



Linear Inequalities

Question1

The sum of all the roots of the equation $(x - 1)^2 - 5|x - 1| + 6 = 0$, is :

JEE Main 2026 (Online) 21st January Morning Shift

Options:

A.

3

B.

1

C.

4

D.

5

Answer: C

Solution:

$$|x - 1|^2 - 5|x - 1| + 6 = 0$$

$$\text{Let } |x - 1| = t$$

$$t^2 - 5t + 6 = 0$$

$$(t - 3)(t - 2) = 0$$

$$t = 2, 3$$

$$|x - 1| = 2 \text{ or } |x - 1| = 3$$

$$x - 1 = \pm 2 \text{ or } x - 1 = \pm 3$$

$$\Rightarrow x = 3, -1, 4, -2$$

$$\Rightarrow \text{Sum} = 3 - 1 + 4 - 2$$

$$= 4$$

Question2

Let α and β be the roots of the equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :

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Options:

A. $(-\infty, \frac{-11}{5}) \cup (5, \infty)$

B. $(-\infty, \frac{-11}{5})$

C. $(-\infty, -3)$

D. $(-\infty, -2) \cup (5, \infty)$

Answer: B

Solution:

$$\because \alpha < 1 < \beta$$

$$f(1) < 0$$

$$\Rightarrow 1 + 2a + (3a + 10) < 0$$

$$\Rightarrow 5a + 11 < 0$$

$$a < \frac{-11}{5}$$

$$\therefore a \in \left(-\infty, \frac{-11}{5}\right)$$

Question3

The number of distinct real solutions of the equation

$$x|x+4| + 3|x+2| + 10 = 0 \text{ is}$$

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Options:

A.

2

B.

3

C.

0

D.

1

Answer: D

Solution:

To find the number of distinct real solutions for the given equation, we need to understand how the expressions inside the absolute values behave. The absolute value expressions are $|x + 4|$ and $|x + 2|$. The signs of $x + 4$ and $x + 2$ change at $x = -4$ and $x = -2$. These two critical points divide the real number line into three intervals:

1. $x \geq -2$
2. $-4 \leq x < -2$
3. $x < -4$

Let's analyze the equation $x|x + 4| + 3|x + 2| + 10 = 0$ for each case separately.

Case 1: When $x \geq -2$

In this interval, both $x + 4$ and $x + 2$ are positive (or zero). So, $|x + 4| = x + 4$ and $|x + 2| = x + 2$.

Substitute these into the equation:

$$x(x + 4) + 3(x + 2) + 10 = 0$$

$$x^2 + 4x + 3x + 6 + 10 = 0$$

$$x^2 + 7x + 16 = 0$$

Now, we check the discriminant D (where $D = b^2 - 4ac$) of this quadratic equation:

$$D = 7^2 - 4(1)(16) = 49 - 64 = -15$$

Since $D < 0$, there are no real roots in this interval.

Case 2: When $-4 \leq x < -2$

Here, $x + 4$ is positive (or zero), but $x + 2$ is negative. So, $|x + 4| = x + 4$ and $|x + 2| = -(x + 2)$.

The equation becomes:

$$x(x + 4) - 3(x + 2) + 10 = 0$$

$$x^2 + 4x - 3x - 6 + 10 = 0$$

$$x^2 + x + 4 = 0$$

Let's check the discriminant for this equation:

$$D = 1^2 - 4(1)(4) = 1 - 16 = -15$$

Again, $D < 0$, so there are no real roots in this interval either.

Case 3: When $x < -4$

In this case, both $x + 4$ and $x + 2$ are negative. So, $|x + 4| = -(x + 4)$ and $|x + 2| = -(x + 2)$.

The equation changes to:

$$-x(x + 4) - 3(x + 2) + 10 = 0$$

$$-x^2 - 4x - 3x - 6 + 10 = 0$$

$$-x^2 - 7x + 4 = 0$$

Multiplying by -1 makes it easier to solve:

$$x^2 + 7x - 4 = 0$$

Let's find the discriminant:

$$D = 7^2 - 4(1)(-4) = 49 + 16 = 65$$

Since $D > 0$, this quadratic equation gives two real roots. Using the quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$, the roots are:

$$x = \frac{-7 \pm \sqrt{65}}{2}$$

Now we must verify if these roots actually lie in our current interval, which is $x < -4$.

We know that $\sqrt{65}$ is slightly larger than 8 (because $\sqrt{64} = 8$). Let's roughly estimate it as 8.06.

For the first root (taking the positive sign):

$$x_1 = \frac{-7 + \sqrt{65}}{2} \approx \frac{-7 + 8.06}{2} \approx 0.53$$

This value is greater than -4 , so it does not lie in the given interval $x < -4$. We reject this root.

For the second root (taking the negative sign):

$$x_2 = \frac{-7 - \sqrt{65}}{2} \approx \frac{-7 - 8.06}{2} \approx -7.53$$

This value is strictly less than -4 , so it falls correctly within our interval.

This means there is exactly 1 valid real solution for the entire equation.

Option D is correct.

Question 4

Let α, β be the roots of the quadratic equation $12x^2 - 20x + 3\lambda = 0, \lambda \in \mathbf{Z}$. If $\frac{1}{2} \leq |\beta - \alpha| \leq \frac{3}{2}$, then the sum of all possible values of λ is :

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Options:

A.

3

B.

6

C.

1

D.

4

Answer: A

Solution:

$$\begin{aligned}
 \frac{1}{2} &\leq |\alpha - \beta| \leq \frac{3}{2} \\
 \frac{1}{4} &\leq |\alpha - \beta|^2 \leq \frac{9}{4} \\
 \frac{1}{4} &\leq (\alpha + \beta)^2 - 4\alpha\beta \leq \frac{9}{4} \\
 \frac{1}{4} &\leq \frac{25}{9} - 4 \times \frac{\lambda}{4} \leq \frac{9}{4} \\
 -\frac{91}{36} &\leq -\lambda \leq -\frac{19}{36} \\
 \frac{19}{36} &\leq \lambda \leq \frac{91}{36} \\
 \lambda &= 1, 2 \\
 \text{Sum} &= 3
 \end{aligned}$$

Question5

A building construction work can be completed by two masons A and B together in 22.5 days. Mason A alone can complete the construction work in 24 days less than mason B alone. Then mason A alone will complete the construction work in :

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Options:

- A.
36 days
- B.
24 days
- C.
30 days
- D.
42 days

Answer: A

Solution:

Let B's time = x days

$\Rightarrow$ As time $= (x - 24)$ day

$$\text{Rate of A} = \frac{1}{x - 24}$$

$$\text{Rate of B} = \frac{1}{x}$$

$$\frac{1}{x - 24} + \frac{1}{x} = \frac{1}{22.5}$$

$$\frac{2x - 24}{x(x - 24)} = \frac{1}{22.5}$$

$$45x - 540 = x^2 - 24x$$

$$x^2 - 69x + 540 = 0$$

$$(x - 60)(x - 9) = 0$$

$$\Rightarrow x = 60$$

$$\text{At time} = 60 - 24$$

$$= 36 \text{ days}$$

Question6

If the domain of the function

$$f(x) = \log_{(10x^2 - 17x + 7)} (18x^2 - 11x + 1)$$

is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then

$90(a + b + c + d + e)$ equals:

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Options:

A.

170

B.

316

C.

177

D.

