



Permutations and Combinations

Question1

The number of ways to distribute 10 identical red pens and 14 identical blue pens among four persons such that each person gets 6 pens, is _____.

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Answer: 206

Solution:

We have to distribute 10 identical red pens and 14 identical blue pens among four persons so that each person gets 6 pens in total.

For every possible way of giving out the red pens, the blue pens will automatically be distributed, because the total number of pens each person receives is fixed at 6. Hence, we only need to count the number of ways to distribute the red pens.

Let the number of red pens received by the four persons be x_1, x_2, x_3, x_4 . Then,

$$x_1 + x_2 + x_3 + x_4 = 10$$

and since each person can receive a maximum of 6 pens, $0 \leq x_i \leq 6$.

The number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 10$ with $x_i \leq 6$ can be obtained by finding the coefficient of x^{10} in the expansion of $(1 + x + x^2 + \dots + x^6)^4$.

We know that:

$$(1 + x + x^2 + \dots + x^6) = \frac{1-x^7}{1-x}$$

Therefore,

$$(1 + x + x^2 + \dots + x^6)^4 = \left(\frac{1-x^7}{1-x}\right)^4 = (1-x^7)^4(1-x)^{-4}$$

Now, we need the coefficient of x^{10} in this expansion. Expanding $(1-x^7)^4$, we get:

$$(1-x^7)^4 = 1 - 4x^7 + \dots$$

Thus, the required coefficient of x^{10} is:

$$\text{Coeff. of } x^{10} \text{ in } (1-x)^{-4} - 4 \times \text{Coeff. of } x^3 \text{ in } (1-x)^{-4}$$

We know that the coefficient of x^r in $(1-x)^{-n}$ is ${}^{n+r-1}C_r$. Hence,

$${}^{13}C_3 - 4 \times {}^6C_3 = 286 - 80 = 206$$

Therefore, the total number of ways = 206.

Question2

Let $S = \{1, 2, 3, \dots, 10\}$. Consider the set

$X = \{R : R \text{ is an equivalence relation on the set } S \text{ such that } R \text{ has exactly 42 elements} \}$

Then the number of elements in X is _____.

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Answer: 2520

Solution:

An **equivalence relation** divides the given set into disjoint groups, called **equivalence classes**. Together, these classes form a complete **partition** of the set.

Here, the set is $S = \{1, 2, 3, \dots, 10\}$. So, we must divide it into parts (subsets) such that the total number of elements across all parts is 10.

For an equivalence relation, if one class has n_i elements, then all possible ordered pairs inside this class are related. Hence, it contributes n_i^2 related pairs to the relation.

Because the total number of related pairs is given as 42, we need to find positive integers $n_1, n_2, \dots, n_k$ satisfying both:

$$n_1 + n_2 + \dots + n_k = 10$$

and

$$n_1^2 + n_2^2 + \dots + n_k^2 = 42$$

Now, we search for combinations of squares that add up to 42, while the corresponding numbers add up to 10.

Possible combinations are:

$$42 = 36 + 4 + 1 + 1 \Rightarrow \{6, 2, 1, 1\}$$

$$42 = 25 + 16 + 1 \Rightarrow \{5, 4, 1\}$$

Hence, the partitions can be $\{6, 2, 1, 1\}$ and $\{5, 4, 1\}$.

Case 1: Partition $\{5, 4, 1\}$

First choose 5 elements out of 10 for one class, then choose 4 out of the remaining 5 for the next class, and the last one forms a single-element class. The number of ways is:

$${}^{10}C_5 \times {}^5C_4 \times {}^1C_1 = 1260$$

Case 2: Partition $\{6, 2, 1, 1\}$

First choose 6 elements out of 10, then 2 from the remaining 4, and the last two each become single-element classes. Since the two single-element groups are identical, we divide by $2!$ to adjust for overcounting. So the number of ways is:

$${}^{10}C_6 \times {}^4C_2 \times \frac{{}^2C_1 \times {}^1C_1}{2} = 1260$$

Therefore, the total number of such equivalence relations is the sum of both cases:

$$1260 + 1260 = 2520$$

Question3

Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S , but 0210222 is NOT in S .

Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x , is equal to _____.

