



Application of Integration

Question 1

Consider the ellipses given by

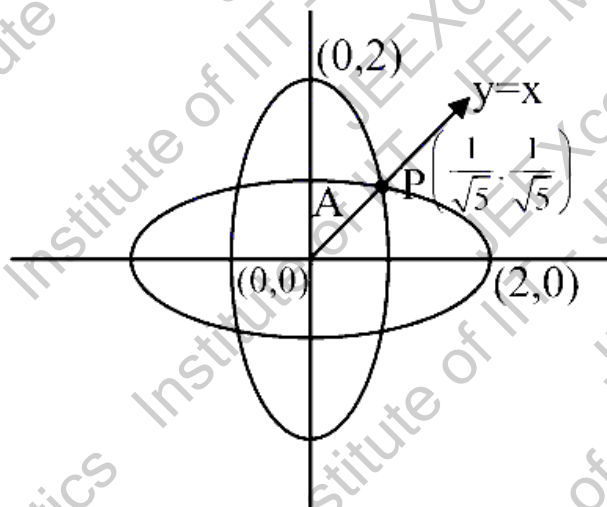
$$x^2 + 4y^2 = 1 \quad \text{and} \quad 4x^2 + y^2 = 1.$$

If α is the area of the common region that lies inside both the given ellipses, then the value of $\cot \alpha$ is _____.

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Answer: 0.75

Solution:



$$A = \int_0^{\frac{1}{\sqrt{5}}} \left(\frac{\sqrt{1-x^2}}{2} - x \right) dx$$

Total required area

$$\alpha = 8A = 8 \left[\int_0^{\frac{1}{\sqrt{5}}} \left(\frac{\sqrt{1-x^2}}{2} - x \right) dx \right]$$

$$\alpha = 8 \left[\frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{x^2}{2} \right]_0^{\frac{1}{\sqrt{5}}}$$

$$\alpha = 8 \left[\frac{1}{4\sqrt{5}} \sqrt{\frac{4}{5}} + \frac{1}{4} \sin^{-1} \frac{1}{\sqrt{5}} - \frac{1}{10} \right]$$

$$\begin{aligned}
 \alpha &= 2 \sin^{-1} \frac{1}{\sqrt{5}} \\
 \cot \alpha &= \cot \left(2 \sin^{-1} \frac{1}{\sqrt{5}} \right) \\
 &= \cot \left(2 \tan^{-1} \frac{1}{2} \right) \\
 &= \cot \left(\tan^{-1} \left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} \right) \right) \\
 &= \cot \left(\tan^{-1} \left(\frac{4}{3} \right) \right) \\
 \cot \alpha &= \frac{3}{4} = 0.75
 \end{aligned}$$

Question2

Consider the curve C_1 given by

$$y = e^{-x} \quad \text{for } x \in [0, 10\pi],$$

and the curve C_2 given by

$$y = e^{-x}(\sin x + \cos x) \quad \text{for } x \in [0, 10\pi].$$

Let n be the total number of points of intersection of the curves C_1 and C_2 .

Suppose that $\alpha_1, \alpha_2, \dots, \alpha_n \in [0, 10\pi]$ are the x -coordinates of the points of intersection of the curves C_1 and C_2 such that

$$\alpha_1 < \alpha_2 < \dots < \alpha_n.$$

Let β be the area of the region enclosed between the curves C_1, C_2 , and the lines $x = \alpha_1$ and $x = \alpha_4$. Then the value of

$$-\frac{1}{\pi} \log_e (\beta - 2e^{-\frac{\pi}{2}})$$

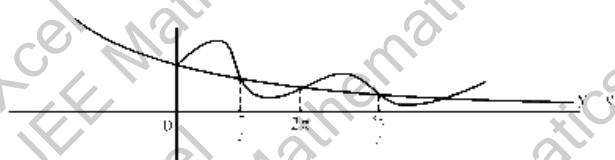
is _____.

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Answer: 2.50

Solution:

$$\alpha_1 = 0 \text{ \& } \alpha_4 = \frac{5\pi}{2}$$



$$\begin{aligned}\beta &= \int_0^{\frac{5\pi}{2}} |e^{-x}(\sin x + \cos x) - e^{-x}| dx \\ \beta &= \int_0^{\frac{\pi}{2}} (e^{-x}(\sin x + \cos x) - e^{-x}) dx + \int_{\frac{\pi}{2}}^{2\pi} (e^{-x} - e^{-x}(\sin x + \cos x)) dx + \int_{2\pi}^{\frac{5\pi}{2}} (e^{-x}(\sin x + \cos x) - e^{-x}) dx \\ \beta &= 2e^{-\frac{\pi}{2}} + e^{-\frac{5\pi}{2}} \\ \text{so } \frac{-1}{\pi} \log \left(\beta - 2e^{-\frac{\pi}{2}} \right) &= \frac{5}{2} = 2.50\end{aligned}$$

Question3

Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$$\left\{ (x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0 \right\}$$

is

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Options:

A.

$$\frac{17}{16} - \log_e 4$$

B.

$$\frac{33}{8} - \log_e 4$$

C.

