



Inverse Trigonometric Functions

Question 1

Considering only the principal values of the inverse trigonometric functions, the value of

$$\cot^{-1}(\cot(-11)) + 10 \sin\left(2 \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10 \sin(2 \tan^{-1}(2))$$

is

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Options:

A.

$$3\pi + 7$$

B.

$$7$$

C.

$$4\pi + 7$$

D.

$$3\pi - 5$$

Answer: C

Solution:

We need to find the value of

$$\cot^{-1}(\cot(-11)) + 10 \sin\left(2 \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10 \sin(2 \tan^{-1}(2))$$

Simplify $\cot^{-1}(\cot(-11))$

The principal value of $\cot^{-1} x$ lies in the interval $(0, \pi)$. We need to find an angle in this range that has the same cotangent as -11 .

Now, -11 radians is less than 0. If we add multiples of π till it falls in $(0, \pi)$, we get $4\pi - 11$, which lies in $(0, \pi)$.

$$\text{Hence, } \cot^{-1}(\cot(-11)) = 4\pi - 11.$$

$$\text{Simplify } 10 \sin\left(2 \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$\text{Let } \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \alpha. \text{ Then, } \cos \alpha = \frac{1}{\sqrt{2}}. \text{ This gives } \alpha = \frac{\pi}{4}.$$

$$\text{Now, } 2\alpha = \frac{\pi}{2}. \sin(2\alpha) = \sin\left(\frac{\pi}{2}\right) = 1.$$

Therefore, this term becomes $10 \times 1 = 10$.

$$\text{Simplify } 10 \sin(2 \tan^{-1}(2))$$

$$\text{Let } \tan^{-1}(2) = \theta, \text{ so } \tan \theta = 2. \text{ Here, } \theta \in \left(0, \frac{\pi}{2}\right).$$

$$\text{We use the double angle formula: } \sin(2\theta) = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \times 2}{1 + 2^2} = \frac{4}{5}.$$

$$\text{Hence, } 10 \sin(2 \tan^{-1}(2)) = 10 \times \frac{4}{5} = 8.$$

Combine all terms

$$\cot^{-1}(\cot(-11)) + 10 \sin\left(2 \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10 \sin(2 \tan^{-1}(2))$$

$$= (4\pi - 11) + 10 + 8$$

$$= 4\pi + 7$$

Final Answer: $4\pi + 7$

Question 2

The total number of real solutions of the equation

$$\theta = \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{6 \tan \theta}{9 + \tan^2 \theta} \right)$$

is

(Here, the inverse trigonometric functions $\sin^{-1} x$ and $\tan^{-1} x$ assume values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $(-\frac{\pi}{2}, \frac{\pi}{2})$, respectively.)

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Options:

A.

1

B.

2

C.

3

D.

5

Answer: C

Solution:

To find the total number of real solutions for the given equation, let's break down the problem step by step:

Define α as:

$$\alpha = \frac{1}{2} \sin^{-1} \left(\frac{6 \tan \theta}{9 + \tan^2 \theta} \right)$$

We need to solve:

$$\theta = \tan^{-1}(2 \tan \theta) - \alpha$$

This can be rearranged as:

$$\theta + \alpha = \tan^{-1}(2 \tan \theta)$$

Taking the tangent of both sides, we have:

$$\tan(\theta + \alpha) = 2 \tan \theta$$

The tangent addition formula gives:

$$\frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha} = 2 \tan \theta \quad \dots (1)$$

Next, consider:

$$\sin 2\alpha = \frac{6 \tan \theta}{9 + \tan^2 \theta} = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

This leads to the equation:

$$3 \tan \theta + 3 \tan \theta \tan^2 \alpha = 9 \tan \alpha + \tan \alpha \tan^2 \theta$$

Simplifying:

$$3(\tan \theta - 3 \tan \alpha) = \tan \alpha \tan \theta (\tan \theta - 3 \tan \alpha)$$

This implies:

$$\tan \theta = \frac{3}{\tan \alpha} \quad \text{or} \quad \tan \theta = 3 \tan \alpha$$

Case I: $\tan \theta = 3 \tan \alpha$

From equation (1):

$$\frac{\tan \theta + \frac{\tan \theta}{3}}{1 - \frac{\tan^2 \theta}{3}} = 2 \tan \theta$$