307

Answer: B

Solution:

$$f(x) = \log_{(10x^2 - 17x + 7)} (18x^2 - 11x + 1)$$

$\log_b a$ is defined if $a > 0$ and $b > 0$ and $b \neq 1$.

$$18x^2 - 11x + 1 > 0$$

$$18x^2 - 9x - 2x + 1 > 0$$

$$9x(2x - 1) - 1(2x - 1) > 0$$

$$(2x - 1)(9x - 1) > 0$$

$$S_1 : x \in (-\infty, 1/9) \cup (1/2, \infty).$$

$$10x^2 - 17x + 7 > 0$$

$$10x^2 - 10x - 7x + 7 > 0$$

$$10x(x - 1) - 7(x - 1) > 0$$

$$(10x - 7)(x - 1) > 0$$

$$S_2 : x \in (-\infty, 7/10) \cup (1, \infty).$$

$$\text{And } 10x^2 - 17x + 7 \neq 1$$

$$10x^2 - 17x + 6 \neq 0$$

$$10x^2 - 12x - 5x + 6 \neq 0$$

$$2x(5x - 6) - 1(5x - 6) \neq 0$$

$$(2x - 1)(5x - 6) \neq 0$$

$$S_3 : x \neq 1/2, 6/5 \Rightarrow x \neq 0.5, 1.2.$$

$$\text{Final solution } x \in [S_1 \cap S_2] - \{0.5, 1.2\}.$$

$$x \in [(-\infty, 1/9) \cup (1/2, \infty)] \cap [(-\infty, 7/10) \cup (1, \infty)] - \{0.5, 1.2\}.$$

$$x \in (-\infty, 1/9) \cup (1/2, 7/10) \cup (1, \infty) - \{1.2\}.$$

Since 0.5 is not included, it need not to be subtracted.

Compare final solution with $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$.

$$a = 1/9, b = 1/2, c = 7/10, d = 1 \text{ and } e = 1.2.$$

$$90(a + b + c + d + e) = 90(1/9 + 1/2 + 7/10 + 1 + 1.2)$$

$$= 10 + 45 + 63 + 90 + 108 = 316.$$

Question 7

The number of the real solutions of the equation: $x|x + 3| + |x - 1| - 2 = 0$ is

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Options:

A.

3

B.

2

C.

5

D.

4

Answer: A

Solution:

given $x|x+3|+|x-1|-2=0$.

critical points $x = -3, 1$.

two critical points means 3 cases.

Case 1 : $x < -3$, $|x+3| = -(x+3)$ and $|x-1| = -(x-1)$. equation become

$$-x(x+3) - (x-1) - 2 = 0$$

$$-x^2 - 3x - x + 1 - 2 = 0$$

$$-x^2 - 4x - 1 = 0 \implies x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

since $x < -3$ so $x = -2 - \sqrt{3}$ is only solution.

Case 2 : $-3 \leq x \leq 1$, $|x+3| = x+3$ and $|x-1| = -(x-1)$. equation become

$$x(x+3) - (x-1) - 2 = 0.$$

$$x^2 + 3x - x + 1 - 2 = 0.$$

$$x^2 + 2x - 1 = 0.$$

$$x = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}.$$

since $-3 \leq x \leq 1$ so $x = -1 + \sqrt{2}, -1 - \sqrt{2}$ both accepted.

Case 3 : $x > 1$, $|x+3| = x+3$ and $|x-1| = x-1$.

equation become

$$x(x+3) + x - 1 - 2 = 0.$$

$$x^2 + 4x - 3 = 0.$$

$$x = \frac{-4 \pm \sqrt{16+12}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}. \text{ Checking positive solution. since } x > 1, \text{ so } x = \sqrt{7} - 2 \approx 0.645, \text{ so no solution.}$$

so final solution is

$$x = -2 - \sqrt{3}, -1 + \sqrt{2}, -1 - \sqrt{2}.$$

Number of solution = 3

Question8

The smallest positive integral value of a , for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to

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Options:

A.

7

B.

3

C.

4

D.

9

Answer: A

Solution:

Given equation

$$x^4 - ax^2 + 9 = 0 \quad \text{--- (1)}$$

let $x^2 = t$, equation (1) become

$$t^2 - at + 9 = 0 \quad \text{--- (2)}$$

If t_1 & t_2 are roots of equation (2) then four distinct roots of equation (1) is $\{-\sqrt{t_1}, -\sqrt{t_2}, \sqrt{t_1}, \sqrt{t_2}\}$

roots of this equation are real and distinct if $D > 0$

$$(-a)^2 - 4 \times 9 > 0$$

$$\Rightarrow a^2 - 36 > 0$$

$$\text{so } a \in (-\infty, -6) \cup (6, \infty)$$

so smallest positive integral value of a is 7 as $a > 6$.

Question9

Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in \mathbb{N} \text{ and } a, b, c \leq 20\}$ be a set of polynomials. Then the number of polynomials in S , which are divisible by $x^2 + 2$, is

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Options:

A.

6

B.

120

C.

20

D.

10

Answer: D

Solution:

$$S = \{x^3 + ax^2 + bx + c : a, b, c \in \mathbb{N} \text{ and } a, b, c \leq 20\} \text{ so } a, b, c \in \{1, 2, 3, \dots, 20\}$$

$$x^3 + ax^2 + bx + c \text{ is divisible by } x^2 + 2$$

$$\therefore x^3 + ax^2 + bx + c = (x^2 + 2)L(x)$$

Let $L(x) = x + n$ because L.H.S is cubic have leading coefficient 1 .

$$\begin{aligned} x^3 + ax^2 + bx + c &= (x^2 + 2)(x + n) \\ &= x^3 + x^2n + 2x + 2n \end{aligned}$$

$$\text{Compare } n = a, b = 2, 2n = c \implies c = 2a$$

so, $c = 2a$ and $b = 2$

Finding the Number of Possible Polynomials

- For b : There is only 1 choice ($b = 2$), which is within the range $[1, 20]$.
- For a and c : Since $c = 2a$, we must ensure $c \leq 20$.
- If $a = 1, c = 2$
- If $a = 2, c = 4$
- ...
- If $a = 10, c = 20$

There are 10 possible values for the pair (a, c).

$$\text{Total polynomials} = (\text{Choices for } b) \times (\text{Choices for } a, c \text{ pairs}) = 1 \times 10 = 10$$

Question 10

If α, β , where $\alpha < \beta$, are the roots of the equation $\lambda x^2 - (\lambda + 3)x + 3 = 0$ such that $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$, then the sum of all possible values of λ is

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Options:

A.

2

B.

6

C.

8

D.

4

Answer: B

Solution:

The given equation is $\lambda x^2 - (\lambda + 3)x + 3 = 0$. We can find the roots by factoring:

$$\lambda x^2 - \lambda x - 3x + 3 = 0$$

$$\Rightarrow \lambda x(x - 1) - 3(x - 1) = 0$$

$$\Rightarrow (x - 1)(\lambda x - 3) = 0$$

The two roots are : $x = 1$, $x = \frac{3}{\lambda}$

We are given that α and β are the roots such that $\alpha < \beta$ and the following condition holds:

$$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$$

Because we don't know if 1 is smaller or larger than $\frac{3}{\lambda}$, we must consider two distinct cases.

Case A : $\alpha = 1$ and $\beta = \frac{3}{\lambda}$

Substitute these into the condition $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$:

$$\frac{1}{1} - \frac{1}{3/\lambda} = \frac{1}{3}$$

$$1 - \frac{\lambda}{3} = \frac{1}{3}$$

$$1 - \frac{\lambda}{3} = \frac{\lambda}{3}$$

$$1 - \frac{1}{3} = \frac{\lambda}{3}$$

$$\frac{2}{3} = \frac{\lambda}{3} \implies \lambda = 2$$

Check the constraint $\alpha < \beta$: If $\lambda = 2$, then $\beta = 3/2 = 1.5$. Since $1 < 1.5$, this value is valid.

