



Application of Integrals

Question 5

If the area of the region

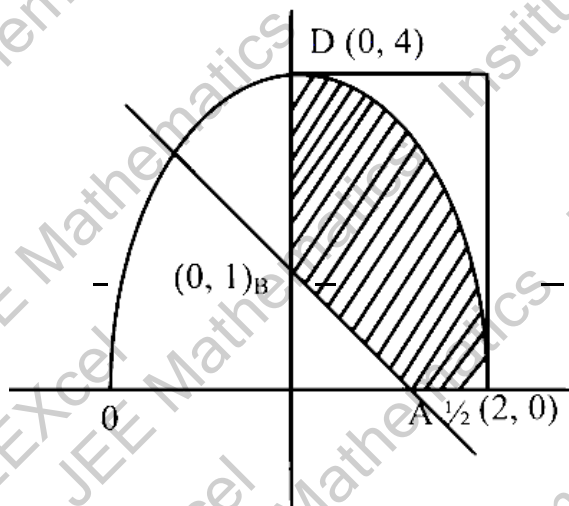
$\{(x, y) : 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbb{N}$, $\gcd(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is: JEE Main 2026 (Online) 21st

January Evening Shift Options:

- A. 73
- B. 85
- C. 67
- D. 91

Answer: A

Solution:



$$\int_0^2 (4 - x^2) dx - \frac{1}{2} \times 1 \times 2$$

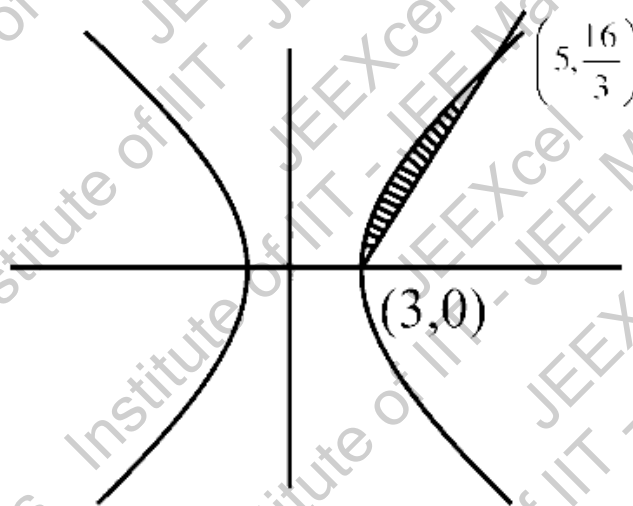
Question2

If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A , then $3(A + 6 \log_e(3))$ is equal to _____.

JEE Main 2026 (Online) 2nd April Evening Shift

Answer: 24

Solution:



$$16x^2 - 9y^2 = 144$$

$$16x^2 - (8x - 24)^2 = 144$$

$$16x^2 - 64(x - 3)^2 = 144 \rightarrow x^2 - 4(x - 3)^2 = 9$$

$$3x^2 - 24x + 45 \rightarrow x^2 - 8x + 15 = 0$$

$$x = 3, 5$$

$$\text{Area} = \int_3^5 \sqrt{\frac{16x^2 - 144}{9}} - \frac{1}{2} \cdot 2 \cdot \frac{16}{3}$$

$$= \frac{4}{3} \int_3^5 \sqrt{x^2 - 9} - \frac{16}{3}$$

$$= \frac{4}{3} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \log_e \left(x + \sqrt{x^2 - 9} \right) \right) \Big|_3^5 - \frac{16}{3}$$

$$= \frac{4}{3} \left(\frac{5}{2} \cdot 4 - \frac{9}{2} \log_e 9 - \frac{3}{2} \cdot 0 + \frac{9}{2} \log_e 3 \right) - \frac{16}{3}$$

$$\text{Area} = 8 - 6 \log_e 3 = A$$

$$\therefore A + 6 \ln 3 = 8$$

$$\Rightarrow 3(A + 6 \ln 3) = 24$$

Question3

Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a function such that

$f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x, x \in \mathbf{R}$. Let the maximum value of f on $\mathbf{R}$ be α . If the area of the region bounded by the curves $g(x) = x^2$ and $h(x) = \beta x^3, \beta > 0$, is α^2 , then $30\beta^3$ is equal to ____.

JEE Main 2026 (Online) 5th April Evening Shift

Answer: 16

Solution:

$$f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x \quad \dots (1)$$

Now replace x by $\frac{\pi}{2} - x$ in (1). This is a standard NCERT step to get a second equation in the same two terms.

$$\Rightarrow f\left(\frac{\pi}{2} - x\right) + 3f(x) = \cos x \quad \dots (2)$$

Now we solve the two linear equations (1) and (2) to find $f(x)$.

Multiply (1) by 3:

$$3f(x) + 9f\left(\frac{\pi}{2} - x\right) = 3 \sin x \quad \dots (3)$$

Subtract (2) from (3):

$$(3f(x) - 3f(x)) + (9 - 1)f\left(\frac{\pi}{2} - x\right) = 3 \sin x - \cos x$$

$$8f\left(\frac{\pi}{2} - x\right) = 3 \sin x - \cos x$$

Now put $x \rightarrow \frac{\pi}{2} - x$ in this result to get $f(x)$:

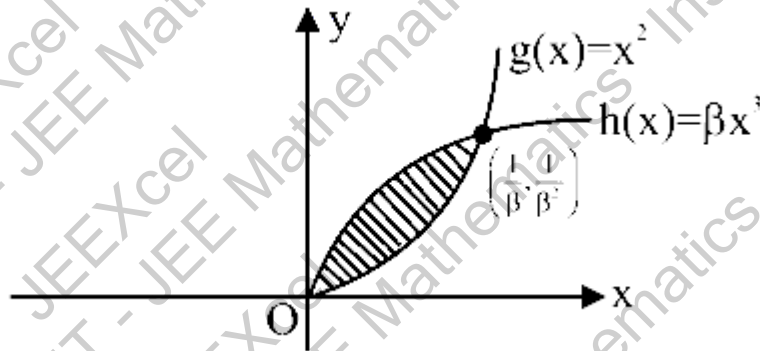
$$8f(x) = 3\cos x - \sin x$$

$$f(x) = \frac{1}{8}(3\cos x - \sin x)$$

To find the maximum value, write $3\cos x - \sin x$ in the form $R\cos(x + \phi)$, where $R = \sqrt{3^2 + (-1)^2} = \sqrt{10}$. So the maximum value of $3\cos x - \sin x$ is $\sqrt{10}$.

$$f_{\max} = \frac{\sqrt{10}}{8}$$

$y = g(x)$ & $y = h(x)$ intersect as shown in the figure



The curves are $g(x) = x^2$ and $h(x) = \beta x^3$ with $\beta > 0$. Their points of intersection come from $x^2 = \beta x^3$:

$$x^2 = \beta x^3 \Rightarrow x^2(1 - \beta x) = 0 \Rightarrow x = 0 \text{ and } x = \frac{1}{\beta}$$

For $0 < \beta x^3 < 1$ (since $\beta x < 1$), so the area between the curves is the integral of $(x^2 - \beta x^3)$ from 0 to $\frac{1}{\beta}$.

$$\therefore \text{Area bounded} = \Delta = \left| \int_0^{\frac{1}{\beta}} (\beta x^3 - x^2) dx \right|$$

$$= \left| \left[\frac{\beta x^4}{4} - \frac{x^3}{3} \right]_0^{\frac{1}{\beta}} \right|$$

$$= \left| \left(\frac{\beta}{4} \cdot \frac{1}{\beta^4} - \frac{1}{3} \cdot \frac{1}{\beta^3} \right) \right|$$

$$= \left| \left(\frac{1}{4\beta^3} - \frac{1}{3\beta^3} \right) \right| = \frac{1}{12\beta^3} = \alpha^2 \text{ (given)}$$

$$\Rightarrow \frac{1}{12\beta^3} = \left(\frac{\sqrt{10}}{8} \right)^2 = \frac{10}{64} = \frac{5}{32}$$

$$\Rightarrow \beta^3 = \frac{1}{12} \cdot \frac{32}{5} = \frac{8}{15}$$

$$\Rightarrow 30\beta^3 = 30 \cdot \frac{8}{15} = 16$$

Question 4

The area of the region, inside the ellipse $x^2 + 4y^2 = 4$ and outside the region bounded by the curves $y = |x| - 1$ and $y = 1 - |x|$, is :

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Options:

A.

$$2\pi - 1$$

B.

$$3(\pi - 1)$$

C.

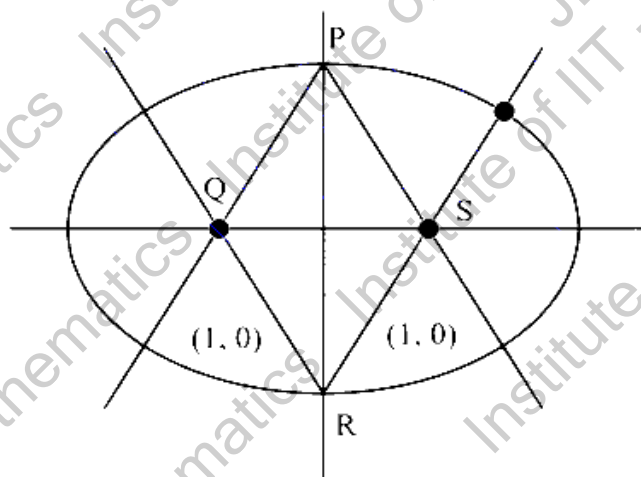
$$2(\pi - 1)$$

D.

$$2\pi - \frac{1}{2}$$

Answer: C

Solution:



$$E: \frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$\text{Area inside } E = 2\pi$$

$$\begin{aligned} \text{Area } PQRS &= (\sqrt{2})^2 \\ &= 2 \end{aligned}$$

$$\begin{aligned}\text{Required area} &= 2\pi - 2 \\ &= 2(\pi - 1)\end{aligned}$$

Question5

If the area of the region

$\{(x, y) : 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$,

$\alpha, \beta \in \mathbb{N}, \gcd(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is:

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Options:

A.

73

B.

85

C.

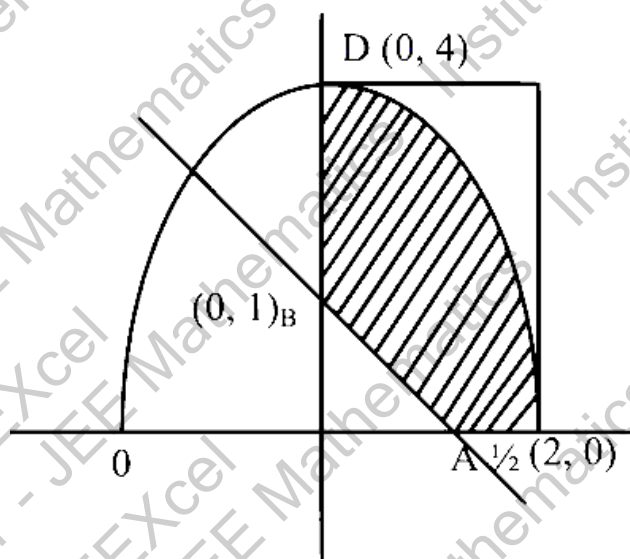
67

D.

91

Answer: A

Solution:



$$\int_0^2 (4 - x^2) dx - \frac{1}{2} \times 1 \times \frac{1}{2}$$

Question6

Let the line $x = -1$ divide the area of the region $\{(x, y) : 1 + x^2 \leq y \leq 3 - x\}$ in the ratio $m : n$, $\gcd(m, n) = 1$. Then $m + n$ is equal to

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Options:

A.

27

B.

28

C.

25

D.

Answer: A**Solution:**

$$m = \int_{-2}^{-1} (3-x) - (1+x^2) dx$$

$$m = 2 + \frac{3}{2} - \frac{7}{3} = \frac{7}{6} \quad M = \int_{-1}^1 (3-x) - (1+x^2) dx = 4 - \frac{1}{2}(0) - \frac{1}{3}(1+1) = 4 - \frac{2}{3} = \frac{10}{3}$$

$$m : M = \frac{7}{6} : \frac{10}{3} = \frac{7}{20}$$

$$m + n = 27$$

Question 7

The area of the region $A = \{(x, y) : 4x^2 + y^2 \leq 8 \text{ and } y^2 \leq 4x\}$ is:

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Options:

A.

$$\pi + \frac{2}{3}$$

B.

$$\frac{\pi}{2} + 2$$

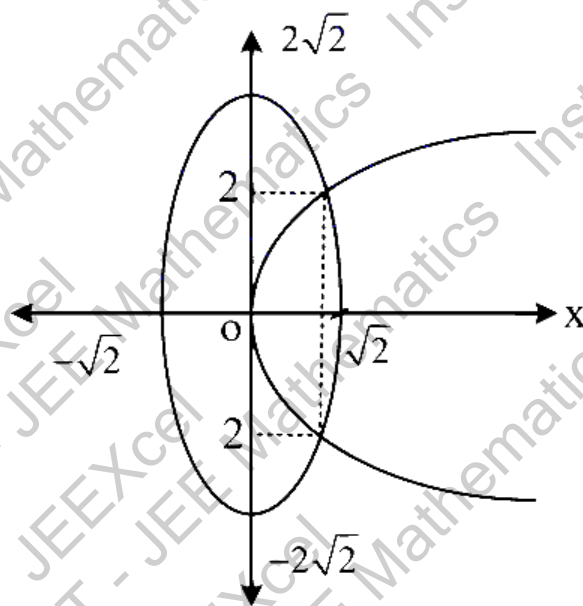
C.

$$\pi + 4$$

D.

$$\frac{\pi}{2} + \frac{1}{3}$$

Answer: A**Solution:**



$$A = 2 \int_0^1 2\sqrt{x} dx + 2 \int_1^{\sqrt{2}} \sqrt{8-4x^2} dx = \frac{8}{3} \left(X^{\frac{3}{2}} \right)_0^1 + 4 \int_1^{\sqrt{2}} \sqrt{2-x^2} dx$$

$$\frac{8}{3} + 4 \times \frac{1}{2} \left[x\sqrt{2-x^2} + 2 \sin^{-1} \left(\frac{x}{\sqrt{2}} \right) \right]_1^{\sqrt{2}}$$

$$\frac{8}{3} + 2 \left[2 \times \frac{\pi}{2} - 1 - 2 \times \frac{\pi}{4} \right] = \frac{8}{3} + 2\pi - 2 - \pi = \pi + \frac{2}{3} \text{ Sq. unit}$$

Question8

The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is:

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Options:

A.

$$\frac{2}{3}(4\pi - 3\sqrt{3})$$

B.

$$\frac{4}{3}(2\pi - \sqrt{3})$$

C.

$$\frac{4}{3}(2\pi - 3\sqrt{3})$$

D.

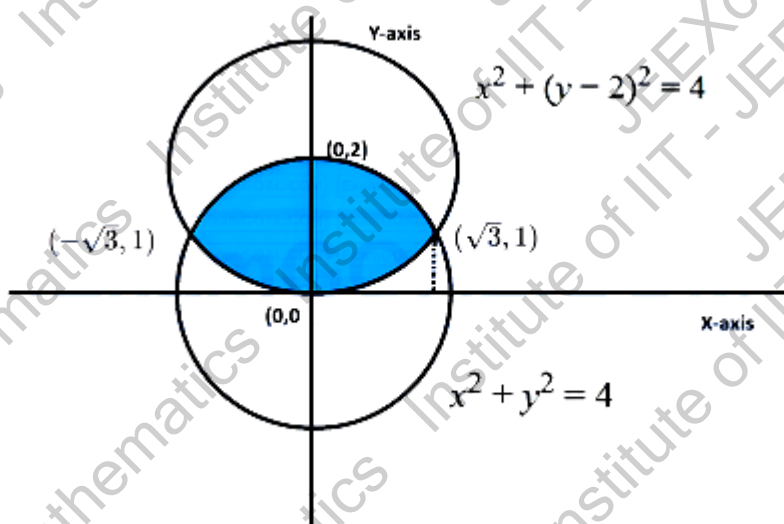
$$\frac{2}{3}(2\pi - 3\sqrt{3})$$

Answer: A

Solution:

$$x^2 + y^2 = 4 \implies C_1(0, 0), r = 2$$

$$x^2 + (y - 2)^2 = 4 \implies C_2(0, 2), r = 2$$



Intersection points:

$$x^2 + y^2 = x^2 + (y - 2)^2$$

$$y^2 = y^2 - 4y + 4 \implies 4y = 4 \implies y = 1$$

$$x^2 + 1 = 4 \implies x^2 = 3 \implies x = \pm\sqrt{3}$$

Points are $(\sqrt{3}, 1)$ and $(-\sqrt{3}, 1)$

For the region between $y = 0$ and $y = 2$:

$$y_{\text{upper}} = \sqrt{4 - x^2} \text{ (from } x^2 + y^2 = 4 \text{)}$$

$$y_{\text{lower}} = 2 - \sqrt{4 - x^2} \text{ (from } x^2 + (y - 2)^2 = 4 \text{)}$$

$$\text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} (y_{\text{upper}} - y_{\text{lower}}) dx$$

$$\text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} \left(\sqrt{4-x^2} - \left(2 - \sqrt{4-x^2} \right) \right) dx$$

$$\text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} \left(2\sqrt{4-x^2} - 2 \right) dx$$

$$\text{Property: } \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \text{ (for even functions)}$$

$$\text{Area} = 2 \int_0^{\sqrt{3}} \left(2\sqrt{4-x^2} - 2 \right) dx$$

$$\text{Area} = 4 \int_0^{\sqrt{3}} \left(\sqrt{4-x^2} - 1 \right) dx$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right)$$

$$\text{Area} = 4 \left[\frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{x}{2} \right) - x \right]_0^{\sqrt{3}}$$

$$\text{Area} = 4 \left[\frac{\sqrt{3}}{2} (1) + 2 \left(\frac{\pi}{3} \right) - \sqrt{3} \right]$$

$$\text{Area} = 4 \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right]$$

$$\text{Area} = \frac{8\pi}{3} - 2\sqrt{3} = \frac{2}{3} (4\pi - 3\sqrt{3})$$

Question9

Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2$, $x + y = 8$ and y -axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2$, $y^2 = x$, $x = 2$, and y -axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to

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Options:

A.

$$\frac{2}{3}(2\sqrt{2} + 1)$$

B.

$$\frac{2}{3}(3\sqrt{2} + 1)$$

C.

