



Limits Continuity and Differentiability

Question 1

Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be an arbitrary function and let $g : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$g(x) = xf(x), \quad \text{for all } x \in \mathbb{R}.$$

Then which of the following statements is (are) TRUE?

JEE Advanced 2026 Paper 1 Online

Options:

A.

The function g is always continuous at $x = 0$

B.

If f is continuous at $x = 0$, then g is differentiable at $x = 0$

C.

If g is differentiable at $x = 0$, then f is continuous at $x = 0$

D.

If g is differentiable at $x = 0$, then $\lim_{x \rightarrow 0} f(x)$ exists

Answer: B

Solution:

(A)

Let

$$f(x) = \begin{cases} \frac{1}{x} \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Now,

$$g(x) = xf(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

When $x \neq 0$, the function $g(x)$ is $\sin\left(\frac{1}{x}\right)$, which keeps oscillating between -1 and 1 as $x \rightarrow 0$. So, $\lim_{x \rightarrow 0} g(x)$ does not exist. Therefore, $g(x)$ is **not continuous** at $x = 0$.

(B)

Assume that $f(x)$ is continuous at $x = 0$. This means $f(0^-) = f(0^+) = f(0)$.

Now consider the derivative of $g(x)$ at $x = 0$:

$$g'(0) = \lim_{h \rightarrow 0} \frac{g(h) - g(0)}{h}$$

Since $g(0) = 0$, this becomes

$$g'(0) = \lim_{h \rightarrow 0} \frac{g(h)}{h} = \lim_{h \rightarrow 0} \frac{hf(h)}{h}$$

$$g'(0) = \lim_{h \rightarrow 0} f(h) = f(0)$$

This limit exists and is finite because $f(x)$ is continuous at $x = 0$. Hence, $g(x)$ is **differentiable** at $x = 0$.

(C)

From the above, if $g(x)$ is differentiable at $x = 0$, then

$$g'(0) = \lim_{h \rightarrow 0} f(h)$$

This shows that the limit of $f(h)$ as $h \rightarrow 0$ exists. However, we cannot say anything about the value of $f(0)$. So, even if $g(x)$ is differentiable at $x = 0$, we cannot conclude that $f(x)$ is continuous at $x = 0$.

(D)

This statement is correct and follows from the reasoning in option (C).

Question2

Consider the function $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow (-\infty, \infty)$ defined by

$$f(x) = (|x| + |x - 1|) \sin x + [x \sin x],$$

where $[x \sin x]$ is the greatest integer less than or equal to $x \sin x$.

Let α be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is NOT continuous, and let β be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is NOT differentiable.

Then the value of $\alpha + \beta$ is _____.

JEE Advanced 2026 Paper 1 Online

Answer: 5

Solution:

Let us write the function as $f(x) = g(x) + h(x)$, where

$$g(x) = (|x| + |x - 1|) \sin x \text{ and } h(x) = [x \sin x].$$

Check continuity of $g(x)$

The expression $|x| + |x - 1|$ is made up of absolute value functions. Absolute value functions are always continuous, and sine is also continuous. Therefore, $g(x)$ is continuous for all x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Check continuity of $h(x) = [x \sin x]$

The greatest integer function $[x \sin x]$ is discontinuous whenever $x \sin x$ takes an integer value, because the value jumps at those points. So we must find where $x \sin x$ equals an integer.

Case (i): $x \sin x = 0 \Rightarrow x = 0$

At $x = 0$, the function $h(x) = [x \sin x]$ does not actually jump, so it is continuous at $x = 0$. Hence, $f(x)$ is continuous there.

Case (ii): $x \sin x = 1 \Rightarrow x = x_1$ and because $x \sin x$ is even, we also have another point $x = -x_1$.

Just before x_1 , $[x \sin x] = 0$; just after x_1 , $[x \sin x] = 1$. So there is a jump at $x = x_1$ and at $x = -x_1$.

Therefore, $f(x)$ is **not continuous** at $x = x_1$ and $x = -x_1$.

Hence, $\alpha = 2$.

Check differentiability

We already know $f(x)$ is not continuous at $x = x_1, -x_1$, so it cannot be differentiable there either.

We must also test $x = 0$ and $x = 1$, since $g(x)$ involves absolute values and these points can cause problems with slope.

At $x = 0$:

$f'(0^-) = f'(0^+) = 1$, hence $f(x)$ is differentiable at $x = 0$.

