta \neq 14$ .
- B. System has a unique solution for  $\alpha = -3, \beta = 14$ .
- C. System has no solution for  $\alpha = 3, \beta = 24$ .
- D. System has infinitely many solutions for  $\alpha = 3, \beta = 14$ .

**Answer: A**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$$

$$= (10 - 3\alpha) - (5 - \alpha) + (3 - 2)$$

$$= 6 - 2\alpha$$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix}$$

$$= 6(10 - 3\alpha) - (50 - \alpha 13) + (30 - 2\beta)$$

$$= 40 - 18\alpha + \alpha\beta - 2\beta$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix}$$

$$= (50 - \alpha\beta) - 6(5 - \alpha) + (\beta - 10)$$

$$= 10 + 6\alpha + \beta - \alpha\beta$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix}$$

$$= (2\beta - 30) - (\beta - 10) + 6(1)$$

$$= \beta - 14$$

for Infinite solution

$$\Delta = 0, \Delta_x = \Delta_y = \Delta_z = 0$$

$x\alpha = 3, x\beta = 14$   
For unique solution  $\alpha \neq 3$   
Ans. Option 1

---

## Question22

Let S be the set of all values of  $\theta \in [-\pi, \pi]$  for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$

$$-x + (\tan \theta)y + \sqrt{7}z = 0$$

$$x + y + (\tan \theta)z = 0$$

has non-trivial solution. Then  $\frac{120}{\pi} \sum_{\theta \in S} \theta$  is equal to  
[8-Apr-2023 shift 2]

Options:

A. 20

B. 40

C. 30

D. 10

Answer: A

Solution:

Solution:

For non trivial solutions  
 $D = 0$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan \theta & \sqrt{7} \\ 1 & 1 & \tan \theta \end{vmatrix} = 0$$

$$\tan^2 \theta - (\sqrt{3} - 1) - \sqrt{3} = 0$$

$$\tan \theta = \sqrt{3}, -1$$

$$\theta = \left\{ \frac{\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{4}, \frac{3\pi}{4} \right\}$$

$$\frac{120}{\pi} (\sum \theta) = \frac{120}{\pi} \times \frac{\pi}{6} = 20 \text{ (Option 1)}$$

---

## Question23

For the system of linear equations

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta$$

which of the following is NOT correct ?

[10-Apr-2023 shift 1]

Options:

- A. The system is inconsistent for  $\alpha = -5$  and  $\beta = 8$
- B. The system has infinitely many solutions for  $\alpha = -6$  and  $\beta = 9$
- C. The system has a unique solution for  $\alpha \neq -5$  and  $\beta = 8$
- D. The system has infinitely many solutions for  $\alpha = -5$  and  $\beta = 9$

**Answer: B**

**Solution:**

**Solution:**

Given

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \alpha z = \beta$$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7\alpha + 35$$

$$\Delta = 7(\alpha + 5)$$

For unique solution  $\Delta \neq 0$

$$\alpha \neq -5$$

For inconsistent & Infinite solution

$$\Delta = 0$$

$$\alpha + 5 = 0 \Rightarrow \alpha = -5$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix}$$

$$\Delta_3 = 7(\beta - 9)$$

For Inconsistent system :-

At least one  $\Delta_1, \Delta_2 \& \Delta_3$  is not zero  $\alpha = -5, \beta = 8$  option (A) True Infinite solution:

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

From here  $\beta - 9 = 0 \Rightarrow \beta = 9$   $\alpha = -5$  & option (D) True

$$\beta = 9$$

Unique solution

$\alpha \neq -5, \beta = 8 \rightarrow$  option (C) True

Option (B) False

For Infinitely many solution  $\alpha$  must be -5 .

## Question 24

Let S be the set of values of  $\lambda$ , for which the system of equations

$$6\lambda x - 3y + 3z = 4\lambda^2 \quad 2x + 6\lambda y + 4z = 1 \quad 3x + 2y + 3\lambda z = \lambda$$

has no solution. Then  $12 \sum_{1 \in S} |\lambda|$  is equal to \_\_\_\_\_.

[10-Apr-2023 shift 2]

**Answer: 24**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$

$$2\lambda(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$$

$$18\lambda^3 - 14\lambda - 4 = 0$$

$$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$$

$$\Rightarrow \lambda = 1, -1/3, -2/3$$

$$\text{For each values of } \lambda, \Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$$

$$12\left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$$

## Question 25

Let  $A$  be a  $2 \times 2$  matrix with real entries such that  $A = \alpha A + I$ , where  $\alpha \in \mathbb{R} - \{-1, 1\}$ . If  $\det(A^2 - A) = 4$ , then the sum of all possible values of  $\alpha$  is equal to :

**[11-Apr-2023 shift 1]**

**Options:**

A. 0

B.  $\frac{5}{2}$

C. 2

D.  $\frac{3}{2}$

**Answer: B**

**Solution:**

**Solution:**

$$A^T = \alpha A + I$$

$$A = \alpha A^T + I$$

$$A = \alpha(\alpha A + I) + I$$

$$A = \alpha^2 A + (\alpha + 1)I$$

$$A(1 - \alpha^2) = (\alpha + 1)I$$

$$A = \frac{I}{1 - \alpha} \dots (1)$$

$$|A| = \frac{1}{(1 - \alpha)^2} \dots (2)$$

$$|A^2 - A| = |A| |A - I| \dots (3)$$

$$A - I = \frac{I}{1 - \alpha} - I = \frac{\alpha}{1 - \alpha} I$$

$$|A - I| = \left( \frac{\alpha}{1 - \alpha} \right)^2 \dots (4)$$

$$\text{Now } |A^2 - A| = 4$$

$$|A| |A - I| = 4$$

$$\Rightarrow \frac{1}{(1 - \alpha)^2} \frac{\alpha^2}{(1 - \alpha)^2} = 4$$

$$\Rightarrow \frac{\alpha}{(1 - \alpha)^2} = \pm 2$$

$$\Rightarrow 2(1 - \alpha)^2 = \pm \alpha$$

$$(C_1) 2(1 - \alpha)^2 = \alpha \quad (C_2) 2(1 - \alpha)^3 = -\alpha$$

$$\alpha_1 + \alpha_2 = \frac{5}{2}$$

## Question 26

If the system of linear equations

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

has infinitely many solutions, then  $\alpha + \beta + 2$  is equal to:

[11-Apr-2023 shift 2]

Options:

A. 3

B. 6

C. 5

D. 4

Answer: D

Solution:

Solution:

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

4 sc condition of Infinite Many solution

$\Delta = 0$  &  $\Delta x, \Delta y, \Delta z = 0$  check.

After solving we get  $\alpha + 13 + 2 = 4$

## Question 27

If 
$$\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81),$$
 then  $\lambda, \frac{\lambda}{3}$  are the roots of the equation

[11-Apr-2023 shift 2]

Options:

A.  $4x^2 - 24x - 27 = 0$

B.  $4x^2 + 24x + 27 = 0$

C.  $4x^2 - 24x + 27 = 0$

D.  $4x^2 + 24x - 27 = 0$

**Answer: C**

**Solution:**

**Solution:**

$$\begin{vmatrix} x+1 & x & x \\ x & x+d & x \\ x & x & x+d^2 \end{vmatrix} = \frac{9}{8}(103x+81)$$

Put  $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8}$$

$$\lambda = \frac{9}{2}$$

$$\frac{\lambda}{3} = \frac{9}{2 \times 3} \Rightarrow \frac{3}{2}$$

$$\frac{\lambda}{3} = \frac{3}{2}$$

Option (C)  $4x^2 - 24x + 27 = 0$

has Root  $\frac{3}{2}, \frac{9}{2}$

## Question 28

Let  $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$ . If  $\sum_{k=1}^n D_k = 96$ , then  $n$  is equal to

**[12-Apr-2023 shift 1]**

**Answer: 6**

**Solution:**

**Solution:**

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$\sum_{k=1}^n D_k = 96 \Rightarrow$$

$$\begin{vmatrix} \sum_{k=1}^n 1 & \sum 2k & \sum (2k-1) \\ n & n^2 + n + 2 & n^2 \\ n & n^2 + n & n^2 + n + 2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2 + n & n^2 \\ n & n^2 + n + 2 & n^2 \\ n & n^2 + n & n^2 + n + 2 \end{vmatrix} = 96$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} n & n^2 + n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n + 2 \end{vmatrix} = 96$$

$$\Rightarrow n(2n + 4) = 96 \Rightarrow n(n + 2) = 48 \Rightarrow n = 6$$

## Question 29

For the system of linear equations

$$2x + 4y + 2az = b$$

$$x + 2y + 3z = 4$$

$$2x - 5y + 2z = 8$$

which of the following is NOT correct?

[13-Apr-2023 shift 1]

Options:

A. It has infinitely many solutions if  $a = 3, b = 8$

B. It has unique solution if  $a = b = 8$

C. It has unique solution if  $a = b = 6$

D. It has infinitely many solutions if  $a = 3, b = 6$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix} = 18(3 - a)$$

$$\Delta_x = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix} = (64 + 19b - 72a)$$

For unique solution  $\Delta \neq 0$

$\Rightarrow a \neq 3$  and  $b \in \mathbb{R}$

For infinitely many solution :

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

$$\Rightarrow a = 3 \therefore \Delta = 0$$

and  $b = 8 \therefore \Delta_x = 0$

## Question30

If the system of equations

$$2x + y - z = 5$$

$$2x - 5y + \lambda z = \mu$$

$$x + 2y - 5z = 7$$

has infinitely many solutions, then  $(\lambda + \mu)^2 + (\lambda - \mu)^2$  is equal to  
[13-Apr-2023 shift 2]

Options:

A. 904

B. 916

C. 912

D. 920

**Answer: B**

**Solution:**

**Solution:**

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 2(25 - 2\lambda) - 1(-10 - \lambda) - 1(4 + 5) = 0$$

$$\Rightarrow 51 - 3\lambda = 0$$

$$\Rightarrow \lambda = 17$$

$$\Delta_x = 0$$

$$\begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 5(25 - 34) - 1(-5\mu - 119) - 1(2\mu + 35) = 0$$

$$\Rightarrow -45 + 5\mu + 119 - 2\mu - 35 = 0$$

$$\Rightarrow 39 + 3\mu = 0 \Rightarrow \mu = -13$$

$$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 4^2 + (30)^2$$

$$= 916$$

## Question31

Let the system of linear equations

$$-x + 2y - 9z = 7$$

$$-x + 3y + 7z = 9$$

$$-2x + y + 5z = 8$$

$$-3x + y + 13z = \lambda$$

has a unique solution  $x = \alpha$ ,  $y = \beta$ ,  $z = \gamma$ . Then the distance of the point  $(\alpha, \beta, \gamma)$  from the plane  $2x - 2y + z = \lambda$  is

**[15-Apr-2023 shift 1]**

**Options:**

- A. 7
- B. 9
- C. 13
- D. 11

**Answer: A**

**Solution:**

**Solution:**

$$\begin{aligned} -x + 2y - 9z &= 7 \quad (1) \\ -x + 3y - 7z &= 9 \quad (2) \\ -2x + y + 5z &= 8 \quad (3) \end{aligned}$$
$$(2) - (1)$$
$$y + 16z = 2 \quad (4)$$
$$(3) - 2 \times (1)$$
$$-3y + 23z = -6 \quad (5)$$
$$3 \times (4) + (5)$$
$$71z = 0 \Rightarrow z = 0$$
$$y = 2$$
$$x = -3$$
$$(-3, 2, 0) \rightarrow (\alpha, \beta, \gamma)$$
$$\text{Put in } -3x + y + 13z = 1$$
$$\lambda = 9 + 2 = 11$$
$$d = \left| \frac{-6 - 4 - 11}{3} \right| = 7$$

