



Question1 Trigonometric Functions and Equations

Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{1}{\sin 118^\circ \sin 119^\circ}$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha} \right)^2$$

is _____.

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Options:

Answer: 3

Solution:

To solve for the value of $\left(\frac{\operatorname{cosec} 1^\circ}{\alpha} \right)^2$, we begin by considering the expression for α :

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{1}{\sin 118^\circ \sin 119^\circ}$$

Using the identity for the product of sines:

$$\sin(x - y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

we can transform each term:

$$\sin 1^\circ \cdot \alpha = \frac{\sin(61^\circ - 60^\circ)}{\sin 60^\circ \sin 61^\circ} + \frac{\sin(63^\circ - 62^\circ)}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{\sin(119^\circ - 118^\circ)}{\sin 118^\circ \sin 119^\circ}$$

This simplifies to:

$$\sin 1^\circ \cdot \alpha = \cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63^\circ + \cdots + \cot 118^\circ - \cot 119^\circ$$

This forms a telescoping series, simplifying to:

$$\sin 1^\circ \cdot \alpha = \cot 60^\circ$$

Therefore:

$$\alpha = \frac{\cot 60^\circ}{\sin 1^\circ} = \frac{1/\sqrt{3}}{\sin 1^\circ} = \frac{\operatorname{cosec} 1^\circ}{\sqrt{3}}$$

Finally, we calculate:

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha} \right)^2 = \left(\frac{\operatorname{cosec} 1^\circ}{\frac{\operatorname{cosec} 1^\circ}{\sqrt{3}}} \right)^2 = 3$$

Question2

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

Then the inradius of the triangle ABC is :

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Options:

Answer: 0.25

Solution:

$$\text{In radius } r = \frac{\Delta}{S} = \left[\frac{a}{2R(\sin A + \sin B + \sin C)} \right]$$

$$r = \frac{a}{\sin\left(\frac{\pi}{2} - 2C\right) \sin\left(\frac{\pi}{2} + C\right) + \sin C}$$

$$= \frac{a}{\cos 2C + \cos C + \sin C}$$

$$= \frac{a}{\cos 2C + \sqrt{1 + \sin 2C}}$$

$$= \frac{\frac{3\sqrt{7}}{16}}{\sqrt{\frac{7}{4}} + \sqrt{\frac{7}{2}}} = \frac{1}{4}$$

$$\Rightarrow r = \frac{1}{4} = 0.25$$

$$\Rightarrow r = 0.25$$

Question 3

Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$.

If $\sin(\alpha + \beta) = \frac{1}{3}$ and $\cos(\alpha - \beta) = \frac{2}{3}$, then the greatest integer less than or equal to

$\left(\frac{\sin \alpha}{\cos \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\cos \alpha}{\sin \beta} + \frac{\sin \beta}{\cos \alpha} \right)^2$ is

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Options:

Answer: 1

Solution:

$$\text{Given, } \sin(\alpha + \beta) = \frac{1}{3}$$

$$\text{and } \cos(\alpha - \beta) = \frac{2}{3}$$

$$\text{Let, } E = \frac{\sin \alpha}{\cos \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\cos \alpha}{\sin \beta} + \frac{\sin \beta}{\cos \alpha}$$

$$= \frac{\sin \alpha}{\cos \beta} + \frac{\cos \alpha}{\sin \beta} + \frac{\cos \beta}{\sin \alpha} + \frac{\sin \beta}{\cos \alpha}$$

$$= \frac{\sin \alpha \sin \beta + \cos \alpha \cos \beta}{\sin \beta \cos \beta} + \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \alpha}$$

$$= \frac{\cos(\alpha - \beta)}{\sin \beta \cos \beta} + \frac{\cos(\alpha - \beta)}{\sin \alpha \cos \alpha}$$

$$= \cos(\alpha - \beta) \left[\frac{2}{2 \sin \beta \cos \beta} + \frac{2}{2 \sin \alpha \cos \alpha} \right]$$

$$= \frac{2}{3} \left[\frac{2}{\sin 2\beta} + \frac{2}{\sin 2\alpha} \right]$$

$$= \frac{4}{3} \left[\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha} \right]$$

$$= \frac{4}{3} \left[\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha \sin 2\beta} \right]$$

$$= \frac{4 \times 2}{3} \left[\frac{2 \sin \left(\frac{2\alpha + 2\beta}{2} \right) \cos \left(\frac{2\alpha - 2\beta}{2} \right)}{2 \sin 2\alpha \sin 2\beta} \right]$$

$$= \frac{16}{3} \left[\frac{\sin(\alpha + \beta) \cos(\alpha - \beta)}{\cos(2\alpha - 2\beta) - \cos(2\alpha + 2\beta)} \right]$$

$$= \frac{16}{3} \left[\frac{\frac{1}{2} \times \frac{2}{2}}{(2 \cos^2(\alpha - \beta) - 1) - (1 - 2 \sin^2(\alpha + \beta))} \right]$$

$$= \frac{32}{27} \left[\frac{1}{2 \times \frac{4}{9} - 2 + 2 \times \frac{4}{9}} \right]$$

$$= \frac{32}{27} \left[\frac{9}{8 - 18 + 2} \right]$$

$$= \frac{32}{27} \left[\frac{9}{-8} \right]$$

$$= -\frac{4}{3}$$

$$\therefore E^2 = \frac{16}{9} = 1.77$$

$$\Rightarrow [E^2] = 1$$