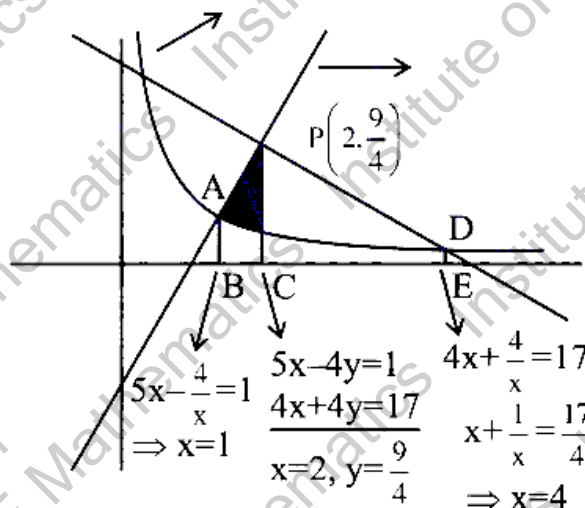
$$\frac{57}{8} - \log_e 4$$

D.

$$\frac{17}{2} - \log_e 4$$

Answer: B

Solution:



$$\text{Area} = \underbrace{\frac{1}{2} \left(1 + \frac{9}{4}\right)}_{\text{Trapezium APBC}} + \underbrace{\frac{1}{4} \left(\frac{9}{4} + \frac{1}{4}\right)}_{\text{Trapezium PDCE}} \cdot 2 - \int_1^4 \frac{dx}{x}$$

$$\text{Area} = \frac{13}{8} + \frac{20}{8} - \log_e 4 = \left(\frac{33}{8} - \log_e 4\right)$$

Question4

Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8}\}$.

If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

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Options:

A. $\frac{17}{2}$

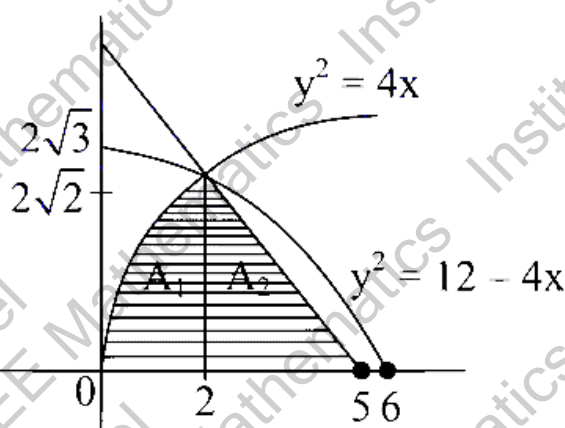
B. $\frac{17}{3}$

C. $\frac{17}{4}$

D. $\frac{17}{5}$

Answer: B

Solution:



Point of intersection of all curves is $(2, 2\sqrt{2})$

$$\text{Area} = A_1 + A_2$$

$$\alpha\sqrt{2} = \int_0^2 2\sqrt{x} dx + \frac{1}{2} \times 3 \times 2\sqrt{2}$$

$$\alpha\sqrt{2} = 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 + 3\sqrt{2}$$

$$\alpha\sqrt{2} = \frac{17\sqrt{2}}{3}$$

$$\alpha = \frac{17}{3}$$

Question 5

The area of the region

$$\{(x, y) : 0 \leq x \leq \frac{9}{4}, \quad 0 \leq y \leq 1, \quad x \geq 3y, \quad x + y \geq 2\} \text{ is}$$

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Options:

A. $\frac{11}{32}$

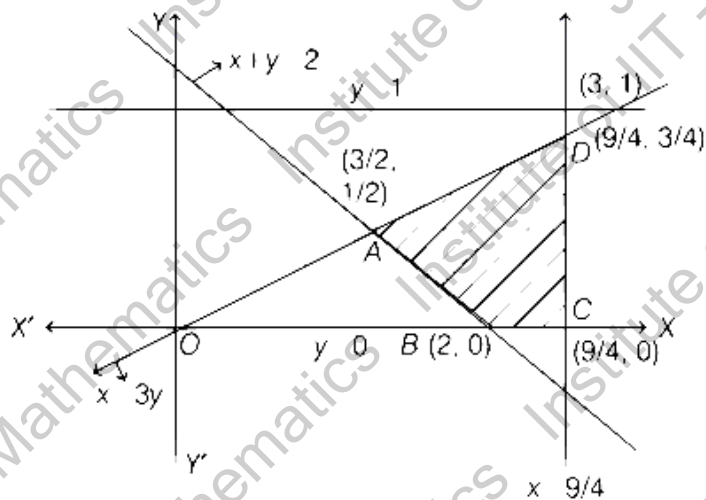
B. $\frac{35}{96}$

C. $\frac{37}{96}$

D. $\frac{13}{32}$

Answer: A

Solution:



Required area = Shaded region

On solving $x + y = 2$ and $x = 3y$, we get

$$A \equiv \left(\frac{3}{2}, \frac{1}{2}\right)$$

On solving $y = 0$ and $x + y = 2$, we get

$$B \equiv (2, 0)$$

On solving $x = \frac{9}{4}$ and $x = 3y$, we get

$$D \equiv \left(\frac{9}{4}, \frac{3}{4}\right)$$

$$\text{and } C \equiv \left(\frac{9}{4}, 0\right)$$

Required area = Area of $\triangle OCD$ - Area of $\triangle OBA$

$$= \frac{1}{2} \times \left(\frac{9}{4} - 0\right) \times \frac{3}{4} - \frac{1}{2} \times (2 - 0) \times \frac{1}{2}$$

$$= \frac{27}{32} - \frac{1}{2}$$

$$= \frac{11}{32}$$

Question 6

Let the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = e^{x-1} - e^{-|x-1|}$$

$$\text{and } g(x) = \frac{1}{2}(e^{x-1} + e^{1-x}).$$

The area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is

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Options:

A. $(2 - \sqrt{3}) + \frac{1}{2}(e - e^{-1})$

B. $(2 + \sqrt{3}) + \frac{1}{2}(e - e^{-1})$

C. $(2 - \sqrt{3}) + \frac{1}{2}(e + e^{-1})$

D. $(2 + \sqrt{3}) + \frac{1}{2}(e + e^{-1})$

Answer: A

Solution:

The given functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = e^{x-1} - e^{-|x-1|}$$
$$= \begin{cases} e^{x-1} - e^{1-x}, & x \geq 1 \\ 0, & x < 1 \end{cases}$$

and $g(x) = \frac{1}{2}(e^{x-1} + e^{1-x})$

For point of intersection of curves $f(x)$ and $g(x)$ put $f(x) = g(x)$

for $x \geq 1$, $e^{x-1} - e^{1-x} = \frac{1}{2}(e^{x-1} + e^{1-x})$

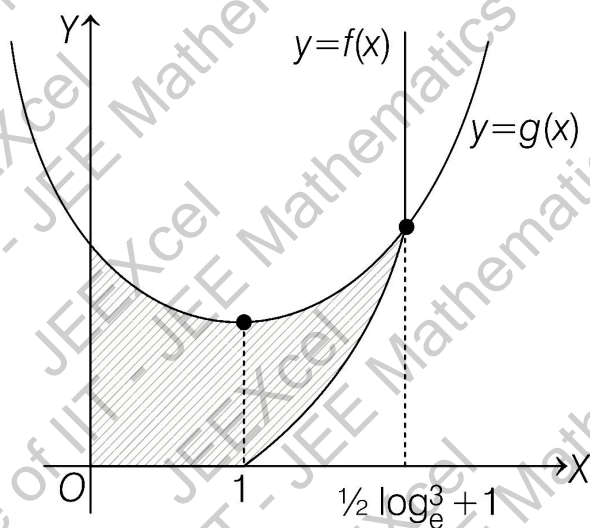
$$\Rightarrow e^{x-1} = 3e^{1-x} \Rightarrow e^{2x} = 3e^2$$

$$\Rightarrow x = \frac{1}{2} \log_e^3 + 1$$

So, required area is

$$\int_0^{1/2 \log_e^3 + 1} (g(x) - f(x)) dx$$
$$= \int_0^{1/2 \log_e^3 + 1} g(x) dx - \int_1^{1/2 \log_e^3 + 1} f(x) dx$$

$$= \frac{1}{2} \int_0^{1/2 \log_e^3 + 1} (e^{x-1} + e^{1-x}) dx - \int_1^{1/2 \log_e^3 + 1} (e^{x-1} - e^{1-x}) dx$$



$$= \frac{1}{2} [e^{x-1} - e^{1-x}]_0^{1/2 \log_e^3 + 1} - [e^{x-1} + e^{1-x}]_1^{1/2 \log_e^3 + 1}$$

$$= \frac{1}{2} \left[\sqrt{3} - \frac{1}{\sqrt{3}} - e^{-1} + e \right] - \left[\sqrt{3} + \frac{1}{\sqrt{3}} - 1 - 1 \right]$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{2} (e - e^{-1}) - \frac{4}{\sqrt{3}} + 2$$

$$= (2 - \sqrt{3}) + \frac{1}{2} (e - e^{-1})$$