## Question32

**Let the system of linear equations**

$$x + y + \alpha z = 2$$

$$3x + y + z = 4$$

$$x + 2z = 1$$

**have a unique solution  $(x^*, y^*, z^*)$ . If  $(\alpha, x^*)$ ,  $(y^*, \alpha)$  and  $(x^*, -y^*)$  are collinear points, then the sum of absolute values of all possible values of  $\alpha$  is**

**[24-Jun-2022-Shift-2]**

**Options:**

- A. 4
- B. 3
- C. 2
- D. 1

**Answer: C**

**Solution:**

Given system of equations

$$x + y + az = 2 \dots (i)$$

$$3x - y - z = 4 \dots (ii)$$

$$x + 2z = 1 \dots (iii)$$

Solving (i), (ii) and (iii), we get

$$x = 1, y = 1, z = 0 \text{ (and for unique solution } a \neq -3)$$

Now,  $(a, 1), (1, a)$  and  $(1, -1)$  are collinear

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(a+1) - 1(0) + 1(-1-a) = 0$$

$$\Rightarrow a^2 - 1 = 0$$

$$\therefore a = \pm 1$$

$$\therefore \text{Sum of absolute values of } a = 1 + 1 = 2$$

## Question33

**The system of equations**

$$-kx + 3y - 14z = 25$$

$$-15x + 4y - kz = 3$$

$$-4x + y + 3z = 4$$

**is consistent for all k in the set**

**[25-Jun-2022-Shift-2]**

**Options:**

A. R

B.  $R - \{-11, 13\}$

C.  $R - \{13\}$

D.  $R - \{-11, 11\}$

**Answer: D**

**Solution:**

**Solution:**

The system may be inconsistent if

$$\begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 0 \Rightarrow k = \pm 11$$

Hence if system is consistent then  $k \in R - \{11, -11\}$ .

## Question34

The ordered pair (a, b), for which the system of linear equations

$$3x - 2y + z = b$$

$$5x - 8y + 9z = 3$$

$$2x + y + az = -1$$

has no solution, is :

[26-Jun-2022-Shift-1]

Options:

A.  $\left(3, \frac{1}{3}\right)$

B.  $\left(-3, \frac{1}{3}\right)$

C.  $\left(-3, -\frac{1}{3}\right)$

D.  $\left(3, -\frac{1}{3}\right)$

Answer: C

Solution:

Solution:

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0 \Rightarrow -14a - 42 = 0 \Rightarrow a = -3$$

Now 3 (equation (1)) - (equation (2)) - 2 (equation (3)) is

$$3(3x - 2y + z - b) - (5x - 8y + 9z - 3) - 2(2x + y + az + 1) = 0$$

$$\Rightarrow -3b + 3 - 2 = 0 \Rightarrow b = \frac{1}{3}$$

So for no solution  $a = -3$  and  $b \neq \frac{1}{3}$

## Question 35

If the system of equations

$$\alpha x + y + z = 5, x + 2y + 3z = 4, x + 3y + 5z = \beta$$

has infinitely many solutions, then the ordered pair  $(\alpha, \beta)$  is equal to :

[26-Jun-2022-Shift-2]

Options:

A.  $(1, -3)$

B.  $(-1, 3)$

C.  $(1, 3)$

D.  $(-1, -3)$

Answer: C

## Solution:

### Solution:

Given system of equations

$$\alpha x + y + z = 5$$

$$x + 2y + 3z = 4, \text{ has infinite solution}$$

$$x + 3y + 5z = \beta$$

$$\therefore \Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0 \Rightarrow \alpha(1) - 1(2) + 1(1) = 0$$

$$\Rightarrow \alpha = 1$$

$$\text{and } \Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5(1) - 1(20 - 3\beta) + 1(12 - 2\beta) = 0$$

$$\Rightarrow \beta = 3$$

$$\text{and } \Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 4 & 3 \\ 1 & \beta & 5 \end{vmatrix} = 0 \Rightarrow (20 - 3\beta) - 5(2) + 1(\beta - 4) = 0$$

$$\Rightarrow -2\beta + 6 = 0$$

$$\Rightarrow \beta = 3$$

Similarly,

$$\Rightarrow \Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 4 \\ 1 & 3 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 3$$

$$\therefore (\alpha, \beta) = (1, 3)$$

## Question 36

Let the system of linear equations

$$x + 2y + z = 2$$

$$\alpha x + 3y - z = \alpha$$

$$-\alpha x + y + 2z = -\alpha$$

be inconsistent. Then  $\alpha$  is equal to:

[27-Jun-2022-Shift-1]

Options:

A.  $\frac{5}{2}$

B.  $-\frac{5}{2}$

C.  $\frac{7}{2}$

D.  $-\frac{7}{2}$

**Answer: D**

**Solution:**

**Solution:**

$$x + 2y + z = 2$$

$$\alpha x + 3y - z = \alpha$$

$$-\alpha x + y + 2z = -\alpha$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 1(6 + 1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha)$$
$$= 7 + 2\alpha$$

$$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0 \text{ for } \alpha = -\frac{7}{2}$$

$$\therefore \text{For no solution } \alpha = -\frac{7}{2}$$

---

## Question 37

Let for some real numbers  $\alpha$  and  $\beta$ ,  $a = \alpha - i\beta$ . If the system of equations  $4ix + (1 + i)y = 0$  and  $8\left(\cos \frac{2\pi}{3} + i\sin \frac{2\pi}{3}\right)x + \bar{a}y = 0$  has more than one solution, then  $\frac{\alpha}{\beta}$  is equal to  
[27-Jun-2022-Shift-2]

**Options:**

A.  $-2\sqrt{3}$

B.  $2 - \sqrt{3}$

C.  $2 + \sqrt{3}$

D.  $-2 - \sqrt{3}$

**Answer: B**

**Solution:**

**Solution:**

Given  $a = \alpha - i\beta$  and

$$4ix + (1 + i)y = 0 \dots\dots (i)$$

$$8\left(\cos \frac{2\pi}{3} + i\sin \frac{2\pi}{3}\right)x + \bar{a}y = 0$$

By (i)

$$\frac{x}{y} = \frac{-(1 + i)}{4i} \dots\dots (iii)$$

By (ii)

$$\frac{x}{y} = \frac{-\bar{a}}{8\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right)} \dots\dots$$

Now by (iii) and (iv)

$$\frac{1+i}{4i} = \frac{\bar{a}}{4(-1+\sqrt{3}i)}$$

$$\Rightarrow \bar{a} = (\sqrt{3}-1) + (\sqrt{3}+1)i$$

$$\Rightarrow \alpha + i\beta = (\sqrt{3}-1) + (\sqrt{3}+1)i$$

$$\therefore \frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$$


---

## Question38

If the system of linear equations

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda| z = 4\lambda - 4$$

where,  $\lambda \in \mathbb{R}$ , has no solution, then

[28-Jun-2022-Shift-1]

Options:

A.  $\lambda = 7$

B.  $\lambda = -7$

C.  $\lambda = 8$

D.  $\lambda^2 = 1$

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda \end{vmatrix} = 7$$

But at  $\lambda = 7$ ,  $D_x = D_y = D_z = 0$

$$P_1 : 2x + 3y - z = -2$$

$$P_2 : x + y + z = 4$$

$$P_3 : x - y + |\lambda| z = 4\lambda - 4$$

So clearly  $5P_2 - 2P_1 = P_3$ , so at  $\lambda = 7$ , system of equation is having infinite solutions.

So  $\lambda = -7$  is correct answer.

---

## Question39

If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$$\alpha x + 5y = \beta + 1,$$

where  $\alpha, \beta, \gamma \in \mathbb{R}$  has infinitely many solutions then the value of  $|9\alpha + 3\beta + 5\gamma|$  is equal to \_\_\_\_\_

[28-Jun-2022-Shift-2]

**Answer: 58**

**Solution:**

**Solution:**

If  $2x - 3y = \gamma + 5$  and  $\alpha x + 5y = \beta + 1$  have infinitely many solutions then

$$\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma+5}{\beta+1}$$

$$\Rightarrow \alpha = -\frac{10}{3} \text{ and } 3\beta + 5\gamma = -28$$

$$\text{So } |9\alpha + 3\beta + 5\gamma| = |-30 - 28| = 58$$

## Question40

**If the system of linear equations**

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

**$x + 4y + \delta z = k$ , where  $\delta, k \in \mathbb{R}$  has infinitely many solutions, then  $\delta + k$  is equal to:**

**[29-Jun-2022-Shift-1]**

**Options:**

A. -3

B. 3

C. 6

D. 9

**Answer: B**

**Solution:**

**Solution:**

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k$$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7\delta - 21 = 0$$

$$\delta = -3$$

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}$$

$$\Rightarrow 6 - k = 0 \Rightarrow k = 6$$

$$\delta + k = -3 + 6 = 3$$

## Question41

**The number of values of  $\alpha$  for which the system of equations:**

$$x + y + z = \alpha$$

$$\alpha x + 2\alpha y + 3z = -1$$

$$x + 3\alpha y + 5z = 4$$

**is inconsistent, is**

**[24-Jun-2022-Shift-1]**

**Options:**

A. 0

B. 1

C. 2

D. 3

**Answer: B**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$$

$$= 1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha)$$

$$= \alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha$$

$$= 3\alpha^2 - 6\alpha + 3$$

For inconsistency  $\Delta = 0$  i.e.  $\alpha = 1$

Now check for  $\alpha = 1$

$$x + y + z = 1 \quad \dots (i)$$

$$x + 2y + 3z = -1 \quad \dots (ii)$$

$$\text{By (ii)} \times 2 - \text{(i)} \times 1$$

$$x + 3y + 5z = -3$$

so equations are inconsistent for  $\alpha = 1$

## Question 42

**The number of  $\theta \in (0, 4\pi)$  for which the system of linear equations**

$$3(\sin 3\theta)x - y + z = 2$$

$$3(\cos 2\theta)x + 4y + 3z = 3$$

$$6x + 7y + 7z = 9$$

**has no solution, is :**

**[25-Jul-2022-Shift-1]**

**Options:**

A. 6

B. 7

C. 8

D. 9

**Answer: B**

**Solution:**

**Solution:**

Given,

$$3(\sin 3\theta)x - y + z = 2$$

$$3(\cos 2\theta)x + 4y + 3z = 3$$

$$6x + 7y + 7z = 9$$

For no solutions determinant of coefficient will be  $= 0$

$$\therefore D = \begin{vmatrix} 3 \sin 3\theta & -1 & 1 \\ 3 \cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 3 \sin 3\theta(28 - 21) + 1(21 \cos 2\theta - 18) + 1(21 \cos 2\theta - 24) = 0$$

$$\Rightarrow 21 \sin 3\theta + 42 \cos 2\theta - 42 = 0$$

$$\Rightarrow \sin 3\theta + 2 \cos 2\theta - 2 = 0$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta + 2(1 - 2 \sin^2 \theta) - 2 = 0$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta - 4 \sin^2 \theta = 0$$

$$\Rightarrow 4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\Rightarrow \sin \theta(4 \sin^2 \theta + 4 \sin \theta - 3) = 0$$

$$\therefore \sin \theta = 0$$

$$\Rightarrow \theta = \pi, 2\pi, 3\pi \text{ when } \theta \in (0, 4\pi)$$

or,

$$4 \sin^2 \theta + 4 \sin \theta - 3 = 0$$

$$\Rightarrow 4 \sin 2\theta + 6 \sin \theta - 2 \sin \theta - 3 = 0$$

$$\Rightarrow 2 \sin \theta(2 \sin \theta + 3) - 1(2 \sin \theta + 3) = 0$$

$$\Rightarrow (2 \sin \theta - 1)(2 \sin \theta + 3) = 0$$

$$\therefore \sin \theta = \frac{1}{2}$$

or,

$$\sin \theta = -\frac{3}{2} \text{ [not possible as } \sin \in [-1, 1] \text{]}$$

$$\therefore \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\therefore \text{Possible values of } \theta = \pi, 2\pi, 3\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$\therefore$  Total 7 values of  $\theta$  possible.