Case B : $\alpha = \frac{3}{\lambda}$ and $\beta = 1$

Substitute these into the condition $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$:

$$\frac{1}{3/\lambda} - \frac{1}{1} = \frac{1}{3}$$

$$\frac{\lambda}{3} - 1 = \frac{1}{3}$$

$$\frac{\lambda}{3} = 1 + \frac{1}{3}$$

$$\frac{\lambda}{3} = \frac{4}{3} \implies \lambda = 4$$

Check the constraint $\alpha < \beta$: If $\lambda = 4$, then $\alpha = 3/4 = 0.75$. Since $0.75 < 1$, this value is valid.

The possible values for λ are 2 and 4 .

$$\text{Sum} = 2 + 4 = 6$$

The sum of all possible values of λ is 6.

Question 11

Let $\alpha, \alpha + 2, \alpha \in \mathbf{Z}$, be the roots of the quadratic equation $x(x + 2) + (x + 1)(x + 3) + (x + 2)(x + 4) + \dots + (x + n - 1)(x + n + 1) = 4n$ for some $n \in \mathbf{N}$. Then $n + \alpha$ is equal to :

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Options:

- A. 0
- B. 1
- C. 2
- D. 3

Answer: C

Solution:

We are given that

the quadratic equation is

$$(x)(x + 2) + (x + 1)(x + 3) + (x + 2)(x + 4) + \dots + (x + n - 1)(x + n + 1) = 4n$$

and the roots are $\alpha, \alpha + 2$ (both integers).

We need to find the value of $n + \alpha$.

Step 1: Expand the general term

The k^{th} term of the left-hand side is:

$$(x + k)(x + k + 2)$$

Expanding this:

$$(x + k)(x + k + 2) = x^2 + 2(k + 1)x + k(k + 2)$$

Step 2: Sum over all k from 0 to $n - 1$

$$\text{LHS} = \sum_{k=0}^{n-1} [x^2 + 2(k + 1)x + k(k + 2)]$$

Let's compute each part separately.

1. For x^2 :

$$\sum_{k=0}^{n-1} x^2 = nx^2$$

2. For $2(k + 1)x$:

$$\sum_{k=0}^{n-1} 2(k + 1)x = 2x \sum_{k=0}^{n-1} (k + 1) = 2x \cdot \frac{n(n+1)}{2} = n(n + 1)x$$

3. For $k(k + 2)$:

$$\sum_{k=0}^{n-1} k(k+2) = \sum_{k=0}^{n-1} (k^2 + 2k) = \sum k^2 + 2 \sum k$$

Now,

$$\sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}, \quad \sum_{k=0}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$$

Therefore,

$$\sum_{k=0}^{n-1} k(k+2) = \frac{(n-1)n(2n-1)}{6} + n(n-1)$$

Simplify:

$$= n(n-1) \left(\frac{2n-1}{6} + 1 \right) = n(n-1) \left(\frac{2n-1+6}{6} \right) = \frac{n(n-1)(2n+5)}{6}$$

Step 3: Combine all terms

$$\text{LHS} = nx^2 + n(n+1)x + \frac{n(n-1)(2n+5)}{6}$$

The equation is

$$nx^2 + n(n+1)x + \frac{n(n-1)(2n+5)}{6} = 4n$$

Step 4: Simplify by dividing both sides by n (since $n > 0$)

$$x^2 + (n+1)x + \frac{(n-1)(2n+5)}{6} = 4$$

$$x^2 + (n+1)x + \left[\frac{(n-1)(2n+5)}{6} - 4 \right] = 0$$

Simplify the constant term:

$$\frac{(n-1)(2n+5)}{6} - 4 = \frac{(n-1)(2n+5)-24}{6} = \frac{2n^2+3n-29}{6}$$

Thus, the equation is

$$x^2 + (n+1)x + \frac{2n^2+3n-29}{6} = 0$$

Step 5: Use relation between roots and coefficients

Let roots be α and $\alpha + 2$.

Then,

Sum of roots:

$$\alpha + (\alpha + 2) = 2\alpha + 2 = -(n+1)$$

so,

$$\alpha = \frac{-(n+1)-2}{2} = -\frac{n+3}{2}$$

Product of roots:

$$\alpha(\alpha + 2) = \frac{2n^2+3n-29}{6}$$

$$\text{Substitute } \alpha = -\frac{n+3}{2}:$$

$$\alpha(\alpha + 2) = \left(-\frac{n+3}{2} \right) \left(-\frac{n-1}{2} \right) = \frac{(n+3)(n-1)}{4}$$

Step 6: Equate the two expressions for product

$$\frac{(n+3)(n-1)}{4} = \frac{2n^2+3n-29}{6}$$

Simplify:

$$3(n+3)(n-1) = 2(2n^2 + 3n - 29)$$

Expand:

$$3(n^2 + 2n - 3) = 4n^2 + 6n - 58$$

$$3n^2 + 6n - 9 = 4n^2 + 6n - 58$$

Simplify:

$$3n^2 + 6n - 9 - 4n^2 - 6n + 58 = 0$$

$$-n^2 + 49 = 0 \implies n^2 = 49$$

$$n = 7 \text{ (since } n \in \mathbb{N})$$

Step 7: Find α and hence $n + \alpha$

$$\alpha = -\frac{n+3}{2} = -\frac{7+3}{2} = -5$$

$$n + \alpha = 7 + (-5) = 2$$

$$\boxed{n + \alpha = 2}$$

Correct Option: C (2)

Question 12

Let α, β be the roots of the equation $x^2 - 3x + r = 0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 + 3x + r = 0$.

If the roots of the equation $x^2 + 6x = m$ are $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$, then m is equal to :

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Options:

A.

-135

B.

-567

C.

135

D.

567

Answer: D

Solution:

For the equation $x^2 - 3x + r = 0$ with roots α, β , by using the standard results:

$$\alpha + \beta = 3$$

$$\frac{\alpha}{2} + 2\beta = -3$$

The first line comes from “sum of roots” $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{-3}{1} = 3$.

The second line comes from the equation $x^2 + 3x + r = 0$, whose roots are $\frac{\alpha}{2}$ and 2β . So, their sum is:

$$\frac{\alpha}{2} + 2\beta = -\frac{3}{1} = -3$$

Now solve the two equations:

$$\alpha + \beta = 3 \Rightarrow \alpha = 3 - \beta$$

Substitute into $\frac{\alpha}{2} + 2\beta = -3$:

$$\frac{3-\beta}{2} + 2\beta = -3$$

Multiply by 2:

$$3 - \beta + 4\beta = -6 \Rightarrow 3 + 3\beta = -6 \Rightarrow 3\beta = -9 \Rightarrow \beta = -3$$

Then $\alpha = 3 - (-3) = 6$.

Now use “product of roots” for $x^2 - 3x + r = 0$:

$$\alpha\beta = r \Rightarrow r = 6 \times (-3) = -18$$

Now consider the equation $x^2 + 6x = m$. Rewrite it in standard form:

$$x^2 + 6x - m = 0$$

So, product of its roots $= \frac{-m}{1} = -m$.

