Coordinate Geometry

Question1

Let (h, k) lie on the circle $C : x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to ____ .

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Answer: 9

Solution:

(h, k) lie on circle $x^2 + y^2 = 1$.

so, $h^2 + k^2 = 1 - - (1)$

$(2h + 1), 3k + 2)$ lies on ellipse This means $x = 2h + 1$ and $y = 3k + 2$ lies on ellipse

Substitute h $= \frac{x-1}{2}$ and k $= \frac{y-2}{3}$ in equation (1)

$$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$$

$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$$

This is shifted ellipse of the form $\frac{(x-x_1)^2}{-2} + \frac{(y-y_1)^2}{12} = 1$a $= 2,$ b $= 3$

eccentricity e $= \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}}$

$e^2 = \frac{5}{9} \Longrightarrow \frac{5}{e^2} = 9$ Ans

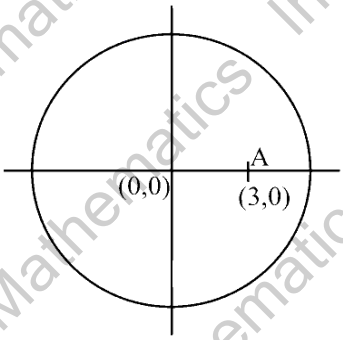
Question2

Let A be the point (3, 0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to _____.

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Answer: 18

Solution:



Let the variable point be $B(h, k)$.

We are told that the circle having AB as diameter touches the fixed circle $x^2 + y^2 = 36$ internally.

If a circle has endpoints of a diameter at $A(3, 0)$ and $B(h, k)$, then its equation is

$(x - h)(x - 3) + (y - k)(y - 0) = 0$

Expanding this, we get

$x^2 + y^2 - (h + 3)x - ky + 3h = 0$

So, the centre of this circle is

Centre $\left(\frac{h+3}{2}, \frac{k}{2}\right)$

Now, the fixed circle $x^2 + y^2 = 36$ has centre $C_1(0, 0)$ and radius $R = 6$.

The variable circle (with diameter AB) has radius equal to half of AB, so

$r = \frac{1}{2}\left(\sqrt{(h - 3)^2 + k^2}\right)$

Since the two circles touch internally, the distance between their centres equals the difference of radii:

$\therefore\ C_1C_2 = |R - r|$

Here, $C_2\ \left(\frac{h+3}{2}, \frac{k}{2}\right)$. So,

$\sqrt{\left(\frac{h+3}{2}\right)^2 + \left(\frac{k}{2}\right)^2} = \left|6 - \frac{1}{2}\left(\sqrt{(h - 3)^2 + k^2}\right)\right|$

On simplifying and writing the locus in terms of a general point (x, y) (i.e., replacing (h, k) by (x, y)), we get

$\sqrt{(x + 3)^2 + y^2} + \sqrt{(x - 3)^2 + y^2} = 12$

This is the standard form of an ellipse: sum of distances from two fixed points (foci) is constant $= 2a$.

So,

$2a = 12 \quad \therefore\ a = 6$

Comparing, the foci are $(-3, 0)$ and $(3, 0)$.

The distance between the foci is 6, so $2c = 6$. Using $c = ae$, we write

$2ae = 6$

Substituting $a = 6$:

$12e = 6$

So,

$e = \frac{1}{2}$

Finally,

$\therefore\ 72e^2 = 72\ \left(\frac{1}{4}\right) = 18$

Question3

Consider the parabola $\mathrm{P} : y^2 = 4kx$ and the ellipse $\mathrm{E} : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let the line segment joining the points of intersection of P and E , be their latus rectums. If the eccentricity of E is e, then $e^2 + 2\sqrt{2}$ is equal to ____ .

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Answer: 3

Solution:

For the parabola

$P:\ y^2 = 4kx$

its latus rectum is the line

$x = k$

and the endpoints of the latus rectum are

$(k,\ 2k)$ and $(k,\ -2k)$.

So, if the line segment joining the points of intersection of the parabola and ellipse is the latus rectum of the parabola, then the two curves intersect at

$(k, \pm 2k)$.

Now for the ellipse

$E:\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$

its latus rectum is the chord through a focus perpendicular to the major axis.

Since the common chord is vertical, the major axis of the ellipse must be along the x-axis.

Hence the foci are

$(\pm ae,\ 0)$,

where

$e = \sqrt{1 - \frac{b^2}{a^2}}$.

The latus rectum of the ellipse is the vertical line through a focus:

$x = ae$.

Its endpoints are

$\left(ae,\ \pm \frac{b^2}{a}\right)$.

But this same segment is also the latus rectum of the parabola, whose endpoints are

$(k, \pm 2k)$.

Therefore, comparing coordinates:

$k = ae$

and

$2k = \frac{b^2}{a}$.

Substitute $k = ae$ into the second equation:

$2ae = \frac{b^2}{a}$.

So,

$b^2 = 2a^2e$.

Now for the ellipse,

$b^2 = a^2(1 - e^2)$.

Hence,

$a^2(1 - e^2) = 2a^2e$.

Dividing by a^2:

$1 - e^2 = 2e$.

So,

$e^2 + 2e - 1 = 0$.

Since eccentricity is positive,

$e = -1 + \sqrt{2}$.

Now calculate:

$e^2 + 2\sqrt{2}$.

First,

$e^2 = (\sqrt{2} - 1)^2 = 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$.

Therefore,

$e^2 + 2\sqrt{2} = 3$.

So the required value is

$\boxed{3}$.

Question4

If the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbf{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

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Options:

A. $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$

B. $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$

C. $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$

D. $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Answer: D

Solution:

$\alpha x + 4y - \sqrt{7} = 0$ touches $3x^2 + 4y^2 = 1$

$\therefore c^2 = a^2m^2 + b^2$

$\frac{7}{16} = \frac{1}{3} \times \frac{\alpha^2}{16} + \frac{1}{4} \Rightarrow \alpha = 3, -3$

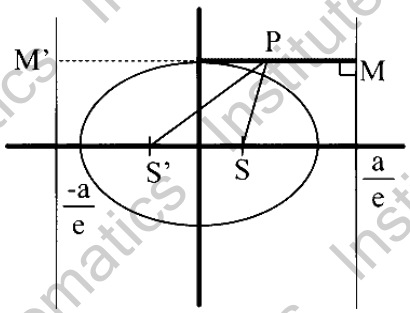
Tangent is $3x + 47 - \sqrt{7} = 0$

Let the point of contact is $P(x_1, y_1)$

$\therefore \frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{\sqrt{7}}$

$\therefore P\left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$

ecc. $= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$



$PS = e(PM) = e\left(\frac{a}{e} - \frac{1}{\sqrt{7}}\right) = \frac{1}{2}\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{7}}\right) = \frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{7}}$

$PS' = e\left(PM'\right) = \frac{1}{2}\left(\frac{a}{e} + \frac{1}{\sqrt{7}}\right) = \frac{1}{2}\left(\frac{1}{\sqrt{7}} + \frac{2}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Question5

Let S and S' be the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\mathrm{P}(\alpha, \beta)$ be a point on the ellipse in the first quadrant. If $(\mathrm{SP})^2 + \left(\mathrm{S}'\mathrm{P}\right)^2 - \mathrm{SP} \cdot \mathrm{S}'\mathrm{P} = 37$, then $\alpha^2 + \beta^2$ is equal to :

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Options:

A.

13

B.

15

C.

11

D.

17

Answer: A

Solution:

$\because$ P lies on ellipse $\Rightarrow \frac{\alpha^2}{25} + \frac{\beta^2}{9} = 1$

$\because PS + PS' = 2a \Rightarrow PS + PS' = 10$

$\because (PS)^2 + \left(PS'\right)^2 - PS.PS' = 37$

$\left(PS + PS'\right) - 3PS.PS' = 37$

$100 - 3PS.PS' = 37$

$3PS.PS' = 63 \Rightarrow PS.PS' = 21$

$\because PS \& PS'$ are $\left(5 \pm \frac{4}{5}.\alpha\right)$

$\therefore PS.PS' = 25 - \frac{16}{25}\alpha^2 = 21$

$\frac{16}{25}\alpha^2 = 4$

$\alpha = \frac{5}{2} \Rightarrow \alpha^2 = \frac{25}{4}$

$\therefore \beta^2 = \frac{27}{4}$

$\therefore \alpha^2 + \beta^2 = \frac{52}{4} = 13$

Question6

Let the line $y - x = 1$ intersect the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at the points A and B . Then the angle made by the line segment AB at the center of the ellipse is :

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Options:

A.

$\pi - \tan^{-1}\left(\frac{1}{4}\right)$

B.

$\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

C.

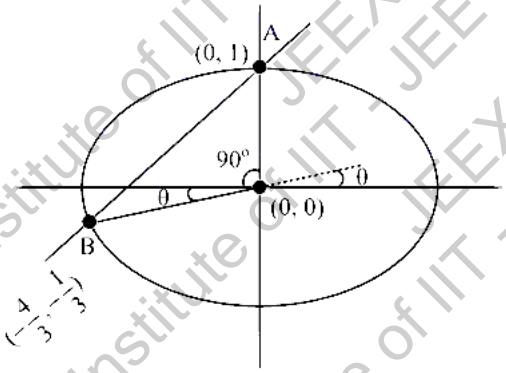
$\frac{\pi}{2} + 2\tan^{-1}\left(\frac{1}{4}\right)$

D.

$\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

Answer: B

Solution:



$y = x + 1$ intersects $\frac{x^2}{2} + y^2 = 1$

$\Rightarrow \frac{x^2}{2} + (x+1)^2 = 1 \Rightarrow x^2 + 2x^2 + 4x = 0$

$\Rightarrow x = 0, \frac{-4}{3}$

$\Rightarrow$ Points A and B are

$(0, 1), \left(\frac{-4}{3}, \frac{-1}{3}\right)$

$\theta = \tan^{-1}\left(\frac{\frac{-1}{3}}{\frac{-4}{3}}\right) = \tan^{-1}\left(\frac{1}{4}\right)$

$\Rightarrow \angle AOB = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Question7

If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and

$4x^2 + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b), then the

value of $ab + 18r^2$ is :

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Options:

A.

53

B.

52

C.

55

D.

51

Answer: C

Solution:

Let the given ellipses be :

$S_1 \equiv x^2 + 2y^2 - 6x - 12y + 23 = 0$
$S_2 \equiv 4x^2 + 2y^2 - 20x - 12y + 35 = 0$

The equation of a curve passing through the intersection of these two ellipses is given by the family of curves $S_1 + \lambda S_2 = 0$:

$\left(x^2 + 2y^2 - 6x - 12y + 23\right) + \lambda\left(4x^2 + 2y^2 - 20x - 12y + 35\right) = 0$

$\Rightarrow (1 + 4\lambda)x^2 + (2 + 2\lambda)y^2 - (6 + 20\lambda)x - (12 + 12\lambda)y + (23 + 35\lambda) = 0$

Condition for circle: coefficient of x^2 = coefficient of y^2

$1 + 4\lambda = 2 + 2\lambda$

$\Rightarrow 2\lambda = 1$

$\Rightarrow \lambda = \frac{1}{2}$

Substitute $\lambda = \frac{1}{2}$:

$3x^2 + 3y^2 - 16x - 18y + \frac{81}{2} = 0$

$\Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0$

$2g = -\frac{16}{3} \Longrightarrow g = -\frac{8}{3}$
$2f = -6 \Longrightarrow f = -3$
Centre $(a, b) = (-g, -f) = \left(\frac{8}{3}, 3\right)$
$a = \frac{8}{3}, b = 3$

$r^2 = g^2 + f^2 - c$
$r^2 = \left(-\frac{8}{3}\right)^2 + (-3)^2 - \frac{27}{2}$
$r^2 = \frac{64}{9} + 9 - \frac{27}{2}$
$r^2 = \frac{128 + 162 - 243}{18}$
$r^2 = \frac{47}{18}$

$ab + 18r^2 = \left(\frac{8}{3}\right)(3) + 18\left(\frac{47}{18}\right)$
$ab + 18r^2 = 8 + 47 = 55$

Question8

Let each of the two ellipses $\mathrm{E}_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$ and $\mathrm{E}_2: \frac{x^2}{\mathrm{A}^2}+\frac{y^2}{\mathrm{B}^2}=1,(\mathrm{A}<\mathrm{B})$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be l_1 and l_2, respectively, such that $2l_1^2=9l_2$. If the distance between the foci of E_1 is 8 , then the distance between the foci of E_2 is

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Options:

A.

$\frac{96}{5}$

B.

$\frac{8}{5}$

C.

$\frac{16}{5}$

D.

$\frac{32}{5}$

Answer: D

Solution:

Given, for ellipse E_1 and E_2 eccentricity is $\frac{4}{5}$.

$E_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad a>b$

distance between foci $=8$

$2ae=8 \Longrightarrow ae=4$

$\Rightarrow a \times \frac{4}{5}=4 \Longrightarrow a=5$

$\Rightarrow e=\sqrt{1-\frac{b^2}{a^2}}=\frac{4}{5}$

$\Rightarrow 1-\frac{b^2}{5^2}=\frac{16}{25}$

$\Rightarrow \frac{b^2}{5^2}=1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow b=3$

Length of latus rectum for $E_1, l_1=\frac{2b^2}{a}$

$l_1=2 \times \frac{3^2}{5}=\frac{18}{5}$

For $E_2: \frac{x^2}{A^2}+\frac{y^2}{B^2}=1 \quad (A<B)$

eccentricity $e=\dfrac{4}{5}$

as $A<B, e=\sqrt{1-\dfrac{A^2}{B^2}}=\dfrac{4}{5}$

$1-\frac{A^2}{B^2}=\frac{16}{25}$

$\Rightarrow \frac{A^2}{B^2}=1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow A^2=\frac{9}{25}B^2$

If l_2 is latus rectum of $E_2, l_2=\frac{2A^2}{B}$

$l_2=\frac{2}{B}\left(\frac{9}{25}B^2\right)=\frac{18}{25}B$

It is given that $2l_1^2=9l_2$

$2\left(\frac{18}{5}\right)^2 = 9 \times \frac{18}{25} B$

$\Rightarrow 2 \times \frac{18}{9} = B$

$\Rightarrow B = 4$

distance between foci of E_2 is $2Be$

$= 2 \times 4 \times \frac{4}{5} = \frac{32}{5}$

Question9

Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be 30 . If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $\left(a^2 + b^2\right)$ is equal to

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Options:

A.

276

B.

516

C.

256

D.

496

Answer: D

Solution:

length of latus rectum of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ is given as:

$l = \frac{2b^2}{a} = 30 \Rightarrow b^2 = 15a$

It is given that eccentricity is the maximum value of the function $f(t) = -t^2 + 2t - \frac{3}{4}$

max value of quadratic function $ax^2 + bx + c$ is given as $\frac{-D}{4a} = \frac{4ac - b^2}{4a}$

$f(t)_{\max} = \frac{4(-1)\left(-\frac{3}{4}\right) - (2)^2}{4(-1)} = \frac{3-4}{-4} = \frac{1}{4}$

so eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{4}$

squaring

$1 - \frac{b^2}{a^2} = \frac{1}{16}$

put value of $b^2 = 15a$

$\frac{a^2 - 15a}{a^2} = \frac{1}{16}$

$16a^2 - 240a = a^2$

$15a^2 - 240a = 0$

$a(15a - 240) = 0$

since $a \neq 0, a = \frac{240}{15} = 16$

Now, value of $a^2 + b^2 = a^2 + 15a \left\{b^2 = 15a\right\}$

$= (16)^2 + 15 \times 16$

$= 256 + 240 = 496$

Question10

An ellipse has its center at $(1,-2)$, one focus at $(3,-2)$ and one vertex at $(5,-2)$. Then the length of its latus rectum is :

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Options:

A.

6

B.

$6\sqrt{3}$

C.

$\frac{16}{\sqrt{3}}$

D.

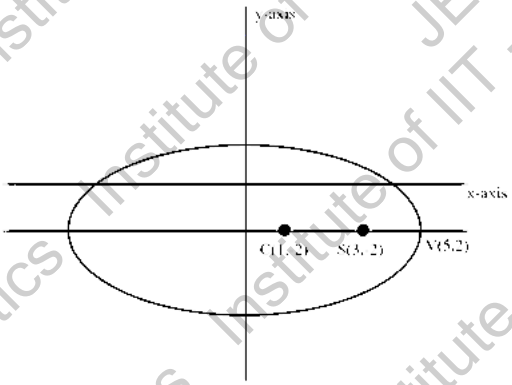
$4\sqrt{3}$

Answer: A

Solution:

Given

- Center of ellipse : $(1,-2)$
- Focus : $(3,-2)$
- Vertex : $(5,-2)$



The center, focus, and vertex all have the same y-coordinate (-2), which means the major axis of the ellipse is horizontal (parallel to the x-axis).

Find a (semi-major axis)

Distance from center to vertex :

$a = |5-1| = 4$

Find c (distance from center to focus)

$c = |3-1| = 2$

Find b using ellipse relation

$c^2 = a^2 - b^2$

$\Rightarrow 2^2 = 4^2 - b^2$

$\Rightarrow 4 = 16 - b^2$

$\Rightarrow b^2 = 12$

$\Rightarrow b = 2\sqrt{3}$

Length of latus rectum

For an ellipse :

Length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2(12)}{4} = 6$

Question11

Let an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a < b$, pass through the point (4, 3) and have eccentricity $\frac{\sqrt{5}}{3}$.

Then the length of its latus rectum is :

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Options:

A.

$\frac{4\sqrt{5}}{3}$

B.

$2\sqrt{5}$

C.

$\frac{7\sqrt{5}}{3}$

D.

$\frac{8\sqrt{5}}{3}$

Answer: D

Solution:

Since $a < b$, the major axis is along the y-axis. So for the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

we use the eccentricity formula

$e = \sqrt{1 - \frac{a^2}{b^2}}$.

Given

$e = \frac{\sqrt{5}}{3}$,

so

$e^2 = \frac{5}{9}$.

Hence,

$1 - \frac{a^2}{b^2} = \frac{5}{9}$

which gives

$\frac{a^2}{b^2} = 1 - \frac{5}{9} = \frac{4}{9}$.

Therefore,

$a^2 = \frac{4}{9}b^2$.

Now the ellipse passes through $(4, 3)$, so it satisfies the equation:

$\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1$.

That is,

$\frac{16}{a^2} + \frac{9}{b^2} = 1$.

Substitute $a^2 = \frac{4}{9}b^2$:

$\frac{16}{\frac{4}{9}b^2} + \frac{9}{b^2} = 1$.

Simplifying,

$16 \cdot \frac{9}{4b^2} + \frac{9}{b^2} = 1$

$\frac{36}{b^2} + \frac{9}{b^2} = 1$

$\frac{45}{b^2} = 1$

so

$b^2 = 45$.

Then

$a^2 = \frac{4}{9} \cdot 45 = 20$.

For an ellipse with major axis along the y-axis, the length of latus rectum is

$\frac{2a^2}{b}$.

So,

Length of latus rectum $= \frac{2 \cdot 20}{\sqrt{45}} = \frac{40}{3\sqrt{5}}$.

Rationalising,

$\frac{40}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{8\sqrt{5}}{3}$.

Therefore, the correct answer is

$$\boxed{\frac{8\sqrt{5}}{3}}$$

So, Option D is correct.

Question12

Let $\mathrm{P}(3\cos\alpha, 2\sin\alpha), \alpha \neq 0$, be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, Q be a point on the circle $x^2 + y^2 - 14x - 14y + 82 = 0$ and R be a point on the line $x + y = 5$ such that the centroid of the triangle PQR is $\left(2 + \cos\alpha, 3 + \frac{2}{3}\sin\alpha\right)$. Then the sum of the ordinates of all possible points R is:

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Options:

A.

6

B.

2

C.

4

D.

8

Answer: D

Solution:

Centroid :

Let Q be a point on the given circle. First write the circle in standard form:

$x^2 + y^2 - 14x - 14y + 82 = 0 \Rightarrow (x-7)^2 + (y-7)^2 = 16$

So we can take

$Q(7 + 4\cos\theta,\ 7 + 4\sin\theta)$

Point R lies on the line $x + y = 5$. Let its ordinate be y, then its abscissa is $5 - y$, so

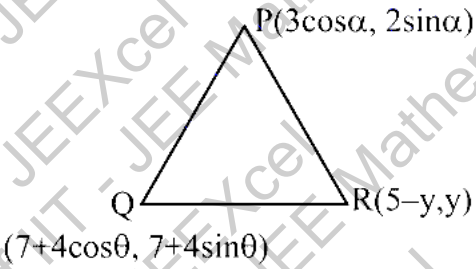
$R(5 - y,\ y)$

Now use the centroid formula. For triangle PQR, centroid is

$\left(\frac{x_P+x_Q+x_R}{3}, \frac{y_P+y_Q+y_R}{3}\right)$

Given $P(3\cos\alpha,\ 2\sin\alpha)$, so the centroid becomes

$$\left(\frac{3\cos\alpha + 7 + 4\cos\theta + 5 - y}{3}, \frac{2\sin\alpha + 7 + 4\sin\theta + y}{3}\right)$$
$$= \left(\cos\alpha + 2, \frac{2}{3}\sin\alpha + 3\right)$$



On comparison

Compare the x-coordinates:

$\frac{3\cos\alpha+7+4\cos\theta+5-y}{3} = \cos\alpha + 2 \Rightarrow 4\cos\theta = y - 6$

So,

$\cos\theta = \frac{y-6}{4}$

Compare the y-coordinates:

$\frac{2\sin\alpha+7+4\sin\theta+y}{3} = \frac{2}{3}\sin\alpha + 3 \Rightarrow 4\sin\theta = 2 - y$

So,

$\sin\theta = \frac{2-y}{4}$

Now use the identity $\sin^2\theta + \cos^2\theta = 1$:

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow (y-6)^2 + (y-2)^2 = 16$

Simplify:

$y^2 - 8y + 12 = 0$

This quadratic gives two possible values of y (ordinates of R). The sum of roots is 8, so

$\Rightarrow$ Sum of ordinates $= 8$

Question13

Let a focus of the ellipse $\mathrm{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be $\mathrm{S}(4, 0)$ and its eccentricity be $\frac{4}{5}$. If the point $\mathrm{P}(3, \alpha)$ lies on E and O is the origin, then the area of $\triangle\mathrm{POS}$ is equal to:

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Options:

A.

12/5

B.

14/5

C.

24/5

D.

48/5

Answer: C

Solution:

For the ellipse

$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

a focus is $S(4, 0)$ and eccentricity is

$e = \frac{4}{5}$

We need the area of $\triangle POS$, where $P(3, \alpha)$ lies on the ellipse and $O = (0, 0)$.

Since the focus is on the x-axis, the ellipse has major axis along the x-axis.

For such an ellipse, focus is at $(\pm ae, 0)$.

Given one focus is $(4, 0)$, so

$ae = 4$

and $e = \frac{4}{5}$. Therefore,

$a \cdot \frac{4}{5} = 4$

$a = 5$

Now,

$c = ae = 4$

and for ellipse,

$b^2 = a^2 - c^2$

So,

$b^2 = 25 - 16 = 9$

Hence the ellipse is

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

Since $P(3, \alpha)$ lies on the ellipse, substitute $x = 3$:

$\frac{3^2}{25} + \frac{\alpha^2}{9} = 1$

$\frac{9}{25} + \frac{\alpha^2}{9} = 1$

$\frac{\alpha^2}{9} = 1 - \frac{9}{25} = \frac{16}{25}$

$\alpha^2 = \frac{144}{25}$

$\alpha = \pm \frac{12}{5}$

Now find the area of triangle with vertices

$O(0, 0), \quad P(3, \alpha), \quad S(4, 0)$

Using base OS:

$OS = 4$

Since OS lies on the x-axis, the height from P to OS is $|\alpha|$.

So area is

$\frac{1}{2} \times 4 \times |\alpha| = 2|\alpha|$

$= 2 \times \frac{12}{5} = \frac{24}{5}$

Therefore, the area of $\triangle POS$ is

$\boxed{\frac{24}{5}}$

So, the correct option is:

Option C

Question14

Let $x = 9$ be a directrix of an ellipse E , whose centre is at the origin and eccentricity is $\frac{1}{3}$. Let $\mathrm{P}(\alpha, 0)$, $\alpha > 0$, be a focus of E and AB be a chord passing through P . Then the locus of the mid point of AB is :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

$9y^2 = 8x(1 - x)$

B.

$3y^2 = 4x(1 - x)$

C.

$9y^2 = 8x(x - 1)$

D.

$3y^2 = 4x(x - 1)$

Answer: A

Solution:

Since the centre of the ellipse is at the origin and one directrix is $x = 9$, the major axis is along the x-axis.

Let the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

with eccentricity

$e = \frac{1}{3}$

For an ellipse with major axis along the x-axis, the directrices are

$x = \pm \frac{a}{e}$

Given one directrix is $x = 9$, so

$\frac{a}{e} = 9$

Since $e = \frac{1}{3}$,

$a = 9 \cdot \frac{1}{3} = 3$

Also,

$c = ae = 3 \cdot \frac{1}{3} = 1$

So the focus $P(\alpha, 0)$ is

$P = (1, 0)$

Hence $\alpha = 1$.

Now,

$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 9 \cdot \frac{8}{9} = 8$

Therefore the ellipse is

$\frac{x^2}{9} + \frac{y^2}{8} = 1$

Let the midpoint of chord AB passing through the focus $(1, 0)$ be (h, k).

For the ellipse

$\frac{x^2}{9} + \frac{y^2}{8} = 1$

the equation of the chord whose midpoint is (h, k) is given by the NCERT midpoint formula:

$\frac{xx_1}{9} + \frac{yy_1}{8} = \frac{x_1^2}{9} + \frac{y_1^2}{8}$

So here the chord with midpoint (h, k) is

$\frac{xh}{9} + \frac{yk}{8} = \frac{h^2}{9} + \frac{k^2}{8}$

Since this chord passes through the focus $(1, 0)$, substitute $(x, y) = (1, 0)$:

$\frac{h}{9} = \frac{h^2}{9} + \frac{k^2}{8}$

Multiply by 72:

$8h = 8h^2 + 9k^2$

So,

$9k^2 = 8h - 8h^2 = 8h(1 - h)$

Replacing (h, k) by (x, y) for the locus of the midpoint, we get

$9y^2 = 8x(1 - x)$

Hence the required locus is

$\boxed{9y^2 = 8x(1 - x)}$

So the correct option is **A**.

Question15

Let $\frac{x^2}{f(a^2+7a+3)} + \frac{y^2}{f(3a+15)} = 1$ represent an ellipse with major axis along y-axis, where f is a strictly decreasing positive function on $\mathbf{R}$. If the set of all possible values of a is $\mathbf{R} - [\alpha, \beta]$, then $\alpha^2 + \beta^2$ is equal to :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

28

B.

40

C.

61

D.

24

Answer: B

Solution:

Since the ellipse has its major axis along the y-axis, the denominator under y^2 must be bigger than the denominator under x^2.

So we need

$f(3a+15) > f\left(a^2+7a+3\right)$.

Given that f is a strictly decreasing positive function, a bigger function value happens when the input is smaller. So,

$3a+15 < a^2+7a+3$.

Bring all terms to one side:

$a^2+7a+3-3a-15 > 0$

$a^2+4a-12 > 0$.

Complete the square:

$a^2+4a-12 = (a+2)^2-16$.

So the inequality becomes

$(a+2)^2-16 > 0 \;\Rightarrow\; (a+2)^2 > 16$.

Taking square roots,

$|a+2| > 4 \;\Rightarrow\; a+2 > 4$ or $a+2 < -4$.

So,

$a > 2$ or $a < -6$.

That means the allowed values are

$a \in \mathbf{R} - [-6, 2]$.

Hence, $\alpha = -6$ and $\beta = 2$. Therefore,

$\alpha^2+\beta^2 = (-6)^2+(2)^2 = 36+4 = 40$.

Question16

For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2\sec^2\theta = 8$ be e_1 and l_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2\sec^2\theta + y^2 = 6$ be e_2 and l_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta+1\right)$, then $\left(\frac{l_1 l_2}{e_1 e_2}\right)\tan^2\theta$ is equal to

JEE Main 2026 (Online) 28th January Morning Shift

Answer: 8

Solution:

Given hyperbola

$x^2 - y^2\sec^2\theta = 8$

Convert it into standard form.

$\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1$

Eccentricity: $e_1^2 = 1 + \frac{8\cos^2\theta}{8} = 1+\cos^2\theta$ latus rectum

$l_1 = \frac{2\times 8\cos^2\theta}{\sqrt{8}} = 4\sqrt{2}\cos^2\theta$

Given ellipse

$x^2\sec^2\theta + y^2 = 6$

Convert into standard form.

$\frac{x^2}{6\cos^2\theta}+\frac{y^2}{6}=1$

Since $\theta\in(0,\pi/2)\Rightarrow\cos^2\theta<1$ so major axis is y -axis. eccentricity:

$e_2^2=1-\frac{6\cos^2\theta}{6}=\sin^2\theta$

Latus rectum

$l_2=\frac{2\times6\cos^2\theta}{\sqrt{6}}=2\sqrt{6}\cos^2\theta$

It is given that

$e_1^2=e_2^2\left(\sec^2\theta+1\right)$

Substituting values of e_1^2 and e_2^2.

$1+\cos^2\theta=\sin^2\theta\left(\frac{1}{\cos^2\theta}+1\right)$

$\Rightarrow1+\cos^2\theta=\sin^2\theta\left(\frac{1+\cos^2\theta}{\cos^2\theta}\right)$

$\Rightarrow\tan^2\theta=1\Rightarrow\theta=\frac{\pi}{4}$ as $\theta\in(0,\pi/2)$

$\Rightarrow\frac{l_1\cdot l_2}{e_1\cdot e_2}=\frac{\left(4\sqrt{2}\cos^2\frac{\pi}{4}\right)\left(2\sqrt{6}\cos^2\frac{\pi}{4}\right)}{\sqrt{\left(1+\cos^2\frac{\pi}{4}\right)\left(\sin\frac{\pi}{4}\right)}}$

$\because\frac{l_1\cdot l_2}{e_1\cdot e_2}=\frac{4\sqrt{2}\times\frac{1}{2}\times2\sqrt{6}\times\frac{1}{2}}{\sqrt{\left(1+\frac{1}{2}\right)}\left(\frac{1}{\sqrt{2}}\right)}=\frac{4\sqrt{3}}{\frac{\sqrt{3}}{2}}=8$

Question17

Let the foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If the eccentricity of the hyperbola is 5 , then the length of its latus rectum is :

JEE Main 2026 (Online) 21st January Morning Shift

Options:

A.

$\frac{96}{\sqrt{5}}$

B.

$24\sqrt{5}$

C.

12

D.

16

Answer: A

Solution:

$E:\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$

$H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

For con-focal E and H

$\because\quad A^2-B^2=a^2+b^2$

(Given that $A^2=36$ and $B^2=16$)

$36 - 16 = a^2 + b^2$

$a^2 + b^2 = 20 \quad \ldots(i)$

$e_H = 5$

$\Rightarrow \quad 25 = 1 + \frac{b^2}{a^2}$

$\Rightarrow \quad 25a^2 = 20 \Rightarrow a^2 = \frac{4}{5} \Rightarrow a = \frac{2}{\sqrt{5}}$

Also $a^2 + b^2 = 20$

$b^2 = 20 - \frac{4}{5} \Rightarrow \frac{96}{5}$

$\mathrm{L(LR)} == 2 \times \frac{b^2}{a} = \frac{2 \times \frac{96}{5}}{\frac{2}{\sqrt{5}}} = \frac{96}{\sqrt{5}}$

Question18

If the line $\alpha x + 2y = 1$, where $\alpha \in \mathbb{R}$, does not meet the hyperbola $x^2 - 9y^2 = 9$, then a possible value of α is :

JEE Main 2026 (Online) 22nd January Morning Shift

Options:

A.

0.6

B.

0.7

C.

0.8

D.

0.5

Answer: C

Solution:

$\alpha x + 2y = 1, \alpha \in R$

$x^2 - 9y^2 = 9$

$y = \frac{1 - \alpha x}{2}$

$4x^2 - 9\left(1 + \alpha^2 x^2 - 2\alpha x\right) = 36$

$\Delta < 0$

$1620\alpha^2 - 720 > 0$

$\alpha^2 > 0.55$

$\alpha = 0.8$

Question19

Let $\mathrm{P}(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S′. If the length of its latus rectum is 8 , then the square of the area of $\Delta \mathrm{PSS}'$ is equal to :

JEE Main 2026 (Online) 22nd January Evening Shift

Options:

A.

4200

B.

1462

C.

900

D.

2700

Answer: D

Solution:

$P(10, 2\sqrt{15})$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \frac{100}{a^2} - \frac{60}{b^2} = 1 \ldots\ldots\ldots\ldots\ldots (1)$

$\because$ Length of latus rectum $= 8$

$\frac{2 \cdot b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4 \ldots\ldots\ldots (2)$

From (1) \& (2)

$\frac{100}{a^2} - \frac{60}{4a} = 1$

$400 - 60a = 4a^2$

$4a^2 + 60a - 400 = 0$

$a^2 + 15a - 100 = 0$

$a = 5\& -20(\text{ rejected })$

$\Rightarrow b = \sqrt{20}$

$\therefore$ Hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

$\therefore$ Focal length $SS^1 = 2ae = 2 \cdot 5 \cdot \left(\sqrt{1 + \frac{4}{5}}\right) = 6\sqrt{5}$

$\therefore$ Area of $\triangle PSS^1 = \frac{1}{2} \cdot 6\sqrt{5} \cdot 2\sqrt{15} = 30\sqrt{3} = A$

$\therefore A^2 = 2700$

Question20

Let the domain of the function $f(x) = \log_3 \log_5 \log_7 \left(9x - x^2 - 13\right)$ be the interval (m, n). Let the hyperbola $\frac{x^2}{\mathrm{a}^2} - \frac{y^2}{\mathrm{b}^2} = 1$ have eccentricity $\frac{\mathrm{n}}{3}$ and the length of the latus rectum $\frac{8\,\mathrm{m}}{3}$. Then $\mathrm{b}^2 - \mathrm{a}^2$ is equal to :

JEE Main 2026 (Online) 23rd January Morning Shift

Options:

A.

7

B.

9

C.

11

D.

5

Answer: A

Solution:

$\log_5\left(\log_7\left(9x-x^2-13\right)\right)>0$

$\Rightarrow \quad 9x-x^2-13>7$

$\Rightarrow \quad x \in (4,5)=(m,n)$

$\Rightarrow \quad \text{eccentricity } =\frac{5}{3}$

$L(LR)=\frac{32}{3}$

$\frac{25}{9}=1+\frac{b^2}{a^2} \qquad \ldots(1)$

$\frac{2b^2}{a}=\frac{32}{3} \Rightarrow b^2=\frac{16a}{3}$

Stultify in (1)

$\frac{16}{9}=\frac{16a}{3a^2} \Rightarrow a=3$

$b^2=16$

$\therefore b^2-a^2=16-9$

$=7$

Question21

Let PQ be a chord of the hyperbola $\frac{x^2}{4}-\frac{y^2}{b^2}=1$, perpendicular to the x -axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is

JEE Main 2026 (Online) 23rd January Evening Shift

Options:

A.

$2\sqrt{3}$

B.

$\frac{11}{5}$

C.

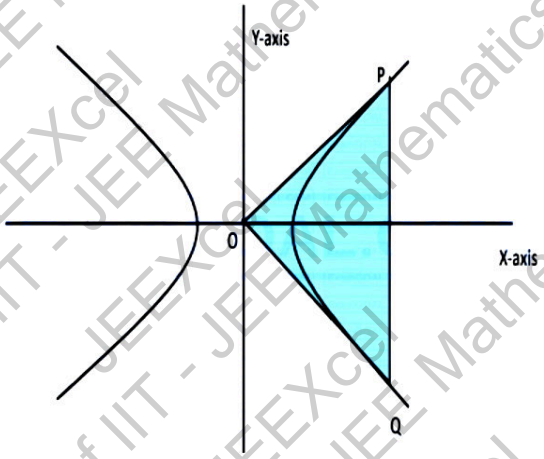
$\frac{8\sqrt{3}}{5}$

D.

$\frac{9}{5}$

Answer: C

Solution:



PQ is a chord perpendicular to x -axis of hyperbola $\frac{x^2}{4}-\frac{y^2}{b^2}=1$

Eccentricity, $e=\sqrt{3}$

$e=\sqrt{1+\frac{b^2}{4}}=\sqrt{3}$

$1+\frac{b^2}{4}=3$

$\frac{b^2}{4}=2 \Longrightarrow b^2=8$

equation become $\frac{x^2}{4}-\frac{y^2}{8}=1$ - (1)

let coordinate of point $P\left(x_1, y_1\right)$

PQ is perpendicular to x -axis so $Q\left(x_1,-y_1\right)$

It is given that OPQ is equilateral triangle where O is origin

so, $\angle POQ=60^{\circ} \Longrightarrow \angle POX=30^{\circ}$

$\tan 30^{\circ}=$ slope of $\mathrm{OP}=\frac{y_1}{x_1}$

$\frac{y_1}{x_1}=\frac{1}{\sqrt{3}} \Longrightarrow y_1=\frac{x_1}{\sqrt{3}}$

Point $P\left(x_1, y_1\right)$ satisfy hyperbola.

$\frac{x_1^2}{4}-\frac{y_1^2}{8}=1$

$\frac{x_1^2}{4}-\frac{x_1^2}{24}=1\left\{y_1=\frac{x_1}{\sqrt{3}}\right\}$

$6x_1^2-x_1^2=24$

$x_1^2=\frac{24}{5}$

Now, area of $\triangle OPQ=\frac{1}{2} \times$ base $PQ \times$ height $\left(x_1\right)$

$=\frac{1}{2} \times 2y_1 \times x_1=\frac{x_1^2}{\sqrt{3}}$

$=\frac{24}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{8\sqrt{3}}{5}$

Question22

Let the ellipse $E: \frac{x^2}{144}+\frac{y^2}{169}=1$ and the hyperbola $H: \frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have the same foci. If e and L

respectively denote the eccentricity and the length of the latus rectum of H, then the value of $24(e + L)$ is :

JEE Main 2026 (Online) 28th January Evening Shift

Options:

A.

296

B.

126

C.

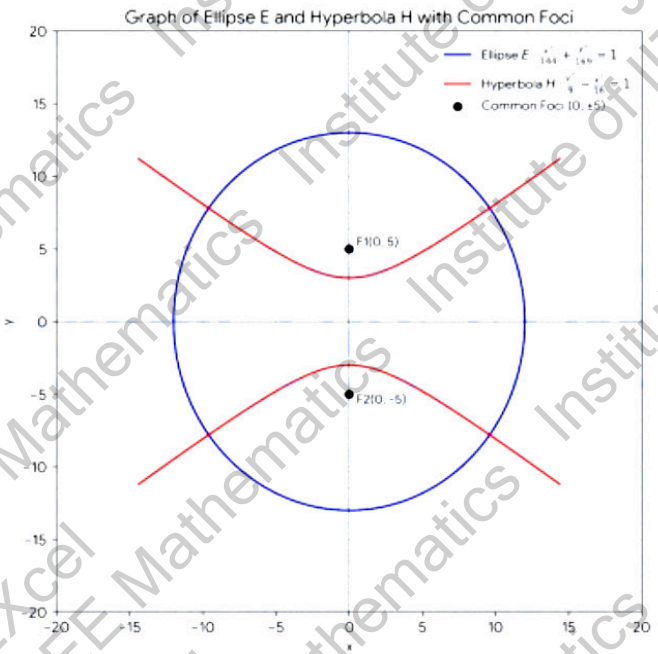
67

D.

148

Answer: A

Solution:



The equation of the ellipse is :

$E : \frac{x^2}{144} + \frac{y^2}{169} = 1$

Comparing this with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have :

- $a^2 = 144 \Rightarrow a = 12$
- $b^2 = 169 \Rightarrow b = 13$

Since $b > a$, the ellipse is vertically oriented. The distance from the center to the foci (c) is given by :

$c^2 = b^2 - a^2$

$c^2 = 169 - 144 = 25 \Longrightarrow c = 5$

The foci of the ellipse are at $(0, \pm 5)$.

The equation of the hyperbola is :

$H : \frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$ which is equivalent to $\frac{y^2}{\lambda^2} - \frac{x^2}{16} = 1$

This is a vertical hyperbola.

Let $B^2 = \lambda^2$ and $A^2 = 16$.

Since it shares the same foci as the ellipse, its focal distance c is also 5 . For a hyperbola:

$$c^2 = A^2 + B^2$$

$25 = 16 + \lambda^2 \Longrightarrow \lambda^2 = 9 \Longrightarrow \lambda = 3$

Calculate Eccentricity (e) and Latus Rectum (L) of H

- Eccentricity (e):

$e = \frac{c}{B} = \frac{5}{3}$

Length of Latus Rectum (L): For a vertical hyperbola $\frac{y^2}{B^2} - \frac{x^2}{A^2} = 1$, the length of the latus rectum is:

$L = \frac{2A^2}{B} = \frac{2 \times 16}{3} = \frac{32}{3}$

We need to find $24(e + L)$:

$e + L = \frac{5}{3} + \frac{32}{3} = \frac{37}{3}$

$\therefore 24(e + L) = 24 \times \frac{37}{3} = 8 \times 37 = 296$

Question23

Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the midpoint of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$. Then the area of the triangle OPQ equals :

JEE Main 2026 (Online) 2nd April Evening Shift

Options:

A.

$\frac{3}{2}$

B.

$\frac{5}{2}$

C.

$\frac{7}{2}$

D.

$\frac{9}{2}$

Answer: C

Solution:

Let

$P(x_1, y_1), \quad Q(x_2, y_2)$

be two points on the rectangular hyperbola

$xy = 12$

So, both points satisfy

$x_1y_1 = 12 \quad \text{and} \quad x_2y_2 = 12$

Also, the midpoint of PQ is given as

$\left(\frac{1}{2}, -\frac{1}{2}\right)$

Using the midpoint formula,

$\frac{x_1+x_2}{2} = \frac{1}{2}, \qquad \frac{y_1+y_2}{2} = -\frac{1}{2}$

Hence,

$x_1 + x_2 = 1, \qquad y_1 + y_2 = -1$

Now we need the area of triangle OPQ, where $O = (0, 0)$.

The area of triangle formed by points $O(0,0)$, $P(x_1, y_1)$, $Q(x_2, y_2)$ is

Area $= \frac{1}{2}|x_1y_2 - x_2y_1|$

So our aim is to find $|x_1y_2 - x_2y_1|$.

Since x_1 and x_2 are the x-coordinates of points on the hyperbola, and their sum is 1, let us use

$y_1 = \frac{12}{x_1}, \qquad y_2 = \frac{12}{x_2}$

Now using

$y_1 + y_2 = -1$

we get

$\frac{12}{x_1} + \frac{12}{x_2} = -1$

$12\left(\frac{x_1+x_2}{x_1x_2}\right) = -1$

Since $x_1 + x_2 = 1$,

$12\left(\frac{1}{x_1x_2}\right) = -1$

So,

$x_1x_2 = -12$

Thus x_1 and x_2 are roots of the quadratic equation

$t^2 - (x_1 + x_2)t + x_1x_2 = 0$

$t^2 - t - 12 = 0$

$(t - 4)(t + 3) = 0$

Hence,

$x_1 = 4, \quad x_2 = -3$

or vice versa.

Then corresponding y-coordinates are

$y_1 = \frac{12}{4} = 3, \qquad y_2 = \frac{12}{-3} = -4$

So the points are

$P(4, 3), \qquad Q(-3, -4)$

Now area of triangle OPQ is

Area $= \frac{1}{2}|4(-4) - (-3)(3)|$

$= \frac{1}{2}|-16 + 9|$

$= \frac{1}{2} \cdot 7$

$= \frac{7}{2}$

Therefore, the required area is

$\boxed{\frac{7}{2}}$

So, the correct option is:

Option C

Question24

Let $\mathrm{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$. If the line $x=\alpha$ intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$, where O is the origin, then α^2 equals

JEE Main 2026 (Online) 4th April Evening Shift

Options:

A.

12

B.

16

C.

24

D.

25

Answer: B

Solution:

The given equation of the hyperbola is:

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Let the uniform eccentricity of the hyperbola be e.

According to the problem, the distance between the foci is 6. For a standard hyperbola, the coordinates of the foci are $(\pm ae, 0)$, so the distance between them is given by $2ae$.

$2ae = 6$

$ae = 3 \quad \ldots(i)$

The distance between the directrices is given as $\frac{8}{3}$. The equations of the directrices are $x = \pm\frac{a}{e}$, so the distance between them is $2\frac{a}{e}$.

$2\frac{a}{e} = \frac{8}{3}$

$\frac{a}{e} = \frac{4}{3} \quad \ldots(ii)$

To find the value of a^2, we can multiply equations (i) and (ii):

$(ae)\cdot\left(\frac{a}{e}\right) = 3\times\frac{4}{3}$

$a^2 = 4$

We know the fundamental relationship between a, b, and e for a hyperbola is:

$b^2 = a^2(e^2-1)$

$b^2 = a^2e^2 - a^2$

Using the value $ae = 3$, we have $a^2e^2 = 9$. Substituting a^2e^2 and a^2 into the equation gives:

$b^2 = 9 - 4 = 5$

Now, substituting a^2 and b^2 back into the standard equation, the hyperbola is:

$\frac{x^2}{4}-\frac{y^2}{5}=1$

The line $x=\alpha$ intersects the hyperbola at points A and B. Substitute $x=\alpha$ into the equation of the hyperbola to find the y-coordinates of points A and B:

$\frac{\alpha^2}{4}-\frac{y^2}{5}=1$

$\frac{y^2}{5} = \frac{\alpha^2}{4} - 1$

$\frac{y^2}{5} = \frac{\alpha^2-4}{4}$

$y^2 = \frac{5(\alpha^2-4)}{4}$

$y = \pm\frac{\sqrt{5(\alpha^2-4)}}{2}$

So, the coordinates of points A and B are $\left(\alpha, \frac{\sqrt{5(\alpha^2-4)}}{2}\right)$ and $\left(\alpha, -\frac{\sqrt{5(\alpha^2-4)}}{2}\right)$.

These points lie vertically above and below each other, forming the base of $\triangle AOB$ along the line $x = \alpha$.

Length of the base AB = Distance between A and B = $2 \times \frac{\sqrt{5(\alpha^2-4)}}{2} = \sqrt{5(\alpha^2-4)}$.

The perpendicular height of $\triangle AOB$ from the origin $O(0,0)$ to the line $x = \alpha$ is simply $|\alpha|$.

We are given that the area of $\triangle AOB$ is $4\sqrt{15}$. Using the area of a triangle formula:

Area $= \frac{1}{2} \times$ base $\times$ height

$4\sqrt{15} = \frac{1}{2} \cdot \sqrt{5(\alpha^2-4)} \cdot |\alpha|$

Squaring both sides to remove the square root and the absolute value:

$(4\sqrt{15})^2 = \frac{1}{4} \cdot (\alpha^2) \cdot 5(\alpha^2-4)$

$240 = \frac{5}{4}\alpha^2(\alpha^2-4)$

$240 \times \frac{4}{5} = \alpha^2(\alpha^2-4)$

$192 = \alpha^4 - 4\alpha^2$

$\alpha^4 - 4\alpha^2 - 192 = 0$

Let $\alpha^2 = t$. This transforms our equation into a quadratic equation:

$t^2 - 4t - 192 = 0$

Splitting the middle term:

$t^2 - 16t + 12t - 192 = 0$

$t(t-16) + 12(t-16) = 0$

$(t-16)(t+12) = 0$

So, $t = 16$ or $t = -12$.

Since $t = \alpha^2$ and the square of a real number cannot be negative, we disregard -12.

Thus, we have:

$\alpha^2 = 16$

Option B is the correct answer.

Question25

Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are $(3, 5)$ and $(3, -4)$, then the length of its latus rectum is :

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Options:

A.

$11/3$

B.

$17/3$

C.

$15/2$

D.

$17/2$

Answer: C

Solution:

For a hyperbola, the eccentricity $e > 1$.

We are given:

$6e^2 - 11e + 3 = 0$

Solve this quadratic:

$6e^2 - 9e - 2e + 3 = 0$

$3e(2e - 3) - 1(2e - 3) = 0$

$(2e - 3)(3e - 1) = 0$

So,

$e = \frac{3}{2}$ or $e = \frac{1}{3}$

Since for a hyperbola $e > 1$, we take

$e = \frac{3}{2}$

Now the foci are $(3, 5)$ and $(3, -4)$.

These lie on the vertical line $x = 3$, so the transverse axis is vertical.

The distance between the foci is

$|5 - (-4)| = 9$

Hence,

$2c = 9 \Rightarrow c = \frac{9}{2}$

For a hyperbola,

$e = \frac{c}{a}$

So,

$a = \frac{c}{e} = \frac{\frac{9}{2}}{\frac{3}{2}} = 3$

Now use the relation

$c^2 = a^2 + b^2$

Thus,

$b^2 = c^2 - a^2 = \left(\frac{9}{2}\right)^2 - 3^2 = \frac{81}{4} - 9 = \frac{81-36}{4} = \frac{45}{4}$

Length of latus rectum of a hyperbola is

$\frac{2b^2}{a}$

Therefore,

Length of latus rectum $= \frac{2 \cdot \frac{45}{4}}{3} = \frac{45}{6} = \frac{15}{2}$

Hence, the correct answer is

$\boxed{\frac{15}{2}}$

So, Option C is correct.

Question26

If the eccentricity e of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passing through $(6, 4\sqrt{3})$, satisfies $15\left(e^2 + 1\right) = 34e$, then the length of the latus rectum of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$ is:

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Options:

A.

10

B.

20

C.

25

D.

30

Answer: A

Solution:

For the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

we are given that it passes through the point $(6, 4\sqrt{3})$.

So, substituting $x = 6$ and $y = 4\sqrt{3}$:

$\frac{36}{a^2} - \frac{(4\sqrt{3})^2}{b^2} = 1$

Since

$(4\sqrt{3})^2 = 48$

we get

$\frac{36}{a^2} - \frac{48}{b^2} = 1 \quad \ldots (1)$

Now for the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

its eccentricity is

$e = \sqrt{1 + \frac{b^2}{a^2}}$

So,

$e^2 = 1 + \frac{b^2}{a^2}$

Given:

$15(e^2 + 1) = 34e$

Substitute $e^2 = 1 + \frac{b^2}{a^2}$ if needed, but first solve this equation in e:

$15e^2 - 34e + 15 = 0$

Factorising:

$15e^2 - 25e - 9e + 15 = 0$

$5e(3e - 5) - 3(3e - 5) = 0$

$(3e - 5)(5e - 3) = 0$

So,

$e = \frac{5}{3} \quad \text{or} \quad e = \frac{3}{5}$

But for a hyperbola, $e > 1$, hence

$e = \frac{5}{3}$

Now,

$e^2 = 1 + \frac{b^2}{a^2}$

So,

$\frac{25}{9} = 1 + \frac{b^2}{a^2}$

$\frac{b^2}{a^2} = \frac{25}{9} - 1 = \frac{16}{9}$

Thus,

$b^2 = \frac{16}{9}a^2$

Substitute this in equation (1):

$\frac{36}{a^2} - \frac{48}{\frac{16}{9}a^2} = 1$

Now,

$\frac{48}{\frac{16}{9}a^2} = 48 \cdot \frac{9}{16a^2} = \frac{27}{a^2}$

So,

$\frac{36}{a^2} - \frac{27}{a^2} = 1$

$\frac{9}{a^2} = 1$

Hence,

$a^2 = 9$

Then,

$b^2 = \frac{16}{9} \cdot 9 = 16$

Now consider the second hyperbola:

$\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$

Substitute $a^2 = 9$ and $b^2 = 16$:

$\frac{x^2}{16} - \frac{y^2}{2(9+1)} = 1$

$\frac{x^2}{16} - \frac{y^2}{20} = 1$

For a hyperbola of the form

$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

the length of the latus rectum is

$\frac{2B^2}{A}$

Here,

$A^2 = 16 \Rightarrow A = 4, \qquad B^2 = 20$

Therefore, length of latus rectum

$= \frac{2 \cdot 20}{4} = 10$

Hence, the required answer is

$\boxed{10}$

So, the correct option is **A**.

Question27

Let e_1 and e_2 be two distinct roots of the equation $x^2 - ax + 2 = 0$. Let the sets $\{a \in \mathbb{R} : e_1$ and e_2 are the eccentricities of hyperbolas $\} = (\alpha, \beta)$, and $\{a \in \mathbb{R} : e_1$ and e_2 are the eccentricities of an ellipse and a hyperbola, respectively $\} = (\gamma, \infty)$.

Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

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Options:

A.

18

B.

22

C.

26

D.

34

Answer: C

Solution:

For the quadratic equation

$x^2 - ax + 2 = 0$

let its two distinct roots be e_1 and e_2.

We are given that these roots are to be interpreted as eccentricities.

Recall:

- For a hyperbola, eccentricity $e > 1$
- For an ellipse, eccentricity satisfies \$0

Also, since e_1, e_2 are roots of the quadratic, by Vieta's formulas,

$e_1 + e_2 = a, \qquad e_1 e_2 = 2$

and since the roots are distinct real roots,

$a^2 - 8 > 0 \quad \Rightarrow \quad |a| > 2\sqrt{2}$

Now we analyse the two sets.

1. Both e_1 and e_2 are eccentricities of hyperbolas

This means both roots must satisfy

$e_1 > 1, \qquad e_2 > 1$

Since

$e_1 e_2 = 2 > 0$

the two roots have the same sign.

If both are hyperbola eccentricities, both must be positive and greater than 1.

Now, for both roots of

$x^2 - ax + 2 = 0$

to be greater than 1, we use the NCERT-style condition:

For both roots to be greater than 1, we need

1. roots real and distinct:

   $a^2 - 8 > 0$

2. sum positive:

   $a > 0$

3. value of polynomial at $x = 1$ positive, because 1 must lie to the left of both roots:

   $f(1) = 1 - a + 2 = 3 - a > 0$ or < 0?

Let us be careful.

Since the parabola opens upward, if both roots are greater than 1, then $x = 1$ lies outside the roots on the left side, so

$f(1) > 0$

Hence,

$3 - a > 0 \Rightarrow a < 3$

Also, for both roots to be greater than 1, the axis of symmetry should be to the right of 1:

$\frac{a}{2} > 1 \Rightarrow a > 2$

Combining:

$$ 2

But also we need distinct real roots, and in this interval automatically $a > 2\sqrt{2}$ is needed. Hence

$$ 2\sqrt2

So,

$(\alpha, \beta) = (2\sqrt{2}, 3)$

Thus,

$\alpha = 2\sqrt{2}, \qquad \beta = 3$

2. e_1 is eccentricity of an ellipse and e_2 is eccentricity of a hyperbola, respectively

This means one root lies in $(0, 1)$ and the other lies in $(1, \infty)$.

Since

$e_1 e_2 = 2 > 0$

both roots are positive. So the only possibility is:

- one root in $(0, 1)$
- the other root greater than 1

Now let the roots be distinct and positive. Since their product is 2, if one root is less than 1, the other must automatically be greater than 2, hence greater than 1.

So we only need one root to be less than 1.

For that, $x = 1$ must lie between the two roots. Since the parabola opens upward, this happens when

$f(1) < 0$

That is,

$1 - a + 2 < 0$

$3 - a < 0$

$a > 3$

Also for real distinct roots, $a^2 - 8 > 0$, which is already true for $a > 3$.

Hence the set is

$(\gamma, \infty) = (3, \infty)$

So,

$\gamma = 3$

3. Find $\alpha^2 + \beta^2 + \gamma^2$

Now,

$\alpha^2 = (2\sqrt{2})^2 = 8$

$\beta^2 = 3^2 = 9$

$\gamma^2 = 3^2 = 9$

Therefore,

$\alpha^2 + \beta^2 + \gamma^2 = 8 + 9 + 9 = 26$

So the correct answer is

$\boxed{26}$

Hence, **Option C**.

Question28

The eccentricity of an ellipse E with centre at the origin O is $\frac{\sqrt{3}}{2}$ and its directrices are $x = \pm\frac{4\sqrt{6}}{3}$. Let $\mathrm{H}: \frac{x^2}{\mathrm{a}^2} - \frac{y^2}{\mathrm{b}^2} = 1$ be a hyperbola whose eccentricity is equal to the length of semi-major axis of E , and whose length of latus rectum is equal to the length of minor axis of E . Then the distance between the foci of H is :

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Options:

A.

$\frac{4\sqrt{2}}{\sqrt{7}}$

B.

$\frac{4\sqrt{2}}{7}$

C.

$\frac{4}{\sqrt{7}}$

D.

$\frac{8}{7}$

Answer: D

Solution:

For the ellipse E, let its equation be

$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, \quad A > B$

since its major axis is along the x-axis because the directrices are of the form $x =$ constant.

Given eccentricity,

$e = \frac{\sqrt{3}}{2}$

For an ellipse with major axis along the x-axis, the directrices are

$x = \pm\frac{A}{e}$

We are given

$x = \pm\frac{4\sqrt{6}}{3}$

So,

$\frac{A}{e} = \frac{4\sqrt{6}}{3}$

Substituting $e = \frac{\sqrt{3}}{2}$,

$A = \frac{\sqrt{3}}{2} \cdot \frac{4\sqrt{6}}{3}$

$A = \frac{4\sqrt{18}}{6} = \frac{4 \cdot 3\sqrt{2}}{6} = 2\sqrt{2}$

Thus the semi-major axis of ellipse E is

$A = 2\sqrt{2}$

Now for the ellipse,

$B^2 = A^2(1 - e^2)$

Since

$A^2 = (2\sqrt{2})^2 = 8$

and

$e^2 = \frac{3}{4}$

we get

$B^2 = 8\left(1 - \frac{3}{4}\right) = 8 \cdot \frac{1}{4} = 2$

Hence

$B = \sqrt{2}$

So the length of minor axis of E is

$2B = 2\sqrt{2}$

Now consider the hyperbola

$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Its eccentricity is equal to the length of semi-major axis of E, so

$e_H = 2\sqrt{2}$

For a hyperbola,

$e_H^2 = 1 + \frac{b^2}{a^2}$

Thus,

$(2\sqrt{2})^2 = 1 + \frac{b^2}{a^2}$

$8 = 1 + \frac{b^2}{a^2}$

$\frac{b^2}{a^2} = 7 \quad \Rightarrow \quad b^2 = 7a^2$

Also, the length of latus rectum of the hyperbola is equal to the length of minor axis of E.

For the hyperbola, length of latus rectum is

$\frac{2b^2}{a}$

So,

$\frac{2b^2}{a} = 2\sqrt{2}$

$\frac{b^2}{a} = \sqrt{2}$

Using $b^2 = 7a^2$,

$\frac{7a^2}{a} = \sqrt{2}$

$7a = \sqrt{2}$

$a = \frac{\sqrt{2}}{7}$

Now for the hyperbola,

$c = ae_H$

So,

$c = \frac{\sqrt{2}}{7} \cdot 2\sqrt{2} = \frac{4}{7}$

Distance between the foci $= 2c$,

$2c = \frac{8}{7}$

Hence, the correct answer is

$\boxed{\frac{8}{7}}$

So, **Option D** is correct.

Question29

Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2 = 16x$. Let the vertex B containing the right angle be $(4, 8)$ and the locus of the centroid of $\triangle ABC$ be a conic

C_0. Then three times the length of latus rectum of C_0 is ____

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Answer: 16

Solution:

Since the right angle is at $B = (4, 8)$, and A, C lie on the parabola

$y^2 = 16x$,

we first parametrize the parabola in the standard NCERT form.

For $y^2 = 4ax$, we have $4a = 16 \Rightarrow a = 4$.

So any point on the parabola can be written as

$(at^2, 2at) = (4t^2, 8t)$.

Thus let

$A = (4t_1^2, 8t_1), \qquad C = (4t_2^2, 8t_2)$.

Also, the point $B = (4, 8)$ lies on the parabola and corresponds to parameter $t = 1$, since

$(4 \cdot 1^2, 8 \cdot 1) = (4, 8)$.

So B is the point with parameter 1.

Now, for the triangle ABC to be right-angled at B, the slopes of BA and BC must satisfy

$m_{BA}\, m_{BC} = -1$.

For parabola $y^2 = 4ax$, the slope of chord joining parameter points t_1 and t_2 is

$\frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2}$.

So slope of chord joining parameter 1 and t_1 is

$m_{BA} = \frac{2}{1 + t_1}$,

and slope of chord joining parameter 1 and t_2 is

$m_{BC} = \frac{2}{1 + t_2}$.

Since these are perpendicular,

$\frac{2}{1 + t_1} \cdot \frac{2}{1 + t_2} = -1$.

Hence

$\frac{4}{(1 + t_1)(1 + t_2)} = -1$

which gives

$(1 + t_1)(1 + t_2) = -4$.

So,

$t_1 t_2 + t_1 + t_2 + 1 = -4$

or

$t_1 t_2 + t_1 + t_2 = -5$.

Now let the centroid of $\triangle ABC$ be $G(h, k)$.

Using centroid formula,

$h = \frac{4t_1^2 + 4 + 4t_2^2}{3} = \frac{4}{3}(t_1^2 + t_2^2 + 1)$,

$k = \frac{8t_1 + 8 + 8t_2}{3} = \frac{8}{3}(t_1 + t_2 + 1)$.

Let

$S = t_1 + t_2, \qquad P = t_1 t_2$.

Then from the perpendicularity condition,

$P + S = -5$.

Also,

$t_1^2 + t_2^2 = S^2 - 2P$.

So

$h = \frac{4}{3}(S^2 - 2P + 1)$.

Since $P = -5 - S$,

$h = \frac{4}{3}\left(S^2 - 2(-5 - S) + 1\right)$

$h = \frac{4}{3}(S^2 + 2S + 11)$.

Also,

$k = \frac{8}{3}(S + 1)$.

From this,

$S + 1 = \frac{3k}{8} \quad \Rightarrow \quad S = \frac{3k}{8} - 1$.

Substitute into h:

$h = \frac{4}{3}\left[\left(\frac{3k}{8} - 1\right)^2 + 2\left(\frac{3k}{8} - 1\right) + 11\right]$.

Now simplify:

$\left(\frac{3k}{8} - 1\right)^2 = \frac{9k^2}{64} - \frac{3k}{4} + 1$

and

$2\left(\frac{3k}{8} - 1\right) = \frac{3k}{4} - 2$.

So inside the bracket,

$\frac{9k^2}{64} - \frac{3k}{4} + 1 + \frac{3k}{4} - 2 + 11 = \frac{9k^2}{64} + 10$.

Hence

$h = \frac{4}{3}\left(\frac{9k^2}{64} + 10\right) = \frac{3k^2}{16} + \frac{40}{3}$.

Therefore,

$h - \frac{40}{3} = \frac{3k^2}{16}$.

Rearranging,

$k^2 = \frac{16}{3}\left(h - \frac{40}{3}\right)$.

So the locus is a parabola:

$k^2 = 4a\left(h - \frac{40}{3}\right)$

with

$4a = \frac{16}{3} \quad \Rightarrow \quad a = \frac{4}{3}$.

Length of latus rectum of this parabola is

$4a = \frac{16}{3}$.

Therefore, three times the length of latus rectum is

$3 \times \frac{16}{3} = 16$.

So the required answer is

$\boxed{16}$.

Question30

Let O be the vertex of the parabola $x^2 = 4y$ and Q be any point on it. Let the locus of the point P , which divides the line segment OQ internally in the ratio $2 : 3$ be the conic C . Then the equation of the chord of C, which is bisected at the point $(1, 2)$, is :

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Options:

A.

$5x - 4y + 3 = 0$

B.

$x - 2y + 3 = 0$

C.

$4x - 5y + 6 = 0$

D.

$5x - y - 3 = 0$

Answer: A

Solution:

Let the vertex of the parabola $x^2 = 4y$ be the origin $O(0, 0)$.

Let Q be any point on the parabola. We can parameterize the coordinates of Q as $(2t, t^2)$ for some real parameter t. Satisfying $x^2 = (2t)^2 = 4t^2$ and $4y = 4(t^2)$.

Let $P(x, y)$ be the point that divides the line segment OQ internally in the ratio $2 : 3$.

Using the section formula, the coordinates of P are given by:

$x = \frac{2 \cdot x_Q + 3 \cdot x_O}{2+3} = \frac{2(2t)+3(0)}{5} = \frac{4t}{5}$

$y = \frac{2 \cdot y_Q + 3 \cdot y_O}{2+3} = \frac{2(t^2)+3(0)}{5} = \frac{2t^2}{5}$

To find the locus of P (the conic C), we eliminate the parameter t.

From the first equation, we have $t = \frac{5x}{4}$.

Substituting this into the second equation:

$y = \frac{2}{5}\left(\frac{5x}{4}\right)^2$

$y = \frac{2}{5} \cdot \frac{25x^2}{16}$

$y = \frac{5x^2}{8}$

$5x^2 = 8y$

So, the equation of the conic C is $5x^2 - 8y = 0$. This is a parabola.

We need to find the equation of the chord of C which is bisected at the point $(x_1, y_1) = (1, 2)$.

The equation of a chord of a conic $S = 0$ bisected at a point (x_1, y_1) is given by $T = S_1$.

Here, $S \equiv 5x^2 - 8y$.

S_1 is the value of S at $(1, 2)$:

$S_1 = 5(1)^2 - 8(2) = 5 - 16 = -11$

T is the expression obtained by replacing x^2 with xx_1 and y with $\frac{y+y_1}{2}$:

$T = 5x(1) - 8\left(\frac{y+2}{2}\right)$

$T = 5x - 4(y + 2)$

$T = 5x - 4y - 8$

Equating $T = S_1$:

$5x - 4y - 8 = -11$

$5x - 4y + 3 = 0$

The equation of the chord is $5x - 4y + 3 = 0$.

Option A

$5x - 4y + 3 = 0$

Question31

Let one end of a focal chord of the parabola $y^2 = 16x$ be $(16, 16)$. If $P(\alpha, \beta)$ divides this focal chord internally in the ratio $5 : 2$, then the minimum value of $\alpha + \beta$ is equal to:

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Options:

A.

5

B.

7

C.

22

D.

16

Answer: B

Solution:

If $P(\alpha, \beta)$ divides focal chord internally in $5 : 2$

If $A(t_1) \& B(t_2)$ are ends of focal chord $A(16, 16)$

$t_1 t_2 = -1$
$2t_2 = -1$
$t_2 = -\frac{1}{2}$
$B = (1, -4)$
$P(\alpha, \beta) = \left(\frac{37}{7}, \frac{12}{7}\right)$ or $\left(\frac{82}{7}, \frac{72}{7}\right)$

Minimum of $\alpha + \beta = \frac{37}{7} + \frac{12}{7} = 7$

Question32

Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90^\circ$. Then the locus of the centroid of such triangles OPA is:

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Options:

A.

$y^2 - 4x + 8 = 0$

B.

$y^2 - 2x + 8 = 0$

C.

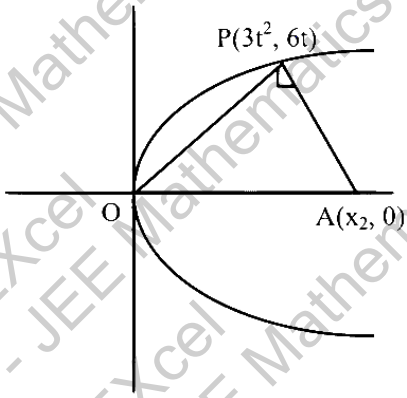
$y^2 - 9x + 6 = 0$

D.

$y^2 - 6x + 4 = 0$

Answer: B

Solution:



$m_{op}m_{PA} = -1\frac{6t}{3t^2} \times \frac{6t}{3t^2 - x} = -1$

$12 = -3t^2 + x$

$x = 12 + 3t^2$

$h = \frac{x + 3t^2 + 0}{3}; k = \frac{0 + 0 + 6t}{3}$

$3h = 12 + 6th2; k = 2t$

$h = 4 + 2t^2; k = 2t$

Substitute t& replace h by x&k by y

$y^2 - 2x + 8 = 0$

Question33

If the chord joining the points $\mathrm{P}_1\left(x_1, y_1\right)$ and $\mathrm{P}_2\left(x_2, y_2\right)$ on the parabola $y^2 = 12x$ subtends a right angle at the vertex of the parabola, then $x_1x_2 - y_1y_2$ is equal to

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Options:

A.

280

B.

288

C.

292

D.

284

Answer: B

Solution:

$P\left(at_1^2, 2at_1\right), Q\left(at_2^2, 2at_2\right)$

$\therefore t_1t_2 = -4, a = 3$

$x_1x_2 - y_1y_2 = a^2 \times \left(t_1t_2\right)^2 - 4a^2t_1t_2 = 288$

Question34

Let the locus of the mid-point of the chord through the origin O of the parabola $y^2 = 4x$ be the curve S . Let P be any point on S . Then the locus of the point, which internally divides OP in the ratio 3 : 1, is :

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Options:

A.

$2x^2 = 3y$

B.

$2y^2 = 3x$

C.

$3y^2 = 2x$

D.

$3x^2 = 2y$

Answer: B

Solution:

Let $M\left(x_1, y_1\right)$ be the locus of the midpoint of the chord of the parabola $y^2 = 4x$

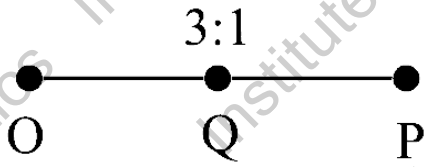
$\therefore$ Equation of the chord is $S_1 = S_{11} \Rightarrow yy_1 - 2\left(x + x_1\right) = y_1^2 - 4x_1$

It passes through $0(0, 0)$

$\therefore -2x_1 = y_1^2 - 4x_1$

Locus of M is $y^2 = 2x$

Let $P\left(\frac{1}{2}t^2, t\right)$ be a point on this locus



$Q(x, y) = \left(\frac{\frac{3t^2}{2} + 0}{4}, \frac{3t + 0}{4}\right) \Rightarrow \left(\frac{3t^2}{8}, \frac{3t}{4}\right)$

$\Rightarrow t^2 = \frac{8x}{3}, t = \frac{4y}{3} \Rightarrow \frac{16y^2}{9} = \frac{8x}{3} \Rightarrow 2y^2 = 3x$

Question35

An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is

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Options:

A.

$2(8-3\sqrt{3})$

B.

$4(6+\sqrt{3})$

C.