## Question 43

**The number of real values of  $\lambda$ , such that the system of linear equations**

$$2x - 3y + 5z = 9$$

$$x + 3y - z = -18$$

$$3x - y + (\lambda^2 - |\lambda|)z = 16$$

**has no solutions, is**

**[25-Jul-2022-Shift-2]**

**Options:**

A. 0

B. 1

C. 2

D. 4

**Answer: C**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - \lambda \end{vmatrix} = 2(3\lambda^2 - 3|\lambda| - 1) + 3(\lambda^2 - |\lambda| - 3) + 5(-1 - 9) \\ = 9\lambda^2 - 9|\lambda| - 43 \\ = 9|\lambda|^2 - 9|\lambda| - 43$$

$\Delta = 0$  for 2 values of  $|\lambda|$  out of which one is -ve and other is +ve

So, 2 values of  $\lambda$  satisfy the system of equations to obtain no solution.

## Question44

**If the system of linear equations,**

$$8x + y + 4z = -2$$

$$x + y + z = 0$$

$$\lambda x - 3y = \mu$$

**has infinitely many solutions, then the distance of the point  $\left(\lambda, \mu, -\frac{1}{2}\right)$  from the plane**

$$8x + y + 4z + 2 = 0 \text{ is :}$$

**[26-Jul-2022-Shift-1]**

**Options:**

A.  $3\sqrt{5}$

B. 4

C.  $\frac{26}{9}$

D.  $\frac{10}{3}$

**Answer: D**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix}$$

$$= 8(3) - 1(-\lambda) + 4(-3 - \lambda)$$

$$= 24 + \lambda - 12 - 4\lambda$$

$$= 12 - 3\lambda$$

So for  $\lambda = 4$ , it is having infinitely many solutions.

$$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$$

$$= -2(3) - 1(-\mu) + 4(-\mu)$$

$$= -6 - 3\mu = 0$$

For  $\mu = -2$

$$\text{Distance of } \left(4, -2, \frac{-1}{2}\right) \text{ from } 8x - y - 4z + 2 = 0 = \frac{32 - 2 - 2 - 2}{\sqrt{64 + 1 + 16}} = \frac{10}{3} \text{ units}$$

## Question 45

Let  $p$  and  $p + 2$  be prime numbers and let  $\Delta =$

$$\begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

Then the sum of the maximum values of  $\alpha$  and  $\beta$ , such that  $p^\alpha$  and  $(p+2)^\beta$  divide  $\Delta$ , is \_\_\_\_\_.

[29-Jul-2022-Shift-1]

**Answer: 4**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & (p+1) & (p+1)(p+2) \\ 1 & (p+2) & (p+2)(p+3) \\ 1 & (p+3) & (p+3)(p+4) \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & p+1 & p^2+3p+2 \\ 0 & 1 & 2p+4 \\ 0 & 1 & 2p+6 \end{vmatrix}$$

$$= 2(p!) \cdot ((p+1)!) \cdot ((p+2)!) \cdot 2$$

$$= 2(p+1) \cdot (p!)^2 \cdot ((p+2)!) \cdot 2$$

$$= 2(p+1)^2 \cdot (p!)^3 \cdot ((p+2)!) \cdot 2$$

∴ Maximum value of  $\alpha$  is 3 and  $\beta$  is 1.

$$\therefore \alpha + \beta = 4$$

## Question 46

If the system of equations

$$x + y + z = 6$$

$$2x + 5y + \alpha z = \beta$$

$$x + 2y + 3z = 14$$

has infinitely many solutions, then  $\alpha + \beta$  is equal to

[29-Jul-2022-Shift-2]

Options:

A. 8

B. 36

C. 44

D. 48

Answer: C

Solution:

Solution:

Given,

$$x + y + z = 6 \dots (1)$$

$$2x + 5y + \alpha z = \beta \dots (2)$$

$$x + 2y + 3z = 14 \dots (3)$$

System of equation have infinite many solutions.

$$\therefore \Delta_x = \Delta_y = \Delta_z = 0 \text{ and } \Delta = 0$$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 - C_3$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 2-\alpha & 5-\alpha & \alpha \\ -2 & -1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -2 + \alpha + 10 - 2\alpha = 0$$

$$\Rightarrow 8 - \alpha = 0$$

$$\Rightarrow \alpha = 8$$

$$\text{Now, } x + y + z = 6$$

$$2x + 5y + 8z = \beta$$

$$x + 2y + 3z = 14$$

$$\therefore \Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & 8 \\ 14 & 2 & 3 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 - 6C_3$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ \beta - 48 & -3 & 8 \\ -4 & -1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -\beta + 48 - 12 = 0$$

$$\Rightarrow \beta = 36$$

$$\therefore \alpha + \beta = 8 + 36 = 44$$

## Question 47

The value of  $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$  is

[26 Feb 2021 Shift 1]

Options:

A.  $(a+2)(a+3)(a+4)$

B.  $-2$

C.  $(a+1)(a+2)(a+3)$

D.  $0$

**Answer: B**

**Solution:**

**Solution:**

$$\text{Given, } A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$\text{Apply } R_2 \rightarrow R_2 - R_1$$

$$\text{Apply } R_3 \rightarrow R_3 - R_1$$

$$A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

Now, expanding along third column,

$$A = 1[4(a+2) - (4a+10)] = 4a+8-4a-10 = -2$$

## Question 48

Let  $A$  be a  $3 \times 3$  matrix with  $\det(A) = 4$ . Let  $R_i$  denote the  $i$ th row of  $A$ . If

a matrix B is obtained by performing the operation  $R_2 \rightarrow 2R_2 + 5R_3$  on 2A, then det (B) is equal to  
[25 Feb 2021 Shift 2]

Options:

A. 16

B. 80

C. 64

D. 128

Answer: C

Solution:

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\text{Then, } 2A = \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{bmatrix}$$

Now, perform the operation  $R_2 \rightarrow 2R_2 + 5R_3$  on 2A, we get

$$B = \begin{bmatrix} 2a & 2b & 2c \\ 4d + 10g & 4e + 10h & 4f + 10i \\ 2g & 2h & 2i \end{bmatrix}$$

Using property of invariance to calculate |B|, apply  $R_2 \rightarrow R_2 - 5R_3$

$$\begin{aligned} |B| &= \begin{vmatrix} 2a & 2b & 2c \\ 4d & 4e & 4f \\ 2g & 2h & 2i \end{vmatrix} = 2 \times 4 \times 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad [\because \det(A) = 4] \\ &= 16 \times \det(A) \\ &= 16 \times 4 = 64 \end{aligned}$$

## Question 49

Let  $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ , where x, y and z are real numbers, such that

$x + y + z > 0$  and  $xyz = 2$ . If  $A^2 = I_3$ , then the value of  $x^3 + y^3 + z^3$  is

.....

[25 Feb 2021 Shift 1]

**Answer: 7**

**Solution:**

**Solution:**

$$\text{Here, } A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$$

$$|A^2| = |I_3| = 1$$

$$\therefore |(x^3 + y^3 + z^3 - 3xyz)^2| = 1$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$$

$$\Rightarrow x^3 + y^3 + z^3 = 1 + 3xyz \quad [\because x + y + z > 0]$$

$$\Rightarrow = 1 + 3(2)$$

$$= 7 \quad [\because xyz = 2]$$

## Question 50

**Consider the following system of equations**

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c$$

**where, a, b and c are real constants. Then, the system of equations [26 Feb 2021 Shift 2]**

**Options:**

A. has a unique solution, when  $5a = 2b + c$

B. has infinite number of solutions when  $5a = 2b + c$

C. has no solution for all a, b and c

D. has a unique solution for all a, b and c

**Answer: B**

**Solution:**

**Solution:**

Given, system of equation can be written as  $AX = B$ , where

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ Then,}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

$$= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6)$$

$$= 20 - 50 + 30 = 0$$

$$\begin{aligned}
 |A_1| &= \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} \\
 &= a(42 - 22) - 2(7b + 11c) \\
 &\quad - 3(-2b - 6c) \\
 &= 20a - 14b - 22c + 6b + 18c \\
 &= 20a - 8b - 4c = 4(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_2| &= \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} \\
 &= 1(7b + 11c) - a(14 + 11) - 3(2c - b) \\
 &= -25a + 10b + 5c = -5(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_3| &= \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} \\
 &= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6) \\
 &= 20 - 50 + 30 = 0
 \end{aligned}$$

$$\begin{aligned}
 |A_1| &= \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} \\
 &= a(42 - 22) - 2(7b + 11c) \\
 &\quad - 3(-2b - 6c) \\
 &= 20a - 14b - 22c + 6b + 18c \\
 &= 20a - 8b - 4c = 4(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_2| &= \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} \\
 &= 1(7b + 11c) - a(14 + 11) - 3(2c - b) \\
 &= -25a + 10b + 5c = -5(5a - 2b - c)
 \end{aligned}$$

$$\begin{aligned}
 |A_3| &= \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} \\
 &= -10a + 4b + 2c \\
 &= -2(5a - 2b - c)
 \end{aligned}$$

For infinite solution,  
 $|A| = |A_1| = |A_2| = |A_3| = 0$   
 $\Rightarrow 5a - 2b - c = 0 \Rightarrow 5a = 2b + c$

## Question51

The following system of linear equations

$$2x + 3y + 2z = 9$$

$$3x + 2y + 2z = 9$$

$$x - y + 4z = 8$$

[25 Feb 2021 Shift 2]

Options:

A. does not have any solution

B. has a unique solution

C. has infinitely many solutions

D. has a solution  $(\alpha, \beta, \gamma)$  satisfying  $\alpha + \beta^2 + \gamma^3 = 12$

**Answer: B**

### Solution:

#### Solution:

The given system of equations is non-homogeneous and it can be written as,

$$\begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 8 \end{bmatrix}$$

i.e.,  $AX = B$

Now,  $|A| = 2(8 + 2) - 3(12 - 2) + 2(-3 - 2)$   
 $= 20 - 30 - 10 = -20 \neq 0$

$\therefore |A| \neq 0$ , then this system have unique solution.

---

## Question52

If the system of equations

$$kx + y + 2z = 1$$

$$-2x - 2y - 4z = 3$$

$3x - y - 2z = 2$  has infinitely many solutions, then  $k$  is equal to .....