The roots are given as $2\alpha + \beta + 2r$ and $\alpha - 2\beta - \frac{r}{2}$. Substitute $\alpha = 6, \beta = -3, r = -18$:

First root:

$$2\alpha + \beta + 2r = 2(6) + (-3) + 2(-18) = 12 - 3 - 36 = -27$$

Second root:

$$\alpha - 2\beta - \frac{r}{2} = 6 - 2(-3) - \frac{-18}{2} = 6 + 6 + 9 = 21$$

Now product of roots:

$$= (2\alpha + \beta + 2r) \left(\alpha - 2\beta - \frac{r}{2} \right)$$

$$\Rightarrow -m = (-27)(21)$$

$$\Rightarrow m = 567$$

Question 13

If the set of all solutions of $|x^2 + x - 9| = |x| + |x^2 - 9|$ is $[\alpha, \beta] \cup [\gamma, \infty)$, then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to:

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Options:

A.

9

B.

18

C.

36

D.

72

Answer: B

Solution:

We need to solve

$$|x^2 + x - 9| = |x| + |x^2 - 9|.$$

We will do it by considering intervals where the expressions inside modulus change sign.

The critical points are obtained from:

- $x = 0$ from $|x|$
- $x = \pm 3$ from $|x^2 - 9|$
- roots of $x^2 + x - 9 = 0$, but for NCERT-style interval splitting here, it is enough to use the main sign-change points $-3, 0, 3$ and then check sign of $x^2 + x - 9$ in each interval.

Now solve interval-wise.

1. For $x \leq -3$

Here,

- $|x| = -x$
- $|x^2 - 9| = x^2 - 9$
- Also $x^2 + x - 9 > 0$, so $|x^2 + x - 9| = x^2 + x - 9$

Thus,

$$x^2 + x - 9 = -x + (x^2 - 9)$$

$$x^2 + x - 9 = x^2 - x - 9$$

$$x = -x$$

$$2x = 0 \Rightarrow x = 0$$

But $0 \not\leq -3$, so no solution here.

2. For $-3 \leq x \leq 0$

Here,

- $|x| = -x$
- $|x^2 - 9| = 9 - x^2$

Now we check $x^2 + x - 9$ on this interval.

At $x = -3$,

$$x^2 + x - 9 = 9 - 3 - 9 = -3 < 0$$

At $x = 0$,

$$x^2 + x - 9 = -9 < 0$$

So throughout this interval,

$$|x^2 + x - 9| = -(x^2 + x - 9) = -x^2 - x + 9.$$

Hence equation becomes

$$-x^2 - x + 9 = -x + (9 - x^2).$$

Simplify RHS:

$$-x^2 - x + 9 = -x + 9 - x^2.$$

Both sides are identical, so this is true for every $x \in [-3, 0]$.

So all $x \in [-3, 0]$ are solutions.

3. For $0 \leq x \leq 3$

Here,

- $|x| = x$
- $|x^2 - 9| = 9 - x^2$

Also on this interval,

$$x^2 + x - 9$$

At $x = 0$, it is $-9 < 0$, and at $x = 3$, it is $9 + 3 - 9 = 3 > 0$.

So the sign changes inside this interval.

Let us find the root:

$$x^2 + x - 9 = 0$$

$$x = \frac{-1 \pm \sqrt{37}}{2}.$$

The positive root is

$$\frac{-1 + \sqrt{37}}{2}.$$

So split further.

$$(i) \text{ For } 0 \leq x \leq \frac{-1 + \sqrt{37}}{2}$$

Here $x^2 + x - 9 \leq 0$, so

$$|x^2 + x - 9| = -x^2 - x + 9.$$

Equation:

$$-x^2 - x + 9 = x + (9 - x^2)$$

$$-x^2 - x + 9 = -x^2 + x + 9$$

$$-x = x$$

$$2x = 0 \Rightarrow x = 0.$$

So only $x = 0$, already included in previous interval.

(ii) For $\frac{-1+\sqrt{37}}{2} \leq x \leq 3$

Here $x^2 + x - 9 \geq 0$, so

$$|x^2 + x - 9| = x^2 + x - 9.$$

Equation:

$$x^2 + x - 9 = x + (9 - x^2)$$

$$x^2 - 9 = 9 - x^2$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3.$$

In this interval, only $x = 3$ is valid.

So from $[0, 3]$, only $x = 0, 3$ are solutions.

4. For $x \geq 3$

Here,

- $|x| = x$
- $|x^2 - 9| = x^2 - 9$
- $x^2 + x - 9 > 0$, so $|x^2 + x - 9| = x^2 + x - 9$

Thus,

$$x^2 + x - 9 = x + (x^2 - 9)$$

$$x^2 + x - 9 = x^2 + x - 9$$

This is true for every $x \geq 3$.

So all $x \in [3, \infty)$ are solutions.

Complete solution set

Combining all intervals:

$$[-3, 0] \cup [3, \infty).$$

Thus,

$$\alpha = -3, \quad \beta = 0, \quad \gamma = 3.$$

Now,

$$\alpha^2 + \beta^2 + \gamma^2 = (-3)^2 + 0^2 + 3^2 = 9 + 0 + 9 = 18.$$

So the correct answer is

Hence, **Option B** is correct.

Question 14

If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$, $\lambda \neq -2$, has two positive roots, then the number of possible integral values of λ is :

JEE Main 2026 (Online) 4th April Evening Shift

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

For the quadratic equation

$$(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0, \quad \lambda \neq -2$$

to have **two positive roots**, let us use the NCERT conditions for a quadratic equation.

If the roots are α and β , then for both roots to be positive, we need:

1. Discriminant ≥ 0 so that roots are real,
2. Sum of roots > 0 ,
3. Product of roots > 0 .

1. Identify coefficients

Comparing with $ax^2 + bx + c = 0$, we get

$$a = \lambda + 2, \quad b = -3\lambda, \quad c = 4\lambda$$

2. Condition for real roots

Discriminant:

$$D = b^2 - 4ac$$

So,

$$D = (-3\lambda)^2 - 4(\lambda + 2)(4\lambda)$$

$$D = 9\lambda^2 - 16\lambda(\lambda + 2)$$

$$D = 9\lambda^2 - 16\lambda^2 - 32\lambda$$

$$D = -7\lambda^2 - 32\lambda$$

For real roots,

$$-7\lambda^2 - 32\lambda \geq 0$$

$$-\lambda(7\lambda + 32) \geq 0$$

Multiplying by -1 reverses the inequality:

$$\lambda(7\lambda + 32) \leq 0$$

This gives

$$-\frac{32}{7} \leq \lambda \leq 0$$

So possible integral values from this condition are

$$\lambda = -4, -3, -2, -1, 0$$

But given $\lambda \neq -2$, so remaining are

$$\lambda = -4, -3, -1, 0$$

3. Condition for sum of roots positive

Sum of roots is

$$\alpha + \beta = -\frac{b}{a} = \frac{3\lambda}{\lambda + 2}$$

For two positive roots,

$$\frac{3\lambda}{\lambda + 2} > 0$$

Since $3 > 0$, this means

$$\frac{\lambda}{\lambda + 2} > 0$$

This is positive when numerator and denominator have same sign:

- $\lambda > 0$ and $\lambda + 2 > 0$, or
- $\lambda < 0$ and $\lambda + 2 < 0$

So,

$$\lambda > 0 \quad \text{or} \quad \lambda < -2$$

From our shortlisted values $-4, -3, -1, 0$, this keeps only

$$\lambda = -4, -3$$

4. Condition for product of roots positive

Product of roots is

$$\alpha\beta = \frac{c}{a} = \frac{4\lambda}{\lambda + 2}$$

For positive roots,

$$\frac{4\lambda}{\lambda+2} > 0$$

Again,

$$\frac{\lambda}{\lambda+2} > 0$$

This gives the same condition:

$$\lambda > 0 \quad \text{or} \quad \lambda < -2$$

Among the shortlisted values, again we get

$$\lambda = -4, -3$$

5. Check both values

- For $\lambda = -4$:

$$D = -7(16) - 32(-4) = -112 + 128 = 16 > 0$$

So roots are real and distinct.