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Answer: 762

Solution:

Given & Asked

- Digits allowed: $\{0, 1, 2\}$; 7-digit numbers; first digit $\neq 0$
- Count numbers where at least one of 0 or 1 appears exactly twice.

Concept (in 1–2 lines)

Use combinatorics with Inclusion–Exclusion. Count:

- A : exactly two 0's (leading digit not 0),
 - B : exactly two 1's (leading digit not 0),
- then $|A \cup B| = |A| + |B| - |A \cap B|$.

Formulas Used

- Combinations: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
- Inclusion–Exclusion: $|A \cup B| = |A| + |B| - |A \cap B|$

Step-by-Step Solution

1) Count $|A|$: exactly two zeros

- Zeros cannot be in the first place. Choose their positions among the last 6: $\binom{6}{2}$.
- Remaining 5 positions are filled with 1 or 2: 2^5 .

$$|A| = \binom{6}{2} \cdot 2^5 = 15 \cdot 32 = 480.$$

2) Count $|B|$: exactly two ones (first digit $\neq 0$)

Count by first digit:

- First digit = 1: choose the other 1 among remaining 6: $\binom{6}{1} = 6$; fill other 5 by 0/2: 2^5 .
$$6 \cdot 2^5 = 6 \cdot 32 = 192.$$
- First digit = 2: choose positions of two 1's among remaining 6: $\binom{6}{2} = 15$; fill other 4 by 0/2: $2^4 = 16$.
$$15 \cdot 16 = 240.$$
- First digit = 0 is not allowed.

Hence,

$$|B| = 192 + 240 = 432.$$

3) Count $|A \cap B|$: exactly two 0's and two 1's

- Place two 0's among last 6 positions: $\binom{6}{2}$.
- From remaining 5 positions (including the first), choose two for 1's: $\binom{5}{2}$.
- Remaining 3 positions are 2's.

$$|A \cap B| = \binom{6}{2} \binom{5}{2} = 15 \cdot 10 = 150.$$

4) Apply Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B| = 480 + 432 - 150 = \boxed{762}.$$

Quick Tip

Don't forget the leading-digit restriction (no leading 0). Also subtract the overlap where both 0 and 1 appear exactly twice to avoid double counting.

Final Answer

762

Question 4

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$ and $Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$.

Let $n(A)$ denote the number of elements in a set A .

If $n(X) = {}^m C_6$, then the value of m is _____

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Answer: 20

Solution:

$$|a - b| \geq 2 \text{ or } |b - a| = 2$$

Total

$$a = 1 \quad b = 3, 4, 5, 6 \quad 8$$

$$a = 2 \quad b = 4, 5, 6 \quad 6$$

$$a = 3 \quad b = 5, 6 \quad 4$$

$$a = 4 \quad b = 6 \quad 2$$

$$\text{sum} = 20$$

$$n(X) = {}^{20}C_6 = {}^mC_6$$

$$m = 20$$

Question 5

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$ and $Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$.

Let $n(A)$ denote the number of elements in a set A .

If the value of $n(Y) + n(Z)$ is k^2 , then $|k|$ is _____.

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Answer: 36

Solution:

given $|a - b| \geq 2$ so if

$$a = 1 \quad b = 3, 4, 5, 6 \rightarrow 4 \times 2 = 8$$

$$a = 1 \quad b = 4, 5, 6 \rightarrow 3 \times 2 = 6$$

$$a = 1 \quad b = 5, 6 \rightarrow 2 \times 2 = 4$$

$$a = 1 \quad b = 6 \rightarrow 2 \times 1 = 2$$

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i.e. Total elements in X is ${}^{20}C_6$

Now for $n(Y)$, range of R has exactly one element i.e. second elements must be constant in R and since R must have 6 element so it is not possible to satisfy both condition so $n(Y) = 0$.

for $n(z)$

$$\begin{aligned} 1 &\rightarrow 3, 4, 5, 6 \\ 2 &\rightarrow 4, 5, 6 \\ 3 &\rightarrow 1, 5, 6 \\ 4 &\rightarrow 1, 2, 6 \\ 5 &\rightarrow 1, 2, 3 \\ 6 &\rightarrow 1, 2, 3, 4 \end{aligned}$$

no. of relation that are function will be

$$\begin{aligned} &= {}^4C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^4C_1 \\ &= (4 \times 3 \times 3)^2 = k^2 \\ \text{i.e. } k &= 36 \end{aligned}$$

Question6

A group of 9 students, $s_1, s_2, \dots, s_9$, is to be divided to form three teams X, Y , and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X , and s_2 cannot be selected for the team Y . Then the number of ways to form such teams, is _____.