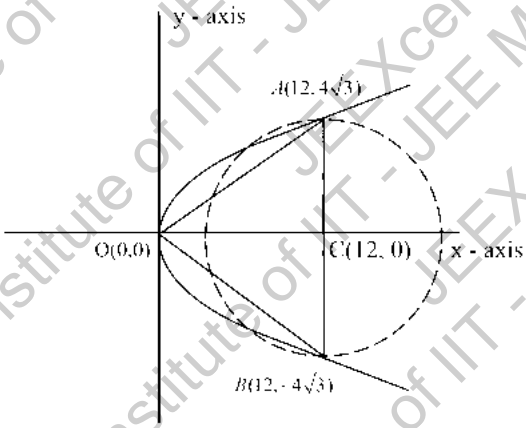
$4(3-\sqrt{3})$

D.

$2(3+\sqrt{3})$

Answer: C

Solution:



$y^2 = 4x \Longrightarrow a = 1$

Let $A = \left(t^2, 2t\right)$ and $B = \left(t^2, -2t\right)$ due to symmetry.

Triangle OAB is equilateral:

Slope of $OA = \tan\left(30^\circ\right) = \frac{1}{\sqrt{3}}$

$\frac{2t-0}{t^2-0} = \frac{1}{\sqrt{3}} \Longrightarrow \frac{2}{t} = \frac{1}{\sqrt{3}} \Longrightarrow t = 2\sqrt{3}$

Coordinates:

$A = (12, 4\sqrt{3})$
$B = (12, -4\sqrt{3})$

Circle with diameter AB :

Center $C = \left(\frac{12+12}{2}, \frac{4\sqrt{3}-4\sqrt{3}}{2}\right) = (12, 0)$

Radius $r = \frac{1}{2}\sqrt{(12-12)^2 + (4\sqrt{3} - (-4\sqrt{3}))^2} = 4\sqrt{3}$

Minimum distance from origin $O(0,0)$:

$d = |OC - r|$

$OC = \sqrt{12^2 + 0^2} = 12$

$d = 12 - 4\sqrt{3} = 4(3 - \sqrt{3})$

Question36

Let the image of parabola $x^2 = 4y$, in the line $x - y = 1$ be $(y + a)^2 = b(x - c)$, $a, b, c \in$ N. Then $a + b + c$ is equal to

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Options:

A.

12

B.

8

C.

6

D.

4

Answer: C

Solution:

Let parametric point on parabola $x^2 = 4y$ is $\left(2t, t^2\right)$.

Key concept: image of point (h, k) in the line $ax + by + c = 0$ is given as $\frac{x-h}{a} = \frac{y-k}{b} = \frac{-2(ah+bk+c)}{a^2+b^2}$

So, Image of point in line $x - y - 1 = 0$ is given as

$\frac{x-2t}{1} = \frac{y-t^2}{-1} = \frac{-2\left(2t-t^2-1\right)}{1^2+(-1)^2}$

equating

$$\frac{x - 2t}{1} = \frac{-2\left(2t - t^2 - 1\right)}{2}$$

$x = -2t + t^2 + 1 + 2t \Rightarrow x = t^2 + 1$

equating

$$\frac{y - t^2}{-1} = \frac{-2\left(2t - t^2 - 1\right)}{2}$$

$y - t^2 = 2t - t^2 - 1 \Rightarrow y = 2t - 1$

from (1) and (2) eliminate parameter t to get

image of curve.

so, from (2) put $t = \frac{y+1}{2}$ in equation (1).

$x = \left(\frac{y+1}{2}\right)^2 + 1$

$\frac{(y+1)^2}{4} = x - 1$

$(y+1)^2 = 4(x-1)$

Compare with given image of curve $(y+a)^2 = b(x-c)$

to find value of a, b, &c.

$a = 1, b = 4, c = 1$

$a + b + c = 1 + 4 + 1 = 6.$

Question37

Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3}, \frac{4}{3}\right)$, then $(BC)^2$ is equal to :

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Options:

A.

89

B.

80

C.

32

D.

41

Answer: B

Solution:



Identify the Parabola Parameters The parabola is $y^2 = 8x$. Comparing this with the standard form $y^2 = 4ax$, we find $4a = 8$, so $a = 2$.

- Focus A : The focus is $(a, 0)$, which is $(2, 0)$.

1. **Define Parametric Points for B and C :** Let the two distinct points of intersection be B a C. Using parametric coordinates $\left(at^2, 2at\right)$:

- Point $B = \left(2t_1^2, 4t_1\right)$

1. **Use the Centroid Formula :** The centroid $G(x, y)$ of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is:

$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

3. **Use the Centroid Property :** The centroid of triangle ABC with vertices $A(2, 0)$, $B\left(2t_1^2, 4t_1\right)$, and $C\left(2t_2^2, 4t_2\right)$ is given as $\left(\frac{7}{3}, \frac{4}{3}\right)$.

- For the y-coordinate :

$\frac{4t_1+4t_2+0}{3} = \frac{4}{3} \Longrightarrow 4\left(t_1 + t_2\right) = 4 \Longrightarrow t_1 + t_2 = 1$

- For the x-coordinate :

$\frac{2t_1^2+2t_2^2+2}{3} = \frac{7}{3} \Longrightarrow 2\left(t_1^2 + t_2^2\right) = 5 \Longrightarrow t_1^2 + t_2^2 = \frac{5}{2}$

4. **Find the Relationship between t_1 and t_2 :**

Using the identity $\left(t_1 + t_2\right)^2 = t_1^2 + t_2^2 + 2t_1t_2$:

$(1)^2 = \frac{5}{2} + 2t_1t_2 \Longrightarrow 1 - \frac{5}{2} = 2t_1t_2 \Longrightarrow 2t_1t_2 = -\frac{3}{2} \Longrightarrow t_1t_2 = -\frac{3}{4}$

Now, calculate $\left(t_1 - t_2\right)^2$:

$\left(t_1 - t_2\right)^2 = \left(t_1 + t_2\right)^2 - 4t_1t_2 = 1^2 - 4\left(-\frac{3}{4}\right) = 1 + 3 = 4$

5. Calculate $(BC)^2$ The distance formula for $(BC)^2$ is :

$(BC)^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2$

$\Rightarrow (BC)^2 = \left[2\left(t_2^2 - t_1^2\right)\right]^2 + \left[4\left(t_2 - t_1\right)\right]^2$

$\Rightarrow (BC)^2 = 4(t_2 - t_1)^2(t_2 + t_1)^2 + 16(t_2 - t_1)^2$

Substitute the values $\left(t_1 - t_2\right)^2 = 4$ and $\left(t_1 + t_2\right) = 1$:

$(BC)^2 = 4(4)(1)^2 + 16(4) = 16 + 64 = 80$

Question38

Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is :

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Options:

A.

2

B.

4

C.

5

D.

8

Answer: B

Solution:

parabola passes through $(1,-1)$ so

$-1 = 1 + p + q$

$p + q = -2$

$q = -2 - p \quad \ldots(1)$

Distance from x-axis

$\frac{-D}{4a} = \frac{-(p^2-4q)}{4(1)} = \frac{4q-p^2}{4}$

$= \frac{4(-2-p)-p^2}{4}$ from (1)

$= \frac{-8-4p-p^2}{4} = \frac{-(p^2+4p+8)}{4} = \frac{-((p+2)^2+4)}{4}$

Minimum at $p = -2 \to q = 0$

$p^2 + q^2 = 4 + 0 = 4$

Question39

Let the directrix of the parabola $\mathrm{P}: y^2 = 8x$, cut x-axis at the point A . Let $\mathrm{B}(\alpha, \beta), \alpha > 1$, be a point on P such that the slope of AB is $3/5$. If BC is a focal chord of P, then six times the area of $\triangle ABC$ is :

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Options:

A.

80

B.

160

C.

174

D.

192

Answer: B

Solution:

For the parabola

$y^2 = 8x$

we compare it with the standard form

$y^2 = 4ax$

So,

$4a = 8 \implies a = 2$

Hence:

- focus $S = (2, 0)$
- directrix is

$x = -2$

Since the directrix cuts the x-axis at point A, we get

$A = (-2, 0)$

Now let $B(\alpha, \beta)$ be a point on the parabola such that the slope of AB is $\frac{3}{5}$.

The slope of AB is

$\frac{\beta - 0}{\alpha - (-2)} = \frac{\beta}{\alpha + 2}$

Given this is $\frac{3}{5}$, so

$\frac{\beta}{\alpha+2} = \frac{3}{5}$

Thus,

$5\beta = 3(\alpha + 2)$

Also, since B lies on the parabola $y^2 = 8x$,

$\beta^2 = 8\alpha$

Now substitute from (1). From (1),

$\beta = \frac{3(\alpha+2)}{5}$

Put into (2):

$\left(\frac{3(\alpha+2)}{5}\right)^2 = 8\alpha$

$\frac{9(\alpha+2)^2}{25} = 8\alpha$

$9(\alpha + 2)^2 = 200\alpha$

$9(\alpha^2 + 4\alpha + 4) = 200\alpha$

$9\alpha^2 + 36\alpha + 36 = 200\alpha$

$9\alpha^2 - 164\alpha + 36 = 0$

Now solve:

$9\alpha^2 - 164\alpha + 36 = 0$

Discriminant:

$D = 164^2 - 4 \cdot 9 \cdot 36 = 26896 - 1296 = 25600$

$\sqrt{D} = 160$

So,

$\alpha = \frac{164 \pm 160}{18}$

Hence,

$\alpha = \frac{324}{18} = 18 \quad \text{or} \quad \alpha = \frac{4}{18} = \frac{2}{9}$

Given $\alpha > 1$, we take

$\alpha = 18$

Then from $\beta^2 = 8\alpha$,

$\beta^2 = 144 \implies \beta = \pm 12$

Since slope is $\frac{3}{5} > 0$, β must be positive. Therefore

$B = (18, 12)$

Now BC is a focal chord of the parabola.

For parabola $y^2 = 4ax$, a point in parametric form is

$(at^2,\ 2at)$

Here $a = 2$, so point is

$(2t^2,\ 4t)$

For point $B = (18, 12)$,

$2t^2 = 18 \implies t^2 = 9 \implies t = 3$

and

$4t = 12 \implies t = 3$

So B corresponds to parameter $t = 3$.

If BC is a focal chord, then the other endpoint corresponds to parameter $-\dfrac{1}{t}$.

Thus for $t = 3$, point C corresponds to parameter $-\dfrac{1}{3}$.

So,

$C = \left(2\left(\frac{1}{9}\right),\ 4\left(-\frac{1}{3}\right)\right) = \left(\frac{2}{9},\ -\frac{4}{3}\right)$

Now find area of $\triangle ABC$.

Coordinates are:

$A = (-2, 0), \quad B = (18, 12), \quad C = \left(\frac{2}{9}, -\frac{4}{3}\right)$

Using determinant formula,

Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

So,

Area $= \frac{1}{2}\left[-2\left(12 + \frac{4}{3}\right) + 18\left(-\frac{4}{3}\right) + \frac{2}{9}(0 - 12)\right]\Big|$

Now simplify:

$12 + \frac{4}{3} = \frac{40}{3}$

So,

$-2 \cdot \frac{40}{3} = -\frac{80}{3}$

Also,

$18 \cdot \left(-\frac{4}{3}\right) = -24$

and

$\frac{2}{9}(-12) = -\frac{24}{9} = -\frac{8}{3}$

Therefore,

Area $= \frac{1}{2}\left|-\frac{80}{3} - 24 - \frac{8}{3}\right|$

$= \frac{1}{2}\left|-\frac{88}{3} - 24\right| = \frac{1}{2}\left|-\frac{88}{3} - \frac{72}{3}\right|$

$= \frac{1}{2}\left|-\frac{160}{3}\right| = \frac{80}{3}$

Hence,

$6 \times \text{Area} = 6 \cdot \frac{80}{3} = 160$

Therefore, the required answer is

$\boxed{160}$

So, the correct option is **B**.

Question40

Let chord PQ of length $3\sqrt{13}$ of the parabola $y^2 = 12x$ be such that the ordinates of points P and Q are in the ratio 1:2. If the chord PQ subtends an angle α at the focus of the parabola, then $\sin\alpha$ is equal to :

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Options:

A.

$\frac{3}{5}$

B.

$\frac{4}{5}$

C.

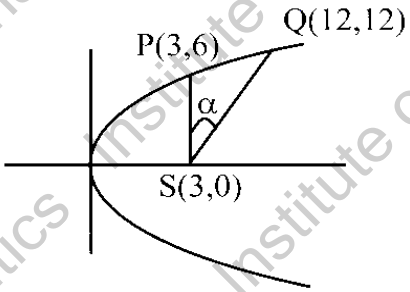
$\frac{5}{13}$

D.

$\frac{12}{13}$

Answer: A

Solution:



For the parabola $y^2 = 12x,$ we have $4a = 12 \Rightarrow a = 3.$

So, the parametric point on $y^2 = 4ax$ is $(at^2, 2at)$.

Let the ordinate (y-coordinate) of P be in ratio $1 : 2$ with Q.

So we take $P(3t^2, 6t)$ and $Q\big(3(2t)^2, 6(2t)\big) = \big(12t^2, 12t\big)$.

Now find PQ :

$PQ = \sqrt{(12t^2 - 3t^2)^2 + (12t - 6t)^2}$

$= \sqrt{(9t^2)^2 + (6t)^2} = \sqrt{81t^4 + 36t^2}.$

Given $PQ = 3\sqrt{13} \Rightarrow PQ^2 = 117$.

So, $81t^4 + 36t^2 = 117$

$\Rightarrow 9t^4 + 4t^2 - 13 = 0$

$\Rightarrow (t^2 - 1)(9t^2 + 13) = 0.$

Since $t^2 \geq 0,\ 9t^2 + 13 \neq 0,$ so $t^2 = 1 \Rightarrow t = 1.$

Hence $P(3, 6),\ Q(12, 12).$

Focus of $y^2 = 12x$ is $S(a, 0) = (3, 0).$

Slope of $QS = \dfrac{12 - 0}{12 - 3} = \dfrac{12}{9} = \dfrac{4}{3}.$

Line PS is vertical since $P(3, 6)$ and $S(3, 0)$ have same x.

So the angle α at the focus is the angle between QS and the vertical.

If slope of QS is $\dfrac{4}{3},$ then the angle it makes with the vertical has

$\tan\alpha = \dfrac{3}{4}.$

$\Rightarrow \sin\alpha = \dfrac{3}{5}.$

Question41

Let one root of the quadratic equation in x :

$$\left(k^2 - 15k + 27\right)x^2 + 9(k - 1)x + 18 = 0$$

be twice the other. Then the length of the latus rectum of the parabola $y^2 = 6kx$ is equal to:

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A.

4

B.

6

C.

8

D.

12

Answer: D

Solution:

Let the roots of the quadratic equation

$\left(k^2-15k+27\right)x^2+9(k-1)x+18=0$

be α and β.

Given that one root is twice the other, let

$\alpha=2\beta$

So the roots are in the ratio $2:1$.

For a quadratic equation $ax^2+bx+c=0$:

$\alpha+\beta=-\frac{b}{a}, \qquad \alpha\beta=\frac{c}{a}$

Here,

$a=k^2-15k+27, \qquad b=9(k-1), \qquad c=18$

So,

$\alpha+\beta=\frac{-9(k-1)}{k^2-15k+27}$

and

$\alpha\beta=\frac{18}{k^2-15k+27}$

Now let the roots be $2t$ and t.

Then,

$\alpha+\beta=3t, \qquad \alpha\beta=2t^2$

Using the identity:

$(\alpha+\beta)^2=9t^2$

and

$\alpha\beta=2t^2$

So,

$(\alpha+\beta)^2=\frac{9}{2}\alpha\beta$

Now substitute the values from the quadratic:

$\left(\frac{-9(k-1)}{k^2-15k+27}\right)^2=\frac{9}{2}\cdot\frac{18}{k^2-15k+27}$

Simplify:

$\frac{81(k-1)^2}{(k^2-15k+27)^2}=\frac{81}{k^2-15k+27}$

Cancel 81:

$\frac{(k-1)^2}{(k^2-15k+27)^2}=\frac{1}{k^2-15k+27}$

Cross-multiply:

$(k-1)^2=k^2-15k+27$

Now expand:

$k^2-2k+1=k^2-15k+27$

Cancel k^2:

$-2k+1=-15k+27$

$13k=26$

$k=2$

Now the parabola is

$y^2 = 6kx$

Substitute $k = 2$:

$y^2 = 12x$

Compare with standard form:

$y^2 = 4ax$

So,

$4a = 12 \implies a = 3$

Length of latus rectum of the parabola $y^2 = 4ax$ is

$4a$

Therefore,

$4a = 12$

Hence, the required length is

$\boxed{12}$

So, the correct option is:

Option D

Question42

Let O be the vertex of the parabola $y^2 = 4x$ and its chords OP and OQ are perpendicular to each other. If the locus of the mid-point of the line segment PQ is a conic C , then the length of its latus rectum is :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

1

B.

2

C.

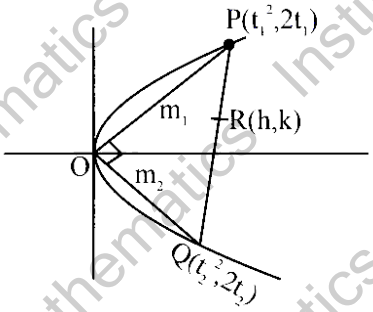
4

D.

8

Answer: B

Solution:



$t_1t_2 = -4$
$2h = t_1^2 + t_2^2 \quad 2k = 2(t_1 + t_2)$
$2h = k^2 - 2t_1t_2$
$2h = k^2 + 8$
$k^2 = 2h - 8$
$= 2((h - 4)$
L.R. $= 2$

Question43

Let PQ and MN be two straight lines touching the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the points A and B respectively. Let O be the centre of the circle and $\angle AOB = \pi/3$. Then the locus of the point of intersection of the lines PQ and MN is :

JEE Main 2026 (Online) 21st January Morning Shift

Options:

A.

$x^2 + y^2 - 18x - 12y - 25 = 0$

B.

$x^2 + y^2 - 12x - 18y - 25 = 0$

C.

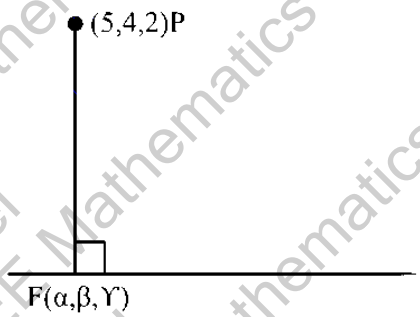
$3(x^2 + y^2) - 12x - 18y - 25 = 0$

D.

$3(x^2 + y^2) - 18x - 12y + 25 = 0$

Answer: C

Solution:



$x^2+y^2-14x-6y-3=0$

$\tan 60^\circ = \dfrac{4}{\sqrt{h^2+k^2-4h-6k-3}}$

squaring both side

$3\left(h^2+k^2-4h-6k-3\right)=16$

To get locus replace h, k by x and y;

$3x^2+3y^2-12x-18y-9-16=0$
$3x^2+3y^2-12x-18y-25=0$

Question44

Let the set of all values of r, for which the circles $(x+1)^2+(y+4)^2=r^2$ and $x^2+y^2-4x-2y-4=0$ intersect at two distinct points be the interval (α,β). Then $\alpha\beta$ is equal to

JEE Main 2026 (Online) 22nd January Morning Shift

Options:

A.

21

B.

24

C.

20

D.

25

Answer: D

Solution:

$|r_1-r_2|<c_1c_2<r_1+r_2$
$r-3<\sqrt{9+25}<r+3$
$r\in(\sqrt{34}-3,\sqrt{34}+3)$
$\alpha\beta=25$

Question45

Let a circle of radius 4 pass through the origin O , the points $\mathrm{A}(-\sqrt{3}a,0)$ and $\mathrm{B}(0,-\sqrt{2}b)$, where a and b are real parameters and $ab\neq 0$. Then the locus of the centroid of $\triangle\mathrm{OAB}$ is a circle of radius

JEE Main 2026 (Online) 24th January Morning Shift

Options:

A.

$\frac{7}{3}$

B.

$\frac{11}{3}$

C.

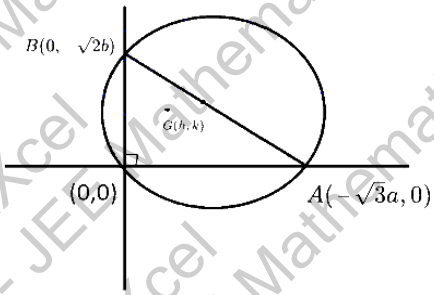
$\frac{5}{3}$

D.

$\frac{8}{3}$

Answer: D

Solution:



Given a circle passes through $O(0,0)$, $A(-\sqrt{3}a, 0)$, and $B(0, -\sqrt{2}b)$. Since A lies on the x -axis and B lies on the y -axis, $\angle AOB = 90^\circ$. Thus AB is the diameter of the circle.

Diameter $d = 2\times$ radius $= 2(4) = 8$.

Using distance formula for AB :

$$\sqrt{(-\sqrt{3}a - 0)^2 + (0 - (-\sqrt{2}b))^2} = 8$$
$$3a^2 + 2b^2 = 64 \quad ------(1)$$

Let the centroid of $\triangle OAB$ be $G(h,k)$.

$$h = \frac{0 - \sqrt{3}a + 0}{3} \Longrightarrow a = \frac{-3h}{\sqrt{3}} = -\sqrt{3}h$$
$$k = \frac{0 + 0 - \sqrt{2}b}{3} \Longrightarrow b = \frac{-3k}{\sqrt{2}}$$

Substitute a and b into (Eq. 1):

$$3(-\sqrt{3}h)^2 + 2\left(\frac{-3k}{\sqrt{2}}\right)^2 = 64$$
$$3\left(3h^2\right) + 2\left(\frac{9k^2}{2}\right) = 64$$
$$9h^2 + 9k^2 = 64$$
$$h^2 + k^2 = \frac{64}{9}$$

The locus is a circle $x^2 + y^2 = \left(\frac{8}{3}\right)^2$.

Radius $= \frac{8}{3}$

Question46

Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point $(2,3)$ and is farthest from the center of C_2, is $x + ay + b = 0$, then $a - b$ is equal to

JEE Main 2026 (Online) 28th January Morning Shift

Options:

A.

-6

B.

10

C.

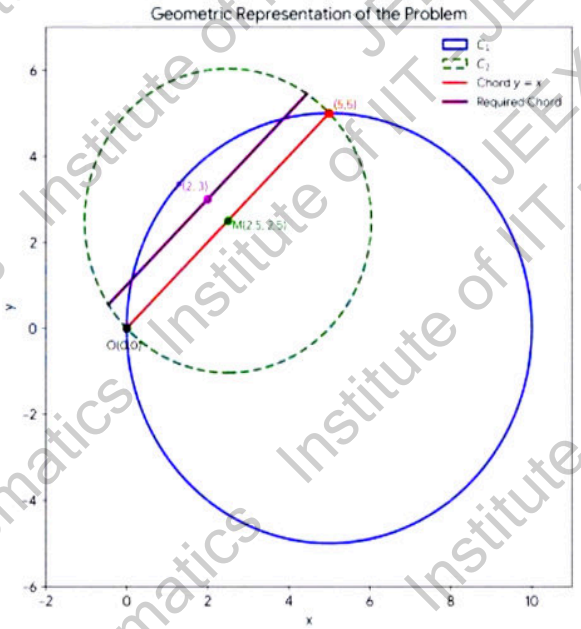
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D.

-2

Answer: D

Solution:



To find the value of $a - b$,

Determine the equation of circle C_1

The circle C_1 has a diameter of 10 , so its radius is $R = 5$. It passes through the origin (0,0) and is contained in the closed half-plane $x \geq 0$.

Let the center of C_1 be (h, k). Since it passes through $(0, 0)$, we have $h^2 + k^2 = R^2 = 25$.

For the circle to lie entirely in the region $x \geq 0$ while passing through $(0, 0)$, the distance from the center to the y-axis must be equal to the radius, and the center must have a non-negative x-coordinate.

Thus, $h = R = 5$.

Substituting $h = 5$ into $h^2 + k^2 = 25$ gives $25 + k^2 = 25$, which implies $k = 0$.

The equation of C_1 is:

$(x - 5)^2 + y^2 = 25$

Find the center of circle C_2

The line $y = x$ is a chord of C_1. We find the intersection points of $y = x$ and C_1 :

$(x - 5)^2 + x^2 = 25$

$\Rightarrow x^2 - 10x + 25 + x^2 = 25$

$\Rightarrow 2x^2 - 10x = 0$

$\Rightarrow 2x(x - 5) = 0$

The intersection points are $(0,0)$ and $(5,5)$. These are the endpoints of the diameter of circle C_2.

The center M of C_2 is the midpoint of the chord:

$M = \left(\frac{0+5}{2}, \frac{0+5}{2}\right) = (2.5, 2.5)$

Determine the equation of the chord of C_2

We are looking for the equation of a chord of C_2 passing through $P(2,3)$ that is farthest from the center $M(2.5, 2.5)$.

In any circle, the chord passing through a point P that is farthest from the center is the one perpendicular to the radius (or segment) MP.

The slope of the segment MP is:

$m_{MP} = \frac{3-2.5}{2-2.5} = \frac{0.5}{-0.5} = -1$

The slope m of the required chord is the negative reciprocal of m_{MP} :

$m = -\frac{1}{-1} = 1$

Using the point-slope form for the line through $P(2,3)$ with slope $m = 1$:

$y - 3 = 1(x - 2)$

$\Rightarrow y - 3 = x - 2 \Rightarrow x - y + 1 = 0$

Calculate $a - b$

The equation is given in the form $x + ay + b = 0$. Comparing this to $x - y + 1 = 0$, we identify:

$a = -1$
$b = 1$

Finally, we calculate the required value:

$a - b = -1 - 1 = -2$

Question47

Let the circle $x^2 + y^2 = 4$ intersect x-axis at the points A$(a, 0)$, $a > 0$ and B$(b, 0)$. Let $P(2\cos\alpha, 2\sin\alpha)$, $0 < \alpha < \frac{\pi}{2}$ and $Q(2\cos\beta, 2\sin\beta)$ be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on :

JEE Main 2026 (Online) 28th January Evening Shift

Options:

A.

$x^2 + y^2 - 4x - 4 = 0$

B.

$x^2 + y^2 - 4x - 4y = 0$

C.

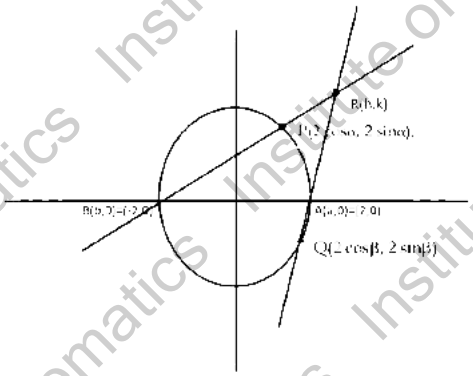
$x^2 + y^2 - 4x - 4y - 4 = 0$

D.

$x^2 + y^2 - 4y - 4 = 0$

Answer: D

Solution:



The circle is $x^2+y^2=2^2$ (radius $r=2$).

- Intersection with **x**-axis: Setting $y=0$ gives $x^2=4 \Rightarrow x=\pm 2$.
- Since $a>0, A=(2,0)$.
- Therefore, $B=(-2,0)$.
- Points P and Q :
- $P=(2\cos\alpha, 2\sin\alpha)$
- $Q=(2\cos\beta, 2\sin\beta)$

Let point of intersection of AQ .& BP be $R(h,k)$.

So, we need to find locus of points of intersection of $AQ \& BP$.

Slope of $BP=$ slope of BR

$$\frac{k-0}{h-(-2)}=\frac{2\sin\alpha-0}{2\cos\alpha+2}$$

$$\frac{k}{h+2}=\frac{\sin\alpha}{\cos\alpha+1} \Rightarrow \frac{k}{h+2}=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$$

$$\frac{k}{h+2}=\tan\frac{\alpha}{2} \qquad \ldots(i)$$

Similarly, slope of $QA=$ slope of QR

$$\frac{2\sin\beta-0}{2\cos\beta-2}=\frac{k-0}{h-2}$$

$$\frac{\sin\beta}{\cos\beta-1}=\frac{k}{h-2} \Rightarrow \frac{2\sin\frac{\beta}{2}\cos\frac{\beta}{2}}{-2\sin^2\frac{\beta}{2}}=\frac{k}{h-2}$$

$$-\cot\frac{\beta}{2}=\frac{k}{h-2}$$

$$\cot\frac{\beta}{2}=\frac{k}{2-h} \Rightarrow \tan\frac{\beta}{2}=\frac{2-h}{k}$$

Given,

$$\alpha-\beta=\frac{\pi}{2} \Rightarrow \frac{\alpha}{2}-\frac{\beta}{2}=\frac{\pi}{4}$$

$$\tan\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)=\tan\frac{\pi}{4}$$

$$\frac{\tan\frac{\alpha}{2}-\tan\frac{\beta}{2}}{1+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}=1$$

Substitute

$\tan\frac{\alpha}{2}=\frac{k}{h+2}, \quad \tan\frac{\beta}{2}=\frac{2-h}{k}$

$$\frac{\frac{k}{h+2}+\frac{2-h}{k}}{1+\left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)}=1$$

$$\frac{k}{h+2}+\frac{h-2}{k}=1+\left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)$$

$$\frac{k^2+h^2-4}{(h+2)k}=\frac{4k}{(h+2)k}$$

$$\frac{k^2+h^2-4}{(h+2)k}=\frac{4k}{(h+2)k}$$

$$k^2+h^2-4=4k$$

$$k^2+h^2-4k-4=0$$

Replace $(h,k) \rightarrow (x,y)$

$$x^2+y^2-4y-4=0$$

Question48

Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k-1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to :

JEE Main 2026 (Online) 2nd April Evening Shift

Options:

A.

10

B.

27

C.

18

D.

34

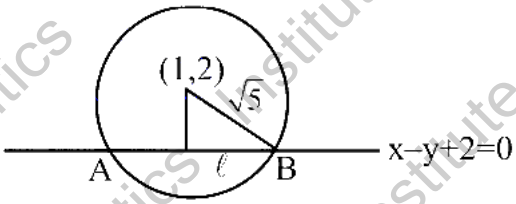
Answer: C

Solution:

$x + (k-1)y + 3 = 0$
$2x + k^2y - 4 = 0$
$\left(\frac{1}{1-k}\right)\left(\frac{2}{k^2}\right) = 1$
$2 = k^2 - k^3$
$k^3 - k^2 + 2 = 0$
$k = -1$

Solving :

$2(x - 2y + 3) = 0$
$2x + y - 4 = 0$
$-5y + 10 = 0$
$y = 2$



$x = 1$

Centre $(1, 2)$

$r = \sqrt{5}$

So circle is $(x-1)^2 + (y-2)^2 = 5$

Chord $x - y + 2 = 0$

$p = \frac{1}{\sqrt{2}}$

$\ell = \sqrt{5 - \frac{1}{2}} = \frac{3}{\sqrt{2}}$

$AB = \frac{6}{\sqrt{2}} \quad \therefore AB^2 = 18$

Question49

Suppose that two chords, drawn from the point $(1, 2)$ on the circle $x^2 + y^2 + x - 3y = 0$ are bisected by the y-axis. If the other ends of these chords are R and S , and the mid point of the line segment RS is (α, β), then $6(\alpha + \beta)$ is equal to :

JEE Main 2026 (Online) 4th April Morning Shift

Options:

A.

1

B.

3

C.

4

D.

6

Answer: B

Solution:

The given circle is

$x^2 + y^2 + x - 3y = 0$

and the point from which the two chords are drawn is

$P(1, 2)$

We are told that these chords are bisected by the y-axis.

Let the other end of one such chord be $R(x, y)$.

Since the chord joining $P(1, 2)$ and $R(x, y)$ is bisected by the y-axis, its midpoint must lie on the y-axis.

So the x-coordinate of the midpoint is 0.

Midpoint of $P(1, 2)$ and $R(x, y)$ is

$\left(\frac{1+x}{2}, \frac{2+y}{2}\right)$

Since this lies on the y-axis,

$\frac{1+x}{2} = 0$

So,

$x = -1$

Thus, the other end of each such chord must lie on the line

$x = -1$

Now find where this line cuts the circle.

Substitute $x = -1$ in

$x^2 + y^2 + x - 3y = 0$

We get

$(-1)^2 + y^2 + (-1) - 3y = 0$

$1 + y^2 - 1 - 3y = 0$

$y^2 - 3y = 0$

$y(y-3)=0$

So,

$y=0 \quad \text{or} \quad y=3$

Hence the two points are

$R(-1,0), \quad S(-1,3)$

Now midpoint of RS is

$(\alpha,\beta)=\left(\frac{-1+(-1)}{2},\frac{0+3}{2}\right)$

$(\alpha,\beta)=\left(-1,\frac{3}{2}\right)$

Therefore,

$\alpha+\beta=-1+\frac{3}{2}=\frac{1}{2}$

So,

$6(\alpha+\beta)=6\cdot\frac{1}{2}=3$

Hence the correct answer is

$\boxed{3}$

So, the correct option is **Option B**.

Question50

Let P be a moving point on the circle $x^2+y^2-6x-8y+21=0$. Then, the maximum distance of P from the vertex of the parabola $x^2+6x+y+13=0$ is equal to:

JEE Main 2026 (Online) 5th April Morning Shift

Options:

A.

8

B.

10

C.

12

D.

9

Answer: C

Solution:

First, find the vertex of the parabola.

The parabola is

$x^2+6x+y+13=0$

Rewrite it by completing the square:

$x^2+6x+y+13=0$

$x^2+6x+9+y+13-9=0$

$(x+3)^2+y+4=0$

$y=-(x+3)^2-4$

So, the vertex of the parabola is

$(-3, -4)$

Now, find the centre and radius of the circle.

Given circle:

$x^2 + y^2 - 6x - 8y + 21 = 0$

Complete the squares:

$x^2 - 6x + y^2 - 8y + 21 = 0$

$(x^2 - 6x + 9) + (y^2 - 8y + 16) + 21 - 9 - 16 = 0$

$(x - 3)^2 + (y - 4)^2 - 4 = 0$

$(x - 3)^2 + (y - 4)^2 = 4$

Hence, centre of the circle is

$(3, 4)$

and radius is

$r = 2$

Now we need the maximum distance of a point P on this circle from the vertex $(-3, -4)$.

Let $V = (-3, -4)$ and centre of circle be $C = (3, 4)$.

First find VC:

$VC = \sqrt{(3+3)^2 + (4+4)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

For a point moving on a circle, the maximum distance from an external point is

$VC + r$

So,

Maximum distance $= 10 + 2 = 12$

Therefore, the correct answer is

$\boxed{12}$

So, **Option C** is correct.

Question51

Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S , then the maximum value of $(\mathrm{PR} + \mathrm{PS})^2$ is :

JEE Main 2026 (Online) 5th April Evening Shift

Options:

A.

10

B.

20

C.

25

D.

5

Answer: B

Solution:

Vertex of the parabola

$y = x^2 - 6x + 12$

is found by completing the square:

$y = (x-3)^2 + 3$

So, the vertex is

$P = (3, 3)$

Now the circle is

$x^2 + y^2 - 2x - 4y + 3 = 0$

Complete squares:

$(x-1)^2 + (y-2)^2 = 2$

Hence, the centre of the circle is

$C = (1, 2)$

and radius is

$r = \sqrt{2}$

Now,

$PC = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$

Since the line through P cuts the circle at R and S, the line must intersect the circle in a chord RS.

Let the perpendicular distance from the centre C to this line be d.

Then chord length

$RS = 2\sqrt{r^2 - d^2} = 2\sqrt{2 - d^2}$

Also, since P lies on the line outside the circle, and R, S are the intersection points on opposite sides along the line, we have

$PR + PS = RS$

only if P lies between them, which is not possible here because P is outside the circle.

So let us use power of a point.

If PR and PS are the two secant segments from P, then

$PR \cdot PS = PC^2 - r^2 = 5 - 2 = 3$

Let the line make an angle such that the foot from C to the line gives distance d.

If M is the midpoint of chord RS, then

$CM = d, \qquad PM = \sqrt{PC^2 - d^2} = \sqrt{5 - d^2}$

because in right triangle with hypotenuse PC.

Also

$MR = MS = \sqrt{r^2 - d^2} = \sqrt{2 - d^2}$

Since P is outside the circle, along the secant line,

$PR = PM - MR, \qquad PS = PM + MR$

Therefore,

$PR + PS = 2PM = 2\sqrt{5 - d^2}$

So

$(PR + PS)^2 = 4(5 - d^2)$

To maximize this, we must minimize d^2.

The minimum possible value of d is 0, when the line passes through the centre C.

Hence maximum value is

$(PR + PS)^2_{\max} = 4 \times 5 = 20$

Therefore, the required maximum value is

$\boxed{20}$

So the correct option is **B**.

Question52

Let C be a circle having centre in the first quadrant and touching the x-axis at a distance of 3 units from the origin. If the circle C has an intercept of length $6\sqrt{3}$ on y-axis, then the length of the chord of the circle C on the line $x - y = 3$ is :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

8

B.

6

C.

$6\sqrt{2}$

D.

$8\sqrt{2}$

Answer: C

Solution:

Since the circle touches the x-axis at a point which is 3 units from the origin, the point of contact is $(3, 0)$.

The centre lies in the first quadrant and the radius is perpendicular to the tangent at the point of contact. So, the centre must be of the form $(3, r)$, where r is the radius.

The circle has an intercept of length $6\sqrt{3}$ on the y-axis. This means the chord cut by the line $x = 0$ has length $6\sqrt{3}$.

For a circle of radius r, if the distance of a chord from the centre is d, then chord length is $2\sqrt{r^2 - d^2}$.

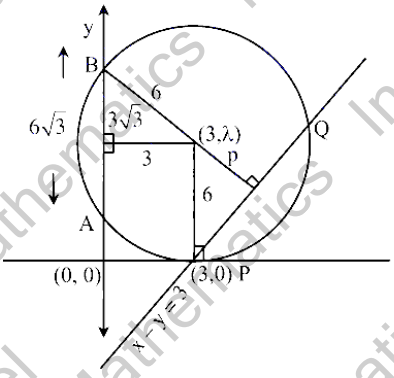
Here, the distance from centre $(3, r)$ to the y-axis $(x = 0)$ is $d = 3$. So the intercept on the y-axis is

$2\sqrt{r^2 - 3^2} = 6\sqrt{3}$

So,

$\sqrt{r^2 - 9} = 3\sqrt{3} \Rightarrow r^2 - 9 = 27 \Rightarrow r^2 = 36 \Rightarrow r = 6$

Hence, centre of the circle is $(3, 6)$ and radius is 6 units.



Now we need the length of the chord cut by the line $x - y = 3$.

First find the perpendicular distance p from the centre $(3,6)$ to the line $x-y-3=0$:

$p=\frac{|1\cdot 3+(-1)\cdot 6-3|}{\sqrt{1^2+(-1)^2}}=\frac{|3-6-3|}{\sqrt{2}}=\frac{6}{\sqrt{2}}$

Now chord length on this line is

$PQ=2\sqrt{6^2-p^2}\quad\therefore p=\frac{6}{\sqrt{2}}$

$$\text{p}=\frac{|3-6-3|}{\sqrt{2}}$$

$$\text{PQ}=2\sqrt{36-\frac{36}{2}}$$

$$=2\times\frac{6}{\sqrt{2}}=6\sqrt{2}$$

Question53

If P is a point on the circle $x^2+y^2=4, Q$ is a point on the straight line $5x+y+2=0$ and $x-y+1=0$ is the perpendicular bisector of PQ , then 13 times the sum of abscissa of all such points P is ____ .

JEE Main 2026 (Online) 21st January Evening Shift

Answer: 2

Solution:

$P\,(x_1y_1)$ and point $Q\,(x_2,y_2)$

Mid point of PQ$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Substitute M into $x-y+1=0$

$x_1+x_2-y_1-y_2+2=0..(i)$

Slope of PQ is Perpendicular to slope of bisector line So, slope of $PQ=-1$

$y_2=x_1-x_2+y_1.....(ii)$

$Q\,(x_2,y_2)$ lie on $5x+y+2=0$

So, $5x_2+y_2+2=0$ ..... (iii)

Substitute (iii) in (i)

$x_2=\frac{-x_1-y_1-2}{4}....(iv)$

Substitute (iii) in (ii)

$x_2=y_1-1.....(v)$

From (iv) and (v)

$x_1=2-5y$

(x_1,y_1) lie on circle

$x_1^2+y_1^2=4$

Pt $x_1=2-5y_1$

$y_1=0,-\frac{10}{13}$

So, $x_1=2,\frac{-24}{13}$

So, $2+\left(-\frac{24}{13}\right)=\frac{2}{13}$

So, $13\times\frac{2}{13}=2$

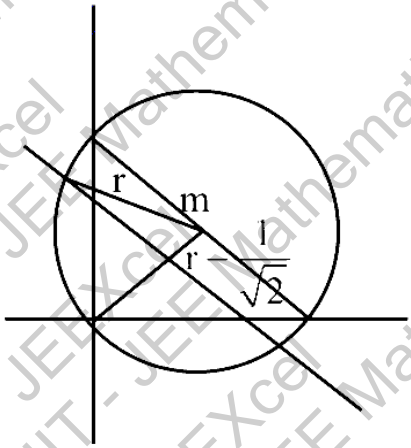
Question54

Let a circle C have its centre in the first quadrant, intersect the coordinate axes at exactly three points and cut off equal intercepts from the coordinate axes. If the length of the chord of C on the line $x + y = 1$ is $\sqrt{14}$, then the square of the radius of C is _____.

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Answer: 8

Solution:



$$r^2 = \left(\frac{\sqrt{14}}{2}\right)^2 + \left(r - \frac{1}{\sqrt{2}}\right)^2$$

$$r^2 = \frac{7}{2} + r^2 - \sqrt{2}r + \frac{1}{2}$$

$$\sqrt{2}r = 4$$

$$r = 2\sqrt{2}$$

$$r^2 = 8$$

Question55

Let the centre of the circle $x^2 + y^2 + 2\,gx + 2fy + 25 = 0$ be in the first quadrant and lie on the line $2x - y = 4$. Let the area of an equilateral triangle inscribed in the circle be $27\sqrt{3}$. Then the square of the length of the chord of the circle on the line $x = 1$ is ____.

JEE Main 2026 (Online) 6th April Morning Shift

Answer: 80

Solution:

For the circle

$x^2 + y^2 + 2gx + 2fy + 25 = 0,$

the centre is

$(-g, -f)$

and radius is

$r = \sqrt{g^2 + f^2 - 25}$.

We are given that the centre lies in the first quadrant, so

$(-g, -f)$

is in the first quadrant.

Also, the centre lies on the line

$2x - y = 4$.

So if the centre is $(-g, -f)$, then

$2(-g) - (-f) = 4$

that is,

$-2g + f = 4$.

Now use the information about the equilateral triangle inscribed in the circle.

For an equilateral triangle of side a,

Area $= \frac{\sqrt{3}}{4}a^2$.

Given area is

$27\sqrt{3}$.

So,

$\frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

which gives

$a^2 = 108 \quad \Rightarrow \quad a = 6\sqrt{3}$.

For an equilateral triangle inscribed in a circle, the circumradius is

$R = \frac{a}{\sqrt{3}}$.

Hence,

$r = \frac{6\sqrt{3}}{\sqrt{3}} = 6$.

Therefore,

$r^2 = 36$.

So,

$g^2 + f^2 - 25 = 36$

which gives

$g^2 + f^2 = 61$.

Now solve the system

$-2g + f = 4$

and

$g^2 + f^2 = 61$.

From the first equation,

$f = 4 + 2g$.

Substitute into the second:

$g^2 + (4 + 2g)^2 = 61$

$g^2 + 16 + 16g + 4g^2 = 61$

$5g^2 + 16g - 45 = 0$.

Solve:

$5g^2 + 16g - 45 = 0$.

Discriminant:

$\Delta = 16^2 - 4 \cdot 5 \cdot (-45) = 256 + 900 = 1156 = 34^2$.

Thus,

$g = \frac{-16 \pm 34}{10}$.

So,

$g = \frac{18}{10} = \frac{9}{5}$ or $g = \frac{-50}{10} = -5$.

Now centre is $(-g, -f)$ and must be in the first quadrant.

If $g = \frac{9}{5}$, then

$f = 4 + 2 \cdot \frac{9}{5} = \frac{38}{5}$,

so centre is

$(-\frac{9}{5}, -\frac{38}{5})$,

not in first quadrant.

Hence reject.

If $g = -5$, then

$f = 4 + 2(-5) = -6$.

So centre is

$(5, 6)$,

which is in the first quadrant.

Thus the circle is

$x^2 + y^2 - 10x - 12y + 25 = 0$,

with centre $(5, 6)$ and radius 6.

Now we need the chord cut by the line

$x = 1$.

Distance of the line $x = 1$ from the centre $(5, 6)$ is

$|5 - 1| = 4$.

If a chord is at distance d from the centre in a circle of radius r, then chord length is

$2\sqrt{r^2 - d^2}$.

So here chord length is

$2\sqrt{6^2 - 4^2} = 2\sqrt{36 - 16} = 2\sqrt{20} = 4\sqrt{5}$.

Therefore, the square of the chord length is

$(4\sqrt{5})^2 = 80$.

Hence, the required answer is

$\boxed{80}$.

Question56

Let the line $x - y = 4$ intersect the circle $\mathrm{C} : (x - 4)^2 + (y + 3)^2 = 9$ at the points Q and R . If $\mathrm{P}(\alpha, \beta)$ is a point on C such that $\mathrm{PQ} = \mathrm{PR}$, then $(6\alpha + 8\beta)^2$ is equal to ____ .

JEE Main 2026 (Online) 6th April Evening Shift

Answer: 18

Solution:

Since $PQ = PR$, the point P must lie on the perpendicular bisector of the chord QR.

Now, the points Q and R are the intersection points of the line

$x - y = 4$

with the circle

$(x-4)^2 + (y+3)^2 = 9.$

So, QR is a chord of the circle lying on the line $x - y = 4$.

For a circle, the perpendicular bisector of any chord passes through the centre.

The centre of the circle is

$(4, -3)$.

The slope of the line $x - y = 4$ is 1, so the slope of the perpendicular bisector is -1.

Hence the perpendicular bisector of chord QR passing through $(4, -3)$ is

$y + 3 = -1(x - 4)$

$y + 3 = -x + 4$

$x + y = 1.$

Therefore, point $P(\alpha, \beta)$ lies on both:

1. the circle:

$(\alpha - 4)^2 + (\beta + 3)^2 = 9$

2. the line:

$\alpha + \beta = 1.$

Now we need $(6\alpha + 8\beta)^2$.

Using $\alpha + \beta = 1$, let

$\beta = 1 - \alpha.$

Substitute into the circle:

$(\alpha - 4)^2 + ((1 - \alpha) + 3)^2 = 9$

$(\alpha - 4)^2 + (4 - \alpha)^2 = 9$

Since $(4 - \alpha)^2 = (\alpha - 4)^2$,

$2(\alpha - 4)^2 = 9$

$(\alpha - 4)^2 = \frac{9}{2}.$

Now,

$6\alpha + 8\beta = 6\alpha + 8(1 - \alpha) = 8 - 2\alpha.$

So,

$(6\alpha + 8\beta)^2 = (8 - 2\alpha)^2 = 4(\alpha - 4)^2.$

Using $(\alpha - 4)^2 = \frac{9}{2}$,

$(6\alpha + 8\beta)^2 = 4 \cdot \frac{9}{2} = 18.$

Hence, the required value is

$\boxed{18}$.

Question57

Consider the circle C : $x^2 + y^2 - 6x - 8y - 11 = 0$. Let a variable chord AB of the circle C subtend a right angle at the origin. If the locus of the foot of the perpendicular drawn from the origin on the chord AB is the circle $x^2 + y^2 - \alpha x - \beta y - \gamma = 0$, then $\alpha + \beta + 2\gamma$ is equal to ____ .

JEE Main 2026 (Online) 8th April Evening Shift

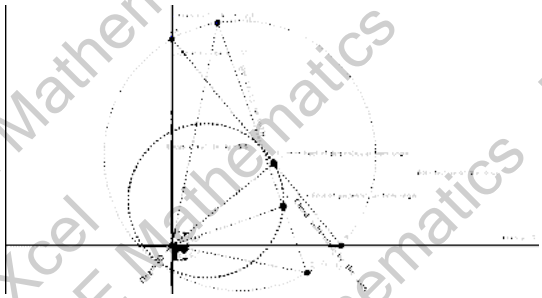
Answer: 18

Solution:

Assumptions :

1. $\boldsymbol{F}(h,k)$ is the foot of the $\perp$ drop from origin on the variable chord $\boldsymbol{AB}$
2. The end-points of the chord $\boldsymbol{AB}$ are

$\boldsymbol{A}$	$\boldsymbol{B}$
$(\boldsymbol{x}_1,\boldsymbol{y}_1)$	$(\boldsymbol{x}_2,\boldsymbol{y}_2)$



Circle	Equation	Transform	Centre	Radius
$\boldsymbol{C}$	$\boldsymbol{x}^2+\boldsymbol{y}^2-6\boldsymbol{x}-8\boldsymbol{y}-11=0$	$(\boldsymbol{x}-3)^2+(\boldsymbol{y}-4)^2=36$	$(3,4)$	6

We will now form the line equations of the $\perp$ drop $\boldsymbol{OF}$ and the variable chord $\boldsymbol{AB}$:

$l_{OF}: y=\left(\frac{k}{h}\right)x \Rightarrow l_{AB}: y=-\left(\frac{h}{k}\right)x+c \ldots . (i)$

The foot of the $\perp$ on line $\boldsymbol{l_{AB}}$ defined in equation (i) is $\boldsymbol{F}(\boldsymbol{h},\boldsymbol{k})$:

$k=-\left(\frac{h}{k}\right)\cdot(\boldsymbol{h})+c \Rightarrow c=\frac{h^2+k^2}{k} \Rightarrow l_{AB}: \boldsymbol{hx}+\boldsymbol{ky}=h^2+k^2 \ldots \ldots (ii)$

Equation (ii) which gives the equation of the variable chord $\boldsymbol{AB}$ cuts the circle at $\boldsymbol{A}(\boldsymbol{x}_1,\boldsymbol{y}_1)$ and $\boldsymbol{B}(\boldsymbol{x}_2,\boldsymbol{y}_2)$ is substituted in the equation of the circle $\boldsymbol{C}$ from chart 1 to form 2 quadratics in $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively :

Quadratic in x :

- $\boldsymbol{x}^2+\boldsymbol{y}^2-6\boldsymbol{x}-8\boldsymbol{y}-11=0 \Rightarrow \boldsymbol{k}^2\boldsymbol{x}^2+\boldsymbol{k}^2\boldsymbol{y}^2-6\boldsymbol{k}^2\boldsymbol{x}-8\boldsymbol{k}^2\boldsymbol{y}-11\boldsymbol{k}^2=0$
- $\boldsymbol{k}^2\boldsymbol{x}^2+\left(\boldsymbol{h}^2+\boldsymbol{k}^2-\boldsymbol{hx}\right)^2-6\boldsymbol{k}^2\boldsymbol{x}-8\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2-\boldsymbol{hx}\right)-11\boldsymbol{k}^2=0$
- $\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)\boldsymbol{x}^2-2\boldsymbol{x}\left\{\boldsymbol{h}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)+3\boldsymbol{k}^2-4\boldsymbol{hk}\right\}+\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-8\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{k}^2=0 \leftarrow$ Quadratic in $\boldsymbol{x}$
- $\boldsymbol{x}_1\boldsymbol{x}_2=\frac{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-8\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{k}^2}{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)} \ldots .$ (iii) where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the roots of the quadratic

Quadratic in y :

- $\boldsymbol{x}^2+\boldsymbol{y}^2-6\boldsymbol{x}-8\boldsymbol{y}-11=0 \Rightarrow \boldsymbol{h}^2\boldsymbol{x}^2+\boldsymbol{h}^2\boldsymbol{y}^2-6\boldsymbol{h}^2\boldsymbol{x}-8\boldsymbol{h}^2\boldsymbol{y}-11\boldsymbol{h}^2=0$
- $\left(\boldsymbol{h}^2+\boldsymbol{k}^2-\boldsymbol{ky}\right)^2+\boldsymbol{h}^2\boldsymbol{y}^2-6\boldsymbol{h}\left(\boldsymbol{h}^2+\boldsymbol{k}^2-\boldsymbol{ky}\right)-8\boldsymbol{h}^2\boldsymbol{y}-11\boldsymbol{h}^2=0$
- $\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)\boldsymbol{y}^2-2\boldsymbol{y}\left\{\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)+4\boldsymbol{h}^2-3\boldsymbol{hk}\right\}+\left(\boldsymbol{h}^2+k^2\right)^2-6\boldsymbol{h}\left(\boldsymbol{h}^2+k^2\right)-11\boldsymbol{h}^2=0$ Quadratic in $\boldsymbol{y}$
- $\boldsymbol{y}_1\boldsymbol{y}_2=\frac{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-6\boldsymbol{h}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{h}^2}{\boldsymbol{h}^2+\boldsymbol{k}^2} \ldots . (\boldsymbol{iv})$ where $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are the roots of the quadratic

The variable chord $\boldsymbol{AB}$ with end-points $\boldsymbol{A}(\boldsymbol{x}_1,\boldsymbol{y}_1)$ and $\boldsymbol{B}(\boldsymbol{x}_2,\boldsymbol{y}_2)$ subtend a right $\measuredangle$ at origin.

We now infer from equations (iii) and (iv) :

$\left(\frac{y_1}{x_1}\right)\cdot\left(\frac{y_2}{x_2}\right)=-1$

$\Rightarrow \frac{\left\{\frac{\left(h^2+k^2\right)^2-6h\left(h^2+k^2\right)-11h^2}{h^2+k^2}\right\}}{\left\{\frac{\left(h^2+k^2\right)^2-8k\left(h^2+k^2\right)-11k^2}{\left(h^2+k^2\right)}\right\}}=-1$

$\Rightarrow \frac{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-6\boldsymbol{h}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{h}^2}{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-8\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{k}^2}=-1$

$\Rightarrow \left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-6\boldsymbol{h}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{h}^2=-\left\{\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-8\boldsymbol{k}\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)-11\boldsymbol{k}^2\right\}$

- $\Rightarrow 2\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)^2-\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)(6\boldsymbol{h}+8\boldsymbol{k})-11\left(\boldsymbol{h}^2+\boldsymbol{k}^2\right)=0$
- $\Rightarrow 2\left(h^2+k^2\right)\cdot\left(h^2+k^2-3h-4k-\frac{11}{2}\right)=0$

- $\Rightarrow h^2+k^2-3h-4k-\frac{11}{2}=0 \leftarrow h^2+k^2 \neq 0$[$\because F(h,k)$ is not the origin]
- $\Rightarrow x^2+y^2-3x-4y-\frac{11}{2}=0 \ldots\ldots (v) \leftarrow$ Locus of $F(h,k)$

The circle $C: x^2+y^2-\alpha x-\beta y-\lambda=0$ be the locus of $F(h,k)$ which must be equivalent to equation (v) :

$\therefore x^2+y^2-\alpha x-\beta y-\lambda \equiv x^2+y^2-3x-4y-\frac{11}{2}$

$\Rightarrow$

α	β	λ	2λ	$\alpha+\beta+2\lambda$
3	4	$\frac{11}{2}$	11	18

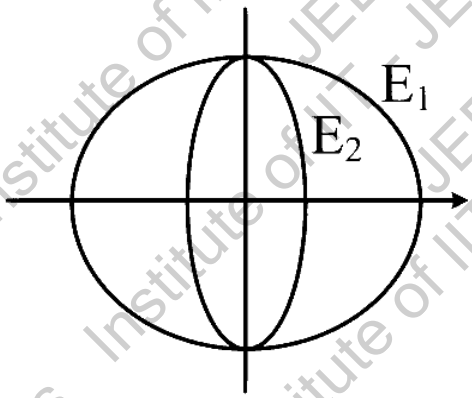
Question58

Let $\mathrm{E}_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ be an ellipse. Ellipses E_i 's are constructed such that their centres and eccentricities are same as that of E_1, and the length of minor axis of E_i is the length of major axis of $E_{i+1}(i \geq 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$, is equal to _____.

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Answer: 54

Solution:



$E_1=\frac{x^2}{9}+\frac{y^2}{4} \Rightarrow e=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$

$E_2: \frac{x^2}{a^2}+\frac{y^2}{4}=1$

$e=\frac{\sqrt{5}}{3}=\sqrt{1-\frac{a^2}{4}} \Rightarrow \frac{5}{9}=1-\frac{a^2}{4}$

$a^2=\frac{16}{9}$

$E_2: \frac{x^2}{16}+\frac{y^2}{4}=1$

$E_3: \frac{x^2}{\frac{16}{9}}+\frac{y^2}{b^2}=1$

$e=\frac{\sqrt{5}}{3}=\sqrt{1-\frac{b^2}{16}} \Rightarrow b^2=\frac{64}{81}$

$$E_3 = \frac{x^2}{\frac{16}{9}} + \frac{y^2}{\frac{64}{81}} = 1$$

$$A_1 = \pi \times 3 \times 2 \Rightarrow 6\pi$$

$$A_2 = \pi \times \frac{4}{3} \times 2 = \frac{8\pi}{3}$$

$$A_3 = \pi \times \frac{4}{3} \times \frac{8}{9} = \frac{32\pi}{81}$$

$$\sum_{i=1}^{\infty} A_i = 6\pi + \frac{8\pi}{3} + \frac{32\pi}{81} + \ldots \infty \Rightarrow \frac{6\pi}{1 - \frac{4}{9}} \Rightarrow \frac{54\pi}{5}$$

$$\therefore \frac{5}{\pi} \sum_{i=1}^{\infty} A_i \Rightarrow \frac{5}{\pi} \times \frac{54\pi}{5} = 54$$

Question59

Let $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $H : \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a - A = 2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to :

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Options:

A. 10

B. 7

C. 9

D. 8

Answer: D

Solution:

We are given an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b,$

and a hyperbola

$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1.$

It is stated that "the distance between the foci of E and the foci of H is $2\sqrt{3}$." A natural interpretation is that the foci of the ellipse are separated by

$2\sqrt{a^2 - b^2},$

and those of the hyperbola by

$2\sqrt{A^2 + B^2},$

and each distance is equal to $2\sqrt{3}$. That is, we have

$2\sqrt{a^2 - b^2} = 2\sqrt{3} \quad \Longrightarrow \quad a^2 - b^2 = 3,$

and

$2\sqrt{A^2 + B^2} = 2\sqrt{3} \quad \Longrightarrow \quad A^2 + B^2 = 3.$

We are also given that

$a - A = 2,$

and that the ratio of the eccentricities is

$\frac{e_E}{e_H} = \frac{1}{3},$

where the eccentricity of the ellipse is

$e_E = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{3}}{a},$

and the eccentricity of the hyperbola is

$e_H = \frac{\sqrt{A^2+B^2}}{A} = \frac{\sqrt{3}}{A}$.

Thus the ratio becomes

$\frac{e_E}{e_H} = \frac{\sqrt{3}/a}{\sqrt{3}/A} = \frac{A}{a} = \frac{1}{3}$.

This implies

$a = 3A$.

Now, using the condition $a - A = 2$ together with $a = 3A$, we get

$3A - A = 2 \implies 2A = 2 \implies A = 1$.

Thus,

$a = 3$.

Next, for the ellipse we have

$a^2 - b^2 = 3 \implies 9 - b^2 = 3 \implies b^2 = 6$.

For the hyperbola,

$A^2 + B^2 = 3 \implies 1 + B^2 = 3 \implies B^2 = 2$.

The length of the latus rectum is given by the following formulas:

For the ellipse:

$L_E = \frac{2b^2}{a}$,

For the hyperbola:

$L_H = \frac{2B^2}{A}$.

Substitute the computed values:

For the ellipse:

$L_E = \frac{2\times 6}{3} = \frac{12}{3} = 4$,

For the hyperbola:

$L_H = \frac{2\times 2}{1} = 4$.

The sum of the lengths of the latus rectums is then

$L_E + L_H = 4 + 4 = 8$.

Thus, the answer is

8.

Question60

The length of the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is :

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Options:

A. $\frac{2}{3}\sqrt{15}$

B. $\frac{1}{3}\sqrt{15}$

C. $\sqrt{15}$

D. $\frac{5}{3}\sqrt{15}$

Answer: A

Solution:

$T = S_1$

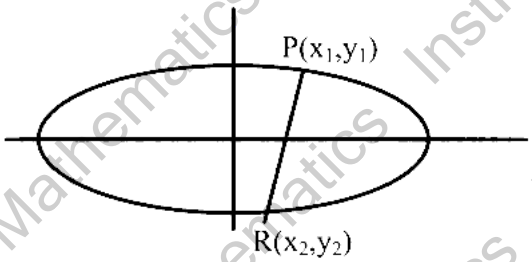
$\frac{x \cdot 1}{4} + \frac{y \cdot 1/2}{2} = \frac{1}{4} + \frac{1}{8}$

$x + y = \frac{3}{2}$

solve with ellipse

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{2}\,|x_2 - x_1|$



$y_2 = \frac{3}{2} - x_2$

$y_1 = \frac{3}{2} - x_1$

$y_2 - y_1 = x_2 - x_1$

$x^2 + 2y^2 = 4$

$x^2 + 2\left(\frac{3}{2} - x\right)^2 = 4$

$6x^2 - 12x + 1 = 0$

$x_1 + x_2 = 2$

$x_1 x_2 = 1/6$

$|x_2 - x_1| = \sqrt{(x_2 + x_1)^2 - 4x_1x_2}$

$= \sqrt{4 - 4/6}$

$PR = \sqrt{2} \cdot 2 \cdot \frac{\sqrt{5}}{\sqrt{2}\sqrt{3}} = \frac{2}{3}\sqrt{15}$

Question61

Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

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Options:

A. $\frac{1-2\sqrt{2}}{\sqrt{3}}$

B. $\frac{1-\sqrt{3}}{\sqrt{2}}$

C. $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

D. $\frac{3-2\sqrt{2}}{3\sqrt{2}}$

Answer: C

Solution:

Product of focal distances $= (a + ex_1)(a - ex_1)$
$= a^2 - e^2x_1^2 = a^2 - e^2(3)$
$= a^2 - 3e^2 = \frac{7}{4} \Rightarrow a^2 = \frac{7}{4} + 3e^2$
$\Rightarrow 4a^2 = 7 + 12e^2$
& $\left(\sqrt{3}, \frac{1}{2}\right)$ lines on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$\therefore \frac{3}{a^2} + \frac{1}{4b^2} = 1$
$\frac{3}{a^2} + \frac{1}{4(a^2)(1-e^2)} = 1$
$12(1-e^2) + 1 = 4a^2(1-e^2)$
$13 - 12e^2 = (7 + 12e^2)(1-e^2)$
$\Rightarrow 13 - 12e^2 = 7 - 7e^2 + 12e^2 - 12e^4$
$\Rightarrow 12e^4 - 17e^2 + 6 = 0$
$\therefore e^2 = \frac{17 \pm \sqrt{289 - 288}}{24} = \frac{17 \pm 1}{24} = \frac{3}{4} \& \frac{2}{3}$
$\therefore e = \frac{\sqrt{3}}{2} \& \sqrt{\frac{2}{3}}$
$\therefore$ difference $== \frac{\sqrt{3}}{2} - \sqrt{\frac{2}{3}} = \frac{3 - 2\sqrt{2}}{2\sqrt{3}}$

Question62

The equation of the chord, of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid-point is $(3, 1)$ is :

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Options:

A. $5x + 16y = 31$

B. $48x + 25y = 169$

C. $4x + 122y = 134$

D. $25x + 101y = 176$

Answer: B

Solution:

Equation of chord with given middle point
$T = S_1$
$\Rightarrow \frac{3x}{25} + \frac{y}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$
$48x + 25y = 144 + 25$
$48x + 25y = 169$ Ans.

Question63

If the midpoint of a chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $(\sqrt{2}, 4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

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Options:

A.

B.

18

C.

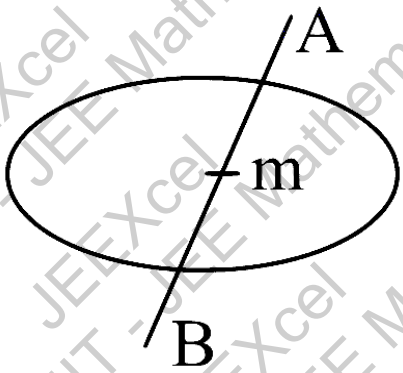
22

D.

20

Answer: C

Solution:



If m $\left(\sqrt{2}, \frac{4}{3}\right)$ than equation of of AB is

$T = S_1$

$\frac{x\sqrt{2}}{9} + \frac{y}{4}\left(\frac{4}{3}\right) = \frac{(\sqrt{2})^2}{9} + \frac{\left(\frac{4}{3}\right)^2}{4}$

$\frac{\sqrt{2}x}{9} + \frac{y}{3} = \frac{2}{9} + \frac{4}{9}$

$\sqrt{2}x + 3y = 6 \Rightarrow y = \frac{6 - \sqrt{2}x}{3}$ put in ellipse

So, $\frac{x^2}{9} + \frac{(6 - \sqrt{2}x)^2}{9 \times 4} = 1$

$4x^2 + 36 + 2x^2 - 12\sqrt{2}x = 36$

$6x^2 - 12\sqrt{2}x = 0$

$6x(x - 2\sqrt{2}) = 0$

$x = 0 \& x = 2\sqrt{2}$

So $y = 2 \quad y = \frac{2}{3}$

Length of chord $= \sqrt{(2\sqrt{2} - 0)^2 + \left(\frac{2}{3} - 2\right)^2}$

$= \sqrt{8 + \frac{16}{9}}$

$= \sqrt{\frac{88}{9}} = \frac{2}{3}\sqrt{22}$ so $\alpha = 22$

Question64

Let the ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$ and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals :

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Options:

A.

$\frac{24\sqrt{6}}{5}$

B.

$\frac{18\sqrt{6}}{5}$

C.

$6\sqrt{6}$

D.

$\frac{12\sqrt{6}}{5}$

Answer: A

Solution:

$2ae = 4$

$2a\left(\frac{1}{\sqrt{3}}\right) = 4$

$\Rightarrow a = 2\sqrt{3}$

$\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

Now $\frac{2b^2}{a} \cdot \frac{2A^2}{B} = \frac{32}{\sqrt{3}} \Rightarrow 2\left(\frac{8}{2\sqrt{3}}\right)\frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B^2} = \frac{1}{3} \Rightarrow 1 - \frac{2B}{B^2} = \frac{1}{3} \Rightarrow B = 3$

$\Rightarrow A^2 = 6$

$\frac{x^2}{12} + \frac{y^2}{8} = 1 \ldots\ldots (1)$

$\frac{x^2}{6} + \frac{y^2}{9} = 1 \ldots\ldots (2)$

On solving (1) & (2) we get

$(x, y) \equiv \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

The four points are vertices of rectangle and its area $=$

$\frac{24\sqrt{6}}{5}$

Question65

If $\alpha x + \beta y = 109$ is the equation of the chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, whose mid point is $\left(\frac{5}{2}, \frac{1}{2}\right)$. then $\alpha + \beta$ is equal to :

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Options:

A.

37

B.

46

C.

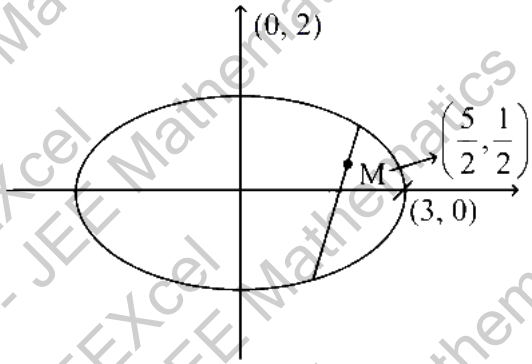
72

D.