[25 Feb 2021 Shift 1]

**Answer: 21**

### Solution:

Given equations,  $kx + y + 2z = 1$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

For infinitely many solutions,

$$\Delta = \Delta x = \Delta y = \Delta z = 0$$

$$\text{Here, } \Delta y = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$$\Rightarrow k(-8 + 6) - 1(-12 - 4) + 2(9 + 4) = 0$$

$$\Rightarrow -2k + 16 + 26 = 0$$

$$\Rightarrow 2k = 42$$

$$\therefore k = 21$$

---

## Question53

Let  $A$  and  $B$  be  $3 \times 3$  real matrices, such that  $A$  is symmetric matrix and  $B$  is skew-symmetric matrix. Then, the system of linear equations

$(A^2B^2 - B^2A^2)X = 0$ , where  $X$  is a  $3 \times 1$  column matrix of unknown variables and  $O$  is a  $3 \times 1$  null matrix, has

[24 Feb 2021 Shift 2]

**Options:**

- A. no solution
- B. exactly two solutions
- C. infinitely many solutions
- D. a unique solution

**Answer: C**

**Solution:**

**Solution:**

Given, A be a  $3 \times 3$  matrix. A is symmetric and B is skew-symmetric.

$$\therefore A^T = A, B^T = -B$$

$$\text{Let } A^2B^2 - B^2A^2 = P$$

$$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$$

$$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$$

$$= B^2A^2 - A^2B^2 = -(A^2B^2 - B^2A^2) = -P$$

$$P^T = -P$$

P is skew-symmetric.  $\therefore |P| = 0$

Hence,  $PX = 0$  have infinite solutions.

## Question54

**For the system of linear equations**

**$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$ , consider the following statements**

- (A) The system has unique solution, if  $k \neq 2, k \neq -2$**
- (B) The system has unique solution, if  $k = -2$**
- (C) The system has unique solution, if  $k = 2$**
- (D) The system has no solution, if  $k = 2$**
- (E) The system has infinite number of solutions, if  $k \neq -2$**

**Which of the following statements are correct?**

**[24 Feb 2021 Shift 2]**

**Options:**

- A. (C) and (D)
- B. (B) and (E)
- C. (A) and (E)
- D. (A) and (D)

**Answer: D**

**Solution:**

**Solution:**

$$\text{Given, } x - 2y + 0z = 1$$

$$x - y + kx = -2$$

$$0x + ky + 4z = 6$$

$$\text{Here, } \Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(4) \\ = -4 - k^2 + 8 = 4 - k^2$$

$$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(-8 - 6k) \\ = -4 - k^2 - 16 - 12k = -k^2 - 12k - 20$$

If  $\Delta \neq 0$ , then it has unique solution i.e.  $4 - k^2 \neq 0$   
 $\Rightarrow k \neq \pm 2$  for unique solution.

Also at  $k = 2$

$$\Delta_x = -2^2 - 12 \times 2 - 20 = -48 \neq 0$$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

$\Rightarrow k \neq \pm 2$  for unique solution. Also at  $k = 2$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

## Question 55

**The system of linear equations :**

**$3x - 2y - kz = 10$ ;  $2x - 4y - 2z = 6$ ;  $x + 2y - z = 5m$  is inconsistent if :**

**24 Feb 2021 Shift 1**

**Options:**

A.  $k = 3, m = \frac{4}{5}$

B.  $k \neq 3, m \in \mathbb{R}$

C.  $k \neq 3, m \neq \frac{4}{5}$

D.  $k = 3, m \neq \frac{4}{5}$

**Answer: D**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 24 + 2(0) - k(8) = 0 \Rightarrow k = 3$$

$$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$$

$$= 10(8) - 2(-10m + 6) - 3(12 + 20m) \\ = 8(4 - 5m)$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$$

$$= 3(-6 + 10m) - 10(0) - 3(10m - 6) \\ = 0$$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$$

$$= 3(-20m - 12) - 2(6 - 10m) + 10(8)$$

$$= -40m + 32 = 8(4 - 5m)$$

For inconsistent,

$$k = 3m \neq \frac{4}{5}$$

## Question 56

Let  $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$ , where  $\alpha \in \mathbb{R}$ . Suppose  $Q = [q_{ij}]$  is a matrix

satisfying  $PQ = kI_3$  for some non-zero  $k \in \mathbb{R}$ . If  $q_{23} = -\frac{k}{8}$  and  $|Q| = \frac{k^2}{2}$ , then  $\alpha^2 + k^2$  is equal to 24  
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**Answer: 17**

**Solution:**

**Solution:**

Given that

$$PQ = kI$$

$$|P| \cdot |Q| = k^3$$

$$\Rightarrow |P| = 2k \neq 0 \Rightarrow P \text{ is an invertible matrix}$$

$$\therefore PQ = kI$$

$$\therefore Q = kP^{-1}I$$

$$\therefore Q = \frac{\text{adj.} P}{2}$$

$$\therefore q_{23} = -\frac{k}{8}$$

$$\therefore \frac{(3\alpha + 4)}{2} = -\frac{k}{8} \Rightarrow k = 4$$

$$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$$

Put value of  $k$  in (i)... we get  $\alpha = -1$

$$\therefore \alpha^2 + k^2 = 1 + 16 = 17.$$

## Question 57

The solutions of the equation

$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix}$$

$= 0$ , ( $0 < x < \pi$ ), are  
[18 Mar 2021 Shift 1]

**Options:**

A.  $\frac{\pi}{12}, \frac{\pi}{6}$

B.  $\frac{\pi}{6}, \frac{5\pi}{6}$

C.  $\frac{5\pi}{12}, \frac{7\pi}{12}$

D.  $\frac{7\pi}{12}, \frac{11\pi}{12}$

**Answer: D**

**Solution:**

**Solution:**

$$\text{Given, } \begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

( $0 < x < \pi$ )

Applying  $R_1 \rightarrow R_1 + R_2$ ,

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4 \sin 2x & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 - C_2$ ,

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 + \cos^2 x & \cos^2 x \\ 0 & 4 \sin 2x & 1 + 4 \sin 2x \end{vmatrix} = 0$$

$$\Rightarrow 2 + 8 \sin 2x - 4 \sin 2x = 0 \text{ (expanding along } C_1 \text{)}$$

$$\Rightarrow 4 \sin 2x = -2 \Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \Rightarrow 2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

[Note You can also solve by applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$ ]

## Question 58

If  $A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$  and  $\det \left( A^2 - \frac{1}{2} I \right) = 0$ , then a possible value of  $\alpha$  is

[17 Mar 2021 Shift 1]

**Options:**

A.  $\frac{\pi}{2}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{4}$

D.  $\frac{\pi}{6}$

**Answer: C**

**Solution:**

**Solution:**

$$A = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$$

$$\text{and } \det \left( A^2 - \frac{1}{2}I \right) = 0$$

$$\therefore A^2 = \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix} \begin{pmatrix} 0 & \sin \alpha \\ \sin \alpha & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha \end{pmatrix}$$

$$\Rightarrow \frac{I}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\left( A^2 - \frac{1}{2}I \right) = \begin{pmatrix} \sin^2 \alpha - 1/2 & 0 \\ 0 & \sin^2 \alpha - 1/2 \end{pmatrix}$$

$$\therefore \det \left( A^2 - \frac{1}{2}I \right) = \begin{vmatrix} \sin^2 \alpha - 1/2 & 0 \\ 0 & \sin^2 \alpha - 1/2 \end{vmatrix}$$

$$\left( \sin^2 \alpha - \frac{1}{2} \right)^2 = 0$$

$$\sin \alpha = \pm \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4} \text{ is one possibility.}$$

## Question 59

If  $x, y, z$  are in arithmetic progression with common difference

$d, x \neq 3d$ , and the determinant of the matrix

$$\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$$

is zero, then

the value of  $k^2$  is

[17 Mar 2021 Shift 2]

**Options:**

A. 72

B. 12

C. 36

D. 6

**Answer: A**

**Solution:**

**Solution:**

Method (I)

Given,  $x, y$  and  $z$  are in AP with common difference  $= d$  $\therefore x =$  First term $y =$  Second term of AP  $=$  First term  $+ \text{Common difference}$  $\Rightarrow y = x + d \dots (i)$ and  $z =$  Third term of AP  $=$  Second term  $+ \text{Common difference}$  $\Rightarrow z = (x + d) + d = x + 2d \dots (ii)$ Also, given  $x \neq 3d \dots (iii)$ 

$$\text{and } \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_1 + R_3 - 2R_2$ , we have

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k - 6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

 $\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$  (Expanding along  $R_2$ )Either  $k - 6\sqrt{2} = 0$  or  $3z - 5x = 0$  $\Rightarrow k = 6\sqrt{2}$  or  $3(x + 2d) - 5x = 0$  [from Eq. (ii)] $\Rightarrow x = 3d$  which is not possible as in Eq. (iii). $\therefore k = 6\sqrt{2}$  is only one solution.Hence,  $k^2 = (6\sqrt{2})^2$  $\Rightarrow k^2 = 72$ **Question60**

If  $1, \log_{10}(4^x - 2)$  and  $\log_{10}\left(4^x + \frac{18}{5}\right)$  are in arithmetic progression for a real number  $x$ , then the value of the determinant

$$\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} \text{ is}$$

**[17 Mar 2021 Shift 2]****Answer: 2****Solution:**Given  $1, \log_{10}(4^x - 2), \log_{10}\left(4^x + \frac{18}{5}\right)$  are in A.P.

$$\therefore 2\log_{10}(4^x - 2) = 1 + \log_{10}\left(4^x + \frac{18}{5}\right) = \log_{10}10 + \log_{10}\left(4^x + \frac{18}{5}\right)$$

$$\Rightarrow \log_{10}(4^x - 2)^2 = \log_{10}\left(10 \times \left(4^x + \frac{18}{5}\right)\right)$$

$$\Rightarrow (4^x - 2)^2 = 10 \times 4^x + 36$$

$$\Rightarrow (4^x)^2 - 4(4^x) + 4 = 10 \times 4^x + 36$$

$$\Rightarrow (4^x)^2 - 14(4^x) - 32 = 0 \Rightarrow (4^x - 16)(4^x + 2) = 0$$

$\Rightarrow 4^x = 16$  or  $4^x = -2$  (Rejected because  $4^x > 0, \forall x \in \mathbb{R}$ )  
 $\Rightarrow 4^x = 4^2 \Rightarrow x = 2$

$$\therefore \begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 2$$

## Question61

Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ , such that  $AB = B$  and

$a + d = 2021$ , then the value of  $ad - bc$  is equal to  
**[17 Mar 2021 Shift 2]**

**Answer: 2020**

**Solution:**

**Solution:**

Given,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

i.e.  $B \neq 0$

and  $AB = B$

$$\Rightarrow AB - B = 0 \Rightarrow B(A - I) = 0$$

$$\Rightarrow |(A - I)B| = 0$$

$$\therefore B \neq 0$$

$$\therefore |A - I| = 0 \Rightarrow \begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$$

$$\Rightarrow (a-1)(d-1) - bc = 0 \Rightarrow ad - bc = 2020$$

## Question62

The maximum value of  $f(x) =$

$$\begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in \mathbb{R} \text{ is}$$

**[16 Mar 2021 Shift 2]**

**Options:**

A.  $\sqrt{7}$

B.  $\frac{3}{4}$

C.  $\sqrt{5}$

D. 5

**Answer: C**

**Solution:**

**Solution:**

$$\text{Given, } f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

On applying  $C_1 \rightarrow C_1 + C_2$ , we get

$$f(x) = \begin{vmatrix} \sin^2 x + 1 + \cos^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x + \cos^2 x & \cos^2 x & \cos 2x \\ \sin^2 x + \cos^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

On applying  $R_1 \rightarrow R_1 - R_2$

$$f(x) = \begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = -1(2 \sin 2x - \cos 2x)$$

As, we know that, if  $f(\theta) = A \sin \theta + B \cos \theta$

$$\text{Then, } -\sqrt{A^2 + B^2} \leq f(\theta) \leq \sqrt{A^2 + B^2}$$

Here, we have,  $f(x) = \cos 2x - 2 \sin 2x$

$$-\sqrt{2^2 + 1^2} \leq f(x) \leq \sqrt{2^2 + 1^2}$$

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

So, maximum value of  $f(x)$  is  $\sqrt{5}$ .

