



Vector Algebra

Question1

For real numbers $\alpha, \beta, \gamma, \delta$ and μ , consider the matrix

$$M = \begin{bmatrix} \alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}} \\ \gamma & \delta & \mu \end{bmatrix}.$$

Suppose that $MM^T = I$, where M^T is the transpose of the matrix M , and I is the 3×3 identity matrix. Let

$$\vec{u} = \alpha \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \gamma \hat{k}, \quad \vec{v} = \frac{1}{\sqrt{2}} \hat{i} + \beta \hat{j} + \delta \hat{k} \quad \text{and} \quad \vec{w} = -\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \mu \hat{k}.$$

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) The value of $\gamma^2 + \delta^2$ is	(1) 0
(Q) If $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$ for some real numbers x, y and z , then the value of x is	(2) 1
(R) The value of $\vec{u} \cdot (\vec{v} \times \vec{w})$ is	(3) $\frac{1}{\sqrt{2}}$
(S) The value of $\vec{u} \times (\vec{v} \times \vec{w})$ is	(4) $\frac{1}{\sqrt{3}}$
	(5) $\frac{5}{6}$

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Options:

A.

(P) $\rightarrow$ (5), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (2), (S) $\rightarrow$ (1)

B.

(P) $\rightarrow$ (4), (Q) $\rightarrow$ (5), (R) $\rightarrow$ (1), (S) $\rightarrow$ (2)

C.

$$(P) \rightarrow (5), (Q) \rightarrow (3), (R) \rightarrow (2), (S) \rightarrow (1)$$

D.

$$(P) \rightarrow (5), (Q) \rightarrow (4), (R) \rightarrow (1), (S) \rightarrow (2)$$

Answer: A

Solution:

Given the matrix

$$M = \begin{bmatrix} \alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}} \\ \gamma & \delta & \mu \end{bmatrix},$$

and $MM^T = I$, this means that the rows of M are orthonormal vectors (that is, each has unit length and they are mutually perpendicular).

Let

$$\vec{u} = \alpha\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \gamma\hat{k}, \quad \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \beta\hat{j} + \delta\hat{k}, \quad \vec{w} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \mu\hat{k}.$$

Length condition for each row (unit vectors)

For $\vec{u}$, the condition $|\vec{u}| = 1$ gives

$$\alpha^2 + \frac{1}{3} + \gamma^2 = 1$$

Thus,

$$\alpha^2 + \gamma^2 = \frac{2}{3}$$

For $\vec{v}$, the condition $|\vec{v}| = 1$ gives

$$\frac{1}{2} + \beta^2 + \delta^2 = 1$$

Therefore,

$$\beta^2 + \delta^2 = \frac{1}{2}$$

For $\vec{w}$, the condition $|\vec{w}| = 1$ gives

$$\frac{1}{2} + \frac{1}{3} + \mu^2 = 1$$

Hence,

$$\mu^2 = \frac{1}{6}$$

Using the third row's condition

The third row of M also has unit length, so

$$\gamma^2 + \delta^2 + \mu^2 = 1$$

Substitute $\mu^2 = \frac{1}{6}$:

$$\gamma^2 + \delta^2 = 1 - \frac{1}{6} = \frac{5}{6}$$

So, for (P), the value of $\gamma^2 + \delta^2 = \frac{5}{6}$.

Finding x when $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$

Taking the dot product with $\vec{u}$ on both sides:

$$x(\vec{u} \cdot \vec{u}) + y(\vec{v} \cdot \vec{u}) + z(\vec{w} \cdot \vec{u}) = \vec{u} \cdot \hat{j}$$

Since the vectors are mutually perpendicular, $\vec{u} \cdot \vec{v} = 0$ and $\vec{u} \cdot \vec{w} = 0$. So,

$$x(1) = \frac{1}{\sqrt{3}}$$

Therefore, $x = \frac{1}{\sqrt{3}}$.

Value of $|\vec{u} \cdot (\vec{v} \times \vec{w})|$

For an orthogonal matrix, the determinant is ± 1 , meaning the scalar triple product equals 1 in magnitude. Hence,

$$|\vec{u} \cdot (\vec{v} \times \vec{w})| = 1.$$

Value of $|\vec{u} \times (\vec{v} \times \vec{w})|$

Using vector identity $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$, and since the vectors are perpendicular, both $\vec{u} \cdot \vec{v} = 0$ and $\vec{u} \cdot \vec{w} = 0$. Thus,

$$\vec{u} \times (\vec{v} \times \vec{w}) = 0,$$

and its magnitude is 0.