At $x = 1$:

$f'(1^-) = \cos 1$ and $f'(1^+) = \cos 1 + 2 \sin 1$

Since these two are not equal, $f(x)$ is **not differentiable** at $x = 1$.

Therefore, the points of non-differentiability are $x = x_1, -x_1$, and 1.

Hence, $\beta = 3$.

Final Answer:

$\alpha + \beta = 5$

Question 3

For a real number α , let $[\alpha]$ denote the greatest integer less than or equal to α . For a finite set S , let $|S|$ denote the number of elements in the set S .

Consider the functions $f : (-3, 3) \rightarrow (-\infty, \infty)$ and $g : (-3, 3) \rightarrow (-\infty, \infty)$ defined by

$$f(x) = [x^3] \log_e (1 + \sin^2(\pi(x - [x])))$$

and

$$g(x) = x^3 \sin^2(\pi \log_e(1 + x - [x])).$$

Let

$$A = \{x \in (-3, 3) : f \text{ is discontinuous at } x\}$$

and

$$B = \{x \in (-3, 3) : g \text{ is discontinuous at } x\}.$$

Then the value of $|A| + 2|B| - |A \cap B|$ is _____.

JEE Advanced 2026 Paper 2 Online

Answer: 56

Solution:

We are given:

$$f(x) = [x^3] \log (1 + \sin^2(\pi(x - [x])))$$

and

$$g(x) = x^3 \sin^2(\pi \log_e(1 + x - [x])),$$

for $x \in (-3, 3)$.

Simplify the expressions

Since $(x - [x])$ represents the fractional part of x , we can replace it by $\{x\}$. Then, because $\sin(\pi(x - [x])) = \sin(\pi x)$, we have

$$f(x) = [x^3] \log(1 + \sin^2 \pi x).$$

Here, $[x^3]$ is the greatest integer less than or equal to x^3 , and the fraction part repeats every integer shift in x . So $f(x)$ behaves differently in each interval between consecutive integers.

Determine where $f(x)$ is discontinuous

The expression $\log(1 + \sin^2 \pi x)$ is always continuous and periodic. The only chance of discontinuity comes from $[x^3]$ because it jumps whenever x^3 crosses an integer.

Since $x \in (-3, 3)$, x^3 takes values from -27 to 27 . The integers in this range are from -26 to 26 , a total of 53 integers. Whenever x^3 equals one of these integers, $[x^3]$ changes value.

At points where x^3 is an integer, $f(x)$ is likely discontinuous, except possibly where $\log(1 + \sin^2 \pi x) = 0$. That happens when $\sin \pi x = 0$, i.e., when x is an integer.

Therefore, $f(x)$ is continuous at $x = -2, -1, 0, 1, 2$ but discontinuous where x^3 is an integer and x is not an integer. So, out of 53 points, 5 are continuous, giving:

$$|A| = 53 - 5 = 48.$$

Determine where $g(x)$ is discontinuous

We have

$$g(x) = x^3 \sin^2(\pi \log(1 + \{x\})).$$

Here $\{x\} = x - [x]$, which is continuous everywhere except at integers. The function $\log(1 + \{x\})$ is defined for $\{x\} \in [0, 1)$, so it is continuous within all open intervals between integers.

At integer values of x , $\{x\}$ jumps from 1^- to 0, leading to discontinuities. So $g(x)$ is discontinuous only at integer points within $(-3, 3)$, i.e., $x = -2, -1, 0, 1, 2$.

However, checking $x = 0$, we find $g(0) = 0$, and the limit from both sides is also 0, making it continuous at $x = 0$. Hence, discontinuities are at $x = \pm 1, \pm 2$.

Thus, $|B| = 4$.

Intersection of discontinuities

Since $f(x)$ is continuous at $x = \pm 1, \pm 2$, none of these belong to both A and B .

Hence, $|A \cap B| = 0$.

Final computation

$$|A| + 2|B| - |A \cap B| = 48 + 8 - 0 = 56.$$

Question 4

Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α , define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x .

Then which one of the following statements is TRUE?

JEE Advanced 2025 Paper 2 Online

Options:

A.

For $\alpha = 2, \lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

B.

For $\alpha = 2, \lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

C.

For $\alpha = 3, \lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

D.