## Question63

Let  $\alpha, \beta, \gamma$  be the real roots of the equation,

$x^3 + ax^2 + bx + c = 0$ , ( $a, b, c \in \mathbb{R}$  and  $a, b \neq 0$ ). If the system of equations (in  $u, v, w$ ) given by  $\alpha u + \beta v + \gamma w = 0$ ,  $\beta u + w + \alpha w = 0$ ;

$\gamma u + \alpha v + \beta w = 0$  has non-trivial solution, then the value of  $\frac{a^2}{b}$  is

[18 Mar 2021 Shift 1]

**Options:**

A. 5

B. 3

C. 1

D. 0

**Answer: B**

## Solution:

### Solution:

Given,  $\alpha, \beta, \gamma$  are the real roots of  $x^3 + ax^2 + bx + c = 0$ , where  $a, b, c \in \mathbb{R}$  and  $a, b \neq 0$

$\therefore \alpha + \beta + \gamma =$  Sum of roots taken one at a time  $= -a$

$\alpha\beta + \beta\gamma + \gamma\alpha =$  Sum of roots taken two at a time  $= b$

$\alpha\beta\gamma =$  Product of roots  $= -c$

Also, given system of equations in  $u, v, w$

$$\left. \begin{aligned} \alpha u + \beta v + \gamma w &= 0 \\ \beta u + \gamma v + \alpha w &= 0 \\ \gamma u + \alpha v + \beta w &= 0 \end{aligned} \right\}$$

has non-trivial solution.

$$\therefore \Delta = 0 \Rightarrow \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\beta\gamma - \alpha^2) - \beta(\beta^2 - \gamma\alpha) + \gamma(\alpha\beta - \gamma^2) = 0 \text{ (expanding along } R_1 \text{)}$$

$$\Rightarrow \alpha\beta\gamma - \alpha^3 - \beta^3 + \alpha\beta\gamma + \alpha\beta\gamma - \gamma^3 = 0$$

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

Then (using standard result),

Either  $\alpha + \beta + \gamma = 0$  or  $\alpha = \beta = \gamma$

If  $\alpha + \beta + \gamma = 0$ , then  $-a = 0$

$\Rightarrow a = 0$  which is not possible according to given condition.

$\therefore \alpha + \beta + \gamma = 0$  (not possible)

Now,

$$\alpha + \beta + \gamma = -a$$

$$\Rightarrow \alpha + \alpha + \alpha = -a \quad (\because \alpha = \beta = \gamma)$$

$$\Rightarrow a = -3\alpha \dots (i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = b$$

$$\Rightarrow b = 3\alpha^2 \dots (ii)$$

Using Eqs. (i) and (ii),

$$\frac{a^2}{b} = 3$$

## Question64

Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$$

has a non-trivial solution. Then which of the following is true?

[18 Mar 2021 Shift 2]

### Options:

A.  $\mu = 6, \lambda \in \mathbb{R}$

B.  $\lambda = 2, \mu \in \mathbb{R}$

C.  $\lambda = 3, \mu \in \mathbb{R}$

D.  $\mu = -6, \lambda \in \mathbb{R}$

**Answer: A**

### Solution:

**Solution:**

Given, system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0$$

For non-trivial solution,  $\Delta = 0$

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 4(-3-2) - \lambda(6-\mu) + 2(4+\mu) = 0$$

$$\Rightarrow -\lambda(6-\mu) - 2(6-\mu) = 0$$

$$\Rightarrow (6-\mu)(\lambda+2) = 0$$

$$\Rightarrow \lambda = -2 \text{ and } \mu \in \mathbb{R} \text{ or } \mu = 6 \text{ and } \lambda \in \mathbb{R}.$$

**Question65**

The system of equations  $kx + y + z = 1$ ,  $x + ky + z = k$  and  $x + y + zk = k^2$  has no solution, if  $k$  is equal to  
[17 Mar 2021 Shift 1]

**Options:**

A. 0

B. 1

C. -1

D. -2

**Answer: D**

**Solution:****Solution:**

$$kx + y + z = 1$$

$$x + ky + z = k$$

$$x + y + kz = k^2$$

For this set of equations to have no solution,  $\Delta = 0$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = k(k^2 - 1) - 1(k - 1) + (1 - k)$$

$$= k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2$$

Now,  $\Delta = 0$

$$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)(k^2 + k - 2) = 0$$

$$\Rightarrow (k-1)(k-1)(k+2) = 0$$

$$\therefore k = 1, -2$$

$$\left. \begin{array}{l} x + y + z = 1 \\ \text{If } k = 1, \quad x + y + z = 1 \\ \quad \quad \quad x + y + z = 1 \end{array} \right\}$$

There are same equations and they will have infinite solutions.

So,  $K = -2$

## Question66

Let

$$f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix},$$

$x \in [0, \pi]$  Then the maximum value of  $f(x)$  is equal to  
[27 Jul 2021 Shift 1]

**Answer: 6**

**Solution:**

**Solution:**

$$\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}$$

$$(R_1 \rightarrow R_1 - R_2 \text{ \& } R_2 \rightarrow R_2 - R_3)$$

$$-2(\cos^2 x) + 2(2 + 2 \cos 2x + \sin^2 x)$$

$$4 + 4 \cos 2x - 2(\cos^2 x - \sin^2 x)$$

$$f(x) = 4 + 2 \cos 2x$$

$$f(x)_{\max} = 4 + 2 = 6$$

## Question67

Let

$$M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}.$$

Define  $f : M \rightarrow \mathbb{Z}$ , as  $f(A) = \det(A)$ , for all  $A \in M$  where  $\mathbb{Z}$  is set of all integers. Then the number of  $A \in M$  such that  $f(A) = 15$  is equal to

[25 Jul 2021 Shift 1]

**Answer: 16**

**Solution:**

$$|A| = ad - bc = 15$$

$$\text{where } a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}$$

$$\text{Case I } ad = 9 \text{ \& } bc = -6$$

$$\text{For } ad \text{ possible pairs are } (3, 3), (-3, -3)$$

$$\text{For } bc \text{ possible pairs are } (3, -2), (-3, 2), (-2, 3), (2, -3)$$

$$\text{So total matrix} = 2 \times 4 = 8$$

Case II  $ad = 6$  &  $bc = -9$   
 Similarly total matrix  $= 2 \times 4 = 8$   
 $\Rightarrow$  Total such matrices are  $= 16$

## Question 68

The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$$

$= 0$  in the interval

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$  is:

[25 Jul 2021 Shift 2]

Options:

- A. 4
- B. 1
- C. 2
- D. 3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

Apply :  $R_1 \rightarrow R_1 - R_2$  &  $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 (\sin x + 2 \cos x) = 0$$

$$\therefore x = \frac{\pi}{4}$$

## Question 69

Let  $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$ ,  $a \in \mathbb{R}$  be written as  $P + Q$  where  $P$  is a symmetric

matrix and  $Q$  is skew symmetric matrix. If  $\det(Q) = 9$ , then the modulus of the sum of all possible values of determinant of  $P$  is equal to :

[20 Jul 2021 Shift 1]

**Options:**

- A. 36
- B. 24
- C. 45
- D. 18

**Answer: A**

**Solution:**

**Solution:**

$$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in \mathbb{R}$$

$$\text{and } P = \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$$

$$\text{and } Q = \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$$

$$\text{As, } \det(Q) = 9$$

$$\Rightarrow (a-3)^2 = 36$$

$$\Rightarrow a = 3 \pm 6$$

$$\therefore a = 9, -3$$

$$\therefore \det(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$$

$$= 0 - \frac{(a-3)^2}{4} = 0, \text{ for } a = -3$$

$$= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12), \text{ for } a = 9$$

$$\therefore \text{Modulus of the sum of all possible values of } \det(P) = |-36| + |0| = 36 \text{ Ans.}$$

$$\Rightarrow \text{Option (1) is correct}$$

## Question 70

Let  $A = [a_{ij}]$  be a  $3 \times 3$  matrix, where  $a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i - j| = 1 \\ 2x + 1 & , \text{ otherwise} \end{cases}$

Let a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = \det(A)$  Then the sum of maximum and minimum values of  $f$  on  $\mathbb{R}$  is equal to:  
[20 Jul 2021 Shift 1]

**Options:**

A.  $-\frac{20}{27}$

B.  $\frac{88}{27}$

C.  $\frac{20}{27}$

D.  $-\frac{88}{27}$

**Answer: D**

**Solution:**

**Solution:**

$$A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$$

$$|A| = 4x^3 - 4x^2 - 4x = f(x)$$

$$f'(x) = 4(3x^2 - 2x - 1) = 0$$

$$\Rightarrow x = 1; x = -\frac{1}{3}$$

$$\therefore f(1) = -4; f\left(-\frac{1}{3}\right) = \frac{20}{27}$$

$$\text{Sum} = -4 + \frac{20}{27} = -\frac{88}{27}$$

## Question 71

Let a, b, c, d be in arithmetic progression with common difference  $\lambda$ . If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2 \text{ then value of } \lambda^2 \text{ is equal to } \underline{\hspace{2cm}}.$$

**[20 Jul 2021 Shift 1]**

**Answer: 1**

**Solution:**

**Solution:**

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$$

$$C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = 2$$

$$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \lambda \begin{vmatrix} x - 2\lambda & 1 & x + a \\ 2\lambda - 1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$$

$$\Rightarrow 1(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$$

$$\Rightarrow \lambda^2 = 1$$


---

## Question 72

For real numbers  $\alpha$  and  $\beta$ , consider the following system of linear equations :

$x + y - z = 2$ ,  $x + 2y + \alpha z = 1$ ,  $2x - y + z = \beta$  If the system has infinite solutions, then  $\alpha + \beta$  is equal to \_\_\_\_\_ .

[27 Jul 2021 Shift 1]

**Answer: 5**

**Solution:**

**Solution:**

For infinite solutions

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = 3(2 + \alpha) = 0$$

$$\Rightarrow \alpha = -2$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$$1(1 + 2\beta) - 2(1 + 4) - (\beta - 2) = 0$$

$$\beta - 7 = 0$$

$$\beta = 7$$

$$\therefore \alpha + \beta = 5 \text{ Ans.}$$


---

## Question 73

The values of  $a$  and  $b$ , for which the system of equations

$$2x + 3y + 6z = 8$$

$$x + 2y + az = 5$$

$$3x + 5y + 9z = b$$

has no solution, are :

[25 Jul 2021 Shift 1]

**Options:**

- A.  $a = 3, b \neq 13$
- B.  $a \neq 3, b \neq 13$
- C.  $a \neq 3, b = 3$
- D.  $a = 3, b = 13$

**Answer: A**

**Solution:**

**Solution:**

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If  $a = 3, b \neq 13$ , no solution.