- For $\lambda = -3$:

$$D = -7(9) - 32(-3) = -63 + 96 = 33 > 0$$

So roots are real and distinct.

Hence both values give **two positive roots**.

Therefore, the number of possible integral values of λ is

2

So, the correct option is **B**.

Question 15

Let $\lim_{x \rightarrow 2} \frac{(\tan(x-2))(rx^2 + (p-2)x - 2p)}{(x-2)^2} = 5$ for some $r, p \in \mathbf{R}$. If the set of all possible values of q , such that the roots of the equation $rx^2 - px + q = 0$ lie in $(0, 2)$, be the interval $(\alpha, \beta]$, then $4(\alpha + \beta)$ equals :

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Options:

A.

11

B.

13

C.

17

D.

21

Answer: C

Solution:

We are given

$$\lim_{x \rightarrow 2} \frac{\tan(x-2)(rx^2 + (p-2)x - 2p)}{(x-2)^2} = 5.$$

We have to first find the relation between r and p , and then determine all values of q for which the roots of

$$rx^2 - px + q = 0$$

lie in $(0, 2)$.

Step 1: Simplify the limit

Let

$$f(x) = rx^2 + (p-2)x - 2p.$$

Then the given limit becomes

$$\lim_{x \rightarrow 2} \frac{\tan(x-2) f(x)}{(x-2)^2} = 5.$$

Now, as $x \rightarrow 2$,

$$\tan(x-2) \sim x-2.$$

So the limit becomes

$$\lim_{x \rightarrow 2} \frac{(x-2)f(x)}{(x-2)^2} = \lim_{x \rightarrow 2} \frac{f(x)}{x-2}.$$

For this limit to be finite, we must have

$$f(2) = 0.$$

Now,

$$f(2) = r(2)^2 + (p-2)(2) - 2p = 4r + 2p - 4 - 2p = 4r - 4.$$

So,

$$4r - 4 = 0 \implies r = 1.$$

Then

$$f(x) = x^2 + (p-2)x - 2p.$$

Since $f(2) = 0$, factor $(x-2)$:

$$x^2 + (p-2)x - 2p = (x-2)(x+p).$$

Hence the limit becomes

$$\lim_{x \rightarrow 2} \frac{\tan(x-2)(x-2)(x+p)}{(x-2)^2} = \lim_{x \rightarrow 2} \frac{\tan(x-2)}{x-2} (x+p).$$

Using

$$\lim_{t \rightarrow 0} \frac{\tan t}{t} = 1,$$

we get

$$1 \cdot (2 + p) = 5.$$

So,

$$p + 2 = 5 \implies p = 3.$$

Thus,

$$r = 1, \quad p = 3.$$

Step 2: Form the quadratic equation

Now the equation becomes

$$x^2 - 3x + q = 0.$$

We want both roots to lie in $(0, 2)$.

Let the roots be α_1, α_2 . Then

$$\alpha_1 + \alpha_2 = 3, \quad \alpha_1 \alpha_2 = q.$$

Since both roots lie in $(0, 2)$, let us find the condition on q .

Step 3: Use condition for both roots in $(0, 2)$

Let

$$f(x) = x^2 - 3x + q.$$

Since coefficient of x^2 is positive, parabola opens upward.

For both roots to lie in $(0, 2)$, we need:

1. Real roots

So discriminant must be non-negative:

$$\Delta = 9 - 4q \geq 0 \implies q \leq \frac{9}{4}.$$

2. Both roots positive

Since product of roots is q ,

$$q > 0.$$

3. Both roots less than 2

Now check value at $x = 2$:

$$f(2) = 4 - 6 + q = q - 2.$$

If both roots are less than 2, then for an upward parabola, point $x = 2$ lies to the right of both roots, so we need

$$f(2) > 0.$$

Thus,

$$q - 2 > 0 \implies q > 2.$$

Now combine:

$$2 < q \leq \frac{9}{4}$$

So the set of all possible values of q is

$$(\alpha, \beta] = (2, \frac{9}{4}].$$

Hence,

$$\alpha = 2, \quad \beta = \frac{9}{4}.$$

Therefore,

$$4(\alpha + \beta) = 4\left(2 + \frac{9}{4}\right) = 4\left(\frac{17}{4}\right) = 17.$$

Final Answer

17

So, the correct option is **Option C**.

Question 16

If the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to _____

JEE Main 2025 (Online) 23rd January Morning Shift

Answer: 117

Solution:

To solve the given problem, we start with the quadratic equation:

$$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$$

Given that the roots are equal (let's assume both roots are 1), we know the sum of the roots, $\alpha + \beta$, is twice the value of one root, which leads us to:

$$\alpha + \beta = 2$$

Using the formula for the sum of roots for a quadratic equation, $\alpha + \beta = -\frac{b(c-a)}{a(b-c)}$, we set this equal to 2:

$$-\frac{b(c-a)}{a(b-c)} = 2$$

Solving for this:

$$-bc + ab = 2ab - 2ac \Rightarrow ab + bc - 2ac = ab \Rightarrow bc - 2ac = 0 \Rightarrow b = 2c$$

Given that $a + c = 15$ and $b = \frac{36}{5}$, substitute these into the equation:

$$2ac = 15b \Rightarrow 2ac = 15 \times \frac{36}{5} = 108 \Rightarrow ac = 54$$

Now, using the equation $a + c = 15$ and $ac = 54$, find $a^2 + c^2$:

$$a^2 + c^2 = (a + c)^2 - 2ac = 15^2 - 2 \times 54 = 225 - 108 = 117$$

Therefore, $a^2 + c^2$ is equal to 117.

Question 17

If the set of all $a \in \mathbb{R} - \{1\}$, for which the roots of the equation $(1-a)x^2 + 2(a-3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to .

JEE Main 2025 (Online) 2nd April Evening Shift

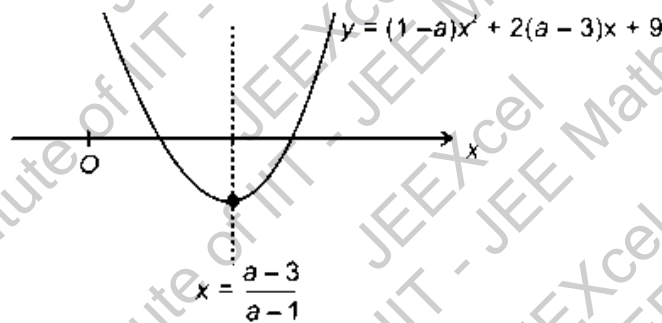
Answer: 7

Solution:

$$f(x) = (1-a)x^2 + 2(a-3)x + 9, f(0) = 9 > 0$$

$$D \geq 0 \Rightarrow 4(a-3)^2 \geq 4(1-a) \cdot 9$$

$$\Rightarrow a \in (-\infty, -3] \cup [0, \infty) \dots (i)$$



$$x_1 + x_2 = \frac{-2(a-3)}{1-a}, x_1 x_2 = \frac{9}{1-a}$$

$$x_1 + x_2 > 0 \Rightarrow \frac{a-3}{a-1} > 0 \Rightarrow a \in (-\infty, 1) \cup (3, \infty) \dots (ii)$$

$$x_1 x_2 > 0 \Rightarrow 1-a > 0 \Rightarrow a \in (-\infty, 1) \dots (iii)$$

$\Rightarrow$ Intersection of (i), (ii) and (iii)

$$a \in (-\infty, -3] \cup [0, 1)$$

$$\Rightarrow \alpha = 3, \beta = 0, \gamma = 1 \Rightarrow 2\alpha + \beta + \gamma = 7$$

Question 18

Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos \theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals :

JEE Main 2025 (Online) 22nd January Evening Shift

Options:

- A. 27
- B. 17
- C. 25
- D. 24

Answer: C

Solution:

To find the sum of the fourth powers of the roots α_θ and β_θ of the quadratic equation $2x^2 + (\cos \theta)x - 1 = 0$, we start analyzing the expression $\alpha_\theta^4 + \beta_\theta^4$.

The equation can be rewritten with its roots using:

$$\alpha + \beta = -\frac{\cos \theta}{2}, \quad \alpha\beta = -\frac{1}{2}$$

We need to calculate $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(-\frac{\cos \theta}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = \frac{\cos^2 \theta}{4} + 1$$

$$\alpha^2\beta^2 = (\alpha\beta)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

Substitute these into:

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = \left(\frac{\cos^2 \theta}{4} + 1\right)^2 - \frac{1}{2}$$

Maximize and minimize $\left(\frac{\cos^2 \theta}{4} + 1\right)^2$:

Zero of $\cos \theta$ leads to:

$$\left(\frac{0^2}{4} + 1\right)^2 = 1$$

Max value $\cos^2 \theta = 1$:

$$\left(\frac{1}{4} + 1\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

Substitute back:

$$\text{Max: } \frac{25}{16} - \frac{1}{2} = \frac{25}{16} - \frac{8}{16} = \frac{17}{16}$$

$$\text{Min: } 1 - \frac{1}{2} = \frac{1}{2}$$

Finally, compute $16(M + m)$:

$$16\left(\frac{17}{16} + \frac{1}{2}\right) = 16\left(\frac{17}{16} + \frac{8}{16}\right) = 16 \times \frac{25}{16} = 25$$

Question 19

The product of all the rational roots of the equation

$$(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3, \text{ is equal to}$$

JEE Main 2025 (Online) 24th January Morning Shift

Options:

- A. 7
- B. 21
- C. 28
- D. 14

Answer: D

Solution:

To solve the given equation, start by rewriting the expression:

$$(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$$

First, simplify the second part of the expression:

$$(x - 4)(x - 5) = x^2 - 9x + 20$$

Now the equation becomes:

$$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3$$

Introduce a substitution for simplification:

$$\text{Let } t = x^2 - 9x$$

Thus, the equation transforms to:

$$t^2 + 22t + 121 - t - 20 - 3 = 0$$

Simplify further:

$$t^2 + 21t + 98 = 0$$

Factor the quadratic:

$$(t + 14)(t + 7) = 0$$

This gives:

$$t = -7 \quad \text{or} \quad t = -14$$

Address each case where $t = x^2 - 9x$:

$$x^2 - 9x = -7$$

$$x^2 - 9x + 7 = 0$$

Solving this quadratic equation, we find the roots:

$$x = \frac{9 \pm \sqrt{81 - 4 \times 7}}{2} = \frac{9 \pm \sqrt{53}}{2}$$

$$x^2 - 9x = -14$$

$$x^2 - 9x + 14 = 0$$

Solving this quadratic equation:

$$x = \frac{9 \pm \sqrt{81 - 4 \times 14}}{2} = \frac{9 \pm \sqrt{25}}{2}$$

$$x = \frac{9 \pm 5}{2} = 7 \quad \text{or} \quad x = 2$$

The rational roots from the second equation are 7 and 2. Thus, the product of all the rational roots is:

$$7 \times 2 = 14$$

Question 20

The number of real solution(s) of the equation

$$x^2 + 3x + 2 = \min\{|x - 3|, |x + 2|\} \text{ is :}$$

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 2

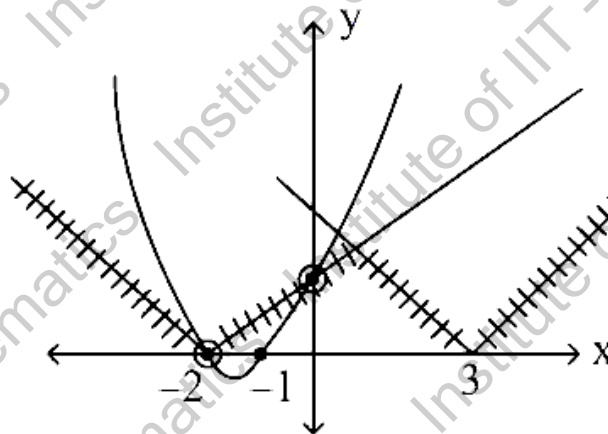
B. 3

C. 1

D. 0

Answer: A

Solution:



Only 2 solutions.

Question 21

The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. $6(2 - \sqrt{2})$

B. $3(3 - \sqrt{2})$

C. $3(2 - \sqrt{2})$

D. $6(3 - \sqrt{2})$

Answer: A

Solution:

To find the sum of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$:

Case I: $x \geq \frac{3}{2}$

For $x \geq \frac{3}{2}$, the expression $|2x - 3|$ becomes $2x - 3$. Thus, the equation becomes:

$$x^2 + 2x - 3 - 4 = 0$$

Simplifying gives:

$$x^2 + 2x - 7 = 0$$

Solving this quadratic equation, we find:

$$x = 2\sqrt{2} - 1$$

Case II: $x < \frac{3}{2}$

For $x < \frac{3}{2}$, the expression $|2x - 3|$ becomes $-(2x - 3) = -2x + 3$. The equation therefore becomes:

$$x^2 + 3 - 2x - 4 = 0$$

Simplifying gives:

$$x^2 - 2x - 1 = 0$$

Solving this quadratic equation, we obtain:

$$x = 1 - \sqrt{2}$$

Sum of the Squares of the Roots

The sum of the squares of the roots is:

$$(2\sqrt{2} - 1)^2 + (1 - \sqrt{2})^2$$

Calculating each term:

$$(2\sqrt{2} - 1)^2 = (2\sqrt{2})^2 - 2 \cdot 2\sqrt{2} \cdot 1 + 1^2 = 8 - 4\sqrt{2} + 1 = 9 - 4\sqrt{2}$$

$$(1 - \sqrt{2})^2 = 1^2 - 2 \cdot 1 \cdot \sqrt{2} + (\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$$

Adding these results:

$$(9 - 4\sqrt{2}) + (3 - 2\sqrt{2}) = 12 - 6\sqrt{2}$$

This simplifies to $6(2 - \sqrt{2})$.

Thus, the sum of the squares of the roots is $6(2 - \sqrt{2})$.

Question 22

Let $f : \mathbb{R} - \{0\} \rightarrow (-\infty, 1)$ be a polynomial of degree 2, satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(K) = -2K$, then the sum of squares of all possible values of K is :

JEE Main 2025 (Online) 28th January Evening Shift

Options:

A.

9

B.

1

C.

6

D.

7

Answer: C

Solution:

as $f(x)$ is a polynomial of degree two let it be

$$f(x) = ax^2 + bx + c \quad (a \neq 0)$$

on satisfying given conditions we get

$$C = 1 \text{ \& } a = \pm 1$$

$$\text{hence } f(x) = 1 \pm x^2$$

also range $\in (-\infty, 1]$ hence

$$f(x) = 1 - x^2$$

$$\text{now } f(k) = -2k$$

$$1 - k^2 = -2k \rightarrow k^2 - 2k - 1 = 0$$

$$\begin{aligned} \text{let roots of this equation be } \alpha \text{ \& } \beta \text{ then } \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 4 - 2(-1) = 6 \end{aligned}$$

Question23

The number of solutions of the equation

$$\left(\frac{9}{x} - \frac{9}{\sqrt{x}} + 2\right) \left(\frac{2}{x} - \frac{7}{\sqrt{x}} + 3\right) = 0 \text{ is :}$$

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

3

B.

2

C.

1

D.

4

Answer: D

Solution:

Consider $\frac{1}{\sqrt{x}} = \alpha \quad x > 0$

$$(9\alpha^2 - 9\alpha + 2)(2\alpha^2 - 7\alpha + 3) = 0$$

$$(3\alpha - 2)(3\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0$$

$$\alpha = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 3$$

$$x = 9, 4, \frac{9}{4}, \frac{1}{9}$$

So, no. of solutions = 4

Question24

If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a - 5)x + 15 = 3a$ has no real root, is the interval (α, β) , and $X = \{x \in \mathbf{Z}; \alpha < x < \beta\}$, then $\sum_{x \in X} x^2$ is equal to:

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

2139

B.

2119

C.

2109

D.

2129

Answer: A

Solution:

$$(a - 5)^2 - 8(15 - 3a) < 0$$

$$a^2 + 14a + 25 - 120 < 0$$

$$a^2 + 14a - 95 < 0$$

$$(a + 19)(a - 5) < 0$$

$$a \in (-19, 5)$$

$$\therefore -19 < x < 5$$

$$\therefore \sum_{x \in X} x^2 = (1^2 + 2^2 + \dots + 4^2) + (1^2 + 2^2 + \dots + 18^2)$$

$$= \frac{4 \times 5 \times 9}{6} + \frac{18 \times 19 \times 37}{6}$$

$$= 30 + 2109$$

$$= 2139$$

Question 25

Let $P_n = \alpha^n + \beta^n$, $n \in \mathbb{N}$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. $x^2 + x - 1 = 0$

B. $x^2 - x + 1 = 0$

C. $x^2 + x + 1 = 0$

D. $x^2 - x - 1 = 0$

Answer: A

Solution:

Given:

$$P_{10} = 123$$

$$P_9 = 76$$

$$P_8 = 47$$

$$P_1 = 1$$

We know that:

$$P_n = \alpha^n + \beta^n$$

According to Newton's identities, we have the relation:

$$P_{10} = P_9 + P_8$$

This implies:

$$P_{10} - P_9 - P_8 = 0$$

From $P_1 = 1$, it follows that:

$$\alpha + \beta = 1$$

$$\alpha\beta = 1$$

Now, we are tasked with finding the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. For such a quadratic equation:

Using the relationship between roots and coefficients, the equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

Substitute the known sum and product of α and β :

$$x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

Given that $\alpha + \beta = 1$ and $\alpha\beta = 1$, we can simplify:

$$x^2 - \left(\frac{1}{1}\right)x + \frac{1}{1} = 0$$

This simplifies to:

$$x^2 + x - 1 = 0$$

Thus, the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

$$x^2 + x - 1 = 0$$

Question 26

Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

- A. 4
- B. 3
- C. 5
- D. 7

Answer: C

Solution:

$$x^2 + 3x - 1 = 0 \begin{matrix} \nearrow \gamma \\ \searrow \delta \end{matrix} \Rightarrow x^2 - 1 = -3x$$

$$\begin{aligned} \Rightarrow P^n &= \gamma^n + \delta^n \\ P_{25} - P_{23} &= (\gamma^{25} - \gamma^{23}) + (\delta^{25} - \delta^{23}) \\ &= \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1) \\ &= \gamma^{23}(-3\gamma) + \delta^{23}(-3\delta) = -3[\gamma^{24} + \delta^{24}] \end{aligned}$$

$$\Rightarrow \frac{P_{25} - P_{23}}{P_{24}} = (-3)$$

Similarly,

$$x^2 + \sqrt{3}x - 16 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad Q_n = \alpha^n + \beta^n$$

$$\begin{aligned} \Rightarrow Q_{25} + \sqrt{3}Q_{24} &= (\alpha^{25} + \sqrt{3}\alpha^{24}) + (\beta^{25} + \sqrt{3}\beta^{24}) \\ &= \alpha^{23}(\alpha^2 + \sqrt{3}\alpha) + \beta^{23}(\beta^2 + \sqrt{3}\beta) \\ &= \alpha^{23}(16) + 16\beta^{23} \end{aligned}$$

$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2 \cdot Q_{23}} = \frac{16(\alpha^{23} + \beta^{23})}{2(\alpha^{23} + \beta^{23})} = 8$$

$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \frac{(P_{25} - P_{23})}{P_{24}} = 8 + (-3) = 5$$

Question 27

Let the equation $x(x+2)(12-k) = 2$ have equal roots. Then the distance of the point $(k, \frac{k}{2})$ from the line $3x + 4y + 5 = 0$ is

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

- A. 15
- B. 12
- C. $5\sqrt{3}$
- D. $15\sqrt{5}$

Answer: A**Solution:**

Given the equation $x(x+2)(12-k) = 2$, we want it to have equal roots. To achieve this, we need to manipulate it into a quadratic form in terms of x :

$$x^2 + 2x - \frac{2}{12-k} = 0$$

For this quadratic equation to have equal (repeated) roots, the discriminant D must be zero. The discriminant D for the equation $ax^2 + bx + c = 0$ is given by:

$$D = b^2 - 4ac$$

Substituting $a = 1$, $b = 2$, and $c = -\frac{2}{12-k}$ into the discriminant formula, we get:

$$4 - 4\left(-\frac{2}{12-k}\right) = 0$$

Simplifying the expression:

$$1 + \frac{2}{12-k} = 0$$

Solving for k :

$$\frac{2}{12-k} = -1 \Rightarrow 2 = -(12-k) \Rightarrow 2 = -12+k \Rightarrow k = 14$$

Now, consider the point $(k, \frac{k}{2})$, which becomes $(14, 7)$. We need to find its distance from the line $3x + 4y + 5 = 0$. The formula for the distance d from a point (x_1, y_1) to a line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Substituting $(x_1, y_1) = (14, 7)$ and the line coefficients $A = 3$, $B = 4$, $C = 5$:

$$d = \frac{|3(14) + 4(7) + 5|}{\sqrt{3^2 + 4^2}}$$

Calculating the numerator:

$$3 \times 14 + 4 \times 7 + 5 = 42 + 28 + 5 = 75$$

And the denominator:

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Therefore, the distance is:

$$d = \frac{75}{5} = 15$$

Thus, the distance is 15.