For $\alpha = 3, \lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Answer: C

Solution:

$$e^{x_0} + x_0 = 0$$

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

$$g(x) = x - \frac{\alpha(e^x + x)}{3(e^x + 1)}, g(x) + e^{x_0} = x + e^{x_0} - \frac{\alpha}{3} \left(\frac{e^x + x}{e^x + 1} \right)$$

$$g(x) + e^{x_0} = (x - x_0) - \frac{\alpha}{3} \left(\frac{e^x + x}{e^x + 1} \right)$$

$$\Rightarrow \text{As very clear } g(x_0) + x_0 \Rightarrow 0$$

$$\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| \frac{0^+}{0^+} = \lim_{x \rightarrow x_0} \left| \frac{g'(x)}{1} \right| = |g'(x_0)|$$

$$g'(x) = 1 - \frac{\alpha}{3} \left(\frac{(e^x + 1)(e^x + 1) - (e^x + x)e^x}{(e^x + 1)^2} \right)$$

$$g'(x_0) = 1 - \frac{\alpha}{3} \left(\frac{(e^{x_0} + 1)^2}{(e^{x_0} + 1)^2} \right) = 1 - \frac{\alpha}{3}$$

$$\Rightarrow \lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = |g'(x_0)| = \left| 1 - \frac{\alpha}{3} \right|$$

Question5

Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

JEE Advanced 2025 Paper 1 Online

Options:

A.

The function f is **NOT** differentiable at $x = 0$

B.

There is a positive real number δ , such that f is a decreasing function on the interval $(0, \delta)$

C.

For any positive real number δ , the function f is **NOT** an increasing function on the interval $(-\delta, 0)$

D.

$x = 0$ is a point of local minima of f

Answer: C

Solution:

The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by:

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin \frac{1}{x}, & \text{if } x \neq 0, \\ 2, & \text{if } x = 0. \end{cases}$$

Explanation:

(A) To determine differentiability at $x = 0$, consider the right-hand derivative (RHD):

$$\lim_{h \rightarrow 0} \frac{(2 - 2h^2 - h^2 \sin \frac{1}{h}) - 2}{h} = 0$$

Similarly, for the left-hand derivative (LHD) at $x = 0$, we also find it equals 0. Therefore, f is differentiable at $x = 0$.

(B) For $x \in (0, \delta)$, the derivative of f is:

$$f'(x) = -(4x + 2x \sin \frac{1}{x}) + \cos \frac{1}{x}$$

Thus:

$$f'(x) = -(2x(4 - \sin \frac{1}{x})) + \cos \frac{1}{x}$$

Since $\cos \frac{1}{x}$ oscillates, we cannot assert that $f(x)$ is strictly decreasing on $(0, \delta)$.

(C) For $x \in (-\delta, 0)$ and any $\delta > 0$, $f(x)$ is not increasing since $\cos \frac{1}{x}$ oscillates between -1 and 1.

(D) Evaluating at $x = 0$:

$$f(0) = 2$$

For small h :

$$f(0 + h) < 2 \quad \text{and} \quad f(0 - h) < 2$$

Therefore, $x = 0$ is a local maximum, not a local minimum.

Question 6

Let $\mathbb{R}$ denote the set of all real numbers. For a real number x , let $[x]$ denote the greatest integer less than or equal to x . Let n denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) The minimum value of n for which the function $f(x) = \left[\frac{10x^3 - 45x^2 + 60x + 35}{n} \right]$ is continuous on the interval $[1, 2]$, is	(1) 8
(Q) The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	(2) 9
(R) The smallest natural number n which is greater than 5, such that $x = 3$ is a point of local minima of $h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$ is	(3) 5
(S) Number of $x_0 \in \mathbb{R}$ such that $l(x) = \sum_{k=0}^4 \left(\sin \left x - k \right + \cos \left x - k + \frac{1}{2} \right \right)$ $x \in \mathbb{R}$, is NOT differentiable at x_0 , is	(4) 6

List-I	List-II
	(5) 10

JEE Advanced 2025 Paper 1 Online

Options:

A.

(P) $\rightarrow$ (1) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (2) (S) $\rightarrow$ (5)

B.

(P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)

C.

(P) $\rightarrow$ (5) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)

D.