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Answer: 665

Solution:

x	y	z
2	3	4
$\overline{S_1}$	$\overline{S_2}$	

C-i) When x does not contain S_1 , but contains S_2

$${}^7C_1 \times \frac{7!}{3!4!} = 245$$

for x for y,z

C-ii) When x does not contain S_1 , S_2 and y does not contain S_2

$$\text{i.e. } {}^7C_2 \times \frac{6!}{3!3!} = 420$$

for x for y,z

so total No. of ways 665

Question7

The number of 4-digit integers in the closed interval $[2022, 4482]$ formed by using the digits 0, 2, 3, 4, 6, 7 is _____.

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Answer: 569

Solution:



This is a 4-digit integer whose 0th digit called D, 10th digit called C, 100th digit called B and 1000th digit called A.

Given range for possible number is 2022 to 4482.

So, position A can have digits 2, 3 or 4.

0, 6 and 7 can't put in A as number starts with 0, 6 or 7 don't fall in the range between 2022 to 4482.

So, Position A can be filled with 2, 3 or 4 in ${}^3C_1 = 3$ ways.

Position B can be filled by one of 0, 2, 3, 4, 6, 7

So, total possible ways for position B = ${}^6C_1 = 6$ ways

Similarly, Position C can be filled in ${}^6C_1 = 6$ ways

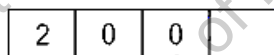
Position D can be filled in ${}^6C_1 = 6$ ways

$\therefore$ Total possible integer numbers starting with 2 or 3 or 4

$$= 3 \times 6 \times 6 \times 6 = 3 \times 216 = 648$$

Now let's find those numbers which start with 2 and 4 but don't fall in the range between 2022 to 4482.

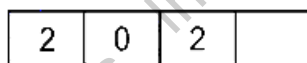
Situation 1 :



Any one of 6 digits among 0, 2, 3, 4, 6, 7 can be present here.

$\therefore$ Total possible numbers starting with 200 are = 6

Situation 2 :



This place can be filled with only "0" which produces number less than 2022.

$\therefore$ Total possible number in this case = 1

Situation 3 :



This place can be filled with 6 or 7 which will produce number more than 4482

Those two places can be filled with any one of 6 digits.

$\therefore$ Total numbers in this case = $2 \times 6 \times 6 = 72$

∴ Total number that don't fall in the range 2022 to 4482 which starts with 2 or 4 are

$$= 6 + 1 + 72 = 79$$

∴ Total numbers falls in the range 2022 to 4482

$$= 648 - 79$$

$$= 569$$

Question8

An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that no two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is

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Answer: 495

Solution:

Let the engineer visits the factory first time after x_1 days to 1 June, second time after x_2 days to first visit and so on.

$$\therefore x_1 + x_2 + x_3 + x_4 + x_5 = 11$$

where $x_1, x_5 \geq 0$ and $x_2, x_3, x_4 \geq 1$ according to the requirement of the question.

Now, let $x_2 = a + 1$, $x_3 = b + 1$ and $x_4 = c + 1$ where $a, b, c \geq 0$

∴ New equation will be

$$x_1 + a + b + c + x_5 = 8$$

Now, the number of all possible ways in which the engineer can made visits is equals to the non-negative integral solution of equation

$x_1 + a + b + c + x_5 = 8$, and it is equal to

$${}^{8+5-1}C_{5-1} = {}^{12}C_4 = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2} = 495$$

Question9

In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is

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Answer: 1080

Solution:

The groups of persons can be made only in 2, 2, 1, 1

$\therefore$ So the number of required ways is equal to number of ways to distribute the 6 distinct objects in group sizes 1, 1, 2 and 2

$$\begin{aligned} &= \frac{6!}{(2!)^2(1!)^2(2!)(2!)} (4!) \\ &= 360 \times 3 = 1080 \end{aligned}$$