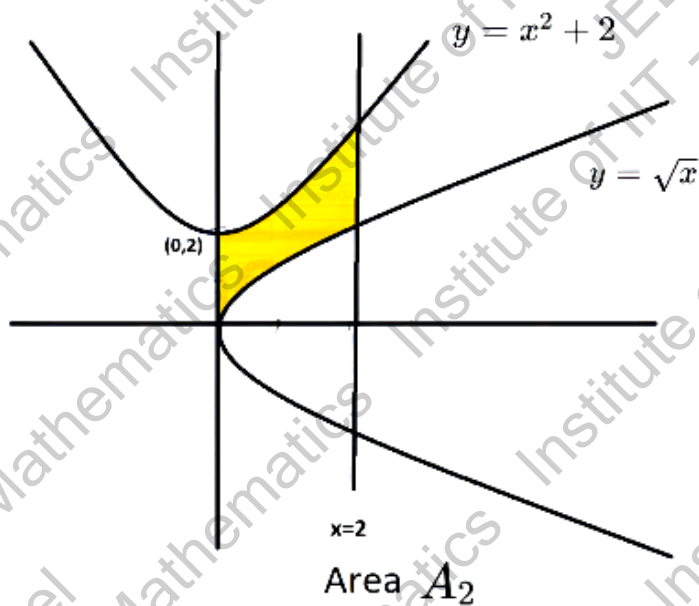
$$\frac{2}{3}(\sqrt{2} + 1)$$

D.

$$\frac{2}{3}(4\sqrt{2} + 1)$$

Answer: A

Solution:



Area A_1 enclosed by curves

$y = x^2 + 2$, $x + y = 8$, y -axis in first quadrant as shown in graph.

Solve $y = x^2 + 2$ and $x + y = 8$ to find point of intersection.

Substituting $y = 8 - x$ in $y = x^2 + 2$

$$8 - x = x^2 + 2$$

$$x^2 + x - 6 = 0$$

$$x^2 + 3x - 2x - 6 = 0$$

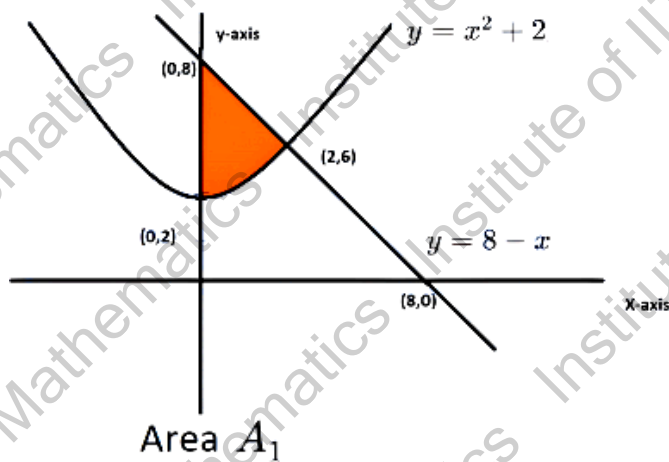
$$x(x + 3) - 2(x + 3) = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3, 2$$

Since area in first quadrant, point of intersection at $y = 8 - 2 = 6 \Rightarrow (2, 6)$.

$$\begin{aligned} A_1 &= \int_0^2 [(8 - x) - (x^2 + 2)] dx \\ &= \int_0^2 (-x^2 - x + 6) dx \\ &= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x \right]_0^2 \\ &= \left[-\frac{8}{3} - \frac{4}{2} + 12 - \{0 - 0 - 0\} \right] \\ &= \frac{-16 - 12 + 72}{6} = \frac{72 - 28}{6} = \frac{44}{6} \\ A_1 &= \frac{22}{3} \end{aligned}$$



Now, A_2 is area enclosed by curves

$$y = x^2 + 2, y^2 = x, x = 2$$

Solve $y = x^2 + 2$ and $y = \sqrt{x}$

$$y = y^4 + 2$$

$$y^4 - y + 2 = 0$$

$$D = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$$

discriminant (D) is negative. This means curves do not intersect.

So A_2 is bounded by curves y -axis, 2, $y = x^2 + 2$ and $y = \sqrt{x}$ in first quadrant is:

$$\begin{aligned} A_2 &= \int_0^2 (x^2 + 2 - \sqrt{x}) dx \\ &= \left[\frac{x^3}{3} + 2x - \frac{2x^{3/2}}{3} \right]_0^2 \\ &= \left[\frac{8}{3} + 2 \times 2 - \frac{2 \times 2\sqrt{2}}{3} - \{0 + 0 - 0\} \right] \\ &= \left[\frac{8}{3} + 4 - \frac{4\sqrt{2}}{3} \right] = \left[\frac{20}{3} - \frac{4\sqrt{2}}{3} \right] \\ A_1 - A_2 &= \frac{22}{3} - \left(\frac{20}{3} - \frac{4\sqrt{2}}{3} \right) \\ &= \frac{22}{3} - \frac{20}{3} + \frac{4\sqrt{2}}{3} = \frac{2}{3} + \frac{4\sqrt{2}}{3} \\ &= \frac{2}{3}(2\sqrt{2} + 1) \end{aligned}$$

Question 10

Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by $x = 0$, $x = 1$, $y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then $(f(0) + f(1))$ is equal to

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Options:

A.

12

B.

14

C.

9

D.

Answer: D

Solution:

Given Curves $y^2 = x$, $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$ and $x = 1$

The area of region bounded by these curve is $f(\alpha)$.

To find $f(0) + f(1)$.

for $\alpha = 0$

$$y = |0 \times x - 5| - |1 - 0 \times x| + 0 \times x^2$$

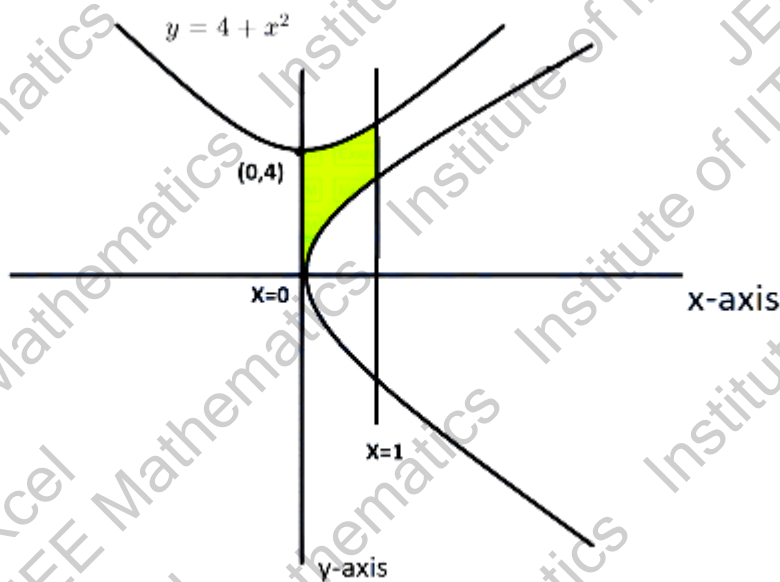
$$y = 5 - 1 = 4$$

so the region bounded by $x = 0$, $x = 1$, $y = \sqrt{x}$

and $y = 4$ is shown in graph.

$$f(0) = \int_0^1 (4 - \sqrt{x}) dx$$

$$f(0) = \left[4x - \frac{2}{3} x^{3/2} \right]_0^1 = 4 - \frac{2}{3} = \frac{10}{3}$$



If, When $\alpha = 1$, the second equation becomes:

$$y = |x - 5| - |1 - x| + x^2$$

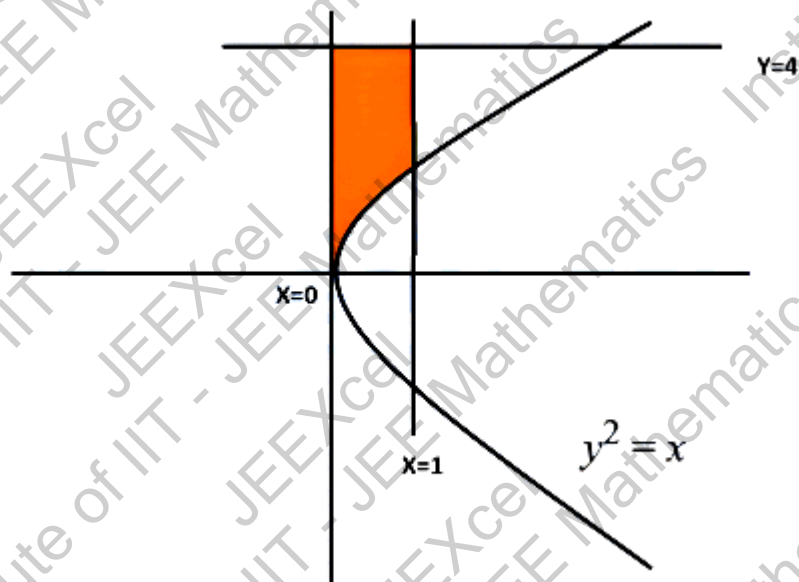
For the interval $0 \leq x \leq 1$:

$$|x - 5| = -(x - 5) = 5 - x$$

$$|1 - x| = 1 - x$$

$$y = (5 - x) - (1 - x) + x^2 = 4 + x^2$$

The region is bounded by $x = 0$, $x = 1$, $y = \sqrt{x}$, and $y = 4 + x^2$ as shown in graph is.



$$f(1) = \int_0^1 (4 + x^2 - \sqrt{x}) dx$$

$$f(1) = \left[4x + \frac{x^3}{3} - \frac{2}{3}x^{3/2} \right]_0^1 = 4 + \frac{1}{3} - \frac{2}{3} = 4 - \frac{1}{3} = \frac{11}{3}$$

$$\text{So, } (f(0) + f(1)) = \frac{10}{3} + \frac{11}{3} = \frac{21}{3} = 7$$

Question11

The area of the region $R = \{(x, y) : xy \leq 8, 1 \leq y \leq x^2, x \geq 0\}$ is

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Options:

A.

$$\frac{2}{3}(20 \log_e(2) + 9)$$

B.

$$\frac{1}{3}(40 \log_e(2) + 27)$$

C.

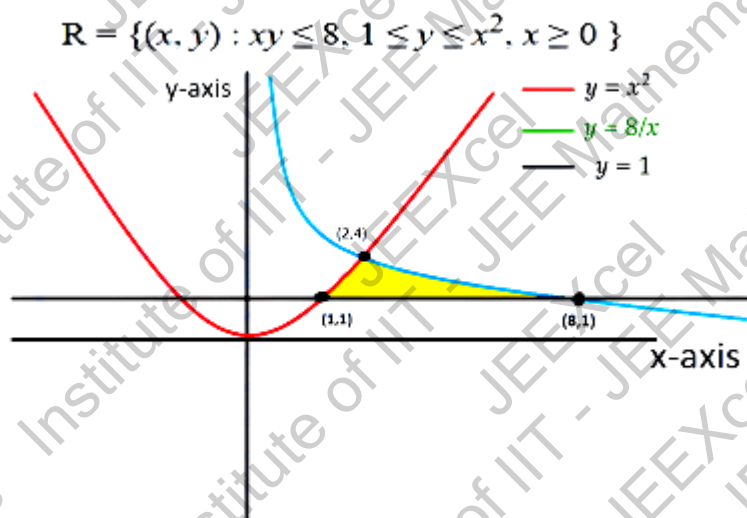
$$\frac{1}{3}(49 \log_e(2) - 15)$$

D.

$$\frac{2}{3}(24 \log_e(2) - 7)$$

Answer: D

Solution:



The region is bounded by three curves in the first quadrant ($x \geq 0$):

- 1. $y = 1$: A horizontal line.
- 2. $y = x^2$: A parabola opening upwards.
- 3. $y = \frac{8}{x}$: A rectangular hyperbola.

Find the Intersection Points

We need to know where these curves meet to set our integration limits:

- Parabola and Line ($y = x^2$ and $y = 1$) : $x^2 = 1 \implies x = 1$ (since $x \geq 0$). Point: $(1, 1)$.
- Parabola and Hyperbola ($y = x^2$ and $y = \frac{8}{x}$) : $x^2 = \frac{8}{x} \implies x^3 = 8 \implies x = 2$. Point: $(2, 4)$.
- Hyperbola and Line ($y = \frac{8}{x}$ and $y = 1$) : $\frac{8}{x} = 1 \implies x = 8$. Point: $(8, 1)$.

Understanding the Inequalities

- $x \geq 0$: The region is located in the first and fourth quadrants (but limited to the first quadrant by $1 \leq y$).
- $1 \leq y \leq x^2$: This gives us a lower bound of the horizontal line $y = 1$ and an upper bound of the parabola $y = x^2$.
- $xy \leq 8$: This can be rewritten as $y \leq \frac{8}{x}$. This represents the area below the rectangular hyperbola.

Setting up the Integrals

The region starts at $x = 1$ and ends at $x = 8$. Because the "upper" boundary changes at $x = 2$, we must split the area into two parts:

- Part 1 (from $x = 1$ to $x = 2$): Bounded above by the parabola $y = x^2$ and below by $y = 1$.
- Part 2 (from $x = 2$ to $x = 8$): Bounded above by the hyperbola $y = \frac{8}{x}$ and below by $y = 1$.

$$\text{Area} = \int_1^2 (x^2 - 1) dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx$$

$$= \left[\frac{x^3}{3} - x\right]_1^2 + [8 \ln x - x]_2^8$$

$$= \left[\frac{8}{3} - 2 - \left\{\frac{1}{3} - 1\right\}\right] + [(8 \ln 8 - 8) - \{8 \ln 2 - 2\}]$$

$$= \frac{7}{3} - 1 + 8 \ln 2^3 - 8 \ln 2 - 6$$

$$= \frac{7}{3} - 7 + 24 \ln 2 - 8 \ln 2 \quad \{\ln a^m = m \ln a\}$$

$$= -\frac{14}{3} + 16 \ln 2$$

$$= \frac{2}{3} [24 \ln 2 - 7]$$

Question 12

Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x$, $\alpha > 0$ and P_1 , then α is equal to :

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Options:

A.

12

B.

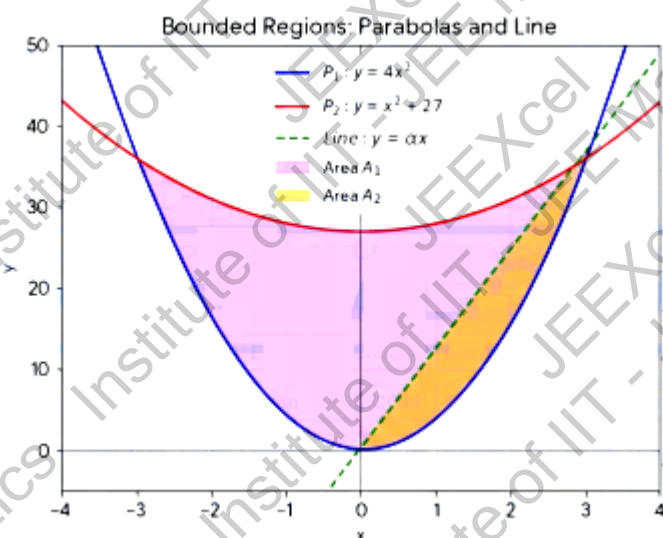
15

C.

8

D.

6

Answer: A**Solution:**

1. Calculate the Area A_1 between P_1 and P_2

The two parabolas are given by:

- $P_1 : y = 4x^2$
- $P_2 : y = x^2 + 27$

Find the intersection points: Setting the equations equal to each other:

$$4x^2 = x^2 + 27$$

$$\Rightarrow 3x^2 = 27$$

$$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$

Find the intersection points : Setting the equations equal to each other :

$$4x^2 = x^2 + 27$$

$$3x^2 = 27, \quad x^2 = 9 \Rightarrow x = \pm 3$$

Calculate the area: On the interval $[-3, 3]$, the parabola P_2 lies above P_1 . The area A_1 is:

$$A_1 = \int_{-3}^3 (x^2 + 27 - 4x^2) dx = \int_{-3}^3 (27 - 3x^2) dx$$

$$A_1 = [27x - x^3]_{-3}^3$$

$$A_1 = (27(3) - 3^3) - (27(-3) - (-3)^3)$$

$$A_1 = (81 - 27) - (-81 + 27) = 54 - (-54) = 108$$

Calculate the Area A_2 between P_1 and the line $y = \alpha x$

Find the intersection points:

$$4x^2 = \alpha x \implies x(4x - \alpha) = 0$$

The points are $x = 0$ and $x = \frac{\alpha}{4}$.