## Question 74

The values of  $\lambda$  and  $\mu$  such that the system of equations  $x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$  has no solution, are :  
[22 Jul 2021 Shift 2]

**Options:**

- A.  $\lambda = 3, \mu = 5$
- B.  $\lambda = 3, \mu \neq 10$
- C.  $\lambda \neq 2, \mu = 10$
- D.  $\lambda = 2, \mu \neq 10$

**Answer: D**

**Solution:**

**Solution:**

$$\begin{aligned} x + y + z &= 6 \dots (i) \\ 3x + 5y + 5z &= 26 \dots (ii) \\ x + 2y + \lambda z &= \mu \dots (iii) \\ 5 \times (i) - (ii) &\Rightarrow 2x = 4 \Rightarrow x = 2 \\ \therefore \text{from (i) and (iii)} \\ y + z &= 4 \dots (iv) \\ 2y + \lambda z &= \mu - 2 \\ (v) - 2 \times (iv) & \\ \Rightarrow (\lambda - 2)z &= \mu - 10 \\ \Rightarrow z &= \frac{\mu - 10}{\lambda - 2} \text{ \& } y = 4 - \frac{\mu - 10}{\lambda - 2} \\ \therefore \text{For no solution } \lambda &= 2 \text{ and } \mu \neq 10. \end{aligned}$$

## Question 75

The value of  $k \in \mathbb{R}$ , for which the following system of linear equations

$$3x - y + 4z = 3$$

$$x + 2y - 3z = -2$$

$$6x + 5y + kz = -3$$

has infinitely many solutions, is :

[20 Jul 2021 Shift 2]

Options:

A. 3

B. -5

C. 5

D. -3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$$

$$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$$

$$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$$

## Question 76

Let  $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$ , where  $[t]$  denotes the greatest integer

less than or equal to  $t$ . If  $\det(A) = 192$ , then the set of values of  $x$  is the interval

[27 Aug 2021 Shift 2]

Options:

A. [68, 69]

B. [62, 63]

C. [65, 66]

D. [60, 61]

Answer: B

## Solution:

### Solution:

$$\text{Given, } A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$$

$$A = \begin{pmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{pmatrix} \quad (\because [x+n] = n + [x], n \in \mathbb{I})$$

Applying  $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix}$$

$$\det(A) = 1([x] + 4 + [x] + 2) - 1(-[x])$$

$$= 3[x] + 6$$

$$\because \det(A) = 192$$

$$\Rightarrow 3[x] + 6 = 192$$

$$\Rightarrow [x] = 62$$

$$\Rightarrow 62 \leq x < 63$$

$$\Rightarrow x \in [62, 63)$$

## Question 77

Let  $A(a, 0)$ ,  $B(b, 2b + 1)$  and  $C(0, b)$ ,  $b \neq 0$ ,  $|b| \neq 1$ , be points such that the area of  $\triangle ABC$  is 1 sq. unit, then the sum of all possible values of  $a$  is [27 Aug 2021 Shift 2]

### Options:

A.  $\frac{-2b}{b+1}$

B.  $\frac{2b}{b+1}$

C.  $\frac{2b^2}{b+1}$

D.  $\frac{-2b^2}{b+1}$

### Answer: D

### Solution:

#### Solution:

$A(a, 0)$ ,  $B(b, 2b + 1)$ ,  $C(0, b)$

$$\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 1$$

$$\Rightarrow \frac{1}{2}[a(b+1) + b^2] = \pm 1$$

$$\Rightarrow a = \frac{2-b^2}{b+1} \text{ or } \frac{-2-b^2}{b+1}$$

Sum of all possible values of  $a = \frac{-2b^2}{b+1}$

---

## Question 78

If the following system of linear equations  $2x + y + z = 5$ ,  $x - y + z = 3$  and  $x + y + az = b$  has no solution, then  
[31 Aug 2021 Shift 1]

Options:

A.  $a = -\frac{1}{3}$  and  $b \neq \frac{7}{3}$

B.  $a \neq \frac{1}{3}$  and  $b = \frac{7}{3}$

C.  $a \neq -\frac{1}{3}$  and  $b = \frac{7}{3}$

D.  $a = \frac{1}{3}$  and  $b \neq \frac{7}{3}$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + (1+1) \\ = 1 - 3a$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1$$

$$(b-3) + 5(1+1) = 7 - 3b$$

$$\text{Now, } z = \frac{\Delta_3}{\Delta}$$

If  $\Delta = 0$  and  $\Delta_3 \neq 0$ , then no solution

$$1 - 3a = 0$$

$$\Rightarrow 7 - 3b \neq 0$$

$$a = \frac{1}{3}, b \neq \frac{7}{3}$$

---

## Question 79

If  $\alpha + \beta + \gamma = 2\pi$ , then the system of equations  
 $x + (\cos \gamma)y + (\cos \beta)z = 0$   
 $(\cos \gamma)x + y + (\cos \alpha)z = 0$   
 $(\cos \beta)x + (\cos \alpha)y + z = 0$   
has :  
[31 Aug 2021 Shift 2]

Options:

- A. no solution
- B. infinitely many solution
- C. exactly two solutions
- D. a unique solution

**Answer: B**

**Solution:**

**Solution:**

Given  $\alpha + \beta + \gamma = 2\pi$

$$\Delta = \begin{vmatrix} 1 & \cos \gamma & \cos \beta \\ \cos \gamma & 1 & \cos \alpha \\ \cos \beta & \cos \alpha & 1 \end{vmatrix}$$

$$= 1 - \cos^2 \alpha - \cos \gamma (\cos \gamma - \cos \alpha \cos \beta) + \cos \beta (\cos \alpha \cos \gamma - \cos \beta)$$

$$= 1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma$$

$$= -\cos(\alpha + \beta) \cos(\alpha - \beta) - \cos \gamma (\cos(2\pi - (\alpha + \beta)) - 2 \cos \alpha \cos \beta)$$

$$= -\cos(2\pi - \gamma) \cos(\alpha - \beta) - \cos \gamma (\cos(\alpha + \beta) - 2 \cos \alpha \cos \beta)$$

$$= -\cos \gamma \cos(\alpha - \beta) + \cos \gamma (\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= -\cos \gamma \cos(\alpha - \beta) + \cos \gamma \cos(\alpha - \beta)$$

$$= 0$$

## Question80

If the system of linear equations

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solution, then  $\alpha + \beta - \alpha\beta$  is equal to

[27 Aug 2021 Shift 1]

**Answer: 5**

**Solution:**

**Solution:**

Given, system of equation

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

has infinitely many solutions,

if  $\Delta = 0$ ,  $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\text{Now, } \Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 2(-\beta + 3) - 1(\beta + 3) - 1(3 + 3) = 0$$

$$\Rightarrow \beta = -1$$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & -1 \\ \alpha & -1 & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 3(1+3) - 1(-\alpha+3) - 1(3\alpha+3) = 0$$

$$\Rightarrow 12 + \alpha - 3 - 3\alpha - 3 = 0$$

$$\Rightarrow \alpha = 3$$

$$\text{Also, } \Delta_2 = \begin{vmatrix} 2 & 3 & -1 \\ 1 & \alpha & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-\alpha+3) - 3(-1+3) - 1(3-3\alpha) = 0$$

$$\Rightarrow \alpha = 3$$

$$\text{and } \Delta_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & \alpha \\ 3 & 3 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 2(-3-3\alpha) - 1(3-3\alpha) + 3(3+3) = 0$$

$$\Rightarrow -3\alpha + 9 = 0$$

$$\Rightarrow \alpha = 3$$

$$\therefore \alpha = 3, \beta = -1$$

$$\text{So, } \alpha + \beta - \alpha\beta = 3 - 1 - 3(-1) = 5$$

## Question81

Let  $|\lambda|$  be the greatest integer less than or equal to  $\lambda$ . The set of all values of  $\lambda$  for which the system of linear equations  $x + y + z = 4$ ,  $3x + 2y + 5z = 3$ ,  $9x + 4y + (28 + |\lambda|)z = |\lambda|$  has a solution is [27 Aug 2021 Shift 2]

**Options:**

A.  $\mathbb{R}$

B.  $(-\infty, -9) \cup (-9, \infty)$

C.  $[-9, -8]$

D.  $(-\infty, -9) \cup [-8, \infty)$

**Answer: A**

**Solution:**

**Solution:**

Given, system of equations

$$x + y + z = 4$$

$$3x + 2y + 5z = 3$$

$$9x + 4y + (28 + |\lambda|)z = |\lambda|$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + |\lambda| \end{vmatrix}$$

$$= 1(56 + 2|\lambda| - 20) - 1(84 + 3|\lambda| - 45) + 1(-6)$$

$$= -(|\lambda| + 9)$$

If  $\Delta \neq 0$  i.e.  $|\lambda| + 9 \neq 0$ , then system of equation has unique solution.

If  $|\lambda| + 9 = 0$ , then  $\Delta_1 = \Delta_2 = \Delta_3 = 0$ , the system of equation has infinite solution.

$$\Rightarrow \lambda \in \mathbb{R}$$

## Question82

Let  $\theta \in \left(0, \frac{\pi}{2}\right)$ . If the system of linear equations

$$(1 + \cos^2\theta)x + \sin^2\theta y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + (1 + \sin^2\theta)y + 4 \sin 3\theta z = 0$$

$$\cos^2\theta x + \sin^2\theta y + (1 + 4 \sin 3\theta)z = 0$$

has a non-trivial solution, then the value of  $\theta$  is

[26 Aug 2021 Shift 1]

Options:

A.  $\frac{4\pi}{9}$

B.  $\frac{7\pi}{18}$

C.  $\frac{\pi}{18}$

D.  $\frac{5\pi}{18}$

Answer: B

Solution:

Solution:

$$\Rightarrow \begin{vmatrix} 1 + \cos^2\theta & \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta \end{vmatrix} = 0$$

Applying  $R_1 \rightarrow R_1 - R_3$ , we get

$$\begin{vmatrix} 1 & 0 & -1 \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta \end{vmatrix} = 0$$

Applying  $C_1 \rightarrow C_1 - C_3$ , we get

$$\begin{vmatrix} 1 & 0 & 0 \\ \cos^2\theta & 1 + \sin^2\theta & 4 \sin 3\theta + \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 1 + 4 \sin 3\theta + \cos^2\theta \end{vmatrix} = 0$$

$$\Rightarrow (1 + \sin^2\theta)(1 + 4 \sin 3\theta + \cos^2\theta) - \sin^2\theta(4 \sin 3\theta + \cos^2\theta) = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + \cos^2\theta + \sin^2\theta + 4 \sin^2\theta \sin 3\theta + \sin^2\theta \cos^2\theta - 4 \sin^2\theta \sin 3\theta - \sin^2\theta \cos^2\theta = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + \cos^2\theta + \sin^2\theta = 0$$

$$\Rightarrow 1 + 4 \sin 3\theta + 1 = 0$$

$$\Rightarrow 4 \sin 3\theta + 2 = 0$$

$$\Rightarrow \sin 3\theta = \frac{-1}{2}$$

$$\Rightarrow 3\theta = \left(\pi + \frac{\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{7\pi}{18}$$