(P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

Answer: B

Solution:

(P) Let $g(x) = 10x^3 - 45x^2 + 60x + 35$

$g'(x) = 30(x^2 - 3x + 2) = 30(x-1)(x-2)$



$\Rightarrow g(x)$ is decreasing in $[1, 2]$

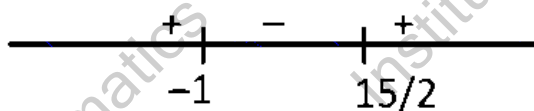
Now, $f(1) = \left\lfloor \frac{g(1)}{n} \right\rfloor = \left\lfloor \frac{60}{n} \right\rfloor, f(2) = \left\lfloor \frac{55}{n} \right\rfloor$

For $f(x)$ to be continuous in $[1, 2]$ its integral value should remain same in whole interval for $n = 9, f(1) = 6, f(2) = 6$

(P) $\rightarrow$ (2)

(Q)

$g'(x) = (2n^2 - 13n - 15)(3x^2 + 3)$
 $= (2n - 15)(n + 1)(3x^2 + 3) > 0$



for $g(x)$ to be increasing $\min n = 8$

$\Rightarrow Q \rightarrow 1$ matching

(R) $h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$

$h(3) = 0$ for $n > 5$

at $n = 6 \Rightarrow n(3 + \delta) > h(3)$

$n(3 - \delta) > n(3)$

$\Rightarrow h(x)$ has local minima at $x = 3$ for $n = 6$

$R \rightarrow 4$ matching

$$(S) \quad I(x) = \sin|x| + \cos\left|x + \frac{1}{2}\right| + \sin|x-1| + \cos\left|x - \frac{1}{2}\right| + \dots + \sin|x-4| + \cos\left|x - \frac{7}{2}\right|$$

as $\sin|x-a|$ is non differentiable at $x=a$

but $\cos|x-a|$ remains differentiable at $x=a$

$\Rightarrow$ given $I(x)$ is non-differentiable at

$$x_0 = 0, 1, 2, 3, 4 \text{ (5 points)}$$

$(S) \rightarrow (3)$ matching

$(P) \rightarrow (2)(Q) \rightarrow (1)(R) \rightarrow (4)(S) \rightarrow (3)$

Question7

Let $k \in \mathbb{R}$. If $\lim_{x \rightarrow 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

JEE Advanced 2024 Paper 2 Online

Options:

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2}{x} (\sin(\sin kx) + \cos x + x - 1) &= 6 \\ \lim_{x \rightarrow 0} \frac{\sin(\sin kx) \cdot \sin kx}{(\sin kx)kx} \cdot k + 1 - \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \cdot x &= 3 \\ k + 1 = 3 &\Rightarrow k = 2 \end{aligned}$$

Question8

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

JEE Advanced 2024 Paper 2 Online

Options:

- A. $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.
- B. $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

C. The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

D. $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

Answer: D

Solution:

Option-A : $f(x) = x^2 \sin \frac{\pi}{x^2}$

$f(x) = 0 \Rightarrow \sin \frac{\pi}{x^2} = 0 \Rightarrow \frac{\pi}{x^2} = n\pi, n \in \mathbb{N}$

$x^2 = \frac{1}{n} \Rightarrow x = \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{n}} \geq \frac{1}{10^{10}} \Rightarrow 10^{10} \geq \sqrt{n} \Rightarrow n \leq 10^{20}$, finite number of solutions

Option-B : $x = \frac{1}{\sqrt{n}} \Rightarrow \frac{1}{\sqrt{n}} > \frac{1}{\pi} \Rightarrow \pi > \sqrt{n} \Rightarrow n < \pi^2$, Number of solutions is 9

Option-C : $x = \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}} < \frac{1}{10^{10}} \Rightarrow \sqrt{n} > 10^{10} \Rightarrow n > 10^{20}$, Infinite number of solutions

Option-D : $\frac{1}{\pi^2} < \frac{1}{\sqrt{n}} < \frac{1}{\pi} \Rightarrow \sqrt{n} \in (\pi, \pi^2) \Rightarrow n \in (\pi^2, \pi^4)$, Definitely more than 25 solutions

Question9

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x|x| \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \text{ and } g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + dg(x), x \in \mathbb{R}.$$

Match each entry in List-I to the correct entry in List-II.

List-I	List-II
(P) If $a = 0, b = 1, c = 0$, and $d = 0$, then	(1) h is one-one.
(Q) If $a = 1, b = 0, c = 0$, and $d = 0$, then	(2) h is onto.
(R) If $a = 0, b = 0, c = 1$, and $d = 0$, then	(3) h is differentiable on $\mathbb{R}$.
(S) If $a = 0, b = 0, c = 0$, and $d = 1$, then	(4) the range of h is $[0, 1]$.
	(5) the range of h is $\{0, 1\}$.