58

Answer: D

Solution:



Equation of chord $T = S_1$

$\frac{5}{2}\left(\frac{x}{9}\right) + \frac{1}{2}\left(\frac{y}{4}\right) = \frac{25}{36} + \frac{1}{16}$

$\Rightarrow \frac{5x}{18} + \frac{y}{8} = \frac{100+9}{144} = \frac{109}{144}$

$\Rightarrow 40x + 18y = 109$

$\Rightarrow \alpha = 40, \beta = 18$

$\Rightarrow \alpha + \beta = 58$

Question66

If S and S' are the foci of the ellipse $\frac{x^2}{18} + \frac{y^2}{9} = 1$ and P be a point on the ellipse, then $\min(SP \cdot S'P) + \max(SP \cdot S'P)$ is equal to :

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Options:

A. $3(6 + \sqrt{2})$

B. $3(1 + \sqrt{2})$

C. 27

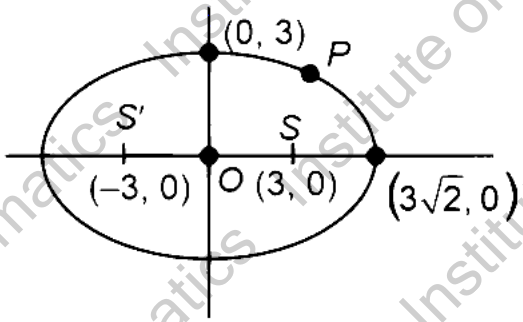
D. 9

Answer: C

Solution:

$a = 3\sqrt{2}, b = 3$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$



$PS \cdot PS' = 2a = 6\sqrt{2}$

$\frac{PS + PS'}{2} \geq \sqrt{PS \cdot PS'}$

$\Rightarrow (PS \times PS') \max = 18$

Minima happens when P lies on major axis

$\Rightarrow \quad P = (3\sqrt{2}, 0)$

$PS = (3\sqrt{2} - 3) \cdot PS' = (3\sqrt{2} + 3)$

$(PS \cdot PS') \min = 9$

$(PS \cdot PS') \min + (PS \cdot PS') \max = 27$

Option (1)

Question67

If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

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Options:

A. $\frac{3}{\sqrt{19}}$

B. $\frac{\sqrt{3}}{16}$

C. $\frac{4}{\sqrt{17}}$

D. $\frac{\sqrt{5}}{7}$

Answer: C

Solution:

The length of the minor axis is equal to one-fourth of the distance between the foci. Mathematically, this can be expressed as:

$2b = \frac{1}{4}(2ae)$

This simplifies to:

$b = \frac{ae}{4}$

Given that the relationship between b, a, and e is:

$\frac{b^2}{a^2} = 1 - e^2$

Substitute $b = \frac{ae}{4}$ into the equation:

$\left(\frac{ae}{4}\right)^2 = a^2 \cdot \left(1 - e^2\right)$

Expanding and simplifying gives:

$\frac{a^2e^2}{16} = a^2(1 - e^2)$

Divide both sides by a^2:

$\frac{e^2}{16} = 1 - e^2$

Rearrange and solve for e^2:

$\frac{e^2}{16} + e^2 = 1$

$\frac{17e^2}{16} = 1$

Solve for e:

$e^2 = \frac{16}{17}$

$e = \frac{4}{\sqrt{17}}$

Thus, the eccentricity of the ellipse is:

$\frac{4}{\sqrt{17}}$

Question68

A line passing through the point $P(\sqrt{5}, \sqrt{5})$ intersects the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ at A and B such that $(PA) \cdot (PB)$ is maximum. Then $5\left(PA^2 + PB^2\right)$ is equal to :

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Options:

A. 290

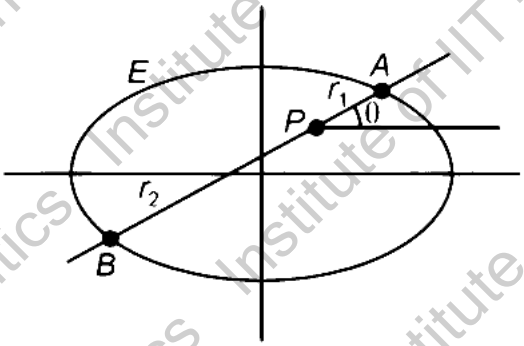
B. 377

C. 338

D. 218

Answer: C

Solution:



$\frac{x - \sqrt{5}}{\cos\theta} = \frac{y - \sqrt{5}}{\sin\theta} = r$

$x = r\cos\theta + \sqrt{5}$

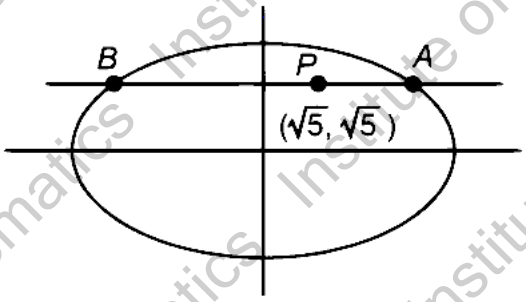
$y = r\sin\theta + \sqrt{5}$

$25x^2 + 36y^2 = 900, (x, y)$ lie on E

$r^2\left[25\cos^2\theta + 36\sin^2\theta\right] + r[50\sqrt{5}\cos\theta + 72\sqrt{5}\sin\theta]$
$+25[5] + 36[5] = 900$

$r_1 r_2 = \frac{900 - 305}{25 + 11\sin^2\theta}, (r_1 r_2)_{\max} = \frac{595}{25} = \frac{119}{5}$

$|r_1 \cdot r_2|_{\max} = \left(\frac{595}{25}\right) = \left(\frac{119}{5}\right)$ at $\theta = 0$



$\frac{x^2}{36}+\frac{y^2}{25}=1$

At $y=\sqrt{5}$

$\frac{x^2}{36}+\frac{1}{5}=1$

$\Rightarrow \frac{x^2}{36}=\frac{4}{5} \Rightarrow x=\pm\frac{12}{\sqrt{5}}$

$PA^2+PB^2=(PA+PB)^2-2PA\cdot PB=\left(\frac{24}{\sqrt{5}}\right)^2-2\cdot\frac{119}{5}$

$5\left(PA^2+PB^2\right)=5\left(\frac{24^2}{5}-\frac{2.119}{5}\right)=24^2-238$

$=338.$

Question69

Let C be the circle of minimum area enclosing the ellipse $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity $\frac{1}{2}$ and foci $(\pm 2,0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length $2a$ is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is :

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. $8(3+\sqrt{2})$

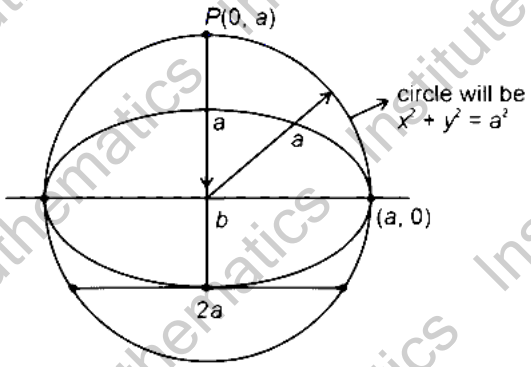
B. $8(2+\sqrt{3})$

C. $6(3+\sqrt{2})$

D. $6(2+\sqrt{3})$

Answer: B

Solution:



Area $=\frac{1}{2}\times(a+b)\cdot 2a=a(a+b)$

Since, $e=\frac{1}{2}ae=2 \Rightarrow a=4, b=2\sqrt{3}$

$\Rightarrow$ Area $=4(4+2\sqrt{3})=8(2+\sqrt{3})$

Question70

The length of the latus-rectum of the ellipse, whose foci are $(2,5)$ and $(2,-3)$ and eccentricity is $\frac{4}{5}$, is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. $\frac{50}{3}$

B. $\frac{18}{5}$

C. $\frac{6}{5}$

D. $\frac{10}{3}$

Answer: B

Solution:

To find the length of the latus rectum of the ellipse, we first recognize that the foci of the ellipse are given as $F_1 : (2,5)$ and $F_2 : (2,-3)$. This indicates that the major axis is aligned along the y-axis.

Calculate the distance between the foci:

$F_1F_2 = 8$

The formula involving the distance between the foci and the eccentricity is:

$F_1F_2 = 2be$

Here, the eccentricity e is given as $\frac{4}{5}$. Thus, we can solve for b:

$8 = 2b \cdot \frac{4}{5} \implies b = \frac{8}{2\times\frac{4}{5}} = 5$

Determine a^2:

Using the relationship between eccentricity, semi-minor axis b, and semi-major axis a:

$e^2 = 1 - \frac{a^2}{b^2} = 1 - \frac{a^2}{25} = \frac{16}{25}$

Solving for a^2:

$1 - \frac{a^2}{25} = \frac{16}{25} \implies \frac{a^2}{25} = \frac{9}{25} \implies a^2 = 9$

Thus, $a = 3$.

Compute the length of the latus rectum:

The formula for the length of the latus rectum L is:

$L = \frac{2a^2}{b}$

Substituting the known values:

$L = \frac{2\times(9)}{5} = \frac{18}{5}$

Therefore, the length of the latus rectum of the ellipse is $\frac{18}{5}$.

Question71

The centre of a circle C is at the centre of the ellipse $\mathrm{E}: \frac{x^2}{\mathrm{a}^2}+\frac{y^2}{\mathrm{b}^2}=1, \mathrm{a}>\mathrm{b}$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle $\mathrm{PF}_1 \mathrm{F}_2$ is 30 and the length of the major axis of E is 17 , then the distance between the foci of E is :

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Options:

A. 12

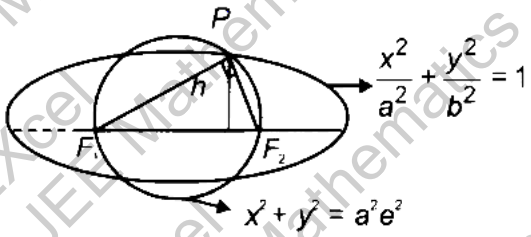
B. 26

C. 13

D. $\frac{13}{2}$

Answer: C

Solution:



$$x^2+\frac{a^2y^2}{b^2}=a^2$$

$$\Rightarrow y^2\left(1-\frac{a^2}{b^2}\right)=a^2\left(e^2-1\right)=a^2\left(1-\frac{b^2}{a^2}-1\right)$$

$$=-b^2$$

$$\Rightarrow \frac{y^2\left(b^2-a^2\right)}{b^2}=-b^2\Rightarrow y^2=\frac{b^4}{(a^2-b^2)}$$

$$\text{Height }=|y|=\frac{b^2}{\sqrt{a^2-b^2}}$$

$$\text{Area }=(2ae)\times\frac{1}{2}\times\frac{b^2}{\sqrt{a^2-b^2}}=30$$

$$=\frac{ab^2e}{a\sqrt{1-\frac{b^2}{a^2}}}=b^2, a=\frac{17}{2}$$

$$\text{Distance between foci }=2ae$$

$$=17\sqrt{1-\frac{b^2}{a^2}}=17\sqrt{1-\frac{30\times4}{289}}=13$$

Question72

Let for two distinct values of p the lines $y=x+\mathrm{p}$ touch the ellipse $\mathrm{E}:\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ at the points A and B . Let the line $y=x$ intersect E at the points C and D . Then the area of the quadrilateral $ABCD$ is equal to :

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Options:

A. 48

B. 20

C. 24

D. 36

Answer: C

Solution:

$E : \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

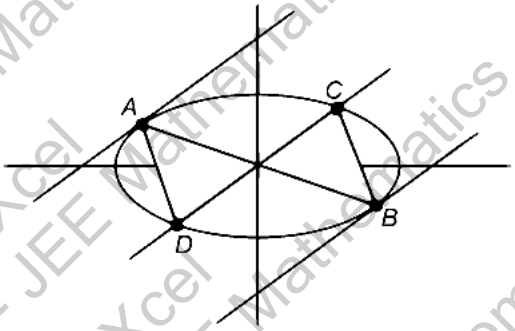
$T : y = mx \pm \sqrt{16m^2 + 9}$
$y = x + p$
$\Rightarrow m = 1$
$\Rightarrow p = \pm\sqrt{16 + 9}$
$= \pm 5$

$T : y = x \pm 5$ will to cut the E at $A\left(-\frac{16}{5}, \frac{9}{5}\right)$

$B\left(\frac{16}{5}, -\frac{9}{5}\right)$

Also, $y = x$ will cut the E at $C\left(\frac{12}{5}, \frac{12}{5}\right)$

$D\left(-\frac{12}{5}, -\frac{12}{5}\right)$



$ABCD$ in not give in cyclic order

$\therefore$ it does not form any quadrilateral

$\therefore$ No option should match

If order is not considered then

Area $= 24$ sq. unit.

Question73

Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If p + q = 126, then the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{n} = 1$ is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

$\frac{1}{\sqrt{2}}$

B.

$\frac{1}{2}$

C.

$\frac{\sqrt{7}}{4}$

D.

$\frac{3}{4}$

Answer: A

Solution:

Total trangles $\Longrightarrow = {}^{h}C_3$

Total auadrilaterals $= {}^{h}C_4 = q$

${}^{n}C_3 + {}^{n}C_4 = 126 \Rightarrow {}^{n+1}C_4 = 126$

$\Rightarrow n + 1 = 9 \Rightarrow n = 8$

$\frac{x^2}{16} + \frac{y^2}{n} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1$

$e = \sqrt{1 - \frac{8}{16}} = \sqrt{\frac{8}{16}} = \frac{1}{\sqrt{2}}$

Question74

Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be 10. If its eccentricity is the minimum value of the function $f(t) = t^2 + t + \frac{11}{12}$, $t \in \mathbb{R}$, then $a^2 + b^2$ is equal to :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

115

B.

120

C.

125

D.

126

Answer: D

Solution:

Given that the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is 10, we have:

$\frac{2b^2}{a} = 10 \quad \Rightarrow \quad 5a = b^2$

Next, consider the function $f(t) = t^2 + t + \frac{11}{12}$. To find its minimum value, we calculate the derivative:

$\frac{df(t)}{dt} = 2t + 1 = 0 \quad \Rightarrow \quad t = \frac{-1}{2}$

Plugging $t = -\frac{1}{2}$ into $f(t)$ gives the minimum value:

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + \frac{11}{12} = \frac{1}{4} - \frac{1}{2} + \frac{11}{12} = \frac{3-6+11}{12} = \frac{8}{12} = \frac{2}{3}$

Thus, the eccentricity e of the ellipse is $\frac{2}{3}$, so $e^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

Using the eccentricity formula for an ellipse:

$e^2 = \frac{1-b^2}{a^2} \quad \Rightarrow \quad \frac{4}{9} = \frac{1-b^2}{a^2}$

Rearranging gives:

$b^2 = a^2\left(1 - \frac{4}{9}\right) = a^2 \cdot \frac{5}{9}$

From equation (1), $b^2 = 5a$. Substituting, we have:

$5a = a^2 \cdot \frac{5}{9} \quad \Rightarrow \quad a^2 = 9a$

Solving for a,

$a = 9, \quad$ and therefore $b^2 = 5a = 45 \quad \Rightarrow \quad b = \sqrt{45} = 3\sqrt{5}$

Finally, calculate $a^2 + b^2$:

$a^2 + b^2 = 81 + 45 = 126$

Question75

Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

78

B.

68

C.

70

D.

74

Answer: C

Solution:

$E : \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle $(1, 2)$, radius

$r = \sqrt{1 + 4 + 11}$

$r = 4$

$\because$ E pass from centre $(1, 2)$

$\therefore \frac{3}{4} + P = 1$

$P = \frac{1}{4}$ $\quad \therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of C(h, k)

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

Question76

Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their ecentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to _____ .

JEE Main 2025 (Online) 24th January Evening Shift

Answer: 55

Solution:

$\frac{2b^2}{a} = 15\sqrt{2}$

$1 + \frac{b^2}{a^2} = \frac{5}{2}$

$a = 5\sqrt{2}$

$b = 5\sqrt{3}$

$\frac{2A^2}{B} = 12\sqrt{5}$

$2a \cdot 2B = 100\sqrt{10}$

$2.5\sqrt{2}.2B = 100\sqrt{10}$

$B = 5\sqrt{5}$

$A = 5\sqrt{6}$

$e_2^2 = 1 + \frac{A^2}{B^2}$

$= 1 + \frac{150}{125}$

$e_2^2 = 1 + \frac{30}{25}$

$25e_2^2 = 55$

Question77

Let the product of the focal distances of the point $\mathbf{P}(4, 2\sqrt{3})$ on the hyperbola $\mathrm{H} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be 32 . Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to_____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 120

Solution:

To find $p^2 + q^2$ for the hyperbola $\mathrm{H} : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, given that the product of the focal distances from a point $\mathbf{P}(4, 2\sqrt{3})$ to the foci is 32, we follow these steps:

Verify the Point on the Hyperbola:

The point $P(4, 2\sqrt{3})$ satisfies the hyperbola equation:

$\frac{16}{a^2} - \frac{12}{b^2} = 1 \quad \ldots (\mathrm{i})$

Product of Focal Distances:

The distances from P to the foci S and S' are:

$SP = e\left(4 - \frac{a}{e}\right), \quad S'P = e\left(4 + \frac{a}{e}\right)$

Therefore, the product:

$SP \cdot S'P = 16e^2 - a^2 = 32$

Relating e, a, and b:

Using the identity for eccentricity:

$e^2 = 1 + \frac{b^2}{a^2}$

Substituting in the product equation gives:

$16\left(1 + \frac{b^2}{a^2}\right) - a^2 = 32 \quad \ldots (\mathrm{ii})$

Solving for a^2 and b^2:

From equations (i) and (ii), solve for a^2 and b^2:

$a^2 = 8, \quad b^2 = 12$

Calculate $p^2 + q^2$:

The conjugate axis length is $p = 2b$ and the latus rectum is $q = \frac{2b^2}{a}$. Thus:

$p^2 + q^2 = (2b)^2 + \left(\frac{2b^2}{a}\right)^2$

Substituting $b^2 = 12$ and $a^2 = 8$ gives:

$p^2 + q^2 = 4b^2 + \frac{4b^4}{a^2} = 48 + \frac{4 \times 144}{8}$

$p^2 + q^2 = 48 \cdot \left(1 + \frac{12}{8}\right) = 120$

Therefore, $p^2 + q^2 = 120$.

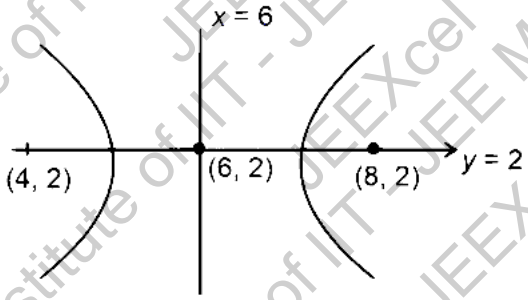
Question78

If the equation of the hyperbola with foci $(4, 2)$ and $(8, 2)$ is $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, then $\alpha + \beta + \gamma$ is equal to_____.

JEE Main 2025 (Online) 3rd April Evening Shift

Answer: 141

Solution:



Given $\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1$

$\Rightarrow b^2x^2 - a^2y^2 - 12xb^2 + 4ya^2 + 36b^2 - 4a^2 - a^2b^2 = 0$

Comparing $\frac{b^2}{a^2} = 3 \Rightarrow e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow e^2 = 4$

$\Rightarrow e = 2$

Similarly, $2ae = 4$

$\Rightarrow a = 1 \Rightarrow b = \sqrt{3}$

$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 108 - 4 - 3 = 0$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$

$\Rightarrow \alpha = 36, \beta = 4, \gamma = 101$

$\Rightarrow \alpha + \beta + \gamma = 141$

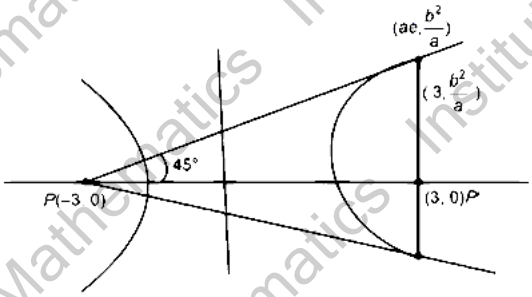
Question79

Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at $\mathrm{P}(-3, 0)$. If the latus ractum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta, \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is _____ .

JEE Main 2025 (Online) 7th April Morning Shift

Answer: 1944

Solution:



$ae = 3, \tan 45^\circ = ae = 3\frac{\frac{b^2}{a}}{6} \Rightarrow \frac{b^2}{a} = 6 \ldots$ (i) ..... (i)

$a\sqrt{\left(1+\frac{b^2}{a^2}\right)} = 3$

$a^2 + b^2 = 9$ ..... (ii)

$\Rightarrow a^2 - 6a + 9 = 0 \Rightarrow a = 3(\sqrt{2}-1)$

$\Rightarrow a^2b^2 = 9(3-2\sqrt{2}) \cdot 6 \cdot 3(\sqrt{2}-1)$

$= 162(5\sqrt{2}-7)$

$\Rightarrow \alpha = 162 \times 5, \beta = 162 \times 7$

$\Rightarrow \alpha + \beta = 162 \times 12 = 1944$

Question80

Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be $2a$ and $2b$, respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then $4p$ is equal to _____.

JEE Main 2025 (Online) 7th April Evening Shift

Answer: 189

Solution:

Given:

Transverse axis length: $2a$

Conjugate axis length: $2b$

One focus at $(-5, 0)$

Directrix given by $5x + 9 = 0$

The equations used are as follows:

Relationship between focus and directrix:

The focal length $ae = 5$

The directrix gives $\frac{a}{e} = \frac{9}{5}$

Solving these equations, we get:

$ae = 5, \quad \frac{a}{e} = \frac{9}{5}$

From $ae = 5$, we have $a = \frac{5}{e}$.

Substituting $a = 3$ and $e = \frac{5}{3}$.

Finding b:

Use the relationship $b^2 = a^2(e^2 - 1)$:

Given $a = 3$ and $e = \frac{5}{3}$, $b = 4$

Equation of the hyperbola:

The standard equation, after substituting values of a and b, is:

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

Focal distances product calculation:

For any point $(\alpha, 2\sqrt{5})$ that lies on the hyperbola:

$\frac{\alpha^2}{9} - \frac{(2\sqrt{5})^2}{16} = 1$

Solving this gives:

$\alpha^2 = 9 \times \frac{36}{16}$

Product of focal distances $PF_1 \cdot PF_2$**:**

$PF_1 \cdot PF_2 = (e\alpha - a)(e\alpha + a) = e^2\alpha^2 - a^2$

Substituting the values:

$P = e^2\alpha^2 - a^2 = \frac{25}{9} \cdot 9 \cdot \frac{9}{4} - 9 = \frac{189}{4}$

Finally, to find $4p$:

$4p = 4 \times \frac{189}{4} = 189$

Question81

Let the foci of a hyperbola be $(1, 14)$ and $(1, -12)$. If it passes through the point $(1, 6)$, then the length of its latus-rectum is :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

A. $\frac{25}{6}$

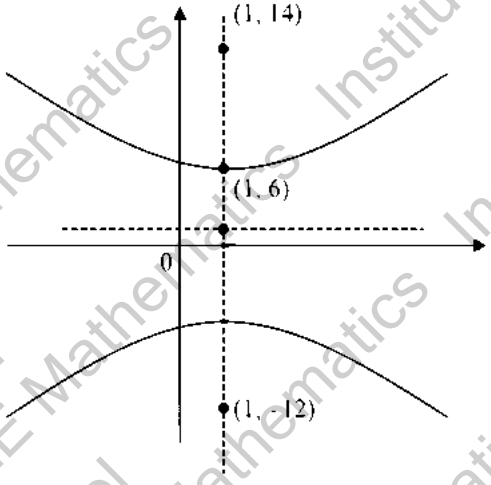
B. $\frac{144}{5}$

C. $\frac{288}{5}$

D. $\frac{24}{5}$

Answer: C

Solution:



$be = 13,\ b = 5$

$a^2 = b^2(e^2 - 1)$

$= b^2e^2 - b^2$

$= 169 - 25 = 144$

$\ell(LR) = \frac{2a^2}{b} = \frac{2 \times 144}{5} = \frac{288}{5}$

Question82

Let one focus of the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H , then $9\left(e^2 + l\right)$ is equal to :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 12

B. 14

C. 15

D. 16

Answer: D

Solution:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Directrix : $x = \frac{9}{\sqrt{10}} = \frac{a}{e}$ ..... (i)

Focus : $(\sqrt{10}, 0) \equiv (ae, 0)$

$ae = \sqrt{10}$ ..... (ii)

(i) × (ii)

$\Rightarrow a^2 = 9 \Rightarrow a = 3$

Substitute in (ii)

$e = \frac{\sqrt{10}}{3}$

Now $e^2 = 1 + \frac{b^2}{a^2}$

$\frac{10}{9} = 1 + \frac{b^2}{a}$

$\Rightarrow b = 1$

$I = \frac{2b^2}{a} = \frac{2 \times 1}{3} = \frac{2}{3}$

$a\left[e^2 + l\right] = 9\left[\frac{10}{9} + \frac{2}{3}\right] = 10 + 6$

$= 16$

Question83

Let the sum of the focal distances of the point $P(4, 3)$ on the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $8\sqrt{\frac{5}{3}}$. If for H , the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2 + 6\,m$ is equal to :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 187

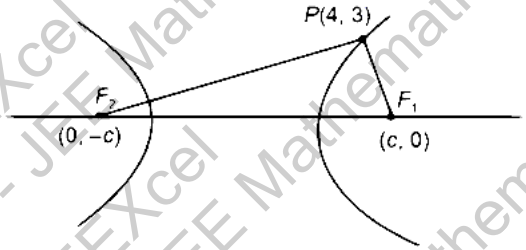
B. 184

C. 186

D. 185

Answer: D

Solution:



$\sqrt{(c+4)^2+9}+\sqrt{(c-4)^2+9}=8\sqrt{\frac{5}{3}}$

Solving, $c=\frac{5}{\sqrt{6}}=\text{al}\Rightarrow a^2\left(1+\frac{b^2}{a^2}\right)=\frac{25}{6}\Rightarrow a^2+b^2=\frac{25}{6}$

$\frac{16}{a^2}-\frac{9}{b^2}=1$

$16b^2-9d^2=a^2b^2\Rightarrow 16\left(\frac{25}{6}-a^2\right)-9a^2=9a^2b^2$

$PF_1+PF_2=8\sqrt{\frac{5}{3}}\Rightarrow a^2=\frac{5}{2},b^2=\frac{5}{3}$

$|PF_1-PF_2|=2a$

$\frac{64.5}{3}=4a^2+4m\Rightarrow m=\frac{80}{3}-a^2$

$6m=160-6a^2$

$9\ell^2=9\left(\frac{2b^2}{a}\right)^2=\frac{36b^4}{a^2}$

$9\ell^2+6m=\frac{36\left(\frac{25}{9}\right)}{\frac{5}{2}}+160-6\left(\frac{5}{2}\right)$

$=\frac{72\times 5}{9}+160-15$

$=160+40-15=185$

Question84

Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2}+\frac{y^2}{25}=1$ and the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$, respectively. If $b<5$ and $e_1e_2=1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

$\frac{4}{5}$

B.

$\frac{3}{5}$

C.

$\frac{\sqrt{7}}{4}$

D.

$\frac{\sqrt{3}}{2}$

Answer: B

Solution:

Let's find the eccentricities of the given ellipse and hyperbola, and then determine the eccentricity of an ellipse that passes through all four foci.

Step 1: Find e_1 for the Ellipse

The equation of the ellipse is:

$\frac{x^2}{b^2} + \frac{y^2}{25} = 1$

The eccentricity e_1 is given by:

$e_1^2 = 1 - \frac{b^2}{25}$

Step 2: Find e_2 for the Hyperbola

The equation of the hyperbola is:

$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$

The eccentricity e_2 is given by:

$e_2^2 = 1 + \frac{b^2}{16}$

Step 3: Using the Product $e_1e_2 = 1$

Given:

$e_1e_2 = 1$

Thus:

$\left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

Expanding gives:

$1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{400} = 1$

Simplifying:

$\frac{9b^2}{400} = \frac{b^4}{400}$

Thus:

$b^2 = 9$

Step 4: Determine Eccentricities e_1 and e_2

Substitute $b^2 = 9$:

For the ellipse:

$e_1^2 = 1 - \frac{9}{25} = \frac{16}{25}$

$e_1 = \frac{4}{5}$

For the hyperbola:

$e_2 = \frac{5}{4}$

Step 5: Find the Eccentricity of the New Ellipse

The new ellipse's equation is:

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

The eccentricity e is:

$e = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Thus, the eccentricity of the ellipse that passes through all four foci is $\frac{3}{5}$.

Question85

Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R . If the circle C with centre at $(-1, -1)$ passes through the points P, Q and R, then the area of $\triangle PQR$ is :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

A. 4

B. 6

C. 5

D. 7

Answer: B

Solution:

The given parabola is $y = x^2 + px - 3$.

Intersection with the y-axis:

At $x = 0$, we find $y = -3$.

Thus, the parabola intersects the y-axis at the point $(0, -3)$.

Circle Equation:

We are given the circle has its center at $(-1, -1)$ and it passes through the points where the parabola intersects the axes. The radius can be found using the distance from the center to any given point the circle passes through. Using $(0, -3)$:

Radius $= \sqrt{(0+1)^2 + (-3+1)^2} = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

Therefore, the equation of the circle is:

$(x+1)^2 + (y+1)^2 = 5$

This simplifies to:

$x^2 + 2x + 1 + y^2 + 2y + 1 = 5$

$x^2 + y^2 + 2x + 2y + 2 = 5$

$x^2 + y^2 + 2x + 2y - 3 = 0$

Intersection with the x-axis:

When $y = 0$, solving the quadratic $x^2 + 2x - 3 = 0$ gives:

$(x+3)(x-1) = 0$

So, $x = -3$ or $x = 1$.

Thus, the intersection points on the x-axis are $(-3, 0)$ and $(1, 0)$.

Vertices of Triangle $\triangle PQR$:

The vertices of the triangle formed are $P = (0, -3)$, $Q = (-3, 0)$, and $R = (1, 0)$.

Area of $\triangle PQR$:

Use the determinant formula to find the area of the triangle:

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & -3 & 1 \\ -3 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Calculate the determinant:

$= \frac{1}{2}((0)(0)(1) + (-3)(1)(1) + (1)(-3)(1) - (1)(0)(1) - (-3)(1)(0) - (0)(-3)(1))$

Simplify:

$= \frac{1}{2}(0 - 3 + 3 + 0) = \frac{1}{2}(12) = 6$

Thus, the area of $\triangle PQR$ is 6.

Question86

Let $\mathrm{P}(4, 4\sqrt{3})$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to :

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Options:

A. $\frac{34\sqrt{3}}{3}$

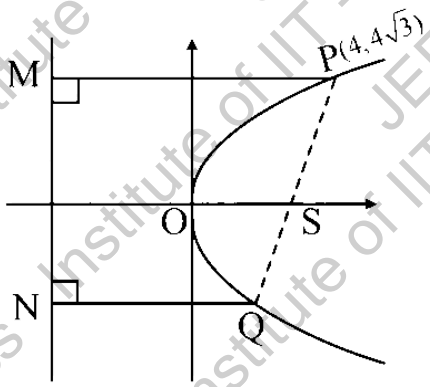
B. $\frac{343\sqrt{3}}{8}$

C. $17\sqrt{3}$

D. $\frac{263\sqrt{3}}{8}$

Answer: B

Solution:



$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a.4$

$4a = 12$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q

is $t_2 = -\frac{\sqrt{3}}{2} \Rightarrow \mathrm{Q}\left(\frac{9}{4}, -3\sqrt{3}\right)$

Area of trapezium PQNM

$= \frac{1}{2}\mathrm{MN} \cdot (\mathrm{PM} + \mathrm{QN})$

$= \frac{1}{2}\mathrm{MN} \cdot (\mathrm{PS} + \mathrm{QS})$

$= \frac{1}{2}\mathrm{MN} \cdot \mathrm{PQ}$

$= \frac{1}{2}7\sqrt{3} \cdot \frac{49}{4} = (343)\frac{\sqrt{3}}{8} = 3$

Question87

If the line $3x - 2y + 12 = 0$ intersects the parabola $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$

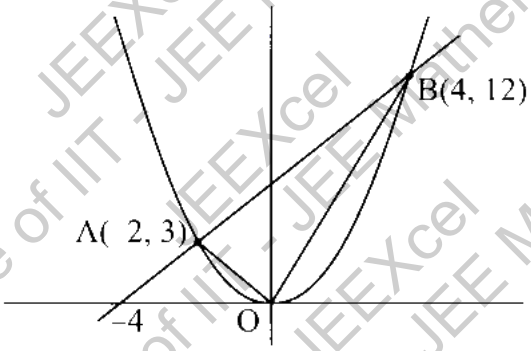
B. $\tan^{-1}\left(\frac{9}{7}\right)$

C. $\tan^{-1}\left(\frac{11}{9}\right)$

D. $\tan^{-1}\left(\frac{4}{5}\right)$

Answer: B

Solution:



$3x - 2y + 12 = 0$

$4y = 3x^2$

$\therefore 2(3x + 12) = 3x^2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

$m_{OA} = -3/2,\ m_{OB} = 3$

$\tan\theta = \left(\frac{\frac{-3}{2} - 3}{1 + \frac{9}{2}}\right) = \frac{9}{7}$

$\theta = \tan^{-1}\left(\frac{9}{7}\right)$ (angle will be acute)

Question88

Let the shortest distance from $(a, 0), a > 0$, to the parabola $y^2 = 4x$ be 4 . Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is :

JEE Main 2025 (Online) 23rd January Evening Shift

Options:

A. $x^2 + y^2 - 8x + 7 = 0$

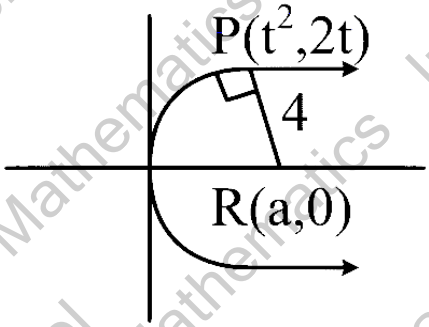
B. $x^2 + y^2 - 6x + 5 = 0$

C. $x^2 + y^2 - 4x + 3 = 0$

D. $x^2 + y^2 - 10x + 9 = 0$

Answer: B

Solution:



Normal at P

$y + tx = 2t + t^3$

$\uparrow$

$(a, 0)$

$at = 2t + t^3$

$a = 2 + t^2$

$R\left(2 + t^2, 0\right)$

$PR = 4 \Rightarrow 4 + 4t^2 = 16$

$4t^2 = 12 \Rightarrow t^2 = 3$

$a = 5, R(5, 0)$

Focus $(1, 0)$

$(1, 0) \& (5, 0)$ will be the end points of diameter

$\Rightarrow Eq^n$ of circle is

$(x - 1)(x - 5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

Question89

If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 6

B. 8

C. 7

D. 9

Answer: D

Solution:

Equation of axis

$y-3=2\left(x-\frac{3}{2}\right)$

$y-2x=0$

foot of directrix

$$\begin{aligned} & y-2x=0 \\ & \& \quad \Rightarrow(0,0) \\ & 2y+x=0 \end{aligned}$$

Focus $=(3,6)$

$PS^2=P^2$

$(x-3)^2+(y-6)^2=\left(\frac{x+2y}{\sqrt{5}}\right)^2$

$4x^2+y^2-4xy-30x-60y+225=0$

$\Rightarrow \alpha=4, \beta=1, \gamma=4 \Rightarrow \alpha+\beta+\gamma=9$

Question90

Let ABCD be a trapezium whose vertices lie on the parabola $y^2=4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point $(1,0)$, then the area of $ABCD$ is

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. $\frac{75}{8}$

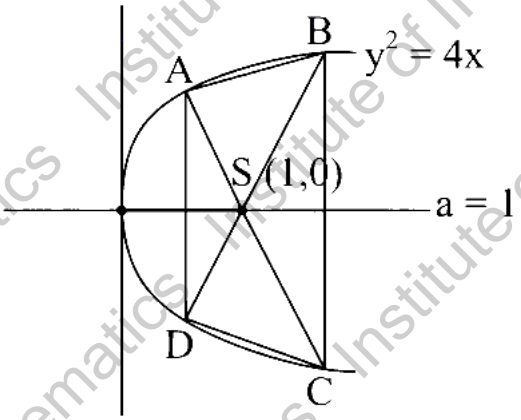
B. $\frac{125}{8}$

C. $\frac{25}{2}$

D. $\frac{75}{4}$

Answer: D

Solution:



A $\left(at_1^2, 2at_1\right)$&C $\left(\frac{a}{t_1^2}, -\frac{2a}{t_1}\right)$

Length AC $=a\left(t_1+\frac{1}{t_1}\right)^2=\frac{25}{4}, t_1+\frac{1}{t_1}=\pm\frac{5}{2}$

$\Rightarrow t_1=2$ or $\frac{1}{2}$, A $\left(\frac{1}{2}, 1\right)$, D $\left(\frac{1}{4}, -1\right)$, B$(4,4)$, C$(4,-4)$

So, area of trapezium $=\frac{1}{2}(8+2)\left(4-\frac{1}{4}\right)=\frac{75}{4}$

Question91

Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersect at the points A and B, then $(AB)^2$ is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

384

B.

392

C.

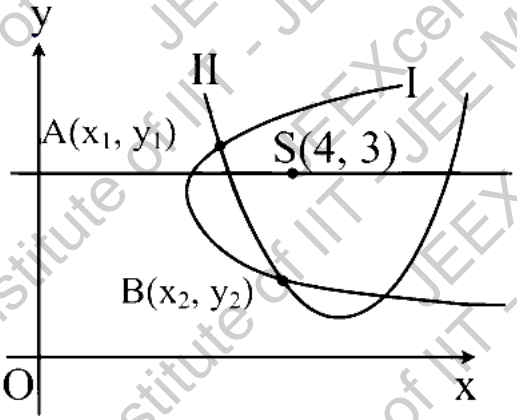
96

D.

192

Answer: D

Solution:



Let intersection points of these two parabolas are $A(x_1, y_1)$& $B(x_2, y_2)$

$\because$ equation of parabola I and II are given below

$\therefore (x-4)^2 + (y-3)^2 = x^2$ ..... (1)
$\&(x-4)^2 + (y-3)^2 = y^2$ ..... (2)

Here $A(x_1, y_1)\&B(x_2, y_2)$ will satisfy the equation

Also from equations (1) & (2), we get $x = y$ .....(3)

Put $x = y$ in equation (1)

We get $x^2 - 14x + 25 = 0$

$x_1 + x_2 = 14$
$x_1 x_2 = 25$
$\therefore AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
$= 2(x_1 - x_2)^2$
$= 2\left[(x_1 + x_2)^2 - 4x_1 x_2\right]$
$= 192$

Question92

Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to:

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 15

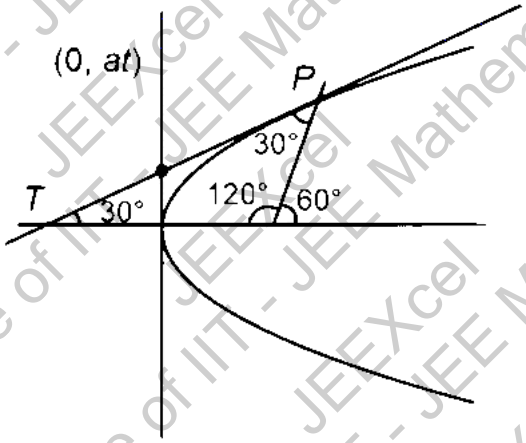
B. 25

C. 20

D. 30

Answer: A

Solution:



$PT : ty = x + at^2$
$PS = PT$
$M_t = \frac{1}{t} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
$t = \sqrt{3}$
$\alpha = at = \sqrt{3} \quad (a = 1)$
$\therefore 5\alpha^2 = 15$

Question93

Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio $m : n, \gcd(m, n) = 1$, then $m^2 + n^2$ is equal to :

JEE Main 2025 (Online) 2nd April Evening Shift

Options:

A. 17

B. 37

C. 10

D. 26

Answer: A

Solution:

Given the parabola $y^2 = 16x$, the point P on the focal chord PQ is $(1, -4)$. We need to determine how the focus divides the chord.

First, express the coordinates for the point P using the parametric form of the parabola. The general parametric equations for the parabola are:

$P(at^2, 2at) \equiv P(4t^2, 8t) \equiv (1, -4)$

The equation $8t = -4$ leads to solving for t:

$t = -\frac{1}{2}$

Now, find the coordinates of point Q using the relation $t_1 \cdot t_2 = -1$. Therefore:

$Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

Given the focus S of the parabola is at $(4, 0)$, we can find the lengths of PS and QS:

The length PS is calculated as:

$PS = a + at^2$

And QS is given by:

$QS = a + \frac{a}{t^2} = \frac{at^2 + a}{t^2}$

Since the focus divides the chord in a specific ratio $m : n$, and $\gcd(m, n) = 1$, we find:

$\frac{PS}{QS} = t^2 = \frac{1}{4} = \frac{m}{n}$

Thus, the squares of m and n sum up to:

$m^2 + n^2 = 17$

Question94

The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

A. $\frac{7\sqrt{2}}{16}$

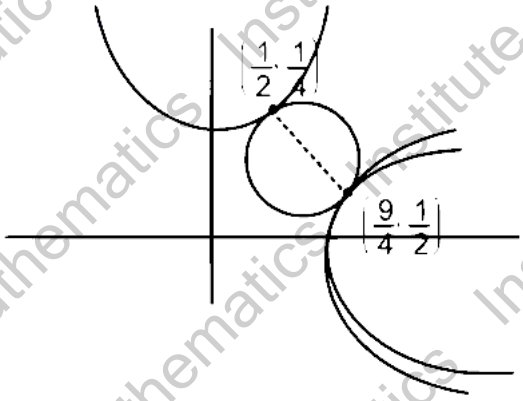
B. $\frac{7\sqrt{2}}{8}$

C. $\frac{7\sqrt{2}}{2}$

D. $\frac{7\sqrt{2}}{4}$

Answer: B

Solution:



The circle will have centre on $x = y$ line since parabolas are symmetric about $y = x$ line

The slope of tangent at closest point $y^2 = x - 2$

$\Rightarrow 2y\left(\frac{dy}{dx}\right) = 1 \Rightarrow y = \frac{1}{2} \Rightarrow$ point will be $\left(\frac{9}{4}, \frac{1}{2}\right)$

Similarly, on $x^2 = y - 2 \Rightarrow 2x = \frac{dy}{dx} = 1 \Rightarrow \left(\frac{1}{2}, \frac{9}{4}\right)$

$$2r = \sqrt{\left(\frac{9}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{9}{4}\right)^2} = \sqrt{2} \cdot \left|\frac{9}{4} - \frac{1}{2}\right|$$

$$= \frac{14}{8}\sqrt{2} = \frac{7\sqrt{2}}{4}$$

$$\Rightarrow r = \frac{7\sqrt{2}}{8}$$

Question95

The axis of a parabola is the line $y = x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point $(1, k)$ lies on the parabola, then a possible value of k is :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 8

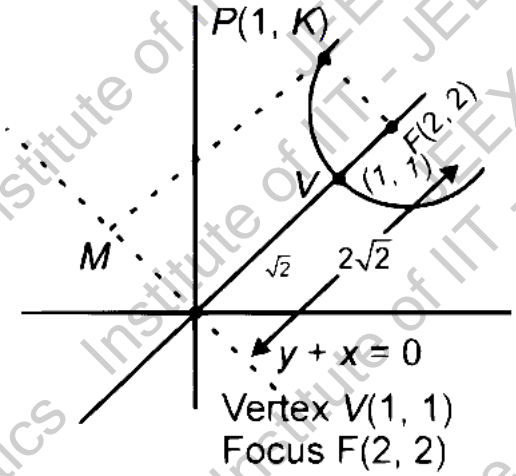
B. 3

C. 9

D. 4

Answer: C

Solution:



Equation of directrix

$\Rightarrow y = -x$

By definition of parabola,

$PM = PF$

$$\left|\frac{1+K}{\sqrt{2}}\right| = \sqrt{(1-2)^2 + (K-2)^2}$$

$$\frac{(1+K)^2}{2} = 1 + K^2 + 4 - 4K$$

$$1 + K^2 + 2K = 10 + 2K^2 - 8K$$

$$K^2 - 10K + 9 = 0$$

$$(K-9)(K-1) = 0$$

$\therefore \quad K = 1$ or $K = 9$

Question96

A line passing through the point $\mathrm{A}(-2,0)$, touches the parabola $\mathrm{P}: y^2=x-2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is :

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 3

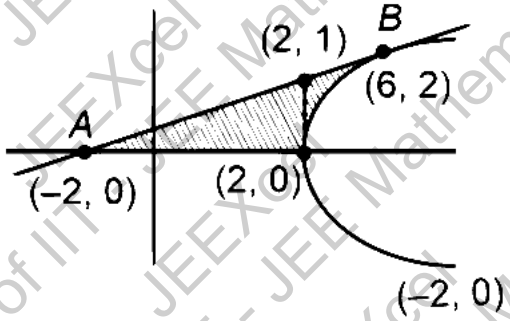
B. $\frac{7}{3}$

C. $\frac{8}{3}$

D. 2

Answer: C

Solution:



$y^2=4\left(\frac{1}{4}\right)(x-2)$

$y=m(x-2)+\frac{1}{4m}$ passes through $(-2,0)$

$\Rightarrow 0=-4m+\frac{1}{4m} \Rightarrow 16m^2=1$

$\Rightarrow m=\pm\frac{1}{4}$

$m=\frac{1}{4}$ in first quadrant $\Rightarrow$ contact point $(6,2)$

$\Rightarrow$ Area $=\frac{1}{2}\times(1)\times 4+\int_2^6\left[\left(\frac{x+2}{4}\right)-\sqrt{x-2}\right]dx$

$=2+\frac{2}{3}=\frac{8}{3}$

Question97

Let P be the parabola, whose focus is $(-2,1)$ and directrix is $2x+y+2=0$. Then the sum of the ordinates of the points on P, whose abscissa is -2, is

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. $\frac{5}{2}$

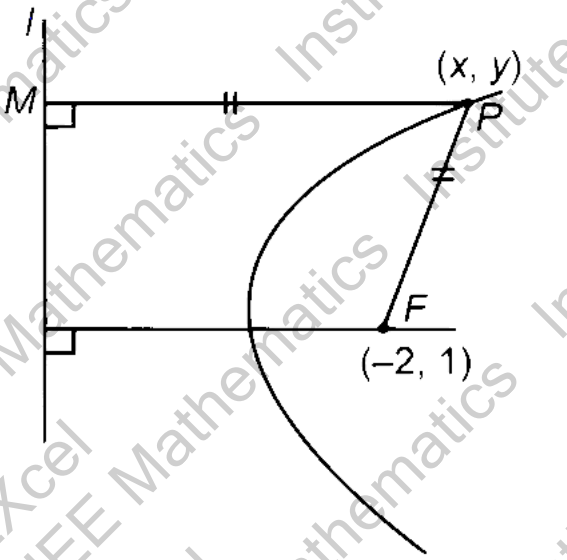
B. $\frac{3}{2}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$

Answer: B

Solution:



$PM = PF$

$\Rightarrow \quad \frac{|2x+y+2|}{\sqrt{5}} = \sqrt{(x+2)^2+(y-1)^2}$

Now abscissa of P is $-2 \Rightarrow x = -2$

$\frac{y-2}{\sqrt{5}} = \sqrt{0+(y-1)^2} \Rightarrow \frac{|y-2|}{\sqrt{5}} = |y-1|$

$\Rightarrow (y-2)^2 = 5(y-1)^2$

$\Rightarrow 4y^2 - 6y + 1 = 0 \Rightarrow$ Sum of ordinates

$= -\left(\frac{-6}{4}\right) = \frac{3}{2}$

Question98

The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5 . If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and $\lambda_2, \lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____.

JEE Main 2025 (Online) 23rd January Evening Shift

Answer: 15

Solution:

$y^2 = 4(x+4)$

Equation of circle

$(x+3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is $\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle, we get

$\lambda = -\frac{7}{6}, 1$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

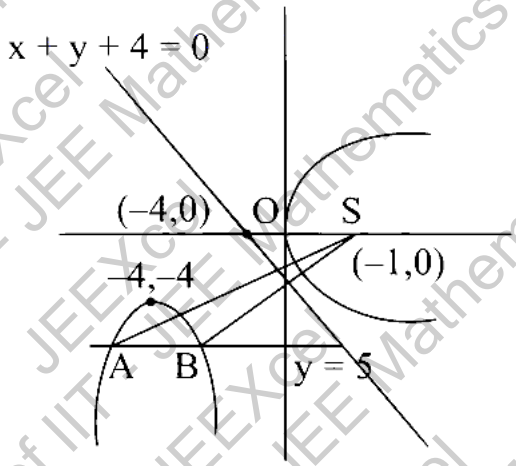
Question99

Let A and B be the two points of intersection of the line $y+5=0$ and the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If d denotes the distance between A and B, and a denotes the area of $\triangle SAB$, where S is the focus of the parabola $y^2=4x$, then the value of $(a+d)$ is _____.

JEE Main 2025 (Online) 28th January Evening Shift

Answer: 14

Solution:



Area $=\frac{1}{2}\times 4\times 5=10=a$

$6=4$

So a + d = 14

Question100

Let $y^2=12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ)=\frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2+64y^2-\alpha x-64\sqrt{3}y=\beta$, then $\beta-\alpha$ is equal to _____.

JEE Main 2025 (Online) 29th January Evening Shift

Answer: 1328

Solution:

$y^2=12x \quad a=3 \quad SP\times SQ=\frac{147}{4}$

Let P $\left(3t^2,6t\right)$ and $t_1t_2=-1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2},\frac{-6}{t}\right)$

$S(3,0)$

$SP\times SQ=PM_1\times QM_2$

(dist. from directrix)

$= (3+3t^2)\left(3+\frac{3}{t^2}\right) = \frac{147}{4}$

$\Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12}$

$t^2 = \frac{3}{4}, \frac{4}{3}$

$t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$

considering $t = \frac{-\sqrt{3}}{2}$

$P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$

Hence, diametric circle:

$(x-4)\left(x-\frac{9}{4}\right) + (y+3\sqrt{3})(y-4\sqrt{3}) = 0$

$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$

$\Rightarrow \alpha = 400, \beta = 1728$

$\beta - \alpha = 1328$

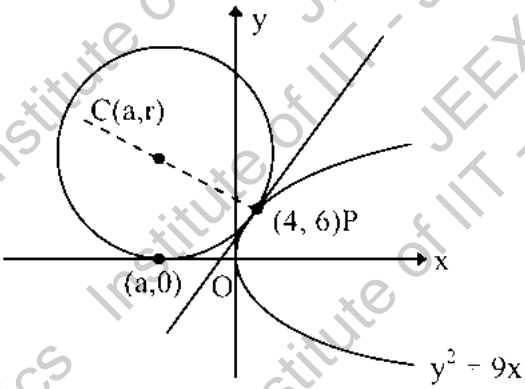
Question101

Let r be the radius of the circle, which touches x - axis at point $(a, 0), a < 0$ and the parabola $y^2 = 9x$ at the point $(4, 6)$. Then r is equal to _____.

JEE Main 2025 (Online) 8th April Evening Shift

Answer: 30

Solution:



$(x-a)^2 + (y-r)^2 = r^2$

$(4-a)^2 + (6-r)^2 = r^2$

$16 + a^2 - 8a + 36 + r^2 - 12r = r^2$

$a^2 - 8a - 12r + 52 = 0$

Tangent to parabola at $(4, 6)$ is

$6.4 = 9.\left(\frac{x+4}{2}\right)$ i.e. $3x - 4y + 12 = 0$

This is also tangent to the circle

$\therefore \quad CP = r$

$\frac{3a - 4r + 12}{5} = \pm r$

$3a + 12 = 4r \pm 5r \begin{cases} ar \\ -r \end{cases}$ ..... (1)

equation of circle is

$(x-a)^2 + (y-r)^2 = r^2$

satsty $P(4, 6) \Rightarrow a^2 - 8a - 12r + 52 = 0$ ..... (2)

From equation (1)

If $a + 4 = 3r$ then $a = +6$ (rejected)

If $3a + 12 = -r$ then $a = -14$ and $r = 30$

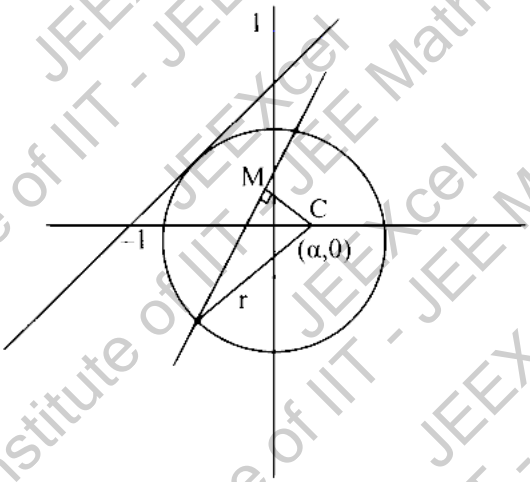
Question102

Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____.

JEE Main 2025 (Online) 23rd January Morning Shift

Answer: 19

Solution:



$x - y + 1 = 0$

p = r

$\frac{\alpha - 0 + 1}{\sqrt{2}} = r \Rightarrow (\alpha + 1)^2 = 2r^2$ .... (1)

now $\left(\frac{-3\alpha + 0 - 1}{\sqrt{9+4}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = r^2$

$\Rightarrow (3\alpha + 1)^2 + 4 = 13r^2 \ldots\ldots (2)$

(1) & (2) $\Rightarrow (3\alpha + 1)^2 + 4 = 13\frac{(\alpha + 1)^2}{2}$

$\Rightarrow 18\alpha^2 + 12\alpha + 2 + 8 = 13\alpha^2 + 26\alpha + 13$

$\Rightarrow 5\alpha^2 - 14\alpha - 3 = 0$

$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$

$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$

$\Rightarrow \alpha = \frac{-1}{5}, 3$

$\therefore \quad r = 2\sqrt{2}$

How $\alpha e = 3$ and $2\alpha = 4\sqrt{2}$

$\alpha^2 e^2 = 9 \Rightarrow \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$

$\alpha^2\left(1 + \frac{\beta^2}{\alpha^2}\right) = 9$

$\alpha^2 + \beta^2 = 9$

$\therefore \beta^2 = 1$

$\therefore 2\alpha^2 + 3\beta^2 = 2(8) + 3(1) = 19$

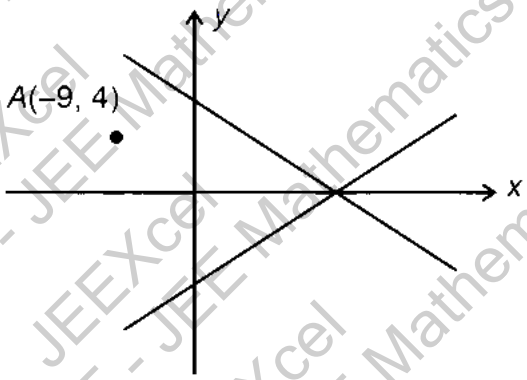
Question103

The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____.

JEE Main 2025 (Online) 2nd April Morning Shift

Answer: 768

Solution:



$\because x + y = 3$ and $x - y = 3$ are tangents

$\therefore$ Both circle centre will lie on x-axis

$\therefore (x - a)^2 + y^2 = r^2$

Hence centre is $C(\alpha, 0)$

$r = \sqrt{(\alpha + 9)^2 + 16}$ .... (1)

Also $\dfrac{\alpha - 3}{\sqrt{2}} = r$ .... (2)

$\sqrt{(\alpha + 9)^2 + 16} = \left|\dfrac{\alpha - 3}{\sqrt{2}}\right|$

$\Rightarrow \quad \alpha = -5$ or -37

$r = \left|\dfrac{-5 - 3}{\sqrt{2}} \text{ or } \dfrac{-37 - 3}{\sqrt{2}}\right|$

$= 4\sqrt{2}$ or $20\sqrt{2}$

$r_1^2 - r_2^2 = |32 - 800| = 768$

Question104

Let C be the circle $x^2 + (y - 1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x -axis and y -axis respectively. Let the straight line $x + y = 3$ touch the curves C, E_1 and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ respectively. Given that P is the mid point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to _____.

JEE Main 2025 (Online) 4th April Morning Shift

Answer: 46

Solution:

Solving the line $x + y = 3$, and the circle $x^2 + (y-1)^2 = 2$

Substitute $y = 3 - x$:

$x^2 + (3 - x - 1)^2 = 2$
$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow x = 1 \Rightarrow y = 2$

So, $P = (x_1, y_1) = (1, 2) \Rightarrow x_1 y_1 = 1 \cdot 2 = 2$

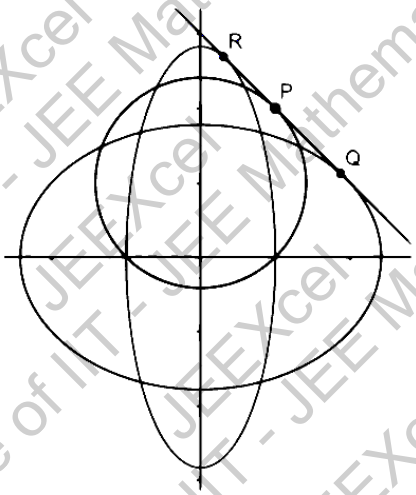
Use midpoint condition

Let $Q = (x_2, y_2), R = (x_3, y_3)$.

Since P is the midpoint of QR:

$$x_2+x_3=2 x_1=2, y_2+y_3=2 y_1=4

$$So, we can write: $x_3 = 2 - x_2, y_3 = 4 - y_2$



Given,

$PQ = \frac{2\sqrt{2}}{3} \Rightarrow PQ^2 = (x_2 - 1)^2 + (y_2 - 2)^2 = \frac{8}{9}$

Let's denote: $x_2 = a, y_2 = b, x_3 = 2 - a, y_3 = 4 - b$

$(a-1)^2 + (b-2)^2 = \dfrac{8}{9}$

$\Rightarrow a^2 - 2a + 1 + b^2 - 4b + 4 = \dfrac{8}{9}$

$\Rightarrow a^2 + b^2 - 2a - 4b + 5 = \dfrac{8}{9}$

$\Rightarrow 9a^2 + 9b^2 - 18a - 36b + 37 = 0$

Hence, $a = \frac{5}{3}, b = \frac{4}{3}$

$x_1y_1 + x_2y_2 + x_3y_3 = 2 + ab + (2 - a)(4 - b)$
$9\left(x_1y_1 + x_2y_2 + x_3y_3\right) = 9(10 + 2ab - 2b - 4a)$
$= 90 + 18ab - 18b - 36a = 46$

Question105

A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2, 5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

A. 10

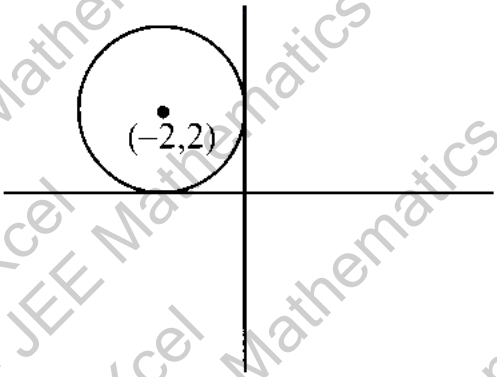
B. 12

C. 14

D. 15

Answer: D

Solution:



$S_1 : (x+2)^2 + (y-2)^2 = 2^2$

$S_2 : (x-2)^2 + (y-5)^2 = r^2$

Both circle intersect at two points

$\therefore |r_1 - r_2| < c_c c_2 < r_1 + r_2$
$|r-2| < 5 < 2 + r$
$\Rightarrow 3 < r < 7$
$r \in (3, 7)$
$\alpha = 3, \beta = 7$
$3\beta - 2\alpha = 15$

Question106

Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{\text{th}}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

JEE Main 2025 (Online) 24th January Morning Shift

Options:

A. $4 - \sqrt{3}$

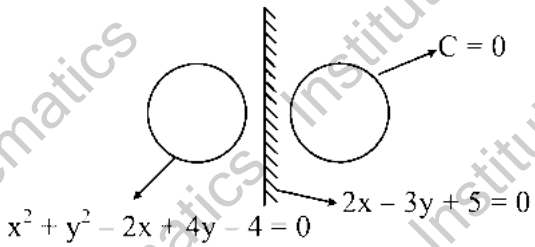
B. 3

C. 4

D. $3 + \sqrt{3}$

Answer: C

Solution:



Centre $(1,-2), r=3$

Reflection of $(1,-2)$ about $2x-3y+5=0$

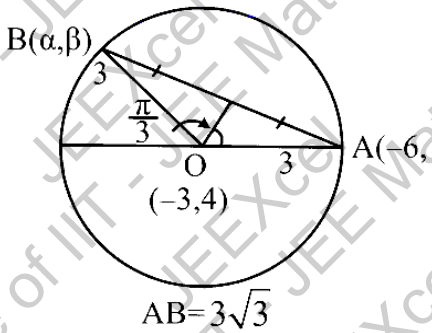
$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2(2+6+5)}{13}=-2$

$x=-3, y=4$

Equation of circle ' C '

$C:(x+3)^2+(y-4)^2=9$

A.T.Q.



$\ell(\text{arcAB})=\frac{1}{6}\times 2\pi \text{r}$

$\text{r}\theta=\frac{1}{6}\times 2\pi \text{r}$

$\theta=\frac{\pi}{3}$

$(\alpha+6)^2+(\beta-4)^2=27$

$\frac{(\alpha+3)^2\pm(\beta-4)^2=9}{(\alpha+6)^2-(\alpha+3)^2=18}$

$\Rightarrow 6\alpha=-9$

$\Rightarrow \alpha=\frac{-3}{2}, \beta=\left(4-\frac{3\sqrt{3}}{2}\right)$

$\therefore \beta-\sqrt{3}\alpha$

$\left(4-\frac{3\sqrt{3}}{2}\right)+\frac{3\sqrt{3}}{2}$

$=4$

Question107

Let the equation of the circle, which touches x-axis at the point $(a,0), a>0$ and cuts off an intercept of length b on $y-axis$ be $x^2+y^2-\alpha x+\beta y+\gamma=0$. If the circle lies below $x-axis$, then the ordered pair $\left(2a, b^2\right)$ is equal to

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. $\left(\alpha, \beta^2+4\gamma\right)$

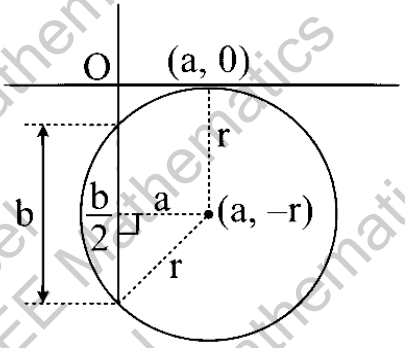
B. $\left(\alpha, \beta^2-4\gamma\right)$

C. $(\gamma, \beta^2 - 4\alpha)$

D. $(\gamma, \beta^2 + 4\alpha)$

Answer: B

Solution:



By pytogorus $r^2 = a^2 + \frac{b^2}{4} = P^2$

$r = \sqrt{\frac{4a^2+b^2}{4}}$

Equation of circle is $(x-\alpha)^2 + (y-\beta)^2 = r^2$

$x^2 + y^2 - 2ax - 2py + \alpha^2 + p^2 - r^2 = 0$
comparision $x^2 + y^2 - \alpha x + \beta y + r = 0$
$-\alpha = -2a, \beta = -2p, r = a^2$
$\Rightarrow 2a = \alpha, 4a^2 + b^2 = 4p^2$
$\alpha^2 + b^2 = 4p^2$
$\alpha^2 + b^2 = \beta^2$

So, $(2a, b^2) = (\alpha, \beta^2 - 4r)$

Question108

Let the line x+y=1 meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid-point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ABCD is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

$\sqrt{14}$

B.

$3\sqrt{7}$

C.

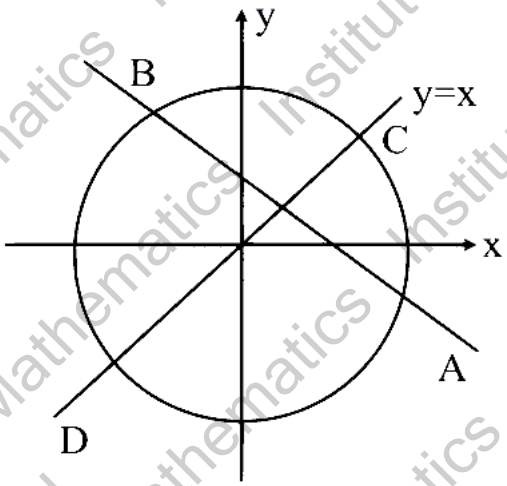
$2\sqrt{14}$

D.

$5\sqrt{7}$

Answer: C

Solution:



By solving $x = y$ with circle

We get

$C(\sqrt{2}, \sqrt{2})$
$D(-\sqrt{2}, -\sqrt{2})$

By solving $x + y = 1$ with circle $x^2 + y^2 = 4$ we set

A $\left(\frac{1+\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right)$

& B $\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$

$\therefore$ Area of Quadrilateral ACBD

$= 2\times$ Area of $\triangle$BCD

$$= 2 \times \frac{1}{2}\begin{vmatrix} \sqrt{2} & \sqrt{2} & 1 \\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{vmatrix}$$

$= 2\sqrt{14}$

Question109

Let a circle C pass through the points (4, 2) and (0, 2), and its centre lie on 3x + 2y + 2 = 0. Then the length of the chord, of the circle C, whose mid-point is (1, 2), is:

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

$4\sqrt{2}$

B.

$2\sqrt{2}$

C.

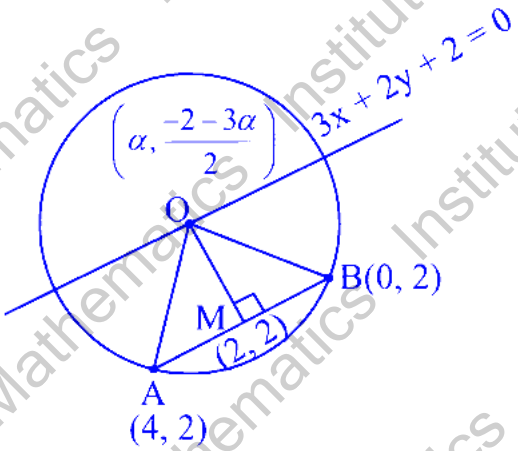
$2\sqrt{3}$

D.

$\sqrt{3}$

Answer: C

Solution:



$M_{AB} = 0 \Rightarrow$ OM is vertical

$\Rightarrow \alpha = 2$

$\therefore$ Centre $(0) \equiv (2, -4)$

$r = OA = \sqrt{(2-4)^2 + (2+4)^2} = \sqrt{40}$

mid point of chord is $N \equiv (1, 2) \quad \therefore ON = \sqrt{37}$

$\therefore$ length of chord $= 2\sqrt{r^2 - (ON)^2}$

$= 2\sqrt{40 - 37} = 2\sqrt{3}$

Question110

If the four distinct points $(4, 6), (-1, 5), (0, 0)$ and $(k, 3k)$ lie on a circle of radius r, then $10k + r^2$ is equal to

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. 34

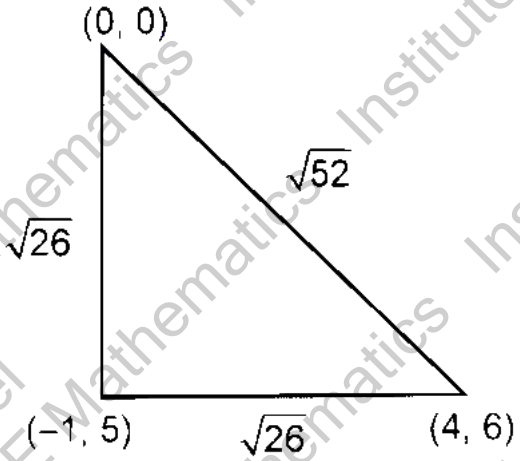
B. 32

C. 35

D. 33

Answer: C

Solution:



$2r = \sqrt{52}$

$4r^2 = 52$

$2r^2 = 26$
$r^2 = 13$
Eq^n of circle is
$x(x-4) + y(y-6) = 0$
$k(k-4) + 3k(3k-6) = 0$
$k^2 - 4k + 9k^2 - 18k = 0$
$10k^2 = 22k$
$10k = 22$
$\therefore \quad 10k + r^2 = 35$

Question111

Let C_1 be the circle in the third quadrant of radius 3 , that touches both coordinate axes. Let C_2 be the circle with centre $(1, 3)$ that touches C_1 externally at the point (α, β). If $(\beta - \alpha)^2 = \frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 22

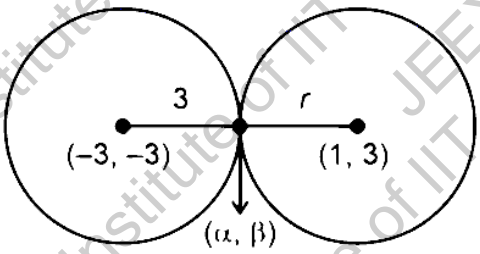
B. 13

C. 9

D. 31

Answer: A

Solution:



$\sqrt{16 + 36} = r + 3$
$\Rightarrow r = \sqrt{52} - 3$
$\Rightarrow \frac{3 - 3r}{-r + 3} = \alpha, \beta = \frac{9 - 3r}{3 + r}$
$\Rightarrow \frac{(9 - 3r - 3 + 3r)^2}{(r + 3)^2}$
$\Rightarrow \frac{36}{12} = \frac{9}{13} = \frac{m}{n} \Rightarrow m + n = 22$

Question112

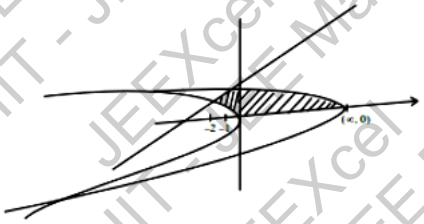
Let the area of the region {(x, y) : x − 2y + 4 ≥ 0, x + 2 y^2 ≥ 0, x + 4 y^2 ≤ 8, y ≥ 0} be m/n , where m and n are coprime numbers.

Then m + n is equal to__

[27-Jan-2024 Shift 1]

Answer: 119

Solution:



$$A = \int_0^1 [(8-4y^2)-(-2y^2)]dy + \int_1^{3/2} [(8-4y^2)-(2y-4)]dy$$

$$= \left[8y - \frac{2y^3}{3}\right]_0^1 + \left[12y - y^2 - \frac{4y^3}{3}\right]_1^{3/2} = \frac{107}{12} = \frac{m}{n}$$

$$\therefore m+n = 119$$

Question113

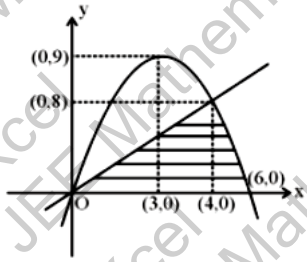
If the area of the region $\{(x, y) : 0 \le y \le \min\{2x, 6x - x^2\}\}$ is A, then 12A is equal to.....

[27-Jan-2024 Shift 2]

Answer: 304

Solution:

We have



$$A = \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2)\,dx$$

$$A = \frac{76}{3}$$

12A = 304

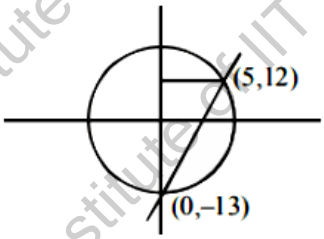
Question114

The area (in sq. units) of the part of circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$ where α, β are coprime numbers. Then α + β is equal to

[29-Jan-2024 Shift 1]

Answer: 171

Solution:



$$\text{Area} = \int_{-13}^{12} \sqrt{169 - y^2}\,dy - \frac{1}{2} \times 25 \times 5$$

$$= \frac{\pi}{2} \times \frac{169}{2} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{12}{13}$$

$$\therefore \alpha + \beta = 171$$

Question115

The area (in square units) of the region bounded by the parabola $y^2 = 4(x - 2)$ and the line $y = 2x - 8$

[30-Jan-2024 Shift 1]

Options:

A.

8

B.

9

C.

6

D.

7

Answer: B

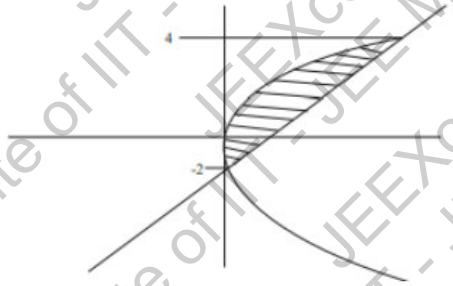
Solution:

Let $X = x - 2$

$y^2 = 4x, \quad y = 2(x+2) - 8$

$y^2 = 4x, \quad y = 2x - 4$

$$A = \int_{-2}^{4} \frac{y^2}{4} - \frac{y+4}{2}$$



$= 9$

Question116

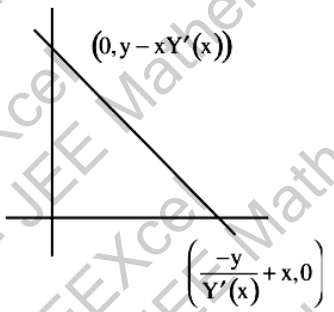
Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $-y^2/2Y(x) + 1$, $Y'(x) \neq 0$. If Y(1) = 1, then 12Y(2) equals_____

[30-Jan-2024 Shift 2]

Answer: 20

Solution:

$$A = \frac{1}{2}\left(\frac{-y}{Y'(x)} + x\right)(y - xY'/x) = \frac{-y^2}{2Y'(x)} + 1$$



$$\Rightarrow (-y + xY'(x))(y - xY'(x)) = -y^2 + 2Y'(x)$$

$$-y^2 + xyY'(x) + xyY'(x) - x^2[Y'(x)]^2 = -y^2 + 2Y'(x)$$

$$2xy - x^2Y'(x) = 2$$

$$\frac{dy}{dx} = \frac{2xy - 2}{x^2}$$

$\frac{dy}{dx} - \frac{2}{x}y = \frac{-2}{x^2}$

I.F. $= e^{-2\ln x} = \frac{1}{x^2}$

$y \cdot \frac{1}{x^2} = \frac{2}{3}x^{-3} + c$

Put $x = 1, y = 1$

$1 = \frac{2}{3} + c \Rightarrow c = \frac{1}{3}$

$Y = \frac{2}{3} \cdot \frac{1}{X} + \frac{1}{3}X^2$

$\Rightarrow 12Y(2) = \frac{5}{3} \times 12 = 20$

Question117

The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x - 2y + 4 = 0$ and the positive coordinate axes is____

[30-Jan-2024 Shift 2]

Answer: 5

Solution:

$\frac{x^2}{4} = x - 1$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

Exclose area (w.r.t. y-axis) $= \int_0^3 x\,dy -$ Area of Δ.

$= \int_0^3 ((y-2)^2 + 1)dy - \frac{1}{2} \times 1 \times 2$

$= \int_0^3 (y^2 - 4y + 5)dy - 1$

$= \left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1$

$= 9 - 18 + 15 - 1 = 5$

Question118

The area of the region

$\left\{(x, y) : y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \ne 3\right\}$

is

[31-Jan-2024 Shift 1]

Options:

A.

16/3

B.

64/3

C.

8/3

D.

32/3

Answer: D

Solution:

$y^2 \le 4x, x < 4$

$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$

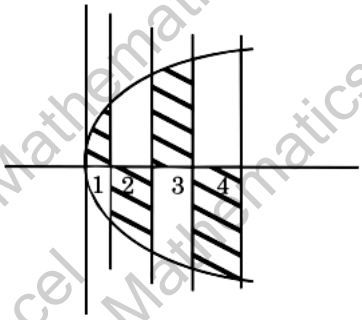
Case $-I : y > 0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$

$x \in (0, 1) \cup (2, 3)$

Case $-$ II : y < 0

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0, x \in (1, 2) \cup (3, 4)$



Area $= 2\int_0^4 \sqrt{x}\, dx$

$= 2 \cdot \frac{2}{3}[x^{3/2}]_0^4 = \frac{32}{3}$

Question119

The area of the region enclosed by the parabola $y = 4x - x^2$ and $3y = (x - 4)^2$ is equal to

[31-Jan-2024 Shift 2]

Options:

A.

32/9

B.

4

C.

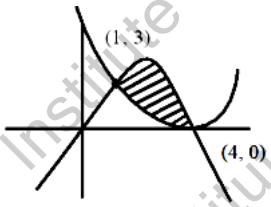
6

D.

14/3

Answer: C

Solution:



$$\text{Area} = \left| \int_1^4 \left[(4x - x^2) - \frac{(x-4)^2}{3} \right] \right| dx.$$

$$\text{Area} = \left| \left(\frac{4x^2}{2} - \frac{x^3}{3} - \frac{(x-4)^3}{9} \right)_1^4 \right|$$

$$= \left| \left(\frac{64}{2} - \frac{64}{3} - \frac{4}{2} + \frac{1}{3} - \frac{27}{9} \right) \right|$$

$$\Rightarrow (27 - 21) = 6$$

Question120

Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

[27-Jan-2024 Shift 1]

Options:

A.

2/13

B.

3/13

C.

5/13

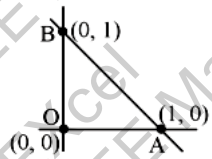
D.

1/13

Answer: C

Solution:

(2k, 3k) will lie on circle whose diameter is AB



$(x-1)(x)+(y-1)(y)=0$

$x^2+y^2-x-y=0$ .....(i)

Satisfy (2k, 3k) in (i)

$(2k)^2+(3k)^2-2k-3k=0$

$13k^2-5k=0$

$k=0, k=\frac{5}{13}$

hence $k=\frac{5}{13}$

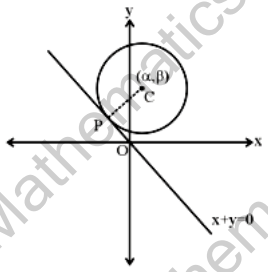
Question121

Consider a circle $(x-\alpha)^2+(y-\beta)^2=50$, where $\alpha, \beta > 0$. If the circle touches the line $y+x=0$ at the point P, whose distance from the origin is $4\sqrt{2}$ then $(\alpha+\beta)^2$ is equal to.....

[27-Jan-2024 Shift 2]

Answer: 100

Solution:



$S:(x-\alpha)^2+(y-\beta)^2=50$

$CP=r$

$\left|\frac{\alpha+\beta}{\sqrt{2}}\right|=5\sqrt{2}$

$\Rightarrow(\alpha+\beta)^2=100$

Question122

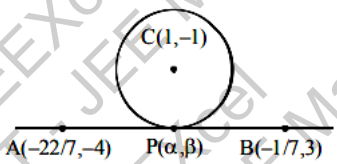
Equation of two diameters of a circle are $2x-3y=5$ and $3x-4y=7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point P(α, β). Then $17\beta-\alpha$ is equal to___

[29-Jan-2024 Shift 1]

Answer: 2

Solution:

Centre of circle is (1, −1)



Equation of AB is $7x - 3y + 10 = 0$ .....(i)

Equation of CP is $3x + 7y + 4 = 0$ .....(ii)

Solving (i) and (ii)

$\alpha = \frac{-41}{29}, \beta = \frac{1}{29}$

$\therefore 17\beta - \alpha = 2$

Question123

If the circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

[30-Jan-2024 Shift 1]

Options:

A.

$5 < r < 9$

B.

$0 < r < 7$

C.

$3 < r < 7$

D.

$1/2 < r < 7$

Answer: C

Solution:

If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < \sqrt{9 + 16} < r + 2$

$|r - 2| < 5$ and $r + 2 > 5$

$-5 < r - 2 < 5$

$r > 3$ .....(2)

$-3 < r < 7$ .....(1)

From (1) and (2)

$3 < r < 7$

Question124

Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}(\sqrt{63}/8)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals ____

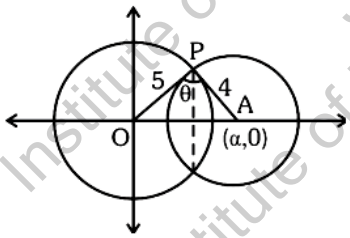
[30-Jan-2024 Shift 2]

Answer: 1575

Solution:

$C_1 : x^2 + y^2 = 25,\ C_2 : (x - \alpha)^2 + y^2 = 16$

$5 < \alpha < 9$



$\theta = \sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$

$\sin\theta = \frac{\sqrt{63}}{8}$

Area of $\triangle OAP = \frac{1}{2} \times \alpha\left(\frac{\beta}{2}\right) = \frac{1}{2} \times 5 \times 4\sin\theta$

$\Rightarrow \alpha\beta = 40 \times \frac{\sqrt{63}}{8}$

$\alpha\beta = 5 \times \sqrt{63}$

$(\alpha\beta)^2 = 25 \times 63 = 1575$

Question125

If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is

[31-Jan-2024 Shift 1]

Options:

A.

$\sqrt{20}$

B.

4

C.

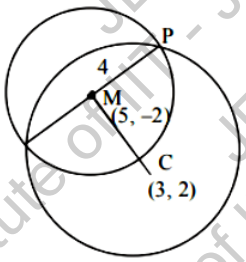
6

D.

$3\sqrt{2}$

Answer: C

Solution:



$2x + 3y = 12$

$3x - 2y = 5$

$13x = 39$

$x = 3, y = 2$

Center of given circle is $(5, -2)$

Radius $\sqrt{25 + 4 - 13} = 4$

$\therefore CM = \sqrt{4 + 16} = 5\sqrt{2}$

$\therefore CP = \sqrt{16 + 20} = 6$

Question126

Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of OA + OB, where O is the origin, is equal to

[31-Jan-2024 Shift 2]

Options:

A.

12

B.

18

C.

20

D.

24

Answer: B

Solution:

$(y-2)=m(x-8)$

$\Rightarrow$ x -intercept

$\Rightarrow \left(\frac{-2}{m}+8\right)$

$\Rightarrow$ y -intercept

$\Rightarrow (-8m+2)$

$\Rightarrow OA+OB=\frac{-2}{m}+8-8m+2$

$f'(m)=\frac{2}{m^2}-8=0$

$\Rightarrow m^2=\frac{1}{4}$

$\Rightarrow m=\frac{-1}{2}$

$\Rightarrow f\left(\frac{-1}{2}\right)=18$

$\Rightarrow$ Minimum $=18$

Question127

The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid point is (1, 2/5), is equal to :

[27-Jan-2024 Shift 1]

Options:

A.

√1691/5

B.

√2009/5

C.

√1741/5

D.

√1541/5

Answer: A

Solution:

Equation of chord with given middle point.

$T = S_1$

$\frac{x}{25} + \frac{y}{40} = \frac{1}{25} + \frac{1}{100}$

$\frac{8x+5y}{200} = \frac{8+2}{200}$

$y = \frac{10-8x}{5}$ .....(i)

$\frac{x^2}{25} + \frac{(10-8x)^2}{400} = 1$ (put in original equation)

$\frac{16x^2+100+64x^2-160x}{400} = 1$

$4x^2 - 8x - 15 = 0$

$x = \frac{8 \pm \sqrt{304}}{8}$

$x_1 = \frac{8+\sqrt{304}}{8}; x_2 = \frac{8-\sqrt{304}}{8}$

Similarly, $y = \frac{10-18 \pm \sqrt{304}}{5} = \frac{2 \pm \sqrt{304}}{5}$

$y_1 = \frac{2-\sqrt{304}}{5}; y_2 = \frac{2+\sqrt{304}}{5}$

Distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{\frac{4\times304}{64} + \frac{4\times304}{25}} = \frac{\sqrt{1691}}{5}$

Question128

If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is__

[29-Jan-2024 Shift 1]

Answer: 432

Solution:

Putting $y^2 = 3x^2$ in both the conics

We get $x^2 = b$ and $\frac{b}{16} + \frac{3}{b} = 1$

$\Rightarrow b = 4, 12$ ($b = 4$ is rejected because curves coincide)

$\therefore b = 12$

Hence points of intersection are

$(\pm\sqrt{12}, \pm 6) \Rightarrow$ area of rectangle $= 432$

Question129

If the length of the minor axis of ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is :

[30-Jan-2024 Shift 1]

Options:

A.

√5/3

B.

√3/2

C.

1/√3

D.

2/√5

Answer: D

Solution:

$2b = ae$

$\frac{b}{a} = \frac{e}{2}$

$e = \sqrt{1 - \frac{e^2}{4}}$

$e = \frac{2}{\sqrt{5}}$

Question130

Let A(α,0) and B(0,β) be the points on the line 5x + 7y = 50. Let the point P divide the line segment AB internally in the ratio 7 : 3. Let 3x − 25 = 0 be a directrix of the ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to

[30-Jan-2024 Shift 2]

Options:

A.

25/3

B.

32/9

C.

25/9

D.

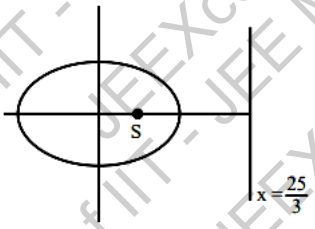
32/5

Answer: D

Solution:

$A = (10, 0)$

$B = \left(0, \frac{50}{7}\right)$ $\Big\}$ $P = (3, 5)$



$ae = 3$

$\frac{a}{e} = \frac{25}{3}$

$a = 5$

$b = 4$

Length of LR $= \frac{2b^2}{a} = \frac{32}{5}$

Question131

Let P be a parabola with vertex (2, 3) and directrix 2x + y = 6. Let an ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ of eccentricity $1/\sqrt{2}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E, is

[31-Jan-2024 Shift 2]

Options:

A.

385/8

B.

347/8

C.

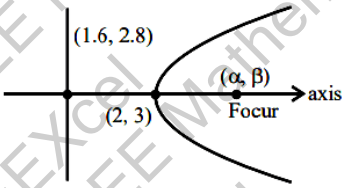
512/25

D.

656/25

Answer: D

Solution:



Slope of axis $= \frac{1}{2}$

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow 2y-6=x-2$

$\Rightarrow 2y-x-4=0$

$2x+y-6=0$

$4x+2y-12=0$

$\alpha+1.6=4 \Rightarrow \alpha=2.4$

$\beta+2.8=6 \Rightarrow \beta=3.2$

Ellipse passes through (2.4, 3.2)

$\Rightarrow \frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1$ .....(1)

Also $1-\frac{b^2}{a^2}=\frac{1}{2}=\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow a^2=2b^2$

Put in (1) $\Rightarrow b^2=\frac{328}{25}$

$\Rightarrow\left(\frac{2b^2}{a}\right)^2=\frac{4b^2}{a^2}\times b^2=4\times\frac{1}{2}\times\frac{328}{25}=\frac{656}{25}$

Question132

Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b,$ which passes through the foci of the hyperbola. If $e_1e_2=1$, then the length of the chord of the ellipse parallel to the x-axis and passing through (0, 2) is :

[27-Jan-2024 Shift 2]

Options:

A.

$4\sqrt{5}$

B.

$8\sqrt{5}/3$

C.

$10\sqrt{5}/3$

D.

$3\sqrt{5}$

Answer: C

Solution:

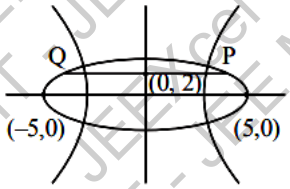
$H: \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad e_1 = \frac{5}{4}$

$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$

Also, ellipse is passing through $(\pm 5, 0)$

$\therefore a = 5$ and $b = 3$

$E: \frac{x^2}{25} + \frac{y^2}{9} = 1$



End point of chord are $\left(\pm \frac{5\sqrt{5}}{3}, 2\right)$

$\therefore L_{PQ} = \frac{10\sqrt{5}}{3}$

Question133

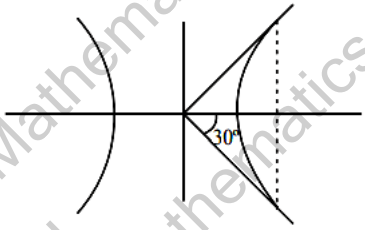
Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\pi/3$ at the centre of the hyperbola. If b^2 is equal to $l/m\,(1 + \sqrt{n})$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to____

[30-Jan-2024 Shift 1]

Answer: 182

Solution:

LR subtends 60° at centre



$\Rightarrow \tan 30^\circ = \frac{b^2/a}{ae} = \frac{b^2}{a^2 e} = \frac{1}{\sqrt{3}}$

$\Rightarrow e = \frac{\sqrt{3}b^2}{9}$

Also, $e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$

$\Rightarrow b^4 = 3b^2 + 27$

$\Rightarrow b^4 - 3b^2 - 27 = 0$

$\Rightarrow b^2 = \frac{3}{2}(1 + \sqrt{13})$

$\Rightarrow \ell = 3, m = 2, n = 13$

$\Rightarrow \ell^2 + m^2 + n^2 = 182$

Question134

Let P be a point on the hyperbola H : $\frac{x^2}{9}-\frac{y^2}{4}=1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

[30-Jan-2024 Shift 2]

Options:

A.

18

B.

26

C.

22

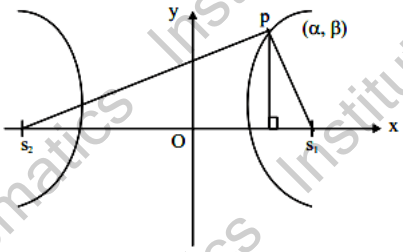
D.

20

Answer: C

Solution:

$\frac{x^2}{9}-\frac{y^2}{4}=1$



$a^2=9,\ b^2=4$

$b^2=a^2(e^2-1)\Rightarrow e^2=1+\frac{b^2}{a^2}$

$e^2=1+\frac{4}{9}=\frac{13}{9}$

$e=\frac{\sqrt{13}}{3}\Rightarrow s_1s_2=2ae=2\times 3\times\sqrt{\frac{13}{3}}=2\sqrt{13}$

Area of $\Delta PS_1S_2=\frac{1}{2}\times\beta\times S_1S_2=2\sqrt{13}$

$\Rightarrow\frac{1}{2}\times\beta\times(2\sqrt{13})=2\sqrt{13}\Rightarrow\beta=2$

$\frac{\alpha^2}{9}-\frac{\beta^2}{4}=1\Rightarrow\frac{\alpha^2}{9}-1=1\Rightarrow\alpha^2=18\Rightarrow\alpha=3\sqrt{2}$

Distance of P from origin $=\sqrt{\alpha^2+\beta^2}$

$=\sqrt{18+4}=\sqrt{22}$

Question135

If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is 15/8 times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

[31-Jan-2024 Shift 1]

Options:

A.

$7\sqrt{\frac{2}{5}}-\frac{8}{3}$

B.

$14\sqrt{\frac{2}{5}}-\frac{4}{3}$

C.

$14\sqrt{\frac{2}{5}}-\frac{16}{3}$

D.

$7\sqrt{\frac{2}{5}}+\frac{8}{3}$

Answer: A

Solution:

$\frac{x^2}{9}+\frac{y^2}{25}=1$

$a=3, b=5$

$e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

$\therefore$ foci $=(0, \pm be)=(0, \pm 4)$

$\therefore e_H=\frac{4}{5}\times\frac{15}{8}=\frac{3}{2}$

Let equation hyperbola

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\therefore B\cdot e_H=4$

$\therefore B=\frac{8}{3}$

$\therefore A^2=B^2(e_H{}^2-1)=\frac{64}{9}\left(\frac{9}{4}-1\right)\therefore A^2=\frac{80}{9}$

$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$

Directrix : $y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$

$PS=e\cdot PM=\frac{3}{2}\left|\frac{14}{3}\cdot\sqrt{\frac{2}{5}}-\frac{16}{9}\right|$

$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Question136

Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b be $(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$ equals

[31-Jan-2024 Shift 1]

Answer: 51

Solution:

$$\text{focii} \equiv (\pm 5, 0);\ \frac{2b^2}{a}=\sqrt{50}$$

$$a=5\ \ b^2=\frac{5\sqrt{2}a}{2}$$

$$b^2=a^2(1-e^2)=\frac{5\sqrt{2}a}{2}$$

$$\Rightarrow a(1-e^2)=\frac{5\sqrt{2}}{2}$$

$$\Rightarrow \frac{5}{e}(1-e^2)=\frac{5\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2}-\sqrt{2}e^2=e$$

$$\Rightarrow \sqrt{2}e^2+e-\sqrt{2}=0$$

$$\Rightarrow \sqrt{2}e^2+2e-e-\sqrt{2}=0$$

$$\Rightarrow \sqrt{2}e(e+\sqrt{2})-1(1+\sqrt{2})=0$$

$$\Rightarrow (e+\sqrt{2})(\sqrt{2}e-1)=0$$

$$\therefore e\neq -\sqrt{2};\ e=\frac{1}{\sqrt{2}}$$

$$\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$$

$$a=5\sqrt{2}$$

$$b=5$$

$$a^2b^2=b^2(e_1{}^2-1)\Rightarrow e_1{}^2=51$$

Question137

If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

[27-Jan-2024 Shift 1]

Options:

A.

16

B.

24

C.

20

D.

36

Answer: C

Solution:

Equation of normal to parabola

$y = mx - 2m - m^3$

this normal passing through center of circle (2, 8)

$8 = 2m - 2m - m^3$

$m = -2$

So point P on parabola $\Rightarrow (am^2, -2am) = (4, 4)$

And C = (2, 8)

$PC = \sqrt{4+16} = \sqrt{20}$

$d^2 = 20$

Question138

Let P(α, β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is (1, 5/4). Then (α − 28) (β − 8) is equal to____

[29-Jan-2024 Shift 2]

Answer: 192

Solution:

Parabola is $x^2 = 8y$

Chord with mid point (x_1, y_1) is $T = S_1$

$\therefore xx_1 - 4(y + y_1) = x_1^2 - 8y_1$

$\therefore (x_1, y_1) = \left(1, \frac{5}{4}\right)$

$\Rightarrow x - 4\left(y + \frac{5}{4}\right) = 1 - 8 \times \frac{5}{4} = -9$

$\therefore x - 4y + 4 = 0$ .....(i)

(α, β) lies on (i) & also on $y^2 = 4x$

$\therefore \alpha - 4\beta + 4 = 0$ .....(ii)

& $\beta^2 = 4\alpha$..... (iii)

Solving (ii) & (iii)

$\beta^2 = 4(4\beta - 4) \Rightarrow \beta^2 - 16\beta + 16 = 0$

$\therefore \beta = 8 \pm 4\sqrt{3}$ and $\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28 + 16\sqrt{3}, 8 + 4\sqrt{3})$ & $(28 - 16\sqrt{3}, 8 - 4\sqrt{3})$

$\therefore (\alpha - 28)(\beta - 8) = (\pm 16\sqrt{3})(\pm 4\sqrt{3})$

$= 192$

Question139

The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (−x, y) where y > 0 is :

[30-Jan-2024 Shift 1]

Options:

A.

88

B.

122

C.

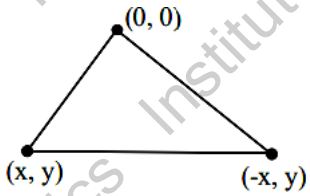
92

D.

108

Answer: D

Solution:



Area of Δ

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \left|\frac{1}{2}(xy + xy)\right| = |xy|$$

$$\text{Area } (\Delta) = |xy| = |x(-2x^2 + 54)|$$

$$\frac{d(\Delta)}{dx} = |(-6x^2 + 54)| \Rightarrow \frac{d\Delta}{dx} = 0 \text{ at } x = 3$$

$$\text{Area } = 3(-2 \times 9 + 54) = 108$$

Question140

Let C : $x^2 + y^2 = 4$ and C′: $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C′ intersect at two distinct points, is R − [a, b], then the point (8a + 12, 16b − 20) lies on the curve :

[1-Feb-2024 Shift 1]

Options:

A.

$x^2 + 2y^2 - 5x + 6y = 3$

B.

$5x^2 - y = -11$

C.

$x^2 - 4y^2 = 7$

D.

$6x^2 + y^2 = 42$

Answer: D

Solution:

$x^2 + y^2 = 4$

C(0, 0)

$r_1 = 2$

$C'(2\lambda, 0)$ $r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$2 - \sqrt{4\lambda^2 - 9} < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$

True $\lambda \in R$... (1)

$4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{4\lambda^2 - 9}$

$5 < 4\sqrt{4\lambda^2 - 9}$ and $\lambda^2 \geq \frac{9}{4}$

$\frac{25}{16} < 4\lambda^2 - 9$ $\quad \lambda \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

$\frac{169}{64} < \lambda^2$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$ .....(2)

from (1) and (2) $\lambda \in$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \Rightarrow R - \left[-\frac{13}{8}, \frac{13}{8}\right]$

as per question $a = -\frac{13}{8}$ and $b = \frac{13}{8}$

required point is $(-1, 6)$ with satisfies option (4)

Question141

Let the line L : $\sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to____

[1-Feb-2024 Shift 1]

Answer: 72

Solution:

$x^2+y^2=3$ and $x^2=2y$

$y^2+2y-3=0 \Rightarrow (y+3)(y-1)=0$

$y=-3$ or $y=1$

$y=1 x=\sqrt{2} \Rightarrow P(\sqrt{2}, 1)$

p lies on the line

$\sqrt{2}x+y=\alpha$

$\sqrt{2}(\sqrt{2})+1=\alpha$

$\alpha=3$

For circle C_1

Q_1 lies on y axis

Let $Q_1(0, \alpha)$ coordinates

$R_1=2\sqrt{3}$ (Given

Line L act as tangent

Apply P = r (condition of tangency)

$\Rightarrow \left|\frac{\alpha-3}{\sqrt{3}}\right|=2\sqrt{3}$

$\Rightarrow |\alpha-3|=6$

$\alpha-3=6$ or $\alpha-3=-6$

$\Rightarrow \alpha=9 \ \alpha=-3$

$$\triangle PQ_1Q_2=\frac{1}{2}\begin{vmatrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{vmatrix}$$

$=\frac{1}{2}(\sqrt{2}(12))=6\sqrt{2}$

$(\triangle PQ_1Q_2)^2=72$

Question142

Let the locus of the mid points of the chords of circle $x^2 + (y - 1)^2 = 1$ drawn from the origin intersect the line x + y = 1 at P and Q. Then, the length of PQ is :

[1-Feb-2024 Shift 2]

Options:

A.

1/√2

B.

√2

C.

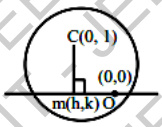
1/2

D.

1

Answer: A

Solution:

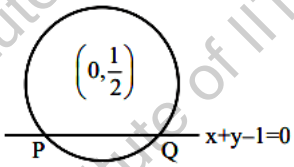


$m_{OM} \cdot m_{CM} = -1$

$\frac{k}{h} \cdot \frac{k-1}{h} = -1$

$\therefore$ locus is $x^2 + y(y-1) = 0$

$x^2 + y^2 - y = 0$



$p = \left|\frac{1/2}{\sqrt{2}}\right|$ $p = \frac{1}{2\sqrt{2}}$

$PQ = 2\sqrt{r^2 - p^2}$

$= 2\sqrt{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\sqrt{2}}$

Question143

The area enclosed by the curves xy + 4y = 16 and x + y = 6 is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

$28 - 30\log_e 2$

B.

$30 - 28\log_e 2$

C.

$30 - 32\log_e 2$

D.

$32 - 30\log_e 2$

Answer: C

Solution:

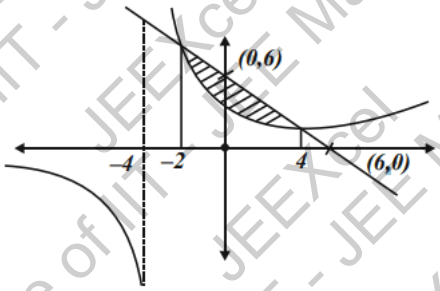
$xy + 4y = 16$

$y(x+4) = 16$ .....(1)

$x + y = 6$ .....(2)

on solving, (1) & (2)

we get $x = 4, x = -2$



$$\text{Area} = \int_{-2}^{4} \left((6-x) - \left(\frac{16}{x+4}\right)\right) dx$$

$$= 30 - 32\ln 2$$

Question144

The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to :

[1-Feb-2024 Shift 2]

Answer: None

Solution:

$ky^2 = 2(y-x) \quad 2y^2 = kx$

Point of intersection $\rightarrow$

$$ky^2 = \left(y - \frac{2y^2}{k}\right)$$

$$y = 0 \qquad ky = 2\left(1 - \frac{2y}{k}\right)$$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2+4}$$

$$A = \int_0^{\frac{2k}{k^2+4}} \left(\left(y - \frac{ky^2}{2}\right) - \left(\frac{2y^2}{k}\right)\right) \cdot dy$$

$$= \frac{y^2}{2} - \left(\frac{k}{2} + \frac{2}{k}\right) \cdot \frac{y^3}{3}\Big|_0^{\frac{2k}{k^2+4}}$$

$$= \left(\frac{2k}{k^2+4}\right)^2 \left[\frac{1}{2} - \frac{k^2+4}{2k} \times \frac{1}{3} \times \frac{2k}{k^2+4}\right]$$

$$= \frac{1}{6} \times 4 \times \left(\frac{1}{k + \frac{4}{k}}\right)^2$$

$A \cdot M \geq G \cdot M \quad \frac{\left(k+\frac{4}{k}\right)}{2} \geq 2$

$k+\frac{4}{k} \geq 4$

Area is maximum when $k = \frac{4}{k}$

$k = 2, -2$

Question145

Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4:3 as P moves on the ellipse, is :

[1-Feb-2024 Shift 2]

Options:

A.

11/19

B.

13/21

C.

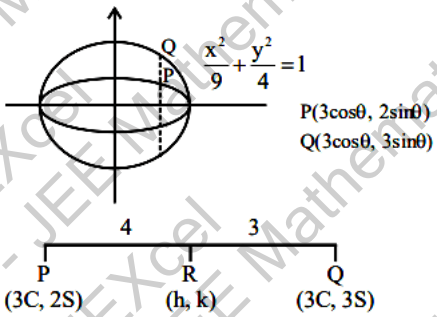
√139/23

D.

√13/7

Answer: D

Solution:



$h = 3\cos\theta$

$k = \frac{18}{7}\sin\theta$

$\therefore \text{ locus } = \frac{x^2}{9}+\frac{49y^2}{324}=1$

$e = \sqrt{1-\frac{324}{49\times 9}} = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$

Question146

For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2 \operatorname{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2 \operatorname{cosec}^2\theta + y^2 = 5$, then the value of θ is :

[1-Feb-2024 Shift 1]

Options:

A.

$\pi/6$

B.

$5\pi/12$

C.

$\pi/3$

D.

$\pi/4$

Answer: C

Solution:

$$e_h = \sqrt{1 + \sin^2\theta}$$

$$e_c = \sqrt{1 - \sin^2\theta}$$

$$e_h = \sqrt{7} e_c$$

$$1 + \sin^2\theta = 7(1 - \sin^2\theta)$$

$$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

Question147

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ be an ellipse, whose eccentricity is $1/\sqrt{2}$ and the length of the latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

[1-Feb-2024 Shift 1]

Options:

A.

3

B.

7/2

C.

3/2

D.

5/2

Answer: C

Solution:

$$e = \frac{1}{\sqrt{2}} = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{1}{2} = 1 - \frac{b^2}{a^2}$$

$$\frac{2b^2}{a} = 14$$

$$e_H = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$(e_H)^2 = \frac{3}{2}$$

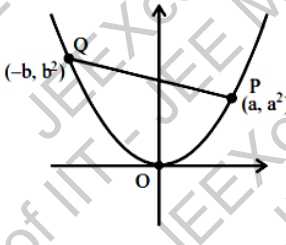
Question148

Three points O(0, 0), P(a, a^2), Q(−b, b^2), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of S_1/S_2 is m/n, gcd (m, n) = 1, then m + n is

[1-Feb-2024 Shift 2]

Answer: 7

Solution:



$$S_2 = 1/2\begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix} = 1/2(ab + a^2b)$$

PQ:- $y - a^2 = \frac{a^2 - b^2}{a+b}(x - a)$

$y - a^2 = (a-b)x - (a-b)a$

$y = (a-b)x + ab$

$S_1 = \int_{-b}^{a} ((a-b)x + ab - x^2)\,dx$

$= (a-b)\frac{x^2}{2} + (ab)x - \frac{x^3}{3}\Big|_{-b}^{a}$

$= \frac{(a-b)^2(a+b)}{2} + ab(a+b) - \frac{(a^3+b^3)}{3}$

$\frac{S_1}{S_2} = \frac{\frac{(a-b)^2}{2} + ab - \frac{(a^2+b^2-ab)}{3}}{\frac{ab}{2}}$

$= \frac{3(a-b)^2 + 6ab - 2(a^2+b^2-ab)}{3ab}$

$= \frac{1}{3}\left[\frac{a}{b} + \frac{b}{a} + 2\right]$

$= \frac{4}{3} = \frac{m}{n}$ m + n = 7

Question149

Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the
[24-Jan-2023 Shift 1]

Options:

A. directrix $4x = 3$

B. directrix $4x = -3$

C. Length of latus rectum -

D. Length of latus rectum 2

Answer: A

Solution:

$y^2 = 24x$
$a = 6$
$xy = 2$
$AB \equiv ty = x + 6t^2$ . . . (1)
$AB \equiv T = S_1$
$kx + hy = 2hk$ . . . (2)
From (1) and (2)

$\Rightarrow$ then locus is $y^2 = -3x$
Therefore directrix is $4x = 3$

Question150

For some a, b, c $\in \mathbb{N}$, let f (x) = ax − 3 and g(x) = x^b + c, x $\in \mathbb{R}$. If (fog)$^{-1}$(x) = $\left(\frac{x-7}{2}\right)^{1/3}$ then (fog) (ac) + (gof) (b) is equal to _____.
[25-Jan-2023 Shift 1]

Answer: 2039

Solution:

Let fog (x) = h(x)

$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)\frac{1}{3}$

$\Rightarrow h(x) = fog(x) = 2x^3 + 7$

$fog\ (x) = a(x^b + c) - 3$

$\Rightarrow a = 2, b = 3, c = 5$

$\Rightarrow\ fog\ (ac) = fog(10) = 2007$

$g(f(x) = (2x - 3)^3 + 5.$

$\Rightarrow gof(b) = gof(3) = 32$

$\Rightarrow$ sum = 2039

Question151

The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is :
[24-Jan-2023 Shift 1]

Options:

A. $\frac{25}{3}$

B. $\frac{22}{3}$

C. 9

D. $\frac{23}{3}$

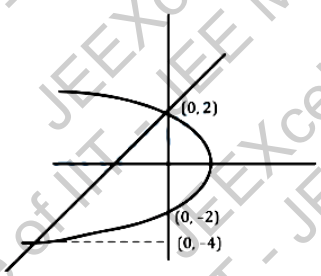
Answer: C

Solution:

$y^2 + 4x = 4$

$y^2 = -4(x - 1)$

$A = \int_{-4}^{2}\left(\frac{4-y^2}{4} - \frac{y-2}{2}\right)dy = 9$



Question152

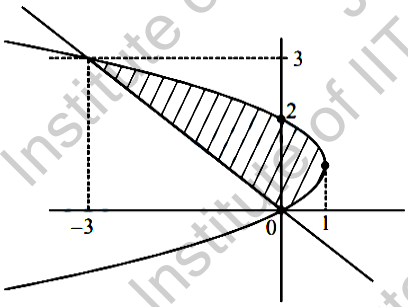
If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8A is equal to
[24-Jan-2023 Shift 2]

Answer: 36

Solution:

$y^2 - 2y = -x$
$\Rightarrow y^2 - 2y + 1 = -x + 1$
$(y-1)^2 = -(x-1)$
$y = -x$

Points of intersection
$x^2 + 2x = -x$
$x^2 + 3x = 0$
$x = 0, -3$



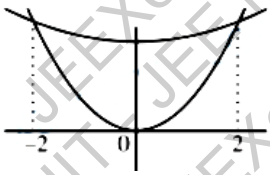
$A = \int_0^3 (-y^2 + 2y + y)\,dy$
$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$
$8A = 36$

Question153

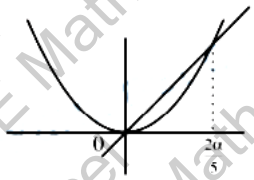
It the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x$, $\alpha > 0$, then α^3 is equal to _____.
[25-Jan-2023 Shift 1]

Answer: 600

Solution:



Abscissa of point of intersection of $2y = 5x^2$ and $y = x^2 + 6$ is ± 2



$$\text{Area} = 2\int_0^2 \left(x^2+6-\frac{5x^2}{2}\right)dx = \int_0^{\frac{2\alpha}{5}} \left(\alpha x-\frac{5x^2}{2}\right)dx$$

$$\Rightarrow \int_0^{\frac{2\alpha}{5}} \left(\alpha x-\frac{5x^2}{2}\right)dx = 16$$

$$\Rightarrow \alpha^3 = 600$$

Question154

Let Δ be the area of the region

$\{(x, y) \in \mathbb{R}^2 : x^2+y^2 \le 21, y^2 \le 4x, x \ge 1\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

[29-Jan-2023 Shift 1]

Options:

A. $2\sqrt{3}-\frac{1}{3}$

B. $\sqrt{3}-\frac{2}{3}$

C. $2\sqrt{3}-\frac{2}{3}$

D. $\sqrt{3}-\frac{4}{3}$

Answer: C

Solution:

$$\text{Area } 2\int_1^3 2\sqrt{x}\,dx + 2\int_3^{\sqrt{21}} \sqrt{21-x^2}\,dx$$

$$\Delta = \frac{8}{3}(3\sqrt{3}-1) + 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right) - 6\sqrt{3}$$

$$\frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right) = \frac{2\sqrt{3}-\frac{8}{3}}{2}$$

$$= \sqrt{3}-\frac{4}{3}$$

Question155

Let $A = \left\{(x, y) \in \mathbb{R}^2 : y \ge 0, 2x \le y \le \sqrt{4-(x-1)^2}\right\}$ and

$B = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \le y \le \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$ Then the ratio of the area of A to the area of B is

[29-Jan-2023 Shift 1]

Options:

A. $\frac{\pi-1}{\pi+1}$

B. $\frac{\pi}{\pi-1}$

C. $\frac{\pi}{\pi+1}$

D. $\frac{\pi+1}{\pi-1}$

Answer: A

Solution:

$y^2+(x-1)^2=4$
shaded portion = circular (OABC)
$-$ Ar(ΔOAB)
$=\frac{\pi(4)}{4}-\frac{1}{2}(2)(1)$
$A=(\pi-1)$
Area B = Ar($\triangle$AOB)+ Area of arc of circle (ABC)
$=\frac{1}{2}(1)(2)+\frac{\pi(2)^2}{4}=\pi+1$
$\frac{A}{B}=\frac{\pi-1}{\pi+1}$

Question156

The area of the region$A=\left\{(x, y):|\cos x-\sin x|\leq y\leq \sin x, 0\leq x\leq \frac{\pi}{2}\right\}$

[29-Jan-2023 Shift 2]

Options:

A. $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$

B. $\sqrt{5}+2\sqrt{2}-4.5$

C. $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$

D. $\sqrt{5}-2\sqrt{2}+1$

Answer: D

Solution:

$|\cos x-\sin x|\leq y\leq \sin x$
Intersection point of $\cos x-\sin x=\sin x$
$\Rightarrow \tan x=\frac{1}{2}$
Let $\psi=\tan^{-1}\frac{1}{2}$
So, $\tan\psi=\frac{1}{2}$, $\sin\psi=\frac{1}{\sqrt{5}}$, $\cos\psi=\frac{2}{\sqrt{5}}$

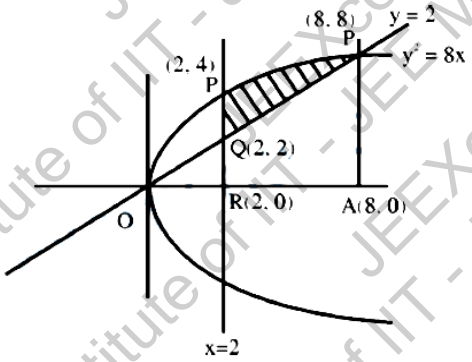
$$\text{Area}=\int_{\psi}^{\pi/2}(\sin x-|\cos x-\sin x|)\,dx$$
$$=\int_{\psi}^{\pi/4}(\sin x-(\cos x-\sin x))\,dx+\int_{\pi/4}^{\pi/2}(\sin x-(\sin x-\cos x))\,dx$$
$$=\int_{\psi}^{\pi/4}(2\sin x-\cos x)\,dx+\int_{\pi/4}^{\pi/2}\cos x\,dx$$
$$=[-2\cos x-\sin x]_{\psi}^{\pi/4}+[\sin x]_{\pi/4}^{\pi/2}$$
$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\cos\psi+\sin\psi+\left(1-\frac{1}{\sqrt{2}}\right)$$
$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\left(\frac{2}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}\right)+1-\frac{1}{\sqrt{2}}$$
$$=\sqrt{5}-2\sqrt{2}+1$$

Question157

Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines y = x and x = 2, which lies in the first quadrant. Then the value of 3α is equal to _____.
[30-Jan-2023 Shift 1]

Answer: 22

Solution:



$y = x$
$y^2 = 8x$
Solving it
$x^2 = 8x$
$\therefore\ x = 0, 8$
$\therefore\ y = 0, 8$
$x = 2$ will intersect occur at
$y^2 = 16 \Rightarrow\ y = \pm 4$
$\therefore$ Area of shaded

$$= \int_2^8 (\sqrt{8x} - x)\,dx = \int 2^8 (2\sqrt{2}\sqrt{x} - x)\,dx$$

$$= \left[2\sqrt{2}\cdot\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^8$$

$$= \left(\frac{4\sqrt{2}}{3}\cdot 2^{9/2} - 32\right) - \left(\frac{4\sqrt{2}}{3}\cdot 2^{9/2} - 2\right)$$

$$= \frac{128}{3} - 32 - \frac{16}{3} + 2 = \frac{112 - 90}{3} = \frac{22}{3} = A$$

$\therefore 3A = 22$

Question158

Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y) : 0 \le y \le (x - q)^2, 0 \le x \le q\}$ is
[30-Jan-2023 Shift 2]

Options:

A. 243

B. 25

C. $\frac{125}{3}$

D. 164

Answer: A

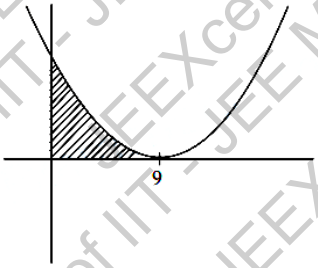
Solution:

$x^2 - px + \frac{5p}{4} = 0$

$D = p^2 - 5p = p(p-5)$

$\therefore q = 9$

$0 \le y \le (x-9)^2$



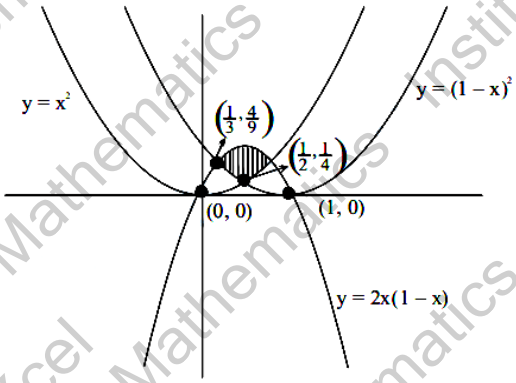
$\text{Area} = \int_0^9 (x-9)^2\,dx = 243$

Question159

Let A be the area of the region
$\{(x, y) : y \ge x^2, y \ge (1-x)^2, y \le 2x(1-x)\}$.
Then 540A is equal to
[30-Jan-2023 Shift 2]

Answer: 25

Solution:



$A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} (2x - 2x^2 - (1-x)^2)\,dx$

$= 2[2x^2 - x^3 - x]_{1/3}^{1/2}$

$\therefore A = \frac{5}{108} \Rightarrow 540A = \frac{5}{108} \times 540 = 25$

Question160

Let for $x \in R$

$f(x) = \frac{x+|x|}{2}$ and $g(x) = \begin{cases} x & ,x<0 \\ x^2, & x \geq 0 \end{cases}$

Then area bounded by the curve $y = (fog)(x)$ and the lines $y = 0$, $2y - x = 15$ is equal to _____.
[31-Jan-2023 Shift 1]

Answer: 72

Solution:

$$f(x) = \frac{x+|x|}{2} = \begin{bmatrix} x & x \geq 0 \\ 0 & x < 0 \end{bmatrix}.$$

$$g(x) = \begin{bmatrix} x^2 & x \geq 0 \\ x & x < 0 \end{bmatrix}.$$

$$fog(x) = f[g(x)] = \begin{bmatrix} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{bmatrix}.$$

$$fog(x) = \begin{bmatrix} x^2 & x \geq 0 \\ 0 & x < 0 \end{bmatrix}.$$

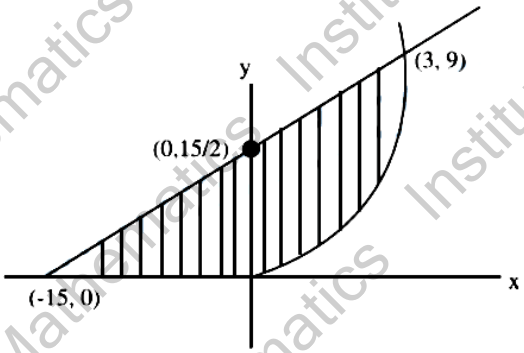
$2y - x = 15$

$$\frac{3}{0}\left(\frac{x+15}{2} - x^2\right)dx + \frac{1}{2} \times \frac{15}{2} \times 15$$

$$\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\Big|_0^3 + \frac{225}{4}$$

$$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4} = \frac{99 - 36 + 225}{4}$$

$$= \frac{288}{4} = 72$$



Question161

Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:
[25-Jan-2023 Shift 2]

Options:

A. $4\sqrt{6} + \frac{44}{3}$

B. $4\sqrt{6} + \frac{28}{3}$

C. $4\sqrt{6} - \frac{44}{3}$

D. $4\sqrt{6} - \frac{28}{3}$

Answer: B

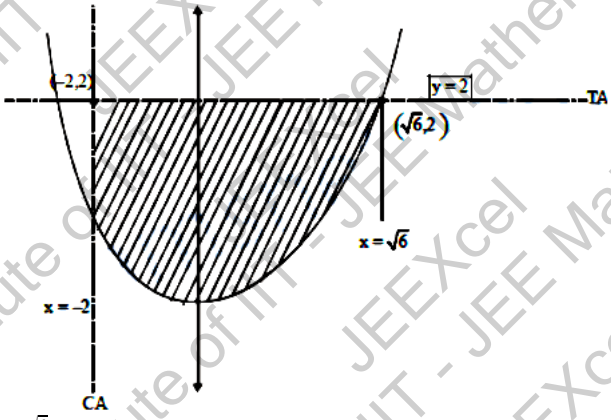
Solution:

$16(x^2+4x) - (y^2-4y) + 44 = 0$

$16(x+2)^2 - 64 - (y-2)^2 + 4 + 44 = 0$

$16(x+2)^2 - (y-2)^2 = 16$

$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$



$A = \int_{-2}^{\sqrt{6}} (2 - (x^2-4))\,dx$

$A = \int_{-2}^{\sqrt{6}} (6 - x^2)\,dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$

$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$

$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}$

$A = 4\sqrt{6} + \frac{28}{3}$

Question162

Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its lat us rectum is
[31-Jan-2023 Shift 2]

Options:

A. 2

B. 3

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

$2ae = |(1+\sqrt{2}) - (1+\sqrt{2})| = 2\sqrt{2}$

$ae = \sqrt{2}$

$a = 1$

$\Rightarrow b = 1 \quad \because e = \sqrt{2} \Rightarrow$ Hyperbola is rectangular

$\Rightarrow L \cdot R = \frac{2b^2}{a} = 2$

Question163

The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to
[24-Jan-2023 Shift 2]

Options:

A. $\frac{\pi}{4}$

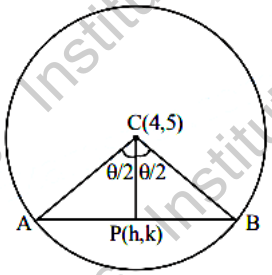
B. $\frac{3\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

Solution:



$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h-4)^2 + (k-5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x-4)^2 + (y-5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \theta_2 = \frac{\pi}{2}$

Question164

The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to
[24-Jan-2023 Shift 2]

Options:

A. $\frac{\pi}{4}$

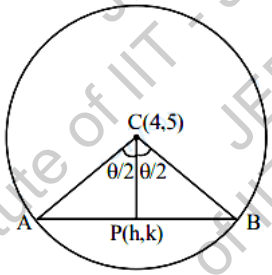
B. $\frac{3\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

Solution:



$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h-4)^2 + (k-5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x-4)^2 + (y-5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \theta_2 = \frac{\pi}{2}$

Question165

The points of intersection of the line $ax + by = 0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :
[25-Jan-2023 Shift 1]

Options:

A. $x^2 + y^2 + 5x + 5y + 12 = 0$

B. $x^2 + y^2 + 3x + 5y + 8 = 0$

C. $x^2 + y^2 + 3x + 3y + 4 = 0$

D. $x^2 + y^2 - 5x - 5y + 12 = 0$

Answer: A

Solution:

Only possibility $\alpha = 0, \beta = 1$
$\therefore$ equation of circle $x^2 + y^2 - x - y = 0$
Image of circle in $x + y + 2 = 0$ is
$x^2 + y^2 + 5x + 5y + 12 = 0$

Question166

Points P(−3, 2), Q(9, 10) and R(α, 4) lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line $2x - ky = 1$, then k is equal to _____.
[25-Jan-2023 Shift 2]

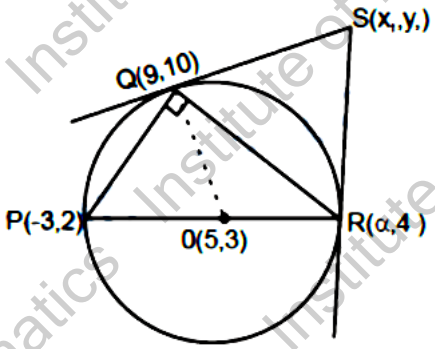
Answer: 3

Solution:

$m_{PQ} \cdot m_{QR} = -1$

$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \alpha = 13$

$m_{OP} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$



Equation of QS

$y - 10 = -\frac{4}{7}(x - 9)$

$\Rightarrow 4x + 7y = 106 \ldots (1)$

$m_{OR} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x - 13)$

$\Rightarrow 8x + y = 108$

Solving eq. (1) & (2)

$x_1 = \frac{25}{2} y_1 = 8$

$S(x_1, y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow k = 3$

Question167

Let the tangents at the points A(4, −11) and B(8, −5) on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to
[29-Jan-2023 Shift 1]

Options:

A. $\frac{3\sqrt{3}}{4}$

B. $2\sqrt{13}$

C. $\sqrt{13}$

D. $\frac{2\sqrt{13}}{3}$

Answer: D

Solution:

Equation of tangent at A$(4, -11)$ on circle is

$\Rightarrow 4x - 11y - 3\left(\frac{x+4}{2}\right) + 10\left(\frac{y-11}{2}\right) - 15 = 0$

$\Rightarrow 5x - 12y - 152 = 0 \ldots\ldots$ (1)

Equation of tangent at B $(8, -5)$ on circle is

$\Rightarrow 8x - 5y - 3\left(\frac{x+8}{2}\right) + 10\left(\frac{y-5}{2}\right) - 15 = 0$

$\Rightarrow 13x - 104 = 0 \Rightarrow x = 8$

put in (1) $\Rightarrow y = \frac{28}{3}$

$r = \left| \frac{3.8 + \frac{2.28}{3} - 34}{\sqrt{13}} \right| = \frac{2\sqrt{13}}{3}$

Question168

A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point S(α, β), then $4\alpha - 7\beta$ is equal to _____.
[29-Jan-2023 Shift 2]

Answer: 11

Solution:

The given line is polar or P$(2, \beta)$ w.r.t. given circle

$x^2 + y^2 - 4x - 6y - 3 = 0$

Chord or contact

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$\Rightarrow (\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0 \ldots$ (i)

$\because$ But the equation of chord of contact is given as : $x + y - 3 = 0 \ldots$ (ii)

comparing the coefficients

$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = -\left(\frac{2\alpha + 3\beta + 3}{-3}\right)$

On solving $\alpha = -6$

$\beta = -5$

Now $4\alpha - 7\beta = 11$

Question169

Let $y = x + 2$, $4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x - h)^2 + (y - k)^2 = r^2$. Then $h + k$ is equal to :
[30-Jan-2023 Shift 1]

Options:

A. 5

B. $5(1 + \sqrt{2})$

C. 6

D. $5\sqrt{2}$

Answer: A

Solution:

$L_1 : y = x + 2$, $L_2 : 4y = 3x + 6$, $L_3 : 3y = 4x + 1$

Bisector of lines L_2&L_3

$$\frac{4x - 3y + 1}{5} = \pm\left(\frac{3x - 4y + 6}{5}\right)$$

(+) $4x - 3y + 1 = 3x - 4y + 6$

$x + y = 5$

Centre lies on Bisector of $4x - 3y + 1 = 0$&

(0) $3x - 4y + 6 = 0$

$\Rightarrow h + k = 5$

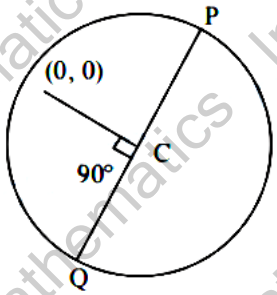
Question170

Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____.
[30-Jan-2023 Shift 2]

Answer: 24

Solution:

$\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}$; $PC = \sqrt{7}$



$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$

$= 2(5 + 7) = 24$

Question171

Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3, 2). Let C_2 be the image of C_1 in T . Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :
[31-Jan-2023 Shift 1]

Options:

A. $2(2 + \sqrt{2})$

B. $4(1 + \sqrt{2})$

C. $3+2\sqrt{2}$

D. $2(1+\sqrt{2})$

Answer: B

Solution:

$C = (2, 3), r = \sqrt{2}$

Centre of $G = A = 2 + 4\left(\frac{1}{\sqrt{2}}\right),$

$3 + \frac{4}{\sqrt{2}} = (2 + 2\sqrt{2}, 3 + 2\sqrt{2})$

$A(2 + 2\sqrt{2}, 3 + 2\sqrt{2})$

$B(4 + 2\sqrt{2}, 1 + 2\sqrt{2})$

$\frac{x-(2+2\sqrt{2})}{1} = \frac{y-(3+2\sqrt{2})}{-1} = 2$

$\therefore$ area of trapezium:

$\frac{1}{2}(4+4\sqrt{2})2 = 4(1+\sqrt{2})$

Question172

The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$ is equal to:

[31-Jan-2023 Shift 2]

Options:

A. $(8, \infty)$

B. $(4, \infty)$

C. $(0, 4]$

D. $(2, 12]$

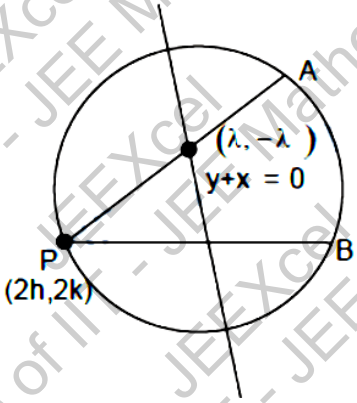
Answer: A

Solution:

$x^2 + y^2 - \frac{(1+a)x}{2} - \frac{(1-a)y}{2} = 0$

Centre $\left(\frac{1+a}{4}, \frac{1-a}{4}\right) \Rightarrow (h, k)$

$P\left(\frac{1+a}{2}, \frac{1-a}{2}\right) \Rightarrow (2h, 2k)$



Equation of chord $\Rightarrow T = S_1$

$\Rightarrow (x-y)\lambda - \frac{2h(x+\lambda)}{2} - \frac{(2k)(y-\lambda)}{2}$

$= 2\lambda^2 - 2h(\lambda) + 2k\lambda$

Now, $\lambda(2h, 2k)$ satisfies the chord

$\therefore (2h-2k)\lambda - h(x+\lambda) - k(y-\lambda)$

$\Rightarrow 2\lambda^2 + 4k\lambda - 4h\lambda + h\lambda - k\lambda + hx + ky = 0$

$\Rightarrow 2\lambda^2 + \lambda(3k-3h) + ky + hx = 0$

$\Rightarrow D > 0$

$\Rightarrow 9(k-h)^2 - 8(ky+hx) > 0$

$\Rightarrow 9(k-h)^2 - 8(2k^2+2h^2) > 0$

$\Rightarrow -7k^2 - 7h^2 - 18kh > 0$

$\Rightarrow 7k^2 + 7h^2 + 18kh < 0$

$\Rightarrow 7\left(\frac{1-a}{4}\right)^2 + 7\left(\frac{1+a}{4}\right)^2 + 18\left(\frac{1-a^2}{16}\right) < 0$

$\Rightarrow 7\left[\frac{2(1+a^2)}{16}\right] + \frac{18(1-a^2)}{16} < 0, \quad a^2 = t$

$\Rightarrow \frac{7}{8}(1+t) + \frac{18(1-t)}{16} < 0$

$\Rightarrow \frac{14+14t+18-18t}{16} < 0$

$\Rightarrow 4t > 32$

$t > 8 \quad a^2 > 8$

Question173

The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x + \lambda y = 4$ and $\lambda x + (1-\lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is
[24-Jan-2023 Shift 2]

Options:

A. $\sqrt{6}$

B. $2\sqrt{2}$

C. 2

D. 4

Answer: B

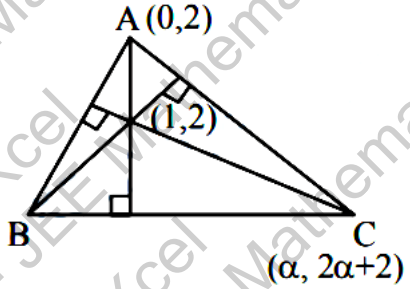
Solution:

AB : $(\lambda+1)x + \lambda y = 4$

AC : $\lambda x + (1-\lambda)y + \lambda = 0$

Vertex A is on y-axis

$\Rightarrow x = 0$



So $y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda-1}$

$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda-1}$

$\Rightarrow \lambda = 2$

AB : $3x + 2y = 4$

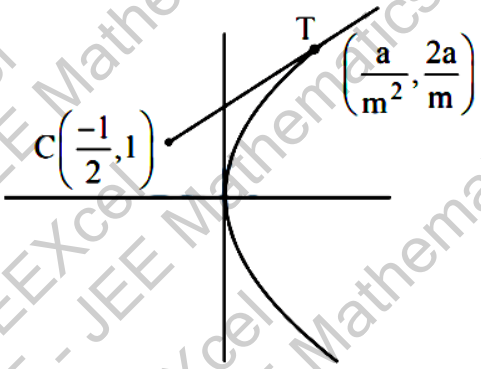
AC : $2x - y + 2 = 0$

$\Rightarrow A(0, 2)$ Let $C(\alpha, 2\alpha+2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right) = -1$

$\left(\frac{2\alpha}{\alpha-1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$

So $C\left(-\frac{1}{2}, 1\right)$



Let Equation of tangent be $y = mx + \frac{3}{2m}$

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

So tangent which touches in first quadrant at T is

$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\equiv \left(\frac{3}{2}, 3\right)$

$\Rightarrow CT = \sqrt{4+4} = 2\sqrt{2}$

Question174

The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is
[24-Jan-2023 Shift 2]

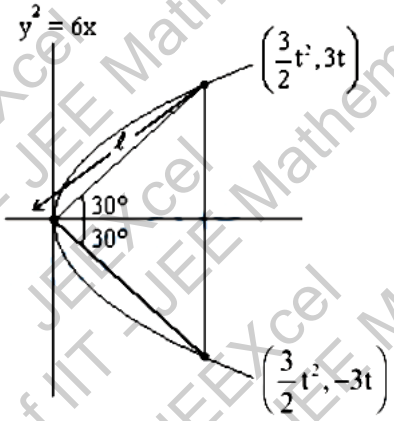
Answer: 432

Solution:

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right) + P(B)P\left(\frac{R}{B}\right) + P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10} + \frac{1}{3}\times\frac{5}{10} + \frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$\Rightarrow \lambda = 6$



$\tan 30^\circ = 3t = \frac{3}{2}t^2$

$\frac{1}{\sqrt{3}} = \frac{2}{t}$

$t = 2\sqrt{3}$

$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$

$t^2 = 18^2 + (6\sqrt{3})^2$

$= 324 + 108$

$= 432$

Question175

Let a tangent to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is
[24-Jan-2023 Shift 1]

Answer: 7

Solution:

Equation of tangent at point $P(4\cos\theta, 3\sin\theta)$ is $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$ So A is $(4\sec\theta, 0)$ and point B is $(0, 3\operatorname{cosec}\theta)$

Length $AB = \sqrt{16\sec^2\theta + 9\operatorname{cosec}^2\theta}$

$= \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta} \geq 7$

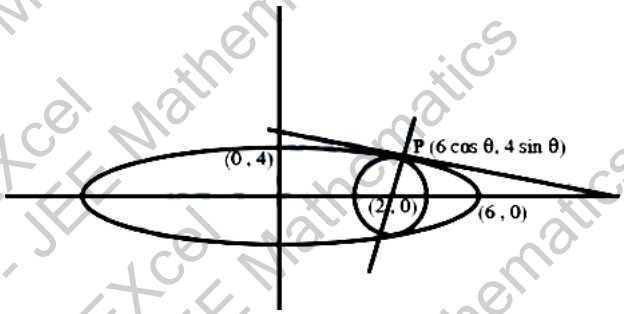
Question176

Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $= \frac{x^2}{36} + \frac{y^2}{16} = 1$.

If (1, α) lies on C, then $10\alpha^2$ is equal to____
[24-Jan-2023 Shift 1]

Answer: 118

Solution:



Equation of normal of ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at any point $P(6\cos\theta, 4\sin\theta)$ is

$3\sec\theta x - 2\operatorname{cosec}\theta y = 10$ this normal is also the normal of the circle passing through the point (2, 0) So,
$6\sec\theta = 10$ or $\sin\theta = 0$ (Not possible)

$\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$ so point $P = \left(\frac{18}{5}, \frac{16}{5}\right)$

So the largest radius of circle
$\sqrt{320}$

So the equation of circle $(x-2)^2 + y^2 = \frac{64}{5}$

Passing it through $(1, \alpha)$

Then $\alpha^2 = \frac{59}{5}$

$10\alpha^2 = 118$

Question177

If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:
[29-Jan-2023 Shift 2]

Options:

A. $\frac{9}{\sqrt{5}}$

B. $5\sqrt{3}$

C. $\frac{3}{2}\sqrt{5}$

D. $3\sqrt{5}$

Answer: D

Solution:

$y^2 = 3x$

Tangent $P(x_1, y_1)$ is parallel to $x + 2y = 1$

Then slope at $P = -\frac{1}{2}$

$2y\frac{dy}{dx} = 3$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2y} = -\frac{1}{2}$

$\Rightarrow y_1 = -3$

Coordinates of $P(3, -3)$

Similarly $Q\left(\frac{4}{\sqrt{3}}, \frac{1}{\sqrt{5}}\right), R\left(-\frac{4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

Area of $\triangle PQR$

$= \frac{1}{2}[3 \quad -3 \quad 1$

$= \frac{1}{2}\left[3\left(\frac{2}{\sqrt{5}}\right) + 3\left(\frac{8}{\sqrt{5}}\right) + 0\right] = \frac{30}{2\sqrt{5}} = 3\sqrt{5}$

Question178

If the maximum distance of normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$, $b < 2$, from the origin is 1 , then the eccentricity of the ellipse is:
[31-Jan-2023 Shift 1]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{4}$

Answer: B

Solution:

Equation of normal is

$2x\sec\theta - by\operatorname{cosec}\theta = 4 - b^2$

Distance from $(0, 0) = \dfrac{4 - b^2}{\sqrt{4\sec^2\theta + b^2\operatorname{cosec}^2\theta}}$

Distance is maximum if

$4\sec^2\theta + b^2\operatorname{cosec}^2\theta$ is minimum

$\Rightarrow \tan^2\theta = \dfrac{b}{2}$

$\Rightarrow \dfrac{4 - b^2}{\sqrt{4 \cdot \dfrac{b+2}{2} + b^2 \cdot \dfrac{b+2}{b}}} = 1$

$\Rightarrow 4 - b^2 = b + 2 \Rightarrow b = 1 \Rightarrow e = \dfrac{\sqrt{3}}{2}$

Question179

If P(h, k) be point on the parabola $x = 4y^2$, which is nearest to the point Q(0, 33), then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :
[30-Jan-2023 Shift 1]

Options:

A. 2

B. 4

C. 8

D. 6

Answer: D

Solution:

Equation of normal

$y = -tx + 2at + at^3$

$y = -tx + \dfrac{2}{16}t + \dfrac{1}{16}t^3$

It passes through $(0, 33)$

$33 = \dfrac{t}{8} + \dfrac{t^3}{16}$

$t^3 + 2t - 528 = 0$

$(t - 8)(t^2 + 8t + 66) = 0$

$t = 8$

$P(at^2, 2at) = \left(\dfrac{1}{16} \times 64, 2 \times \dfrac{1}{16} \times 8\right) = (4, 1)$

Parabola :

$y^2 = 4(x + y)$

$\Rightarrow y^2 - 4y = 4x$

$\Rightarrow (y - 2)^2 = 4(x + 1)$

Equation of directix :-

$x + 1 = -1$

$x = -2$

Distance of point $= 6$

Ans. : (4)

Question180

The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is
[25-Jan-2023 Shift 1]

Options:

A. $\frac{1}{3}$

B. 5

C. $\frac{14}{3}$

D. $5\sqrt{3}$

Answer: B

Solution:

For
$y^2 = \frac{x}{2}$, $T : y = mx + \frac{1}{8m}$
For tangent to $y^2 + 1 = x$
$\Rightarrow \left(mx + \frac{1}{8m}\right)^2 + 1 = x$
$D = 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$
$\therefore T : x - 2\sqrt{2}y + 1 = 0$
$d = \left|\frac{6 + 8 + 1}{\sqrt{9}}\right| = 5$

Question181

Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to
[30-Jan-2023 Shift 2]

Options:

A. 64

B. 76

C. 81

D. 72

Answer: D

Solution:

$y = mx + \frac{4}{m}$
$\frac{\left|\frac{4}{m}\right|}{\sqrt{1 + m^2}} = 2\sqrt{2} \therefore m = \pm 1$
Let $m = 1$, $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
R(4, 8)
Point of contact on circle Q(−2, 2)
$\therefore (QR)^2 = 36 + 36 = 72$

Question182

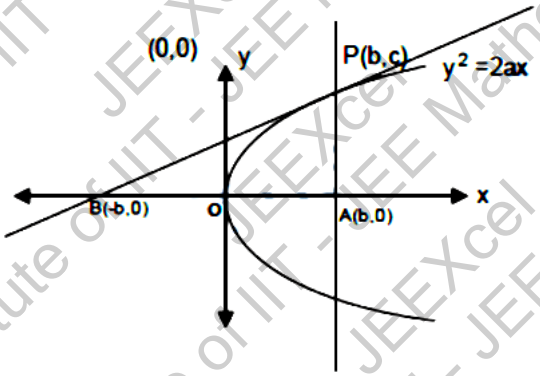
Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point P(b, c), $b, c \in N$, on the parabola $y^2 = 2ax$ and the lines $x = b$, $y = 0$ is 16 unit ^{2}, then $\sum_{a \in S} a$ is equal to

_____.

[31-Jan-2023 Shift 2]

Answer: 146

Solution:



As P(b, c) lies on parabola so $c^2 = 2ab$ . . . (1)

Now equation of tangent to parabola $y^2 = 2ax$ in point form is $yy_1 = 2a\frac{(x + x_1)}{2}$, $(x_1, y_1) = (b, c)$

$\Rightarrow yc = a(x + b)$

For point B, put $y = 0$, now $x = -b$

So, area of $\triangle PBA$, $\frac{1}{2} \times AB \times AP = 16$

$\Rightarrow \frac{1}{2} \times 2b \times c = 16$

$\Rightarrow bc = 16$

As b and c are natural number so possible values of (b, c) are (1, 16), (2, 8), (4, 4), (8, 2) and (16, 1)

Now from equation (1) $a = \frac{c^2}{2b}$ and $a \in N$, so values of (b, c) are (1, 16), (2, 8) and (4, 4) now values of are 128,16 and 2 .

Hence sum of values of a is 146.

Question183

The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x + a}{y - 2} = 0$, $y(1) = 0$ is 4π. Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

[1-Feb-2023 Shift 1]

Options:

A. $2\sqrt{3}$

B. $\frac{2\sqrt{3}}{3}$

C. 2

D. $\frac{4\sqrt{3}}{3}$

Answer: D

Solution:

$\frac{dy}{dx} + \frac{x + a}{y - 2} = 0$

$\frac{dy}{dx} = \frac{x + a}{2 - y}$

$(2 - y)\,dy = (x + a)\,dx$

$2y\frac{-y}{2} = \frac{x^2}{2} + ax + c$

$a + c = -\frac{1}{2}$ as $y(1) = 0$

$X^2 + y^2 + 2ax - 4y - 1 - 2a = 0$

$\pi r^2 = 4\pi$

$r^2 = 4$

$4 = \sqrt{a^2 + 4 + 1 + 2a}$

$(a + 1)^2 = 0$

P, Q = $(0, 2 \pm \sqrt{3})$

Equation of normal at P, Q are $y - 2 = \sqrt{3}(x - 1)$

$y - 2 = -\sqrt{3}(x - 1)$

$R = \left(1 - \frac{2}{\sqrt{3}}, 0\right)$

$S = \left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$RS = \frac{4}{\sqrt{3}} = 4\frac{\sqrt{3}}{3}$

Question184

Let A be the area bounded by the curve $y = x\,|\,x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to _____.
[1-Feb-2023 Shift 1]

Answer: 62

Solution:

$A = \int_{-1}^{0} (x^2 - 3x)\,dx + \int_{0}^{2} (3x - x^2)\,dx$

$\Rightarrow A = \frac{x^3}{3} - \frac{3x^2}{2}\Big|_{-1}^{0} + \frac{3x^2}{2} - \frac{x^3}{3}\Big|_{0}^{2}$

$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$

$\therefore 12A = 62$

Question185

If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :
[1-Feb-2023 Shift 2]

Options:

A. $A^{30} - A^{25} = 2I$

B. $A^{30} + A^{25} + A = I$

C. $A^{30} + A^{25} - A = I$

D. $A^{30} = A^{25}$

Answer: C

Solution:

$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^{25} = A$

$A^{25} - A = 0$

Question186

Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to :
[1-Feb-2023 Shift 2]

Options:

A. -3

B. 9

C. -9

D. 3

Answer: C

Solution:

$3x^2 - 4y^2 = 36 \quad 3x + 2y = 1$

$m = -\frac{3}{2}$

$m = +\frac{\sec\theta 3}{\sqrt{12}\cdot\tan\theta}$

$\Rightarrow \frac{3}{\sqrt{12}} \times \frac{1}{\sin\theta} = \frac{-3}{2}$

$\sin\theta = -\frac{1}{\sqrt{3}}$

$(\sqrt{12}\cdot \sec\theta, 3\tan\theta)$

$\left(\sqrt{12}\cdot\frac{\sqrt{3}}{\sqrt{2}}, -3\times\frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{6}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$

Question187

If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3 , then the length of this chord is equal to _____.
[1-Feb-2023 Shift 2]

Answer: 16

Solution:

$y^2 = 8x + 4y + 4$
$(y-2)^2 = 8(x+1)$
$y^2 = 4ax$
$a = 2,\ x = x+1,\ Y = y-2$
focus $(1, 2)$
$y - 2 = m(x-1)$
Put $(3, 0)$ in the above line $m = -1$
Length of focal chord $= 16$

Question188

The line x = 8 is the directrix of the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3\,PQ)^2$ is equal to _____.
[1-Feb-2023 Shift 2]

Answer: 39

Solution:

$\frac{a}{e} = 8 \ldots (1)$
$ae = 2 \ldots (2)$
$8e = \frac{2}{e}$
$e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$
$a = 4$
$b^2 = a^2(1 - e^2)$
$= 16\left(\frac{3}{4}\right) = 12$
$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$
$\sin\theta = \frac{1}{2}$
$\theta = 30^\circ$
$P(2\sqrt{3}, \sqrt{3})$
$Q\left(\frac{8}{\sqrt{3}}, 0\right)$
$(3\,PQ)^2 = 39$

Question189

If the area of the region $S = \{(x, y) : 2y - y^2 \le x^2 \le 2y,\ x \ge y\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to _____.
[6-Apr-2023 shift 1]

Answer: 5

Solution:

$x^2+y^2-2y \geq 0\ x^2-2y \leq 0, x \geq y$

Hence required area $= \frac{1}{2} \times 2 \times 2 - \int_0^2 \frac{x^2}{2} dx - \left(\frac{\pi}{4} - \frac{1}{2} \right)$

$= \frac{7}{6} - \frac{\pi}{4} \Rightarrow n = 5$

Question190

The area bounded by the curves $y = |x-1| + |x-2|$ and $y = 3$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 5

B. 4

C. 6

D. 3

Answer: B

Solution:

$y = |x-1| + |x-2|$

$A = \frac{1}{2}[1+3][2]$

$= 4$

Ans. Option 2

Question191

The area of the region $\{(x, y) : x^2 \leq y \leq 8 - x^2, y \leq 7\}$ is.
[8-Apr-2023 shift 1]

Options:

A. 24

B. 21

C. 20

D. 18

Answer: C

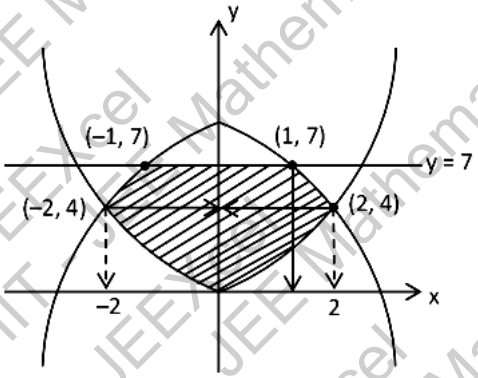
Solution:

$y \geq x^2\ \ y \leq 8 - x^2\ \ y \leq 7$

$x^2 = 8 - x^2$

$x^2 = 4$

$x = \pm 2$



$$2\left(1.7+\int_1^2 (8-2x^2)\,dx\right)-2\int_0^1 (x^2)\,dx$$

$$=2\left[7+\left(8x-\frac{2x^3}{3}\right)_1^2\right]-2\left(\frac{x^3}{3}\right)_0^1$$

$$=2\left[7+\left(16-\frac{16}{3}\right)-\left(8-\frac{2}{3}\right)\right]-2\left(\frac{1}{3}\right)$$

$$=2\left[7+\frac{32}{3}-\frac{22}{3}\right]=2\left[7+\frac{10}{3}\right]\frac{-2}{3}$$

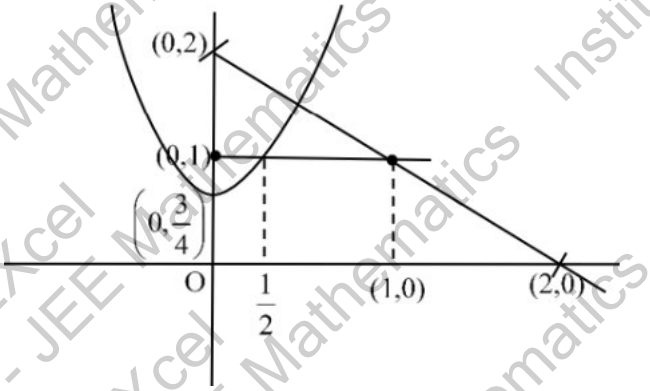
$$=\frac{60}{3}=20$$

Question192

Let the area enclosed by the lines x + y = 2, y = 0x = 0 and the curve f (x) = min $\left\{x^2+\frac{3}{4}, 1+[x]\right\}$ where [x] denotes the greatest integer ≤x, be A. Then the value of 12A is _____.
[8-Apr-2023 shift 2]

Answer: 17

Solution:



$$\int_0^{\frac{1}{2}}\left(x^2+\frac{3}{4}\right)dx+\frac{1}{2}\times\left(\frac{3}{2}+\frac{1}{2}\right)\times 1$$

$$=\left[\frac{x^3}{3}+\frac{3x}{4}\right]_0^{\frac{1}{2}}+1$$

$$A=\frac{1}{24}+\frac{3}{8}+1$$

$$12A=\frac{1}{2}+\frac{36}{8}+12$$

$$=\frac{1}{2}+\frac{9}{2}+12$$

$$=5+12$$

$$=17$$

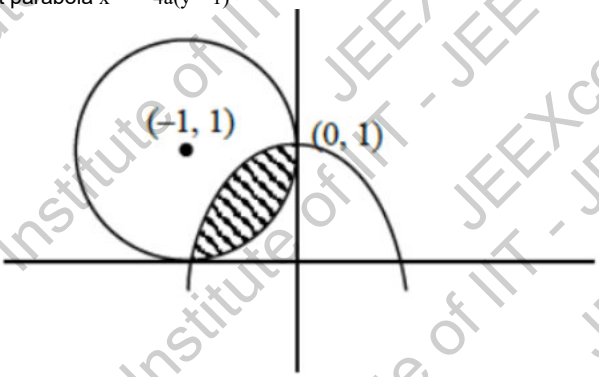
Question193

Let y = p(x) be the parabola passing through the points (−1, 0), (0, 1) and (1, 0). If the area of the region $\{(x, y) : (x+1)^2 + (y-1)^2 \le 1, y \le p(x)\}$ is A, then $12(\pi - 4A)$ is equal to _____.
[10-Apr-2023 shift 1]

Answer: 16

Solution:

There can be infinitely many parabolas through given points.
Let parabola $x^2 = -4a(y-1)$



Passes through $(1, 0)$

$\therefore b = -4a(-1) \Rightarrow a = \frac{1}{4}$

$\therefore x^2 = -(y-1)$

Now area covered by parabola $= \int_{-1}^{0} (1 - x^2)\,dx$

$= \left(x - \frac{x^3}{3}\right)_1^b = (0-0) - \left\{-1 + \frac{1}{3}\right\}$

$= \frac{2}{3}$

Required Area = Area of sector − { Area of square - Area covered by Parabola }

$= \frac{\pi}{4} - \left\{1 - \frac{2}{3}\right\}$

$= \frac{\pi}{4} - \frac{1}{3}$

$\therefore 12(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right]$

$= 12\left[\pi - \pi + \frac{4}{3}\right]$

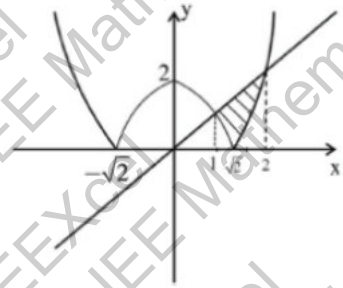
$= 16$

Question194

If the area of the region $\{(x, y) : |x^2 - 2| \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____.
[10-Apr-2023 shift 2]

Answer: 27

Solution:



$A = \int_1^{\sqrt{2}} (x - (2 - x^2))\,dx + \int_{\sqrt{2}}^{2} (x - (x^2 - 2))\,dx$

$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$

$= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2}$

$6A = -16\sqrt{2} + 27 \therefore 6A + 16\sqrt{2} = 27$

Ans. 27

Question195

Area of the region $\{(x, y) : x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is :
[11-Apr-2023 shift 1]

Options:

A. $\pi + \frac{8}{3}$

B. $2\pi + \frac{16}{3}$

C. $2\pi - \frac{16}{3}$

D. $\pi - \frac{8}{3}$

Answer: C

Solution:

$x^2 + (y-2)^2 \le 2^2$ and $x^2 \ge 2y$

Solving circle and parabola simultaneously:

$2y + y^2 - 4y + 4 = 4$

$y^2 - 2y = 0$

$y = 0, 2$

Put $y = 2$ in $x^2 = 2y \rightarrow x = \pm 2$

$\Rightarrow (2, 2)$ and $(-2, 2)$

$= 2 \times 2 - \frac{1}{4} \cdot \pi \cdot 2^2 = 4 - \pi$

Required area $= 2\left[\int_0^2 \frac{x^2}{2}\,dx - (4 - \pi)\right]$

$= 2\left[\frac{4}{3} + \pi - 4\right]$

$= 2\left[\pi - \frac{8}{3}\right]$

$= 2\pi - \frac{16}{6}$

Question196

The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1, -1)$ is
[12-Apr-2023 shift 1]

Options:

A. $\frac{23}{4}$

B. $\frac{19}{4}$

C. $\frac{31}{4}$

D. $\frac{27}{4}$

Answer: D

Solution:

equation of tangent: $y + 1 = 3(x + 1)$
i.e. $y = 3x + 2$
Point of intersection with curve $(2, 8)$
So Area $= \int_{-1}^{2} ((3x + 2) - x^3)\,dx = \frac{27}{4}$

Question197

The area of the region enclosed by the curve f(x) = max{sin x, cos x}, $-\pi \le x \le \pi$ and the x-axis is [13-Apr-2023 shift 1]

Options:

A. $2\sqrt{2}(\sqrt{2} + 1)$

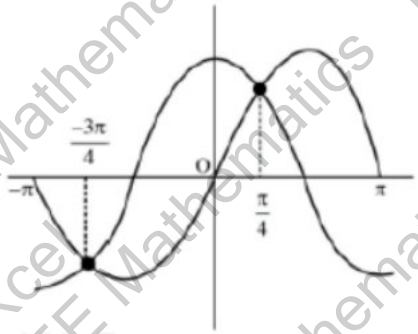
B. $4(\sqrt{2})$

C. 4

D. $2(\sqrt{2} + 1)$

Answer: C

Solution:



Area =

$$\left| \int_{-\pi}^{\frac{-3\pi}{4}} \sin x\,dx \right| + \left| \int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x\,dx \right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x\,dx + \int_{\frac{\pi}{4}}^{\pi} \sin x\,dx = 4$$

Question198

The area of the region $\{(x \cdot y) : x^2 \le y \le |x^2 - 4|, y \ge 1\}$ is: [13-Apr-2023 shift 2]

Options:

A. $\frac{3}{4}(4\sqrt{2}+1)$

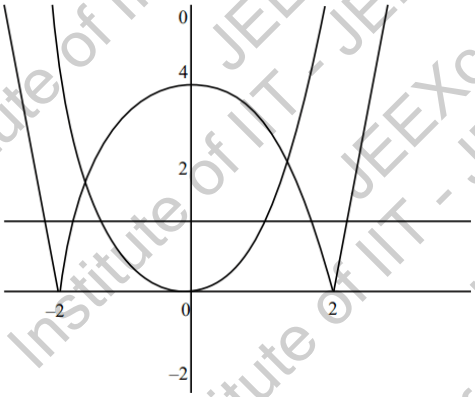
B. $\frac{4}{3}(4\sqrt{2}-1)$

C. $\frac{3}{4}(4\sqrt{2}-1)$

D. $\frac{4}{3}(4\sqrt{2}+1)$

Answer: B

Solution:



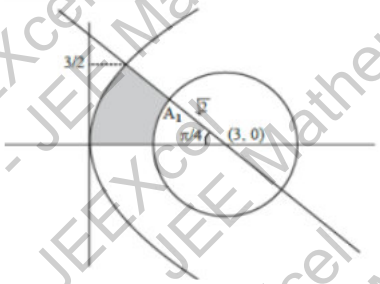
Required area $= 2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}[4\sqrt{2}-1]$

Question199

If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x-3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to _____.
[15-Apr-2023 shift 1]

Answer: 42

Solution:



$y^2 = \frac{3x}{2}$, $x + y = 3$, $y = 0$

$2y^2 = 3(3-y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

$(2y-3)(y+2) = 0$; $y = 3/2$

$$\text{Area} \left(\int_0^{\frac{3}{2}} (x_R - x_2)\,dy \right) - A_1$$

$$\int_0^{\frac{3}{2}} \left((3-y) - \frac{2y^2}{3} \right) dy - \frac{\pi}{8}(2)$$

$$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9} \right)_0^{\frac{3}{2}} - \frac{\pi}{4}$$

$$4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4} \right] = \frac{21}{2} = 10.50$$

$$\therefore 4(4A + \pi) = 42$$

Question200

Let H_n : $\frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$, $n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then 21l is equal to _____.
[11-Apr-2023 shift 1]

Answer: 306

Solution:

$$Hn \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$$

$$e = \sqrt{\frac{2n+4}{n+1}}$$

$n = 48$ (smallest even value for which $e \in Q$)

$$e = \frac{10}{7}$$

$$a^2 = n+1 \quad b^2 = n+3$$

$$= 49 \quad = 51$$

$$l = \text{length of LR} = \frac{2b^2}{a}$$

$$L = 2 \cdot \frac{51}{7}$$

$$l = \frac{102}{7}$$

$$21\ell = 306$$

Question201

Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to _____.
[11-Apr-2023 shift 2]

Answer: 116

Solution:

$\because P(3, \alpha)$ lies on $y^2 = 12x$

$\Rightarrow \alpha = \pm 6$

But, $\left.\frac{dy}{dx}\right|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6$ ($\alpha = -6$ reject)

Now, hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$, normal at

$Q(\alpha - 1, \alpha + 2)$ is $\frac{9x}{5} + \frac{36y}{8} = 45$

$\Rightarrow 2x + 5y - 50 = 0$

Now, distance of $(6, -4)$ from $2x + 5y - 50 = 0$ is equal to

$$\left|\frac{2(6) - 5(4) - 50}{\sqrt{2^2 + 5^2}}\right| = \frac{58}{\sqrt{29}}$$

$\Rightarrow$ Square of distance $= 116$

Question202

Let m_1, and m_2 be the slopes of the tangents drawn from the point P(4, 1) to the hyperbola H : $\frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____.
[13-Apr-2023 shift 1]

Answer: 8

Solution:

Equation of tangent to the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$y = mx \pm \sqrt{a^2 - b^2m^2}$

passing through $(4, 1)$

$1 = 4m \pm \sqrt{25 - 16m^2} \Rightarrow 4m^2 - m - 3 = 0$

$\Rightarrow m = 1, \frac{-3}{4}$

Equation of tangent with positive slopes $1 \& \frac{3}{4}$

$4y = 3x - 16$

$y = x - 3$ } with positive intercept on x-axis.

$\alpha = \frac{16}{3}, \beta = 3$

Intersection points:

$Q : (-4, -7)$

$P : (4, 1)$

$PQ^2 = 128$

$\frac{P^2}{\alpha\beta} = \frac{128}{16} = 8$

Question203

The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x and y axes are a and b respectively, then $|6a| + |5b|$ is equal to _____.
[13-Apr-2023 shift 2]

Answer: 12

Solution:

$2ae = 4$

$2a\left(\frac{3}{2}\right) = 4$

$\Rightarrow a = \frac{4}{3}$

$e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow \frac{9}{4} = 1 + b^2\left(\frac{9}{16}\right)$

$\Rightarrow b^2 = \left(\frac{5}{4}\right)\left(\frac{16}{9}\right) = \frac{20}{9}$

slope of tangent $m = \frac{3}{2}$ equation of tangent is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = \frac{3}{2}x \pm \sqrt{\frac{16}{9}\left(\frac{9}{4}\right) - \frac{20}{9}}$

$\Rightarrow y = \frac{3x}{2} \pm \frac{4}{3}$

$y = 0 \Rightarrow a = \pm\frac{8}{9}$

$x = 0 \Rightarrow b = \pm\frac{4}{3}$

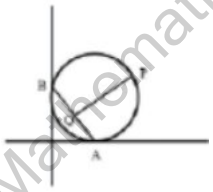
$|6a| + |5b| = \frac{16}{3} + \frac{20}{3} = 12$

Question204

A circle passing through the point P(α · β) in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then value of αβ is _____.
[6-Apr-2023 shift 1]

Answer: 121

Solution:



Let equation of circle is $(x-a)^2 + (y-a)^2 = a^2$
which is passing through $P(\alpha, \beta)$
then $(\alpha - a)2 + (\beta - a)^2 = a^2$
$\alpha^2 + \beta^2 - 2\alpha a - 2\beta\alpha + a^2 = 0$
Here equation of AB is $x + y = a$
Let $Q(\alpha', \beta')$ be foot of perpendicular of P on AB

$\frac{\alpha' - \alpha}{1} = \frac{\beta' - \beta}{1} = \frac{-(\alpha + \beta - a)}{2}$

$PQ^2 = (\alpha' - \alpha) + (\beta' - \beta) = \frac{1}{4}(\alpha + \beta - a)^2 + \frac{1}{4}(\alpha + \beta - a)^2$

$121 = \frac{1}{2}(\alpha + \beta - a)^2$

$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$
$242 = 2\alpha\beta$
$\Rightarrow \alpha\beta = 121$

Question205

If the tangents at the points P and Q are the circle $x^2 + y^2 - 2x + y = 5$ meet at the point R $\left(\frac{9}{4}, 2\right)$, then the area of the triangle PQR is :
[6-Apr-2023 shift 2]

Options:

A. $\frac{5}{4}$

B. $\frac{13}{4}$

C. $\frac{5}{8}$

D. $\frac{13}{8}$

Answer: C

Solution:

$x^2 + y^2 - 2x + y = 5$

with resperct to RPQ is x C.O.C eq^n of C.O.C is $xT = 0$

$\frac{9}{4}x + 2y - \left(x + \frac{9}{4}\right) + \frac{1}{2}(y + 2) - 5 = 0$

$\frac{5}{4}x + \frac{5}{2}y - \frac{25}{4} = 0$

$5x + 10y - 25 = 0$

$x + 2y = 5$

Area $= \frac{1}{2}(P')(PQ)$ $(PQ) = 2\sqrt{r^2 - p^2} = \sqrt{5}$

$= \frac{1}{2}\left[\frac{\sqrt{5}}{4}\right](\sqrt{5})$ $P' = \frac{\frac{9}{4} + 4 - 5}{\sqrt{5}}$

$= \frac{5}{8}$ $= \left(\frac{5}{4\sqrt{5}}\right) = \frac{\sqrt{5}}{4}$

Method II

area $= \frac{RL^3}{R^2 + L^2}$

$R = \frac{5}{2}$

$L = \sqrt{\frac{81}{16} + 4 - \frac{9}{2} + 2 - 5}$

$= \frac{5}{4}$

area $= = \frac{5}{8}$

Ans. Option 3

Question206

Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is.
[8-Apr-2023 shift 2]

Options:

A. $-\frac{1}{2}$

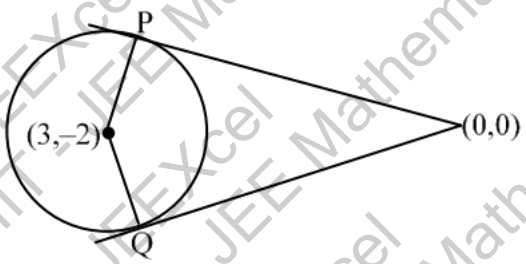
B. $\frac{5}{2}$

C. 1

D. $\frac{3}{2}$

Answer: A

Solution:



Circumcircle of $\triangle OPQ$

$(x-0)(x-3)+(y-0)(y+2)=0$

$x^2+y^2-3x+2y=0$

passes through $\left(\alpha, \frac{1}{2}\right)$

$\therefore \alpha^2+\frac{1}{4}-3\alpha+1=0$

$\Rightarrow \alpha^2-3\alpha+\frac{5}{4}=0 \Rightarrow 4\alpha^2-12\alpha+5=0$

$\Rightarrow 4\alpha^2-10\alpha-2\alpha+5=0$

$(2\alpha-1)(2\alpha-5)=0 \therefore \alpha=\frac{1}{2}, \frac{5}{2}$ Ans.

Question207

If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2+4y^2=36$ is r, then $12r^2$ is equal to
[11-Apr-2023 shift 2]

Options:

A. 69

B. 72

C. 115

D. 92

Answer: D

Solution:

C(2, 0)

Ellipse $x^2+4y^2=36$

$\frac{x^2}{36}+\frac{y^2}{9}=1$

Equation of Normal at $P(6\cos\theta, 3\sin\theta)$ is $(6\sec\theta)x-(3\operatorname{cosec}\theta)y=27$

It passes through (2, 0)

$\Rightarrow \sec\theta=\frac{27}{12}=\frac{9}{4}$

$\cos\theta\frac{4}{9}, \sin\theta=\frac{\sqrt{65}}{9}$

$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$

$\overline{P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)C(2,0)}$

$\gamma=\sqrt{\left(\frac{8}{3}-2\right)^2+\left(\frac{\sqrt{65}}{3}\right)^2}=\frac{\sqrt{69}}{3}$

Value of $12\gamma^2 = \left(\frac{\sqrt{69}}{3}\right)^2 \times 12$

$\Rightarrow \frac{12 \times 69}{9} = 92$

Question208

Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to _____.
[12-Apr-2023 shift 1]

Answer: 7

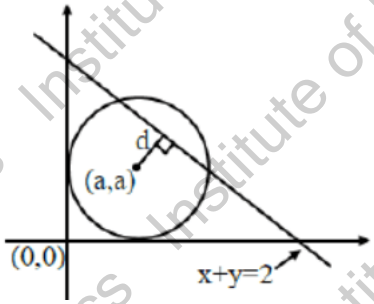
Solution:

Circle $(x-a)^2 + (y-a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

Intercept $= 2$

$\Rightarrow 2\sqrt{a^2 - d^2} = 2$



Where d = perpendicular distance of centre from line $x + y = 2$

$\Rightarrow 2\sqrt{a^2 - \left(\frac{a + a - 2}{\sqrt{2}}\right)^2} = 2$

$\Rightarrow a^2 - \frac{(2a-2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$

$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$

$\therefore r_1 + r_2 = 4$ and $r_1 r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1 r_2 = (r_1 + r_2)^2 - 3r_1 r_2$

$= 16 - 9 = 7$

Question209

Let the tangent and normal at the point $(3\sqrt{3}, 1)$ on the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x = 2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2 - \beta^2$ is equal to
[13-Apr-2023 shift 1]

Options:

A. $\frac{304}{5}$

B. 60

C. $\frac{314}{5}$

D. 61

Answer: A

Solution:

Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$

$y=4$

$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

$y=-8$

$x^2+y^2+4y-32=0$

$hx+ky+2(y+k)-32=0$

$k=-2$

$hx+2k-32=0$

$hx=36$

$\alpha=h=\frac{36}{2\sqrt{5}}$

$\beta=k=-2$

$\alpha^2-\beta^2=\frac{304}{5}$

Question210

Let the centre of a circle C be (α, β) and its radius r < 8. Let 3x + 4y = 24 and 3x − 4y = 32 be two tangents and 4x + 3y = 1 be a normal to C. Then (α − β + r) is equal to
[13-Apr-2023 shift 2]

Options:

A. 5

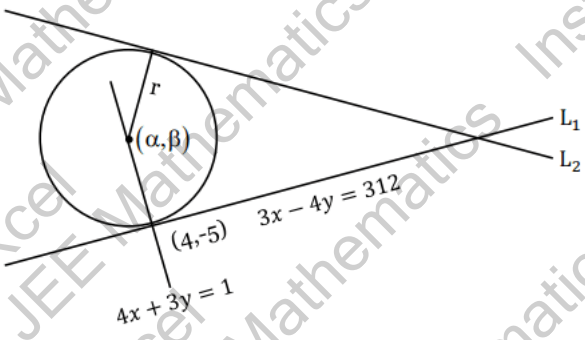
B. 6

C. 7

D. 9

Answer: C

Solution:



Centre lies on normal

$\Rightarrow 4\alpha+3\beta=1$

Distance of (α, β) from L_1 and L_2 are equal

$\left|\frac{3\alpha+4\beta-24}{5}\right|=\left|\frac{3\alpha-4\beta-32}{5}\right|$

$3\alpha+4\beta-24=-3\alpha+4\beta+32$

$\Rightarrow 6\alpha=56$

$\Rightarrow \alpha=\frac{28}{3},\ \beta=\frac{-109}{3}$

$r=\sqrt{\left(\frac{28}{3}-4\right)^2+\left(\frac{-109}{3}+5\right)^2}>8$

(reject)

$3\alpha+4\beta-24=-3\alpha-4\beta-32$

$8\beta=-8$

$\beta=-1,\ \alpha=1$

$\gamma = \sqrt{(4-1)^2 + (-5+1)^2} = 5$

$\alpha - \beta + \gamma = 7$

Question211

The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is
[15-Apr-2023 shift 1]

Options:

A. 4

B. 1

C. 3

D. 2

Answer: C

Solution:

$C_1\left(9, \frac{15}{2}\right) r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$

$C_2(3, 3) r_2 = 5$

$C_1C_2 = \sqrt{6^2 + \frac{81}{4}} = \frac{15}{2}$

$r_1 + r_2 = \frac{15}{2}$

$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

Number of common tangents = 3

Question212

Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is
[8-Apr-2023 shift 1]

Options:

A. 325

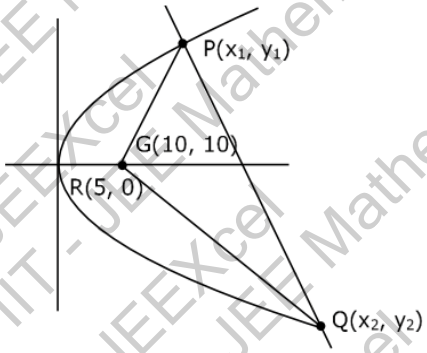
B. 346

C. 296

D. 317

Answer: A

Solution:



$y^2 = 20x$, $y = mx + c$

$y^2 = 20\left(\frac{y-c}{m}\right)$

$y^2 - \frac{20y}{m} + \frac{20c}{m} = 0$ $\quad \frac{y_1+y_2+y_3}{3} = 10$

$\frac{20}{m} = 30$

$m = 2/3$

and $c - m = 6$

$c = \frac{2}{3} + 6 \Rightarrow \frac{20}{3} = c$

$y^2 - 30y + \frac{20 \times 20/3}{2/3} = 0 \Rightarrow \quad y^2 - 30y + 200 = 0$

$y = 10, y = 20$

$y = 20, x = 20$ $\quad P(5, 10); (20, 20)Q$

$\frac{20+5+x}{3} = 10 \Rightarrow x = 5$ $\quad P^2 = 15^2 + 10^2 = 225 + 100 = 325$

Question213

The ordinates of the points P and Q on the parabola with focus (3.0) and directrix $x = -3$ are in the ratio 3 : 1:. If R(α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to _____.
[8-Apr-2023 shift 2]

Answer: 16

Solution:

Parabola is $y^2 = 12x$

Let $Q(3t^2, 6t)$

so $P(27t^2, 18t)$

$R(\alpha, \beta) = (at_1t_2, a(t_1 + t_2))$

$= (3t \cdot 3t, 3(t + 3t))$

$R(\alpha, \beta) = (9t^2, 12t)$

$R(\alpha, \beta) = (9t^2, 12t)$

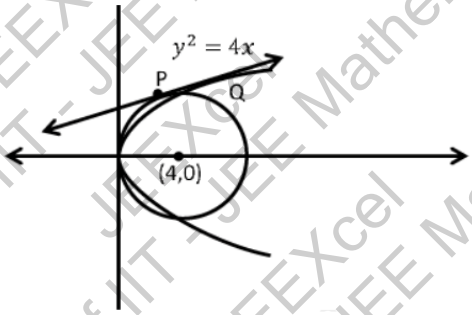
$\frac{\beta^2}{\alpha} = \frac{(12t)^2}{9t^2} = \frac{144}{9} = 16$

Question214

Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to _____.
[10-Apr-2023 shift 1]

Answer: 32

Solution:



$y^2 = 4x$

$(x-4)^2 + y^2 = 16$

Let equation of tangent of parabola

$y = mx + 1/m$

Now equation 1 also touches the circle

$\therefore \left| \frac{4m + 1/m}{\sqrt{1+m^2}} \right| = 4$

$(4m + 1/m)^2 = 16 + 16m^2$

$16m^4 + 8m^2 + 1 = 16m^2 + 16m^4$

$8m^2 = 1$

$m^2 = 1/8 \quad \{m^4 = 0\}(m \to \infty)$

For distinct points consider only $m^2 = 1/8$.

Point of contact of parabola

$P(8, 4\sqrt{2})$

$\therefore PQ = \sqrt{S_1} \Rightarrow (PQ)^2 = S_1$

$= 16 + 32 - 16 = 32$

Question215

Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100 , making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM : MQ = 3 : 1. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ ?
[13-Apr-2023 shift 1]

Options:

A. (3, 33)

B. (6, 29)

C. (−6, 45)

D. (−3, 43)

Answer: D

Solution:

$9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow P(81, 54) \& Q(1, -6)$

$M(21, 9)$

$\Rightarrow L \text{ is } (y-9) = \frac{-4}{3}(x-21)$

$3y - 27 = -4x + 84$
$4x + 3y = 111$

Question216

Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____:
[6-Apr-2023 shift 2]

Answer: 2

Solution:

$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow e$

$H: x^2 - y^2 = \frac{1}{2} \Rightarrow e' = \sqrt{2}$

$e = \frac{1}{\sqrt{2}}$

$\because e^2 = \frac{1}{2}$

$1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

$a^2 = 2b^2$

E&H are at right angle
they are confocal
Focus of Hyperbola = focus of ellipse

$\left(\pm \frac{1}{\sqrt{2}} \cdot \sqrt{2}, 0\right) = \left(\pm \frac{a}{\sqrt{2}}, 0\right)$

$a = \sqrt{2}$

$\because a^2 = 2b^2 \Rightarrow b^2 = 1$

Length of LR $= \frac{2b^2}{a} = \frac{2(1)}{\sqrt{2}} = \sqrt{2}$

Square of LR $= 2$

Question217

Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x-and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then $m - n$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. 16

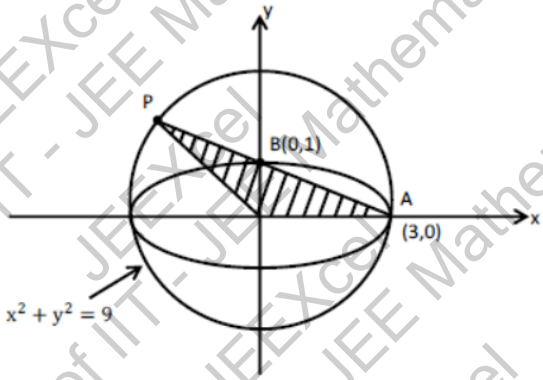
B. 15

C. 18

D. 17

Answer: D

Solution:



Equation of line AB or AP is

$\frac{x}{3} + \frac{y}{1} = 1$

$x + 3y = 3$

$x = (3 - 3y)$

Intersection point of line AP & circle is $P(x_0, y_0)$

$x^2 + y^2 = 9 \Rightarrow (3 - 3y)^2 + y^2 = 9$

$\Rightarrow 3^2(1 + y^2 - 2y) + y^2 = 9$

$\Rightarrow 5y^2 - 9y = 0 \Rightarrow y(5y - 9) = 0$

$\Rightarrow y = 9/5$

Hence $x = 3(1 - y) = 3\left(1 - \frac{9}{5}\right) = 3\left(\frac{-4}{5}\right)$

$x = \frac{-12}{5}$

$P(x_0, y_0) = \left(\frac{-12}{5}, \frac{9}{5}\right)$

Area of $\triangle AOP$ is $= \frac{1}{2} \times OA \times$ height

Height $= 9/5$, $OA = 3$

$= \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10} = \frac{m}{n}$

Compare both side $m = 27$, $n = 10 \Rightarrow m - n = 17$

Therefore, correct answer is option-D

Question218

Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle.
[10-Apr-2023 shift 2]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

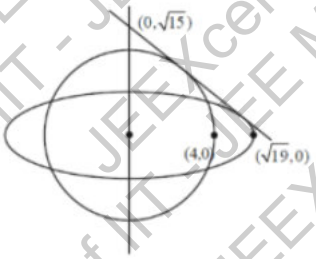
C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

$\frac{x^2}{19} + \frac{y^2}{15} = 1$



Let tang be

$y = mx \pm \sqrt{19m^2 + 15}$

$mx - y \pm \sqrt{19m^2 + 15} = 0$

Parallel from $(0, 0) = 4$

$\left| \frac{\pm \sqrt{19m^2 + 15}}{\sqrt{m^2 + 1}} \right| = 4$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$m = \pm \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ with x-axis

Required angle $\frac{\pi}{3}$.

Question219

Consider ellipse $E_k : kx^2 + k^2y^2 = 1$, k = 1, 2, ..., 20. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

[11-Apr-2023 shift 1]

Options:

A. 3320

B. 3210

C. 3080

D. 2870

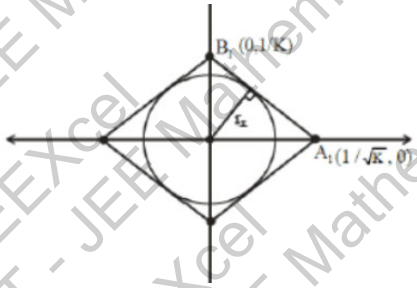
Answer: C

Solution:

K $Kx^2 + K^2y^2 = 1$

$\frac{x^2}{1/K} + \frac{y^2}{1/K^2} = 1$

Now



Equation of

A_1B_2; $\frac{x}{1/\sqrt{K}} + \frac{y}{1/K} = 1 \Rightarrow \sqrt{K}x + Ky = 1$

$r_K = \perp r$ distance of $(0, 0)$ from line A_1B_1

$$r_K = \left| \frac{(0+0-1)}{\sqrt{K+K^2}} \right| = \frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2} = K + K^2 \Rightarrow \sum_{k=1}^{20} \frac{1}{r_K^2} = \sum_{K=1}^{20} (K + K^2)$$

$$= \sum_{K=1}^{20} K + \sum_{K=1}^{20} K^2$$

$$= \frac{20 \times 21}{2} + \frac{20.21.41}{6}$$

$$= 210 + 10 \times 7 \times 41$$

$$= 210 + 2870$$

$$= 3080$$

Question220

Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R, and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then p + q is equal to
[12-Apr-2023 shift 1]

Options:

A. 137

B. 143

C. 157

D. 147

Answer: C

Solution:

Let $R(2\cos\theta, 3\sin\theta)$
As $OP \perp OR$

$$\text{so } \frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$$

$$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$$

$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right)$ or $R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$

Now $= \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$

$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{144}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$

$= \frac{13}{144}$

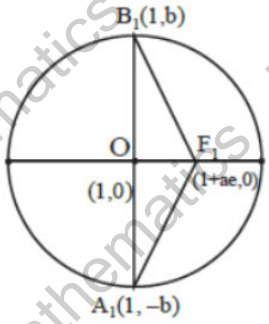
$\Rightarrow p + q = 157$

Question221

Let an ellipse with centre (1, 0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to _____
[15-Apr-2023 shift 1]

Answer: 9

Solution:



L.R. $= \frac{2b^2}{a} = \frac{1}{2}$

$4b^2 = a \ldots$ (i)

Ellipse $\frac{(x-1)^2}{a^2} + \frac{y^2}{b^2} = 1$

$m_{B_1F_1} = \frac{1}{\sqrt{3}}$

$\frac{b}{ae} = \frac{1}{\sqrt{3}}$

$3b^2 = a^2e^2 = a^2 - b^2$

$4b^2 = a^2 \ldots$ (ii)

From (i) and (ii)

$a = a^2$

$\therefore a = 1$

$b^2 = \frac{1}{4}$

$((2a) + (2b))^2 = 9$

Question222

Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to
[24-Jun-2022-Shift-1]

Options:

A. 16

B. 88/5

C. 72

D. −8

Answer: A

Solution:

For tangent to parabola $y = x^2$ at $(2, 4)$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = 4$$

Equation of tangent is $y - 4 = 4(x - 2)$

$$\Rightarrow 4x - y - 4 = 0$$

Family of circle can be given by

$$(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$$

As it passes through $(0, 6)$

$$2^2 + 2^2 + \lambda(-10) = 0$$

$$\Rightarrow \lambda = \frac{4}{5}$$

Equation of circle is

$$(x-2)^2 + (y-4)^2 + \frac{4}{5}(4x - y - 4) = 0$$

$$\Rightarrow (x^2 + y^2 - 4x - 8y + 20) + \left(\frac{16}{5}x - \frac{4}{5}y - \frac{16}{5}\right) = 0$$

$$A = -4 + \frac{16}{5},\ C = 20 - \frac{16}{5}$$

So, $A + C = 16$

Question223

Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. -2

B. -4

C. 2

D. 4

Answer: D

Solution:

$$\frac{x^2}{\left(\frac{b^2}{a^2}\right)} - \frac{y^2}{b^2} = 1$$

Tangent in slope form $\Rightarrow y = mx \pm \sqrt{\frac{b^2}{a^2}m^2 - b^2}$

i.e., same as $y = \frac{\lambda x}{2} - \frac{\mu}{2}$

Comparing coefficients,

$m = \frac{\lambda}{2}$, $\frac{b^2}{a^2}m^2 - b^2 = \frac{\mu^2}{4}$

Eliminating m, $\frac{b^2}{a^2} \cdot \frac{\lambda^2}{4} - b^2 = \frac{\mu^2}{4}$

$\Rightarrow \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$

Question224

If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals
[24-Jun-2022-Shift-1]

Answer: 2929

Solution:

$\because (\alpha, \beta)$ lies on the given ellipse, $25\alpha^2 + 4\beta^2 = 1 \quad \ldots (i)$

Tangent to the parabola, $y = mx + \frac{1}{m}$ passes through (α, β). So, $\alpha m^2 - \beta m + 1 = 0$ h

$m_1 + 4m_1 = \frac{\beta}{\alpha}$ and $m_1 \cdot 4m_1 = \frac{1}{\alpha}$

Gives that $4\beta^2 = 25\alpha \quad \ldots (ii)$

from (i) and (ii)

$25(\alpha^2 + \alpha) = 1 \quad \ldots (iii)$

Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$

$= 25(2\alpha + 1)^2 + 2500(2\alpha + 1)^2$

$= 2525(4\alpha^2 + 4\alpha + 1)$ from equation (iii)

$= 2525\left(\frac{4}{25} + 1\right)$

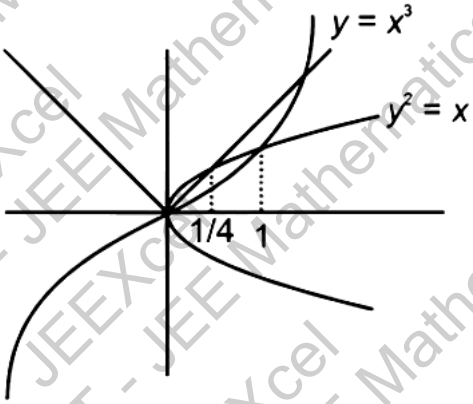
$= 2929$

Question225

Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1, R_2. If $\max\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to
[24-Jun-2022-Shift-1]

Answer: 19

Solution:



$C_1 : y = x^3$

$C_2 : y^2 = x$

and $C_3 = y = 2|x|$

C_1 and C_2 intersect at $(1, 1)$

C_2 and C_3 intersect at $\left(\frac{1}{4}, \frac{1}{2}\right)$

Clearly $R_1 = \int_0^{1/4} (\sqrt{x} - 2x)dx = \frac{2}{3}\left(\frac{1}{8}\right) - \frac{1}{16} = \frac{1}{48}$

and $R_1 + R_2 = \int_0^1 (\sqrt{x} - x^3)dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

So, $\frac{R_1 + R_2}{R_1} = \frac{5/12}{1/48} \Rightarrow 1 + \frac{R_2}{R_1} = 20$

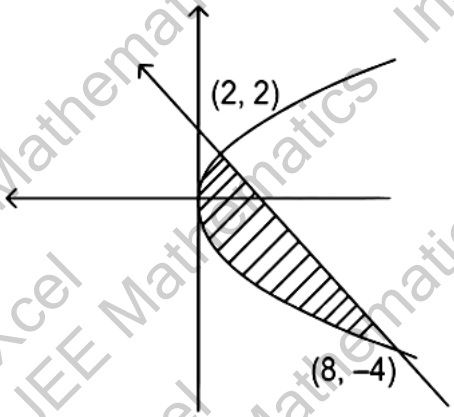
$\Rightarrow \frac{R_2}{R_1} = 19$

Question226

The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is ___
[24-Jun-2022-Shift-2]

Answer: 18

Solution:



The required area $= \int_{-4}^{2} \left(4 - y - \frac{y^2}{2}\right) dy$

$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$

= 18 square units

Question227

The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is
[25-Jun-2022-Shift-2]

Options:

A. $\frac{1}{3}$

B. $\frac{1}{6}$

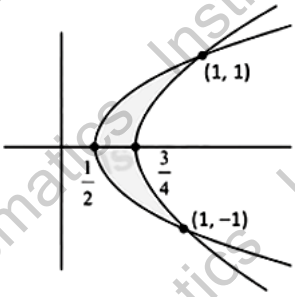
C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: A

Solution:

Area of the shaded region



$$= 2\int_0^1 \left(\frac{y^2+3}{4} - \frac{y^2+1}{2} \right) dy$$

$$= 2\int_0^1 \left(\frac{1}{4} - \frac{y^2}{4} \right) dy$$

$$= 2\left[\frac{1}{4} - \frac{1}{12} \right] = \frac{1}{3}$$

Question228

The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is :
[26-Jun-2022-Shift-1]

Options:

A. $4(2\sqrt{3} + \sqrt{6} - 4)$

B. $4(4\sqrt{3} + \sqrt{6} - 4)$

C. $8(4\sqrt{3}+3\sqrt{6}-9)$

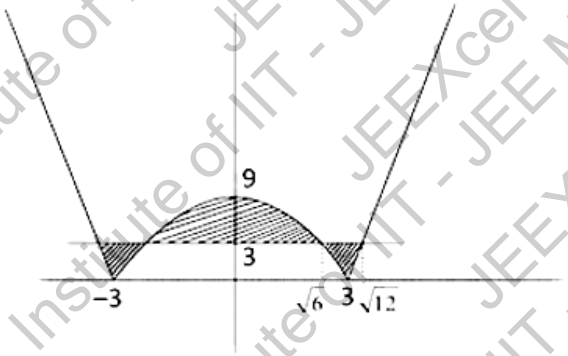
D. $8(4\sqrt{3}+\sqrt{6}-9)$

Answer: D

Solution:

$y=3$ and $y=|x^2-9|$

Intersect in first quadrant at $x=\sqrt{6}$ and $x=\sqrt{12}$



Required area

$$=2\left[\frac{2}{3}(6\times\sqrt{6})+\int_{\sqrt{6}}^{3}(3-(9-x^2))dx+\int_{3}^{\sqrt{12}}(3-(x^2-9))dx\right]$$

$$=2\left[4\sqrt{6}+.\left(\frac{x^3}{3}-6x\right)|_{\sqrt{6}}^{3}+.\left(12x-\frac{x^3}{3}\right)|_{3}^{\sqrt{12}}\right]$$

$$=2[4\sqrt{6}+(4\sqrt{6}-9)+(8\sqrt{12}-27)]$$

$$=2[8\sqrt{6}+16\sqrt{3}-36]=8[2\sqrt{6}+4\sqrt{3}-9]$$

Question229

The area of the region bounded by $y^2=8x$ and $y^2=16(3-x)$ is equal to:
[26-Jun-2022-Shift-2]

Options:

A. $\frac{32}{3}$

B. $\frac{40}{3}$

C. 16

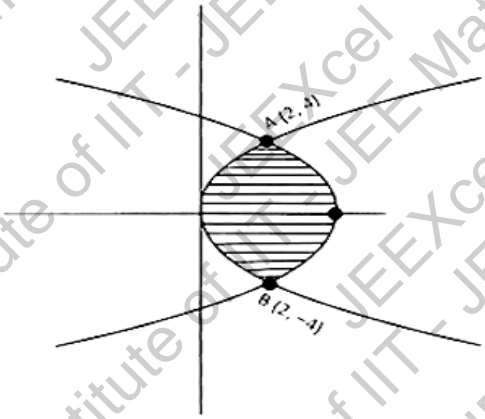
D. 19

Answer: C

Solution:

$c_1 : y^2 = 8x$

$c_2 : y^2 = 16(3-x)$



Solving c_1 and c_2

$48 - 16x = 8x$

$x = 2$

$\therefore y = \pm 4$

$\therefore$ Area of shaded region

$$= 2\int_0^4 \left\{ \left(\frac{48-y^2}{16} \right) - \left(\frac{y^2}{8} \right) \right\} dy$$

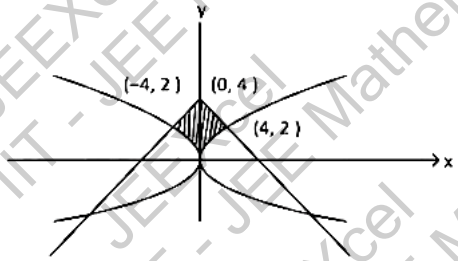
$$= \frac{1}{8}[48y - y^3]_0^4 = 16$$

Question230

Let
$A_1 = \{(x, y): |x| \le y^2, |x| + 2y \le 8\}$ and
$A_2 = \{(x, y): |x| + |y| \le k\}$.
If $27(\text{Area } A_1) = 5(\text{Area } A_2)$, then k is equal to :
[27-Jun-2022-Shift-1]

Answer: 6

Solution:

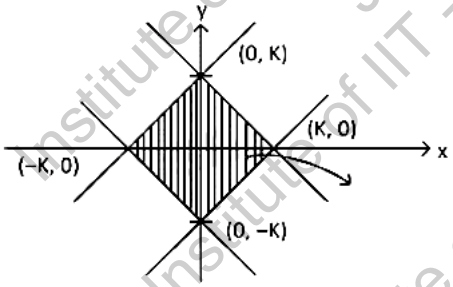


Required area (above x-axis)

$A_1 = 2\int_0^4 \left(\frac{8-x}{2} - \sqrt{x}\right) dx$

$= 2\left(16 - \frac{16}{4} - \frac{8}{3/2}\right) = \frac{40}{3}$

and $A_2 = 4\left(\frac{1}{2} \cdot k^2\right) = 2k^2$



$\therefore 27 \cdot \frac{40}{3} = 5 \cdot (2k^2)$

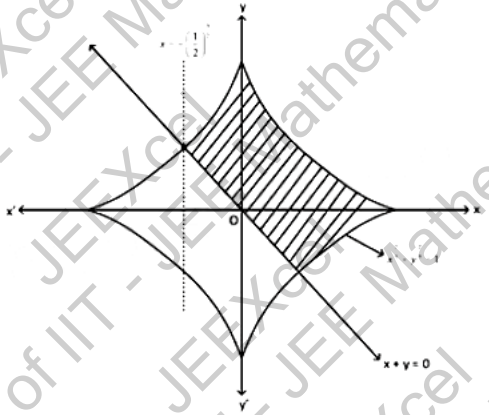
$\Rightarrow k = 6$

Question231

If the area of the region $\left\{(x, y) : x^{\frac{2}{3}} + y^{\frac{2}{3}} \leq 1, x + y \geq 0, y \geq 0\right\}$ is A, then $\frac{256A}{\pi}$ is equal to_____
[27-Jun-2022-Shift-2]

Answer: 36

Solution:



∴ Area of shaded region

$$= \int_{-\left(\frac{1}{2}\right)}^{\frac{3}{2}} \left(1 - x\frac{2}{3}\right) + x \Bigg) dx + \int_{0}^{\frac{3}{2}} \left(1 - x\frac{2}{3}\right) dx$$

$$= \int_{-\left(\frac{1}{2}\right)}^{\frac{3}{2}} dx + \int_{-\left(\frac{1}{2}\right)}^{\frac{3}{2}} x\, dx$$

Let $x = \sin^3\theta$

$\therefore dx = 3\sin^2\theta\cos\theta\, d\theta$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin^2\theta\cos^4\theta d\theta + \left(0 + \frac{1}{16}\right)$$

$$= \frac{9\pi}{64} + \frac{1}{16} - \frac{1}{16} = \frac{36\pi}{256} = A$$

$$\therefore \frac{256A}{\pi} = 36$$

Question232

The area of the region S = {(x, y) : $y^2 \le 8x$, $y \ge \sqrt{2}x$, $x \ge 1$} is [28-Jun-2022-Shift-1]

Options:

A. $\frac{13\sqrt{2}}{6}$

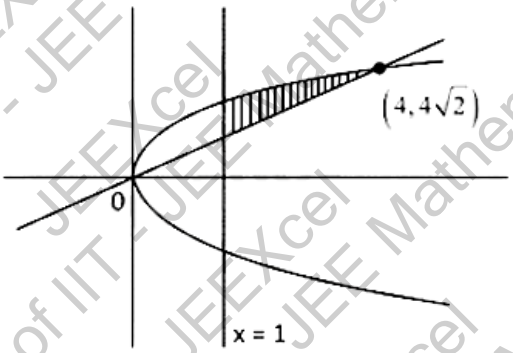
B. $\frac{11\sqrt{2}}{6}$

C. $\frac{5\sqrt{2}}{6}$

D. $\frac{19\sqrt{2}}{6}$

Answer: B

Solution:



Required area

$$= \int_1^4 (\sqrt{8x} - \sqrt{2}x)\,dx$$

$$= \frac{2\sqrt{8}}{3}x\frac{3}{2} - \frac{x^2}{\sqrt{2}}\Big|_1^4$$

$$= \frac{16\sqrt{3}}{3} - \frac{16}{\sqrt{2}} - \frac{2\sqrt{8}}{3} + \frac{1}{\sqrt{2}}$$

$$= \frac{11\sqrt{2}}{6} \text{ sq. units}$$

Question233

The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - \left|x + 1\right|$ and the x-axis is : [28-Jun-2022-Shift-2]

Options:

A. $\frac{9}{4}$

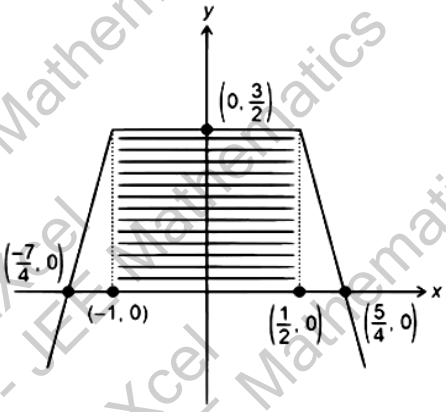
B. $\frac{45}{16}$

C. $\frac{27}{8}$

D. $\frac{63}{16}$

Answer: C

Solution:



$$y = \begin{cases} 2x - \frac{7}{2} & x < -1 \\ \frac{3}{2} & -1 \le x \le \frac{1}{2} \\ \frac{5}{2} - 2x & x > \frac{1}{2} \end{cases}.$$

$y = 3 - \left|x - \frac{1}{2}\right| - \left|x + 1\right|$

Area of shaded region (required area)

$= \frac{1}{2}\left(3 + \frac{3}{2}\right) \cdot \frac{3}{2} = \frac{27}{8}$

Question234

The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to:
[29-Jun-2022-Shift-1]

Options:

A. $\frac{16\sqrt{2}}{6}$

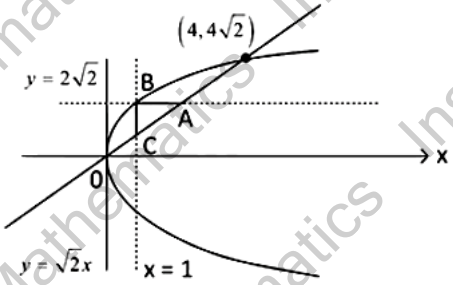
B. $\frac{11\sqrt{2}}{6}$

C. $\frac{13\sqrt{2}}{6}$

D. $\frac{5\sqrt{2}}{6}$

Answer: C

Solution:



$A(2, 2\sqrt{2})$, $B(1, 2\sqrt{2})$, $C(1, \sqrt{2})$

Area $= \int_0^{4\sqrt{2}} \left(\frac{y}{\sqrt{2}} - \frac{y^2}{8}\right) dy -$ area (ΔBAC)

$= \left[\frac{y^2}{2\sqrt{2}} - \frac{y^3}{24}\right]_0^{4\sqrt{2}} - \frac{1}{2} \times AB \times BC$

$= 8\sqrt{2} - \frac{32 \times 4\sqrt{2}}{24} - \frac{1}{2} \times 1 \times \sqrt{2}$

$= 8\sqrt{2} - \frac{16\sqrt{2}}{3} - \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{6}(48 - 32 - 3) = \frac{13\sqrt{2}}{6}$

Question235

For real numbers a, b(a > b > 0), let

Area $\{(x, y) : x^2 + y^2 \le a^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\} = 30\pi$ and

Area $\{(x, y) : x^2 + y^2 \le b^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\} = 18\pi$

Then, the value of $(a - b)^2$ is equal to____
[29-Jun-2022-Shift-2]

Answer: 12

Solution:

given $\pi a^2 - \pi ab = 30\pi$ and $\pi ab - \pi b^2 = 18\pi$
on subtracting, we get $(a - b)^2 = a^2 - 2ab + b^2 = 12$

Question236

Let the hyperbola H : $\frac{x^2}{a^2} - y^2 = 1$ and the ellipse E : $3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12(e_H^2 + e_E^2)$ is equal to
[24-Jun-2022-Shift-2]

Answer: 42

Solution:

$\because H : \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

$\therefore$ Length of latus rectum $= \frac{2}{a}$

$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$

Length of latus rectum $= \frac{6}{2} = 3$

$\because \frac{2}{a} = 3 \Rightarrow a = \frac{2}{3}$

$\therefore 12(e_H^2 + e_E^2) = 12\left(1 + \frac{9}{4}\right) + \left(1 - \frac{3}{4}\right) = 42$

Question237

Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\frac{5}{4}$. If the equation of the normal at the point

$\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ **on the hyperbola is** $8\sqrt{5}x + \beta y = \lambda$**, then** $\lambda - \beta$ **is equal to____**

[25-Jun-2022-Shift-2]

Answer: 85

Solution:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\left(e = \frac{5}{4}\right)$

So, $b^2 = a^2\left(\frac{25}{16} - 1\right) \Rightarrow b = \frac{3}{4}a$

Also $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ lies on the given hyperbola

So, $\frac{64}{5a^2} - \frac{144}{25\left(\frac{9a^2}{16}\right)} = 1 \Rightarrow a = \frac{8}{5}$ and $b = \frac{6}{5}$

Equation of normal

$\frac{64}{25}\left(\frac{x}{\frac{8}{\sqrt{5}}}\right) + \frac{36}{25}\left(\frac{y}{\frac{12}{5}}\right) = 4$

$\Rightarrow \frac{8}{5\sqrt{5}}x + \frac{3}{5}y = 4$

$\Rightarrow 8\sqrt{5}x + 15y = 100$

So, $\beta = 15$ and $\lambda = 100$

Gives $\lambda - \beta = 85$

Question238

The normal to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ **at the point** $(8, 3\sqrt{3})$ **on it passes through the point :**

[26-Jun-2022-Shift-2]

Options:

A. $(15, -2\sqrt{3})$

B. $(9, 2\sqrt{3})$

C. $(-1, 9\sqrt{3})$

D. $(-1, 6\sqrt{3})$

Answer: C

Solution:

Given hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$

$\because$ It passes through $(8, 3\sqrt{3})$

$\because \frac{64}{a^2} - \frac{27}{9} = 1 \Rightarrow a^2 = 16$

Now, equation of normal to hyperbola

$\frac{16x}{8} + \frac{9y}{3\sqrt{3}} = 16 + 9$

$\Rightarrow 2x + \sqrt{3}y = 25 \ldots\ldots$ (i)

$(-1, 9\sqrt{3})$ satisfies (i)

Question239

Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 12

Solution:

Equation of L_1 is

$\frac{x \sec\theta}{4} - \frac{y \tan\theta}{2} = 1 \ldots$ (i)

Equation of line L_2 is

$\frac{x \tan\theta}{2} + \frac{y \sec\theta}{4} = 0 \ldots .$ (ii)

$\because$ Required point of intersection of L_1 and L_2 is (x_1, y_1) then

$\frac{x_1 \sec\theta}{4} - \frac{y_1 \tan\theta}{2} - 1 = 0 \ldots\ldots$ (iii)

and $\frac{y_1 \sec\theta}{4} - \frac{x_1 \tan\theta}{2} = 0 \ldots\ldots$ (iv)

From equations (iii) and (iv)

$\sec\theta = \frac{4x_1}{x_1^2 + y_1^2}$ and $\tan\theta = \frac{-2y_1}{x_1^2 + y_1^2}$

$\therefore$ Required locus of (x_1, y_1) is

$(x^2+y^2)^2 = 16x^2 - 4y^2$

$\therefore \alpha = 16, \beta = -4$

$\therefore \alpha + \beta = 12$

Question240

Let the eccentricity of the hyperbola H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$. If y = 2x + c is a tangent to the hyperbola H, then the value of c^2 is equal to

[28-Jun-2022-Shift-1]

Options:

A. 18

B. 20

C. 24

D. 32

Answer: B

Solution:

$1 + \frac{b^2}{a^2} = \frac{5}{2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{2}$

$\frac{2b^2}{a} = 6\sqrt{2} \Rightarrow 2 \cdot \frac{3}{2} \cdot a = 6\sqrt{2}$

$\Rightarrow a = 2\sqrt{2}, b^2 = 12$

$c^2 = a^2m^2 - b^2 = 8 \cdot 4 - 12 = 20$

Question241

Let a > 0, b > 0. Let e and I respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and I' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}l$ and $(e')^2 = \frac{11}{8}l'$, then the value of 77a + 44b is equal to :
[28-Jun-2022-Shift-2]

Options:

A. 100

B. 110

C. 120

D. 130

Answer: D

Solution:

$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then

$e^2 = \frac{11}{14}l$ (l be the length of LR)

$\Rightarrow a^2 + b^2 = \frac{11}{7}b^2a$..... (i)

and $e'^2 = \frac{11}{8}l'$ (l' be the length of LR of conjugate hyperbola)

$\Rightarrow a^2 + b^2 = \frac{11}{4}a^2b$

By (i) and (ii)

$7a = 4b$

then by (i)

$\frac{16}{49}b^2 + b^2 = \frac{11}{7}b^2 \cdot \frac{4b}{7}$

$\Rightarrow 44b = 65$ and $77a = 65$

$\therefore 77a + 44b = 130$

Question242

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $a > 0$, $b > 0$, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4(2\sqrt{2} + \sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to [29-Jun-2022-Shift-1]

Answer: 88

Solution:

$2a + 2b = 4(2\sqrt{2} + \sqrt{14})$..... (1)

$1 + \frac{b^2}{a^2} = \frac{11}{14}$..... (2)

$\Rightarrow \frac{b^2}{a^2} = \frac{7}{4}$..... (3)

and $a + b = 4\sqrt{2} + 2\sqrt{14}$

By (3) and (4)

$\Rightarrow a + \frac{\sqrt{7}}{2}a = 4\sqrt{2} + 2\sqrt{14}$

$\Rightarrow \frac{a(2+\sqrt{7})}{2} = 2\sqrt{2}(2+\sqrt{7})$

$\Rightarrow a = 4\sqrt{2} \Rightarrow a^2 = 32$ and $b^2 = 56$

$\Rightarrow a^2 + b^2 = 32 + 56 = 88$

By (3) and (4)

$\Rightarrow a + \frac{\sqrt{7}}{2}a = 4\sqrt{2} + 2\sqrt{14}$

$\Rightarrow \frac{a(2+\sqrt{7})}{2} = 2\sqrt{2}(2+\sqrt{7})$

$\Rightarrow a = 4\sqrt{2} \Rightarrow a^2 = 32$ and $b^2 = 56$

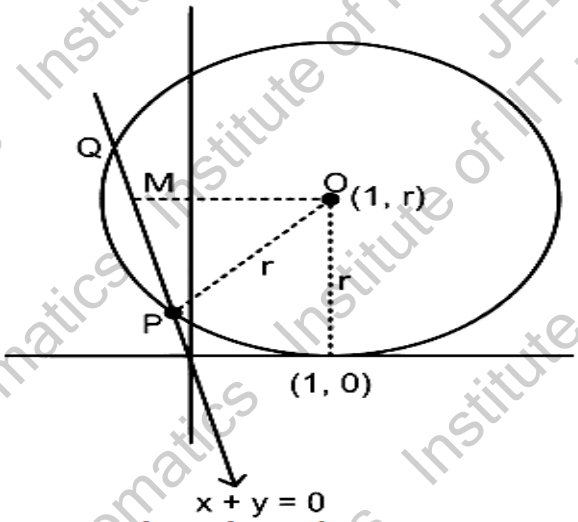
$\Rightarrow a^2 + b^2 = 32 + 56 = 88$

Question243

Let a circle C : $(x-h)^2 + (y-k)^2 = r^2$, k > 0, touch the x-axis at (1, 0). If the line x + y = 0 intersects the circle C at P and Q such that the length of the chord PQ is 2 , then the value of h + k + r is equal to [24-Jun-2022-Shift-2]

Answer: 7

Solution:



Here, $OM^2 = OP^2 - PM^2$

$\left(\frac{|1+r|}{\sqrt{2}}\right)^2 = r^2 - 1$

$\therefore r^2 - 2r - 3 = 0$

$\therefore r = 3$

$\therefore$ Equation of circle is

$(x-1)^2 + (y-3)^2 = 3^2$

$\therefore h = 1, k = 3, r = 3$

$\therefore h + k + r = 7$

Question244

Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 = 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (−1, 2) and L_2 at (3, −6), then the equation of the circle C is :
[25-Jun-2022-Shift-1]

Options:

A. $(x-1)^2 + (y-2)^2 = 4$

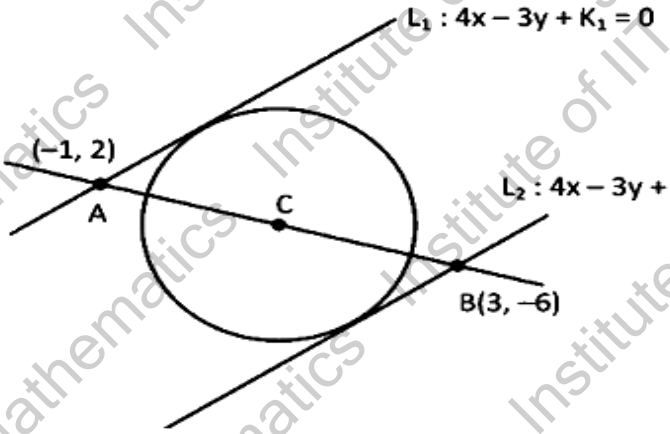
B. $(x+1)^2 + (y-2)^2 = 4$

C. $(x-1)^2 + (y+2)^2 = 16$

D. $(x-1)^2 + (y-2)^2 = 16$

Answer: C

Solution:



Co-ordinate of centre $C \equiv \left(\frac{3+(-1)}{2}, \frac{-6-2}{2}\right) \equiv (1, -2)$

L_1 is passing through A

$\Rightarrow -4 - 6 - K_1 = 0$

$\Rightarrow K_1 = 10$

L_2 is passing through B

$\Rightarrow 12 - 18 + K_2 = 0$

$\Rightarrow K_2 = -30$

Equation of $L_1 : 4x - 3y - 10 = 0$

Equation of $L_2 = 4x - 3y - 30 = 0$

Diameter of circle $= \left| \frac{10+30}{\sqrt{4^2+(-3)^2}} \right| = 8$

$\Rightarrow$ Radius $= 4$

Equation of circle $(x-1)^2+(y+2)^2=16$

Question245

Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to
[25-Jun-2022-Shift-1]

Answer: 7

Solution:

Let $P(x_1, y_1) \& Q(x_2, y_2)$

$\therefore$ Roots of $2x^2 - rx + p = 0$ are x_1, x_2

and roots of $x^2 - sx - q = 0$ are y_1, y_2

$\therefore$ Equation of circle $\equiv (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow x^2-(x_1+x_2)x+x_1x_2+y^2-(y_1+y_2)y+y_1y_2=0$

$\Rightarrow x^2-\frac{r}{2}x+\frac{p}{2}+y^2+sy-q=0$

$\Rightarrow 2x^2+2y^2-rx+2sy+p-2q=0$

Compare with $2x^2+2y^2-11x-14y-22=0$

We get $r=11,\ s=7,\ p-2q=-22$

$\Rightarrow 2r+s+p-2q=22+7-22=7$

Question246

A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is :
[25-Jun-2022-Shift-2]

Options:

A. $y = \sqrt{2}x$

B. $x = \sqrt{2}y$

C. $y^2 - x^2 = 2xy$

D. $x^2 - y^2 = 2xy$

Answer: D

Solution:

Let the centre be (h, k)

So, $|h| = \left|\frac{h+k}{\sqrt{2}}\right|$

$\Rightarrow 2h^2 = h^2 + k^2 + 2hk$

Locus will be $x^2 - y^2 = 2xy$

Question247

Let C be a circle passing through the points A(2, −1) and B(3, 4). The line segment AB s not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2 + (y-1)^2 = \frac{13}{2}$, then r^2 is equal to: [26-Jun-2022-Shift-1]

Options:

A. 32

B. $\frac{65}{2}$

C. $\frac{61}{2}$

D. 30

Answer: B

Solution:

Equation of perpendicular bisector of AB is

$$y - \frac{3}{2} = -\frac{1}{5}\left(x - \frac{5}{2}\right) \Rightarrow x + 5y = 10$$

Solving it with equation of given circle,

$$(x-5)^2\left(\frac{10-x}{5} - 1\right)^2 = \frac{13}{2}$$

$$\Rightarrow (x-5)^2\left(1 + \frac{1}{25}\right) = \frac{13}{2}$$

$$\Rightarrow x - 5 = \pm\frac{5}{2} \Rightarrow x = \frac{5}{2} \text{ or } \frac{15}{2}$$

But $x \neq \frac{5}{2}$ because AB is not the diameter.

So, centre will be $\left(\frac{15}{2}, \frac{1}{2}\right)$

Now, $r^2 = \left(\frac{15}{2} - 2\right)^2 + \left(\frac{1}{2} + 1\right)^2$

$= \frac{65}{2}$

Question248

The set of values of k, for which the circle C : $4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C, is
[27-Jun-2022-Shift-2]

Options:

A. an empty set

B. $\left(6, \frac{65}{9}\right]$

C. $\left[\frac{80}{9}, 10\right)$

D. $\left(9, \frac{92}{9}\right]$

Answer: D

Solution:

$C : 4x^2 + 4y^2 - 12x + 8y + k = 0$

$\because \left(1, -\frac{1}{3}\right)$ lies on or inside the C

then $4 + \frac{4}{9} - 12 - \frac{8}{3} + k \le 0$

$\Rightarrow k \le \frac{92}{9}$

Now, circle lies in 4th quadrant centre $\equiv \left(\frac{3}{2}, -1\right)$

$\therefore r < 1 \Rightarrow \sqrt{\frac{9}{4} + 1 - \frac{k}{4}} < 1$

$\Rightarrow \frac{13}{4} - \frac{k}{4} < 1$

$\Rightarrow \frac{k}{4} > \frac{9}{4}$

$\Rightarrow k > 9$

$\therefore k \in \left(9, \frac{92}{9}\right)$

Question249

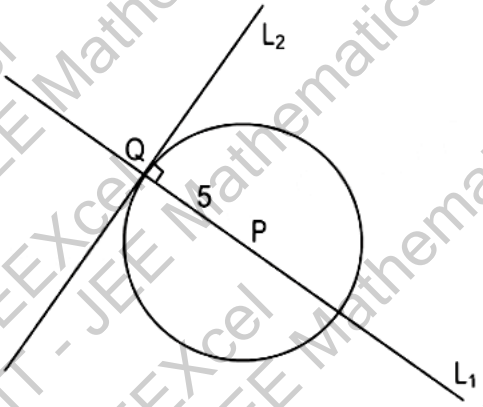
Let a circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects the line $L_2 = 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is____
[27-Jun-2022-Shift-2]

Answer: 11

Solution:

$L_1 : 4x + 3y + 2 = 0$

$L_2 : 3x - 4y - 11 = 0$



Since circle C touches the line L_2 at Q intersection point Q of L_1 and L_2, is $(1, -2)$

$\because$P lies of L_1

$\therefore P\left(x, -\frac{1}{3}(2+4x)\right)$

Now, $PQ = 5 \Rightarrow (x-1)^2 + \left(\frac{4x+2}{3} - 2\right)^2 = 25$

$\Rightarrow (x-1)^2\left[1 + \frac{16}{9}\right] = 25$

$\Rightarrow (x-1)^2 = 9$

$\Rightarrow x = 4, -2$

$\because$ Circle lies below the x-axis

$\therefore y = -6$

P(4, −6)

Now distance of P from $5x - 12y + 51 = 0$

$= \left|\frac{20 + 72 + 51}{13}\right| = \frac{143}{13} = 11$

Question250

If the tangents drawn at the points O(0, 0) and P$(1+\sqrt{5}, 2)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to
[28-Jun-2022-Shift-1]

Options:

A. $\frac{3+\sqrt{5}}{2}$

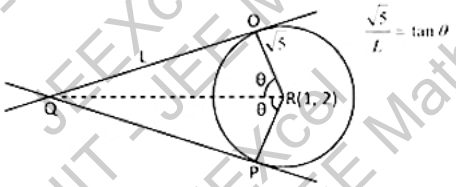
B. $\frac{4+2\sqrt{5}}{2}$

C. $\frac{5+3\sqrt{5}}{2}$

D. $\frac{7+3\sqrt{5}}{2}$

Answer: C

Solution:



$\frac{\sqrt{5}}{t} = \tan\theta$

$\tan 2\theta = 2 \Rightarrow \frac{2\tan\theta}{1 - \tan^2\theta} = 2$

$\tan\theta = \frac{\sqrt{5}-1}{2}$ (as θ is acute)

Area $= \frac{1}{2}L^2 \sin 2\theta = \frac{1}{2} \cdot \frac{5}{\tan^2\theta} \cdot 2\sin\theta\cos\theta$

$= \frac{5\sin\theta\cos\theta}{\sin^2\theta} \cdot \cos^2\theta$

$= 5\cot\theta \cdot \cos^2\theta$

$= 5 \cdot \frac{2}{\sqrt{5}-1} \cdot \frac{1}{1+\left(\frac{\sqrt{5}-1}{2}\right)^2}$

$= \frac{10}{\sqrt{5}-1} \cdot \frac{4}{4+6-2\sqrt{5}}$

$= \frac{40}{2\sqrt{5}(\sqrt{5}-1)^2} = \frac{4\sqrt{5}}{6-2\sqrt{5}}$

$= \frac{4\sqrt{5}(6+2\sqrt{5})}{16}$

$= \frac{\sqrt{5}(3+\sqrt{5})}{2}$

Question251

Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle C : $(x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to
[28-Jun-2022-Shift-1]

Answer: 816

Solution:

Normal are

$y + 2x = \sqrt{11} + 7\sqrt{7}$

$2y + x = 2\sqrt{11} + 6\sqrt{7}$

Center of the circle is point of intersection of normals i.e.

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center

i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$

Question252

If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to_____
[28-Jun-2022-Shift-2]

Answer: 10

Solution:

For $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

Radius $= \sqrt{(\sqrt{2})^2 + (3\sqrt{2})^2 - 14} = \sqrt{6}$

$\Rightarrow$ Diameter $= 2\sqrt{6}$

If this diameter is chord to

$(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$ then

$\Rightarrow r^2 = 6 + \left(\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}\right)^2$

$\Rightarrow r^2 = 6 + 4 = 10$
$\Rightarrow r^2 = 10$

Question253

Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point M (−1, 1) intersect the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N , then the area of the triangle AN B is equal to:
[29-Jun-2022-Shift-1]

Options:

A. $\frac{1}{2}$

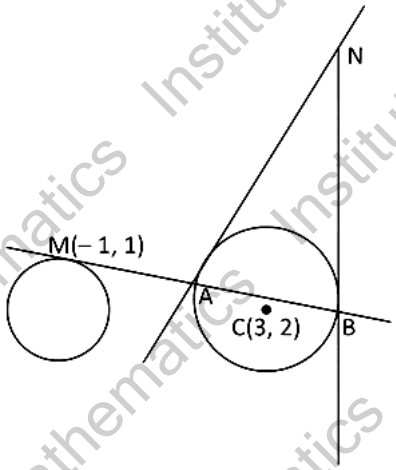
B. $\frac{2}{3}$

C. $\frac{1}{6}$

D. $\frac{5}{3}$

Answer: C

Solution:



Equation of tangent at point M is
$T = 0$
$\Rightarrow xx_1 + yy_1 = 2$
$\Rightarrow -x + y = 2$
$\Rightarrow y = x + 2$
Putting this value to equation of circle C_2,
$(x-3)^2 + (y-2)^2 = 5$
$\Rightarrow (x-3)^2 + x^2 = 5$
$\Rightarrow x^2 - 6x + 9 + x^2 = 5$
$\Rightarrow 2x^2 - 6x + 4 = 0$
$\Rightarrow x^2 - 3x + 2 = 0$
$\Rightarrow (x-2)(x-1) = 0$
$\Rightarrow x = 1, 2$
when $x = 1$, $y = 3$
and when $x = 2$, $y = 4$
$\therefore$ Point A(1, 3) and B(2, 4)
Now, equation of tangent at A(1, 3) on circle $(x-3)^2 + (y-2)^2 = 5$ or $x^2 + y^2 - 6x - 4y + 8 = 0$ is

$T = 0$
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + C = 0$
$\Rightarrow x + 3y - 3(x+1) - 2(y+3) + 8 = 0$
$\Rightarrow x + 3y - 3x - 3 - 2y - 6 + 8 = 0$
$\Rightarrow -2x + y - 1 = 0$
$\Rightarrow 2x - y + 1 = 0$
Similarly tangent at B(2, 4) is

$2x+4y-3(x+2)-2(y+4)+8=0$
$\Rightarrow 2x+4y-3x-6-2y-8+8=0$
$\Rightarrow -x+2y-6=0$
$\Rightarrow x-2y+6=0$
Solving equation (1) and (2), we get
$x-2(2x+1)+6=0$
$\Rightarrow x-4x-2+6=0$
$\Rightarrow -3x+4=0$
$\Rightarrow x=\frac{4}{3}$
$\therefore y=2\times\frac{4}{3}+1=\frac{11}{3}$
$\therefore$ Point $N=\left(\frac{4}{3},\frac{11}{3}\right)$
Now area of the triangle ANB

$$=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ \frac{4}{3} & \frac{11}{3} & 1 \end{vmatrix}$$

$=\frac{1}{2}\left[1\left(4-\frac{11}{3}\right)-3\left(2-\frac{4}{3}\right)+1\left(\frac{22}{3}-\frac{16}{3}\right)\right]$
$=\frac{1}{2}\left[\frac{1}{3}-\frac{6}{3}+\frac{6}{3}\right]$
$=\frac{1}{6}$

Question254

Let a triangle ABC be inscribed in the circle $x^2-\sqrt{2}(x+y)+y^2=0$ such that $\angle BAC=\frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the $\triangle ABC$ is equal to :
[29-Jun-2022-Shift-2]

Options:

A. 1

B. $(\sqrt{6}+\sqrt{3})/2$

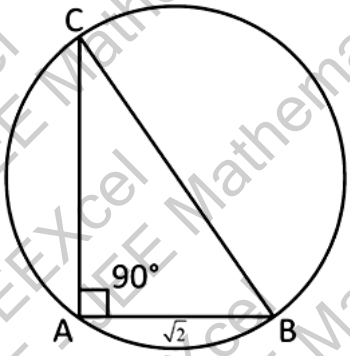
C. $(3+\sqrt{3})/4$

D. $(\sqrt{6}+2\sqrt{3})/4$

Answer: A

Solution:

For equation of circle $x^2+y^2+2gx+2fy+c=0$, center is $(-g,-f)$ and radius $r=\sqrt{g^2+f^2-c}$
Given,
equation of circle is
$x^2-\sqrt{2}(x+y)+y^2=0$
$\Rightarrow x^2+y^2-\sqrt{2}x-\sqrt{2}y=0$
$\Rightarrow x^2+y^2+2\left(-\frac{1}{\sqrt{2}}\right)x+2\left(-\frac{1}{\sqrt{2}}\right)=0$
$\therefore g=-\frac{1}{\sqrt{2}}$ and $f=-\frac{1}{\sqrt{2}}$
$\therefore$ Center $=(-g,-f)=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$
And Radius $=r=\sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2+\left(-\frac{1}{\sqrt{2}}\right)^2-0}$
$=\sqrt{\frac{1}{2}+\frac{1}{2}}=\sqrt{1}=1$



As AB and AC makes an angle 90° then line BC passes through the center of circle and BC is the diameter of the circle.
$\therefore$ Length of $BC = 2r = 2 \times 1 = 2$
$\therefore AC^2 = BC^2 - AB^2$
$= 2^2 - (\sqrt{2})^2$
$= 2$
$\Rightarrow AC = \sqrt{2}$
$\therefore$ Area of right angle triangle ABC
$= \frac{1}{2} \times AC \times AB$
$= \frac{1}{2} \times \sqrt{2}\sqrt{2}$
$= 1$ square unit.

Question255

A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with
[24-Jun-2022-Shift-2]

Options:

A. length of latus rectum 3

B. length of latus rectum 6

C. focus $\left(\frac{4}{3}, 0\right)$

D. focus $\left(0, \frac{3}{4}\right)$

Answer: A

Solution:

According to the question (Let P(x, y))

$2x - y\frac{dx}{dy} = 0$

$\left(\because \text{equation of tangent at } P : y - y = \frac{dy}{dx}(y - x)\right)$

$\therefore 2\frac{dy}{y} = \frac{dx}{x}$

$\Rightarrow 2\ln y = \ln x + \ln c$

$\Rightarrow y^2 = cx$

$\because$ this curve passes through (3, 3)

$\therefore c = 3$

$\therefore$ required parabola $y^2 = 3x$ and L.R $= 3$

Question256

Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$
[24-Jun-2022-Shift-2]

Answer: 10

Solution:

Focus $= (4, 4)$ and vertex $= (3, 2)$

$\therefore$ Point of intersection of directrix with axis of parabola $= A = (2, 0)$

Image of $A(2, 0)$ with respect to line $x + 2y = 6$ is $B(x_2, y_2)$

$$\therefore \frac{x_2 - 2}{1} = \frac{y_2 - 0}{2} = \frac{-2(2 + 0 - 6)}{5}$$

$$\therefore B(x_2, y_2) = \left(\frac{18}{5}, \frac{16}{5}\right)$$

Point B is point of intersection of direction with axes of parabola P_2.

$\therefore x + 2y = \lambda$ must have point $\left(\frac{18}{5}, \frac{16}{5}\right)$

$\therefore x + 2y = 10$

Question257

If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8 | m_1m_2 |$ is equal to :
[25-Jun-2022-Shift-1]

Options:

A. $3 + 4\sqrt{2}$

B. $-5 + 6\sqrt{2}$

C. $-4 + 3\sqrt{2}$

D. $7 + 6\sqrt{2}$

Answer: C

Solution:

Let tangent to $y^2 = x$ be

$$y = mx + \frac{1}{4m}$$

For it being tangent to circle.

$$\left| \frac{\frac{1}{4}m}{\sqrt{1+m^2}} \right| = \sqrt{2}$$

$$\Rightarrow 32m^4 + 32m^2 - 1 = 0$$

$$\Rightarrow m^2 = \frac{-32 \pm \sqrt{(32)^2 + 4(32)}}{64}$$

$$\Rightarrow 8m_1m_2 = -4 + 3\sqrt{2}$$

Question258

Let x = 2t, y = $\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that SA⊥BA, where A is any point on the conic. If k is the ordinate of the centroid of the ΔSAB, then $\lim_{t \to 1}$ k is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\frac{17}{18}$

B. $\frac{19}{18}$

C. $\frac{11}{18}$

D. $\frac{13}{18}$

Answer: D

Solution:

$x = 2t, y = \frac{2}{3}$

$t \longrightarrow 1A \equiv \left(2, \frac{1}{3}\right)$

Given conic is $x^2 = 12y \Rightarrow S \equiv (0, 3)$

Let $B \equiv (0, \beta)$

Given $SA \perp BA$

$$\left(\frac{\frac{1}{3}}{2-3}\right)\left(\frac{\beta - \frac{1}{3}}{-2}\right) = -1$$

$$\Rightarrow \left(\beta - \frac{1}{3}\right)\frac{1}{3} = -2$$

$$\Rightarrow \beta = \frac{1}{3}\left(\frac{-17}{3}\right)$$

Ordinate of centroid $= k = \frac{\beta + \frac{1}{3} + 3}{3}$

$$= \frac{\frac{-17}{9} + \frac{10}{3}}{3} = \frac{13}{18}$$

Question259

Let the normal at the point on the parabola $y^2 = 6x$ pass through the point (5, −8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :
[26-Jun-2022-Shift-1]

Options:

A. −3

B. $-\frac{9}{4}$

C. $-\frac{5}{2}$

D. −2

Answer: B

Solution:

Let $P(at^2, 2at)$ where $a = \frac{3}{2}$

T : $yt = x + at^2$ So point Q is $\left(-a, at - \frac{a}{t}\right)$

$N : y = -tx + 2at + at^3$ passes through $(5, -8)$

$$-8 = -5t + 3t + \frac{3}{2}t^3$$

$$\Rightarrow 3t^3 - 4t + 16 = 0$$

$$\Rightarrow (t + 2)(3t^2 - 6t + 8) = 0$$

$$\Rightarrow t = 2$$

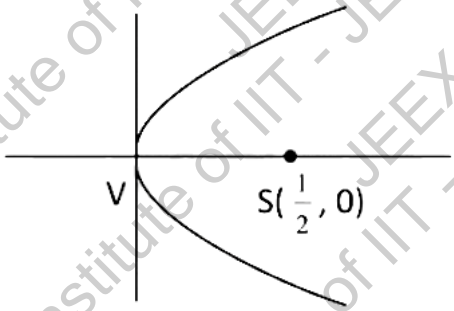
So ordinate of point Q is $-\frac{9}{4}$.

Question260

A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to___
[27-Jun-2022-Shift-1]

Answer: 63

Solution:



Let the equation of circle be

$x\left(x - \frac{1}{2}\right) + y^2 + \lambda y = 0$

$\Rightarrow x^2 + y^2 - \frac{1}{2}x + \lambda y = 0$

$\text{Radius} = \sqrt{\frac{1}{16} + \frac{\lambda^2}{4}} = 2$

$\Rightarrow \lambda^2 = \frac{63}{4}$

$\Rightarrow \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{\lambda}{2}\right)^2 = 4$

$\because$ This circle and parabola $y - \alpha = \left(x - \frac{1}{4}\right)^2$ touch each other, so

$\alpha = -\frac{\lambda}{2} + 2$

$\Rightarrow \alpha - 2 = -\frac{\lambda}{2}$

$\Rightarrow (\alpha - 2)^2 = \frac{\lambda^2}{4} = \frac{63}{16}$

$\Rightarrow (4\alpha - 8)^2 = 63$

Question261

If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$, then $a + b + c + d + k$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. 575

B. −575

C. 576

D. −576

Answer: D

Solution:

Given vertex is (5, 4) and directrix $3x + y - 29 = 0$
Let foot of perpendicular of (5, 4) on directrix is (x_1, y_1)

$$\frac{x_1 - 5}{3} = \frac{y_1 - 4}{1} = \frac{-(-10)}{10}$$

$\therefore (x_1, y_1) \equiv (8, 5)$
So, focus of parabola will be S = (2, 3)
Let P(x, y) be any point on parabola, then

$$(x-2)^2 + (y-3)^2 = \frac{(3x + y - 29)^2}{10}$$

$\Rightarrow 10(x^2 + y^2 - 4x - 6y + 13) = 9x^2 + y^2 + 841 + 6xy - 58y - 174x$
$\Rightarrow x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$
and given parabola
$x^2 + ay^2 + bxy + cx + d y + k = 0$
$\therefore a = 9, b = -6, c = 134, d = -2, k = -711$
$\therefore a + b + c + d + k = -576$

Question262

If vertex of a parabola is (2, −1) and the equation of its directrix is 4x − 3y = 21, then the length of its latus rectum is :
[28-Jun-2022-Shift-2]

Options:

A. 2

B. 8

C. 12

D. 16

Answer: B

Solution:

Vertex of Parabola : (2, −1)
and directrix : $4x - 3y = 21$
Distance of vertex from the directrix

$$a = \left| \frac{8 + 3 - 21}{\sqrt{25}} \right| = 2$$

$\therefore$ length of latus rectum $= 4a = 8$

Question263

Let $P : y^2 = 4ax, a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B. Then the value of a for which A, B and S are collinear is
[29-Jun-2022-Shift-2]

Options:

A. 8 only

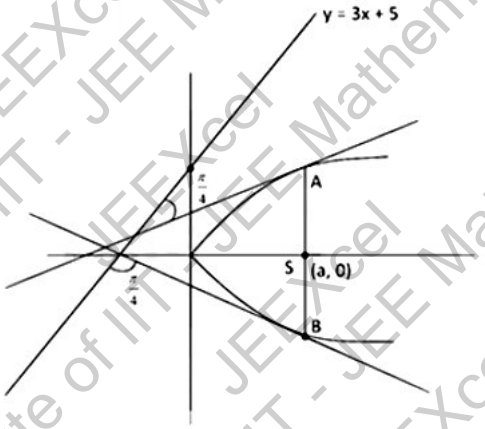
B. 2 only

C. $\frac{1}{4}$ only

D. any $a > 0$

Answer: D

Solution:



$$\tan\frac{\pi}{4} = \left|\frac{m-3}{1+3m}\right|$$

$$\Rightarrow \frac{m-3}{1+3m} = \pm 1$$

$$\Rightarrow \frac{m-3}{1+3m} = +1 \text{ and } \frac{m-3}{1+3m} = -1$$

$$\Rightarrow m-3 = 1+3m \text{ and } m-3 = -1-3m$$

$$\Rightarrow 2m = -4 \text{ and } 4m = 2$$

$$m = -2 \text{ and } m = \frac{1}{2}$$

We know, Equation of tangent to the parabola $y^2 = 4m$ is $y = mx + \frac{a}{m}$ and point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

∴ Equation of tangent

$$y = -2x - \frac{a}{2}$$

and $y = \frac{x}{2} + 2a$

∴ Point of contact A and B are

$$A\left(\frac{a}{(-2)^2}, \frac{2a}{-2}\right) = A\left(\frac{a}{4}, -a\right)$$

$$B\left(\frac{a}{\left(\frac{1}{2}\right)^2}, \frac{2a}{\left(\frac{1}{2}\right)}\right) = B(4a, 4a)$$

As points A, B and S are colinear so area of triangle formed by those 3 points are zero.

$$\text{Area of } \triangle ABS = \frac{1}{2}\begin{vmatrix} \frac{a}{4} & -a & 1 \\ 4a & 4a & 1 \\ a & 0 & 1 \end{vmatrix}$$

$$= \frac{a}{4}(4a-0) + a(4a-a) + 1(0-4a^2)$$

$$= a^2 + 3a^2 - 4a^2 = 0$$

∴ Area of triangle is independent of value of a.

So, for all value of $a > 0$ (already given a must be greater than 0) point A, B and S will be collinear.

Question264

Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, $a > 2$, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is :

[24-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

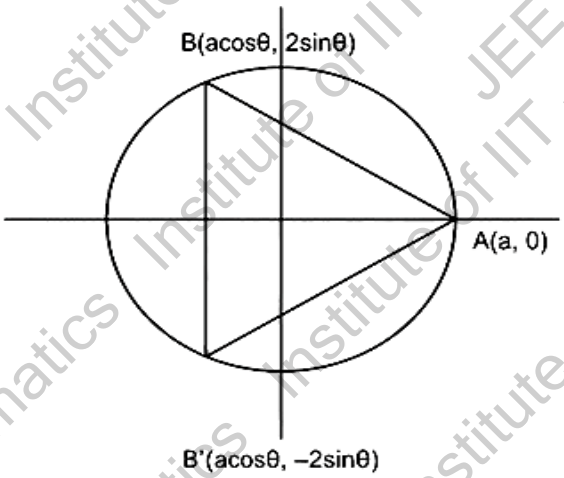
C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{3}}{4}$

Answer: A

Solution:

Given ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,\ a>2$



$\therefore$ Let A(θ) be the area of $\triangle ABB'$

Then $A(\theta)=\frac{1}{2}4\sin\theta(a+a\cos\theta)$

$A'(\theta)=a(2\cos\theta+2\cos^2\theta)$

For maxima $A'(\theta)=0$

$\Rightarrow\cos\theta=1,\ \cos\theta=\frac{1}{2}$

But for maximum area $\cos\theta=\frac{1}{2}$

$\therefore A(\theta)=6\sqrt{3}$

$\Rightarrow 2\frac{\sqrt{3}}{2}\left(a+\frac{a}{2}\right)=6\sqrt{3}$

$\Rightarrow a=4$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{4}{16}}=\frac{\sqrt{3}}{2}$

Question265

The line $y = x + 1$ meets the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to:
[25-Jun-2022-Shift-2]

Options:

A. 20

B. 12

C. 11

D. 8

Answer: A

Solution:

Let point $(a, a+1)$ as the point of intersection of line and ellipse.

So, $\frac{a^2}{4} + \frac{(a+1)^2}{2} = 1 \Rightarrow a^2 + 2(a^2 + 2a + 1) = 4$

$\Rightarrow 3a^2 + 4a - 2 = 0$

If roots of this equation are α and β.

So, $P(\alpha, \alpha+1)$ and $Q(\beta, \beta+1)$

$PQ = 4r^2 = (\alpha - \beta)^2 + (\alpha - \beta)^2$

$\Rightarrow 9r^2 = \frac{9}{4}(2(\alpha - \beta)^2)$

$= \frac{9}{2}[(\alpha + \beta)^2 - 4\alpha\beta]$

$= \frac{9}{2}\left[\left(-\frac{4}{3}\right)^2 + \frac{8}{3}\right]$

$= \frac{1}{2}[16 + 24] = 20$

Question266

Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin 0 , has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and I respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{1}{e^2}$ is equal to____
[26-Jun-2022-Shift-1]

Answer: 4

Solution:

Let $y = mx + c$ is the common tangent

So $c = \frac{1}{m} = \pm\frac{3}{2}\sqrt{1+m^2} \Rightarrow m^2 = \frac{1}{3}$

So equation of common tangents will be $y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{3}$, which intersects at $Q(-3, 0)$

Major axis and minor axis of ellipse are 12 and 6.

So eccentricity

$e^2 = 1 - \frac{1}{4} = \frac{3}{4}$ and length of latus rectum $= \frac{2b^2}{a} = 3$

Hence, $\frac{l}{e^2} = \frac{3}{3/4} = 4$

Question267

If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to : [26-Jun-2022-Shift-2]

Options:

A. 6

B. 9

C. 10

D. 12

Answer: B

Solution:

$C_1 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ and $C_2 : x^2 + y^2 = 12$

Let $y = mx \pm \sqrt{16m^2 + 9}$ be any tangent to C_1 and if this is also tangent to C_2 then

$$\left|\frac{\sqrt{16m^2+9}}{\sqrt{m^2+1}}\right| = \sqrt{12}$$

$\Rightarrow 16m^2 + 9 = 12m^2 + 12$

$\Rightarrow 4m^2 = 3 \Rightarrow 12m^2 = 9$

Question268

The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :
[26-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: C

Solution:

Let $P(2\cos\theta, \sqrt{2}\sin\theta)$ be any point on ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ and Q(4, 3) and let (h, k) be the mid point of PQ then $h = \frac{2\cos\theta + 4}{2}$, $k = \frac{\sqrt{2}\sin\theta + 3}{2}$

$\therefore \cos\theta = h - 2, \sin\theta = \frac{2k-3}{\sqrt{2}}$

$\therefore (h-2)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1$

$\Rightarrow \frac{(x-2)^2}{1} + \frac{\left(y-\frac{3}{2}\right)^2}{\frac{1}{2}} = 1$

$\therefore e = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$

Question269

Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b, be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to :
[27-Jun-2022-Shift-1]

Options:

A. 29

B. 31

C. 32

D. 34

Answer: B

Solution:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\Rightarrow \frac{\left(-4\sqrt{\frac{2}{5}}\right)^2}{a^2}+\frac{32}{b^2}=1$

$\Rightarrow \frac{32}{5a^2}+\frac{9}{b^2}=1 \ldots.$ (i)

$a^2(1-e^2)=b^2$

$a^2\left(1-\frac{1}{16}\right)=b^2$

$15a^2=16b^2 \Rightarrow a^2=\frac{16b^2}{15}$

From (i)

$\frac{6}{b^2}+\frac{9}{b^2}=1 \Rightarrow b^2=15 \& a^2=16$

$a^2+b^2=15+16=31$

Question270

Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a^2 > b^2$. If e is the eccentricity of the ellipse E , then the value of $\frac{1}{e^2}$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. $1+\sqrt{2}$

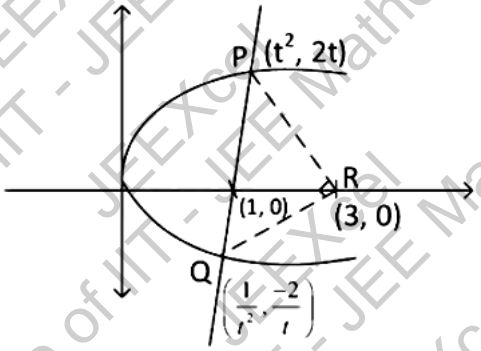
B. $3+2\sqrt{2}$

C. $1+2\sqrt{3}$

D. $4+5\sqrt{3}$

Answer: B

Solution:



As $\angle PRQ = \frac{\pi}{2}$

$$\left(\frac{\frac{2}{t}}{3-\frac{1}{t^2}}\right)\cdot\left(\frac{-2t}{3-t^2}\right) = -1$$

$\Rightarrow t = \pm 1$

$\therefore P \equiv (1, 2) \& Q(1, -2)$

$\therefore$ for ellipse $\frac{1}{a^2} + \frac{4}{b^2} = 1$ and $ae = 1$

$$\Rightarrow \frac{1}{a^2} + \frac{4}{a^2(1-e^2)} = 1$$

$$\Rightarrow 1 + \frac{4}{(1-e^2)} = \frac{1}{e^2}$$

$$\Rightarrow (5-e^2)e^2 = 1-e^2$$

$$\Rightarrow e^4 - 6e^2 + 1 = 0$$

$$\Rightarrow e^2 = \frac{1}{3-2\sqrt{2}} \Rightarrow \frac{1}{e^2} = 3 + 2\sqrt{2}$$

Question271

The odd natural number a, such that the area of the region bounded by y = 1, y = 3, x = 0, $x = y^a$ is $\frac{364}{3}$, is equal to:
[26-Jul-2022-Shift-1]

Options:

A. 3

B. 5

C. 7

D. 9

Answer: B

Solution:

a is a odd natural number and

$\left|\int_1^3 y^a dy\right| = \frac{364}{3}$

$\Rightarrow \left| \frac{1}{a+1}(y^{a+1})_1^3 \right| = \frac{364}{3}$

$\Rightarrow \frac{3^{a+1}-1}{a+1} = \pm \frac{364}{3}$

Solving with (-) sign,

$\frac{3^{a+1}-1}{a+1} = \frac{364}{3} \Rightarrow (a=5)$

Solving with (+) sign,

$\frac{3^{a+1}-1}{a+1} = \frac{-364}{3}$, No a exist

$\therefore (a=5)$

Question272

The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is
[26-Jul-2022-Shift-2]

Options:

A. $\frac{2}{3}(\sqrt{2}+1)$

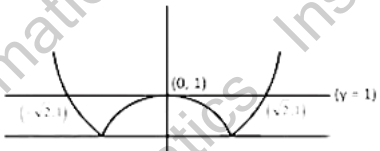
B. $\frac{4}{3}(\sqrt{2}-1)$

C. $2(\sqrt{2}-1)$

D. $\frac{8}{3}(\sqrt{2}-1)$

Answer: D

Solution:



Area $= 2\int_0^{\sqrt{2}} (1 - |x^2 - 1|)dx$

$2\left[\int_0^1 (1-(1-x^2))dx + \int_1^{\sqrt{2}} (2-x^2)dx\right]$

$= 2\left[\left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}\right]$

$= 2\left(\frac{4\sqrt{2}-4}{3}\right) = \frac{8}{3}(\sqrt{2}-1)$

Question273

The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. $\frac{1}{3}(2 - 12\sqrt{3} + 8\pi)$

B. $\frac{1}{3}(2 - 12\sqrt{3} + 6\pi)$

C. $\frac{1}{3}(4 - 12\sqrt{3} + 8\pi)$

D. $\frac{1}{3}(4-12\sqrt{3}+6\pi)$

Answer: C

Solution:

$\cos\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

Area of the required region

$= \frac{2}{3}\left(4\times\frac{1}{2}\right) + 4^2\times\frac{\pi}{6} - \frac{1}{2}\times 4\times 2\sqrt{3}$

$= \frac{4}{3} + \frac{8\pi}{3} - 4\sqrt{3} = \frac{1}{3}\{4-12\sqrt{3}+8\pi\}$

Question274

The area of the region enclosed by $y \le 4x^2$, $x^2 \le 9y$ and $y \le 4$, is equal to : [27-Jul-2022-Shift-2]

Options:

A. $\frac{40}{3}$

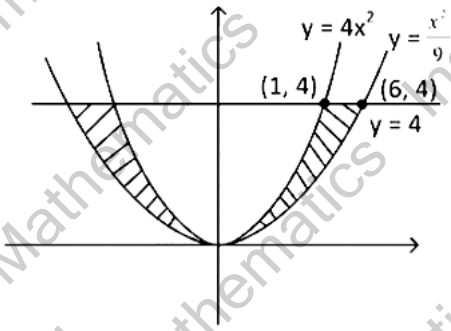
B. $\frac{56}{3}$

C. $\frac{112}{3}$

D. $\frac{80}{3}$

Answer: D

Solution:



$y \le 4x^2$, $x^2 \le 9y$, $y \le 4$

So, required area

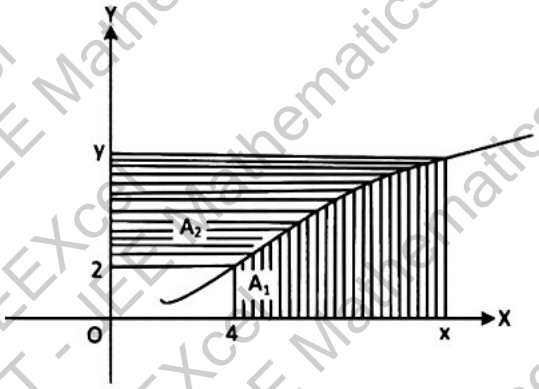
$A = 2\int_0^4 \left(3\sqrt{y} - \frac{1}{2}\sqrt{y}\right)dy$

$= 2\cdot\frac{5}{2}\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$

$= \frac{10}{3}[4^{3/2} - 0] = \frac{80}{3}$

Question275

Consider a curve y = y(x) in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does NOT pass through the point.



[27-Jul-2022-Shift-2]

Options:

A. (6, 21)

B. (8, 9)

C. (10, −4)

D. (12, −15)

Answer: C

Solution:

$A_1 + A_2 = xy - 8 \& A_1 = 2A_2$

$A_1 + \frac{A_1}{2} = xy - 8$

$A_1 = \frac{2}{3}(xy - 8)$

$\int_4^x f(x)dx = \frac{2}{3}(xf(x) - 8)$

Differentiate w.r.t. x

$f(x) = \frac{2}{3}\{xf'(x) + f(x)\}$

$\frac{2}{3}xf'(x) = \frac{1}{3}f(x)$

$2\int \frac{f'(x)}{f(x)}dx = \int \frac{dx}{x}$

$2\ln f(x) = \ln x + \ln c$

$f^2(x) = cx$

Which passes through (4, 2)

$4 = c \times 4 \Rightarrow c = 1$

Equation of required curve

$y^2 = x$

Equation of normal having slope (−6) is

$y = -6x - 2\left(\frac{1}{4}\right)(-6) - \frac{1}{4}(-6)^3$

$y = -6x + 57$

Which does not pass through (10, −4)

Question276

The area enclosed by the curves $y = \log_e(x + e^2)$, $x = \log_e\left(\frac{2}{y}\right)$ and $x = \log_e 2$, above the line $y = 1$ is:

[28-Jul-2022-Shift-2]

Options:

A. $2 + e - \log_e 2$

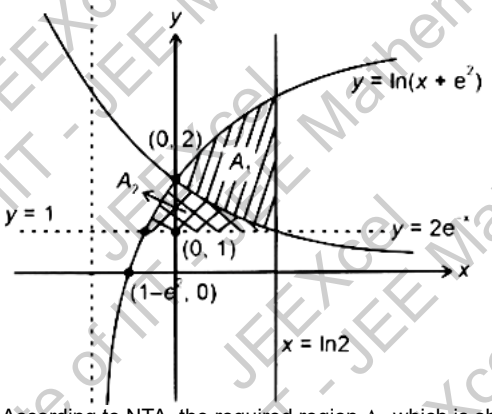
B. $1 + e - \log_e 2$

C. $e - \log_e 2$

D. $1+\log_e 2$

Answer: B

Solution:



According to NTA, the required region A_2 which is shaded in crossed lines and comes out to be

$$A_2=\int_1^2\left(\ln\frac{2}{y}-e^y+e^2\right)dy=1+e-\ln 2$$

But according to us the required region A_1 comes out to be shaded in parallel lines, which can be obtained as

$$A_1=\int_0^{\ln 2}(\ln(x+e^2)-2e^{-x})dx$$

$$=.\{(x+e^2)\ln(x+e^2)-x+2e^{-x}\}|_0^{\ln 2}$$

$$=(\ln 2+e^2)\ln(\ln 2+e^2)-\ln 2+1$$

$$-2e^2-2$$

$$=(\ln 2+e^2)\ln(\ln 2+e^2)-\ln 2-2e^2-1$$

Not given in any option.

The region asked in the question is bounded by three curves

$y=\ln(x+e^2)$

$x=\ln\left(\frac{2}{y}\right)$

$x=\ln 2$

There is only one region which satisfies above requirement and which also lies above line $y=1$

Line $y=1$ may or may not be the boundary of the region.

Question277

The area of the region $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$ is equal to [29-Jul-2022-Shift-1]

Options:

A. $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$

B. $\frac{5\pi}{4}-\frac{3}{2}$

C. $\frac{3\pi}{4}+\frac{3}{2}$

D. $\frac{5\pi}{4}-\frac{1}{2}$

Answer: D

Solution:

$$A=2\left(\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right)-2x\Big|_0^1+\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}-\frac{x^2}{2}+x\Big|_1^2$$

$$=\left(\frac{5\pi}{4}-\frac{1}{2}\right)\text{ sq. units}$$

Question278

Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to
[25-Jul-2022-Shift-1]

Answer: 2

Solution:

$2p + f - 1 = 0 \quad \ldots (1)$
$2 - pf - 4p = 0 \quad \ldots (2)$
$2 = p(f + 4)$
$p = \frac{2}{f+4}$
$2p = 1 - f$
$\frac{4}{f+4} = 1 - f$
$f^2 + 3f = 0$
$f = 0 \text{ or } -3$
Hyperbola $3x^2 - y^2 = 3, x^2 - \frac{y^2}{3} = 1$
$y = mx \pm \sqrt{m^2 - 3}$
It passes $(1, 0)$
$o = m \pm \sqrt{m^2 - 3}$
m tends ∞
It passes $(1, 3)$
$3 = m \pm \sqrt{m^2 - 3}$
$(3 - m)^2 = m^2 - 3$
$m = 2$

Question279

Let the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{\alpha} = \frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:
[25-Jul-2022-Shift-2]

Options:

A. $\frac{32}{9}$

B. $\frac{18}{5}$

C. $\frac{27}{4}$

D. $\frac{27}{10}$

Answer: D

Solution:

Ellipse: $\frac{x^2}{16} + \frac{y^2}{7} = 1$
Eccentricity $= \sqrt{1 - \frac{7}{16}} = \frac{3}{4}$

Foci $\equiv (\pm ae, 0) \equiv (\pm 3, 0)$

Hyperbola : $\frac{x^2}{\left(\frac{144}{25}\right)} - \frac{y^2}{\left(\frac{\alpha}{25}\right)} = 1$

Eccentricity $= \sqrt{1 + \frac{\alpha}{144}} = \frac{1}{12}\sqrt{144 + \alpha}$

Foci $\equiv (\pm ae, 0) \equiv \left(\pm \frac{12}{5} \cdot \frac{1}{12}\sqrt{144 + \alpha}, 0\right)$

If foci coincide then $3 = \frac{1}{5}\sqrt{144 + \alpha} \Rightarrow \alpha = 81$

Hence, hyperbola is $\frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$

Length of latus rectum $= 2 \cdot \frac{\frac{81}{25}}{\frac{12}{5}} = \frac{27}{10}$

Question280

Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point:
[26-Jul-2022-Shift-1]

Options:

A. (25, 10)

B. (20, 12)

C. (30, 8)

D. (15, 13)

Answer: D

Solution:

Any tangent to $y^2 = 24x$ at (α, β)
$\beta y = 12(x + \alpha)$
Slope $= \frac{12}{\beta}$ and perpendicular to $2x + 2y = 5$

$\Rightarrow \frac{12}{\beta} = 1 \Rightarrow \beta = 12, \alpha = 6$

Hence hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$ and normal is drawn at $(10, 16)$

Equation of normal $\frac{36 \cdot x}{10} + \frac{144 \cdot y}{16} = 36 + 144$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$
This does not pass though $(15, 13)$ out of given option.

Question281

If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point
[26-Jul-2022-Shift-2]

Options:

A. $(-2\sqrt{5}, 6)$

B. $(-\sqrt{5}, 3)$

C. $(\sqrt{5}, -2)$

D. $(2\sqrt{5}, 3\sqrt{6})$

Answer: C

Solution:

Given hyperbola : $\frac{x^2}{6/k} - \frac{y^2}{6} = 1$

Eccentricity $= e = \sqrt{1 + \frac{6}{6/k}} = \sqrt{1+k}$

Directrices : $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}}$

As given : $\frac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}} = 1$

$\Rightarrow k = 2$

Here hyperbola is $\frac{x^2}{3} - \frac{y^2}{6} = 1$

Checking the option gives $(\sqrt{5}, -2)$ satisfies it.

Question282

An ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola H : $\frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H, respectively. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E, then the value of 113l is equal to _____.
[27-Jul-2022-Shift-1]

Answer: 1552

Solution:

Vertices of hyperbola $= (0, \pm 8)$
As ellipse pass through it i.e.,
$0 + \frac{64}{b^2} = 1 \Rightarrow b^2 = 64 \ldots\ldots (1)$
As major axis of ellipse coincide with transverse axis of hyperbola we have $b > a$ i.e.
$e_E = \sqrt{1 - \frac{a^2}{64}} = \frac{\sqrt{64 - a^2}}{8}$
and $e_H = \sqrt{1 + \frac{49}{64}} = \frac{\sqrt{113}}{8}$
$\therefore e_E \cdot e_H = \frac{1}{2} = \frac{\sqrt{64 - a^2}\sqrt{113}}{64}$
$\Rightarrow (64 - a^2)(113) = 32^2$
$\Rightarrow a^2 = 64 - \frac{1024}{113}$
L.R of ellipse $= \frac{2a^2}{b} = \frac{2}{8}\left(\frac{113 \times 64 - 1024}{113}\right) = 1 = \frac{1552}{113}$
$\therefore 113l = 1552$

Question283

A common tangent T to the curves C_1 : $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and C_2 : $\frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to _____.
[27-Jul-2022-Shift-2]

Answer: 20

Solution:

Equation of tangent to ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ and given slope m is : $y=mx+\sqrt{4m^2+9}$.... (i)

For slope m equation of tangent to hyperbola is :

$y=mx+\sqrt{42m^2-143}$

Tangents from (i) and (ii) are identical then

$4m^2+9=42m^2-143$

$\therefore m=\pm 2 (+2$ is not acceptable $)$

$\therefore m=-2.$

Hence, $x_1=\frac{8}{5}$ and $x_2=\frac{84}{5}$

$\therefore |2x_1+x_2|=\left|\frac{16}{5}+\frac{84}{5}\right|=20$

Question284

For the hyperbola H : $x^2-y^2=1$ and the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b>0$, let the

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line $y=\sqrt{\frac{5}{2}}x+K$ be a common tangent of F and H.

Then $4(a^2+b^2)$ is equal to _____.
[28-Jul-2022-Shift-1]

Answer: 3

Solution:

The equation of tangent to hyperbola $x^2-y^2=1$ within slope m is equal to $y=mx\pm\sqrt{m^2-1}$.

And for same slope m, equation of tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y=mx\pm\sqrt{a^2m^2+b^2}$..... (ii)

$\because$ Equation (i) and (ii) are identical

$\therefore a^2m^2+b^2=m^2-1$

$\therefore m^2=\frac{1+b^2}{1-a^2}$

But equation of common tangent is $y=\sqrt{\frac{5}{2}}x+k$

$\therefore m=\sqrt{\frac{5}{2}}\Rightarrow \frac{5}{2}=\frac{1+b^2}{1-a^2}$

$\therefore 5a^2+2b^2=3$..... (i)

eccentricity of ellipse $=\frac{1}{\sqrt{2}}$

$\therefore 1-\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow a^2=2b^2$..... (ii)

From equation (i) and (ii) : $a^2=\frac{1}{2}, b^2=\frac{1}{4}$

$\therefore 4(a^2+b^2)=3$

Question285

Let the hyperbola H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $(2\sqrt{2}, -2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola? [28-Jul-2022-Shift-2]

Options:

A. $(2\sqrt{3}, 3\sqrt{2})$

B. $(3\sqrt{3}, -6\sqrt{2})$

C. $(\sqrt{3}, -\sqrt{6})$

D. $(3\sqrt{6}, 6\sqrt{2})$

Answer: B

Solution:

H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Focus of parabola : (ae, 0)

Directrix : $x = -ae$.

Equation of parabola $= y^2 = 4aex$

Length of latus rectum of parabola $= 4ae$

Length of latus rectum of hyperbola $= \frac{2.b^2}{a}$

as given, $4ae = \frac{2b^2}{a} \cdot e$

$2 = \frac{b^2}{a^2}$..... (i)

$\because$ H passes through $(2\sqrt{2}, -2\sqrt{2}) \Rightarrow \frac{8}{a^2} - \frac{8}{b^2} = 1$..... (ii)

From (i) and (ii) $a^2 = 4$ and $b^2 = 8 \Rightarrow e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$.

Question286

Let the focal chord of the parabola P : $y^2 = 4x$ along the line L : $y = mx + c$, $m > 0$ meet the parabola at the points M and N . Let the line L be a tangent to the hyperbola H : $x^2 - y^2 = 4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is : [29-Jul-2022-Shift-1]

Options:

A. $2\sqrt{6}$

B. $2\sqrt{14}$

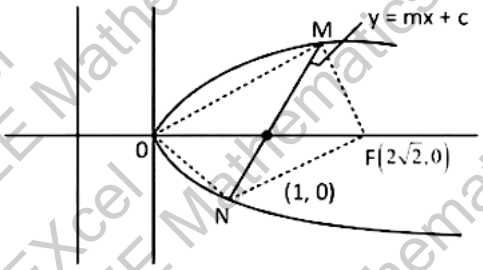
C. $4\sqrt{6}$

D. $4\sqrt{14}$

Answer: B

Solution:

H : $\frac{x^2}{4} - \frac{y^2}{4} = 1$



Focus (ae, 0)

$F(2\sqrt{2}, 0)$

y = mx + c passes through (1, 0)

$0 = m + C$..... (i)

L is tangent to hyperbola

$C = \pm\sqrt{4m^2 - 4}$

$-m = \pm\sqrt{4m^2 - 4}$

$m^2 = 4m^2 - 4$

$m = \frac{2}{\sqrt{3}}$

$C = \frac{-2}{\sqrt{3}}$

$T : y = \frac{2}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

$P : y^2 = 4x$

$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$

$y^2 - 2\sqrt{3}y - 4 = 0$

Area

$$\frac{1}{2}\begin{vmatrix} 0 & 0 \\ x_1 & y_1 \\ 2\sqrt{2} & 0 \\ x_2 & y_2 \\ 0 & 0 \end{vmatrix}$$

Question287

Let the locus of the centre (α, β), β > 0, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line y = 4 is:
[25-Jul-2022-Shift-1]

Options:

A. $\frac{32\sqrt{2}}{3}$

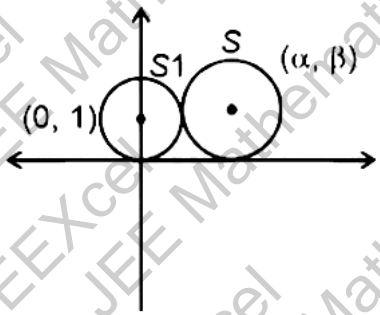
B. $\frac{40\sqrt{2}}{3}$

C. $\frac{64}{3}$

D. $\frac{32}{3}$

Answer: C

Solution:



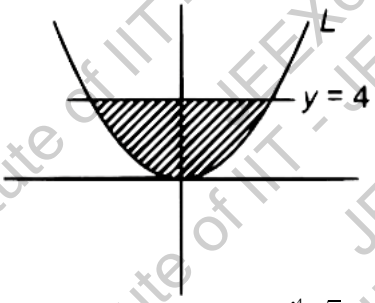
Radius of circle S touching x-axis and centre (α, β) is $|\beta|$. According to given conditions

$\alpha^2 + (\beta - 1)^2 = (|\beta| + 1)^2$

$\alpha^2 + \beta^2 - 2\beta + 1 = \beta^2 + 1 + 2|\beta|$

$\alpha^2 = 4\beta$ as $\beta > 0$

$\therefore$ Required louse is $L : x^2 = 4y$



The area of shaded region $= 2\int_0^4 2\sqrt{y}\,dy$

$= 4 . \left[\frac{y^{3/2}}{\frac{3}{2}}\right]_0^4$

$= \frac{64}{3}$ square units.

Question288

If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2(3 - \sqrt{3})x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6}$, $k > 0$, touch internally at the point $P(\alpha, \beta)$, then $(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2$ is equal to
[25-Jul-2022-Shift-2]

Answer: 25

Solution:

The circle $x^2 + y^2 + 6x + 8y + 16 = 0$ has centre $(-3, -4)$ and radius 3 units,

The circle $x^2 + y^2 + 2(3 - \sqrt{3})x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6}$, $k > 0$ has centre $(\sqrt{3} - 3, \sqrt{6} - 4)$ and radius $\sqrt{k + 34}$

$\because$ These two circles touch internally hence

$\sqrt{3 + 6} = |\sqrt{k + 34} - 3|$

Here, $k = 2$ is only possible ($\because k > 0$)

Equation of common tangent to two circles is $2\sqrt{3}x + 2\sqrt{6}y + 16 + 6\sqrt{3} + 8\sqrt{6} + k = 0$

$\because k = 2$ then equation is

$x + \sqrt{2}y + 3 + 4\sqrt{2} + 3\sqrt{3} = 0$..... (i)

$\because (\alpha, \beta)$ are foot of perpendicular from $(-3, -4)$

To line (i) then

$\frac{\alpha + 3}{1} = \frac{\beta + 4}{\sqrt{2}} = \frac{-(-3 - 4\sqrt{2} + 3 + 4\sqrt{2} + 3\sqrt{3})}{1 + 2}$

$\therefore \alpha + 3 = \frac{\beta + 4}{\sqrt{2}} = -\sqrt{3}$

$\Rightarrow (\alpha + \sqrt{3})^2 = 9$ and $(\beta + \sqrt{6})^2 = 16$

$\therefore (\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2 = 25$

Question289

If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, g, c $\in \mathbb{R}$ passes through the point (6, 1) and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is
[27-Jul-2022-Shift-1]

Options:

A. $\sqrt{11}$

B. 4

C. 3

D. $2\sqrt{23}$

Answer: D

Solution:

Circle : $x^2 + y^2 - 2gx + 6y - 19c = 0$
It passes through h(6, 1)
$\Rightarrow 36 + 1 - 12g + 6 - 19c = 0$
$= 12g + 19c = 43 \ldots\ldots (1)$
Line $x - 2cy = 8$ passes through centre
$\Rightarrow g + 6c = 8 \ldots\ldots$ (2)
From (1) & (2)
$g = 2, c = 1$
$C : x^2 + y^2 - 4x + 6y - 19 = 0$
xint $= 2\sqrt{g^2 - C}$
$= 2\sqrt{4 + 19}$
$= 2\sqrt{23}$

Question290

A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA : AP is equal to:
[27-Jul-2022-Shift-2]

Options:

A. 1 : 4

B. 1 : 5

C. 2 : 5

D. 1 : 3

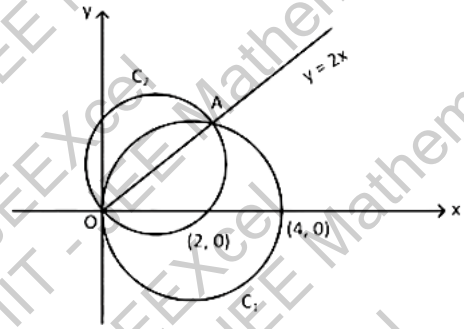
Answer: A

Solution:

Equation of C_1
$x^2 + y^2 - 4x = 0$
Intersection with
$y = 2x$
$x^2 + 4x^2 - 4x = 0$
$5x^2 - 4x = 0$
$\Rightarrow x = 0, \frac{4}{5}$

$y = 0, \frac{8}{5}$

A : $\left(\frac{4}{5}, \frac{8}{5}\right)$



Tangent of C_2 at A $\left(\frac{4}{5}, \frac{8}{5}\right)$

$x + 2y = 4 \Rightarrow P : (4, 0), Q : (0, 2)$

QA : AP = 1 : 4

Question291

Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R.. Then the area of the triangle PQR (in unit ^{2}) is :
[28-Jul-2022-Shift-1]

Options:

A. 2

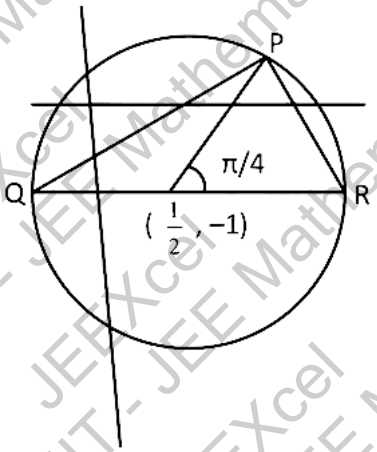
B. $2\sqrt{2}$

C. $8 \sin\left(\frac{\pi}{8}\right)$

D. $8 \cos\left(\frac{\pi}{8}\right)$

Answer: B

Solution:



QR = 2r = 4

$P = \left(\frac{1}{2} + 2\cos\frac{\pi}{4}, -1 + 2\sin\frac{\pi}{4}\right)$

$= \left(\frac{1}{2} + \sqrt{2}, -1 + \sqrt{2}\right)$

Area of $\triangle PQR = \frac{1}{2} \times 4 \times \sqrt{2} = 2\sqrt{2}$ sq. units

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Question292

Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O(0, 0). Then the area of the triangle OAB is :
[28-Jul-2022-Shift-2]

Options:

A. $\frac{3\sqrt{3}}{2}$

B. $\frac{3\sqrt{3}}{4}$

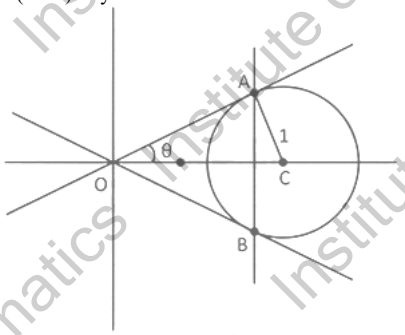
C. $\frac{3}{2\sqrt{3}}$

D. $\frac{3}{4\sqrt{3}}$

Answer: B

Solution:

$x^2 + y^2 - 4x + 3 = 0$
$\Rightarrow (x-2)^2 + y^2 = 1$



$AO = \sqrt{(OC)^2 - (AC)^2}$
$= \sqrt{4-1} = \sqrt{3}$
$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$
Also, AO = BO
Area of $\Delta OAB = \frac{1}{2} \cdot OA \cdot OB \sin 60^\circ$
$= \frac{1}{2} \times \sqrt{3} \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

Question293

Let a line L pass through the point of intersection of the lines $bx + 10y - 8 = 0$ and $2x - 3y = 0$, $b \in R - \left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17(x^2 + y^2) = 16$, then the eccentricity of the ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is :
[29-Jul-2022-Shift-1]

Options:

A. $\frac{2}{\sqrt{5}}$

B. $\sqrt{\frac{3}{5}}$

C. $\frac{1}{\sqrt{5}}$

D. $\sqrt{\frac{2}{5}}$

Answer: A

Solution:

$L_1 : bx + 10y - 8 = 0, L_2 : 2x - 3y = 0$

then $L : (bx + 10y - 8) + \lambda(2x - 3y) = 0$

$\because$ It passes through (1, 1)

$\therefore b + 2 - \lambda = 0 \Rightarrow \lambda = b + 2$

and touches the circle $x^2 + y^2 = \frac{16}{17}$

$$\left| \frac{8^2}{(2\lambda + b)^2 + (10 - 3\lambda)^2} \right| = \frac{16}{17}$$

$\Rightarrow 4\lambda^2 + b^2 + 4b\lambda + 100 + 9\lambda^2 - 60\lambda = 68$

$\Rightarrow 13(b + 2)^2 + b^2 + 4b(b + 2) - 60(b + 2) + 32 = 0$

$\Rightarrow 18b^2 = 36$

$\therefore b^2 = 2$

$\therefore$ Eccentricity of ellipse : $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is

$\therefore e = \sqrt{1 - \frac{2}{5}} = \sqrt{\frac{3}{5}}$

Question294

Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line $y = x + 1$ be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to _____.
[29-Jul-2022-Shift-1]

Answer: 12

Solution:

$c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$

Then centre $= (1, 3)$ and radius $(r) = \sqrt{10 - \alpha}$

Image of (1, 3) w.r.t. line $x - y + 1 = 0$ is (2, 2)

$c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$

or $x^2 + y^2 + 2gx + 2fy + \frac{38}{5} = 0$

Then $(-g, -f) = (2, 2)$

$\therefore g = f = -2$..... (i)

Radius of $c_2 = r = \sqrt{4 + 4 - \frac{38}{5}} = \sqrt{10 - \alpha}$

$\Rightarrow \frac{2}{5} = 10 - \alpha$

$\therefore \alpha = \frac{48}{5}$ and $r = \sqrt{\frac{2}{5}}$

$\therefore \alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$

Question295

Let AB be a chord of length 12 of the circle $(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$. If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to

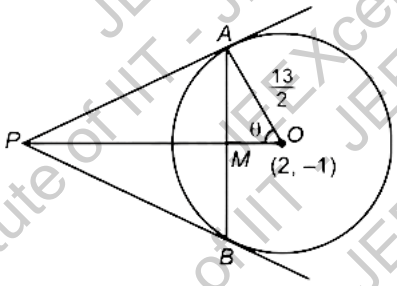
[29-Jul-2022-Shift-2]

Answer: 72

Solution:

Here AM = BM = 6

$QM = \sqrt{\left(\frac{13}{2}\right)^2 - 6^2} = \frac{5}{2}$



$\sin\theta = \frac{12}{13}$

In $\triangle PAO$:

$\frac{PO}{OA} = \sec\theta$

$PO = \frac{13}{2} \cdot \frac{13}{5} = \frac{169}{10}$

$\therefore PM = \frac{169}{10} - \frac{5}{2} = \frac{144}{10} = \frac{72}{5}$

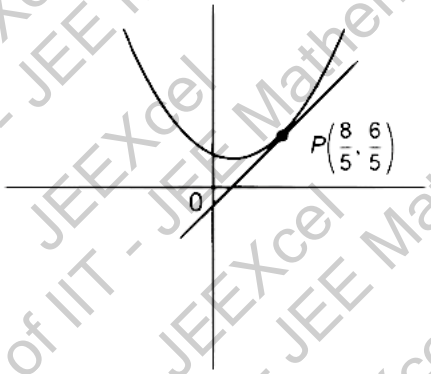
$\therefore 5PM = 72$

Question296

The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii) the y-axis, is equal to
[25-Jul-2022-Shift-1]

Answer: 10

Solution:



Equation of tangent to the parabola at $P\left(\frac{8}{5}, \frac{6}{5}\right)$

$75x \cdot \frac{8}{5} = 160\left(y + \frac{6}{5}\right) - 192$

$\Rightarrow 120x = 160y$

$\Rightarrow 3x = 4y$

Equation of circle touching the given parabola at P can be taken as

$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda(3x - 4y) = 0$

If this circle touches y-axis then

$\frac{64}{25} + \left(y - \frac{6}{5}\right)^2 + \lambda(-4y) = 0$

$\Rightarrow y^2 - 2y\left(2\lambda + \frac{6}{5}\right) + 4 = 0$

$\Rightarrow D = 0$

$\Rightarrow \left(2\lambda + \frac{6}{6}\right)^2 = 4$

$\Rightarrow \lambda = \frac{2}{5}$ or $-\frac{8}{5}$

Radius $= 1$ or 4

Sum of diameter $= 10$

Question297

The tangents at the points A(1, 3) and B(1, −1) on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit ^{2}) of the triangle PAB is:
[25-Jul-2022-Shift-2]

Options:

A. 4

B. 6

C. 7

D. 8

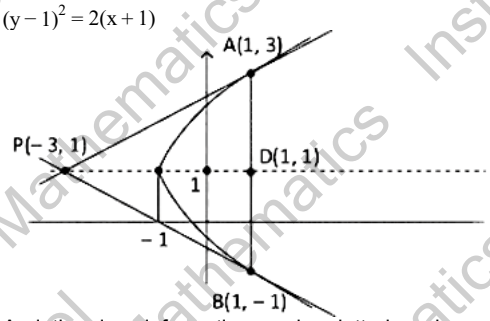
Answer: D

Solution:

Given curve : $y^2 - 2x - 2y = 1$.

Can be written as

$(y - 1)^2 = 2(x + 1)$



And, the given information can be plotted as shown in figure

Tangent at A : $2y - x - 5 = 0$ using $T = 0$ }

Intersection with $y = 1$ is $x = -3$

Hence, point P is $(-3, 1)$

Taking advantage of symmetry

Area of $\triangle PAB = 2 \times \frac{1}{2} \times (1 - (-3)) \times (3 - 1)$

$= 8$ sq. units

Question298

Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a − 1) but does not pass through the

point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is : $\frac{x}{\alpha}+\frac{y}{\beta}=1$, then $\alpha+\beta$ is equal to _____.
[26-Jul-2022-Shift-1]

Answer: 45

Solution:

$\delta'(x)=\frac{4x^2-1}{x}$ so $f(x)$ is decreasing in $\left(0, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right) \Rightarrow a=\frac{1}{2}$

Tangent at $y^2=2x \Rightarrow y=, x+\frac{1}{2m}$

It is passing through $(4, 3)$

$3=4m+\frac{1}{2m} \Rightarrow m=\frac{1}{2}$ or $\frac{1}{4}$

So tangent may be

$y=\frac{1}{2}x+1$ or $y=\frac{1}{4}x+2$

But $y=\frac{1}{2}x+1$ passes through $(-2, 0)$ so rejected.

Equation of normal

$y=-4x-2\left(\frac{1}{2}\right)(-4)+\frac{1}{2}(-4)^3$

or $y=-4x+4+32$

or $\frac{x}{9}+\frac{y}{36}=1$

Question299

Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y=x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval
[26-Jul-2022-Shift-2]

Options:

A. $\left(0, \frac{1}{4}\right)$

B. $\left(\frac{1}{2}, \frac{3}{4}\right)$

C. $\left(\frac{1}{4}, \frac{1}{2}\right)$

D. $\left(\frac{3}{4}, 1\right)$

Answer: C

Solution:

$y=mx+2a+\frac{1}{m^2}$ (Equation of normal to $x^2=4ay$ in slope form) through $(1, -1)$.

$4m^3+6m^2+1=0$

$\Rightarrow m \simeq -1.6$

Slope of normal $\simeq \frac{-8}{5}=\tan\theta$

$\Rightarrow \cos\theta \simeq \frac{-5}{\sqrt{89}}, \sin\theta \simeq \frac{8}{\sqrt{89}}$

$x_p=1+\cos\theta \simeq 1-\frac{5}{\sqrt{89}} \in \left(\frac{1}{4}, \frac{1}{2}\right)$

Question300

The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is
[26-Jul-2022-Shift-2]

Options:

A. $y = 4(x-2)$

B. $y = 4(x-1)$

C. $y = 4(x+1)$

D. $y = 4(x+2)$

Answer: B

Solution:

Equation of tangent of slope m to $y = x^2$
$y = mx - \frac{1}{4}m^2$
Equation of tangent of slope m to $y = -(x-2)^2$
$y = m(x-2) + \frac{1}{4}m^2$
If both equation represent the same line
$\frac{1}{4}m^2 - 2m = -\frac{1}{4}m^2$
$m = 0, 4$
So, equation of tangent
$y = 4x - 4$

Question301

Let P(a, b) be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of A + B is equal to
[27-Jul-2022-Shift-1]

Options:

A. 0

B. 25

C. 40

D. 65

Answer: D

Solution:

Centre of circle $x^2 + y^2 - 10x - 14y + 65 = 0$ is at $(5, 7)$
Let the equation of tangent to $y^2 = 8x$ is
$yt = x + 2t^2$
which passes through $(5, 7)$
$7t = 5 + 2t^2$
$\Rightarrow 2t^2 - 7t + 5 = 0$
$t = 1, \frac{5}{2}$
$A = 2 \times 1^2 \times 2 \times \left(\frac{5}{2}\right)^2 = 25$
$B = 2 \times 2 \times 1 \times 2 \times 2 \times \frac{5}{2} = 40$
$A + B = 65$

Question302

If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is x + y = a, is 16 , then |a| is equal to :
[27-Jul-2022-Shift-2]

Options:

A. $2\sqrt{2}$

B. $2\sqrt{3}$

C. $4\sqrt{2}$

D. 4

Answer: C

Solution:

Equation of tangent at vertex: $L \equiv x + y - a = 0$
Focus : $F \equiv (a, a)$
Perpendicular distance of L from F
$= \left| \frac{a + a - a}{\sqrt{2}} \right| = \left| \frac{a}{\sqrt{2}} \right|$

Length of latus rectum $= 4 \left| \frac{a}{\sqrt{2}} \right|$

Given 4. $\left| \frac{a}{\sqrt{2}} \right| = 16$

$\Rightarrow |a| = 4\sqrt{2}$

Question303

If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is:
[28-Jul-2022-Shift-1]

Options:

A. (0, 1)

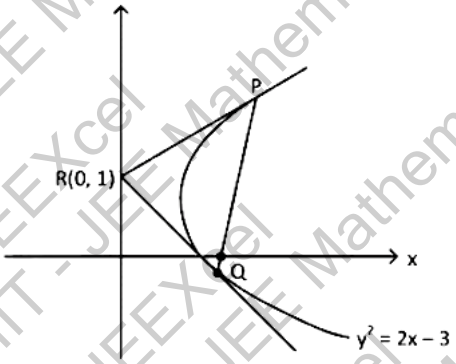
B. (2, −1)

C. (6, 3)

D. (2, 1)

Answer: B

Solution:



Equation of chord PQ
$\Rightarrow y \times 1 = x - 3$
$\Rightarrow x - y = 3$
For point P \& Q
Intersection of PQ with parabola P : (6, 3)Q : (2, −1)
Slope of RQ = −1 \& slope of PQ = 1
Therefore $\angle PQR = 90^\circ \Rightarrow$ Orthocentre is at Q : (2, −1)

Question304

Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then 17r is equal to_____.
[28-Jul-2022-Shift-2]

Answer: 9

Solution:

Given : $y^2 = \frac{-x}{2}$

$cT \equiv y = mx - \frac{1}{8m}$

$\downarrow (2, 0)$

$\Rightarrow m^2 = \frac{1}{16} \Rightarrow m = \pm \frac{1}{4}$

Tangents are $y = \frac{1}{4}x - \frac{1}{2}, y = \frac{-x}{4} + \frac{1}{2}$

$4y = x - 2$ and $4y + x = 2$

If these are also tangent to circle then $d_c = r$

$\Rightarrow \left| \frac{5-2}{\sqrt{17}} \right| = \sqrt{r} \Rightarrow r = \left(\frac{3}{\sqrt{17}} \right)^2$

$\Rightarrow 17r = 17 \cdot \frac{9}{17} = 9$

Question305

The area of the region given by $A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ is :
[25-Jul-2022-Shift-1]

Options:

A. $\frac{31}{8}$

B. $\frac{17}{6}$

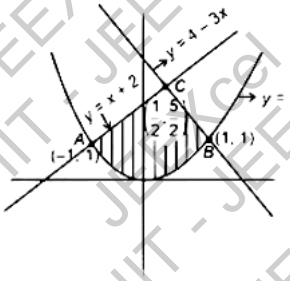
C. $\frac{19}{6}$

D. $\frac{27}{8}$

Answer: B

Solution:

$A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}$.



So area of required region

$$A = \int_{-1}^{\frac{1}{2}} (x + 2 - x^2)dx + \int_{\frac{1}{2}}^{1} (4 - 3x - x^2)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{\frac{1}{2}} + \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$= \left(\frac{1}{8} + 1 - \frac{1}{24}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(4 - \frac{3}{2} - \frac{1}{3}\right) - \left(2 - \frac{3}{8} - \frac{1}{24}\right)$$

$$= \frac{17}{6}$$

Question306

If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} + \frac{y}{2\sqrt{6}} = 1$ on the x-axis and the line $\frac{x}{7} - \frac{y}{2\sqrt{6}} = 1$ on the y-axis, then the eccentricity of the ellipse is
[25-Jul-2022-Shift-2]

Options:

A. $\frac{5}{7}$

B. $\frac{2\sqrt{6}}{7}$

C. $\frac{3}{7}$

D. $\frac{2\sqrt{5}}{7}$

Answer: A

Solution:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} + \frac{y}{2\sqrt{6}} = 1$ on the x-axis So, $a = 7$

and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} - \frac{y}{2\sqrt{6}} = 1$ on the y-axis So, $b = 2\sqrt{6}$

Therefore, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{24}{49}$

$e = \frac{5}{7}$

Question307

The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1, 3) is
[26-Jul-2022-Shift-2]

Options:

A. $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$

B. $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

C. $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$

D. $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Answer: B

Solution:

$2x^2 + 3y^2 = 5$

Equation of tangent having slope m.

$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$

which passes through (1, 3)

$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$

$\frac{5}{2}m^2 + \frac{5}{3} = 9 + m^2 - 6m$

$\frac{3}{2}m^2 + 6m - \frac{22}{3} = 0$

$9m^2 + 36m - 44 = 0$

$m_1 + m_2 = -4,\ m_1 m_2 = -\frac{44}{9}$

$(m_1 - m_2)^2 = 16 + 4 \times \frac{44}{9} = \frac{320}{9}$

Acute angle between the tangents is given by

$\alpha = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \tan^{-1}\left|\frac{\frac{8\sqrt{5}}{3}}{1 - \frac{44}{9}}\right|$

$= \tan^{-1}\left(\frac{24\sqrt{5}}{35}\right)$

$\alpha = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

Question308

If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4 , and l is the length of its major axis, then $\lambda + l$ is equal to _____.
[27-Jul-2022-Shift-1]

Answer: 75

Solution:

Equation of ellipse is : $x^2 + 4y^2 + 2x + 8y - \lambda = 0$

$(x+1)^2 + 4(y+1)^2 = \lambda + 5$

$\frac{(x+1)^2}{\lambda+5}+\frac{(y+1)^2}{\left(\frac{\lambda+5}{4}\right)}=1$

Length of latus rectum $=\frac{2\cdot\left(\frac{\lambda+5}{4}\right)}{\sqrt{\lambda+5}}=4.$

$\therefore\lambda=59$

Length of major axis $=2\cdot\sqrt{\lambda+5}=16=l$

$\therefore\lambda+l=75$.

Question309

Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point R($\sqrt{2}$, $2\sqrt{2}-2$). If S is the focus of the ellipse on its negative major axis, then SP^2+SQ^2 is equal to _____.
[28-Jul-2022-Shift-2]

Answer: 13

Solution:

$E\equiv\frac{x^2}{2}+\frac{y^2}{4}=1$

$T\equiv y=mx\pm\sqrt{2m^2+4}$

$\downarrow(\sqrt{2},2\sqrt{2}-2)$

$\Rightarrow(2\sqrt{2}-2-m\sqrt{2})=\pm\sqrt{2m^2+4}$

$\Rightarrow 2m^2-2m\sqrt{2}(2\sqrt{2}-2)+4(3-2\sqrt{2})=2m^2+4$

$\Rightarrow -2\sqrt{2}m(2\sqrt{2}-2)=4-12+8\sqrt{2}$

$\Rightarrow -4\sqrt{2}m(\sqrt{2}-1)=8(\sqrt{2}-1)$

$\Rightarrow m=-\sqrt{2}$ and $m\to\infty$

$\therefore$ Tangents are $x=\sqrt{2}$ and $y=-\sqrt{2}x+\sqrt{8}$

$\therefore P(\sqrt{2},0)$ and $Q(1,\sqrt{2})$

and $S=(0,-\sqrt{2})$

$\therefore(PS)^2+(QS)^2=4+9=13$

Question310

Let $S=\{(x,y)\in\mathbb{N}\times\mathbb{N}: 9(x-3)^2+16(y-4)^2\le 144\}$ and $T=\{(x,y)\in\mathbb{R}\times\mathbb{R}: (x-7)^2+(y-4)^2\le 36\}$. Then $n(S\cap T)$ is equal to _____.
[29-Jul-2022-Shift-2]

Answer: 27

Solution:

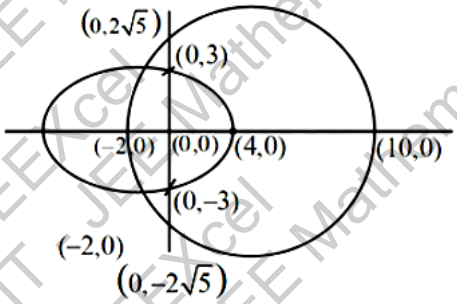
$S:\ \frac{(x-3)^2}{16}+\frac{(y-4)^2}{9}\le 1;\ x,y\in\{1,2,3,.....\}$

$T:(x-7)^2+(y-4)^2\le 36x,y\in R$

Let $x-3=x:y-4=y$

$S:\ \frac{x^2}{16}+\frac{y^2}{9}\le 1;\ x\in\{-2,-1,0,1,.....\}$

T : $(x-4)^2 + y^2 \le 36$; $y \in \{-3, -2, -1, 0, \ldots\ldots$



S ∩ T = (−2, 0), (−1, 0), ... (4, 0) → (7)
(−1, 1), (0, 1), ... (3, 1) → (5)
(−1, −1), (0, −1), ... (3, −1) → (5)
(−1, 2), (0, 2), (1, 2), (2, 2) → (4)
(−1, −2), (0, −2), (1, −2), (2, −2) → (4)
(0, 3)(0, −3) → (2)

Question311

Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (−5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to .....
[2021, 24 Feb. Shift-II]

Answer: 56.25

Solution:

Let P be (h, k), A(5, 0) and B(−5, 0).
Given PA = 3PB
$\Rightarrow PA^2 = 9PB^2$
$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2 + k^2]$
$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$
$\therefore$ Locus of P is $x^2 + y^2 + \frac{25}{2}x + 25 = 0$

Centre $= \left(\frac{-25}{4}, 0\right)$

and $r^2 = \left(\frac{-25}{4}\right)^2 + 0^2 - 25$

$= \frac{625}{16} - 25 = \frac{225}{16}$

$\therefore 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$

Question312

If the point of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$, lie on the curve $y^2 = 3x^2$, then b is equal to
[2021, 16 March Shift-II]

Options:

A. 12

B. 5

C. 6

D. 10

Answer: A

Solution:

Given, equation of ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1\ldots$ (i)

and equation of circle $x^2+y^2=4b$ $\cdots\cdots$ (ii)

From Eqs. (i) and (ii),

$\frac{4b-y^2}{16}+\frac{y^2}{b^2}=1$

$\Rightarrow\ 4b^3-y^2b^2+16y^2=16b^2$

$\Rightarrow\ y^2=\frac{16b^2-4b^3}{16-b^2}$

As, (x, y) lie on $y^2=3x^2$

So, $y^2=3(4b-y^2)$ [from Eq. (ii)]

$\Rightarrow\ 4y^2=12b$

$\Rightarrow\ y^2=3b$

Now, $\frac{16b^2-4b^3}{16-b^2}=3b$

$\Rightarrow\ 16b^2-4b^3=48b-3b^3$

$\Rightarrow b^3-16b^2+48b=0$

$\Rightarrow b(b^2-16b+48)=0$

$\Rightarrow b(b-4)(b-12)=0$

As, $b>4$

So, $b=12$

Question313

If one of the diameters of the circle $x^2+y^2-2x-6y+6=0$ is a chord of another circle ' C ', whose centre is at (2, 1), then its radius is
[2021, 24 Feb. Shift-I]

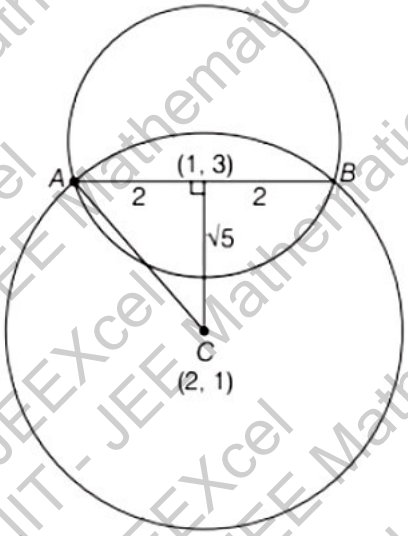
Answer: 3

Solution:

Given, circle $\Rightarrow x^2+y^2-2x-6y+6=0$

Coordinate of C = (1, 3)

Radius $=r=\sqrt{1^2+3^2-6}=\sqrt{4}=2$ units



$CD=\sqrt{(2-1)^2+(1-3)^2}$

$=\sqrt{1+4}=\sqrt{5}$

Now, by using Pythagoras theorem in $\triangle ADC, AC^2=AD^2+CD^2$

$=(2)^2+(\sqrt{5})^2=4+5$

$AC^2=9$

AC $= \sqrt{9} = 3$

$\therefore$ Required radius $= 3$

Question314

For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?
[2021, 24 Feb. Shift-II]

Options:

A. $x^2 + y^2 = 7$

B. $y^2 = \frac{1}{6\sqrt{3}}x$

C. $2x^2 - 18y^2 = 9$

D. $x^2 + 9y^2 = 9$

Answer: D

Solution:

Given, line $x + \sqrt{3}y = 2\sqrt{3}$

and point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

From options, we take the conic

$x^2 + 9y^2 = 9$

Equation of any tangent at (x_1, y_1) is

$cxx_1 + 9 \cdot yy_1 = 9$

$\frac{3\sqrt{3}}{2}x + 9 \times \frac{1}{2} \times y = 9 \left[\because (x_1, y_1) = \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)\right]$

$\Rightarrow \frac{3\sqrt{3}}{2}(x + \sqrt{3}y) = 9$

$\Rightarrow x + \sqrt{3}y = \frac{9 \times 2}{3\sqrt{3}}$

$\Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

Clearly, $x + \sqrt{3}y = 2\sqrt{3}$ is a tangent to the curve $x^2 + 9y^2 = 9$.

Question315

If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to
[2021, 26 Feb. Shift-II]

Options:

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: B

Solution:

Given, circle $x^2 + y^2 = 1$
Let (h, k) be the mid-point of line segment from point (3, 2) to a point on circle. Let point on circle be (a, b).
Then, $h = \frac{a+3}{2}$
$\Rightarrow a = 2h - 3$
$k = \frac{b+2}{2}$
$\Rightarrow b = 2k - 2$
$\because$(a, b) is point on circle, then
$(a)^2 + (b)^2 = 1$
$\Rightarrow (2h-3)^2 + (2k-2)^2 = 1$
$\Rightarrow \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$
Which is an equation of circle with
radius $\frac{1}{2}$.
$\therefore$ Radius $= \frac{1}{2}$.

Question316

Let A(1, 4) and B(1, −5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$, such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on
[2021, 26 Feb. Shift-III]

Options:

A. a straight line

B. a hyperbola

C. an ellipse

D. a parabola

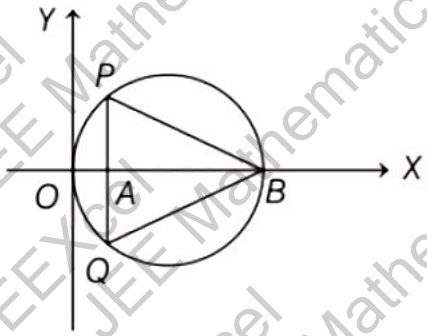
Answer: A

Solution:

Pbe a point on circle
$(x-1)^2 + (y-1)^2 = 1$
So, coordinate of P will be of
form $P(1 + \cos\theta, 1 + \sin\theta)$
Given,
A(1, 4) and B(1, −5)
Then,
$(PA)^2 + (PB)^2$
$= \cos^2\theta + (\sin\theta - 3)^2 + \cos^2\theta + (\sin\theta + 6)^2$
$= 2\cos^2\theta + 2\sin^2\theta - 6\sin\theta + 12\sin\theta + 45 = 47 + 6\sin\theta$
Now,
$(PA)^2 + (PB)^2 = 47 + 6\sin\theta$ is
Maximum, when $\sin\theta = 1$
$\Rightarrow \theta = \frac{\pi}{2}$
$\Rightarrow \cos\theta = 0$
$\therefore$P(1, 2), A(1, 4), B(1, −5)
$\therefore$P, B are collinear points on the line
x = 1

Question317

In the circle given below, let OA = 1 unit, OB = 13 unit and PQ⊥OB.
Then, the area of the triangle PQB (in square units) is



[2021, 26 Feb. Shift-I]

Options:

A. $24\sqrt{2}$

B. $24\sqrt{3}$

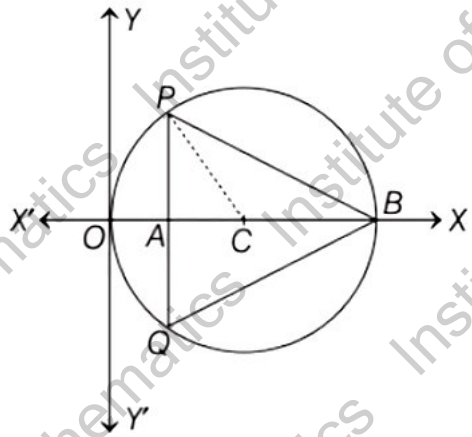
C. $26\sqrt{3}$

D. $26\sqrt{2}$

Answer: B

Solution:

Given, OA = 1 unit, OB = 13 unit Since, OB is diameter of circle. Then, radius (r) = $\frac{13}{2}$ = 6.5 unit



Draw a line joining points P and C, where C is the centre of the given circle.
Then, PC = radius of circle = 6.5 units OC = radius of circle = 6.5 units
Now, AC = OC − OA = 6.5 − 1 = 5.5 units Then, using Pythagoras theorem,

$(PA)^2 = (PC)^2 - (AC)^2$

$= (6.5)^2 - (5.5)^2$

$= (6.5 - 5.5)(6.5 + 5.5)$

$= (1)(12) = 12$

$\therefore$ PA $= \sqrt{12}$

Then, PQ = 2PA = $2\sqrt{12}$

Hence, area of $\triangle PQB = \frac{1}{2} \times$ (Base) x (Height)

$= \frac{1}{2} \times (PQ) \times (AB)$

$= \frac{1}{2} \times (PQ) \times (OB - OA)$

$= \frac{1}{2} \times (2\sqrt{12}) \times (13 - 1)$

$= 12\sqrt{12} = 24\sqrt{3}$ sq. units

Question318

A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to

[2021, 25 Feb. Shift-II]

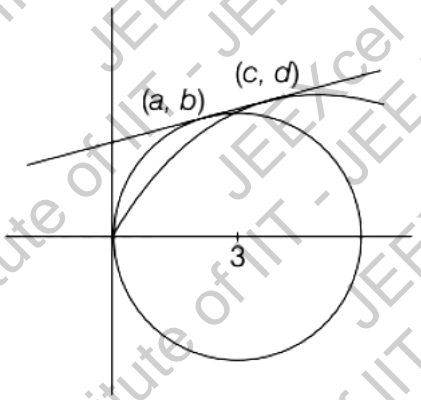
Answer: 9

Solution:

Given, circle $\Rightarrow (x-3)^2+y^2=9$

Parabola $\Rightarrow y^2=4x$

Let equation of common tangent be



$y = mx + \frac{a}{m}$

$\Rightarrow y = mx + \frac{1}{m} \quad (\because y^2=4x)$

$\Rightarrow m^2x - my + 1 = 0$

The above line is tangent to circle.

$\therefore$ Perpendicular from $(3, 0)$ to line $= 3$

$\Rightarrow \left| \frac{(3m^2-0+1)}{\sqrt{m^2+m^4}} \right| = 3$

$\Rightarrow (3m^2+1)^2 = 9(m^2+m^4)$

$\Rightarrow m = \pm \frac{1}{\sqrt{3}}$

Tangent is $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

$\Rightarrow m = \frac{1}{\sqrt{3}}$

or $y = \frac{(-1)}{\sqrt{3}} \times (-\sqrt{3})$ (rejected)

For parabola, point of contact is

$(c, d) = \left(\frac{a}{m^2}, \frac{2a}{n} \right)$

$\therefore (c, d) = (3, 2\sqrt{3})$

Again, solve circle and line equation, we get

$\&(x-3)^2 + \left[\left(\frac{1}{\sqrt{3}}x + \sqrt{3} \right) \right]^2 = 9$

$\Rightarrow x^2 + 9 - 6x + \frac{1}{3}x^2 + 3 + 2x = 9$

$\Rightarrow \frac{4}{3}x^2 - 4x + 3 = 0$

$\Rightarrow x = \frac{3}{2} = a$

$\therefore 2(a+c) = 2\left(\frac{3}{2}+3\right) = 9$

Question319

A tangent is drawn to the parabola $y^2 = 6x$, which is perpendicular to the line $2x + y = 1$. Which of the following points does not lie on it?

[2021, 25 Feb. Shift-I]

Options:

A. $(-6, 0)$

B. $(4, 5)$

C. $(5, 4)$

D. $(0, 3)$

Answer: C

Solution:

Given, parabola $\Rightarrow y^2 = 6x$

$\Rightarrow \; y^2 = 4\left(\frac{3}{2}\right)x \;\; [\because y^2 = 4ax]$

and given, line $\Rightarrow 2x + y = 1$.

$\because$ Equation of any tangent to the parabola having slope mis

$y = mx + 3/2m \;\; [\because a = 3/2]$

Slope of line $2x + y = 1$ is $m_1 = -2$

$\because$ Tangent is perpendicular to this line,

$\therefore$ Slope of tangent $= m_2 = -\frac{1}{m_1} = \frac{1}{2}$

$\therefore$ Equation of tangent will be

$y = \frac{1}{2}x + \frac{3}{2} \times 2$

$\Rightarrow \; y = \frac{x}{2} + 3$

or $2y = x + 6$

or $x - 2y + 6 = 0$

Clearly, on putting the coordinates of point $(5, 4)$, the equation of tangent is not satisfie(d)

$\therefore$ Point $(5, 4)$ does not lie on this tangent.

Question320

If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are
[2021, 24 Feb. Shift-II]

Options:

A. $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

B. $a = 1, b = 0, c = 1$

C. $a = 1, b = 1, c = 0$

D. $a = -1, b = 1, c = 1$

Answer: C

Solution:

Given, curve $\Rightarrow y = ax^2 + bx + c$, $x \in R$ and point $(1, 2)$

$\because$ The given curve passes through $(1, 2)$.

$\therefore \; 2 = a + b + c$

Also, slope of tangent of $y = ax^2 + bx + c$ is $\frac{dy}{dx} = 2ax + b$

$\because$ Tangent passes through origin $(0, 0)$.

$\therefore \; \frac{dy}{dx}|_{(0, 0)} = 2a \times 0 + b = b \quad \cdots\cdots (i)$

According to the question, tangent at origin is $y = x$

$\therefore$ Its slope is 1. $\cdots\cdots (ii)$

From Eqs. (i) and (ii),

$b = 1$

Also, $a + b + c = 2$

cc $\Rightarrow a + c + 1 = 2$
$\Rightarrow a + c = 1$
From the option look for $b = 1$ and $a + c = 1$
The only correct order triplet is $a = 1, b = 1, c = 0$.

Question321

If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the coordinates of P are
[2021, 24 Feb. Shift-II]

Options:

A. (3, 13)

B. (1, 5)

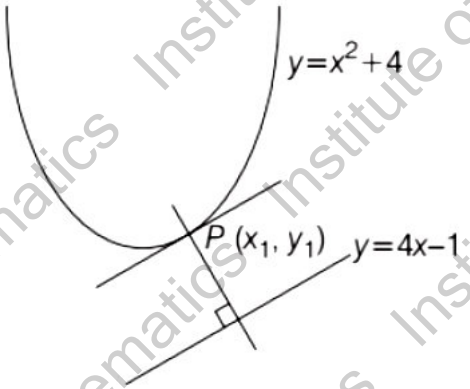
C. (−2, 8)

D. (2, 8)

Answer: D

Solution:

Given, curve $y = x^2 + 4$
and, line $y = 4x - 1$
Here, $y = x^2 + 4$



$\therefore \quad \frac{dy}{dx} = 2x \quad \cdots\cdots\cdot (i)$
and
$y = 4x - 1$

$\frac{dy}{dx} = 4 \quad \cdots\cdots\cdot (ii)$
Let the required point be $P(x_1, y_1)$.

$\therefore \quad \frac{dy}{dx}|_P = 2x_1 \quad \cdots\cdots\cdot (iii)$
$\because$ Slopes will be equal.
$\therefore \quad 2x_1 = 4$ [from Eqs. (ii) and (iii)]

$\Rightarrow x_1 = \frac{4}{2} = 2$
Now, the given point $P(x_1, y_1)$ lies on curve
$y = x^2 + 4,$
$\therefore \quad y_1 = x_1^2 + 4$
$\Rightarrow \quad y_1 = 2^2 + 4 = 8$
Hence, required coordinate of P = (2, 8)

Question322

A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is
[2021, 25 Feb. Shift-II]

Options:

A. $\frac{x^2}{9}-\frac{y^2}{16}=1$

B. $\frac{x^2}{9}-\frac{y^2}{4}=1$

C. $\frac{x^2}{9}-\frac{y^2}{25}=1$

D. $x^2-y^2=9$

Answer: A

Solution:

Given, ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$

or $\frac{x^2}{(5)^2}+\frac{y^2}{(4)^2}=1$

Compare it with standard equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

we get $a=5, b=4$

Now, focus of ellipse $=(\pm e, 0)$ where

$c=\sqrt{a^2-b^2}$

Put the values of a and b, we get

$c=\sqrt{5^2-4^2}=\sqrt{25-16}=\sqrt{9}$

$\therefore$ Focus $=(\pm 3, 0)$

According to question, hyperbola passes through the focus of ellipse.

Let equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Since, it passes through $(\pm 3, 0)$, we get $\frac{(\pm 3)^2}{a^2}-\frac{0}{b^2}=1$, gives $a=\pm 3$ or $a^2=9$

Also, given that product of eccentrcikes is 1.

Now, (Eccentricity of ellipse)

(Eccentricity of hyperbola) $=1 \Rightarrow \left(\sqrt{1-\frac{16}{25}}\right)\left(\sqrt{1+\frac{b^2}{9}}\right)=1$

(using formula of eccentricity of ellipse and hyperbola)

$\Rightarrow \left(\sqrt{\frac{9}{25}}\right)\left(\sqrt{1+\frac{b^2}{9}}\right)=1$

(using formula of eccentricity of ellipse and hyperbola)

$\Rightarrow \left(\sqrt{\frac{9}{25}}\right)\left(\sqrt{1+\frac{b^2}{9}}\right)=1$

Squaring on both sides,

$1+\frac{b^2}{9}=\frac{25}{9} \Rightarrow b^2=16$

Thus, equation of hyperbola is

$\frac{x^2}{9}-\frac{y^2}{16}=1.$

Question323

The locus of the point of intersection of the lines $(\sqrt{3})kx+ky-4\sqrt{3}=0$ and $\sqrt{3}x-y-4(\sqrt{3})k=0$ is a conic, whose eccentricity is
[2021, 25 Feb. Shift-1]

Answer: 2

Solution:

Given, lines are $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ ... (i)

$\sqrt{3}x - y - 4\sqrt{3}k = 0$ ..... (ii)

Multiply Eq. (ii) × k and then adding Eqs. (i) and (ii),

$\sqrt{3}kx + ky - 4\sqrt{3} = 0$

$\sqrt{3}kx - ky - 4\sqrt{3}k^2 = 0$

$(2\sqrt{3}x)k = 4\sqrt{3} + 4\sqrt{3}k^2$

$\therefore\ x = \frac{4\sqrt{3}(1+k^2)}{2\sqrt{3}k} = 2\left(k + \frac{1}{k}\right)$

Subtracting Eq. (i) from Eq. (ii),

$\sqrt{3}kx + ky - 4\sqrt{3} = 0$

$\sqrt{3}kx - ky - 4\sqrt{3}k^2 = 0$

$2ky = 4\sqrt{3} - 4\sqrt{3}k^2$

$y = \frac{4\sqrt{3}(1+k^2)}{2k} = 2\sqrt{3}\left(k + \frac{1}{k}\right)$

We have, $x = 2\left(k + \frac{1}{k}\right)$ and

$cy = 2\sqrt{3}\left(\frac{1}{k} - k\right)$

$\frac{x}{2} = \left(k + \frac{1}{k}\right)$ ..... (iii)

$\frac{y}{2\sqrt{3}} = \left(\frac{1}{k} - k\right)$ ..... (iv)

Squaring and subtracting Eq. (iii) from Eq. (iv),

$\frac{x^2}{4} - \frac{y^2}{12} = \left(k^2 + \frac{1}{k^2} + 2\right) - \left(\frac{1}{k^2} + k^2 - 2\right)$

$\frac{x^2}{4} - \frac{y^2}{12} = 4$

or $\frac{x^2}{16} - \frac{y^2}{48} = 1$

Clearly, this is a hyperbol(a)

$\therefore\ e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{48}{16} = 1 + 3$

$e^2 = 4$

$\therefore\ e = \sqrt{4} = 2(\because e \text{ is positive})$

Question324

The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :
24 Feb 2021 Shift 1

Options:

A. $x = -\frac{a}{2}$

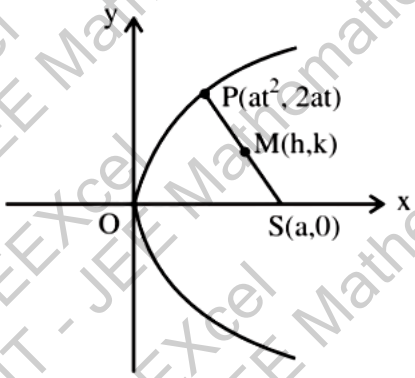
B. $x = \frac{a}{2}$

C. $x = 0$

D. $x = a$

Answer: C

Solution:



$h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2}$

$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

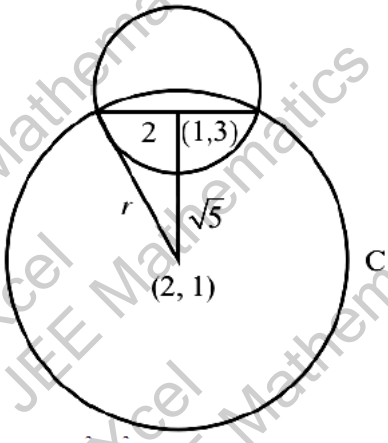
Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

Question325

If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle ' C ', whose center is at (2, 1), then its radius is
24 Feb 2021 Shift 1

Answer: 3

Solution:



Given that $x^2 + y^2 - 2x - 6y + 6 = 0$ center (1, 3) and radius = 2
Distance between (1, 3) and (2, 1) is $\sqrt{5}$
$\therefore (\sqrt{5})^2 + (2)^2 = r^2$

Question326

If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$, intersect each other at an angle of 90°, then which of the following relations is true?

[2021, 25 Feb. Shift-I]

Options:

A. $a + b = c + d$

B. $a - b = c - d$

C. $a - c = b + d$

D. $ab = \frac{c+d}{a+b}$

Answer: B

Solution:

Given, curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1 \because \frac{x^2}{a} + \frac{y^2}{b} = 1$

On differentiating both sides w.r.t. x, we get

$\frac{2x}{a} + \frac{2y}{b} \cdot \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{-bx}{ay} \quad \cdots\cdots (i)$

Also, $\frac{x^2}{c} + \frac{y^2}{d} = 1$

On differentiating both sides w.r.t. x, we get

$\frac{2x}{c} + \frac{2y}{d} \cdot \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{-dx}{cy} \quad \cdots\cdots (ii)$

$\because$ Both the curves intersect each other at 90°.

Question327

Choose the correct statement about two circles whose equations are given below.

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

[2021, 18 March Shift-1]

Options:

A. circles have same centre

B. circles have no meeting point

C. circles have only one meeting point

D. circles have two meeting points

Answer: C

Solution:

Given, equation of circles

$x^2 + y^2 - 10x - 10y + 41 = 0 \quad \cdots\cdots (i)$

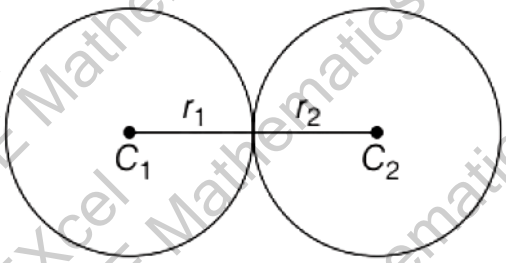
$x^2 + y^2 - 22x - 10y + 137 = 0 \quad \cdots\cdots (ii)$

Centre (C_1) and radius (r_1) of Eq. (i) are

$C_1 = (5, 5)$ and $r_1 = 3$

and centre (C_2) and radius (r_2) of Eq. (ii) are

$C_2 = (11, 5)$ and $r_2 = 3$



We observe that,
C_1C_2 = distance between centres
$= (11-5)^2 + (5-5)^2 = 6 = r_1 + r_2$
∴ Circles Eqs. (i) and (ii) touch each other externally.
Hence, circles have only one meeting point.

Question328

For the four circles M , N , O and P_1 following four equations are given

Circle M : $x^2 + y^2 = 1$
Circle N : $x^2 + y^2 - 2x = 0$
Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$
Circle P : $x^2 + y^2 - 2y = 0$
If the centre of circle M is joined with centre of the circle N , further centre of circle N is joined with centre of the circle 0 , centre of circle 0 is joined with the centre of circle P and lastly, centre of circle P is joined with centreof circle M , then these lines form the sides of a
[2021, 18 March Shift-I]

Options:

A. rhombus

B. square

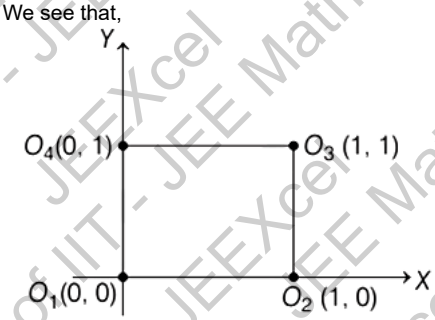
C. rectangle

D. parallelogram

Answer: B

Solution:

Given, M $\Rightarrow x^2 + y^2 = 1$, its centre
$O_1 \equiv (0, 0)$
N $\Rightarrow x^2 + y^2 - 2x = 0$, its centre $O_2 \equiv (1, 0)$
$0 \Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$, its centre
$O_3 \equiv (1, 1)$
P $\Rightarrow x^2 + y^2 - 2y = 0$, its centre $O_4 \equiv (0, 1)$
We see that,



$O_1O_2 = O_2O_3 = O_3O_4 = O_4O_1$ and $O_1O_3 = O_2O_4$
Hence, $O_1O_2O_3O_4$ form a square.

Question329

Let $S_1 \Rightarrow x^2 + y^2 = 9$ and $S_2 \Rightarrow (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points
[2021, 18 March Shift-II]

Options:

A. $(0, \pm\sqrt{3})$

B. $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

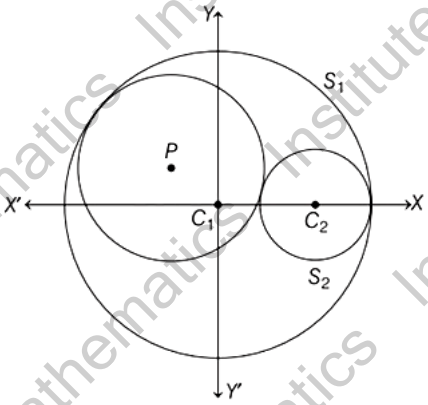
C. $\left(2, \pm\frac{3}{2}\right)$

D. $(1, \pm 2)$

Answer: A

Solution:

Given, $S_1 \Rightarrow x^2 + y^2 = 9$
$S_2 \Rightarrow (x-2)^2 + y^2 = 1$
$\therefore$ $C_1 = (0, 0)$ and $r_1 = 3$
$C_2 = (2, 0)$, $r_2 = 1$
Let centre of variable circle be $P(h, k)$ and radius be r.
From the given data, it is clear that given circles touch each other internally. Hence, variable circle touches S_1 internally and S_2 externally as shown in figure below.



Clearly, $PC_1 = 3 - r$
and $PC_2 = 1 + r$
and $C_1C_2 = 2$
$\therefore$ $PC_1 + PC_2 = 4$
which is greater than C_1C_2.
Hence, locus of P is an ellipse whose foci are C_1 and C_2 and major axis is $2a = 4$.
and $2ae = C_1C_2 = 2$
$\Rightarrow$ $e = \frac{1}{2}$

$\Rightarrow$ $b^2 = 4\left(1 - \frac{1}{4}\right) = 3$
Centre is mid-point of $C_1(0, 0)$ and $C_2(2, 0)$.
$\therefore$ Centre of ellipse is $(1, 0)$.
$\frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$
is the equation of locus of centre of variable circle which is satisfied by $\left(2, \pm\frac{3}{2}\right)$.

Question330

Choose the incorrect statement about the two circles whose equations are given below
$x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 16x - 10y + 80 = 0$
[2021, 17 March Shift-1]

Options:

A. Distance between two centres is the average of radii of both the circles.

B. Both circles' centres lie inside region of one another.

C. Both circles pass through the centre of each other.

D. Circles have two intersection points.

Answer: B

Solution:

$C_1 \Rightarrow x^2 + y^2 - 10x - 10y + 41 = 0$

$(x-5)^2 + (y-5)^2 + 41 = 25 + 25$

$\Rightarrow$ Centre $= (5, 5)$ and Radius $= 3$

$\Rightarrow (x-5)^2 + (y-5)^2 = 3^2$

$C_2 \Rightarrow x^2 + y^2 - 16x - 10y + 80 = 0$

$(x-8)^2 + (y-5)^2 + 80 = 64 + 25$

$\Rightarrow (x-8)^2 + (y-5)^2 = 3^2$

$\Rightarrow$ Centre $= (8, 5)$ and Radius $= 3$

Now, distance between centres

$= \sqrt{(8-5)^2 + (5-5)^2} = 3$

Average radii $= \frac{3+3}{2} = 3$

$\therefore$ Option (a) is correct.

$C_1(8, 5) = (8-5)^2 + (5-5)^2 - 9 = 0$

$C_2(5, 5) = (5-8)^2 + (5-5)^2 - 9 = 0$

Centres of each other lies on circumference of each other. Hence, (b) is incorrect statement.

Question331

The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles equations
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 24x - 10y + 160 = 0$ is .....
[2021, 17 March Shift-1]

Answer: 1

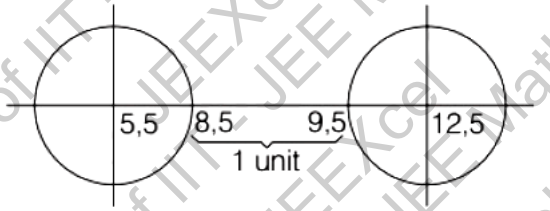
Solution:

$C_1 \Rightarrow x^2 + y^2 - 10x - 10y + 41 = 0$

$\Rightarrow (x-5)^2 + (y-5)^2 = 3^2$

$C_2 \Rightarrow x^2 + y^2 - 24x - 10y + 160 = 0$

$\Rightarrow (x-12)^2 + (y-5)^2 = 3^2$



$\therefore$ Minimum distance $= 1$ unit.

Question332

The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is
[2021, 17 March Shift-I]

Options:

A. $3\sqrt{5}$

B. $5\sqrt{3}$

C. $5\sqrt{4}$

D. $4\sqrt{5}$

Answer: A

Solution:

Given, $2x - y + 1 = 0$ is a tangent to the circle at $(2, 5)$

So, normal at $(2, 5)$ will be $\frac{(y-5)}{(x-2)} = \frac{-1}{2}$

$\Rightarrow 2y - 10 = -x + 2$
$\Rightarrow x + 2y = 12$

Now, it is also given that centre lies on $x - 2y = 4$.

So, coordinates of centre will be the solution of $\begin{cases} x + 2y = 12 \\ x - 2y = 4. \end{cases} \Rightarrow x = 8, y = 2$

Radius will be the distance between $(8, 2)$ and $(2, 5)$

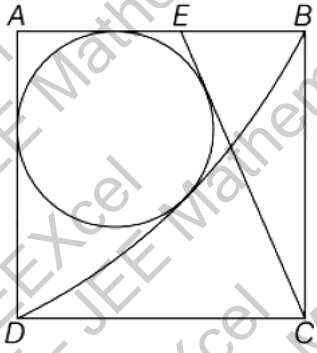
$r^2 = (8-2)^2 + (2-5)^2$
$\Rightarrow r^2 = 36 + 9 \Rightarrow r = \sqrt{45} = 3\sqrt{5}$

Question333

Let ABCD be a square of side of unit length. Let a circle C_1 centred at A with unit radius is drawn. Another circle C_2, which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of E B is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to
[2021, 16 March Shift-1]

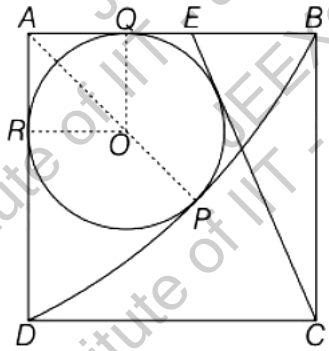
Answer: 1

Solution:



Given, $EB = \alpha + \sqrt{3}\beta$

Let O be the centre of circle C_2.



$OQ^2 + QR^2 = AO^2$

$\Rightarrow$ AO + OP = AP = 1

$\Rightarrow$ AO + r = 1

$OQ^2 + QR^2 = AO^2$

$\Rightarrow$ $r^2 + r^2 = (1 - r)^2$

$\Rightarrow$ $2r^2 = r^2 + 1 - 2r$

$\Rightarrow$ $r^2 + 2r + 1 = 1 + 1$

$\Rightarrow$ $(r + 1)^2 = 2$

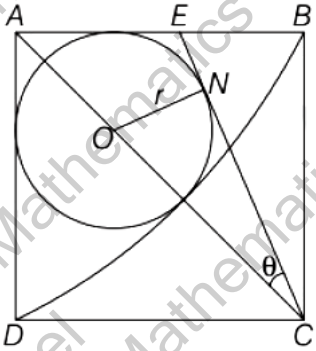
$\Rightarrow$ $r + 1 = \sqrt{2} \Rightarrow r = \sqrt{2} - 1$

As, AO = 1 − r

AC = $\sqrt{2}$

So, OC = AC − AO

= $\sqrt{2} - (1 - \sqrt{2} + 1) = 2(\sqrt{2} - 1)$



In $\triangle OCN$, we have

$\frac{ON}{OC} = \sin\theta$

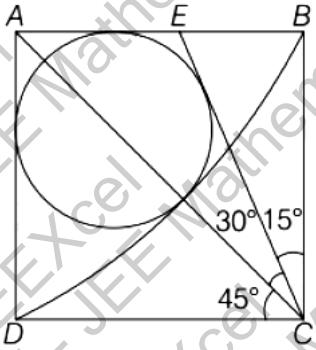
$\Rightarrow \frac{\sqrt{2} - 1}{2(\sqrt{2} - 1)} = \frac{1}{2} = \sin\theta \Rightarrow \theta = \pi/6$

In Δ EBC, we have

$\frac{EB}{BC} = \tan 15^\circ$

$EB = (\tan 15^\circ)BC$

$= (\tan 15^\circ)$ [$\because BC = 1$ unit]



Now,

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \tan 30^\circ = \dfrac{2\tan 15^\circ}{1-\tan^2 15^\circ}$

$\Rightarrow 1-\tan^2 15^\circ = 2\sqrt{3}\tan 15^\circ$

$\Rightarrow \tan^2 15^\circ + 2\sqrt{3}\tan 15^\circ - 1 = 0$

$\tan 15^\circ = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$\tan 15^\circ = -\sqrt{3}+2$

$\therefore EB = 2-\sqrt{3} = \alpha + \sqrt{3}\beta$

$\alpha = 2, \beta = -1$

$\therefore \alpha + \beta = 2 + (-1) = 1$

Question334

Let the lengths of intercepts on X -axis and Y -axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then, the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to
[2021, 16 March Shift-11]

Options:

A. $\sqrt{11}$

B. $\sqrt{7}$

C. $\sqrt{6}$

D. $\sqrt{10}$

Answer: C

Solution:

$x^2 + y^2 + ax + 2ay + c = 0 (a < 0)$

For a circle in standard form

$x^2 + y^2 + 2gx + 2fy + c = 0$

x -intercept $= 2\sqrt{g^2 - c}$

y -intercept $= 2\sqrt{f^2 - c}$

Here, $2g = a \Rightarrow g = a/2$

$c2f = 2a \Rightarrow f = a$

x-intercept $\Rightarrow 2\sqrt{\dfrac{a^2}{4} - c} = 2\sqrt{2}$

$\Rightarrow \dfrac{a^2}{4} - c = 2 \quad \cdots\cdots (i)$

y -intercept $\Rightarrow 2\sqrt{a^2 - c} = 2\sqrt{5}$

$a^2 - c = 5 \quad \cdots\cdots (ii)$

Subtracting Eq. (ii) from Eq. (i), we get

$c\left(\dfrac{3}{4}\right)a^2 = 3$

$\Rightarrow a^2 = 4 \Rightarrow a = \pm 2$

As, $a < 0$, so, $a = -2, c = -1$
So, $x^2 + y^2 - 2x - 4y - 1 = 0$
$(x-1)^2 + (y-2)^2 - 1 = 1 + 4$
$\Rightarrow (x-1)^2 + (y-2)^2 = 6$
Centre $(1, 2)$, Radius $= \sqrt{6}$
Equation of tangent is perpendicular to $x + 2y = 0$.
So, equation of tangent will be
$2x - y + \lambda = 0$
Now, perpendicular distance of this line from $(1, 2)$ will be $\sqrt{6}$ units.
$\frac{|2.1 - 2 + \lambda|}{\sqrt{5}} = \sqrt{6}$
$\Rightarrow \lambda = \pm\sqrt{30}$
So, T $\Rightarrow 2x - y \pm \sqrt{30} = 0$
Now, shortest distance from origin $(0, 0)$
to tangentT will be
$\frac{|2.0 - 0 \pm \sqrt{30}|}{\sqrt{2^2 + 1^2}} = \frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$

Question335

If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0), $a \neq 0$, then a must be greater than
[2021, 16 March Shift-1]

Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. -1

D. 1

Answer: D

Solution:

Given, equation of parabola $\Rightarrow y^2 = 2x$
Equation of normal of parabola, $y^2 = 4ax$ is $tx + y = 2at + at^3$
Here, $4a = 2$
So, $a = 1/2$
So, equation of normal $\Rightarrow tx + y = t + t^3/2$
$t^3 + (2 - 2x)t - 2y = 0$
As, there are three normals which are passing through $(a, 0)$, so there must be three roots of this equation.
$t^3 + (2 - 2a)t - 2.0 = 0$
$\Rightarrow t^3 + (2 - 2a)t = 0$
$\therefore t_1 + t_2 + t_3 = -(2 - 2a) = 2a - 2$
and $t_1t_2 + t_2t_3 + t_3t_1 = 0$
So, $t_1^2 + t_2^2 + t_3^2 > 0$
$(t_1 + t_2 + t_3)^2 - 2(t_1t_2 + t_2t_3 + t_3t_1) > 0$
$\Rightarrow (2a - 2)^2 - 2.0 > 0$
$\Rightarrow a > 1$

Question336

Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then, the equation of tangent to C at P(2, 1) is
[2021, 16 March Shift-II]

Options:

A. $x - y = 1$

B. $2x + y = 5$

C. $x + 3y = 5$

D. $x + 2y = 5$

Answer: A

Solution:

The mirror image of any point (α, β) with respect to line $y = x$ is simply (β, α).
Let (h, k) be the mirror image of a point on parabola $y^2 = 4ax$
Then, (k, h) will be the mirror image of (h, k) and it will lie on parabola.
So, $y^2 = 4x$
$\Rightarrow x^2 = 4y$
Hence, Locus is $x^2 = 4y \quad \cdots\cdots (i)$
For finding equation of tangent differentiate Eq. (i)w.r.t. x
$2x = 4\frac{dy}{dx}$
$\Rightarrow \frac{dy}{dx} = \frac{2x}{4} = \left(\frac{x}{2}\right)$
$\Rightarrow \left(\frac{dy}{dx}\right)_{2,1} = \left(\frac{2}{2}\right) = 1$
$\Rightarrow \frac{y-1}{x-2} = 1 \Rightarrow y - 1 = x - 2$
$y = x - 1$
$\therefore$ Equation of tangent $\Rightarrow y = x - 1$
$\Rightarrow x - y = 1$

Question337

Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$, where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ, such that the sum of intercepts on axes made by this tangent is minimum is equal to [2021, 18 March Shift-II]

Options:

A. $\frac{\pi}{8}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: C

Solution:

Given, equation of ellipse is
$\frac{x^2}{27} + y^2 = 1$
Now, equation of tangent to the ellipse at $P(3\sqrt{3}\cos\theta, \sin\theta)$ is given by
$\frac{3\sqrt{3}\cos\theta \cdot x}{27} + \sin\theta \cdot y = 1$
$\Rightarrow \frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1 \quad \cdots\cdots (i)$
X -intercept of Eq. (i) is
$x = 3\sqrt{3}\sec\theta = OA$ (say)
Y -intercept of Eq. (i) is
$y = \text{cosec}\theta = OB$ (say)
$\therefore$ Sum of intercepts
$= 3\sqrt{3}\sec\theta + \text{cosec}\theta = f(\theta)$ (say)
$\Rightarrow f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\theta \cdot \cot\theta$
Put

$\Rightarrow \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin^2\theta}$

$\Rightarrow \frac{\sin\theta}{\cos^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{3\sqrt{3}}$

$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}}$

$\Rightarrow \theta = \pi/6$

$-\infty \blacktriangleleft \overset{-}{\longrightarrow} \underset{\theta=\pi/6}{|} \overset{+}{\longrightarrow} \blacktriangleright +\infty$

$\because f'(\theta)$ changes sign from negative to positive when moving from left to right

$\therefore f(\theta)$ will attain minima at $\theta = \frac{\pi}{6}$

Question338

Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y}{b} = 1$, then the value of b is equal to
[2021, 17 March Shift-II]

Options:

A. 11

B. 14

C. 16

D. 20

Answer: B

Solution:

Given, equation of parabola is

$y^2 = 4x - 20 \quad \cdots\cdots (i)$

Differentiating Eq. (i) w.r.t. x , we get

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\therefore \left(\frac{dy}{dx}\right)_{(6,2)}$ = Slope of tangent at

$(6, 2) = \frac{2}{2} = 1$

$\therefore$ Equation of tangent is

$y - 2 = 1(x - 6) \Rightarrow x - y - 4 = 0 \quad \cdots\cdots (ii)$

As, we know the condition of tangency to the ellipse,

A straight line $y = mx + c$ will be tangent to the ellipse

$x^2/a^2 + y^2/b^2 = 1$ is

$c^2 = a^2m^2 + b^2$

Then, $16 = 2(1)^2 + b$ [here, $c = 4, m = 1, b = \sqrt{b}$]

$b = 14$

Question339

A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The mid-points of its sides also lie on the same curve. Then, the square of area of ABCD is .....
[2021, 18 March Shift-I]

Answer: 80

Solution:

Let $A\left(t_1, \frac{1}{t_1}\right)$ and $C\left(t_3, \frac{1}{t_3}\right)$ lies on $xy = 1$

and $B\left(t_2, \frac{1}{t_2}\right)$ and $D\left(t_4, \frac{1}{t_4}\right)$ lies on $xy = -1$

as shown in figure.

ABCD is a square and by symmetry, diagonals must intersects at $O(0, 0)$.

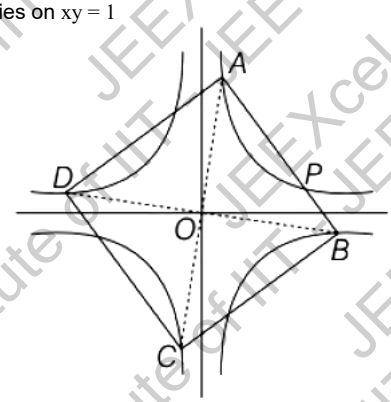
$\angle AOB = 90^\circ$

$\Rightarrow \frac{-1}{t_1^2} \cdot \frac{1}{t_2^2} = -1$

$\Rightarrow t_1^2 t_2^2 = 1 \quad \cdots\cdots (i)$

Also, mid-point of AB is $P\left(\frac{t_1 + t_2}{2}, \frac{t_2 - t_1}{2t_1 t_2}\right)$

lies on $xy = 1$



$\Rightarrow \left(\frac{t_1 + t_2}{2}\right)\left(\frac{t_2 - t_1}{2t_1 t_2}\right) = 1$

$\Rightarrow t_2^2 - t_1^2 = 4t_1 t_2 \quad \cdots\cdots (ii)$

Now, area of square $= (AB)^2$

$= (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2$

$= (t_1 - t_2)^2 + \frac{(t_2 + t_1)^2}{t_1^2 t_2^2}$

$= (t_1 - t_2)^2 + (t_2 + t_1)^2$ [From Eq. (i)]

$= 2(t_1^2 + t_2^2)$

$= 2\sqrt{(t_2^2 - t_1^2)^2 + 4t_1^2 t_2^2}$

$= 2\sqrt{16t_1^2 t_2^2 + 4t_1^2 \cdot t_2^2}$

$= 2\sqrt{20} = \sqrt{80}$

∴ Square of area $= 80$ sq units

Question340

Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point P(4, $\sqrt{6}$) meet the x-axis at Q and latus rectum at R(x_1, y_1), $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of $\triangle$QF R is equal to

[2021, 18 March Shift-II]

Options:

A. $4\sqrt{6}$

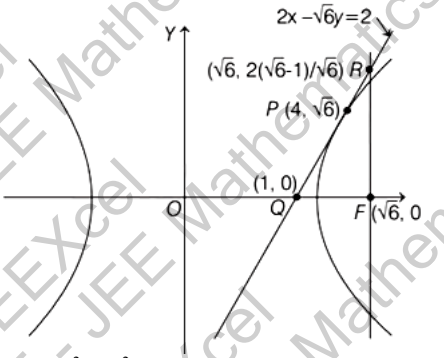
B. $\sqrt{6} - 1$

C. $\frac{7}{\sqrt{6}} - 2$

D. $4\sqrt{6} - 2$

Answer: C

Solution:



Given, $x^2 - 2y^2 = 4$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{2} = 1$

$\Rightarrow \frac{x^2}{(2)^2} - \frac{y^2}{(\sqrt{2})^2} = 1$

Here, $a = 2, b = \sqrt{2}$

$\therefore e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{2}{4}}$

$= \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$

So, Focus $(F) = (\pm ae, 0) = (\pm\sqrt{6}, 0)$

Now, equation of tangent at $P(4, \sqrt{6})$ is

$4x - 2\sqrt{6}y = 4$ $[\because x \cdot x - 2 \cdot y \cdot y = 4, x \cdot 4 - 2 \cdot y \cdot \sqrt{6} = 4]$

$\Rightarrow 2x - \sqrt{6}y = 2 \quad \cdots\cdots (i)$

Putting $y = 0$ in Eq. (i), we get x -intercept of tangent i.e. $x = 1$

$\therefore Q \equiv (1, 0)$

Hence, equation of corresponding latus rectum is $x = \sqrt{6}$.

$\therefore R \equiv \left(\sqrt{6}, \frac{2(\sqrt{6}-1)}{\sqrt{6}}\right)$

[putting $x = \sqrt{6}$ in Eq. (i), we get $\partial y = \frac{2(\sqrt{6}-1)}{\sqrt{6}}$]

$\therefore$ Area of $\triangle OF R = \frac{1}{2} \times (QF) \times (RF)$

$= \frac{1}{2}(\sqrt{6}-1) \times \frac{2(\sqrt{6}-1)}{\sqrt{6}}$

$= \frac{(\sqrt{6}-1)^2}{\sqrt{6}} = \left(\frac{7}{\sqrt{6}} - 2\right)$

Question341

The locus of the mid-points of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

[2021, 16 March Shift-1]

Options:

A. $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

B. $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

C. $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

D. $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Answer: D

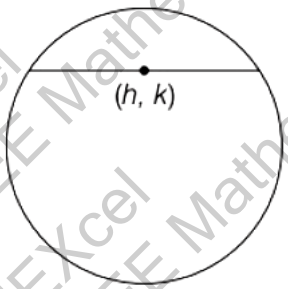
Solution:

Given, equation of circle $\Rightarrow x^2 + y^2 = 25$

Equation of hyperbola $\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

Let the mid-point of the chord of the circle be (h, k).

The equation of a chord through its mid-point is simply $T = S_1$



where T is the equation of tangent and S_1 is the value of S by putting (h, k).

Here,

$hx + ky = h^2 + k^2$

The equation of tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m - b^2}$

Here, $y = mx \pm \sqrt{9m^2 - 16}$

So, $y = \frac{-hx}{k} + \frac{h^2 + k^2}{k}$ ..... (i)

and $y = mx \pm \sqrt{9m^2 - 16}$ ..... (ii)

Eqs. (i) and (ii) are identical, so

$\frac{1}{1} = \frac{-h/k}{m} = \frac{h^2 + k^2}{k\sqrt{9m^2 - 16}} \Rightarrow m = \frac{-h}{k}$

$\Rightarrow 9m^2 - 16 = \frac{(h^2 + k^2)^2}{k^2}$

$\Rightarrow 9\left(\frac{h^2}{k^2}\right) - 16 = \frac{(h^2 + k^2)^2}{k^2}$

$\Rightarrow 9h^2 - 16k^2 = (h^2 + k^2)^2 \begin{cases} h \to x \\ k \to y. \end{cases}$

So, $9x^2 - 16y^2 = (x^2 + y^2)^2$

or $(x^2 + y^2) - 9x^2 + 16y^2 = 0$

Question342

Two tangents are drawn from the point P(−1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the △ABD is equal to
[2021, 27 July Shift-I]

Options:

A. 2

B. $(3\sqrt{2} + 2)$

C. 4

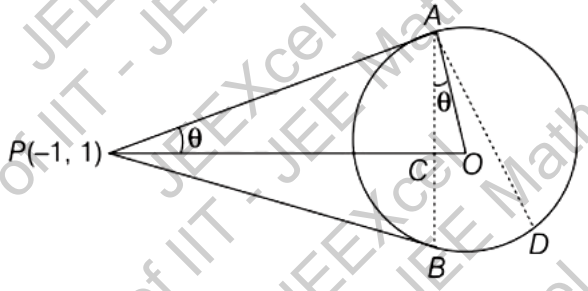
D. $3(\sqrt{2} - 1)$

Answer: C

Solution:

$C : x^2 + y^2 - 2x - 6y + 6 = 0$

$(x - 1)^2 + (y - 3)^2 = 2^2$



$AP = \text{Length of tangent} = \sqrt{S_1}$

$\Rightarrow AP^2 = (-1-1)^2 + (1-3)^2 - 4$

$\Rightarrow AP^2 = 4 + 4 - 4$

$\therefore AP = 2$

$\tan\theta = \frac{AO}{AP} = \frac{2}{2} = 1 \quad (\therefore \theta = 45^\circ)$

In $\triangle ACO$,

$\frac{AC}{AO} = \cos\theta$

$\therefore AC = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

$\therefore \triangle OAC \cong \triangle OAM$

$\Delta OAC \cong DOBC$

$\Delta OBC \cong \triangle ODM$

$\therefore$ Area of $\triangle ABD = 4$ Area of $\triangle ACO$

$= 4 \times \frac{1}{2}(AO \times AC \times \sin\theta)$

$= 2 \times 2 \times \sqrt{2} \times \frac{1}{\sqrt{2}} = 4$

Question343

Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to [2021, 27 July Shift-1]

Options:

A. $\{(4, 0), (0, 6)\}$

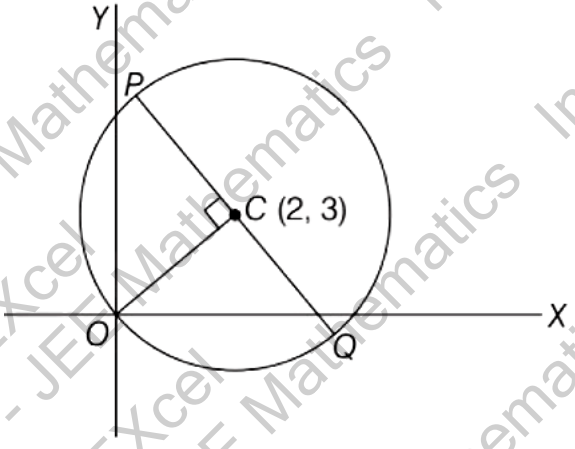
B. $\{(2+2\sqrt{2}, 3-\sqrt{3}), (2-2\sqrt{2}, 3+\sqrt{5})\}$

C. $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})$

D. $\{(-1, 5), (5, 1)\}$

Answer: D

Solution:



Equation of circle is

$(x-2)^2 + (y-3)^2 = (2-0)^2 + (3-0)^2$

$\Rightarrow (x-2)^2 + (y-3)^2 = 13$

Slope of $OC = \frac{3-0}{2-0} = \frac{3}{2}$

$\therefore$ Slope of $PQ = -\frac{2}{3} = \tan\theta$

Points at a distance of $\sqrt{13}$ units from 0

with slope $= -\frac{2}{3}$

Using, $\begin{cases} x_2 = x_1 \pm r\cos\theta \\ y_2 = y_1 \pm r\sin\theta. \end{cases}$

So, $x_2 = 2 \pm \sqrt{13}\left(\frac{3}{\sqrt{13}}\right) = 2 \pm 3$

$\therefore\ x_2 : \{5, -1\}$

$y_2 : 3 \pm \sqrt{13}\left(\frac{-2}{\sqrt{13}}\right) = 3 \pm (-2)$

Question344

Let $A = \{(x, y) \in R \times R |$
$2x^2 + 2y^2 - 2x - 2y = 1\}$,
$B = \{x, y) \in R \times R |$
$4x^2 + 4y^2 - 16y + 7 = 0\}$
$C = \{(x, y) \in R \times R |$
$x^2 + y^2 - 4x - 2y + 5 \le r^2\}$
Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to
[2021, 27 July Shift-1]

Options:

A. $\frac{3+\sqrt{10}}{2}$

B. $\frac{2+\sqrt{10}}{2}$

C. $\frac{3+2\sqrt{5}}{2}$

D. $1 + \sqrt{5}$

Answer: C

Solution:

$A \Rightarrow 2x^2 + 2y^2 - 2x - 2y = 1$

$\Rightarrow\ 2(x^2 - x) + 2(y^2 - y) = 1$

$\Rightarrow\ 2\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right)$

$+2\left(y^2 - y + \frac{1}{4} - \frac{1}{4}\right) = 1$

$\Rightarrow\ 2\left(x - \frac{1}{2}\right)^2 - \frac{1}{2} + 2\left(y - \frac{1}{2}\right)^2 - \frac{1}{2} = 1$

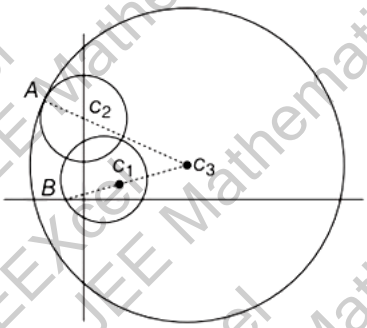
$\Rightarrow\ \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1$

Here, $C_1 = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $r_1 = 1$

$\Rightarrow\ 4x^2 + 4y^2 - 16y + 7 = 0$
$\Rightarrow\ 4(y^2 - 4y + 4 - 4) + 4x^2 + 7 = 0$
$\Rightarrow\ 4(y-2)^2 + 4x^2 + 7 - 16 = 0$
$\Rightarrow\ x^2 + (y-2)^2 = \frac{9}{4}$

Here $C_2 = (0, 2)$ and $r_2 = \frac{3}{2}$

$C \Rightarrow x^2 + y^2 - 4x - 2y + 5 \le r^2$
$(x-2)^2 + (y-1)^2 \le r^2$
Here, $C_3 = (2, 1)$



At $(2, 1)$ circle of B, $S_B = 2^2 + (1-2)^2 - \frac{9}{4}$

$= 4 + 1 - \frac{9}{4} = \frac{11}{4} > 0$

$C_1C_2 = \sqrt{\frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{5}{2}}$

$C_2C_3 = \sqrt{4+1} = \sqrt{5}$

$C_1C_3 = \sqrt{\frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{5}{2}}$

$r_3 \geq C_2C_3 + r_2$ [$\because C_2C_3 > C_1C_2$ and C_1C_3]

$|r| \geq \sqrt{5} + \frac{3}{2} \Rightarrow |r| \geq \frac{3+2\sqrt{5}}{2}$

Question345

Consider a circle C which touches the Y -axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the X -axis. Then the radius of the circle C is equal to
[2021, 27 July Shift-II]

Options:

A. $\sqrt{53}$

B. 9

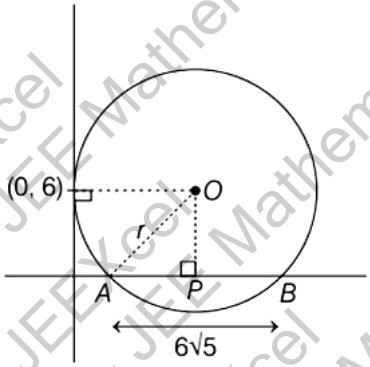
C. 8

D. $\sqrt{82}$

Answer: B

Solution:

Let a circle C which touches the Y -axis at $(0, 6)$ and cuts off an intercept $6\sqrt{5}$ on the X -axis.



Chord AB = $6\sqrt{5}$
So, AP = $3\sqrt{5}$ (perpendicular from centre bisects the chord)
and OP = 6
In $\triangle APO$, $r^2 = 6^2 + (3\sqrt{5})^2 = 36 + 45$
$\Rightarrow r^2 = 81 \Rightarrow r = 9$

Question346

Let the circle
$S : 36x^2 + 36y^2 - 108x + 120y + C = 0$
be such that it neither intersects nor touches the coordinate axes. If the point of intersection of the lines $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then
[2021, 22 July Shift-II]

Options:

A. $\frac{25}{9} < C < \frac{13}{3}$

B. $100 < C < 165$

C. $81 < C < 156$

D. $100 < C < 156$

Answer: D

Solution:

$S \Rightarrow 36x^2 + 36y^2 - 108x + 120y + C = 0$

$x^2 + y^2 - 3x + \left(\frac{10}{3}\right)y + \frac{C}{36} = 0$

$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{3}\right)^2 + \frac{C}{36} - \frac{9}{4} - \frac{25}{9} = 0$

When $x = 0$,

$36y^2 + 120y + C = 0$

Since, it neither touch the coordinate axis nor intersects. So, the roots of this equation must be imaginary.

$cc \Rightarrow D < 0$

$\Rightarrow 120^2 - 4.36C < D$

$\Rightarrow C > \frac{120 \times 120}{4 \times 36}$

$C > 100 \quad \cdots\cdots (i)$

Similarly, when

$y = 0, 36x^2 - 108x + C = 0$

$108^2 - 4.36C < 0$

$\Rightarrow \quad C > 81 \quad \cdots\cdots (ii)$

From Eqs. (i) and (ii),

$\Rightarrow \quad C > 100$

and point of intersection of $\begin{cases} x - 2y = 4 \\ 2x - y = 5. \end{cases}$ is $(2, -1)$.

So, $S_{(2,\,1)} < 0$

$\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 + \frac{C}{36} - \frac{9}{4} - \frac{25}{9} < 0$

$c < 156$

$C \in (100, 156)$

Question347

Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to
[2021, 20 July Shift-II]

Options:

A. 3

B. 11

C. 5

D. 7

Answer: C

Solution:

Given, circle $\Rightarrow x^2 + y^2 + 2x + 4y - 4 = 0$
$\Rightarrow (x^2 + 2x + 1) + (y^2 + 4y + 4)$
$-1 - 4 - 4 = 0$
$\Rightarrow (x + 1)^2 + (y + 2)^2 = 9$
Any point on the given circle be
$[(3\cos\theta - 1), (3\sin\theta - 2)]$
[to find the point coordinates, take $x + 1 = 3\cos\theta$ and $y + 2 = 3\sin\theta$]
Now, circle passes through $(-4, 1)$ and their centres lie on the given circle.
So, the centre coordinate of that circle be $(3\cos\theta - 1, 3\sin\theta - 2)$. Since, it passes through $(-4, 1)$, then radius of this circle be

$$r = \sqrt{(3\cos\theta - 1 + 4)^2 + (3\sin\theta - 2 - 1)^2}$$
$$= \sqrt{9\cos^2\theta + 9 + 18\cos\theta + 9\sin^2\theta + 9 - 18\sin\theta}$$
$$= \sqrt{27 + 18(\cos\theta - \sin\theta)}$$
$$= 3\sqrt{3 + 2(\cos\theta - \sin\theta)}$$

Maximum radius will be when $(\cos\theta - \sin\theta)$ is maximum i.e.

$$\cos\theta - \sin\theta = \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\therefore\ r_1 = r_{max} = 3\sqrt{3 + 2/\sqrt{2}}$
Minimum radius will be when
$(\cos\theta - \sin\theta)$ is minimum i.e. $-\sqrt{2}$.
$\therefore\ r_2 = r_{min} = 3\sqrt{3 - 2\sqrt{2}}$

Given, $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $\frac{r_1^2}{r_2^2} = (a + b\sqrt{2})^2$

$$\Rightarrow \frac{9(3 + 2\sqrt{2})}{g(3 - 2\sqrt{2})} = (a + b\sqrt{2})^2$$
$$\Rightarrow \frac{(3 + 2\sqrt{2})(3 + 2\sqrt{2})}{(3 - 2\sqrt{2})(3 + 2\sqrt{2})} = (a + b\sqrt{2})^2$$
$$\Rightarrow \frac{(3 + 2\sqrt{2})^2}{1} = (a + b\sqrt{2})^2$$

Comparing coefficients, $a = 3, b = 2$
$\therefore\ a + b = 5$

Question348

Let a parabola P be such that its vertex and focus lie on the positive X -axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of $\triangle SOR$ is equal to
[2021, 25 July Shift-I]

Options:

A. $16\sqrt{2}$

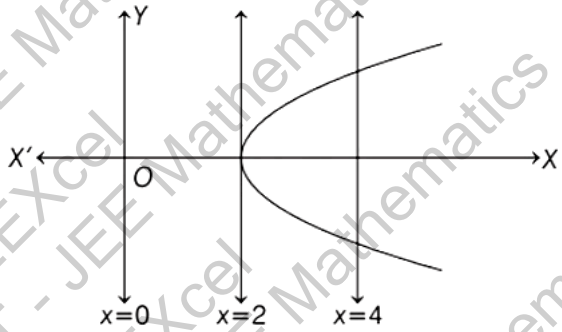
B. 16

C. 32

D. $8\sqrt{2}$

Answer: B

Solution:



Equation of parabola $\Rightarrow y^2 = 4a(x-2)$

$a = 2$

So, $y^2 = 8(x-2)$

Equation of tangent

$\Rightarrow \; y = m(x-2) + \frac{2}{m}$

It pases through $(0, 0)$,

$0 = -2m + \frac{2}{m}$

$\Rightarrow \; m^2 = 1 \Rightarrow m = \pm 1$

Equation $\Rightarrow y = \pm(x-2) \pm 2$

$y = x$ or $y = -x$

If $y = x$

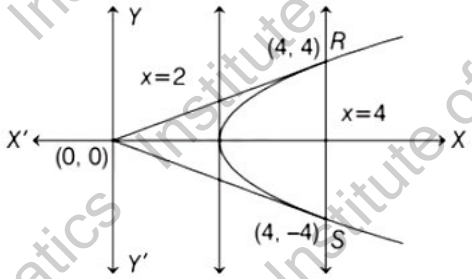
cc $\Rightarrow x^2 = 8(x-2)$

$\Rightarrow x^2 - 8x + 16 = 0$

$\Rightarrow (x-4)^2 = 0 \Rightarrow x = 4$

$y = \pm 4$

So, points are $(4, 4)$, $(4, -4)$ and $(0, 0)$.



$\therefore$ Area $= \left(\frac{1}{2} \times 4 \times 4\right) \times 2 = 16$ sq units

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Question349

The equation of a circle is $Re(z^2) + 2[I\,m(z)]^2 + 2Re(z) = 0$ where, $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to .....

[2021, 25 July Shift-II]

Answer: 1

Solution:

The equation of circle

$Re(z^2) + 2[I\,m(z)]^2 + 2Re(z) = 0$, where $z = x + iy$

cc $\Rightarrow (x^2 - y^2) + 2y^2 + 2x = 0$

$\Rightarrow [\because z^2 = (x^2 - y^2) + 2xyi]$

$\Rightarrow x^2 + y^2 + 2x = 0$

$(x+1)^2 + y^2 = 1$

Centre $= (-1, 0)$

Now, parabola $\Rightarrow x^2 - 6x - y + 13 = 0$

$\Rightarrow \; x^2 - 6x + 9 = y - 4$

$\Rightarrow \; (x-3)^2 = y - 4$

Vertex =
Equation of line passing through centre($-1, 0$) and vertex $(3, 4)$ is
$y - 0 = \frac{4-0}{3+1}(x+1) \Rightarrow y = x + 1$
On comparing, $y = mx + c$
$\Rightarrow$ y-intercept $= 1$

Question350

Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the X -axis at Q and normal at it meet the parabola S at the point R. Then, the area (in square units) of $\triangle PQR$ is equal to
[2021, 20 July Shift-1]

Options:

A. $\frac{25}{2}$

B. $\frac{35}{2}$

C. $\frac{15}{2}$

D. 25

Answer: A

Solution:

$y^2 = 2x$
At $(2, 2)$, equation of tangent
$\Rightarrow y(2) = (x + 2)$
$T_i \Rightarrow 2y = x + 2$
Normal at $(2, 2) \Rightarrow y - 2 = -2(x - 2)$
$N_i \Rightarrow y = -2x + 6$
R is point of intersection of T and line $y = 0$
$R(-2, 0)$
$(2, 2) = \left(\frac{t^2}{2}, t\right) \Leftrightarrow t = 2$
If normal at t_1 meets the parabola again at t_2,
$t_2 = -t_1 - \frac{2}{t_1}$
$-t_2 = -2 - \frac{2}{2} = -3$
$\therefore\ t_2 = 3$
$R : (at_2^2, 2at_2) \Rightarrow \left(\frac{1}{2} \cdot 9, 2 \cdot \frac{1}{2} \cdot 3\right) \Rightarrow \left(\frac{9}{2}, 3\right)$
$\therefore P(2, 2), Q(-2, 1), R\left(\frac{9}{2}, 3\right)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & \frac{9}{2} \\ 2 & 1 & 3 \end{vmatrix}$$

$\Rightarrow \text{Area} = \frac{25}{2}$

Question351

Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to
[2021, 20 July Shift-1]

Answer: 34

Solution:

Equation of circle $\Rightarrow (x+10)^2+y^2=4$ Equation of tangent to circle $x^2+y^2=a^2$ is $y=mx\pm a\sqrt{1+m^2}$

Here, equation of tangent

$y=m(x+10)\pm 2\sqrt{1+m^2}$

$\Rightarrow y=mx+10m\pm 2\sqrt{1+m^2}$

On comparing this equation with $y=mx+c$, we get

$c=10m\pm 2\sqrt{1+m^2}$ ……(i)

This equation is also a focal chord of $y^2=-64x$ whose focus is at $(-16, 0)$. So, it must pass through $(-16, 0)$.

$y=mx+c$

$\Rightarrow 0=-16m+c$

$\therefore c=16m$ ……(ii)

From Eqs. (i) and (ii),

$16m=10m\pm 2\sqrt{1+m^2}$

$\Rightarrow 6m=\pm 2\sqrt{1+m^2}$

$\Rightarrow 9m^2=1+m^2$

$\Rightarrow m=\frac{1}{2\sqrt{2}}$ (as $m>0$)

$\therefore c=16m=16\cdot\left(\frac{1}{2\sqrt{2}}\right)=4\sqrt{2}$

So, $4\sqrt{2}(m+c)=4\sqrt{2}\left(\frac{1}{2\sqrt{2}}+4\sqrt{2}\right)$

$=2+32=34$

Question352

Let P be a variable point on the parabola $y=4x^2+1$. Then, the locus of the mid point of the point P and the foot of the perpendicular drawn from the point P to the line $y=x$ is
[2021, 20 July Shift-II]

Options:

A. $(3x-y)^2+(x-3y)+2=0$

B. $2(3x-y)^2+(x-3y)+2=0$

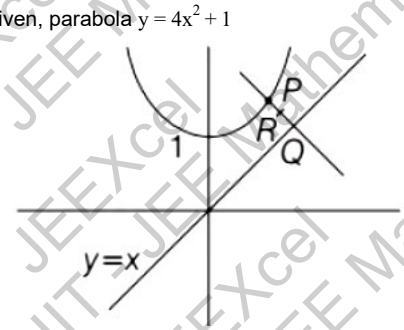
C. $(3x-y)^2+2(x-3y)+2=0$

D. $2(x-3y)^2+(3x-y)+2=0$

Answer: B

Solution:

Given, parabola $y=4x^2+1$



Let R(a, b) be mid-point of line joining point P and Q where PQ is perpendicular to line $y=x$.

Let coordinates of P be P(x, y), Q(q, q) and R(a, b) then, $a=\frac{x+q}{2}$ and $b=\frac{y+q}{2}$

Now, slope of line $y=x$ is $m_1=1$

Slope of line PO be

$\frac{b-q}{a-q}=m_2$ (say)

$\because$ Line $y = x$ and PQ are perpendicular to each other,

$m_1 \cdot m_2 = -1$

$\Rightarrow \quad \frac{b-q}{a-q} = -1 \Rightarrow b - q = q - a$

$\Rightarrow q = \frac{b+a}{2}$

$\therefore a = \frac{x+q}{2}$

$= \frac{x + \left(\frac{b+a}{2}\right)}{2} = \frac{2x+b+a}{4}$

$\Rightarrow \quad x = \frac{4a-b-a}{2} = \frac{3a-b}{2}$ and $b = \frac{y+q}{2} = \frac{y + \left(\frac{b+a}{2}\right)}{2} = \frac{2y+b+a}{4}$

$\Rightarrow \quad y = \frac{3b-a}{2}$

Put (x, y) in equation of parabola as $P(x, y)$ is variable point on parabola

$\frac{3b-a}{2} = 4\left(\frac{3a-b}{2}\right)^2 + 1$

$\frac{(3b-a)}{2} = (3a-b)^2 + 1$

$\Rightarrow \quad (3b-a) = 2(3a-b)^2 + 2$

Replace (a, b) as (x, y)

$\Rightarrow \quad (3y-x) = 2(3x-y)^2 + 2$

or $2(3x-y)^2 + (x-3y) + 2 = 0$

Question353

A ray of light through (2, 1) is reflected at a point P on the Y -axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be

[2021, 27 July Shift-1]

Options:

A. $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$

B. $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

C. $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

D. $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

Answer: C

Solution:

Equation of the reflected ray will be

$L \Rightarrow \frac{y-3}{x-5} = m$

Now, the image of $(2, 1)$ w.r.t. line $x = 0$ should lie on the reflected line.

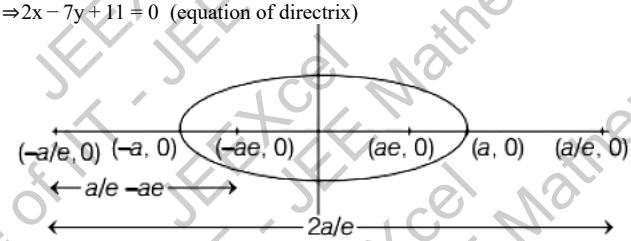
Image of $(2, 1) = (-2, 1)$

So, $\frac{1-3}{-2-5} = m \Rightarrow m = \frac{2}{7}$

So, equation of reflected ray $\Rightarrow \frac{y-3}{x-5} = \frac{2}{7}$

$c7y - 21 = 2x - 10$

$\Rightarrow 2x - 7y + 11 = 0$ (equation of directrix)



Now, $e = \frac{1}{3}$ (given)

So, $\frac{a}{e} - ae = \frac{a}{1/3} - \frac{1}{3}a = \frac{8a}{3}$

$\Rightarrow \frac{8a}{3} = \frac{8}{\sqrt{53}}$ (given)

So, $a = \frac{3}{\sqrt{53}}$

and $\frac{2a}{e} = \left(2 \cdot \frac{3}{\sqrt{53}}\right) \times \left(\frac{3}{1}\right) = \frac{18}{\sqrt{53}}$

Now, another directrix will be parallel to the first directrix and lie at a distance of $\frac{18}{\sqrt{53}}$ units.