---

## Question83

Two fair dice are thrown. The numbers on them are taken as  $\lambda$  and  $\mu$ , and a system of linear equations  $x + y + z = 5$ ,  $x + 2y + 3z = \mu$  and  $x + 3y + \lambda z = 1$  is constructed. If  $p$  is the probability that the system has a unique solution and  $q$  is the probability that the system has no solution, then [26 Aug 2021 Shift 2]

Options:

A.  $p = \frac{1}{6}$  and  $q = \frac{1}{36}$

B.  $p = \frac{5}{6}$  and  $q = \frac{5}{36}$

C.  $p = \frac{5}{6}$  and  $q = \frac{1}{36}$

D.  $p = \frac{1}{6}$  and  $q = \frac{5}{36}$

Answer: B

Solution:

Solution:

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + 3z &= \mu \\ x + 3y + \lambda z &= 1 \end{aligned}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = (2\lambda - 9) + (3 - \lambda) + (3 - 2) = \lambda - 5$$

For unique solution  $\Delta \neq 0$

$$\Rightarrow \lambda \neq 5$$

And  $\Delta_1$  or  $\Delta_2$  or  $\Delta_3 \neq 0$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} = (2 - 3\mu) + (\mu - 1) + 5 = 6 - 2\mu$$

If  $\Delta_3 \neq 0$  and  $\Delta = 0$ , then no solution

$$\mu \neq 3 \text{ and } \lambda = 5$$

$$p = \text{Probability of unique solution} = \frac{5}{6}$$

$$q = \text{Probability of no solution} = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$$

## Question84

Consider the system of linear equations

$$-x + y + 2z = 0$$

$$3x - ay + 5z = 1$$

$$2x - 2y - az = 7$$

Let  $S_1$  be the set of all  $a \in \mathbb{R}$  for which the system is inconsistent and  $S_2$  be the set of all  $a \in \mathbb{R}$  for which the system has infinitely many solutions. If  $n(S_1)$  and  $n(S_2)$  denote the number of elements in  $S_1$  and  $S_2$

**respectively, then**  
**[1 Sep 2021 Shift 2]**

**Options:**

- A.  $n(S_1) = 2$  and  $n(S_2) = 2$
- B.  $n(S_1) = 1$  and  $n(S_2) = 0$
- C.  $n(S_1) = 2$  and  $n(S_2) = 0$
- D.  $n(S_1) = 0$  and  $n(S_2) = 2$

**Answer: C**

**Solution:**

**Solution:**

For in consistent system of equations

[ $\Delta = 0$  and atleast one is non-zero in  $\Delta_1, \Delta_2$  and  $\Delta_3$ ]

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix} = 0$$

$$\Rightarrow a^2 - 7a + 12 = 0$$

$$\Rightarrow a = 3, 4$$

$$\Delta_x = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix} = 15a + 31$$

$$\Delta_x \neq 0 \text{ for } a = 3, 4$$

$$\Rightarrow n(S_1) = 2$$

Now, for infinitely many solutions.

If  $\Delta = 0$  also  $\Delta_x = \Delta_y = \Delta_z = 0$

Which is not possible for any real value of  $a$

$$\Rightarrow n(S_2) = 0$$

---

## Question85

**If the system of linear equations,**

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

**has more than two solutions, then  $\mu - \lambda^2$  is equal to \_\_\_\_\_.**

**[NA Jan. 7, 2020 (II)]**

**Answer: 13**

**Solution:**

$x + y + z = 6$  .....(i)  
 $x + 2y + 3z = 10$  .....(ii)  
 $3x + 2y + \lambda z = \mu$  .....(iii)  
 From (i) and (ii),  
 If  $z = 0 \Rightarrow x + y = 6$  and  $x + 2y = 10$   
 $\Rightarrow y = 4, x = 2$   
 $(2, 4, 0)$   
 If  $y = 0 \Rightarrow x + z = 6$  and  $x + 3z = 10$   
 $\Rightarrow z = 2$  and  $x = 4$   
 $(4, 0, 2)$   
 So,  $3x + 2y + \lambda z = \mu$ , must pass through  $(2, 4, 0)$  and  $(4, 0, 2)$   
 So,  $6 + 8 = \mu \Rightarrow \mu = 14$   
 and  $12 + 2\lambda = \mu$   
 $12 + 2\lambda = 14 \Rightarrow \lambda = 1$   
 So,  $\mu - \lambda^2 = 14 - 1 = 13$

## Question86

Let  $A = [a_{ij}]$  and  $B = [b_{ij}]$  be two  $3 \times 3$  real matrices such that  
 $b_{ij} = (3)^{(i+j-2)} a_{ij}$ , where  $i, j = 1, 2, 3$ . If the determinant of B is 81, then  
 the determinant of A is:  
**[Jan. 7, 2020 (II)]**

**Options:**

- A.  $1/3$
- B. 3
- C.  $1/81$
- D.  $1/9$

**Answer: D**

**Solution:**

**Solution:**

It is given that  $|B| = 81$

$$\therefore |B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{12} & 3^2 a_{13} \\ 3^1 a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

$$\Rightarrow 81 = 3^3 \cdot 3^2 \cdot 3^1 |A|$$

$$\Rightarrow 3^4 = 3^6 |A| \Rightarrow |A| = \frac{1}{9}$$

## Question87

Let two points be  $A(1, -1)$  and  $B(0, 2)$ . If a point  $P(x', y')$  be such that  
 the area of  $\Delta PAB = 5$  sq. units and it lies on the line,  $3x + y - 4\lambda = 0$ ,  
 then a value of  $\lambda$  is:  
**[Jan.8,2020 (I)]**

**Options:**

A. 4

B. 3

C. 1

D. -3

**Answer: B**

**Solution:**

**Solution:**

$$D = \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = 5$$

$$\Rightarrow -2(1 - x') + (y' + x') = \pm 10$$

$$\Rightarrow -2 + 2x' + y' + x' = \pm 10$$

$$\Rightarrow 3x' + y' = 12 \text{ or } 3x' + y' = -8$$

$$\therefore \lambda = 3, -2$$

## Question88

If the matrices  $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ ,  $B = \text{adj } A$  and  $C = 3A$ , then  $\frac{|\text{adj } B|}{|C|}$  is

equal to:

[Jan. 9, 2020 (I)]

**Options:**

A. 8

B. 16

C. 72

D. 2

**Answer: A**

**Solution:**

**Solution:**

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = ((9 + 4) - 1(3 - 4) + 2(-1 - 3))$$

$$= 13 + 1 - 8 = 6$$

$$|\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^4 = (36)^2$$

$$|C| = |3A| = 3^3 \times 6$$

$$\text{Hence, } \frac{|\text{adj } B|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$$

## Question89

The following system of linear equations

$$7x + 6y - 2z = 0$$

$$3x + 4y + 2z = 0$$

$$x - 2y - 6z = 0, \text{ has}$$

[Jan. 9, 2020 (II)]

Options:

A. infinitely many solutions,  $(x, y, z)$  satisfying  $y = 2z$ .

B. no solution.

C. infinitely many solutions,  $(x, y, z)$  satisfying  $x = 2z$ .

D. only the trivial solution.

Answer: C

Solution:

Solution:

The given system of linear equations

$$7x + 6y - 2z = 0 \dots\dots(i)$$

$$3x + 4y + 2z = 0 \dots\dots(ii)$$

$$x - 2y - 6z = 0 \dots\dots(iii)$$

Now, determinant of coefficient matrix

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$
$$= 7(-20) - 6(-20) - 2(-10)$$
$$= -140 + 120 + 20 = 0$$

So, there are infinite non-trivial solutions.

From eqn. (i)  $+3 \times$  (iii); we get

$$10x - 20z = 0 \Rightarrow x = 2z$$

Hence, there are infinitely many solutions  $(x, y, z)$  satisfying  $x = 2z$

---

## Question90

For which of the following ordered pairs  $(\mu, \delta)$ , the system of linear equations

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta$$

is inconsistent?

[Jan. 8, 2020 (I)]

Options:

A. (4,3)

B. (4,6)

C. (1,0)

D. (3,4)

**Answer: A**

**Solution:**

**Solution:**

From the given linear equation, we get

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} \quad (R_3 \rightarrow R_3 - 2R_2 + 3R_1)$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now, let  $P_3 = 4x + 4y + 4z - \delta = 0$ . If the system has solutions it will have infinite solution.

So,  $P_3 \equiv \alpha P_1 + \beta P_2$

Hence,  $3\alpha + \beta = 4$  and  $4\alpha + 2\beta = 4$

$\Rightarrow \alpha = 2$  and  $\beta = -2$

So, for infinite solution  $2\mu - 2 = \delta$

$\Rightarrow$  For  $2\mu \neq \delta + 2$  system is inconsistent

## Question 91

**The system of linear equations**

$$\lambda x + 2y + 2z = 5$$

$$2\lambda x + 3y + 5z = 8$$

$$4x + \lambda y + 6z = 10 \text{ has :}$$

**[Jan. 8, 2020 (II)]**

**Options:**

A. no solution when  $\lambda = 8$

B. a unique solution when  $\lambda = -8$

C. no solution when  $\lambda = 2$

D. infinitely many solutions when  $\lambda = 2$

**Answer: C**

**Solution:**

**Solution:**

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$$

$$D = \lambda^2 + 6\lambda - 16$$

$$D = (\lambda + 8)(2 - \lambda)$$

For no solutions,  $D = 0$

$$\Rightarrow \lambda = -8, 2$$

when  $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$$= 5[18 - 10] - 2[48 - 50] + 2(16 - 30)$$

$$= 40 + 4 - 28 \neq 0$$

There exist no solutions for  $\lambda = 2$

---

## Question92

If the system of linear equations

$$2x + 2ay + az = 0$$

$$2x + 3by + bz = 0$$

$$2x + 4cy + cz = 0$$

where  $a, b, c \in \mathbb{R}$  are non-zero and distinct; has a non-zero solution, then:

[Jan. 7, 2020 (I)]

Options:

A.  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in A.P.

B.  $a, b, c$  are in G.P.

C.  $a + b + c = 0$

D.  $a, b, c$  are in A.P.

**Answer: A**

**Solution:**

**Solution:**

For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$$

$$\Rightarrow -bc + 2ac - ab = 0$$

$$\Rightarrow ab + bc + 2ac$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ in A.P.}$$


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## Question93

Let  $\theta = \frac{\pi}{5}$  and  $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ . If  $B = A + A^4$ , then  $\det(B)$ :

[Sep. 06, 2020 (II)]

Options:

- A. is one
- B. lies in (2,3)
- C. is zero
- D. lies in (1,2)

Answer: D

Solution:

Solution:

$$\therefore A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbb{N}$$

$$\therefore B = A + A^4$$

$$= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} \cos \frac{\pi}{5} + \cos \frac{4\pi}{5} & \sin \frac{\pi}{5} + \sin \frac{4\pi}{5} \\ -\sin \frac{\pi}{5} - \sin \frac{4\pi}{5} & \cos \frac{\pi}{5} + \cos \frac{4\pi}{5} \end{bmatrix}$$

$$\text{Then, } \det(B) = 2 \sin\left(\frac{\pi}{5}\right) \cdot \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= \frac{\sqrt{10 - 2\sqrt{5}}}{2} \approx 2.352 \approx 1.175$$

$$\therefore \det B \in (1, 2)$$

## Question94

If  $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$  then  $B + C$  is equal to :

[Sep. 03, 2020 (I)]

Options:

- A. -1
- B. 1
- C. -3
- D. 9

**Answer: C**

**Solution:**

**Solution:**

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$$
$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ 2x-3 & x-1 & x-1 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} \begin{bmatrix} C_3 \rightarrow C_3 - C_2 \\ C_2 \rightarrow C_2 - C_1 \end{bmatrix}$$
$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ x-1 & 0 & 0 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} [R_2 \rightarrow R_2 - R_1]$$
$$\Rightarrow \Delta = -(x-1)[(x-1)(5x-9) - (x-1)(2x-3)]$$
$$\Rightarrow \Delta = -(x-1)[(5x^2 - 14x + 9) - (2x^2 - 5x + 3)]$$
$$= -3x^3 + 12x^2 - 15x + 6$$

So,  $B + C = -3$

## Question 95

If the minimum and the maximum values of the function  $f : \left[ \frac{\pi}{4}, \frac{\pi}{2} \right] \rightarrow \mathbb{R}$ ,

defined by  $f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 - \sin^2\theta & 1 \\ -\cos^2\theta & -1 - \cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$  are  $m$  and  $M$  respectively, then

the ordered pair  $(m, M)$  is equal to :  
[Sep. 05, 2020 (I)]

**Options:**

- A.  $(0, 2\sqrt{2})$
- B.  $(-4, 0)$
- C.  $(-4, 4)$
- D.  $(0, 4)$

**Answer: B**

**Solution:**

**Solution:**

Applying  $C_2 \rightarrow C_2 - C_1$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix}$$
$$= 4(\cos^2\theta - \sin^2\theta)$$

$$= 4 \cos 2\theta, \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\text{Max. } f(\theta) = M = 0$$

$$\text{Min. } f(\theta) = m = -4$$

$$\text{So, } (m, M) = (-4, 0)$$

## Question96

$$\text{Let } a - 2b + c = 1.$$

$$\text{If } f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}, \text{ then:}$$

[Jan. 9,2020 (II)]

Options:

A.  $f(-50) = 501$

B.  $f(-50) = -1$

C.  $f(50) = -501$

D.  $f(50) = 1$

Answer: D

Solution:

Solution:

$$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$R_1 = R_1 + R_3 - 2R_2$$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$$

## Question97

If  $a + x = b + y = c + z + 1$ , where  $a, b, c, x, y, z$  are non-zero distinct real

numbers, then  $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$  is equal to

[Sep. 05, 2020 (II)]

Options:

A.  $y(b - a)$

B.  $y(a - b)$

C. 0

D.  $y(a - c)$

**Answer: B**

**Solution:**

**Solution:**

Use properties of determinant

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + y \begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$= 0 + y \begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \left[ \begin{array}{l} R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1 \end{array} \right]$$

$$= -y(x-y) = -y(b-a) = y(a-b)$$

## Question98

Let A be a  $3 \times 3$  matrix such that  $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$  and

$B = \text{adj}(\text{adj } A)$ . If  $|A| = \lambda$  and  $|(B^{-1})^T| = \mu$ , then the ordered pair,  $(|\lambda|, \mu)$  is equal to:

[Sep. 03, 2020 (II)]

**Options:**

A.  $\left(3, \frac{1}{81}\right)$

B.  $\left(9, \frac{1}{9}\right)$

C. (3,81)

D.  $\left(9, \frac{1}{81}\right)$

**Answer: A**

**Solution:**

**Solution:**

$$|\text{adj } A| = |A|^2 = 9$$

$$[\because |\text{adj } A| = |A|^{n-1}]$$

$$\Rightarrow |A| = \pm 3 = \lambda \Rightarrow |\lambda| = 3$$

$$\Rightarrow |B| = |\text{adj } A|^2 = 81$$

$$\mu = |(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \frac{1}{81}$$

## Question99

The values of  $\lambda$  and  $\mu$  for which the system of linear equations

$$x + y + z = 2$$

$$x + 2y + 3z = 5$$

$$x + 3y + \lambda z = \mu$$

has infinitely many solutions are, respectively:

[Sep. 06, 2020 (I)]

Options:

A. 6 and 8

B. 5 and 7

C. 5 and 8

D. 4 and 9

**Answer: C**

**Solution:**

**Solution:**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 5$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0 \Rightarrow \mu = 8$$

## Question100

The sum of distinct values of  $\lambda$  for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

has non-zero solutions, is \_\_\_\_\_.

[NA Sep. 06, 2020 (II)]

**Answer: 3**

**Solution:**

For non-zero solution,  $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0$$

$$\Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$$

$$\Rightarrow \lambda = 0, \lambda = 3 \text{ [Distinct values]}$$

Then, the sum of distinct values of  $\lambda = 0 + 3 = 3$ .

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## Question 101

Let  $\lambda \in \mathbb{R}$ . The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

[Sep. 05, 2020 (I)]

Options:

A. exactly one negative value of  $\lambda$

B. exactly one positive value of  $\lambda$

C. every value of  $\lambda$

D. exactly two value of  $\lambda$

Answer: A

Solution:

Solution:

$$\therefore \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$$

$$\Rightarrow \lambda = 3 \text{ or } -\frac{2}{3}$$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3 - \lambda)$$

$$\therefore \text{When } \lambda = -\frac{2}{3}, D_1 \neq 0.$$

Hence, equations will be inconsistent when  $\lambda = -\frac{2}{3}$ .

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## Question 102

If the system of linear equations

$$x + y + 3z = 0$$

$$x + 3y + k^2z = 0$$

$$3x + y + 3z = 0$$

has a non-zero solution  $(x, y, z)$  for some  $k \in \mathbb{R}$ , then  $x + \left(\frac{y}{z}\right)$  is equal to

:  
[Sep. 05, 2020 (II)]

Options:

- A. -3
- B. 9
- C. 3
- D. -9

Answer: A

Solution:

Solution:

Since, system of linear equations has non-zero solution  
 $\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 1(9 - k^2) - 1(3 - 3k^2) + 3(1 - 9) = 0$$

$$\Rightarrow 9 - k^2 - 3 + 3k^2 - 24 = 0$$

$$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, k = \pm 3$$

So, equations are

$$x + y + 3z = 0 \dots\dots(i)$$

$$x + 3y + 9z = 0 \dots\dots(ii)$$

$$3x + y + 3z = 0 \dots\dots(iii)$$

Now, from equation (i) - (ii),

$$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3 \dots\dots(iv)$$

Now, from equation (i) - (iii),

$$-2x = 0 \Rightarrow x = 0$$

$$\text{So, } x + \frac{y}{z} = 0 - 3 = -3$$

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## Question 103

If the system of equations  $x - 2y + 3z = 9$ ,  $2x + y + z = bx - 7y + az = 24$ , has infinitely many solutions, then  $a - b$  is equal to \_\_\_\_\_.

[NA Sep. 04, 2020 (I)]

Answer: 5

Solution:

For infinitely many solutions,

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$$\Rightarrow (a+7) - 2(1-2a) + 3(-15) = 0$$

$$\Rightarrow a = 8$$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$$\Rightarrow (24+7b) - 2(b-48) + 9(-15) = 0$$

$$\Rightarrow b = 3$$

$$\therefore a - b = 5$$

## Question 104

Suppose the vectors  $x_1$ ,  $x_2$  and  $x_3$  are the solutions of the system of linear equations,  $Ax = b$  when the vector  $b$  on the right side is equal to  $b_1$ ,  $b_2$  and  $b_3$  respectively. If

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\text{and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of } A \text{ is equal to:}$$

[Sep. 04, 2020 (II)]

**Options:**

A. 4

B. 2

C.  $\frac{1}{2}$

D.  $\frac{3}{2}$

**Answer: B**

**Solution:**

Given that  $Ax = b$  has solutions  $x_1, x_2, x_3$  and  $b$  is equal to  $b_1, b_2$  and  $b_3$

$$\therefore x_1 + y_1 + z_1 = 1$$

$$\Rightarrow 2y_1 + z_1 = 2 \Rightarrow z_1 = 2$$

Determinant of coefficient matrix

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

## Question 105

If the system of equations

$$x + y + z = 2$$

$$2x + 4y - z = 6$$

$$3x + 2y + \lambda z = \mu$$

has infinitely many solutions, then :

[Sep. 04, 2020 (II)]

Options:

A.  $\lambda + 2\mu = 14$

B.  $2\lambda - \mu = 5$

C.  $\lambda - 2\mu = -5$

D.  $2\lambda + \mu = 14$

Answer: D

Solution:

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \quad [\because \text{Equation has many solutions}]$$

$$\Rightarrow -15 + 6 + 2\lambda = 0 \Rightarrow \lambda = \frac{9}{2}$$

$$\therefore D_z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 2\mu \end{vmatrix} = 0 \Rightarrow \mu = 5$$

$$\therefore 2\lambda + \mu = 14$$

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## Question 106

Let S be the set of all integer solutions, (x, y, z), of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that  $15 \leq x^2 + y^2 + z^2 \leq 150$ . Then, the number of elements in the set S is equal to \_\_\_\_\_.

[NA Sep. 03, 2020 (II)]

Answer: 8

Solution:

The given system of equations

$$x - 2y + 5z = 0 \dots\dots(i)$$

$$-2x + 4y + z = 0 \dots\dots(ii)$$

$$-7x + 14y + 9z = 0 \dots\dots(iii)$$

From equation,  $2 \times (i) + (ii) \Rightarrow z = 0$

Put  $z = 0$  in equation (i), we get  $x = 2y$

$$\because 15 \leq x^2 + y^2 + z^2 \leq 150$$

$$\Rightarrow 15 \leq 4y^2 + y^2 \leq 150$$

$$[\because x = 2y, z = 0]$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$$

$$\Rightarrow 8 \text{ solutions.}$$

## Question 107

Let  $S$  be the set of all  $\lambda \in \mathbb{R}$  for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set  $S$

[Sep. 02, 2020 (I)]

Options:

A. contains more than two elements.

B. is an empty set.

C. is a singleton.

D. contains exactly two elements.

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

For no solution  $\Delta = 0$  and at least one of  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$  is non-zero.

$$\therefore \Delta = 0 \Rightarrow \lambda = 1, -\frac{1}{2} \text{ and } \Delta_1 \neq 0$$

$$\text{Hence, } S = \left\{ 1, -\frac{1}{2} \right\}$$

## Question 108

Let  $A = \{ \mathbf{X} = (x, y, z)^T : \mathbf{P}\mathbf{X} = \mathbf{0} \text{ and } x^2 + y^2 + z^2 = 1 \}$  where

$$P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}, \text{ then the set A :}$$

**[Sep. 02, 2020 (II)]**

**Options:**

- A. is a singleton
- B. is an empty set
- C. contains more than two elements
- D. contains exactly two elements

**Answer: D**

**Solution:**

**Solution:**

$$\because |P| = 1(-3 + 36) - 2(2 + 4) + 1(-18 - 3) = 0$$

Given that  $PX = 0$

$\therefore$  System of equations

$$x + 2y + z = 0; 2x - 3y + 4z = 0$$

and  $x + 9y - z = 0$  has infinitely many solution.

Let  $z = k \in \mathbb{R}$  and solve above equations, we get

$$x = -\frac{11k}{7}, y = \frac{2k}{7}, z = k$$

$$\text{But given that } x^2 + y^2 + z^2 = 1$$

$$\therefore k = \pm \frac{7}{\sqrt{174}}$$

$\therefore$  Two solutions only.