The correct option is

JEE Advanced 2024 Paper 1 Online

Options:

A. (P) $\rightarrow$ (4)(Q) $\rightarrow$ (3)(R) $\rightarrow$ (1)(S) $\rightarrow$ (2)

B. (P) $\rightarrow$ (5)(Q) $\rightarrow$ (2)(R) $\rightarrow$ (4)(S) $\rightarrow$ (3)

C. (P) $\rightarrow$ (5)(Q) $\rightarrow$ (3)(R) $\rightarrow$ (2)(S) $\rightarrow$ (4)

D. (P) $\rightarrow$ (4)(Q) $\rightarrow$ (2)(R) $\rightarrow$ (1)(S) $\rightarrow$ (3)

Answer: C

Solution:

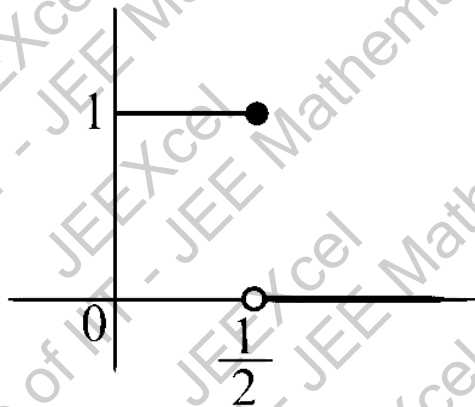
$$f(x) = \begin{cases} x|x| \sin \frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases} \quad g(x) = \begin{cases} 1 - 2x & 0 \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$g\left(\frac{1}{2} - x\right) = \begin{cases} 2x & ; 0 \leq \frac{1}{2} - x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 2x & ; 0 \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1 & ; 0 \leq x \leq \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

(P) Now $a = 0, b = 1, c = 0, d = 0$

$$\therefore h(x) = g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1 & ; 0 \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



Hence Range of $h(x)$ is $\{0, 1\}$

(Q) $a = 1, b = 0, c = 0, d = 0$

$$h(x) = f(x) = \begin{cases} x|x| \sin \frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

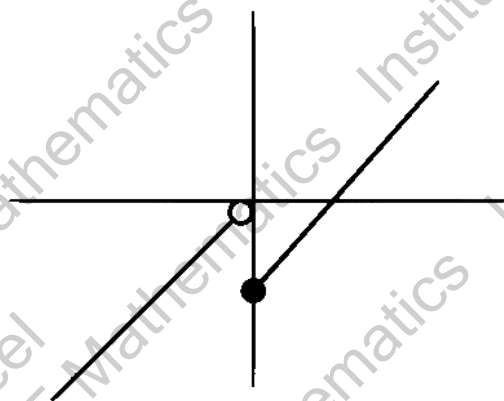
$$RHD = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - 0}{x} = 0$$

$$LHD = \lim_{x \rightarrow 0} \frac{-x^2 \sin \frac{1}{x} - 0}{x} = 0$$

Hence $h(x)$ is differentiable on $\mathbb{R}$

(R) $a = 0, b = 0, c = 1, d = 0$

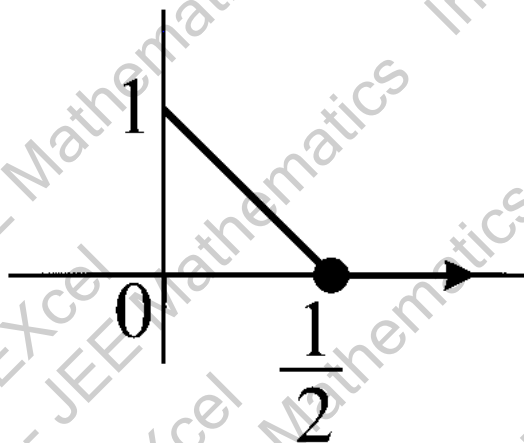
$$h(x) = x - g(x) = \begin{cases} 3x - 1 & ; 0 \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



$\therefore h(x)$ is ONTO

(S) $a = 0, b = 0, c = 0, d = 1$

$$h(x) = g(x) = \begin{cases} 1 - 2x & ; 0 \leq x \leq \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$



Range of $h(x)$ is $[0, 1]$

Question 10

For positive integer n , define

$$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \dots + \frac{25n-7n^2}{7n^2}.$$

Then, the value of $\lim_{n \rightarrow \infty} f(n)$ is equal to :

JEE Advanced 2022 Paper 2 Online

Options:

A. $3 + \frac{4}{3} \log_e 7$

B. $4 - \frac{3}{4} \log_e \left(\frac{7}{3}\right)$

C. $4 - \frac{4}{3} \log_e \left(\frac{7}{3}\right)$

D. $3 + \frac{3}{4} \log_e 7$

Answer: B

Solution: