



Matrices and Determinants

Question 1

Consider the matrix

$$M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}.$$

Let p, q, r, s, a, b, c and d be integers such that

$$M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \quad \text{and} \quad \sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then which of the following statements is (are) TRUE?

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Options:

A.

There exists a 2×2 invertible matrix N with real entries such that

$$MN = N \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

B.

The value of a is 378

C.

For any two given integers m and n , there exist unique integers x and y such that

$$px + qy = m \quad \text{and} \quad rx + sy = n$$

D.

For each positive real number t , the system of linear equations

$$\begin{aligned} (a+t)x + by &= 1 \\ cx + (d+t)y &= -1 \end{aligned}$$

has a unique solution

Answer: A

Solution:

We can write the matrix as

$$M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, let $B = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$. Then $M = B + I$.

Find B^2

$$B^2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This means B is a nilpotent matrix, i.e., its square is a zero matrix.

Expand M^{26} using the Binomial Theorem

$$M^{26} = (B + I)^{26} = {}^{26}C_0 I + {}^{26}C_1 B + {}^{26}C_2 B^2 + \dots + {}^{26}C_{26} B^{26}$$

As $B^2 = 0$, all higher powers of B are zero. So,

$$M^{26} = I + 26B$$

Substitute the values

$$M^{26} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 26 \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix}$$

So, $p = 27$, $q = -26$, $r = 26$, $s = -25$.

Find the sum $D = \sum_{k=1}^{26} M^k$

$$D = M + M^2 + M^3 + \dots + M^{26}$$

Substitute $M = B + I$:

$$D = (B + I) + (B + I)^2 + (B + I)^3 + \dots + (B + I)^{26}$$

Since $(B + I)^k = I + kB$, we have

$$D = (I + B) + (I + 2B) + (I + 3B) + \dots + (I + 26B)$$

Combine like terms:

$$D = 26I + B(1 + 2 + 3 + \dots + 26)$$

The sum $1 + 2 + 3 + \dots + 26 = 351$, so

$$D = 26I + 351B$$

Substitute the matrices:

$$\sum_{k=1}^{26} M^k = \begin{bmatrix} 377 & -351 \\ 351 & -325 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Hence, $a = 377$, $b = -351$, $c = 351$, $d = -325$.

So, option (B) is incorrect.

Check option (C)

Given system of equations:

$$27x - 26y = m$$

$$26x - 25y = n$$

Determinant of coefficients:

$$\begin{vmatrix} 27 & -26 \\ 26 & -25 \end{vmatrix} \neq 0$$

Since the determinant is non-zero, a unique solution exists.

On solving, we get:

$$x = -24n - 25m, \quad y = -23n - 26m$$

Both x and y are integers. Hence, option (C) is correct.

Check option (D)

We have:

$$\begin{vmatrix} a+t & b \\ c & d+t \end{vmatrix} = ad + at + dt + t^2 - bc$$

Substitute $a = 377$, $b = -351$, $c = 351$, $d = -325$:

$$= (377)(-325) + t(52) + t^2 + (351)(351) = t^2 + 52t + 676 = (t + 26)^2$$

Since $(t + 26)^2 \neq 0$ for all $t > 0$, the determinant is non-zero, so a unique solution exists. Option (D) is correct.

Check option (A)

$$\text{Let } N = \begin{bmatrix} j & k \\ l & m \end{bmatrix}.$$

Compute MN :

$$MN = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} j & k \\ l & m \end{bmatrix} = \begin{bmatrix} 2j - l & 2k - m \\ j & k \end{bmatrix}$$

Compute $N \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$:

$$N \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} j & j+k \\ l & l+m \end{bmatrix}$$

Comparing both, we get $j=l$ and $2k-m=j+k \Rightarrow m-k=j$.

Determinant of N :

$$|N| = jm - lk = j(m-k)$$

This is not always zero if $j \neq 0$ and $m \neq k$, which means N can be invertible. Hence, option (A) is correct.

Question2

Which one of the following matrices can be obtained by performing elementary row transformations on the 3×3 identity matrix?

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Options:

A.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

B.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

C.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$$

D.

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

Answer: B

Solution:

Let us assume that the new matrix we get after performing elementary row operations on the identity matrix is A .

This means,

$$A = E_n E_{n-1} \dots E_1 I$$

Here, each E_i represents an elementary matrix (a matrix obtained by performing one elementary row operation on the identity matrix).

Now, the determinant of A can be written as:

$$\det(A) = \prod_{k=1}^n \det(E_k)$$

For an elementary matrix, $\det(E_k)$ can be either 1, -1 , or any non-zero number k . Therefore,

$$\det(A) \neq 0$$

This tells us that any matrix obtained from the identity matrix through elementary row transformations must have a non-zero determinant.

Now, let's check the determinant of each given option:

$$(A) \det \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = 0$$

$$(B) \det \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{pmatrix} = -2 \neq 0$$

$$(C) \det \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{pmatrix} = 0$$

$$(D) \det \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix} = 0$$

Since only option (B) has a non-zero determinant, this means that it can be obtained from the identity matrix through elementary row operations.

The steps to obtain matrix (B) from the 3×3 identity matrix are shown below:

Start with:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Perform $R_2 \rightarrow R_2 + 2R_3$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Next, $R_1 \rightarrow R_1 - R_3$:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, $R_3 \rightarrow -2R_3$:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

Now, $R_3 \rightarrow R_3 + R_2$:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

Next, $R_1 \rightarrow R_1 + R_2$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

Then, $R_3 \rightarrow R_3 + R_1$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Finally, $R_2 \rightarrow R_2 + 2R_1$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

Hence, option (B) can indeed be obtained by performing elementary row transformations on the 3×3 identity matrix.

Question3

Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y , and z , for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$.

Then which of the following statements is (are) TRUE?

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Options:

A.

The determinant of $Q - 2I$ is zero

B.

The determinant of $Q - 6I$ is 12

C.

The determinant of $Q - 3I$ is 15

D.

$$yz = 2$$

Answer: A

Solution:

Let us consider the matrices given:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$$

$$R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where x, y, z, a, b, c, d are non-zero real numbers. We aim to verify properties of matrix Q given the relation $QR = RP$.

Steps and Calculations

Matrix Equation Analysis:

Given $QR = RP$, we equate both sides:

$$\begin{pmatrix} ax + cy & bx + dy \\ az + 4c & bz + 4d \end{pmatrix} = \begin{pmatrix} 2a & 3b \\ 2c & 3d \end{pmatrix}$$

From the matrix equality, we derive the following key equations:

$$ax + cy = 2a$$

$$bx + dy = 3b$$

$$az + 4c = 2c$$

$$bz + 4d = 3d$$

Determinant Condition:

Comparing the equations:

$$\text{Set } az = -2c \text{ and } bz = -d, \text{ resulting in } \frac{a}{b} = \frac{2c}{d}.$$

The requirement $|R| \neq 0$ implies $|P| = |Q|$. So, equating determinants gives us:

$$|P| = |Q| \Rightarrow 6 = 4x - yz \quad (\text{Equation 1})$$

Condition for Non-Trivial Solutions:

From:

$$a(x - z) + cy = 0$$

$$az + 2c = 0$$

implies a non-trivial solution matrix must have:

$$\begin{vmatrix} x - z & y \\ z & 2 \end{vmatrix} = 0 \Rightarrow 2x - yz = 4 \quad (\text{Equation 2})$$

Solve for x, yz :

Solving Equations 1 and 2:

$$4x - yz = 6$$

$$2x - yz = 4$$

Subtract the second from the first:

$$(4x - yz) - (2x - yz) = 6 - 4 \Rightarrow 2x = 2 \Rightarrow x = 1$$

Plugging $x = 1$ back, we find:

$$4(1) - yz = 6 \Rightarrow 4 - yz = 6 \Rightarrow yz = -2$$

Verify Determinants:

For $Q - 2I$:

$$|Q - 2I| = \begin{vmatrix} x-2 & y \\ z & 2 \end{vmatrix} = 2x - yz = 0$$

For $Q - 6I$:

$$|Q - 6I| = \begin{vmatrix} x-6 & y \\ z & -2 \end{vmatrix} = -2x + 12 - yz = 12$$

For $Q - 3I$:

$$|Q - 3I| = \begin{vmatrix} x-3 & y \\ z & 1 \end{vmatrix} = x - 3 - yz = 0$$

Therefore, using the derived values, the statements for the determinants align correctly. The findings are:

$$|Q - 2I| = 0$$

$$|Q - 6I| = 12$$

$$yz = -2$$

These calculations validate the conditions for matrix Q concerning the predefined matrix properties.

Question4

Let $\mathbb{R}^2$ denote $\mathbb{R} \times \mathbb{R}$. Let

$$S = \{(a, b, c) : a, b, c \in \mathbb{R} \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x, y) \in \mathbb{R}^2 - \{(0, 0)\}\}.$$

Then which of the following statements is (are) TRUE?

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Options:

A. $(2, \frac{7}{2}, 6) \in S$

B. If $(3, b, \frac{1}{12}) \in S$, then $|2b| < 1$.

C.

For any given $(a, b, c) \in S$, the system of linear equations

$$ax + by = 1$$

$$bx + cy = -1$$

has a unique solution.

D.

For any given $(a, b, c) \in S$, the system of linear equations

$$(a+1)x + by = 0$$

$$bx + (c+1)y = 0$$

has a unique solution.

Answer: B

Solution:

(A) $ax^2 + 2bxy + cy^2 > 0 \forall (x, y) \in \mathbb{R}^2 - \{(0, 0)\}$

$\Rightarrow ax + 2bxy + cy^2$ must represent pair of imaginary lines and $a, c > 0$.

$$\Rightarrow b^2 < ac$$

(B) $b^2 < 3 \times \frac{1}{12} \Rightarrow |2b| < 1$

(C) since $b^2 \neq ac$

$$\Rightarrow ax + by = 1 \text{ and } bx + cy = -1$$

are not parallel lines.

(D) $ac + a + c > b^2 \Rightarrow$ lines are not parallel.

$\Rightarrow$ Options (B), (C), (D) are Correct.

Question5

Let $M = (a_{ij}), i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j+1$ is divisible by i , otherwise $a_{ij} = 0$. Then which of the following statements is(are) true?

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Options:

A. M is invertible

B. There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

C. The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$, where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

D. The matrix $(M - 2I)$ is invertible, where I is the 3×3 identity matrix

Answer: B

Solution:

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|M| = -1 + 1 = 0 \Rightarrow M \text{ is singular so non-invertible} \Rightarrow [A] \text{ is wrong.}$$

(B) $M \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$$\left. \begin{aligned} a_1 + a_2 + a_3 &= -a_1 \\ a_1 + a_3 &= -a_2 \\ a_2 &= -a_3 \end{aligned} \right\} \Rightarrow a_1 = 0 \text{ and } a_2 + a_3 = 0 \text{ infinite solutions exists [B] is correct.}$$

Option (D) :

$$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$|M - 2I| = 0 \Rightarrow [D] \text{ is wrong}$$

Option (C) :

$$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + y + z = 0$$

$$x + z = 0$$

$$y = 0$$

$\therefore$ Infinite solution

[C] is correct

Question6

For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3 , then which of the following statements is(are) TRUE?

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Options:

A. $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

C. $|(EF)^3| > |EF|^2$

D. Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Answer: A

Solution:

For option (a)

$$PEP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix} = F$$

$$\text{and } P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, option (a) is correct.

For option (b)

$$|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}| \dots (i)$$

$$\because |E| = 0 \text{ and } |F| = 0 \text{ and } |Q| \neq 0$$

$$\therefore |EQ| = |E||Q| = 0$$

$$\text{and } |PFQ^{-1}| = \frac{|P||F|}{|Q|} = 0$$

$$\text{Let } R = EQ + PFQ^{-1} \dots (ii)$$

$$\Rightarrow RQ = EQ^2 + PF = EQ^2 + P^2EP = EQ^2 + EP [\because P^2 = I]$$

$$= E(Q^2 + P)$$

$$\Rightarrow |RQ| = |E(Q^2 + P)|$$

$$\Rightarrow |R||Q| = |E||Q^2 + P| = 0 \quad [\because |E| = 0]$$

$$\Rightarrow |R| = 0 \text{ (as } |Q| \neq 0) \dots \text{ (iii)}$$

From Eqs. (ii) and (iii), we get Eq. (i) is true.

Hence, option (b) is correct.

For option (c)

$$|(EF)^3| > |EF|^2$$

i.e. $0 > 0$ which is false.

For option (d)

$$\because P^2 = I \Rightarrow P^{-1} = P$$

$$\therefore P^{-1}FP = PFP = PPEPP = E$$

$$\text{So, } E + P^{-1}FP = E + E = 2E$$

$$\Rightarrow \text{Tr}(E + P^{-1}FP) = \text{Tr}(2E) = 2\text{Tr}(E) \dots \text{ (iv)}$$

$$\text{and } P^{-1}EP + F$$

$$\Rightarrow PEP + F = 2PEP \quad [\because F = PEP]$$

$$\therefore \text{Tr}(2PEP) = 2\text{Tr}(PEP) = 2\text{Tr}(F) = 2\text{Tr}(E) \dots \text{ (v)}$$

From Eqs. (iv) and (v) option (d) is also correct.

Question 7

For any 3×3 matrix M , let $|M|$ denote the determinant of M . Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE?

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Options:

A. $|FE| = |I - FE| |FGE|$

B. $(I - FE)(I + FGE) = I$

C. $EFG = GEF$

D. $(I - FE)(I - FGE) = I$

Answer: A

Solution:

$$\because I - EF = G^{-1} \Rightarrow G - GEF = I \dots (i)$$

$$\text{and } G - EFG = I \dots (ii)$$

Clearly, $GEF = EFG \rightarrow$ option (c) is correct.

$$\text{Also, } (I - FE)(I + FGE)$$

$$= I - FE + FGE - FEFGE$$

$$= I - FE + FGE - F(G - I)E$$

$$= I - FE + FGE - FGE + FE$$

$$= I \rightarrow \text{option (b) is correct but option (d) is incorrect.}$$

$$\because (I - FE)(I - FGE) = I - FE - FGE + F(G - I)E$$

$$= I - 2FE$$

$$\text{Now, } (I - FE)(-FGE) = -FE$$

$$\Rightarrow |I - FE| |FGE| = |FE|$$

$\rightarrow$ option (a) is correct.

Question 8

Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE?

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Options:

B. $\det M = 1$

C. $M^2 = I$

D. $(\text{adj } M)^2 = I$

Answer: B

Solution:

It is given that matrix M be a 3×3 invertible matrix, such that

$$M^{-1} = \text{adj}(\text{adj } M) \Rightarrow M^{-1} = |M| M$$

$$(\because \text{for a matrix } A \text{ of order 'n' } \text{adj}(\text{adj } A) = |A|^{n-2} A)$$

$$\Rightarrow M^{-1} M = |M| M^2 \Rightarrow M^2 |M| = I \dots (i)$$

$$\because \det(M^2 |M|) = \det(I) = 1$$

$$\Rightarrow |M|^3 |M|^2 = 1 \Rightarrow |M| = 1 \dots (ii)$$

from Eqs. (i) and (ii), we get

$$M^2 = I$$

$$\text{As, } \text{adj } M = |M| M^{-1} = M$$

$$\Rightarrow (\text{adj } M)^2 = M^2 (\text{adj } M)^2 = I$$