Calculate the area: On the interval $[0, \frac{\alpha}{4}]$, the line $y = \alpha x$ lies above $y = 4x^2$. The area A_2 is:

$$A_2 = \int_0^{\alpha/4} (\alpha x - 4x^2) dx$$

$$A_2 = \left[\frac{\alpha x^2}{2} - \frac{4x^3}{3} \right]_0^{\alpha/4}$$

$$A_2 = \frac{\alpha}{2} \left(\frac{\alpha}{4} \right)^2 - \frac{4}{3} \left(\frac{\alpha}{4} \right)^3$$

$$A_2 = \frac{\alpha^3}{32} - \frac{4\alpha^3}{192} = \frac{\alpha^3}{32} - \frac{\alpha^3}{48} = \frac{3\alpha^3 - 2\alpha^3}{96} = \frac{\alpha^3}{96}$$

The problem states that A_1 is six times A_2 :

$$A_1 = 6 \times A_2$$

$$108 = 6 \times \left(\frac{\alpha^3}{96} \right)$$

$$108 = \frac{\alpha^3}{16}$$

$$\alpha^3 = 108 \times 16 = 1728$$

$$\alpha = \sqrt[3]{1728} = 12$$

Question 13

The area of the region $\{(x, y) : y \leq \pi - |x|, y \leq |x \sin x|, y \geq 0\}$ is:

JEE Main 2026 (Online) 4th April Morning Shift

Options:

A. $1 + \frac{\pi^2}{8}$

B.

$$2 + \frac{\pi^2}{4}$$

C.

$$\frac{\pi^2}{8} - 1$$

D.

$$4 + \frac{\pi^2}{2}$$

Answer: B

Solution:

We need the area of the region

$$\{(x, y) : y \leq \pi - |x|, y \leq |x \sin x|, y \geq 0\}.$$

This means the required area is the area under the curve

$$y = \min(\pi - |x|, |x \sin x|)$$

and above the x -axis.

So the area is

$$A = \int \min(\pi - |x|, |x \sin x|) dx$$

over the values of x where the region exists.

Since $y \geq 0$ and $y \leq \pi - |x|$, we must have

$$\pi - |x| \geq 0 \implies |x| \leq \pi.$$

Hence the region lies in $[-\pi, \pi]$.

Also, both functions $\pi - |x|$ and $|x \sin x|$ are even functions, so the region is symmetric about the y -axis. Therefore,

$$A = 2 \int_0^\pi \min(\pi - x, x \sin x) dx.$$

Now for $0 \leq x \leq \pi$, we have $\sin x \geq 0$, so

$$|x \sin x| = x \sin x.$$

Thus,

$$A = 2 \int_0^\pi \min(\pi - x, x \sin x) dx.$$

Now compare $\pi - x$ and $x \sin x$ on $[0, \pi]$.

At $x = 0$,

$$\pi - x = \pi, \quad x \sin x = 0.$$

$$\text{At } x = \pi,$$

$$\pi - x = 0, \quad x \sin x = 0.$$

Check where they meet:

$$x \sin x = \pi - x.$$

It is easy to see that $x = \frac{\pi}{2}$ satisfies this, because

$$\frac{\pi}{2} \sin \frac{\pi}{2} = \frac{\pi}{2} = \pi - \frac{\pi}{2}.$$

Also, on $[0, \pi/2]$, we have $x \sin x \leq \pi - x$, and on $[\pi/2, \pi]$, we have $\pi - x \leq x \sin x$.

So,

$$A = 2 \left[\int_0^{\pi/2} x \sin x \, dx + \int_{\pi/2}^{\pi} (\pi - x) \, dx \right].$$

Now evaluate each integral.

First,

$$\int x \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x.$$

Therefore,

$$\int_0^{\pi/2} x \sin x \, dx = [-x \cos x + \sin x]_0^{\pi/2} = 1.$$

Next,

$$\int_{\pi/2}^{\pi} (\pi - x) \, dx.$$

Let $u = \pi - x$. Then as x goes from $\pi/2$ to π , u goes from $\pi/2$ to 0. Or directly,

$$\int_{\pi/2}^{\pi} (\pi - x) \, dx = \left[\pi x - \frac{x^2}{2} \right]_{\pi/2}^{\pi}.$$

Compute:

$$\text{At } x = \pi,$$

$$\pi^2 - \frac{\pi^2}{2} = \frac{\pi^2}{2}.$$

$$\text{At } x = \frac{\pi}{2},$$

$$\pi \cdot \frac{\pi}{2} - \frac{1}{2} \left(\frac{\pi}{2} \right)^2 = \frac{\pi^2}{2} - \frac{\pi^2}{8} = \frac{3\pi^2}{8}.$$

So,

$$\int_{\pi/2}^{\pi} (\pi - x) \, dx = \frac{\pi^2}{2} - \frac{3\pi^2}{8} = \frac{\pi^2}{8}.$$

Hence,

$$A = 2 \left(1 + \frac{\pi^2}{8} \right) = 2 + \frac{\pi^2}{4}.$$

Therefore, the correct answer is

$$2 + \frac{\pi^2}{4}$$

So, Option B is correct.

Question 14

The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2 = 1$ is equal to :

JEE Main 2026 (Online) 4th April Evening Shift

Options:

A.

$$\frac{1}{3}$$

B.

$$\frac{2}{3}$$

C.

$$\frac{4}{3}$$

D.

$$\frac{5}{3}$$

Answer: C

Solution:

Given curves are

$$x + 3y^2 = 0 \quad \text{and} \quad x + 4y^2 = 1$$

First, write them in the form $x =$:

$$x = -3y^2$$

and

$$x = 1 - 4y^2$$

To find the bounded area, we first find their points of intersection.

Set the two values of x equal:

$$-3y^2 = 1 - 4y^2$$

$$y^2 = 1$$

$$y = \pm 1$$

So the curves intersect at $y = -1$ and $y = 1$.

Now, for a fixed value of y , the right curve is

$$x = 1 - 4y^2$$

and the left curve is

$$x = -3y^2$$

So the horizontal width of the region is

$$(1 - 4y^2) - (-3y^2) = 1 - y^2$$

Hence, area of the enclosed region is

$$A = \int_{-1}^1 (1 - y^2) dy$$

Now evaluate:

$$A = \left[y - \frac{y^3}{3} \right]_{-1}^1$$

$$A = \left(1 - \frac{1}{3} \right) - \left(-1 + \frac{1}{3} \right)$$

$$A = \frac{2}{3} - \left(-\frac{2}{3} \right)$$

$$A = \frac{4}{3}$$

Therefore, the required area is

$$\boxed{\frac{4}{3}}$$

So, the correct option is:

Option C

Question 15

The area of the region $R = \{(x, y) : xy \leq 27, 1 \leq y \leq x^2\}$ is equal to :

JEE Main 2026 (Online) 5th April Morning Shift

Options:

A.

$$78 \log_e 3 - \frac{52}{3}$$

B.

$$54 \log_e 3 - \frac{52}{3}$$

C.

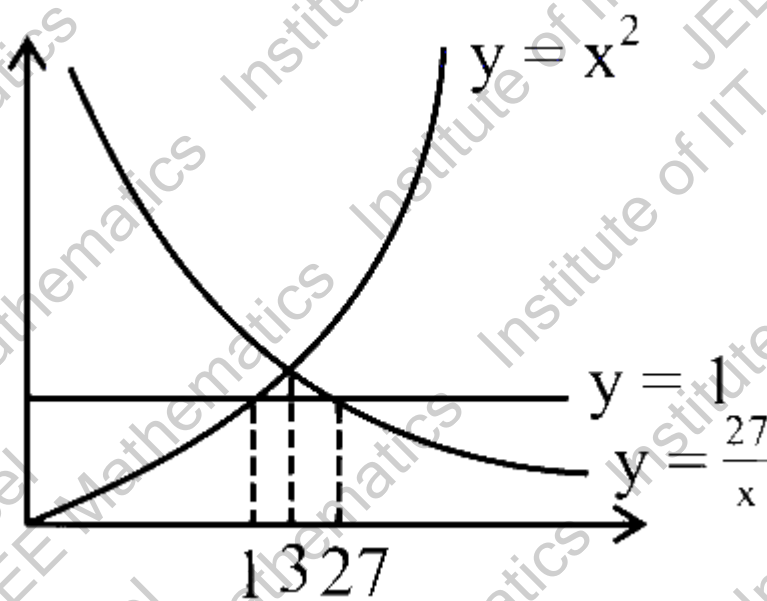
$$54 \log_e 3 - \frac{26}{3}$$

D.

$$54 \log_e 3 + \frac{26}{3}$$

Answer: B

Solution:



$$\begin{aligned}
 \text{Area} &= \int_1^3 (x^2 - 1) dx + \int_3^{27} \left(\frac{27}{x} - 1 \right) dx \\
 &= \left[\frac{x^3}{3} - x \right]_1^3 + [27 \ln x - x]_3^{27} \\
 &= \frac{26}{3} - 2 + 27 \ln 9 - 24 \\
 &= 27 \ln 9 - \frac{26 \times 2}{3} \\
 &= 54 \ln 3 - \frac{52}{3}
 \end{aligned}$$

Question 16

Let e be the base of natural logarithm and let

$f : \{1, 2, 3, 4\} \rightarrow \{1, e, e^2, e^3\}$ and $g : \{1, e, e^2, e^3\} \rightarrow \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$ be two bijective functions such that f is strictly decreasing and g is strictly increasing. If $\phi(x) = [f^{-1} \{g^{-1}(\frac{1}{2})\}]^x$, then the area of the region $R = \{(x, y) : x^2 \leq y \leq \phi(x), 0 \leq x \leq 1\}$ is :

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A. $\frac{3 - \log_e(2)}{3 \log_e(2)}$

B.

$\frac{1}{3 \log_e(2)}$

C.

$3 + \log_e(2)$

D.

$\frac{3 + \log_e(2)}{2 + \log_e(3)}$

Answer: A

Solution:

Since $f : \{1, 2, 3, 4\} \rightarrow \{1, e, e^2, e^3\}$ is bijective and strictly decreasing, we match the largest element of the domain to the smallest element of the codomain.

So,

$$f(1) = e^3, \quad f(2) = e^2, \quad f(3) = e, \quad f(4) = 1$$

Hence,

$$f^{-1}(1) = 4, \quad f^{-1}(e) = 3, \quad f^{-1}(e^2) = 2, \quad f^{-1}(e^3) = 1$$

Now $g : \{1, e, e^2, e^3\} \rightarrow \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$ is bijective and strictly increasing.

Since

and

and

$$\frac{1}{4} < \frac{1}{3} < \frac{1}{2} < 1$$

for g to be strictly increasing, we must assign:

$$g(1) = \frac{1}{4}, \quad g(e) = \frac{1}{3}, \quad g(e^2) = \frac{1}{2}, \quad g(e^3) = 1$$

Therefore,

$$g^{-1}\left(\frac{1}{2}\right) = e^2$$

Now,

$$f^{-1}\left(g^{-1}\left(\frac{1}{2}\right)\right) = f^{-1}(e^2) = 2$$

So,

$$\phi(x) = \left[f^{-1}\left\{g^{-1}\left(\frac{1}{2}\right)\right\}\right]^x = 2^x$$

Thus the region is

$$R = \{(x, y) : x^2 \leq y \leq 2^x, 0 \leq x \leq 1\}$$

Its area is

$$\int_0^1 (2^x - x^2) dx$$

Now,

$$\int 2^x dx = \frac{2^x}{\ln 2}$$

So,

$$\int_0^1 2^x dx = \left[\frac{2^x}{\ln 2}\right]_0^1 = \frac{2-1}{\ln 2} = \frac{1}{\ln 2}$$

Also,

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

Hence area

$$= \frac{1}{\ln 2} - \frac{1}{3}$$

Taking LCM,

$$= \frac{3 - \ln 2}{3 \ln 2}$$

Therefore, the required area is

$$\boxed{\frac{3 - \log_e 2}{3 \log_e 2}}$$

So the correct option is **A**.

Question 17

The area of the region $\{(x, y) : 0 \leq y \leq 6 - x, y^2 \geq 4x - 3, x \geq 0\}$ is :

JEE Main 2026 (Online) 6th April Morning Shift

Options:

A.

8

B.

9

C.

12

D.

15

Answer: B

Solution:

We need to find the area of the region

$$\{(x, y) : 0 \leq y \leq 6 - x, y^2 \geq 4x - 3, x \geq 0\}.$$

Let us understand each condition carefully.

1. From $0 \leq y \leq 6 - x$

This means:

- $y \geq 0$, so the region lies above the x -axis.
- $y \leq 6 - x$, or

$$x + y \leq 6,$$

so the region lies below the line

$$y = 6 - x.$$

2. From $y^2 \geq 4x - 3$

Rewrite it in terms of x :

$$4x - 3 \leq y^2$$

$$4x \leq y^2 + 3$$

$$x \leq \frac{y^2 + 3}{4}.$$

So the region lies to the left of the parabola

$$x = \frac{y^2 + 3}{4}.$$

Also, we are given

$$x \geq 0.$$

So for a fixed y , x lies between 0 and whichever is smaller:

$$x = \frac{y^2 + 3}{4} \quad \text{and} \quad x = 6 - y.$$

So the horizontal width is

$$0 \leq x \leq \min\left(\frac{y^2 + 3}{4}, 6 - y\right).$$

Now we find where these two curves intersect.

3. Point of intersection

Set

$$\frac{y^2 + 3}{4} = 6 - y.$$

Multiply by 4:

$$y^2 + 3 = 24 - 4y$$

$$y^2 + 4y - 21 = 0$$

$$(y+7)(y-3) = 0.$$

So,

$$y = 3 \quad \text{or} \quad y = -7.$$

Since $y \geq 0$, we take

$$y = 3.$$

Now compare the two right boundaries:

- For $0 \leq y \leq 3$, we have

$$\frac{y^2+3}{4} \leq 6-y,$$

so width is

$$\frac{y^2+3}{4}.$$

- For $3 \leq y \leq 6$, we have

$$6-y \leq \frac{y^2+3}{4},$$

so width is

$$6-y.$$

Hence area is

$$A = \int_0^3 \frac{y^2+3}{4} dy + \int_3^6 (6-y) dy.$$

4. Evaluate the integrals

First integral:

$$\begin{aligned} \int_0^3 \frac{y^2+3}{4} dy &= \frac{1}{4} \int_0^3 (y^2+3) dy \\ &= \frac{1}{4} \left[\frac{y^3}{3} + 3y \right]_0^3 = \frac{1}{4} \left(\frac{27}{3} + 9 \right) = \frac{1}{4} (9+9) = \frac{18}{4} = \frac{9}{2}. \end{aligned}$$

Second integral:

$$\begin{aligned} \int_3^6 (6-y) dy &= \left[6y - \frac{y^2}{2} \right]_3^6 \\ &= (36-18) - \left(18 - \frac{9}{2} \right) = 18 - \frac{27}{2} = \frac{9}{2}. \end{aligned}$$

So total area is

$$A = \frac{9}{2} + \frac{9}{2} = 9.$$

Therefore, the correct answer is:

9

So, **Option B** is correct.

Question 18

The area of the region $\{(x, y) : x^2 - 8x \leq y \leq -x\}$ is :

JEE Main 2026 (Online) 6th April Evening Shift

Options:

A.

$$\frac{343}{6}$$

B.

$$\frac{637}{6}$$

C.

$$\frac{437}{6}$$

D.

$$\frac{523}{6}$$

Answer: A

Solution:

For the region

$$\{(x, y) : x^2 - 8x \leq y \leq -x\},$$

the lower curve is

$$y = x^2 - 8x$$

and the upper curve is

$$y = -x.$$

To find the area between the curves, we first find their points of intersection.

Set

$$x^2 - 8x = -x$$

$$x^2 - 7x = 0$$

$$x(x - 7) = 0$$

So, the curves intersect at

$$x = 0 \text{ and } x = 7.$$

Now, on the interval $[0, 7]$, the line $y = -x$ lies above the parabola $y = x^2 - 8x$.

Hence, the required area is

$$\text{Area} = \int_0^7 [(-x) - (x^2 - 8x)] dx.$$

Simplify the integrand:

$$(-x) - (x^2 - 8x) = -x - x^2 + 8x = 7x - x^2.$$

Therefore,

$$\text{Area} = \int_0^7 (7x - x^2) dx.$$

Now integrate:

$$\int (7x - x^2) dx = \frac{7x^2}{2} - \frac{x^3}{3}.$$

Applying the limits from 0 to 7:

$$\text{Area} = \left[\frac{7x^2}{2} - \frac{x^3}{3} \right]_0^7$$

$$= \left(\frac{7 \cdot 49}{2} - \frac{343}{3} \right) - 0$$

$$= \frac{343}{2} - \frac{343}{3}$$

Taking LCM 6:

$$= \frac{1029 - 686}{6} = \frac{343}{6}.$$

Hence, the correct answer is

$$\boxed{\frac{343}{6}}$$

So, the correct option is A.

Question 19

Let $f : (1, \infty) \rightarrow \mathbf{R}$ be a function defined as $f(x) = \frac{x-1}{x+1}$. Let

$f^{i+1}(x) = f(f^i(x))$, $i = 1, 2, \dots, 25$, where $f^1(x) = f(x)$. If

$g(x) + f^{26}(x) = 0$, $x \in (1, \infty)$, then the area of the region bounded by the curves $y = g(x)$, $2y = 2x - 3$, $y = 0$ and $x = 4$ is :

JEE Main 2026 (Online) 8th April Evening Shift

Options:

A.

$$\frac{1}{8} + \log_e 2$$

B.

$$\frac{1}{4} + \log_e 2$$

C.

$$\frac{5}{6} + 3 \log_e 2$$

D.

$$\frac{5}{6} + \log_e 2$$

Answer: A

Solution:

We first focus on the orbit periodicity of $f^n(x)$:

$$f^1(x) = f(x) = \frac{x-1}{x+1}; x \in (1, \infty) \dots\dots (i)$$

$$f^2(x) = f\{f^1(x)\} = f\{f(x)\} = f\left(\frac{x-1}{x+1}\right) = \frac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1} = -\frac{2}{2x} = -\frac{1}{x}; x \in (1, \infty) \dots\dots (ii)$$

$$f^3(x) = f\{f^2(x)\} = f\left(-\frac{1}{x}\right) = \frac{\left(-\frac{1}{x}\right)-1}{\left(-\frac{1}{x}\right)+1} = \frac{1+x}{1-x}; x \in (1, \infty) \dots\dots (iii)$$

$$f^4(x) = f\{f^3(x)\} = f\left(\frac{1+x}{1-x}\right) = \frac{\left(\frac{1+x}{1-x}\right)-1}{\left(\frac{1+x}{1-x}\right)+1} = \frac{2x}{2} = x; x \in (1, \infty) \dots\dots (iv)$$

$$f^5(x) = f\{f^4(x)\} = f(x) = \frac{x-1}{x+1}; x \in (1, \infty) \dots\dots (v)$$

$f(x)$	$f^1(x)$	$f^2(x)$	$f^3(x)$	$f^4(x)$	$f^5(x)$	$f^6(x)$	$f^7(x)$	$f^8(x)$	$f^9(x)$		$f^{11}(x)$		$f^{12}(x)$		$f^{13}(x)$		$f^{14}(x)$		$f^{15}(x)$		$f^{16}(x)$		$f^{17}(x)$		$f^{18}(x)$		$f^{19}(x)$		$f^{20}(x)$	n	Orbit periodicity	Cycle
$\frac{x-1}{x+1}$	$\frac{x-1}{x+1}$	$-\frac{1}{x}$	$\frac{1+x}{1-x}$	x	$\frac{x-1}{x+1}$	$-\frac{1}{x}$	$\frac{1+x}{1-x}$	x	$\frac{x-1}{x+1}$		$\frac{x-1}{x+1}$		$-\frac{1}{x}$	$\frac{1+x}{1-x}$		x		$\frac{x-1}{x+1}$		$-\frac{1}{x}$	$\frac{1+x}{1-x}$		x		$\frac{x-1}{x+1}$		$-\frac{1}{x}$	$\frac{1+x}{1-x}$	x	26	1	6

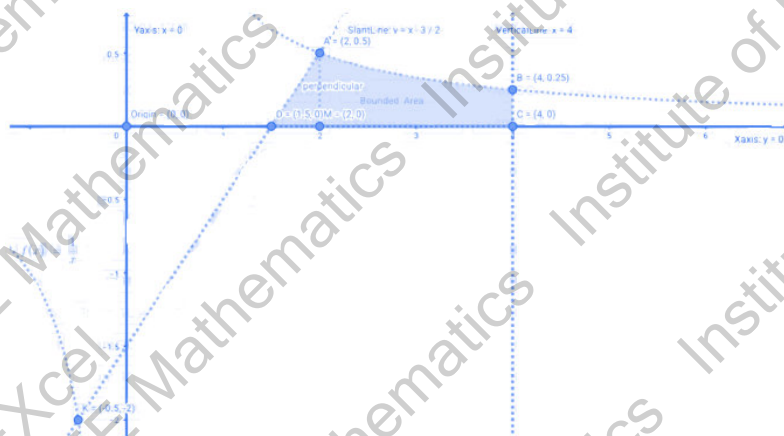
From the nature of table which completes the 25 iterations in 6 cycles with orbit periodicity 4 :

$$f^{25}(x) = f(x) = f^1(x) \Rightarrow f^{26}(x) = f^2(x) = -\frac{1}{x} \dots\dots (vi)$$

Equation (vi) exactly identifies the function $g(x)$ defined under SOW :

$$g(x) + f^{26}(x) = 0 \Rightarrow g(x) = -f^{26}(x) = \frac{1}{x} \dots\dots (vii)$$

Equation (vii) has identified the function $g(x)$ and we can mathematically plot the bounded area between the curves :



$g(x)$	Slant line	Horizontal line	Vertical line
$\frac{1}{x}$	$y = x - \left(\frac{3}{2}\right)$	X-axis	$x = 4$

Pair	$g(x)$	Slant line	Feasible	Intersection Points
1	$\frac{1}{x}$	$y = x - \left(\frac{3}{2}\right)$	Yes	A, K

$$\Rightarrow \frac{1}{x} = x - \left(\frac{3}{2}\right) \Rightarrow 2x^2 - 3x - 2 = 0 \Rightarrow x = \left(2, -\frac{1}{2}\right) \Rightarrow y = \left(\frac{1}{2}, -2\right)$$

Intersections	A	K	Discarded point	Reason
Coordinates	$\left(2, \frac{1}{2}\right)$	$\left(-\frac{1}{2}, -2\right)$	K	Falls outside the scope of bounded area

Pair	$g(x)$	Vertical line	Feasible	Intersection Points
2	$\frac{1}{x}$	$x = 4$	Yes	B

$$\Rightarrow x = 4 \Rightarrow y = 0.25$$

Intersection	B
Coordinate	$(4, 0.25)$

Pair	Horizontal line	Vertical line	Feasible	Intersection point
3	X -axis	$x = 4$	Yes	C

$$\Rightarrow x = 4, y = 0$$

Intersection	C
Coordinate	$(4, 0)$

Pair	Slant line	Horizontal line	Feasible	Intersection point
4	$y = x - \left(\frac{3}{2}\right)$	X-axis	Yes	D

$$\Rightarrow y = 0 \Rightarrow x = \frac{3}{2}$$

Intersection	D
Coordinate	$\left(\frac{3}{2}, 0\right)$

$g(x)$	X-axis	Feasible	Intersection point	Reason
$1/x$	$y = 0$	No	None	X-axis is a horizontal asymptote to $g(x)$
Slant line	Vertical line	Feasible	Intersection point	Reason
$y = x - (3/2)$	$x = 4$	No	$(4, 5/2)$	Intersects much above B and outside the scope of bounded area

Sides	Left boundary	Equation	Top boundary	Equation	Right boundary	Equation	Bottom boundary	Equation
Curves	Slant line	$y = x - (3/2)$	Rectangular Hyperbola	$y = 1/x$	Vertical line	$x = 4$	X-axis	$y = 0$
Vertices	Top left	Coordinates	Top right	Coordinates	Bottom right	Coordinates	Bottom left	Coordinates
Points	A	A $(2, 1/2)$	B	B $(4, 1/4)$	C	C $(4, 0)$	D	D $(3/2, 0)$

We can visualize that both A and D lie on the slant line but A is ahead of D along the X -axis. From A we can drop a $\perp$ on the X -axis at M to form a right angled ΔAMD . From A and D the remaining area upto the vertical $x = 4$ lies under the rectangular hyperbola $g(x) = \frac{1}{x}$:

$$\begin{aligned}
 \text{Area} &= \left(\frac{1}{2}\right) \cdot (DM) \cdot (AM) + \int_2^4 g(x) dx = \left(\frac{1}{2}\right) \cdot (2 - 1 \cdot 5) \cdot \left(\frac{1}{2}\right) + \int_2^4 \left(\frac{1}{x}\right) dx \\
 &= \frac{1}{8} + [\ln |x|]_2^4 \\
 &= \frac{1}{8} + \ln 2
 \end{aligned}$$

Question20

The area of the region, inside the circle $(x - 2\sqrt{3})^2 + y^2 = 12$ and outside the parabola $y^2 = 2\sqrt{3}x$ is :

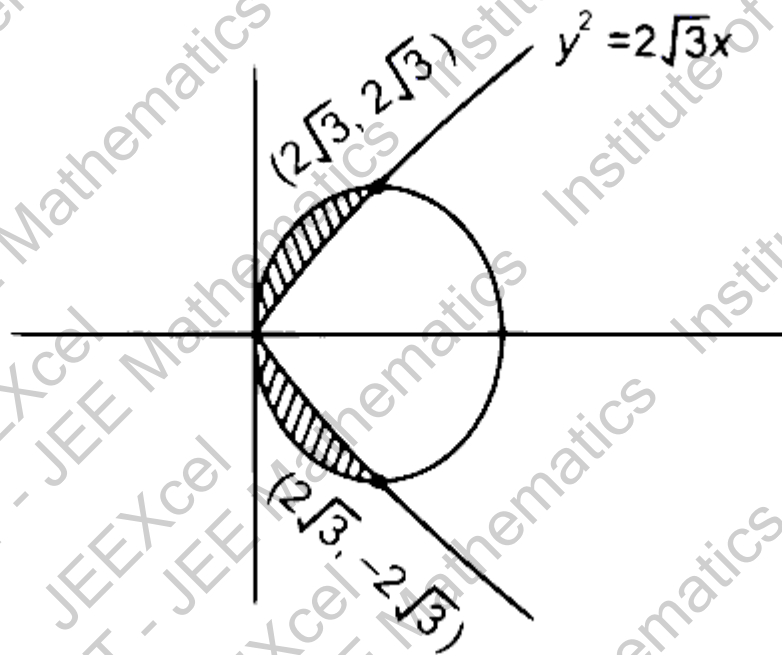
JEE Main 2025 (Online) 22nd January Morning Shift

Options:

- A. $3\pi - 8$
- B. $6\pi - 8$
- C. $3\pi + 8$
- D. $6\pi - 16$

Answer: D

Solution:



$$\begin{aligned}
 \text{Required area} &= 2 \int_0^{2\sqrt{3}} \left(\sqrt{4\sqrt{3}x - x^2} - \sqrt{2\sqrt{3}x} \right) dx \\
 &= 2 \int_0^{2\sqrt{3}} \left(\sqrt{12 - (x - 2\sqrt{3})^2} - \sqrt{2\sqrt{3}x} \right) dx \\
 &= 2 \left[\frac{x-2\sqrt{3}}{2} \sqrt{12 - (x-2\sqrt{3})^2} + \frac{12}{2} \sin^{-1} \left(\frac{x-2\sqrt{3}}{2\sqrt{3}} \right) - \frac{\sqrt{2\sqrt{3}x^{\frac{3}{2}}}}{\frac{3}{2}} \right]_0^{2\sqrt{3}} \\
 &= 2\{3\pi - 8\} \\
 &= 6\pi - 16 \text{ sq. units.}
 \end{aligned}$$

Question 21

The area of the region enclosed by the curves $y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is :

JEE Main 2025 (Online) 22nd January Evening Shift

Options:

A. $\frac{8}{3}$

B. 5

C. 8

D. $\frac{4}{3}$

Answer: A

Solution:

Consider the curves

$$y = x^2 - 4x + 4 = (x - 2)^2$$

and

$$y^2 = 16 - 8x.$$

Notice that the second equation can be rewritten in terms of x :

$$8x = 16 - y^2 \implies x = 2 - \frac{y^2}{8}.$$

Step 1. Find the Intersection Points

To find the points where the curves intersect, substitute

$$y = (x - 2)^2$$

into

$$y^2 = 16 - 8x.$$

Let

$$u = x - 2 \quad \text{so that} \quad y = u^2.$$

Then

$$y^2 = u^4$$

and

$$x = u + 2.$$

Substitute into the second curve:

$$u^4 = 16 - 8(u + 2).$$

Simplify the right side:

$$16 - 8(u + 2) = 16 - 8u - 16 = -8u.$$

Thus, the equation becomes

$$u^4 + 8u = 0 \implies u(u^3 + 8) = 0.$$

In fact, factor by taking out a common factor u :

$$u(u^3 + 8) = 0.$$

Thus, either

$$u = 0 \quad \text{or} \quad u^3 = -8.$$

For $u = 0$:

$$x = u + 2 = 2, \quad y = u^2 = 0.$$

For $u^3 = -8$:

$$u = -2, \quad \text{so} \quad x = -2 + 2 = 0, \quad y = (-2)^2 = 4.$$

The curves intersect at the points $(2, 0)$ and $(0, 4)$.

Step 2. Express the Curves in Terms of y

It is easier to integrate horizontally by expressing x as a function of y .

From the second curve:

$$x = 2 - \frac{y^2}{8}.$$

From the first curve, solving

$$y = (x - 2)^2$$

for x gives

$$x - 2 = \pm\sqrt{y}.$$

Since at the intersection $(0, 4)$ the x -value is less than 2, we take the negative branch:

$$x = 2 - \sqrt{y}.$$

Thus, for a fixed y between 0 and 4, the left boundary is

$$x_{\text{left}} = 2 - \sqrt{y},$$

and the right boundary is

$$x_{\text{right}} = 2 - \frac{y^2}{8}.$$

Step 3. Set Up the Integral for the Area

The horizontal distance between the curves at a given y is

$$\Delta x = x_{\text{right}} - x_{\text{left}} = \left(2 - \frac{y^2}{8}\right) - (2 - \sqrt{y}) = \sqrt{y} - \frac{y^2}{8}.$$

Integrate with respect to y from $y = 0$ to $y = 4$:

$$A = \int_0^4 \left(\sqrt{y} - \frac{y^2}{8} \right) dy.$$

Step 4. Evaluate the Integral

Write the integral as

$$A = \int_0^4 y^{1/2} dy - \frac{1}{8} \int_0^4 y^2 dy.$$

Compute each term:

For the first integral:

$$\int y^{1/2} dy = \frac{2}{3} y^{3/2}.$$

For the second integral:

$$\int y^2 dy = \frac{y^3}{3}.$$

Thus,

$$A = \left[\frac{2}{3} y^{3/2} \right]_0^4 - \frac{1}{8} \left[\frac{y^3}{3} \right]_0^4.$$

Substitute $y = 4$ (note that at $y = 0$ both terms vanish):

Compute $4^{3/2}$:

$$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8.$$

Compute the first term:

$$\frac{2}{3} \times 8 = \frac{16}{3}.$$

Compute the second term:

$$\frac{1}{8} \cdot \frac{64}{3} = \frac{64}{24} = \frac{8}{3}.$$

Therefore, the area is

$$A = \frac{16}{3} - \frac{8}{3} = \frac{8}{3}.$$

Final Answer

The area of the region enclosed by the curves is

$$\frac{8}{3}.$$

Question 22

If the area of the region

$$\{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq a + e^{|x|} - e^{-x}, a > 0\} \text{ is } \frac{e^2 + 8e + 1}{e},$$

then the value of a is :

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Options:

A. 7

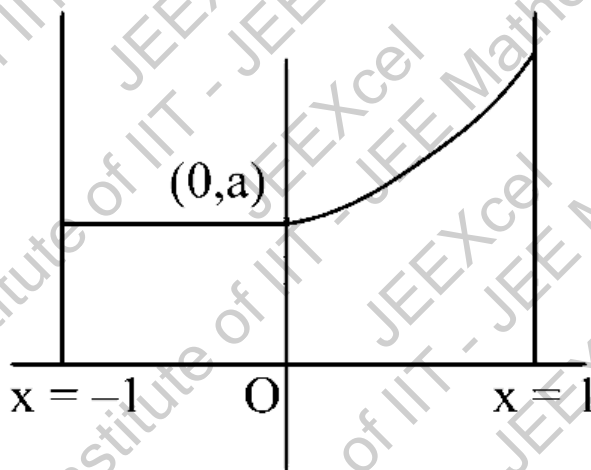
B. 5

C. 6

D. 8

Answer: B

Solution:



required area is $a + \int_0^1 (a + e^x - e^{-x}) dx$

$$a + [a + e^x + e^{-x}]_0^1$$

$$2a + e - 1 + e^{-1} - 1 = e + 8 + \frac{1}{e}$$

$$2a = 10 \Rightarrow a = 5$$

Question 23

The area of the region $\{(x, y) : x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is equal to

JEE Main 2025 (Online) 24th January Morning Shift

Options:

- A. 7
- B. $24/5$
- C. $20/3$
- D. 5

Answer: C

Solution:

$$x^2 + 4x + 2 \leq y \leq |x + 2|$$

The area bounded between

$$y = x^2 + 4x + 2 = (x + 2)^2 - 2$$

and $y = |x + 2|$ is same as area bounded between $y = x^2 - 2$ and $y = |x|$

$$\text{For P.O.I } |x|^2 - 2 = |x|$$

$$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$$

$$\therefore \text{ Required area} = - \int_{-2}^2 (x^2 - 2) dx + \int_{-2}^2 |x| dx$$

$$= -2 \int_0^2 (x^2 - 2) dx + 2 \int_0^2 x \cdot dx$$

$$= -2 \left[\frac{x^3}{3} - 2x \right]_0^2 + 2 \left[\frac{x^2}{2} \right]_0^2$$

$$= -2 \left[\frac{8}{3} - 4 \right] + 2 \left[\frac{4}{2} \right]$$

$$= -2 \times \left(\frac{-4}{3} \right) + 4$$

$$= \frac{20}{3}$$

Question 24

The area of the region enclosed by the curves

$y = e^x$, $y = |e^x - 1|$ and y -axis is :

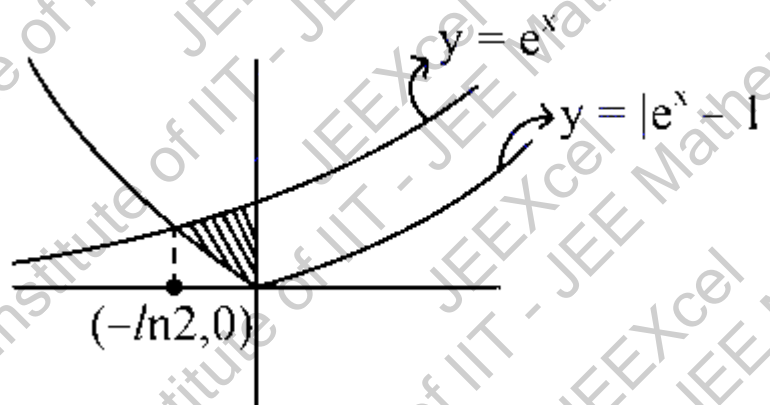
JEE Main 2025 (Online) 24th January Evening Shift

Options:

- A. $1 + \log_e 2$
- B. $\log_e 2$
- C. $1 - \log_e 2$
- D. $2 \log_e 2 - 1$

Answer: C

Solution:



$$\begin{aligned}\text{For Area } & \int_{-\ln 2}^0 [e^x - (1 - e^x)] dx \\ & \int_{-\ln 2}^0 (2e^x - 1) dx = [2e^x - x]_{-\ln 2}^0 \\ & = (2 - (1 + \ln 2)) \\ & = 1 - \ln 2\end{aligned}$$

Question 25

The area (in sq. units) of the region

$$\{(x, y) : 0 \leq y \leq 2|x| + 1, 0 \leq y \leq x^2 + 1, |x| \leq 3\} \text{ is}$$

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. $\frac{32}{3}$

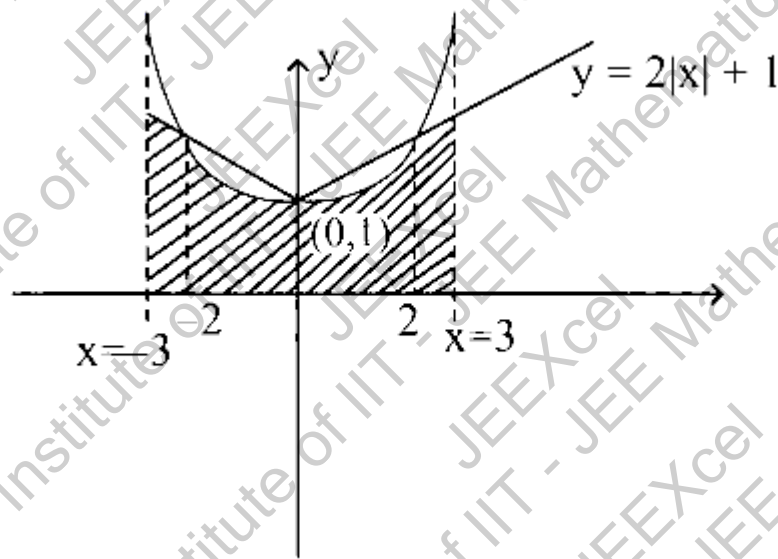
B. $\frac{64}{3}$

C. $\frac{17}{3}$

D. $\frac{80}{3}$

Answer: B

Solution:



$$\begin{aligned}\text{Area} &= 2 \left[\int_0^2 (x^2 + 1) dx + \int_2^3 (2x + 1) dx \right] \\ &\Rightarrow \frac{64}{3} \quad \therefore (2)\end{aligned}$$

Question 26

The area of the region bounded by the curves $x(1 + y^2) = 1$ and $y^2 = 2x$ is:

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Options:

A.

$$\frac{\pi}{4} - \frac{1}{3}$$

B.

$$\frac{\pi}{2} - \frac{1}{3}$$

C.

$$2\left(\frac{\pi}{2} - \frac{1}{3}\right)$$

D.

$$\frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{3}\right)$$

Answer: B

Solution:

$$x(1+y^2) = 1 \quad \dots (1)$$

$$y^2 = 2x \quad \dots (2)$$

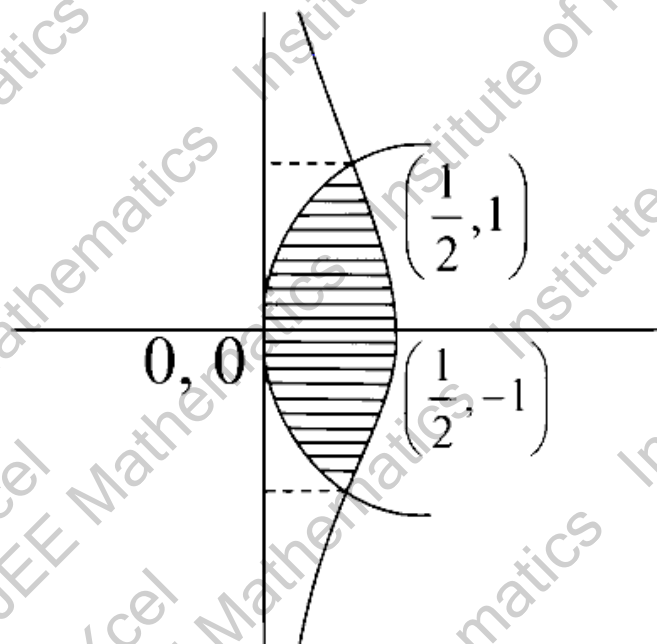
From equation (1) & (2)

$$x(1+2x) = 1 \Rightarrow 2x^2 + x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}, x = -1 \text{ (Reject)}$$

$$\Rightarrow y^2 = 2\left(\frac{1}{2}\right)$$

$$\Rightarrow y = \pm 1$$



$$\begin{aligned}
 \text{Area bounded} &= \int_{-1}^1 \left(\frac{1}{1+y^2} - \frac{y^2}{2} \right) dy \\
 &= \left(\tan^{-1} y - \frac{y^3}{6} \right) \Big|_{-1}^1 \\
 &= \frac{\pi}{2} - \frac{1}{3}
 \end{aligned}$$

Question 27

Let the area of the region

$(x, y) : 2y \leq x^2 + 3, y + |x| \leq 3, y \geq |x - 1|$ be A . Then $6A$ is equal to :

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

14

B.

18

C.

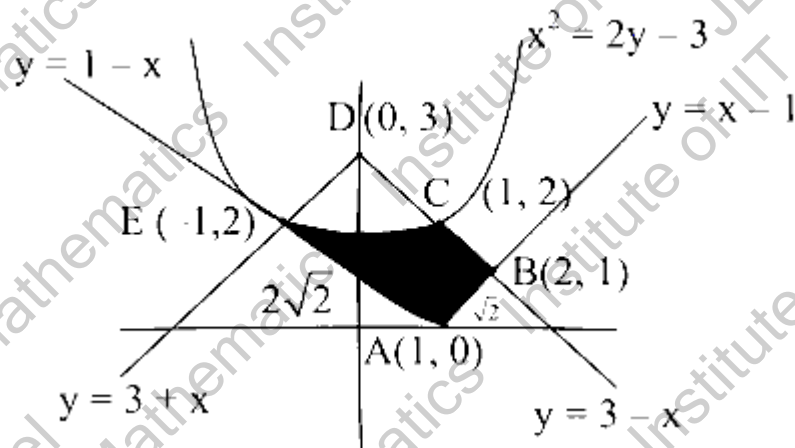
16

D.

12

Answer: A

Solution:



$A \Rightarrow \text{Rectangle ABDE} - \text{Area of region EDC}$

$$A \Rightarrow 4 - 2 \int_0^1 (3 - x) - \left(\frac{x^2 + 3}{2} \right) dx$$

$$A \Rightarrow 4 - 2 \left\{ 3x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{3}{2}x \right\}_0^1$$

$$A \Rightarrow 4 - 2 \left\{ 3 - \frac{1}{2} - \frac{1}{6} - \frac{3}{2} \right\} = \frac{7}{3}$$

So $6A = 14$

Question 28

Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α . If $9\alpha = \beta\pi + \gamma$; β, γ are integers, then the value of $|\beta - \gamma|$ equals:

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

15

B.

18

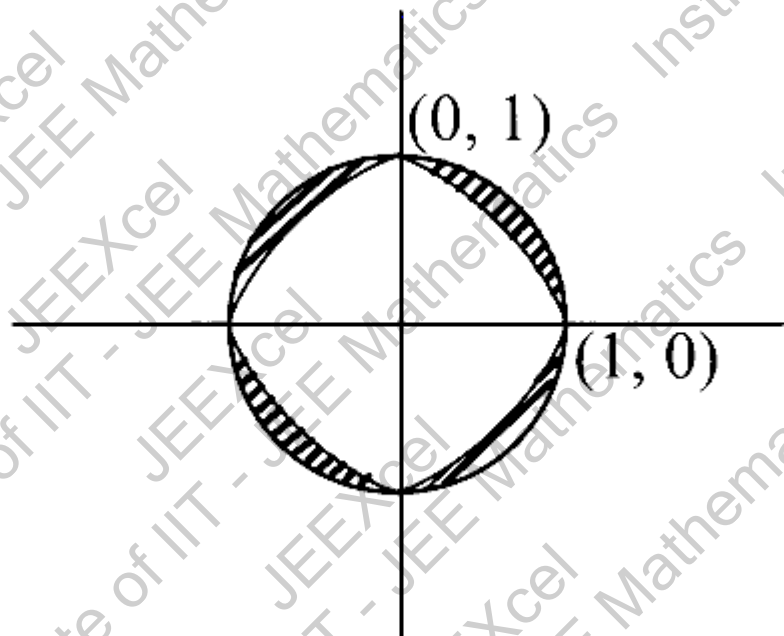
C. 33

D.

Answer: C**Solution:**

$$C_1: |y| = 1 - x^2$$

$$C_2: x^2 + y^2 = 1$$



∴ Required Area

$$= \alpha = 4 \left[\text{Area of circle in 1}^{\text{st}} \text{ quad.} - \int_0^1 (1 - x^2) dx \right]$$

$$= 4 \left[\frac{\pi}{4} - \left[x - \frac{x^3}{3} \right]_0^1 \right]$$

$$\alpha = \pi - \frac{8}{3}$$

$$\therefore 3\alpha = 3\pi - 8$$

$$\therefore 9\alpha = 9\pi - 24$$

$$\therefore \beta = 9, \gamma = -24$$

$$\therefore |\beta - \gamma| = 33$$

Question 29

The area of the region $\{(x, y) : |x - y| \leq y \leq 4\sqrt{x}\}$ is

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. $\frac{512}{3}$

B. $\frac{2048}{3}$

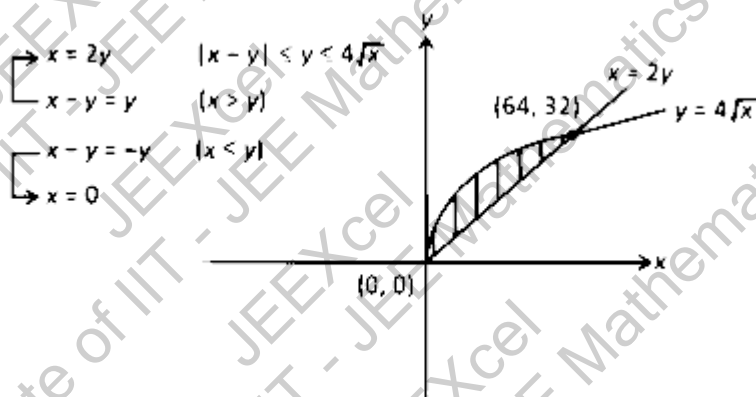
C. 512

D. $\frac{1024}{3}$

Answer: D

Solution:

$$|x - y| \leq y \leq 4\sqrt{x}$$



$$\begin{aligned} \text{Area} &= \int_0^{64} \left(4\sqrt{x} - \frac{x}{2} \right) dx \\ &= \left[\frac{4x^{3/2}}{\frac{3}{2}} - \frac{x^2}{4} \right]_0^{64} \end{aligned}$$

Question 30

Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt \text{ for all } x \in [0, \infty).$$

Then the area of the region bounded by $y = f(x)$ and the coordinate axes is

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. $\sqrt{5}$

B. 2

C. $\sqrt{2}$

D. $\frac{1}{2}$

Answer: D

Solution:

$$\therefore f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$$

$$\text{or, } f(x) = 1 - 2x + e^x \int_0^x e^{-t} f(t) dt$$

on differentiating both sides w.r.t. x we get

$$f'(x) = -2 + e^x \int_0^x e^{-t} f(t) dt + e^x \cdot e^{-x} f(x)$$

$$f'(x) = -2 + f(x) + 2x - 1 + f(x) \{ \text{from eq. (1)} \}$$

$$\therefore f(x) - 2f(x) = 2x - 3$$

$$\text{I.F.} = e^{\int -2dx} = e^{-2x}$$

$$\therefore e^{-2x} \cdot f(x) = \int e^{-2x} (2x - 3) dx$$

$$e^{-2x} \cdot f(x) = (2x - 3) \cdot \frac{e^{-2x}}{-2} - \int 2 \cdot \frac{e^{-2x}}{-2} dx$$

$$e^{-2x} \cdot f(x) = \frac{(2x - 3)e^{-2x}}{-2} + \frac{e^{-2x}}{-2} + c$$

$$f(x) = -x + 1 + c'e^{2x}$$

$$\therefore f(x) = 1 \text{ from eq. (1)}$$

$$\therefore 1 = 0 + 1 + c' \Rightarrow c' = 0$$

$$\therefore f(x) = -x + 1$$

$$\Rightarrow \text{Area} = \frac{1}{2}$$

Question 31

If the area of the region bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α , then 6α equals

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 210

B. 250

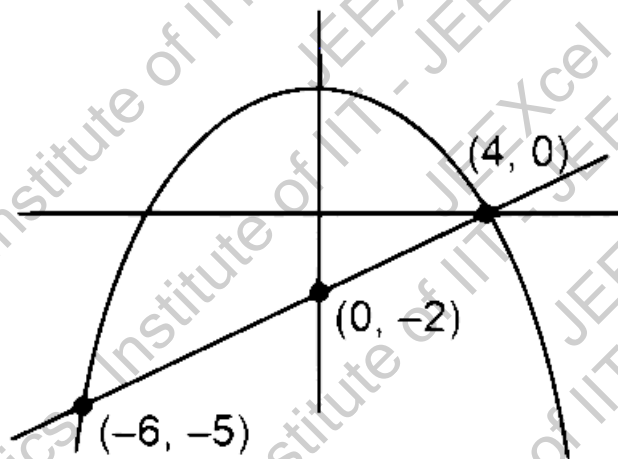
C. 240

D. 220

Answer: B

Solution:

$$y = 4 - \frac{x^2}{4} \text{ and } y = \frac{x-4}{2}$$



$$\begin{aligned} \text{Area} &= \int_{-6}^4 \left(4 - \frac{x^2}{4} - \frac{x}{2} + 2 \right) dx \\ &= \left[6x - \frac{x^3}{12} - \frac{x^2}{4} \right]_{-6}^4 \\ &= 6(4+6) - \left(\frac{64}{12} + \frac{216}{12} \right) - \left(\frac{16}{4} - \frac{36}{4} \right) \\ &= 60 - \frac{70}{3} + 5 \\ \alpha &= \frac{125}{3} \\ 6\alpha &= 6 \times \frac{125}{3} = 250 \end{aligned}$$

Question32

If the area of the region

$\{(x, y) : 1 + x^2 \leq y \leq \min\{x + 7, 11 - 3x\}\}$ is A , then $3A$ is equal to :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

50

B.

46

C.

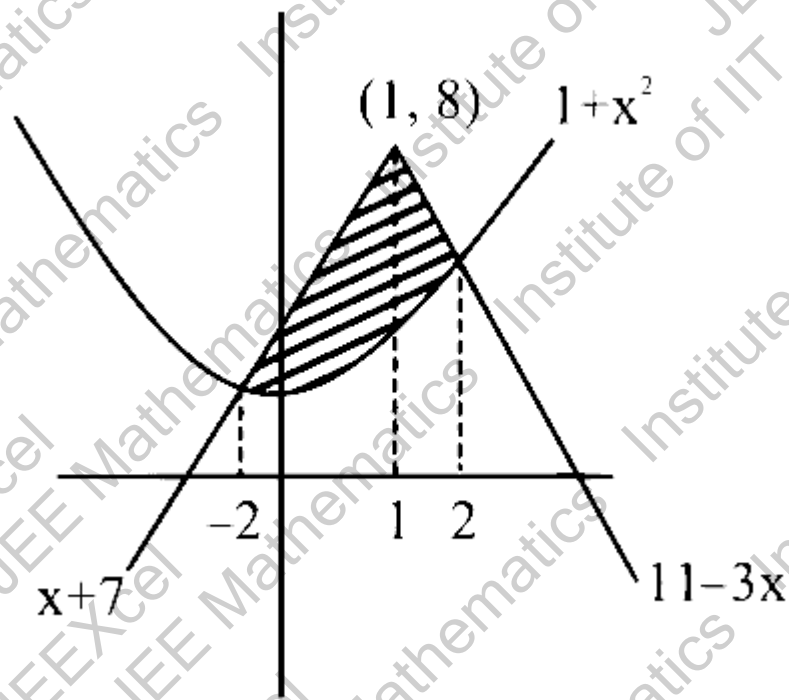
49

D.

47

Answer: A

Solution:



$$\begin{aligned}
 A &= \int_{-2}^1 (x + 7 - x^2 - 1) dx + \int_1^2 (11 + 3x - x^2 - 1) dx \\
 &= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^1 + \left[10x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_1^2 \\
 &= \frac{50}{3} \Rightarrow 3A = 50 \quad \text{Option (1)}
 \end{aligned}$$

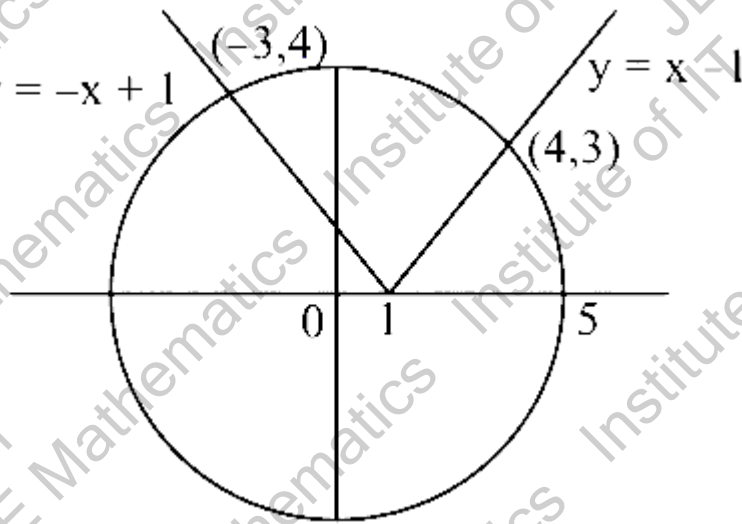
Question 33

If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c)$, $b, c \in \mathbb{N}$, then $b + c$ is equal to _____

JEE Main 2025 (Online) 23rd January Morning Shift

Answer: 77

Solution:



$$x^2 + y^2 = 5$$

$$x^2 + (x - 1)^2 = 25 \Rightarrow x = 4$$

$$x^2 + (-x + 1)^2 = 5 \Rightarrow x = -3$$

$$A = 25\pi - \int_{-3}^4 \sqrt{25 - x^2} dx + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 3 \times 3$$

$$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1} \frac{x}{5} \right]_3^4$$

$$A = 25\pi + \frac{25}{2} - \left[6 + \frac{25}{2} \sin^{-1} \frac{4}{5} + 6 + \frac{25}{2} \sin^{-1} \frac{3}{5} \right]$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2} \cdot \frac{\pi}{2}$$

$$A = \frac{75\pi}{4} + \frac{1}{2}$$

$$A = \frac{1}{4}(75\pi + 2)$$

$$b = 75, c = 2$$

$$b + c = 75 + 2 = 77$$

Question34

If the area of the region

$$\{(x, y) : 4 - x^2 \leq y \leq x^2, y \leq 4, x \geq 0\} \text{ is}$$

$$\left(\frac{80\sqrt{2}}{\alpha} - \beta \right), \alpha, \beta \in \mathbf{N}, \text{ then } \alpha + \beta \text{ is equal to } \underline{\hspace{2cm}}.$$

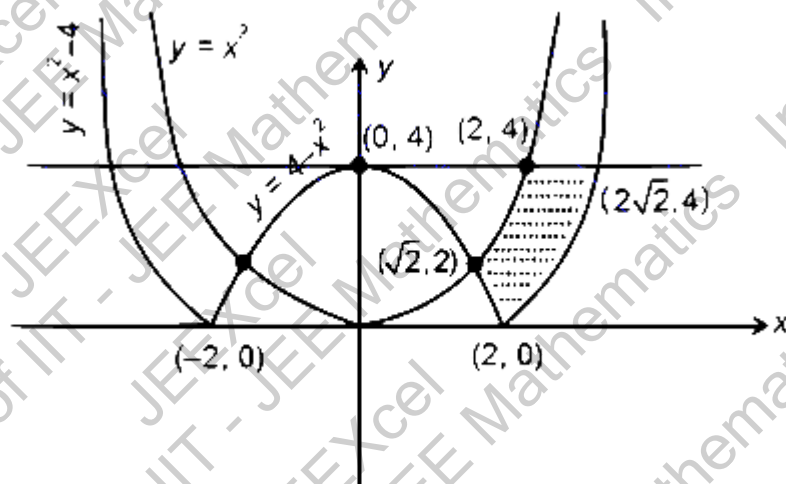
JEE Main 2025 (Online) 2nd April Morning Shift

Answer: 22

Solution:

Area =

$$\int_{\sqrt{2}}^2 (x^2 - (4 - x^2)) dx + (2\sqrt{2} - 2) \times 4 - \int_2^{2\sqrt{2}} (x^2 - 4) dx$$



$$\begin{aligned} &= \left[\frac{2x^3}{3} - 4x \right]_{\sqrt{2}}^2 + 8\sqrt{2} - 8 - \left[\frac{x^3}{3} - 4x \right]_2^{2\sqrt{2}} \\ &= \frac{40\sqrt{2}}{3} - 16 \\ &\Rightarrow \alpha = 6, \beta = 16 \Rightarrow \alpha + \beta = 22 \end{aligned}$$

Question 35

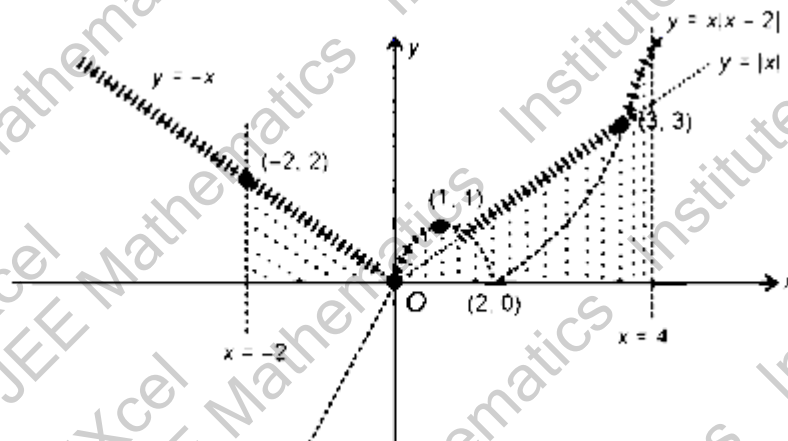
The area of the region bounded by the curve

$y = \max\{|x|, x|x - 2|\}$, the x -axis and the lines $x = -2$ and $x = 4$ is equal to _____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 12

Solution:



Area =

$$\begin{aligned} & \frac{1}{2} \times 2 \times 2 + \int_0^1 (2x - x^2) dx + \int_1^3 x dx + \int_3^4 (x^2 - 2x) dx \\ &= 2 + \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^1 + \left[\frac{x^2}{2} \right]_1^3 + \left[\frac{x^3}{3} - \frac{2x^2}{2} \right]_3^4 \\ &= 12 \text{ square units} \end{aligned}$$

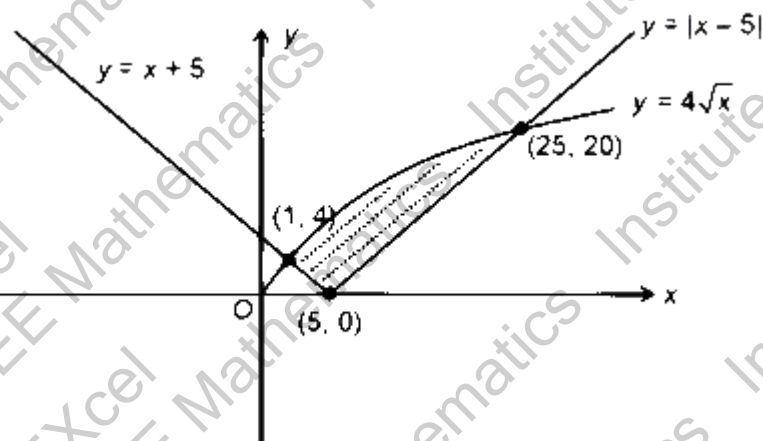
Question 36

If the area of the region $\{(x, y) : |x - 5| \leq y \leq 4\sqrt{x}\}$ is A , then $3A$ is equal to _____.

JEE Main 2025 (Online) 4th April Morning Shift

Answer: 368

Solution:



$$\begin{aligned}
 \text{Area} &= \int_1^{25} 4\sqrt{x} dx - \frac{1}{2} \times (5-1) \cdot 4 - \frac{1}{2} \times (25-5) \times 20 \\
 &= 4 \left[\frac{2}{3} x^{\frac{3}{2}} \right]_1^{25} - 8 - 200 = \frac{368}{3} \\
 \Rightarrow 3A &= 368
 \end{aligned}$$

Question 37

Let the area of the bounded region

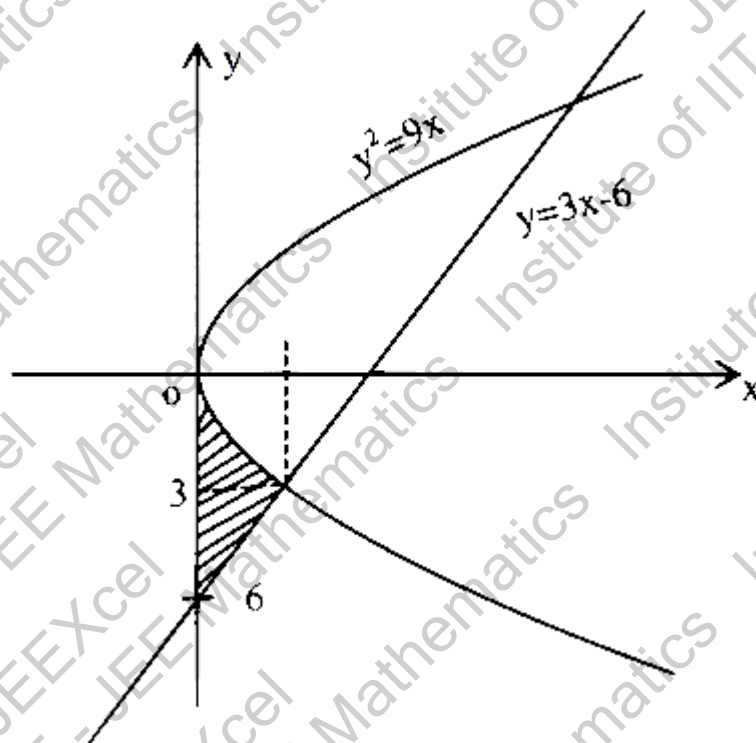
$\{(x, y) : 0 \leq 9x \leq y^2, y \geq 3x - 6\}$ be A . Then $6A$ is equal to _____.

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Answer: 15

Solution:

$$0 \leq 9x \leq y^2 \text{ \& } y \geq 3x - 6$$



$$A = \text{Required Area} = \left[\int_0^1 (-3\sqrt{x}) dx - \int_0^1 (3x - 6) dx \right]$$

$$A = -3 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) \Big|_0^1 - \left(\frac{3x^2}{2} - 6x \right) \Big|_0^1$$

$$A = -2[1 - 0] \left[\frac{3}{2} - 6 \right]$$

$$A = -2 - \frac{3}{2} + 6 = \frac{5}{2} \text{ Sq. unit}$$

$$\therefore 6A = 6 \times \frac{5}{2} = 15$$

Question 38

Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y -axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, x -axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,
[26 Feb 2021 Shift 2]

Options:

A. $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

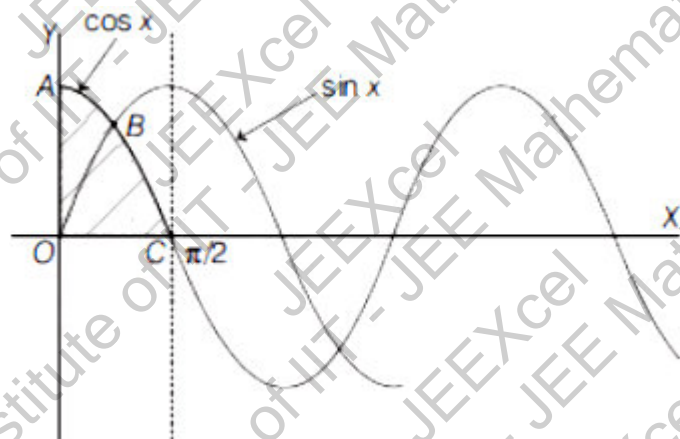
B. $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

C. $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

D. $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

Answer: A

Solution:



A_1 is the area of region OAB.

A_2 is the area of region OBC.

Coordinate of B is $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

$$\begin{aligned} \text{Now, } A_1 &= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx \\ &= [\sin x + \cos x]_0^{\pi/4} \\ &= \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) - (0 + 1) = \sqrt{2} - 1 \end{aligned}$$

$$\begin{aligned} A_2 &= \int_0^{\frac{\pi}{4}} \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx \\ &= [-\cos x]_0^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2} \\ &= \left(-\cos \frac{\pi}{4} + \cos 0\right) + \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{4}\right) \\ &= \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = \sqrt{2} - 1 \end{aligned}$$

$$\begin{aligned} A_2 &= \int_0^{\frac{\pi}{4}} \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx \\ &= [-\cos x]_0^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2} \\ &= \left(-\cos \frac{\pi}{4} + \cos 0\right) + \left(\sin \frac{\pi}{2} - \sin \frac{\pi}{4}\right) \\ &= \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = \sqrt{2} - 1 \end{aligned}$$

$$\text{Now, } A_1 : A_2 = (\sqrt{2} - 1) : (\sqrt{2} - 1)$$

$$A_1 : A_2 = 1 : 1$$

$$\text{and } A_1 + A_2 = (\sqrt{2} - 1) + (\sqrt{2} - 1) = 2(\sqrt{2} - 1)$$

$$A_1 + A_2 = 2 - 1 = 1$$

Therefore,

and $A_1 : A_2 = 1 : \sqrt{2}$,
 $A_1 + A_2 = 1$

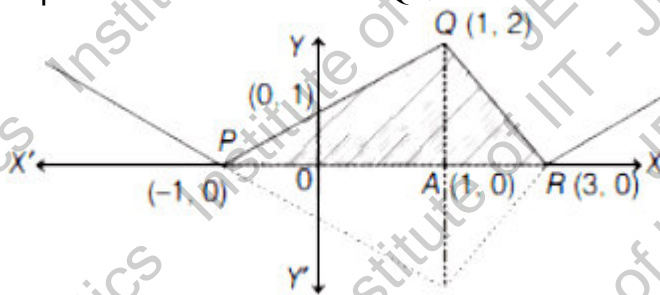
Question39

The area bounded by the lines $y = ||x - 1| - 2|$ is
 [26 Feb 2021 Shift 1]

Answer: 4

Solution:

Given, $y = ||x - 1| - 2|$
 Required area is area of $\triangle PQR$.



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times (\text{Base}) \times (\text{Height}) \\ &= \frac{1}{2} \times (PR) \times (AQ) \\ &= \frac{1}{2} \times 4 \times 2 = 4 \end{aligned}$$

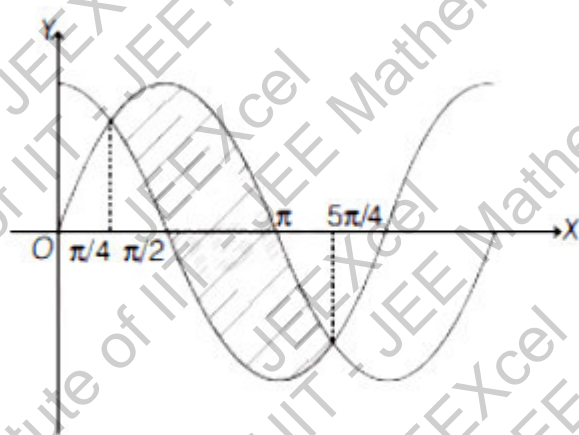
Since, only one curve is given, here assume the area bounded by X-axis. Then, the area will be 4 sq unit.

Question40

The graph of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to
 [25 Feb 2021 Shift 1]

Answer: 64

Solution:



Required area of shaded region

$$\begin{aligned} A &= \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx \\ &= [-\cos x - \sin x]_{\pi/4}^{5\pi/4} \\ &= -\left[\cos \frac{5\pi}{4} + \sin \frac{5\pi}{4}\right] - \left[\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right] \\ &= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] \\ \therefore A &= \frac{4}{\sqrt{2}} = 2\sqrt{2} \\ \Rightarrow A^4 &= (2\sqrt{2})^4 = 64 \end{aligned}$$

Question 41

The area of the region $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is [24 Feb 2021 Shift 2]

Options:

- A. $11\sqrt{3}$ square units
- B. $12\sqrt{3}$ square units
- C. $9\sqrt{3}$ square units

D. $6\sqrt{3}$ square units

Answer: B

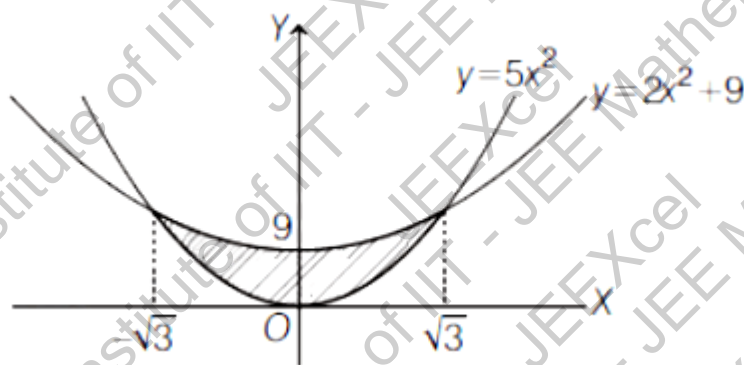
Solution:

Given, $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$

Here, we have two curves $y = 5x^2$ and $y = 2x^2 + 9$, point of intersection of both curves is find by solving both equations i.e.

$$5x^2 = 2x^2 + 9$$

$$\Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$



$$\begin{aligned}\therefore \text{Area} &= \int_{-\sqrt{3}}^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx \\ &= 2 \int_0^{\sqrt{3}} (9 - 3x^2) dx \\ &= 2[9x - x^3]_0^{\sqrt{3}} \\ &= 2[9\sqrt{3} - 3\sqrt{3}] \\ &= 12\sqrt{3} \text{ sq units}\end{aligned}$$

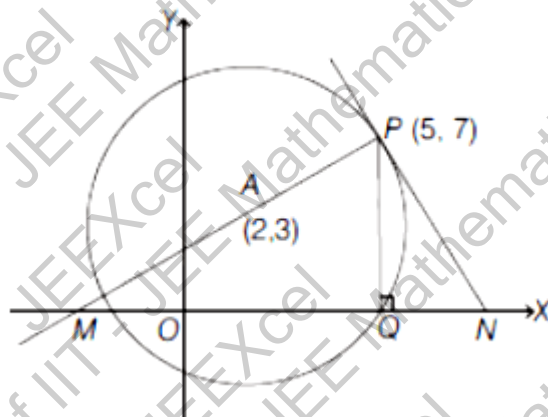
Question42

If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to
[24 Feb 2021 Shift 2]

Answer: 1225

Solution:

Given, circle $(x - 2)^2 + (y - 3)^2 = 5^2$
 $c = (2, 3)$
 $r = 5$



Equation of normal at P (i.e. PA line)

$$\Rightarrow (y - 7) = \left(\frac{7-3}{5-2} \right) (x - 5)$$

$$\Rightarrow 3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0$$

Therefore, $M = \left(-\frac{1}{4}, 0 \right)$ [Put $y = 0$ in above equation]

Now, equation of tangent at P.

$$y - 7 = \frac{-3}{4}(x - 5) \left[\because \text{slope of PN} = \frac{-1}{\text{Slope of PA}} \right]$$

$$\Rightarrow 4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43$$

Therefore, $N = \left(\frac{43}{3}, 0 \right)$ [Put $y = 0$ in above equation]

$$\therefore \text{Area (A)} = \frac{1}{2} \times MN \times PQ$$

$$= \frac{1}{2} \times \left(\frac{43}{3} + \frac{1}{4} \right) \times 7$$

$$= \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\therefore 24A = 24 \times \frac{1}{2} \times \frac{175}{12} \times 7 = 1225$$

But this question is wrong as in question. It is mentioned that the triangle is formed with the positive X-axis which contradicts the solution.