So, let the equation of another directrix be $2x - 7y + \lambda = 0$

Accordingly, $\frac{|\lambda - 11|}{\sqrt{2^2 + 7^2}} = \frac{18}{\sqrt{53}}$

$\Rightarrow |\lambda - 11| = 18$

$\Rightarrow \lambda = 11 \pm 18$

$\Rightarrow \lambda = 29$ or -7

So, equation of another directrix will be

$2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

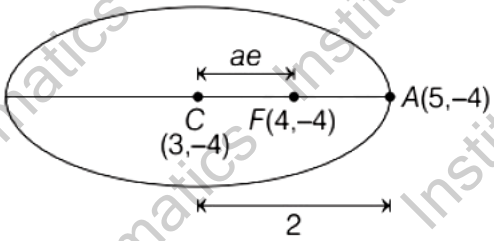
Question354

Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, −4), one focus at (4, −4) and one vertex at (5, −4). If $mx - y = 4$, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to
[2021, 27 July Shift-II]

Answer: 3

Solution:

E be an ellipse whose centre C(3, −4) and focus F (4, −4) and one vertex A(5, −4).



Hence, $a = 2$ and $ae = 1$

$\Rightarrow e = \frac{1}{2}$

Now, $b^2 = a^2 - (ae)^2 = 4 - 1$

$b^2 = 3$

So, equation of ellipse

$E \Rightarrow \frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$

Intersecting with given tangent

$mx - y = 4$

$\Rightarrow y + 4 = mx$

$\frac{x^2 - 6x + 9}{4} + \frac{m^2x^2}{3} = 1$

$\Rightarrow (3 + 4m^2)x^2 - 18x + 15 = 0$

Now, D = 0

$1r \Rightarrow 324 - 4(3 + 4m^2)(15) = 0$

$\Rightarrow 144 - 240m^2 = 0$

$\Rightarrow 5m^2 = 3$

Question355

Let an ellipse E $\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a^2>b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus F $(\alpha, 0)$, $\alpha>0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to
[2021, 25 July Shift-1]

Options:

A. $\frac{8}{3}$

B. $\frac{4}{3}$

C. $\frac{16}{3}$

D. 3

Answer: C

Solution:

$E \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$

$\therefore \frac{3}{2a^2}+\frac{1}{b^2}=1$

$a^2-b^2=a^2e^2$

$\Rightarrow a^2-b^2=\frac{a^2}{3}$

$\Rightarrow 3a^2-3b^2=a^2$

$\Rightarrow 2a^2=3b^2$

$\frac{3}{3b^2}+\frac{1}{b^2}=1$

$\Rightarrow 2=b^2$

$\Rightarrow a^2=3$

Focus $\left(\sqrt{3}\cdot\frac{1}{\sqrt{3}}, 0\right)$

$\Rightarrow$ Focus $=(1, 0)$

Centre $=(1, 0)$, Radius $=\frac{2}{\sqrt{3}}$

$(x-1)^2+(y-0)^2=\left(\frac{2}{\sqrt{3}}\right)^2 \quad \cdots\cdots (i)$

$E \Rightarrow \frac{x^2}{3}+\frac{y^2}{2}=1 \quad \cdots\cdots (ii)$

From Eqs. (i) and (ii)

$x=1, y=\pm\frac{2}{\sqrt{3}}$

$PO=\frac{4}{\sqrt{3}}$

$PQ^2=\frac{16}{3}$

Question356

If a tangent to the ellipse $x^2+4y^2=4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point
[2021, 25 July Shift-II]

Options:

A. $(\sqrt{3}, 0)$

B. $(\sqrt{2}, 0)$

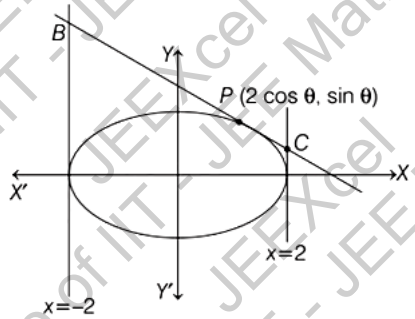
C. (1, 1)

D. (−1, 1)

Answer: A

Solution:

Equation of ellipse $\Rightarrow x^2 + 4y^2 = 4$



$\Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1$

Extremities end of major axis $= (\pm 2, 0)$

Let any point P on ellipse $(2\cos\theta, \sin\theta)$.

Equation of tangent to the ellipse at the point P is $\frac{2\cos\theta\, x}{4} + \frac{y\sin\theta}{1} = 1$

Coordinate of $B\left(-2, \cot\frac{\theta}{2}\right)$ and $C\left(2, \tan\frac{\theta}{2}\right)$.

Equation of circle whose end points of diameter are B and C is

$$(x-2)(x+2) + \left(y - \cot\frac{\theta}{2}\right)\left(y - \tan\frac{\theta}{2}\right) = 0$$

$$\Rightarrow x^2 + y^2 - \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)y - 3 = 0$$

At $y = 0, x = \pm\sqrt{3}$

Hence, circle is passing through the point $(\pm\sqrt{3}, 0)$.

Question357

Let E_1 : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is
[2021, 22 July Shift-II]

Options:

A. $\frac{-1+\sqrt{5}}{2}$

B. $\frac{-1+\sqrt{8}}{2}$

C. $\frac{-1+\sqrt{3}}{2}$

D. $\frac{-1+\sqrt{6}}{2}$

Answer: A

Solution:

$e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{a^2}{c^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{a^2}{c^2} \Rightarrow c = \frac{a^2}{b}$

Also, $b = ce \Rightarrow c = \frac{b}{e}$

$\Rightarrow \frac{b}{e} = \frac{a^2}{b}$

$\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0 \Rightarrow e = \frac{-1+\sqrt{5}}{2}$

Question358

Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to .....

[2021, 20 July, Shift-1]

Answer: 1.25

Solution:

$x^2 + 4y^2 = 5$

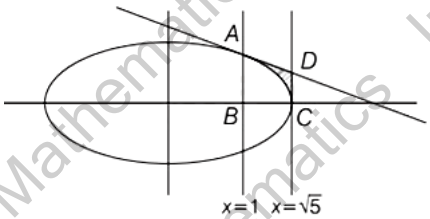
At (1, 1), equation of tangent

$\Rightarrow x \cdot 1 + 4y \cdot 1 = 5$

$\Rightarrow x + 4y = 5$

$\therefore$ A : (1, 1)

D : $\left(\sqrt{5}, \frac{5-\sqrt{5}}{4}\right)$



Area of shaded region

$= \int_1^{\sqrt{5}} \left(\frac{5-x}{4} - \frac{1}{2}\sqrt{5-x^2}\right) dx$

$= \left[\frac{1}{4}\left(5x - \frac{x^2}{2}\right) - \frac{x}{4}\sqrt{5-x^2} - \frac{5}{4}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$

$= \frac{1}{4}\left[5\sqrt{5} - \frac{5}{2} - 0 - \frac{5\pi}{2} - 5 + \frac{1}{2} + 2 + 5\sin^{-1}\frac{1}{\sqrt{5}}\right]$

$= \frac{1}{4}\left[5\sqrt{5} - 5 - 5\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}\right)\right]$

$= \frac{1}{5}\left[5\sqrt{5} - 5 - 5\cos^{-1}\frac{1}{\sqrt{5}}\right]$

$= \alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore \alpha = \frac{5}{4}, \beta = \frac{-5}{4}, \gamma = \frac{-5}{4}$

$\therefore \left|\alpha + \beta + \gamma\right| = \frac{5}{4} = 1.25$

Question359

The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is
[2021,25 July Shift-1]

Options:

A. $16x^2 - 9y^2 + 32x - 36y - 36 = 0$

B. $9x^2 - 16y^2 + 36x - 32y - 144 = 0$

C. $16x^2 - 9y^2 + 32x - 36y - 144 = 0$

D. $9x^2 - 16y^2 + 36x - 32y - 36 = 0$

Answer: A

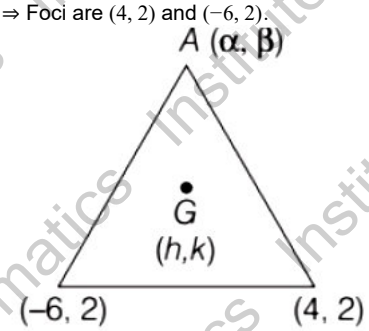
Solution:

Given, hyperbola is
$16x^2 - 9y^2 + 32x - 36y - 164 = 0$
$\Rightarrow 16(x+1)^2 - 9(y-2)^2 = 164 + 16 - 36 = 144$

$\Rightarrow \frac{(x+1)^2}{9} - \frac{(4-2)^2}{16} = 1$

Eccentricity, $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\Rightarrow$ Foci are (4, 2) and (−6, 2).



Let the centroid be (h, k) and A(α, β) be point on hyperbol(a)
So, $h = \frac{\alpha - 6 + 4}{3}$, $k = \frac{\beta + 2 + 2}{3}$
$\Rightarrow \alpha = 3h + 2, \beta = 2k - 4$
(α, β) lies on hyperbola, so
$16(3h + 2 + 1)^2 - 9(3k - 4 - 2)^2 = 144$
$\Rightarrow 144(h+1)^2 - 81(k-2)^2 = 144$
$\Rightarrow 16(h^2 + 2h + 1) - 9(k^2 - 4k + 4) = 16$
$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

Question360

Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to
[2021, 22 July Shift-II]

Options:

A. 12

B. −12

C. 24

D. −24

Answer: D

Solution:

$T : 2x + y = k$
$H : x^2 - y^2 = 3$
$P : y^2 = \alpha x$
If T is tangent, then
$y = mx \pm \sqrt{a^2m^2 - b^2}$
$y = mx \pm \sqrt{m^2 - 1}$
$\Rightarrow y = -2x - k$
On comparing both sides we get $m = -2$ and $k = 3$

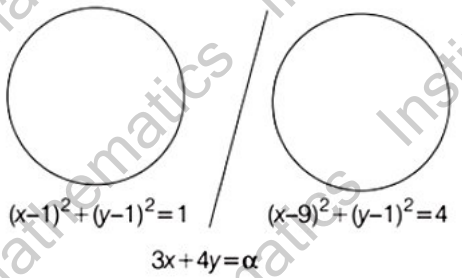
If $2x + y = 3$ is also tangent to $y^2 = \alpha x$,
Then,
$\because m = -2$
$\therefore \frac{\alpha}{-8} = 3$
$\Rightarrow \alpha = -24$

Question361

If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is
[2021, 31 Aug. Shift-1]

Answer: 165

Solution:



There are 2 possible cases
1. Both centres should lie on different sides of line.
2. Line can be tangent also
Centre of both the circles are $(1, 1)$ and $(9, 1)$
$(3 + 4 - \alpha)(27 + 4 - \alpha) < 0$
$(7 - \alpha)(31 - \alpha) < 0$
$\alpha \in (7, 31) \quad \cdots\cdots\cdots (A)$
$P_1 =$ Distance of line from centre $(1, 1)$
$P_1 \geq r_1$
$\frac{|3 + 4 - \alpha|}{5} \geq 1$
$|7 - \alpha| \geq 5$
$\alpha \in (-\infty, 2] \cup [12, \infty) \quad \cdots\cdots\cdots (B)$
Similarly, $P_2 =$ Distance of line from $(9, 1)$
$\frac{|27 + 4 - \alpha|}{5} \geq 2$
$\alpha \in (-\infty, 21] \cup [41, \infty) \quad \cdots\cdots\cdots (C)$
Now, $(A) \cap (B) \cap (C) \Rightarrow \alpha \in [12, 21]$
Sum $= (12 + 13 + 14 + \ldots + 21)$
$= \left(\frac{21 \times 22}{2}\right) - \left(\frac{11 \times 12}{2}\right)$
$= 231 - 66 = 165$

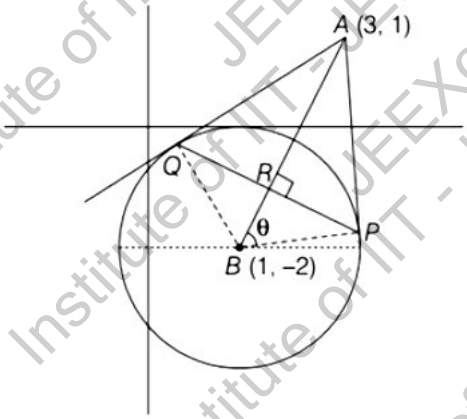
Question362

Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point Az (3, 1). Then $8\left(\frac{\text{area } \Delta \text{ APQ}}{\text{area } \Delta \text{ BPQ}}\right)$ is equal to

[2021, 31 Aug. Shift-II]

Answer: 18

Solution:



Radius $= \sqrt{1+4-1} = 2$

$AB = \sqrt{3^2+2^2} = \sqrt{13}$

In $\triangle ABP$

$AP^2 = AB^2 - BP^2 = 13 - 4 =$

$AP = 3$

$AO = AP = 3$

Let $\angle ABP = \theta, \angle BAP = 90 - \theta$

In $\triangle ABP, \tan\theta = \frac{3}{2}$

$\sin\theta = \frac{3}{\sqrt{13}}, \cos\theta = \frac{2}{\sqrt{13}}$

In $\triangle ARP$,

$\cos(90-\theta) = \frac{AR}{AP} \Rightarrow AR = 3\sin\theta$

In $\triangle BRP$

$\cos\theta = \frac{BR}{BP} \Rightarrow BR = 2\cos\theta$

$= \frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta BPQ)} = \frac{\frac{1}{2} \times PQ \times AR}{\frac{1}{2} \times PQ \times BR}$

$= \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$

$\Rightarrow 8\left(\frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta BPQ)}\right) = 18$

Question363

Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then, the number of elements in the set $S = \{q : q = p^2$ and q is an integer $\}$ is

[2021, 27 Aug. Shift-I]

Answer: 61

Solution:

Given equation,

$x^2 + y^2 + px + (1-p)y + 5 = 0$

Radius, $r = \sqrt{\left(\frac{p}{2}\right)^2 + \left(\frac{1-p}{2}\right)^2 - 5}$

$= \sqrt{\frac{2p^2 - 2p - 19}{4}}$

Now, $r \in (0, 5]$

$0 < r \le 5$

$\Rightarrow 0 < \sqrt{\frac{2p^2 - 2p - 19}{4}} \le 5$

$\Rightarrow 0 < \frac{2p^2 - 2p - 19}{4} \le 25$

$\Rightarrow 2p^2 - 2p - 19 > 0$ and

$2p^2 - 2p - 19 \le 100$

$\Rightarrow 2p^2 - 2p - 19 > 0$ and

$2p^2 - 2p - 19 \le 100$

$\Rightarrow p^2 - p - \frac{19}{2} > 0$ and $2p^2 - 2p - 119 \le 0$

$\Rightarrow p \in \left(-\infty, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \infty\right)$

and $p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1+\sqrt{239}}{2}\right]$

$p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right] \cup \left[\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$

$\Rightarrow p^2 \in \left[\left(\frac{1-\sqrt{39}}{2}\right)^2, \left(\frac{1+\sqrt{239}}{2}\right)^2\right]$

$\because q = p^2, q \in Z$

$\Rightarrow q = 7, 8, 9, 10, ..., 67$

$\Rightarrow$ 61 integers in S

$\therefore n(S) = 61$

Question364

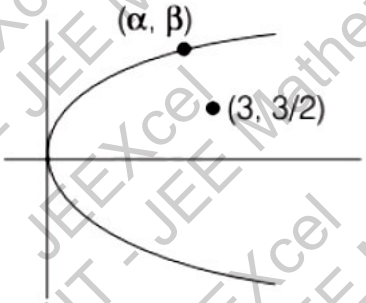
If the point on the curve $y^2 = 6x_1$ nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to

[2021, 20 July Shift-II]

Answer: 9

Solution:

Shortest distance is along the normal.



Equation of normal at $(\boldsymbol{\alpha}, \boldsymbol{\beta})$

$cy - \beta = -\frac{\beta}{2a}(x - \alpha)$

$(\because y^2 = 4ax \Rightarrow 4a = 6 \Rightarrow 2a = 3)$

$\Rightarrow 3y - 3\beta = -\beta(x - \alpha)$

This normal pass through $\left(3, \frac{3}{2}\right)$.

$\frac{9}{2} - 3\beta = -3\beta + \alpha\beta\alpha\beta = \frac{9}{2}$ ····· (i)

Now, from $y^2 = 6\beta^2 = 6\alpha$ ····· (ii)

From Eqs. (i) and (ii),

$c\alpha = \frac{3}{2}\beta = 3$

$\therefore 2(\alpha + \beta) = 2\left(\frac{3}{2} + 3\right) = 2\left(\frac{3+6}{2}\right) = 9$

Question365

Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$ and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to
[2021, 27 Aug. Shift-II]

Answer: 40

Solution:

Equation of circles

$(x-1)^2 + (y-2)^2 + \lambda(4x + 3y - 10) = 0$

$x^2 + y^2 + (2\lambda - 1)2x + \left(\frac{3}{2}\lambda - 2\right)$

$2y + 5 - 10\lambda = 0$

$r = \sqrt{(2\lambda-1)^2 + \left(\frac{3}{2}\lambda - 2\right)^2 - 5 + 10\lambda} = 5$

$\Rightarrow 4\lambda^2 + 1 - 4\lambda + \frac{9}{4}\lambda^2 + 4 - 6\lambda - 5 + 10\lambda = 25$

$\Rightarrow \frac{25}{4}\lambda^2 - 25 = 0$

$\Rightarrow \lambda = \pm 2$

For $\lambda = 2$

$x^2 + y^2 + 6x + 2y - 15 = 0$, $C_1(-3, -1)$

For $\lambda = -2$

$x^2 + y^2 - 10x - 10y + 25 = 0$, $C_2(5, 5)$

$|(\alpha + \beta)(\gamma + \delta)| = |-4 \times 10| = 40$

Question366

Let Z be the set of all integers, $A = \{(x, y) \in Z \times Z : (x-2)^2 + y^2 \leq 4\}$, $B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ and $C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \leq 4\}$ If the total number of relation from $A \cap B$ to $A \cap B$ is 2^p, then the value of p is
[2021,27 Aug. Shift-II]

Options:

A. 16

B. 25

C. 49

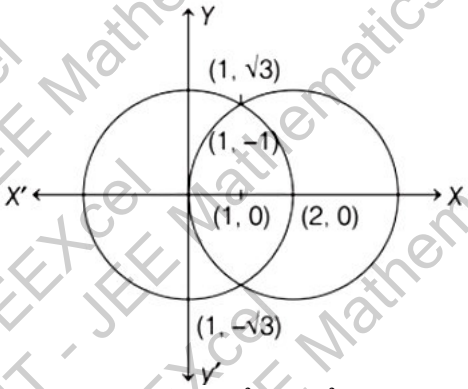
D. 9

Answer: B

Solution:

$A = \{(x, y) \in Z \times Z : (x-2)^2 + y^2 \leq 4\}$

$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$



$C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \le 4\}$
$\therefore A \cap B = \{(0, 0), (1, 0), (2, 0), (1, 1), (1, -1)\}$
$n(A \cap B) = 5$
Similarly, $n(A \cap C) = 5$
Number of relation from $(A \cap B)$ to
$(A \cap C) = 2^{5\times 5} = 2^p$
$\Rightarrow p = 25$

Question367

The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1)(1, 1) is 18 units, is a circle of diameter (d) Then, d^2 is equal to
[2021, 26 Aug. Shift-1]

Answer: 16

Solution:

Let P(x, y) be the required point Then,
$(x^2 + y^2) + [x^2 + (y-1)^2] + [(x-1)^2 + y^2]$
$+ [(x-1)^2 + (y-1)^2] = 18$
$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$
$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$\text{Radius} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{7}{2}} = 2$

$\text{Diameter} = 2 \cdot \text{Radius} = 4$
$d^2 = 16$

Question368

A circle C touches the line x = 2y at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is
[2021, 26 Aug. Shift-11]

Options:

A. $7\sqrt{5}$

B. 15

C. 285

D. $4\sqrt{15}$

Answer: A

Solution:

Circle C touches $x = 2y$ at $(2, 1)$, then equation of circle C is given by

$(x-2)^2 + (y-1)^2 + \lambda(x-2y) = 0$

$\Rightarrow x^2 - 4x + 4 + y^2 - 2y + 1 + \lambda x - 2\lambda y = 0$

$\Rightarrow x^2 + y^2 + (\lambda - 4)x + (-2 - 2\lambda)y + 5 = 0$

Since circles C and C_1 intersect each other at PQ, so PQ is common chor(d)

$\therefore$ Equation of PQ will be $C - C_1 = 0 \Rightarrow x(\lambda - 4) + y(-4 - 2\lambda) + 10 = 0$

Again PQ is diameter of circle C_1.

So, PQ passes through centre of C_1.

Now, equation of C_1 is $x^2 + y^2 + 2y - 5 = 0$

$\therefore$ Centre of C_1 is $(0, -1)$.

So, PQ passes through $(0, -1)$

$\therefore\ 0(\lambda - 4) - 1(-4 - 2\lambda) + 10 = 0 \Rightarrow\ 4 + 2\lambda + 10 = 0 \Rightarrow\ \lambda = -7$

On putting $\lambda = -7$ in Eq. (i), we get equation of circle C as

$x^2 + y^2 - 11x + 12y + 5 = 0$

$\therefore$ Radius of $C = \sqrt{\left(\frac{11}{2}\right)^2 + (-6)^2 - 5}$

$= \sqrt{\frac{121}{4} + 36 - 5}$

$= \sqrt{\frac{245}{4}} = \frac{\sqrt{245}}{2}$

$\therefore$ Diameter of $C = 2 \times$ Radius

$= \sqrt{245} = 7\sqrt{5}$

Question369

The length of the latus rectum of a parabola, whose vertex and focus are on the positive X -axis at a distance R and S(>R) respectively from the origin, is
[2021, 31 Aug. Shift-1]

Options:

A. $4(S + R)$

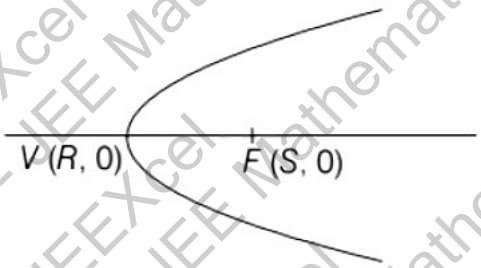
B. $2(S - R)$

C. $4(S - R)$

D. $2(S + R)$

Answer: C

Solution:



Length of LR of $y^2 = 4ax$ is $4a$.

OR Length of LR = 4× [Distance between focus and vertex]

Similarly here

Length of LR = $4(S - R)$

Question370

A tangent line L is drawn at the point (2, −4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then a is equal to
[2021, 31 Aug. Shift-II]

Answer: 2

Solution:

Equation of tangent to parabola $y^2 = 8x$ at $(2, -4)$ is $-4y = 4(x + 2)$
$\Rightarrow x + y + 2 = 0$
Center and radius of circle $x^2 + y^2 = a$ is $(0, 0)$ and $\sqrt{a}$ respectively.
$\because$ Eq. (i) is tangent to the circle.
$\Rightarrow$ Perpendicular distance of Eq. (i) from center $(0, 0) = \sqrt{a}$
$\Rightarrow \left| \frac{0 + 0 + 2}{\sqrt{2}} \right| = \sqrt{a} \Rightarrow a = 2$

Question371

A tangent and a normal are drawn at the point P(2, −4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQBP is a square, then 2a + b is equal to
[2021, 27 Aug. Shift-I]

Options:

A. −16

B. −18

C. −12

D. −20

Answer: A

Solution:

Given, parabola
$y^2 = 8x \quad \cdots\cdots (i)$
Equation of tangent at $P(2, -4)$ is
$-4y = 4(x + 2)$
or $x + y + 2 = 0 \quad \cdots\cdots (ii)$
and Equation of normal to the parabola is
$x - y + C = 0$
$\therefore$ Normal passes through $(2, -4)$
$\therefore \quad C = -6 \quad \cdots\cdots (iii)$
Normal: $x - y = 6$
Equation of directrix of parabola
$x = -2 \quad \cdots\cdots (iv)$
Point of intersection of tangent and normal with directrix are $x = -2$ at $A(-2, 0)$ and $B(-2, -8)$ respectively. $O(a, b)$ and $P(2, -4)$ are given and AQBP is a square.
Mid-point of AB = Mid-point of PQ
$\Rightarrow (-2, -4) = \left(\frac{a + 2}{2}, \frac{b - 4}{2} \right)$
$\Rightarrow a = -6, b = -4$
$\Rightarrow 2a + b = -16$

Question372

If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is
[2021, 27 Aug. Shift-II]

Options:

A. $x + 3 = 0$

B. $x + 1 = 0$

C. $x + 2 = 0$

D. $x + 4 = 0$
(d) $x + 4 = 0$

Answer: B

Solution:

We know that, the locus of the points of intersection of the mutually perpendicular tangents to a parabola is the directrix of the parabol(a)
$\Rightarrow X + A = 0$
$\Rightarrow x - 3 + 4 = 0$
$\Rightarrow x + 1 = 0$

Question373

If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is
[2021, 26 Aug Shift-I]

Options:

A. 5

B. 7

C. $5\sqrt{3}$

D. $3\sqrt{5}$

Answer: D

Solution:

Equation of tangent to $y^2 = 30x$ is
$y = mx + \frac{30}{4m}$
Now, this tangent passes through $(-30, 0)$.
$\therefore 0 = -30m + \frac{30}{4m}$
$\Rightarrow \frac{30}{4m} = 30m \Rightarrow m^2 = \frac{1}{4}$
$\Rightarrow m = \pm \frac{1}{2}$
$\therefore$ Equation of tangent is
$y = \frac{x}{2} + 15$
or
$y = -\frac{x}{2} - 15$
Now equation of circle is
$x^2 + y^2 + 30x + \frac{675}{4} = 0$

Let perpendicular distance of the tangent from the centre $(-15, 0)$ of the circle $= p$

$\therefore\ p = \frac{\left|\frac{-15}{2}+15\right|}{\sqrt{1+\frac{1}{4}}} = 3\sqrt{5}$

$\therefore$ Length of chord $= 2\sqrt{r^2-p^2}$

$= 2\sqrt{\left(15^2+0-\frac{675}{4}\right)-45}$

$= 3\sqrt{5}$

where, r is radius of the given circle.

Question374

The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves?
[2021, 31 Aug. Shift-1]

Options:

A. $x^2 + y^2 = 169$

B. $144x^2 + 25y^2 = 3600$

C. $25x^2 + 12y^2 = 3600$

D. $x^2 + y^2 = 60$

Answer: B

Solution:

$12x\cos\theta + 5y\sin\theta = 60$

$\Rightarrow \frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$

The given equation is the equation of the tangent which passes through $(5\cos\theta, 12\sin\theta)$.

Hence, the given equation is tangent to

$\frac{x^2}{25} + \frac{y^2}{144} = 1$ at $(5\cos\theta, 12\sin\theta)$.

Question375

The locus of mid-points of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is
[2021, 31 Aug. Shift-II]

Options:

A. $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

B. $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

C. $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

D. $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

Answer: C

Solution:

Let $(2\sin\theta, 3\cos\theta)$ be the point on ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and let mid -point of the line segments joining $(-3, -5)$ and $(2\sin\theta, 3\cos\theta)$ will be (h, k).

Then, $\frac{2\sin\theta - 3}{2} = h,\ \frac{3\cos\theta - 5}{2} = k$

$\Rightarrow \sin\theta = \frac{2h+3}{2},\ \cos\theta = \frac{2k+5}{3}$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$\Rightarrow \frac{(4h^2+9+12h)}{4} + \frac{(4k^2+25+20k)}{9} = 1$

$\Rightarrow 36h^2 + 16k^2 + 108h + 80k + 145 = 0$

Locus of (h, k)

$36x^2 + 16y^2 + 108x + 80y + 145 = 0$

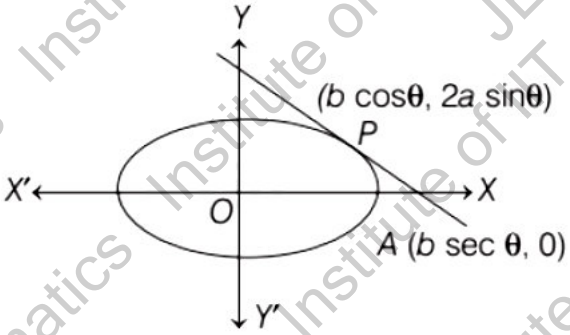
Question376

If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the coordinate axis is kab, then k is equal to
[2021, 27 Aug. Shift-I]

Answer: 2

Solution:

Equation of an ellipse B(0, 2a cosec θ)



$\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$

Equation of tangent at P(b cos θ, 2a sin θ) is

$\frac{x\cos\theta}{b} + \frac{y\sin\theta}{2a} = 1$Point of intersection of tangent and coordinate axis are

A(b sec θ, 0) and B(0, 2acosecθ)

Area of $\Delta OAB = \frac{1}{2}(b\sec\theta)(2a\operatorname{cosec}\theta)$

$= \frac{ab}{\sin\theta\cos\theta} = \frac{2ab}{\sin 2\theta}$

or Area $= 2ab(\operatorname{cosec} 2\theta)$

Area $_{min} = 2ab$

$\therefore k = 2$

Question377

If $x^2 + 9y^2 - 4x + 3 = 0$, x, y ∈ R, then x and y respectively lie in the intervals
[2021, 27 Aug. Shift-I]

Options:

A. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

B. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and [1, 3]

C. [1, 3] and [1, 3]

D. [1, 3] and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Answer: D

Solution:

$x^2+9y^2-4x+3=0$

$\Rightarrow x^2-4x+2^2+9y^2=1$

$\Rightarrow (x-2)^2+9y^2=1$

$\Rightarrow \frac{(x-2)^2}{1}+\frac{y^2}{1/9}=1$

$\Rightarrow \frac{(x-2)^2}{1^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$

This represents on ellipse

$-1 \le x-2 \le 1$ and $-\frac{1}{3} \le y \le \frac{1}{3}$

$\Rightarrow 1 \le x \le 3$ and $-\frac{1}{3} \le y \le \frac{1}{3}$

$\Rightarrow x \in [1, 3]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

Question378

On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$. Let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S′ be the foci of the ellipse and e be its eccentricity. If A is the area of the △SPS then, the value of $(5-e^2)\cdot A$ is
[2021, 26 Aug. Shift-1]

Options:

A. 6

B. 12

C. 14

D. 24

Answer: A

Solution:

Given, equation of ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$

Then, equation of tangent at (x_1, y_1) will be $\frac{xx_1}{8}+\frac{yy_1}{4}=1$

Since, tangent is perpendicular to the line $x+2y=0$, then

$\left(\frac{-x_1}{2y_1}\right)\left(\frac{-1}{2}\right)=-1$

$\Rightarrow x_1=-4y_1$

Also, $\frac{x_1^2}{8}+\frac{y_1^2}{4}=1$

$\Rightarrow \frac{16y_1^2}{8}+\frac{y_1^2}{4}=1 \quad [\because x_1=-4y_1]$

$\Rightarrow \frac{9}{4}y_1^2=1 \Rightarrow y_1^2=\frac{4}{9}$

$\Rightarrow y_1=\pm\frac{2}{3} \Rightarrow y_1=\frac{2}{3}$

$[\because (x_1, y_1)$ lies in second quadrant]

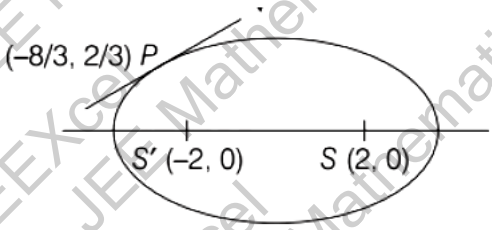
And $x_1=-4y_1=-4\times\frac{2}{3}=\frac{-8}{3}$

$\therefore P\left(\frac{-8}{3}, \frac{2}{3}\right)$

Again, $a^2 - b^2 = a^2e^2$

$8 - 4 = 8e^2$

$e = \frac{1}{\sqrt{2}}$



Now, S and S′ are the foci of the ellipse,

So, S : (ae, 0)

$= \left(2\sqrt{2} \cdot \frac{1}{\sqrt{2}}, 0\right) = (2, 0)$

and S′ : (−ae, 0) $= \left(-2\sqrt{2} \cdot \frac{1}{\sqrt{2}}, 0\right) = (-2, 0)$

Area of $\Delta SPS' = \frac{1}{2} \times 4 \times \frac{2}{3} = \frac{4}{3}$

$\left[\because \text{ base } = 4 \text{ , height } = \frac{2}{3}\right]$

So, $(5 - e^2)A = \left(5 - \frac{1}{2}\right)\frac{4}{3} = \frac{9}{2} \cdot \frac{4}{3} = 6$

Question379

Let A(sec θ, 2 tan θ) and B (sec φ, 2 tan φ), where θ + φ = π/2, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to
[2021, 27 Aug. Shift-II]

Answer: 1.56

Solution:

Let A(sec θ, 2 tan θ) lie on hyperbola

$2x^2 - y^2 = 2$

$\Rightarrow 2(1 + \tan^2\theta) - 4\tan^2\theta = 2$

$\Rightarrow 2\tan^2\theta = 0$

$\Rightarrow \tan\theta = 0$

$\Rightarrow \theta = 0$

Similarly, point B(sec φ, 2 tan φ) lie on hyperbola

$\Rightarrow \tan\varphi = 0$

$\Rightarrow \varphi = 0$

$\Rightarrow \theta + \varphi = 0$

But in question it is given that, $\theta + \varphi = \frac{\pi}{2}$ which is not possible.

Question380

The point P($-2\sqrt{6}$, $\sqrt{3}$) lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to
[2021,26 Aug. Shift-II]

Options:

A. $4\sqrt{3}$

B. 6

C. $6\sqrt{3}$

D. $3\sqrt{6}$

Answer: C

Solution:

$a^2+b^2 = a^2e^2$

$a^2+b^2 = a^2\left(\frac{5}{4}\right)$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\because$ Point $P(-2\sqrt{6}, \sqrt{3})$ lies on given ellips.

$\therefore \frac{24}{4b^2}-\frac{3}{b^2}=1$

$\Rightarrow b^2=3 \Rightarrow a^2=12$

Equation of tangent at P

$\frac{x(-2\sqrt{6})}{12}-\frac{y(\sqrt{3})}{3}=1$

For conjugate axis, put $x = 0$

$\therefore Q(0, -\sqrt{3})$

Equation of normal at P

$\frac{y-\sqrt{3}}{x+2\sqrt{6}}=\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{-12}{-2\sqrt{6}}\right)$

$\Rightarrow y-\sqrt{3}=2\sqrt{12}=4\sqrt{3}$

$\Rightarrow y=5\sqrt{3}$

$\therefore R(0, 5\sqrt{3})$

$\Rightarrow QR=6\sqrt{3}$

Question381

The locus of the mid-points of the chords of the hyperbola $x^2-y^2=4$, which touch the parabola $y^2=8x$, is
[2021, 26 Aug. Shift-II]

Options:

A. $y^3(x-2)=x^2$

B. $x^3(x-2)=y^2$

C. $y^2(x-2)=x^3$

D. $x^2(x-2)=y^3$

Answer: C

Solution:

Let the mid-point of the chord is (h_1k). Then, chord through mid-point (h, k) is

$T = S_1$

$xh-yk=h^2-k^2 \quad \cdots\cdots (i)$

Now, this is also a tangent of $y^2=8x$ The equation of the tangent of slope m to the parabola $y^2=8x$ is given by

$\text{Tangent}: \quad y=mx+\frac{2}{m}$

$\Rightarrow m^2x-my=-2 \quad \cdots\cdots (i)$

Eqs. (i) and (ii) are coincide

$\therefore \frac{h}{m^2}=\frac{-k}{-m}=\frac{h^2-k^2}{-2}$

$\Rightarrow h=km \Rightarrow m=\frac{h}{k}$

$\therefore \frac{k^2}{h}=\frac{h^2-k^2}{-2}$

$\Rightarrow -2k^2=h^3-hk^2$

$h^3 = k^2(h-2)$

Therefore, locus of mid-point of the chords, $x^3 = y^2(x-2)$

Question382

Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point 0 , then $(PQ)^2$ is equal to
[2021, 01 Sep. Shift-11]

Options:

A. $\frac{75}{8}$

B. $\frac{125}{16}$

C. $\frac{25}{2}$

D. $\frac{15}{2}$

Answer: B

Solution:

Vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$

Equation of directrix $y = \frac{1}{2}$

Equation of parabola is $\left(x-\frac{1}{2}\right)^2 = y-\frac{3}{4}$

Point on parabola $P\left(\frac{-1}{2}, \frac{7}{4}\right)$

Equation of normal at $P\left(\frac{-1}{2}, \frac{7}{4}\right)$ is $x = 2y-4$

This normal cuts the parabola at $Q(2, 3)$

$(PQ)^2 = \left(2+\frac{1}{2}\right)^2 + \left(3-\frac{7}{4}\right)^2 = \frac{125}{16}$

Question383

Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then, tan θ is equal to
[2021, 01 Sep. Shift-II]

Options:

A. $\frac{5}{2\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{4}{\sqrt{3}}$

D. 2

Answer: B

Solution:

Given, ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1 \quad \cdots\cdots\cdot(i)$

and circle $x^2+y^2=3 \quad \cdots\cdots\cdot(ii)$

The point of intersection by solving Eqs. (i) and (ii) in first quadrant $(3/2, \sqrt{3}/2)$.

Differentiating Eqs. (i) and (ii) w.r.t. x, we have

Let $m_1=\frac{dy}{dx}=\frac{-x}{9y}$ and $m_2=\frac{dy}{dx}=\frac{-x}{y}$

rAt$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

$m_1=-\frac{1}{3\sqrt{3}}, m_2=-\sqrt{3}$

If angle between both curves is θ, then

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{\frac{-1}{3\sqrt{3}}+\sqrt{3}}{1+\left(\frac{-1}{3\sqrt{3}}\right)(-\sqrt{3})}\right| = \frac{2}{\sqrt{3}}$

Question384

If a hyperbola passes through the point P(10, 16) and it has vertices at (±6, 0), then the equation of the normal to it at P is:
[Jan. 8, 2020 (II)]

Options:

A. $3x + 4y = 94$

B. $2x + 5y = 100$

C. $x + 2y = 42$

D. $x + 3y = 58$

Answer: B

Solution:

Let the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

If a hyperbola passes through vertices at $(\pm 6, 0)$, then

$\therefore a = 6$

As hyperbola passes through the point $P(10, 16)$

$\frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow b^2=144$

$\therefore$ Required hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

Equation of normal is $\frac{36x}{10}+\frac{144y}{16}=36+144$

At $P(10, 16)$ normal is

$\frac{36x}{10}+\frac{144y}{16}=36+144$

$\therefore 2x + 5y = 100$

Question385

A circle touches the y -axis at the point (0,4) and passes through the point (2,0) . Which of the following lines is not a tangent to this circle?
[Jan. 9, 2020 (I)]

Options:

A. $4x - 3y + 17 = 0$

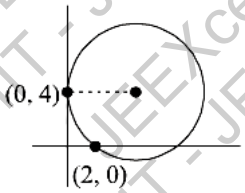
B. $3x - 4y - 24 = 0$

C. $3x + 4y - 6 = 0$

D. $4x + 3y - 8 = 0$

Answer: D

Solution:



Equation of family of circle

$(x-0)^2 + (y-4)^2 + \lambda x = 0$

Passes through the point (2,0) then

$4 + 16 + 2\lambda = 0 \Rightarrow \lambda = -10$

Hence, the equation of circle

$x^2 + y^2 - 10x - 8y + 16 = 0$

$\Rightarrow (x-5)^2 + (y-4)^2 = 25$

Centre (5,4)

$R = \sqrt{\frac{1}{2}\text{ coeff. of } x + \frac{1}{2}\text{ coeff. of } y - \text{ constant}}$

$= \sqrt{25 + 16 - 16} = 5$

Perpendicular distance of $4x + 3y - 8 = 0$ from the centre of circle

$= \left|\frac{20 + 16 - 8}{\sqrt{16+9}}\right| = \frac{28}{5} \neq 5$

Hence, $4x + 3y - 8 = 0$ can not be tangent to the circle.

Question386

If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is _____.
[NA Jan. 9, 2020 (II)]

Answer: 36

Solution:

The given equation of circle

$x^2 - 6x + y^2 + 8 = 0$

$(x-3)^2 + y^2 = 1 \quad \ldots (i)$

So, centre of circle (i) is $C_1(3, 0)$ and radius $r_1 = 1$

And the second equation of circle

$x^2 - 8y + y^2 + 16 - k = 0 (k > 0)$

$x^2 + (y-4)^2 = (\sqrt{k})^2$

So, centre of circle (ii) is $C_2(0, 4)$ and radius $r_2 = \sqrt{k}$

Two circles touches each other when

$C_1C_2 = |r_1 \pm r_2| \Rightarrow 5 = |1 \pm \sqrt{k}|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is either $\sqrt{k} + 1$ or $|\sqrt{k} - 1| (C_1C_2 = 5)$

$\Rightarrow \sqrt{k} + 1 = 5$ or $|\sqrt{k} - 1| = 5$

$\Rightarrow k = 16$ or $k = 36$

Hence, maximum value of k is 36

The given equation of circles $x^2 - 6x + y^2 + 8 = 0$

$\Rightarrow (x-3)^2 + y^2 = 1$

Question387

If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then:
[Jan. 8, 2020 (II)]

Options:

A. $c^2 - 7c + 6 = 0$

B. $c^2 + 7c + 6 = 0$

C. $c^2 + 6c + 7 = 0$

D. $c^2 - 6c + 7 = 0$

Answer: C

Solution:

Slope of tangent of $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - 1 = 0$

$x + y\sqrt{2} = 0$, which is perpendicular to $x - y + c = 0$

At $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ which is tangent of $(x-3)^2 + y^2 = 1$

So, $m = 1 \Rightarrow y = x + c$

Now, distance of (3,0) from $y = x + c$ is $\left|\frac{c+3}{\sqrt{2}}\right| = 1$

$\Rightarrow c = -3 \pm \sqrt{2}$

$\Rightarrow (c+3)^2 = 2$

$\Rightarrow c^2 + 6c + 9 = 2$

$\therefore c^2 + 6c + 7 = 0$

Question388

Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to:
[Jan. 7, 2020 (II)]

Options:

A. $\frac{52}{5}$

B. $\frac{56}{5}$

C. $\frac{64}{5}$

D. $\frac{32}{5}$

Answer: C

Solution:

$L = \sqrt{S_1} = \sqrt{16} = 4$

$R = \sqrt{16 + 4 - 16} = 2$

Length of chord of contact

$= \frac{2LR}{\sqrt{L^2 + R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}}$

Square of length of chord of contact $= \frac{64}{5}$

Question389

If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{\sqrt{2}}, -2\right)$, then the equation of the tangent to it at B is:
[Jan. 9, 2020 (II)]

Options:

A. $2x + y - 24 = 0$

B. $x - 2y + 8 = 0$

C. $x + 2y + 8 = 0$

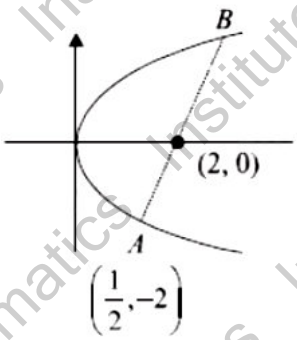
D. $2x - y - 24 = 0$

Answer: B

Solution:

Let parabola $y^2 = 8x$ at point $\left(\frac{1}{2}, -2\right)$ is $(2t^2, 4t)$

$\Rightarrow t = \frac{-1}{2}$



Parameter of other end of focal chord is 2 So, coordinates of B is (8,8)
$\Rightarrow$ Equation of tangent at B is $8y - 4(x + 8) = 0$
$\Rightarrow 2y - x = 8$
$\Rightarrow x - 2y + 8 = 0$

Question390

The locus of a point which divides the line segment joining the point (0,-1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is:
[Jan. 8, 2020 (I)]

Options:

A. $9x^2 - 12y = 8$

B. $9x^2 - 3y = 2$

C. $x^2 - 3y = 2$

D. $4x^2 - 3y = 2$

Answer: A

Solution:

Let point P be $(2t, t^2)$ and Q be (h, k) Using section formula,

$h = \frac{2t}{3}, k = \frac{-2+t^2}{3}$

Hence, locus is $3k + 2 = \left(\frac{3h}{2}\right)^2$

$\Rightarrow 9x^2 = 12y + 8$

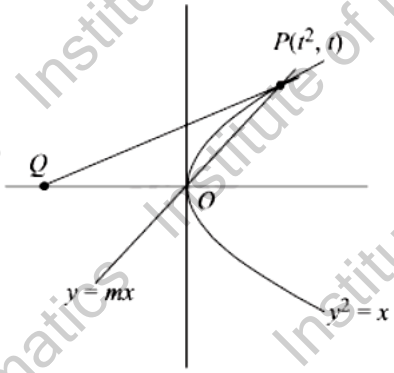
Question391

Let a line y = mx(m > 0) intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x -axis at the point Q, If area (ΔOPQ) = 4 sq. units, then m is equal to _____.
[NA Jan. 8, 2020 (II)]

Answer: 0.5

Solution:

Let the coordinates of $P = P(t^2, t)$



Tangent at $P(t^2, t)$ is $ty = \frac{x+t^2}{2}$

$\Rightarrow 2ty = x + t^2$

$Q(-t^2, 0), O(0, 0)$

Area of $\Delta OPQ = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = 4$

$\Rightarrow |t|^3 = 8$

$t = \pm 2 (t > 0)$

$\therefore 4y = x + 4$ is a tangent $\therefore P$ is (4,2)

Now, $y = mx \quad \therefore m = \frac{1}{2}$

Question392

If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to
[Jan. 9, 2020 (I)]

Options:

A. 16

B. 17

C. 15

D. 14

Answer: A

Solution:

Eccentricity of ellipse

$e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$

Eccentricity of hyperbola

$e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$

Since, the point (e_1, e_2) is on the ellipse

$15x^2 + 3y^2 = k$

Then, $15e_1^2 + 3e_2^2 = k$

$\Rightarrow k = 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right)$

$\Rightarrow k = 16$

Question393

The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, x +6y = 8; then its eccentricity is:
[Jan. 9, 2020 (II)]

Options:

A. $\frac{1}{2}\sqrt{\frac{11}{3}}$

B. $\sqrt{\frac{5}{6}}$

C. $\frac{1}{2}\sqrt{\frac{5}{3}}$

D. $\frac{1}{3}\sqrt{\frac{11}{3}}$

Answer: A

Solution:

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; a > b$

$2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

Comparing with $\equiv y = \frac{-x}{6} + \frac{4}{3}$

$m = \frac{-1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} + \frac{4}{3} = \frac{16}{9} \Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3} = \frac{4}{9}$

$\Rightarrow a^2 = 16 \Rightarrow a = \pm 4$

Now, eccentricity of ellipse $(e) = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}$

Question394

Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and (0, β), then β is equal to:
[Jan. 8, 2020 (I)]

Options:

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{2}{3}$

D. $\frac{\sqrt{2}}{3}$

Answer: D

Solution:

Let P be (x_1, y_1).
So, equation of normal at P is
$\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$
It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$
$\Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$
So, $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in I ^{It} quadrant)
So, $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

Question395

If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:
[Jan. 7, 2020 (I)]

Options:

A. $\sqrt{3}$

B. $3\sqrt{2}$

C. $\frac{3}{\sqrt{2}}$

D. $2\sqrt{3}$

Answer: B

Solution:

$2ae = 6$ and $\frac{2a}{e} = 12$
$\Rightarrow ae = 3$ ..... (i)
and $\frac{a}{e} = 6 \Rightarrow e = \frac{a}{6}$... (ii)
$\Rightarrow a^2 = 18$ [From (i) and (ii)]
$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$
$\therefore$ Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}} = 3\sqrt{2}$

Question396

If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is:
[Jan. 7, 2020 (II)]

Options:

A. $2\sqrt{7}$

B. 4

C. $2\sqrt{5}$

D. $2\sqrt{2}$

Answer: A

Solution:

$3x + 4y = 12\sqrt{2}$
$\Rightarrow 4y = -3x + 12\sqrt{2}$
$\Rightarrow\ y = -\frac{3}{4}x + 3\sqrt{2}$

Now, condition of tangency, $c^2 = a^2m^2 + b^2$
$18 = a^2 \cdot \frac{9}{16} + 9 \Rightarrow a^2 \cdot \frac{9}{16} = 9$
$\Rightarrow a^2 = 16 \Rightarrow a = 4$
Eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore\ ae = \frac{\sqrt{7}}{4} \cdot 4 = \sqrt{7}$
$\therefore$ Focus are $(\pm\sqrt{7}, 0)$
$\therefore$ Distance between foci of ellipse $= 2\sqrt{7}$

Question397

If the length of the chord of the circle, $x^2 + y^2 = r^2 (r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to :
[Sep. 05, 2020 (II)]

Options:

A. $\frac{9}{5}$

B. 12

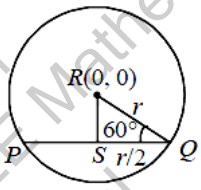
C. $\frac{24}{5}$

D. $\frac{12}{5}$

Answer: D

Solution:

In right ΔRSQ, $\sin 60^\circ = \frac{RS}{r}$



$\Rightarrow RS = r \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r}{2}$

Now equation of PQ is $y - 2x - 3 = 0$

$\therefore \frac{\sqrt{3}r}{2} = \frac{|0+0-3|}{\sqrt{5}}$

$\Rightarrow \frac{\sqrt{3}r}{2} = \frac{3}{\sqrt{5}} \Rightarrow r = \frac{2\sqrt{3}}{5} \Rightarrow r^2 = \frac{12}{5}$

Question398

The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point:
[Sep. 04, 2020 (II)]

Options:

A. (-1,3)

B. (-3,6)

C. (-3,1)

D. (1,-3)

Answer: B

Solution:

We know family of circle be $S_1 + \lambda S_2 = 0$

$x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 4y) = 0$

$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 - 6x - 4\lambda y = 0 \ldots$ (i)

Centre $(-g, -f) = \left(\frac{3}{1+\lambda}, \frac{2\lambda}{\lambda+1}\right)$

Centre lies on $2x - 3y + 12 = 0$, then

$\frac{6}{\lambda+1} - \frac{6\lambda}{\lambda+1} + 12 = 0 \Rightarrow \lambda = -3$

Equation of circle (i),

$-2x^2 - 2y^2 - 6x + 12y = 0$

$\Rightarrow x^2 + y^2 + 3x - 6y = 0 \ldots$ (ii)

Only (-3,6) satisfy equation (ii).

Question399

Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of αβ is _____.
[NA Sep. 04, 2020 (II)]

Answer: 7

Solution:

Let $P(3\cos\theta, 3\sin\theta)$, $Q(-3\cos\theta, -3\sin\theta)$

$\alpha = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right|, \beta = \left|\frac{-3\cos\theta - 3\sin\theta - 2}{\sqrt{2}}\right|$

$\alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right| = \left|\frac{5 + 9\sin 2\theta}{2}\right|$

$\alpha\beta$ is max. when $\sin 2\theta = 1$

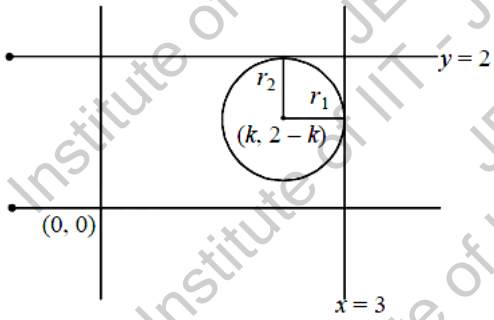
$\therefore \alpha\beta|_{max} = \frac{5+9}{2} = 7$

Question400

The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _____.
[NA Sep. 03, 2020 (I)]

Answer: 3

Solution:



$\Rightarrow$ Radius $(r_1) = 3 - k$

$\because$ Centre lies on $x + y = 2$

Let $x = k$

$\therefore y = 2 - k$

$\Rightarrow$ Centre $= (k, 2 - k)$

Also, radius $(r_2) = 2 - (2 - k)$

$\therefore 3 - k = 2 - (2 - k)$

$\Rightarrow k = \frac{3}{2}$

$r = 3 - \frac{3}{2} = \frac{3}{2}$

Hence, diameter $= 3$.

Question401

The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____.
[NA Sep. 02, 2020 (I)]

Answer: 9

Solution:

The given circle is $x^2 + y^2 - 2x - 4y + 4 = 0$

$\therefore$ Centre of circle $(1, 2)$, $r = 1$

If line cuts circle then $p < r$, where $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$\Rightarrow \left|\frac{3+8-k}{5}\right| < 1 \Rightarrow k \in (6, 16)$

k = 7, 8, 9, 10, 11, 12, 13, 14, 15

Question402

Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :
[Sep. 06, 2020 (I)]

Options:

A. $x + 3 = 0$

B. $2x + 1 = 0$

C. $x + 2 = 0$

D. $x + 2y = 0$

Answer: A

Solution:

$L_1 : y = m_1(x + 1) + \frac{1}{m_1}$ [Tangent to $y^2 = 4(x + 1)$]

$L_2 : y = m_2(x + 2) + \frac{2}{m_2}$ [Tangent to $y^2 = 8(x + 2)$]

$m_1^2(x + 1) - ym_1 + 1 = 0 \ldots$. (i)

$m_2^2(x + 2) - ym_2 + 2 = 0 \ldots$. (ii)

$\because m_2 = -\frac{1}{m_1}$ $(\because L_1 \perp L_2)$

[From(ii)] $\Rightarrow 2m_1^2 + ym_1 + (x + 2) = 0$ .....(iii)

From (i) and (iii),

$\frac{x+1}{2} = \frac{-y}{y} = \frac{1}{x+2} \Rightarrow x + 3 = 0$

Question403

The centre of the circle passing through the point (0,1) and touching the parabola $y = x^2$ at the point (2,4) is:
[Sep. 06, 2020 (II)]

Options:

A. $\left(\frac{-53}{10}, \frac{16}{5}\right)$

B. $\left(\frac{6}{5}, \frac{53}{10}\right)$

C. $\left(\frac{3}{10}, \frac{16}{5}\right)$

D. $\left(\frac{-16}{5}, \frac{53}{10}\right)$

Answer: D

Solution:

Circle passes through $A(0, 1)$ and $B(2, 4)$. So its centre is the point of intersection of perpendicular bisector of AB and normal to the parabola at (2,4) .
Perpendicular bisector of AB;

$y - \frac{5}{2} = -\frac{2}{3}(x - 1) \Rightarrow 4x + 6y = 19$ . . . (i)

Equation of normal to the parabola at (2,4) is,

$y - 4 = -\frac{1}{4}(x - 2) \Rightarrow x + 4y = 18$ . . . (ii)

$\therefore$ From (i) and (ii), $x = -\frac{16}{5}, y = \frac{53}{10}$

$\therefore$ Centre of the circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

Question404

If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to:
[Sep. 05, 2020 (I)]

Options:

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: B

Solution:

Equation tangent to parabola $y^2 = 4x$ with slope m be:

$y = mx + \frac{1}{m}$ . . . (i)

$\because$ Equation of tangent to $x^2 = 4y$ with slope m be :

$y = mx - am^2$ . . . (ii)

From eq. (i) and (ii), $\frac{1}{m} = -m^2 \Rightarrow m = -1$

$\therefore$ Equation tangent : $x + y + 1 = 0$

It is tangent to circle $x^2 + y^2 = c^2$

$\Rightarrow c = \frac{1}{\sqrt{2}}$

Question405

Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. Aline is now drawn through the mid-point M of PN , parallel to its axis which meets the parabola at Q. If the y -intercept of the line N Q is $\frac{4}{3}$, then :
[Sep. 03, 2020 (I)]

Options:

A. PN = 4

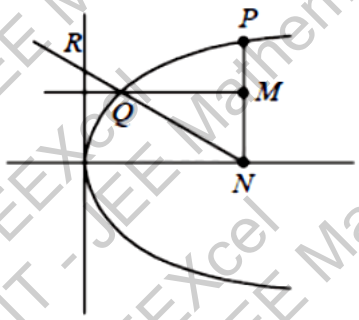
B. M Q = $\frac{1}{3}$

C. M Q = $\frac{1}{4}$

D. PN = 3

Answer: C

Solution:



$\because y^2 = 12x$

$\therefore a = 3$

Let $P(at^2, 2at)$

$\Rightarrow N(at^2, 0) \Rightarrow M(at^2, at)$

$\because$ Equation of QM is $y = at$

So, $y^2 = 4ax \Rightarrow x = \frac{at^2}{4}$

$\Rightarrow Q\left(\frac{at^2}{4}, at\right)$

$y = \frac{-4}{3t}(x - at^2)$

$\because$QN passes through $\left(0, \frac{4}{3}\right)$, then

$\frac{4}{3} = -\frac{4}{3t}(-at^2) \Rightarrow at = 1 \Rightarrow t = \frac{1}{3}$

Now, $MQ = \frac{3}{4}at^2 = \frac{1}{4}$ and $PN = 2at = 2$

Question406

Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is :
[Sep. 03, 2020 (II)]

Options:

A. $8\sqrt{5}$

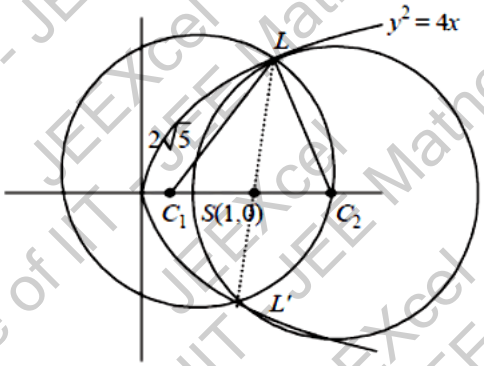
B. 8

C. 12

D. $4\sqrt{5}$

Answer: B

Solution:



Distance between the centres

$= C_1C_2 = 2C_1S = 2\sqrt{20 - 4} = 8$

Question407

The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is:
[Sep. 02, 2020 (II)]

Options:

A. $64\sqrt{3}$

B. $256\sqrt{3}$

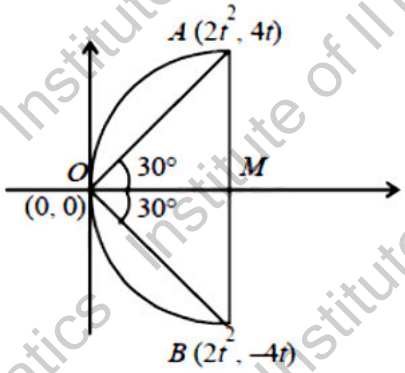
C. $192\sqrt{3}$

D. $128\sqrt{3}$

Answer: C

Solution:

Let A = $(2t^2, 4t)$ and B = $(2t^2, -4t)$



For equilateral triangle ($\angle AOM = 30^\circ$)

$\tan 30^\circ = \frac{4t}{2t^2} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4t}{2t^2} \Rightarrow t = 2\sqrt{3}$

Area $= \frac{1}{2} \cdot 8(2\sqrt{3}) \cdot 2 \cdot 24 = 192\sqrt{3}$

Question408

Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci?
[Sep. 06, 2020 (I)]

Options:

A. $(-2, \sqrt{3})$

B. $(-1, \sqrt{2})$

C. $(-1, \sqrt{3})$

D. (1,2)

Answer: C

Solution:

We know that the locus of the feet of the perpendicular draw from foci to any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the auxiliary circle $x^2+y^2=a^2$

$\therefore$ Auxiliary circle : $x^2+y^2=4$

$(-1, \sqrt{3})$ satisfies the given equation.

Question409

If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies:
[Sep. 06, 2020 (II)]

Options:

A. $e^4+2e^2-1=0$

B. $e^2+e-1=0$

C. $e^4+e^2-1=0$

D. $e^2+2e-1=0$

Answer: C

Solution:

Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(ae, \frac{b^2}{a}\right)$ is

$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$

$\Rightarrow x-ey=\frac{e(a^2-b^2)}{a}$

$\because (0, -b)$ lies on equation (i), then

$be=\frac{e(a^2-b^2)}{a}$

$\Rightarrow ab=a^2e^2 \Rightarrow b=ae^2 \Rightarrow \frac{b^2}{a^2}=e^4$

$\therefore 1-e^2=e^4 \Rightarrow e^4+e^2-1=0$

Question410

If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2+16y^2=144$, then PA + PB is equal to :
[Sep. 05, 2020 (I)]

Options:

A. 16

B. 8

C. 6

D. 9

Answer: B

Solution:

Ellipse: $\frac{x^2}{16}+\frac{y^2}{9}=1$,

$a=4, b=3, c=\sqrt{16-9}=\sqrt{7}$

$\therefore(\pm\sqrt{7}, 0)$ are the foci of given ellipse. So for any point P on it; PA + PB = 2a

$\Rightarrow$PA + PB = 2(4) = 8

Question411

If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, −4), then PQ^2 is equals to : [Sep. 05, 2020 (I)]

Options:

A. 36

B. 48

C. 21

D. 29

Answer: A

Solution:

Ellipse $\equiv \frac{x^2}{5} + \frac{y^2}{4} = 1$

Let a point on ellipse be $(\sqrt{5}\cos\theta, 2\sin\theta)$

$\therefore PQ^2 = (\sqrt{5}\cos\theta)^2 + (-4 - 2\sin\theta)^2$

$= 5\cos^2\theta + 4\sin^2\theta + 16 + 16\sin\theta$

$= 21 + 16\sin\theta - \sin^2\theta$

$= 21 + 64 - (\sin\theta - 8)^2 = 85 - (\sin\theta - 8)^2$

PQ^2 to be maximum when $\sin\theta = 1$

$PQ_{max}^{\ 2} = 85 - 49 = 36$

Question412

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ be a given ellipse, length of whose latus rectum is 10 . If its eccentricity is the maximum value of the function, $\varphi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to: [Sep. 04, 2020 (I)]

Options:

A. 145

B. 116

C. 126

D. 135

Answer: C

Solution:

The given ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$

Length of latus rectum $= \frac{2b^2}{a}$

$\Rightarrow \frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ . . . . . (i)

Now $\varphi(t) = \frac{5}{12} + t - t^2$

$\varphi'(t) = 1 - 2t = 0 \Rightarrow t = \frac{1}{2}$

$\varphi''(t) = -2 < 0 \Rightarrow$ maximum

$\Rightarrow \varphi(t)_{max} = \frac{5}{12} + \frac{1}{2} - \frac{1}{4} = \frac{8}{12} = \frac{2}{3}$

Since, $\varphi(t)_{max} =$ eccentricity

$\Rightarrow e = \frac{2}{3}$

Now, $b^2 = a^2(1 - e^2)$

$5a = a^2\left(1 - \frac{4}{9}\right) \Rightarrow 5a = \frac{5a^2}{9} \Rightarrow a^2 - 9a = 0$

$\Rightarrow a = 9 \Rightarrow a^2 = 81$ and $b^2 = 45$

$\therefore a^2 + b^2 = 81 + 45 = 126$

Question413

Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P(1, β), β > 0 is a point on this ellipse, then the equation of the normal to it at P is : [Sep. 04, 2020 (II)]

Options:

A. $4x - 3y = 2$

B. $8x - 2y = 5$

C. $7x - 4y = 1$

D. $4x - 2y = 1$

Answer: D

Solution:

$\frac{a}{e} = 4 \Rightarrow a = 4 \times \frac{1}{2} = 2$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 4\left(1 - \frac{1}{4}\right) = 4 \times \frac{3}{4} = 3$

So, equation $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow 3x^2 + 4y^2 = 12 \ldots\ldots$ (i)

Now, P(1, β) lies on it

$\Rightarrow 3 + 4\beta^2 = 12 \Rightarrow \beta = \frac{3}{2}$

So, equation of normal at $P\left(1, \frac{3}{2}\right)$

$\Rightarrow \frac{a^2x}{1} - \frac{b^2y}{3/2} = a^2 - b^2 \Rightarrow 4x - 2y = 1$

Question414

A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points? [Sep. 03, 2020 (I)]

Options:

A. $\left(\frac{1}{\sqrt{2}}, 0\right)$

B. $\left(-\sqrt{\frac{3}{2}}, 1\right)$

C. $\left(1, -\frac{1}{\sqrt{2}}\right)$

D. $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

Answer: D

Solution:

The given ellipse:

$\frac{x^2}{4}+\frac{y^2}{3}=1$

$\because c=\sqrt{a^2-b^2}=\sqrt{4-3}=1$

$\therefore$ Foci $=(\pm 1, 0)$

Now for hyperbola:

Given: $2a=\sqrt{2} \Rightarrow a=\frac{1}{\sqrt{2}}$

$\because c^2=a^2+b^2 \Rightarrow 1=\frac{1}{2}+b^2 \Rightarrow b=\frac{1}{\sqrt{2}}$

So, equation of hyperbola is

$\frac{x^2}{\frac{1}{2}}-\frac{y^2}{\frac{1}{2}}=1$

$\Rightarrow 2x^2-2y^2=1$

So, option (d) does not satisfy it.

Question415

Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is:
[Sep. 02, 2020 (I)]

Options:

A. $6(\pi-2)$

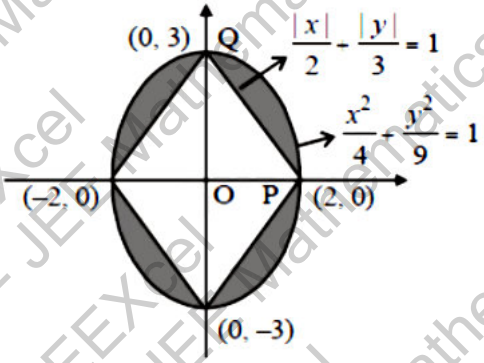
B. $3(\pi-2)$

C. $3(4-\pi)$

D. $6(4-\pi)$

Answer: A

Solution:



$\because$ Area of ellipse $=\pi ab=\pi\times 2\times 3=6\pi$

Required area = Area of ellipse

-4 (Area of triangle OPQ)

$=6\pi-4\left(\frac{1}{2}\times 2\times 3\right)$

$=6\pi-12=6(\pi-2)$ sq. units

Question416

If the line y = mx + c is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?
[Sep. 05, 2020 (II)]

Options:

A. $c^2 = 369$

B. $5m = 4$

C. $4c^2 = 369$

D. $8m + 5 = 0$

Answer: C

Solution:

General tangent to hyperbola in slope form is $y = mx \pm \sqrt{100m^2 - 64}$
and the general tangent to the circle in slope form is $y = mx \pm 6\sqrt{1 + m^2}$
For common tangent,
$36(1 + m^2) = 100m^2 - 64$
$\Rightarrow 100 = 64m^2 \Rightarrow m^2 = \frac{100}{64}$
$\therefore c^2 = 36\left(1 + \frac{100}{64}\right) = \frac{164 \times 36}{64} = \frac{369}{4}$
$\Rightarrow 4c^2 = 369$

Question417

Let P(3, 3) be a point on the hyperbola , $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x -axis at (9,0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :
[Sep. 04, 2020 (I)]

Options:

A. $\left(\frac{9}{2}, 3\right)$

B. $\left(\frac{3}{2}, 2\right)$

C. $\left(\frac{9}{2}, 2\right)$

D. (9,3)

Answer: A

Solution:

$\because$ The equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$\because$ Equation of hyperbola passes through (3,3)
$\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{9} \ldots\ldots$ (i)
Equation of normal at point (3,3) is:
$\frac{x-3}{\frac{1}{a^2} \cdot 3} = \frac{y-3}{-\frac{1}{b^2} \cdot 3}$
$\because$ It passes through (9,0)
$\frac{6}{\frac{1}{a^2}} = \frac{-3}{-\frac{1}{b^2}}$

$\frac{1}{b^2} = \frac{1}{2a^2}$ . . . . (ii)

From equations (i) and (ii),

$a^2 = \frac{9}{2}, b^2 = 9$

$\because$ Eccentricity $= e$, then $e^2 = 1 + \frac{b^2}{a^2} = 3$

$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

Question418

Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :
[Sep. 03, 2020 (II)]

Options:

A. (8,12)

B. $\left(\frac{20}{3}, 12\right)$

C. $\left(\frac{24}{5}, 10\right)$

D. (8,10)

Answer: B

Solution:

Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$

Then, $e_1 = \sqrt{1 - \frac{b^2}{25}}$

The equation of hyperbola, $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Then, $e_2 = \sqrt{1 + \frac{b^2}{16}}$ $G_1e_2 = 1$

$\Rightarrow (e_1e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25 \times 16} = 1$

$\Rightarrow \frac{9}{16 \cdot 25}b^2 - \frac{b^4}{25 \cdot 16} = 0 \Rightarrow b^2 = 9$

$\therefore e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

And, $e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Distance between focii of ellipse
$= \alpha = 2ae_1 = 2(5)(e_1) = 8$

Distance between focii of hyperbola
$= \beta = 2ae_2 = 2(4)(e_2) = 10$

$\therefore (\alpha, \beta) = (8, 10)$

Question419

A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :
[Sep. 02, 2020 (I)]

Options:

A. 6

B. 8

C. 10

D. 5

Answer: A

Solution:

The tangent to the hyperbola at the point (x_1, y_1) is,

$xx_1 - 2yy_1 - 4 = 0$

The given equation of tangent is

$2x - y = 0$

$\Rightarrow \frac{x_1}{2y_1} = 2$

$\Rightarrow x_1 = 4y_1 \ldots .$ (i)

Since, point (x_1, y_1) lie on hyperbola.

$\therefore \frac{x_1^2}{4} - \frac{y_1^2}{2} - 1 = 0$

On solving eqs. (i) and (ii)

$y_1^2 = \frac{2}{7}, x_1^2 = \frac{32}{7}$

$\therefore x_1^2 + 5y_1^2 = \frac{32}{7} + 5 \times \frac{2}{7} = 6$

Question 420

For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:
[Sep. 02, 2020 (II)]

Options:

A. $2\sqrt{6}$

B. $\sqrt{30}$

C. $\frac{2\sqrt{5}}{3}$

D. $\frac{4\sqrt{5}}{3}$

Answer: D

Solution:

Hyperbola: $\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1 \Rightarrow e_1 = \sqrt{1 + \cos^2\theta}$ and Ellipse: $\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1$

$\Rightarrow e_2 = \sqrt{1 - \cos^2\theta} = \sin\theta$

According to the question, $q_1 = \sqrt{5}e_2$

$\Rightarrow 1 + \cos^2\theta = 5\sin^2\theta \Rightarrow \cos^2\theta = \frac{2}{3}$

Now length of latus rectum of ellipse

$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$
