



Differentiation

Question1

Let $\mathbb{R}$ denote the set of all real numbers. Consider the polynomial function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{d^{10}}{dx^{10}} \left((x^2 - 1)^{10} \right), \quad \text{for all } x \in \mathbb{R}$$

Here $\frac{d^{10}}{dx^{10}} \left((x^2 - 1)^{10} \right)$ is the 10th order derivative of the function $(x^2 - 1)^{10}$.

Then which of the following statements is (are) TRUE ?

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Options:

A.

The coefficient of x^8 in the polynomial $f(x)$ is $(-10) \left(\frac{18!}{8!} \right)$

B.

The value of $f(1) + f(-1)$ is equal to $10! \cdot 2^{11}$

C.

The degree of the polynomial $f(x)$ is 10

D.

The constant term of the polynomial $f(x)$ is $-\left(\frac{10!}{5!} \right)$

Answer: A

Solution:

Let us first expand the given function:

$$g(x) = (x^2 - 1)^{10}$$

Using the binomial theorem, we get:

$$g(x) = {}^{10}C_0 x^{20} - {}^{10}C_1 x^{18} + {}^{10}C_2 x^{16} - \dots + (-1)^{10} {}^{10}C_{10}$$

Now, the function is defined as:

$$f(x) = \frac{d^{10}}{dx^{10}} ((x^2 - 1)^{10})$$

This means we need to take the 10th derivative of each term of $g(x)$. When we differentiate $x^{20}, x^{18}, \dots$ ten times, their powers will reduce by 10 at each differentiation step. Hence:

$$f(x) = {}^{10}C_0 (20 \cdot 19 \cdot \dots \cdot 11) x^{10} - {}^{10}C_1 (18 \cdot 17 \cdot \dots \cdot 9) x^8 + {}^{10}C_2 (16 \cdot 15 \cdot \dots \cdot 7) x^6 - \dots - {}^{10}C_5 (10 \cdot 9 \cdot 8 \cdot \dots \cdot 1)$$

(A) The coefficient of x^8 is obtained from the second term:

$$\text{Coefficient of } x^8 = (-1)^{10} {}^{10}C_1 \frac{18!}{8!} = (-10) \left(\frac{18!}{8!} \right)$$

(B) Since $g(x) = (x^2 - 1)^{10}$ is an even function, its even-order derivatives (like the 10th derivative) are also even. Therefore,

$$f(1) = f(-1)$$

The sum can be written as:

$$\begin{aligned} f(1) + f(-1) &= 2 \left[\frac{20!}{10!} - {}^{10}C_1 \frac{18!}{8!} + {}^{10}C_2 \frac{16!}{6!} - {}^{10}C_3 \frac{14!}{4!} + {}^{10}C_4 \frac{12!}{2!} - {}^{10}C_5 (10!) \right] \\ &= 2 \times 10! \left[{}^{10}C_0 {}^{20}C_{10} - {}^{10}C_1 {}^{18}C_{10} + {}^{10}C_2 {}^{16}C_{10} - {}^{10}C_3 {}^{14}C_{10} + {}^{10}C_4 {}^{12}C_{10} - {}^{10}C_5 {}^{10}C_{10} \right] \end{aligned}$$

This expression equals twice the coefficient of x^{10} in $((1+x)^2 - 1)^{10}$. Hence:

$$f(1) + f(-1) = 10! \times 2^{11}$$

(C) After differentiating $(x^2 - 1)^{10}$ ten times, the highest power of x left is x^{10} . Therefore,

$$\text{Degree of } f(x) = 10$$

(D) The constant term comes from the last term where the power of x becomes zero after the 10th derivative, which is:

$$\text{Constant term} = \frac{-(10!)^2}{5! \times 5!}$$

Question2

Let S be the set of all twice differentiable functions f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2 f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of

points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is(are) true?

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Options:

- A. There exists a function $f \in S$ such that $X_f = 0$
- B. For every function $f \in S$, we have $X_f \leq 2$
- C. There exists a function $f \in S$ such that $X_f = 2$
- D. There does NOT exist any function f in S such that $X_f = 1$

Answer: A

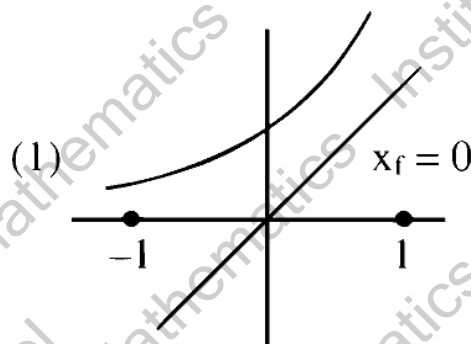
Solution:

S = Set of all twice differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$

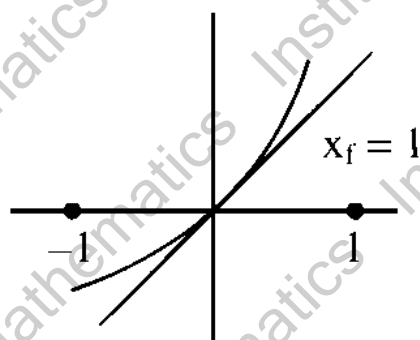
$$\frac{d^2f}{dx^2} > 0 \text{ in } (-1, 1)$$

Graph 'f' is Concave upward.

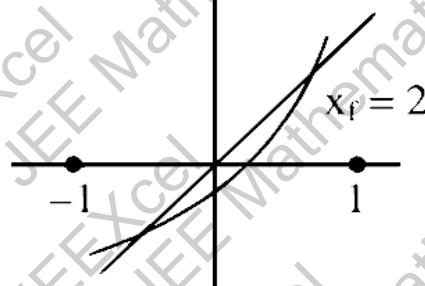
Number of solutions of $f(x) = x \rightarrow x_f$



(2)



(3)



$\Rightarrow$ Graph of $y = f(x)$ can intersect graph of $y = x$ at atmost two points $\Rightarrow 0 \leq x_f \leq 2$