This gives:

$$\tan \theta = 0, \quad \frac{2}{3} = 1 - \frac{\tan^2 \theta}{3} \Rightarrow \tan \theta = 1, -1$$

Thus, we have:

$$\tan \theta = 0, -1, 1 \Rightarrow \theta = \frac{\pi}{4}, -\frac{\pi}{4}, 0$$

Case II: $\tan \theta = \frac{3}{\tan \alpha}$

From equation (1):

$$\frac{\tan \theta + \frac{3}{\tan \theta}}{-2} = 2 \tan \theta$$

Simplifying this results in:

$$\tan \theta + \frac{3}{\tan \theta} = -4 \tan \theta$$

This leads to no viable solutions (as denoted by "No Solution").

So, the real solutions for θ are $\frac{\pi}{4}$, $-\frac{\pi}{4}$, and 0, resulting in a total of 3 real solutions.

Question3

Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan \left(\sin^{-1} \left(\frac{3}{5} \right) - 2 \cos^{-1} \left(\frac{2}{\sqrt{5}} \right) \right)$$

is

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Options:

A. $\frac{7}{24}$

B. $\frac{-7}{24}$

C. $\frac{-5}{24}$

D. $\frac{5}{24}$

Answer: B

Solution:

$$\tan \left(\tan^{-1} \left(\frac{3}{4} \right) - 2 \tan^{-1} \left(\frac{1}{2} \right) \right)$$

$$\tan \left(\tan^{-1} \left(\frac{3}{4} \right) - \tan^{-1} \left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} \right) \right)$$

$$\tan \left(\tan^{-1} \left(\frac{3}{4} \right) - \tan^{-1} \left(\frac{4}{3} \right) \right)$$

$$\tan \left(\tan^{-1} \left(\frac{3}{4} - \frac{4}{3} \right) \right)$$

$$\frac{9 - 16}{24} = \frac{-7}{24}$$

Question4

For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3} \text{ for } 0 < |y| < 3, \text{ is equal to :}$$

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Options:

A. $2\sqrt{3} - 3$

B. $3 - 2\sqrt{3}$

C. $4\sqrt{3} - 6$

D. $6 - 4\sqrt{3}$

Answer: C

Solution:

Concept :

$$\cot^{-1} x = \begin{cases} \pi + \tan^{-1} \frac{1}{x}; x < 0 \\ \tan^{-1} \frac{1}{x}; x > 0 \end{cases}$$

Solution : Given, $0 < |y| < 3$

$$\Rightarrow y \in (-3, 3) - \{0\}$$

$9 - y^2$ is always positive when $y \in (-3, 3) - \{0\}$

And $6y$ is positive when $y \in (0, 3)$

And $6y$ is negative when $y \in (-3, 0)$

$\therefore$ In overall, $\frac{6y}{9-y^2} > 0$ when $y \in (0, 3)$

And $\frac{6y}{9-y^2} < 0$ when $y \in (-3, 0)$

Case - 1 :

When $-3 < y < 0$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \pi + \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow 2 \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{-\pi}{3}$$

$$\Rightarrow \frac{6y}{9-y^2} = \frac{-1}{\sqrt{3}}$$

$$\Rightarrow y^2 - 6\sqrt{3}y - 9 = 0$$

$$\Rightarrow y^2 - 6\sqrt{3}y - 9 = 0$$

$$\Rightarrow y = \frac{6\sqrt{3} \pm \sqrt{108 + 36}}{2} = \frac{6\sqrt{3} \pm 12}{2} = 3\sqrt{3} \pm 6$$

$$\text{as } y \in (-3, 0) \therefore y = 3\sqrt{3} - 6$$

Case-2 : When $0 < y < 3$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow 2 \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \frac{6y}{9-y^2} = \sqrt{3}$$

$$\Rightarrow 6y = 9\sqrt{3} - \sqrt{3}y^2$$

$$\Rightarrow \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}y^2 + 9y - 3y - 9\sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}y(y + 3\sqrt{3}) - 3(y + 3\sqrt{3}) = 0$$

$$\Rightarrow (y + 3\sqrt{3}) - (\sqrt{3}y - 3) = 0$$

$$y \neq -3\sqrt{3} \therefore y = \sqrt{3} \text{ as } y \in (0, 3)$$