Final Matching:

$$(P) \rightarrow (5) \frac{5}{6}$$

$$(Q) \rightarrow (4) \frac{1}{\sqrt{3}}$$

$$(R) \rightarrow (2) 1$$

$$(S) \rightarrow (1) 0$$

Question2

Let $\vec{a}, \vec{b}$ be two vectors, and let P, Q and R be the points with position vectors $\vec{a}, \vec{b}$ and $\vec{a} + \vec{b}$, respectively, with respect to the origin O. If $|\vec{a} + \vec{b}| = \sqrt{21}$, $|\vec{a} - \vec{b}| = 3$, and $\vec{a}$ and $(\vec{a} - \vec{b})$ are perpendicular to each other, then the area of the triangle OPR is :

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Options:

A.

$$\sqrt{3}$$

B.

$$\frac{\sqrt{3}}{2}$$

C.

$$\frac{3\sqrt{3}}{2}$$

D.

$$\frac{3}{2}$$

Answer: C

Solution:

Let us find the area of triangle OPR step by step.

Given:

- Position vector of P is $\vec{a}$
- Position vector of Q is $\vec{b}$
- Position vector of R is $\vec{a} + \vec{b}$

So, for triangle OPR , the two side vectors from O are:

$$\overrightarrow{OP} = \vec{a}, \quad \overrightarrow{OR} = \vec{a} + \vec{b}$$

Hence, area of triangle OPR is

$$\frac{1}{2} |\vec{a} \times (\vec{a} + \vec{b})|$$

Now,

$$\vec{a} \times (\vec{a} + \vec{b}) = \vec{a} \times \vec{a} + \vec{a} \times \vec{b} = \vec{a} \times \vec{b}$$

since $\vec{a} \times \vec{a} = 0$.

Therefore,

$$\text{Area of } \triangle OPR = \frac{1}{2} |\vec{a} \times \vec{b}|$$

So we only need to find $|\vec{a} \times \vec{b}|$.

We are given:

$$|\vec{a} + \vec{b}| = \sqrt{21}$$

Squaring both sides,

$$|\vec{a} + \vec{b}|^2 = 21$$

So,

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 21 \quad (1)$$

Also,

$$|\vec{a} - \vec{b}| = 3$$

Squaring,

$$|\vec{a} - \vec{b}|^2 = 9$$

Thus,

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 9 \quad (2)$$

Now subtract (2) from (1):

$$4\vec{a} \cdot \vec{b} = 12$$

So,

$$\vec{a} \cdot \vec{b} = 3$$

Now use the condition that $\vec{a}$ is perpendicular to $(\vec{a} - \vec{b})$.

Therefore,

$$\vec{a} \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - \vec{a} \cdot \vec{b} = 0$$

Since $\vec{a} \cdot \vec{b} = 3$, we get

$$|\vec{a}|^2 = 3$$

So,

$$|\vec{a}| = \sqrt{3}$$

Now use equation (2):

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 9$$

Substitute $|\vec{a}|^2 = 3$ and $\vec{a} \cdot \vec{b} = 3$:

$$3 + |\vec{b}|^2 - 6 = 9$$

$$|\vec{b}|^2 = 12$$

Now use the identity

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

Substitute the values:

$$|\vec{a} \times \vec{b}|^2 = (3)(12) - 3^2 = 36 - 9 = 27$$

Therefore,

$$|\vec{a} \times \vec{b}| = \sqrt{27} = 3\sqrt{3}$$

Hence area of triangle OPR is

$$\frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} (3\sqrt{3})$$

$$\frac{3\sqrt{3}}{2}$$

So, the correct option is:

Option C

Question 3

Consider the vectors

$$\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}, \quad \text{and} \quad \vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}$$

For two distinct positive real numbers α and β , define

$$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \quad \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}, \quad \text{and} \quad \vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}.$$

If the vectors $\vec{X}$, $\vec{Y}$, and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is _____.

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Answer: -2

Solution:

$$\begin{aligned} [\vec{x}\vec{y}\vec{z}] &= 0 \\ \Rightarrow \begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix} &= 0 \\ \Rightarrow (\alpha^3 + \beta^3 - 1) - (-\alpha\beta - \alpha\beta - \alpha\beta) &= 0 \\ \Rightarrow \alpha^3 + \beta^3 + 3\alpha\beta &= 1 \\ \Rightarrow \alpha^3 + \beta^3 + (-1)^3 &= 3(\alpha)(\beta)(-1) \\ \Rightarrow \alpha + \beta - 1 &= 0 \end{aligned}$$

$$\text{So, } \alpha + \beta - 3 = -2$$

Question4

For any two points M and N in the XY -plane, let $\overrightarrow{MN}$ denote the vector from M to N , and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY -plane. Let S be a point inside the triangle $\triangle PQR$ such that

$$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}.$$

Let E and F be the mid-points of the sides PR and QR , respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is _____.

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Answer: 1.2

Question 5

Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β , and γ , we have

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q}),$$

then the value of γ is _____.

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Answer: 2

Solution:

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$$

taking dot with $(\vec{p} \times \vec{q})$

$$\begin{aligned} \begin{vmatrix} 15 & 10 & 6 \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} &= 0 + 0 + \gamma(\vec{p} \times \vec{q} \cdot \vec{p} \times \vec{q}) \\ \Rightarrow 52 &= 26\gamma \\ \therefore \gamma &= 2 \end{aligned}$$

Question 6

Let $\vec{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\vec{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\vec{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three

vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\vec{OP} \times \vec{OQ}) \cdot \vec{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is

_____.

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Answer: 5

Solution:

$$(\vec{OP} \times \vec{OQ}) \cdot \vec{OR} = 0$$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0$$

$$\alpha + \beta + 1 = 0 \quad \dots (i)$$

Also $(\alpha, \beta, 2)$ lies on $3x + 3y - z + l = 0$

$$\Rightarrow 3\alpha + 3\beta - 2 + l = 0 \Rightarrow l = 2 - 3(\alpha + \beta)$$

$$\text{use (1) in it} \Rightarrow l = 5$$

Question 7

Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let

$S = \left\{ \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1 \text{ and the distance of } (\alpha, \beta, \gamma) \text{ from the plane } P \text{ is } \frac{7}{2} \right\}$. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that

$|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped

determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}} V$ is :

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Answer: 45

Solution:

$$P : \sqrt{3}x + 2y + 3z = 16$$

$$S = \left\{ \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1, \quad d_p = \frac{7}{2} \right\}$$

$$\therefore |\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}| \dots \dots \dots (1)$$

$\vec{u}, \vec{v}, \vec{w}$ are elements of set S and in set S magnitude of vector is 1

$\therefore \vec{u}, \vec{v}, \vec{w}$ are unit vectors and by equation (1) we can say $\vec{u}, \vec{v}, \vec{w}$ are equally inclined and vertices of equilateral triangle also lying on a circle which is intersection of sphere $|\vec{r}| = 1$

Distance from origin to P ,

$$d = \frac{|-16|}{\sqrt{3+4+9}} = \frac{16}{4} = 4$$

$\therefore$ Plane containing $\hat{u}, \hat{v}, \hat{w}$ are at a distance

$$4 - \frac{7}{2} = \frac{1}{2} \text{ from origin and Parallel to } \sqrt{3}x + 2y + 3z = 16.$$

$\therefore$ Equation of the plane is

$$\sqrt{3}x + 2y + 3z = \gamma$$

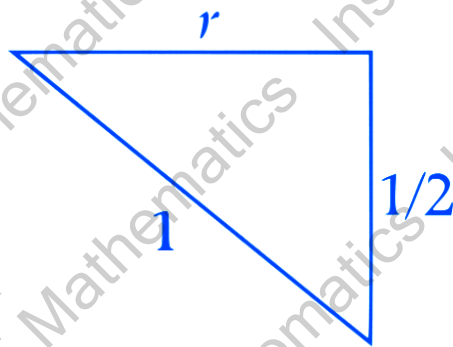
$$\therefore \frac{1}{2} = \frac{\gamma}{4}$$

$$\Rightarrow \gamma = \pm 2$$

$$\sqrt{3}x + 2y + 3z = 2$$

$$\text{Equation of sphere } x^2 + y^2 + z^2 = 1$$

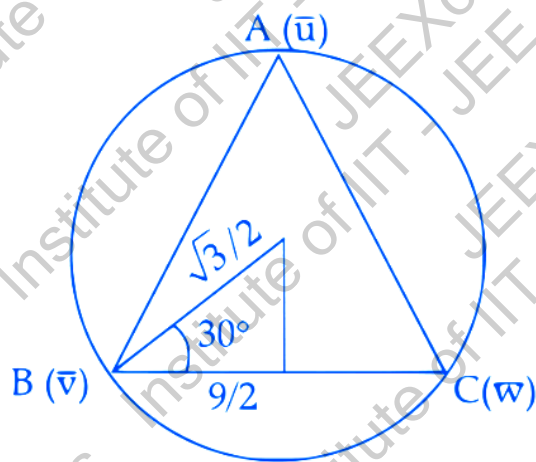
$\therefore$ Radius of circle



$$r = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

then $\frac{a}{2} = \frac{\sqrt{3}}{2} \cos 30^\circ$

$$a = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$$



$\therefore$ Area of triangle

$$= \frac{\sqrt{3}}{2} a^2 = \frac{\sqrt{3}}{2} \times \frac{9}{4} = \frac{9\sqrt{3}}{16}$$

$\therefore$ Volume of Parallelepiped

$$= 2 \times \frac{1}{2} \times \frac{9\sqrt{3}}{16}$$

$$V = \frac{9\sqrt{3}}{16}$$

$\therefore$ Volume of Parallelepiped

$$= 2 \times \frac{1}{2} \times \frac{9\sqrt{3}}{16}$$

$$V = \frac{9\sqrt{3}}{16}$$

$$\therefore \frac{80V}{\sqrt{3}} = \frac{80}{\sqrt{3}} \times \frac{9\sqrt{3}}{16} = 45$$

Question 8

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1, \vec{w} \cdot \vec{w} = 4$

If the volume of the parallelepiped, whose adjacent sides are represented by the vectors, $\vec{u}, \vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is _____.

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Answer: 7

Solution:

Given: $|\vec{u}| = 1, |\vec{v}| = 1$

$\vec{u}$ is not perpendicular to $\vec{v}$ $\vec{u} \cdot \vec{v} \neq 0$

And $\vec{u} \cdot \vec{w} = 1 \quad \vec{v} \cdot \vec{w} = 1 \quad \vec{w} \cdot \vec{w} = 4$

$$\Rightarrow \vec{w} \cdot \vec{w} = |\vec{w}|^2 = 4$$

$$\Rightarrow |\vec{w}| = 2$$

Volume of parallelepiped with $\vec{u}$, $\vec{v}$ and $\vec{w}$ as its sides,

$$= [\vec{u}\vec{v}\vec{w}] = \sqrt{2}$$

$$\text{Now, } [\vec{u}\vec{v}\vec{w}]^2 = \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{v} & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & \vec{w} \cdot \vec{w} \end{vmatrix} = 2$$

$$\Rightarrow \begin{vmatrix} 1 & \vec{u} \cdot \vec{v} & 1 \\ \vec{v} \cdot \vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$\Rightarrow 1(4 - 1) - \vec{u} \cdot \vec{v}(4\vec{u} \cdot \vec{v} - 1) + 1(\vec{u} \cdot \vec{v} - 1) = 2$$

$$\Rightarrow 3 - 4(\vec{u}\vec{v})^2 + \vec{u}\vec{v} + \vec{u}\vec{v} - 1 = 2$$

$$\Rightarrow -4(\vec{u}\vec{v})^2 + 2\vec{u}\vec{v} + 2 = 2$$

$$\Rightarrow -4(\vec{u}\vec{v})^2 + 2\vec{u}\vec{v} = 0$$

$$\Rightarrow 2\vec{u}\vec{v}(-2\vec{u}\vec{v} + 1) = 0$$

$$\Rightarrow \vec{u} \cdot \vec{v} = \frac{1}{2}, \vec{u} \cdot \vec{v} \neq 0$$

$$\text{Now, } |3\vec{u} + 5\vec{v}| = \sqrt{9 + 25 + 30\left(\frac{1}{2}\right)}$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = \sqrt{49}$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = 7$$
