



Properties of Triangle

Question1

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

Let a be the area of the triangle ABC . Then the value of $(64a)^2$ is :

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Options:

Answer: 1008

Solution:

$$\text{Area of } \triangle ABC = \frac{AB \cdot BC \cdot AC}{4R}$$

$$\Rightarrow a = \frac{8 \sin A \cdot \sin B \sin C}{4}$$

$$= 2 \sin \left(\frac{\pi}{2} - 2C \right) \sin \left(\frac{\pi}{2} + C \right) \sin C$$

$$= 2 \cos 2C \cdot \cos C \cdot \sin C$$

$$= \cos 2C \cdot \sin 2C$$

$$= \sqrt{1 - \sin^2 2C} \cdot \sin 2C$$

$$= \sqrt{1 - \frac{9}{6}} \cdot \frac{3}{4}$$

$$\Rightarrow a = \frac{3\sqrt{7}}{16}$$

$$\therefore (64a)^2 = 1008$$

Question2

In a triangle ABC, let $AB = \sqrt{23}$, $BC = 3$ and $CA = 4$. Then the value of $\frac{\cot A + \cot C}{\cot B}$ is _____.

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Options:

Answer: 2

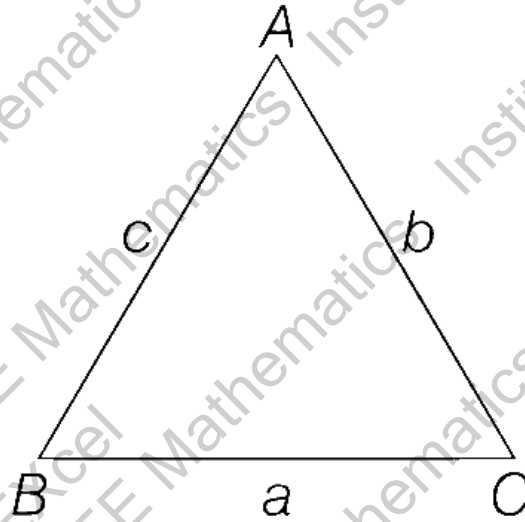
Solution:

Given,

$$AB = \sqrt{23} = c$$

$$BC = 3 = a$$

$$CA = 4 = b$$



Now,

$$\begin{aligned}
 \frac{\cot A + \cot C}{\cot B} &= \frac{\frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}}{\frac{\cos B}{\sin B}} \\
 &= \frac{\frac{b^2 + c^2 - a^2}{2bc(\sin A)} + \frac{a^2 + b^2 - c^2}{2ab(\sin C)}}{\frac{c^2 + a^2 - b^2}{2ac(\sin B)}} \\
 &= \frac{\frac{b^2 + c^2 - a^2}{4\Delta} + \frac{a^2 + b^2 - c^2}{4\Delta}}{\frac{c^2 + a^2 - b^2}{4\Delta}} \\
 &= \frac{b^2 + c^2 - a^2 + a^2 + b^2 - c^2}{c^2 + a^2 - b^2} \\
 &= \frac{2b^2}{a^2 + c^2 - b^2} = \frac{2 \times 16}{9 + 23 - 16} = \frac{32}{16} = 2
 \end{aligned}$$

Question 3

In a triangle PQR, let $a = QR$, $b = RP$, and $c = PQ$. If $|a| = 3$, $|b| = 4$

and $\frac{a \cdot (c-b)}{c \cdot (a-b)} = \frac{|a|}{|a|+|b|}$, then the value of $|a \times b|^2$ is

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Options:

Answer: 108

Solution:

It is given that, in a ΔPQR

$PQ = c$, $QR = a$ and $RP = b$, so $a + b + c = 0$

$$\therefore |a|^2 + a \cdot b + a \cdot c = 0 \dots(i)$$

$$\text{and } a \cdot b + |b|^2 + c \cdot b = 0 \dots(ii)$$

and, also given that $|a| = 3$, $|b| = 4$

$$\text{and } \frac{a \cdot c - a \cdot b}{c \cdot a - c \cdot b} = \frac{|a|}{|a|+|b|} = \frac{3}{7}$$

$$\Rightarrow 7[(a \cdot c) - (a \cdot b)] = 3[(c \cdot a) - (c \cdot b)]$$

from Eqs. (i) and (ii), on putting the values of $a \cdot c$ and $c \cdot b$, we get,

$$\Rightarrow 7[-9 - (a \cdot b) - (a \cdot b)] = 3[-9 - (a \cdot b) + 16 + (a \cdot b)]$$

$$\Rightarrow 7[-9 - 2(a \cdot b)] = 3 \times 7$$

$$\Rightarrow a \cdot b = -6$$

$$\because |a \times b|^2 = (|a||b|)^2 - (a \cdot b)^2$$

$$= 144 - 36 = 108$$

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