Question 43

The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

24 Feb 2021 Shift 1

Options:

A. $24\pi + 3\sqrt{3}$

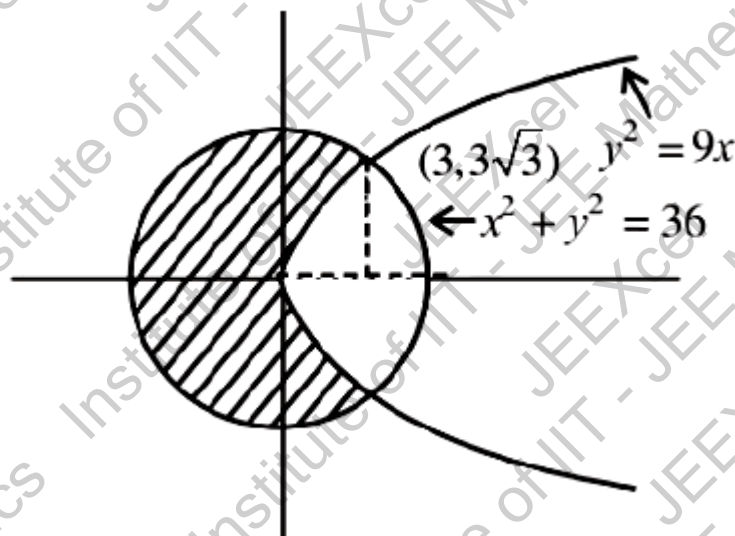
B. $12\pi - 3\sqrt{3}$

C. $24\pi - 3\sqrt{3}$

D. $12\pi + 3\sqrt{3}$

Answer: C

Solution:



Required area

$$\begin{aligned}
 &= \pi \times (6)^2 - 2 \int_0^3 \sqrt{9x} \, dx - 2 \int_3^6 \sqrt{36 - x^2} \, dx \\
 &= 36\pi - 12\sqrt{3} - 2 \left(\frac{x}{2} \sqrt{36 - x^2} + 18 \sin^{-1} \frac{x}{6} \right) \Big|_3^6 \\
 &= 36\pi - 12\sqrt{3} - 2 \left(9\pi - 3\pi - \frac{9\sqrt{3}}{2} \right) \\
 &= 24\pi - 3\sqrt{3}
 \end{aligned}$$

Question 44

Let $g(x) = \int_0^x f(t) \, dt$, where f is continuous function in $[0, 3]$ such that $\frac{1}{3} \leq f(t) \leq 1$ for all $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for all $t \in (1, 3]$.

The largest possible interval in which $g(3)$ lies is
[18 Mar 2021 Shift 2]

Options:

A. $\left[-1, -\frac{1}{2}\right]$

B. $\left[-\frac{3}{2}, -1\right]$

C. $\left[\frac{1}{3}, 2\right]$

D. $[1, 3]$

Answer: C

Solution:

Given, $g(x) = \int_0^x f(t) dt$

$\therefore g(3) = \int_0^3 f(t) dt = \int_0^1 f(t) dt + \int_1^3 f(t) dt$

$\Rightarrow \int_0^1 \frac{1}{3} dt + \int_1^3 0 \cdot dt \leq g(3) \leq \int_0^1 1 dt + \int_1^3 \frac{1}{2} dt$

$\Rightarrow \frac{1}{3} \leq g(3) \leq 1 + 1$

$\Rightarrow \frac{1}{3} \leq g(3) \leq 2$

Question 45

The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\frac{\pi}{8}$

B. $\frac{3\pi}{8}$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{16}$

Answer: C

Solution:

Given, equation of curve is $4y^2 = x^2(4-x)(x-2)$

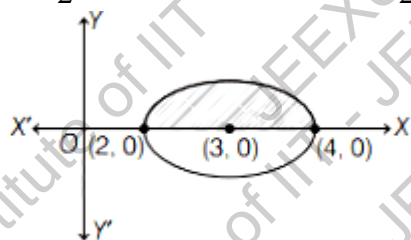
$$\Rightarrow \sqrt{4y^2} = \sqrt{x^2(4-x)(x-2)}$$

$$\Rightarrow |y| = \frac{|x|}{2} \sqrt{(4-x)(x-2)}$$

$$y = \begin{cases} \frac{x}{2} \sqrt{(4-x)(x-2)} & x > 0 \\ -\frac{x}{2} \sqrt{(4-x)(x-2)} & x < 0. \end{cases}$$

Let

$$y_1 = \frac{x}{2} \sqrt{(4-x)(x-2)} \Rightarrow y_2 = -\frac{x}{2} \sqrt{(4-x)(x-2)}$$



and domain $x \in [2, 4]$

$$[\because (4-x)(x-2) \geq 0 \Rightarrow (x-2)(x-4) \leq 0 \Rightarrow 2 \leq x \leq 4]$$

$$\therefore \text{Required area} = \int_2^4 (y_1 - y_2) dx$$

$$= \int_2^4 x \sqrt{(4-x)(x-2)} dx \dots (i)$$

$$\text{Using property, } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

From Eq. (i),

$$\text{Area} = \int_2^4 (6-x) \sqrt{(4-x)(x-2)} dx \dots (ii)$$

$$\text{From Eqs. (i) and (ii), } 2A = 6 \int_2^4 \sqrt{(4-x)(x-2)} dx$$

$$\Rightarrow A = 3 \int_2^4 \sqrt{1 - (x-3)^2} dx$$

$$A = 3 \left[\frac{x-3}{2} \sqrt{1 - (x-3)^2} + \frac{1}{2} \sin^{-1}(x-3) \right]_2^4$$

$$A = \frac{3}{2} \left[\frac{0+\pi}{2} - \frac{0+\pi}{2} \right] \Rightarrow A = \frac{3\pi}{2}$$

$$\Rightarrow A = 3 \cdot \frac{\pi}{2} \cdot (1)^2 = \frac{3\pi}{2}$$

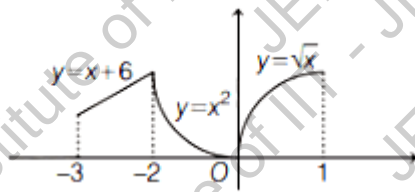
Question 46

Let $f : [-3, 1] \rightarrow \mathbb{R}$ be given as $f(x) = \begin{cases} \min\{(x+6), x^2\} & -3 \leq x \leq 0 \\ \max\{\sqrt{x}, x^2\} & 0 \leq x \leq 1 \end{cases}$

If the area bounded by $y = f(x)$ and x -axis is A , then the value of $6A$ is equal to
[17 Mar 2021 Shift 2]

Answer: 41

Solution:



$A =$ Area bounded by $y = f(x)$ and X -axis.

$$= \int_{-3}^{-2} (x+6) dx + \int_{-2}^0 x^2 dx + \int_0^1 \sqrt{x} dx$$

$$= \left[\frac{x^2}{2} \right]_{-3}^{-2} + 6[x]_{-3}^{-2} + \left[\frac{x^3}{3} \right]_{-2}^0 + \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^1 = \frac{41}{6}$$

$$\therefore 6A = 6 \times \frac{41}{6}$$

$$\Rightarrow 6A = 41$$

Question 47

If the area of the bounded region

$$R = \{(x, y) : \max\{0, \log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2\}$$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :

[27 Jul 2021 Shift 1]

Options:

A. 8

B. 2

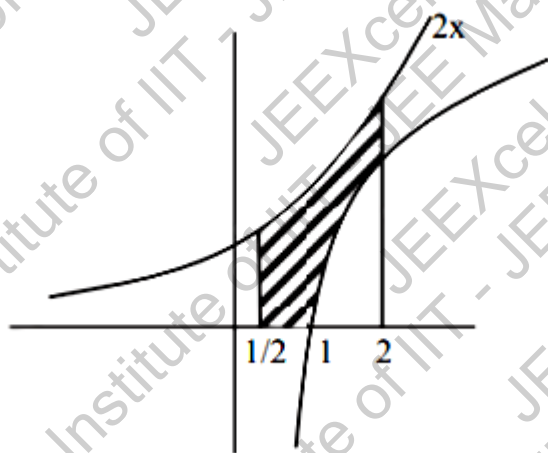
C. 4

D. 1

Answer: B

Solution:

$$R = \left\{ (x, y) : \max\{0, \log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2 \right\}$$



$$\int_{1/2}^2 2^x dx - \int_1^2 \ln x dx$$

$$\Rightarrow \left[\frac{2^x}{\ln 2} \right]_{1/2}^2 - [x \ln x - x]_1^2$$

$$\Rightarrow \frac{(2^2) - 2^{1/2}}{\log_e 2} - (2 \ln 2 - 1)$$

$$\Rightarrow \frac{(2^2 - \sqrt{2})}{\log_e 2} - 2 \ln 2 + 1$$

$$\therefore \alpha = 2^2 - \sqrt{2}, \beta = -2, \gamma = 1$$

$$\Rightarrow (\alpha + \beta + 2\gamma)^2$$

$$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2$$

$$\Rightarrow (\sqrt{2})^2 = 2$$

Question 48

The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :-
[27 Jul 2021 Shift 2]

Options:

A. $\frac{16}{3}$

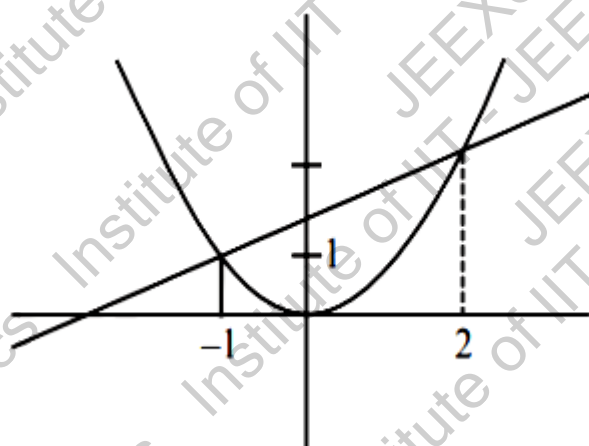
B. $\frac{2}{3}$

C. $\frac{9}{2}$

D. $\frac{4}{3}$

Answer: C

Solution:



$$y - x = 2, x^2 = y$$

$$\text{Now, } x^2 = 2 + x$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x + 1)(x - 2) = 0$$

$$\text{Area} = \int_{-1}^2 (2 + x - x^2)$$

$$= \left| 2x + \frac{x^2}{2} - \frac{x^3}{3} \right|_{-1}^2$$

$$= \left(4 + 2 - \frac{8}{3} \right) - \left(-2 + \frac{1}{2} + \frac{1}{3} \right)$$

$$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$$

Question49

The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

[25 Jul 2021 Shift 1]

Options:

A. $\frac{8}{3}$

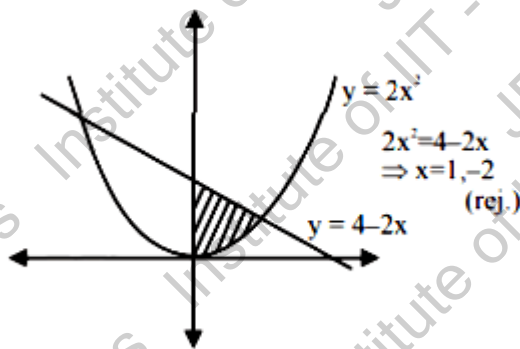
B. $\frac{17}{3}$

C. $\frac{13}{3}$

D. $\frac{7}{3}$

Answer: D

Solution:



$$\text{Required area} = \int_0^1 (4 - 2x - 2x^2) dx = 4x - x^2 - \frac{2x^3}{3} \Big|_0^1 = 4 - 1 - \frac{2}{3} = \frac{7}{3}$$

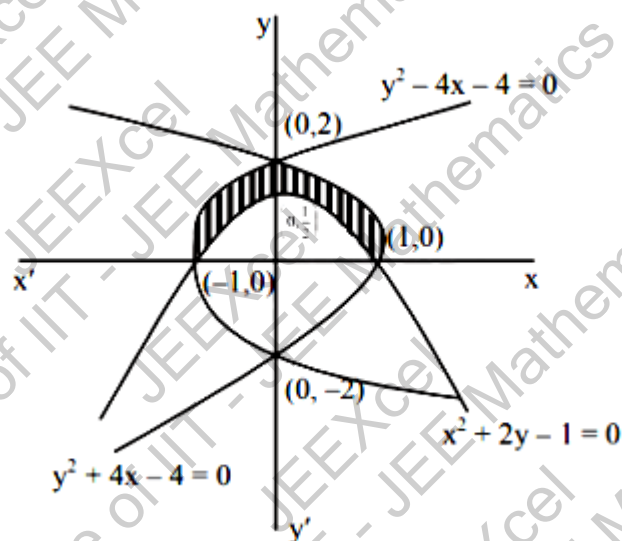
Question 50

The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$ in the upper half plane is _____.

[22 Jul 2021 Shift 2]

Answer: 2

Solution:



Required Area (shaded)

$$= 2 \left[\int_0^2 \left(\frac{4-y^2}{4} \right) dy - \int_0^1 \left(\frac{1-x^2}{2} \right) dx \right]$$

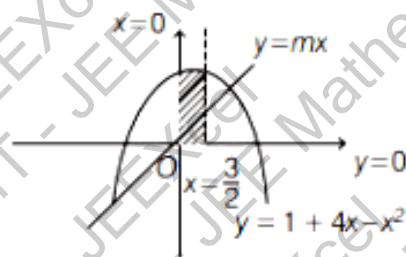
$$= 2 \left[\frac{4}{3} - \frac{1}{3} \right] = (2)$$

Question 51

If the line $y = mx$ bisects the area enclosed by the lines $x = 0$ and $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then $12m$ is equal to
[31 Aug 2021 Shift 2]

Answer: 26

Solution:



According to the question,

$$\frac{1}{2} \int_0^{\frac{3}{2}} (1 + 4x - x^2) dx = \int_0^{\frac{3}{2}} mx dx$$

$$\Rightarrow \frac{1}{2} \left[\left(x + 2x^2 - \frac{x^3}{3} \right) \right]_0^{\frac{3}{2}} = \frac{m}{2} [x]_0^{\frac{3}{2}}$$

$$\Rightarrow \frac{3}{2} + \frac{9}{2} - \frac{9}{8} = \frac{9m}{4}$$

$$\Rightarrow m = \frac{39}{18}$$

$$\Rightarrow 12m = 26$$

Question52

The area of the region bounded by the parabola $(y - 2)^2 = (x - 1)$, the tangent to it at the point whose ordinate is 3 and the X-axis is

[27 Aug 2021 Shift 2]

Options:

A. 9

B. 10

C. 4

D. 6

Answer: A

Solution:

Given parabola

$$(y - 2)^2 = (x - 1)$$

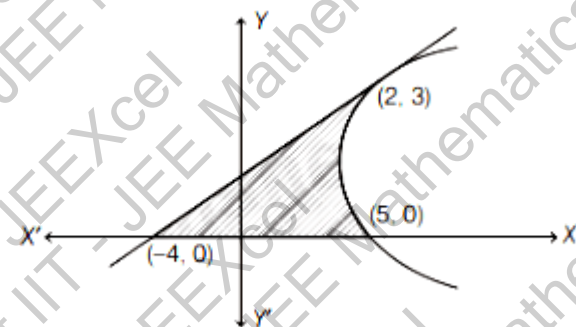
Since, Ordinate = $y = 3$

Then, $x = 2$

Point on parabola (2, 3)

Differentiating Eq. (i) w.r.t. x , we get

$$2(y - 2) \frac{dy}{dx} = 1$$



$$\frac{dy}{dx} = \frac{1}{2(y-2)}$$

At (2, 3)

$$\frac{dy}{dx} = \frac{1}{2}$$

Equation of tangent at (2, 3)

$$y - 3 = \frac{1}{2}(x - 2)$$

$$\text{or } x - 2y + 4 = 0$$

Intersection point of parabola on X-axis is

$$y = 0, x = 5 \text{ i.e. } (5, 0)$$

Intersection point of tangent and X-axis

$$y = 0, x = -4 \text{ i.e. } (-4, 0)$$

$$\text{Area of shaded region} = \int_0^3 [(y-2)^2 + 1 - (2y-4)] dy$$

$$= \int_0^3 (y^2 - 6y + 9) dy$$

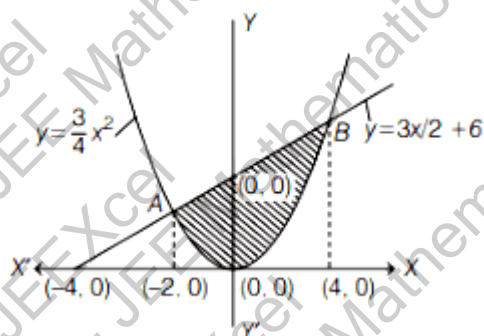
$$= \left(\frac{y^3}{3} - 3y^2 + 9y \right)_0^3 = 9 \text{ sq. unit.}$$

Question 53

The area of the region $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$ is
[26 Aug 2021 Shift 1]

Answer: 27

Solution:



We have, $y = \left(\frac{3}{4}\right)x^2$ and $y = \left(\frac{3x}{2}\right) + 6$

$$\Rightarrow \frac{3x^2}{4} = \frac{3x}{2} + 6$$

$$\Rightarrow 3x^2 = 6x + 24$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow x = -2, 4$$

$$\Rightarrow y = 3, 12$$

A(-2, 3) and B(4, 12)

$$\text{Required area} = \int_{-2}^4 \left(\frac{3x}{2} + 6 \right) - \left(\frac{3x^2}{4} \right) dx$$

$$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4} \right]_{-2}^4$$

$$= [(12 + 24 - 16) - (3 - 12 + 2)]$$

$$= (20 + 7) = 27 \text{ sq units}$$

Question 54

Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by $y = f(x)$, the X- axis and the lines $x = a$ and $x = b$, then 4A is equal to
[26 Aug 2021 Shift 2]

Answer: 114

Solution:

We have, $f(x) = 2x^3 - 3x^2 - 12x$

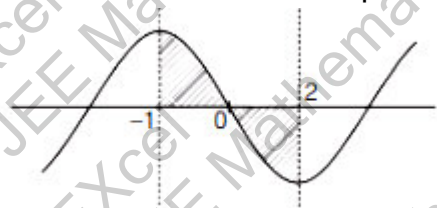
$$\therefore f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$= 6(x+1)(x-2)$$

$$f'(x) = 0$$

$$\Rightarrow x = -1 \text{ and } 2$$

$\therefore x = -1$ and 2 are critical points



$\therefore a = -1$ and $b = 2$

$$\text{Now, required area, } A_0 = \int_{-1}^0 f(x) dx + \int_0^2 -f(x) dx$$

$$= \int_{-1}^0 (2x^3 - 3x^2 - 12x) dx + \int_0^2 (12x + 3x^2 - 2x^3) dx$$

$$= \left[\frac{x^4}{2} - x^3 - 6x^2 \right]_{-1}^0 + \left[6x^2 + x^3 - \frac{x^4}{2} \right]_0^2 = \frac{114}{4}$$

$$\therefore 4A = 114$$

Question 55

The area, enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ and the lines $x = 0$, $x = \frac{\pi}{2}$, is

[1 Sep 2021 Shift 2]

Options:

A. $2\sqrt{2}(\sqrt{2} - 1)$

B. $2(\sqrt{2} + 1)$

C. $4(\sqrt{2} - 1)$

D. $2\sqrt{2}(\sqrt{2} + 1)$

Answer: A

Solution:

$$\text{Area} = \int_0^{\frac{\pi}{2}} ((\cos x + \sin x) - |\cos x - \sin x|) dx$$

$$= \int_0^{\frac{\pi}{4}} ((\cos x + \sin x) - (\cos x - \sin x)) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} ((\cos x + \sin x) - (\sin x - \cos x)) dx$$

$$\begin{aligned}
 &= 2 \int_0^{\frac{\pi}{4}} \frac{1}{4} \sin x \, dx + 2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} \cos x \, dx \\
 &= 2 \left(\frac{-1}{\sqrt{2}} + 1 \right) + 2 \left(1 - \frac{1}{\sqrt{2}} \right) = 2\sqrt{2}(\sqrt{2} - 1)
 \end{aligned}$$

Question 56

The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is:
[Jan. 7, 2020 (I)]

Options:

A. $(24\pi - 1)$

B. $(6\pi - 1)$

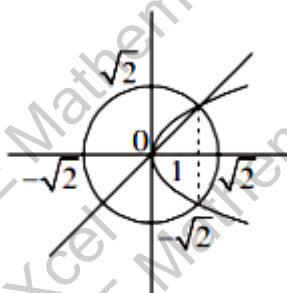
C. $(12\pi - 1)$

D. $(12\pi - 1)/6$

Answer: D

Solution:

Total area – enclosed area between line and parabola



$$\begin{aligned}
 &= 2\pi - \int_0^1 \sqrt{x} - x \, dx \\
 &= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2} \right) \Big|_0^1 \\
 &= 2\pi - \left(\frac{2}{3} - \frac{1}{2} \right) = 2\pi - \left(\frac{1}{6} \right) = \frac{12\pi - 1}{6}
 \end{aligned}$$

Question 57

The area (in sq. units) of the region $\{(x, y) \in \mathbb{R}^2 \mid 4x^2 \leq y \leq 8x + 12\}$ is:
[Jan. 7, 2020 (II)]

Options:

A. $\frac{125}{3}$

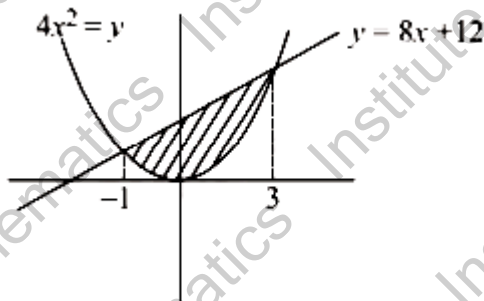
B. $\frac{128}{3}$

C. $\frac{124}{3}$

D. $\frac{127}{3}$

Answer: B

Solution:



Given curves are

$4x^2 = y$ (i)

$y = 8x + 12$ (ii)

From eqns. (i) and (ii),

$$4x^2 = 8x + 12$$

$$\Rightarrow x^2 - x - 3 = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow (x+1)(x-3) = 0$$

$$\Rightarrow x = -1, 3$$

Required area bounded by curves is given by

$$A = \int_{-1}^3 (8x + 12 - 4x^2) dx$$

$$A = \frac{8x^2}{2} + 12x - \frac{4x^3}{3} \Big|_{-1}^3$$

$$\begin{aligned}
 &= (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right) \\
 &= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3}
 \end{aligned}$$

Question 58

For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ ($0 < b < a$) intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2 , and the area of $\triangle OQR = \frac{1}{2}$, then 'a' satisfies the equation:

[Jan. 8, 2020 (I)]

Options:

A. $x^6 - 6x^3 + 4 = 0$

B. $x^6 - 12x^3 + 4 = 0$

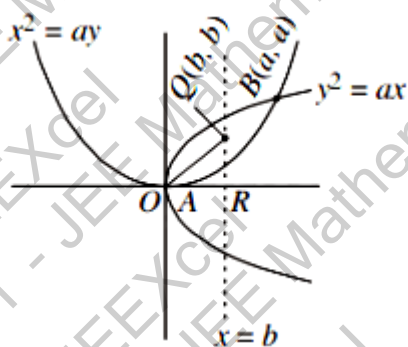
C. $x^6 + 6x^3 - 4 = 0$

D. $x^6 - 12x^3 - 4 = 0$

Answer: B

Solution:

Given eqns. are, $x^2 = ay$ and $y^2 = ax$



After solving, we get $x = a$, $y = a$

Now, coordinates of B is (a, a) and A is $(0, 0)$

Now, coordinates of Q is (b, b)

$$\frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$$

Area bounded by curves and $x = 1$ is

$$\int_0^1 \left(\sqrt{ax}^{1/2} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax}^{1/2} - \frac{x^2}{a} \right) dx$$

$$\Rightarrow \frac{2}{3} \sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$\Rightarrow 4a\sqrt{a} - 2 = a^3$$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0$$

Question 59

The area (in sq. units) of the region

$\{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq |3 - 2x|, \text{ is:}$

[Jan. 8, 2020 (II)]

Options:

A. $\frac{32}{3}$

B. $\frac{34}{3}$

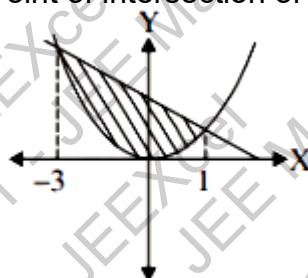
C. $\frac{29}{3}$

D. $\frac{31}{3}$

Answer: A

Solution:

Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by $x^2 + 2x - 3 = 0$



$$\Rightarrow x = -3, 1$$

$$\text{So, required area} = \int_{-3}^1 (\text{line} - \text{parabola}) dx$$

$$= \int_{-3}^1 (3 - 2x - x^2) dx = \left[3x - x^2 - \frac{x^3}{3} \right]_{-3}^1$$

$$= (3)4 - 2 \left(\frac{1^2 - 3^2}{2} \right) - \left(\frac{1^3 + 3^3}{3} \right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$$

Question60

Given:

$$f(x) = \begin{cases} x & , 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \leq 1 \end{cases}$$

and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in \mathbb{R}$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is:

[Jan. 9, 2020 (II)]

Options:

A. $\frac{1}{3} + \frac{\sqrt{3}}{4}$

B. $\frac{\sqrt{3}}{4} - \frac{1}{3}$

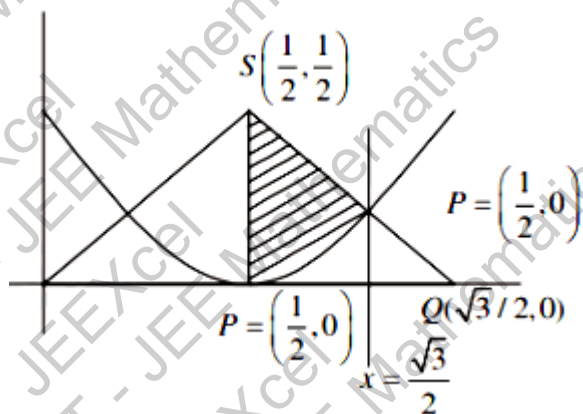
C. $\frac{1}{2} - \frac{\sqrt{3}}{4}$

D. $\frac{1}{2} + \frac{\sqrt{3}}{4}$

Answer: B

Solution:

Coordinates of $P\left(\frac{1}{2}, 0\right)$, $Q\left(\frac{\sqrt{3}}{2}, 0\right)$, $R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$ and $S\left(\frac{1}{2}, \frac{1}{2}\right)$



Required area = Area of trapezium PQRS

$$\begin{aligned}
 &= \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx \\
 &= \frac{1}{2} \left(\frac{\sqrt{3}-1}{2}\right) \left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3} \left(\left(x - \frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2} \\
 &= \frac{\sqrt{3}}{4} - \frac{1}{3}
 \end{aligned}$$

Question 61

The area (in sq. units) of the region

$A = \{(x, y) : (x-1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$, where $[t]$ denotes the greatest integer function, is:

[Sep. 05, 2020 (II)]

Options:

A. $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

B. $\frac{4}{3}\sqrt{2} + 1$

C. $\frac{8}{3}\sqrt{2} - 1$

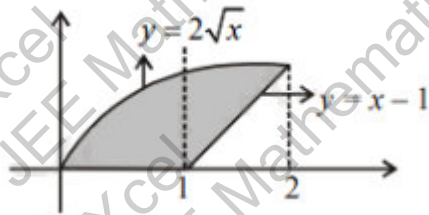
D. $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

Answer: A

Solution:

$[x] = 0$ when $x \in [0, 1)$ and $[x] = 1$ when $x \in [1, 2)$

$$y = \begin{cases} 0 & 0 \leq x < 1 \\ x-1 & 1 \leq x < 2 \end{cases}$$



$$\begin{aligned} \therefore A &= \int_0^2 2\sqrt{x} dx - \frac{1}{2}(1)(1) \\ &= \frac{4x^{3/2}}{3} \Big|_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2} \end{aligned}$$

Question 62

The area (in sq. units) of the region

$$\left\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2 \right\}$$

is

[Sep. 03, 2020 (I)]

Options:

A. $\frac{23}{16}$

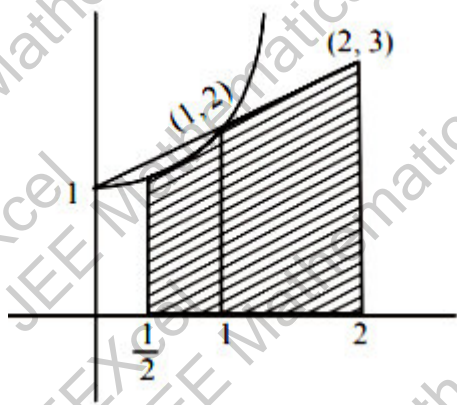
B. $\frac{79}{24}$

C. $\frac{79}{16}$

D. $\frac{23}{6}$

Answer: B

Solution:



$$\begin{aligned}
 \text{Required area} &= \int_{\frac{1}{2}}^1 (x^2 + 1) dx + \int_1^2 (x + 1) dx \\
 &= \left[\frac{x^3}{3} + x \right]_{\frac{1}{2}}^1 + \left[\frac{x^2}{2} + x \right]_1^2 \\
 &= \left[\frac{4}{3} - \frac{13}{24} \right] + \frac{5}{2} = \frac{79}{24}
 \end{aligned}$$

Question 63

The area (in sq. units) of the region

$A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$ is:

[Sep. 06, 2020 (I)]

Options:

A. $\frac{1}{3}$

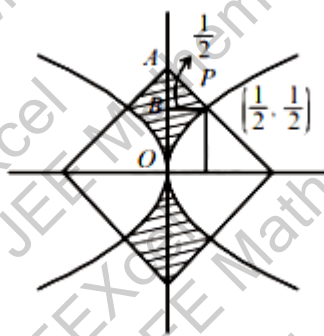
B. $\frac{7}{6}$

C. $\frac{1}{6}$

D. $\frac{5}{6}$

Answer: D

Solution:



$$\text{Required area} = 4 \left[\int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2} \text{area}(\Delta PAB) \right]$$

$$= 4 \left[\frac{2}{3} [y^3]_0^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right] = 4 \left[\frac{2}{3} \times \frac{1}{8} + \frac{1}{8} \right]$$

$$= 4 \times \frac{5}{24} = \frac{5}{6}$$

Question 64

The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{4}{3}$

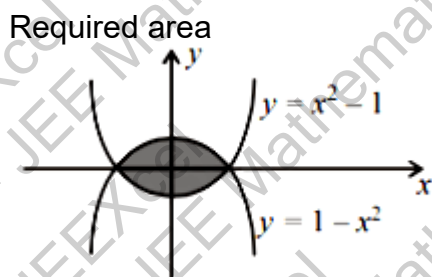
B. $\frac{8}{3}$

C. $\frac{7}{2}$

D. $\frac{16}{3}$

Answer: B

Solution:



$$\text{Area} = 2 \int_0^1 ((1 - x^2) - (x^2 - 1)) dx$$

$$= 4 \int_0^1 (1 - x^2) dx$$

$$= 4 \left(x - \frac{x^3}{3} \right)_0^1 = 4 \left(1 - \frac{1}{3} \right) = 4 \cdot \frac{2}{3} = \frac{8}{3} \text{ sq. units}$$

Question 65

Consider a region $R = (x, y) \in \{R^2 : x^2 \leq y \leq 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

[Sep.02,2020(II)]

Options:

A. $\alpha^3 - 6\alpha^2 + 16 = 0$

B. $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

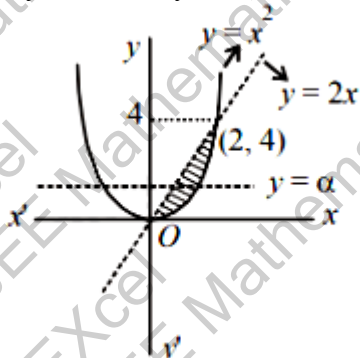
C. $3\alpha^2 - 8\alpha + 8 = 0$

D. $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

Answer: B

Solution:

Let $y = x^2$ and $y = 2x$



According to question

$$\therefore \int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2} \right) dy = \int_{\alpha}^4 \left(\sqrt{y} - \frac{y}{2} \right) dy$$

$$\Rightarrow \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\alpha} - \left[\frac{y^2}{4} \right]_0^{\alpha} = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_{\alpha}^4 - \left[\frac{y^2}{4} \right]_{\alpha}^4$$

$$\Rightarrow \frac{2}{3} \alpha^{3/2} - \frac{\alpha^2}{4} = \frac{2}{3} (8 - \alpha^{3/2}) - \frac{1}{4} (16 - \alpha^2)$$

$$\Rightarrow \frac{4}{3} \alpha^{3/2} - \frac{\alpha^2}{2} = \frac{4}{3}$$

$$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$$

$$\therefore 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$$