Question 28

Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n , for which the given equation has integral roots, is equal to

JEE Main 2025 (Online) 4th April Morning Shift

Options:

- A. 6
- B. 5
- C. 8
- D. 7

Answer: A

Solution:

$x^2 + 4x - n = 0$ has integer roots

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 4n}}{2} = -2 \pm \sqrt{4 + n}$$

For x to be integer $4 + n$ must be perfect squares

$$n \in [20, 100]$$

$$n + 4 \in [24, 104] = S$$

$$\{25, 36, \dots, 10^2\} \in S \Rightarrow 5^2, 6^2, \dots, 10^2 \Rightarrow 6 \text{ values of } n$$

Question 29

Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p + 2)x + (2p + 9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

- A. 5
- B. 0
- C. 20
- D. 9

Answer: A

Solution:

To find the set of all values of $p \in \mathbb{R}$ for which both roots of the equation $x^2 - (p+2)x + (2p+9) = 0$ are negative real numbers, follow these steps:

Discriminant Condition:

The equation's discriminant D must be non-negative for real roots:

$$(p+2)^2 - 4(2p+9) \geq 0$$

Simplifying this:

$$p^2 + 4p + 4 - 8p - 36 \geq 0 \Rightarrow p^2 - 4p - 32 \geq 0$$

This can be factored as:

$$(p-8)(p+4) \geq 0$$

$$\text{Meaning } p \in (-\infty, -4] \cup [8, \infty) \dots\dots (1)$$

Sum of Roots Condition:

The sum of the roots (which is $p+2$) must be negative:

$$p+2 < 0 \Rightarrow p < -2 \dots\dots (2)$$

Product of Roots Condition:

The product of the roots ($2p+9$) must be positive:

$$2p+9 > 0 \Rightarrow p > -\frac{9}{2} \dots\dots (3)$$

Determine the Valid Interval:

Combine the results from conditions (1), (2), and (3). From conditions (1) and (2), we find $p < -2$:

Intersection of $(-\infty, -4]$ and $(-\frac{9}{2}, -2)$ gives:

$$p \in (-\frac{9}{2}, -4]$$

Calculate $\beta - 2\alpha$:

With $\alpha = -\frac{9}{2}$ and $\beta = -4$, compute:

$$\beta - 2\alpha = -4 - 2(-\frac{9}{2}) = -4 + 9 = 5$$

Therefore, the difference $\beta - 2\alpha$ is 5.

Question30

The number of real roots of the equation $x|x-2| + 3|x-3| + 1 = 0$ is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

4

B.

3

C.

2

D.

1

Answer: D

Solution:

(I) $x < 2$

$$-x^2 + 2x - 3x + 9 + 1 = 0$$

$$\Rightarrow x^2 + x - 10 = 0$$

$$\Rightarrow x = \frac{-1 + \sqrt{41}}{2}, \frac{-1 - \sqrt{41}}{2}$$

(II) $2 \leq x < 3$

$$\Rightarrow x^2 - 2x - 3x + 9 + 1 = 0$$

$$\Rightarrow x^2 - 5x + 10 = 0$$

D < 0

(III) $x \geq 3$

$$x^2 - 2x + 3x - 9 + 2 = 0$$

$$\Rightarrow x^2 + x - 8 = 0$$

$$x = \frac{-1 + \sqrt{32}}{2}, \frac{-1 - \sqrt{32}}{2}$$

1 real roots

Question 31

The sum of the squares of the roots of $|x - 2|^2 + |x - 2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x - 3| - 5 = 0$, is

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

24

B.

26

C.

36

D.

30

Answer: C

Solution:

$$|x - 2|^2 + 2|x - 2| - |x - 2| - 2 = 0$$

$$\Rightarrow (|x - 2| + 2)(|x - 2| - 1) = 0$$

$$\Rightarrow |x - 2| = 1$$

$$\Rightarrow x = 2 \pm 1 = 3, 1$$

$$\Rightarrow \text{sum of square of roots} = 9 + 1 = 10$$

$$x^2 - 2|x - 3| - 5 = 0$$

$$\text{Case-I } x - 3 \geq 0$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x - 1)^2 = 0$$

$$\Rightarrow x = 1$$

$$\text{But } x \geq 3$$

$$\Rightarrow x \in \phi$$

$$\text{Case-II } x - 3 < 0$$

$$x^2 + 2x - 11 = 0, D > 0 \Rightarrow \text{Real \& distinct roots}$$

$$f(x) = x^2 + 2x - 11$$

$$f(3) > 0, \frac{-P}{2a} = -1 < 3$$

$$\Rightarrow \text{both roots} < 3, \text{ both roots acceptable}$$

$$\text{Sum of square of roots} = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4 + 22 = 26$$

$$\Rightarrow \text{Final sum} = 10 + 26 = 36$$

Question 32

Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = 3^x - 1 \mid + 3^x - 2 \mid$.

Then S:

[Jan. 8, 2020 (II)]

Options:

A. contains exactly two elements.

B. is a singleton.

C. is an empty set.

D. contains at least four elements.

Answer: B

Solution:

Let $3^x = y$

$$\therefore y(y-1)+2 = |y-1| + |y-2|$$

Case 1: when $y > 2$

$$y^2 - y + 2 = y - 1 + y - 2$$

$$y^2 - 3y + 5 = 0$$

$\therefore D < 0$ [$\therefore$ Equation not satisfy.]

Case 2 : when $1 \leq y \leq 2$

$$y^2 - y^2 + 2 = y - 1 - y + 2$$

$$y^2 - y + 1 = 0$$

$\therefore D < 0$ [$\therefore$ Equation not satisfy.]

Case 3: when $y \leq 1$

$$y^2 - y + 2 = -y + 1 - y + 2$$

$$y^2 + y - 1 = 0$$

$$\therefore y = \frac{-1 + \sqrt{5}}{2}$$

$$= \frac{-1 - \sqrt{5}}{2} \quad [\therefore \text{Equation not Satisfy}]$$

$\therefore$ Only one $-1 + \frac{\sqrt{5}}{2}$ satisfy equation

Question33

If $A = \{x \in \mathbb{R} : |x| < 2\}$ and $B = \{x \in \mathbb{R} : |x - 2| \geq 3\}$; then :
[Jan. 9, 2020 (II)]

Options:

A. $A \cap B = (-2, -1)$

B. $B - A = \mathbb{R} - (-2, 5)$

C. $A \cup B = \mathbb{R} - (2, 5)$

D. $A - B = [-1, 2)$

Answer: B

Solution:

$$A = \{x : x \in (-2, 2)\}$$

$$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$$

$$A \cap B = \{x : x \in (-2, -1]\}$$

$$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$$

$$A - B = \{x : x \in (-1, 2)\}$$

$$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$$

Question34

Consider the two sets :

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real}\}$ and $B = [-3, 5)$

Which of the following is not true?

[Sep. 03, 2020 (I)]

Options:

A. $A - B = (-\infty, -3) \cup (5, \infty)$

B. $A \cap B = \{-3\}$

C. $B - A = (-3, 5)$

D. $A \cup B = \mathbb{R}$

Answer: A

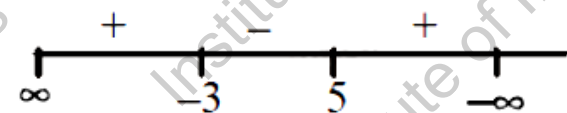
Solution:

$A = \{m \in \mathbb{R} : x^2 - (m+1)x + m + 4 = 0 \text{ has real roots}$

$D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$



$A = \{(-\infty, -3] \cup [5, \infty)\}$

$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup (5, \infty)$

Question 35

The region represented by $\{z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1\}$ is also given by the inequality:

[Sep. 06, 2020 (I)]

Options:

A. $y^2 \geq 2(x+1)$

B. $y^2 \leq 2\left(x + \frac{1}{2}\right)$

C. $y^2 \leq x + \frac{1}{2}$

D. $y^2 \geq x + 1$

Answer: B

Solution:

$$\because |z| \frac{-\operatorname{Re}(z)}{|z|} \leq 1 (\because z = x + iy)$$

$$\Rightarrow \sqrt{x^2 + y^2} - x \leq 1 \Rightarrow \sqrt{x^2 + y^2} \leq 1 + x$$

$$\Rightarrow x^2 + y^2 \leq 1 + x^2 + 2x$$

$$\Rightarrow y^2 \leq 1 + 2x \Rightarrow y^2 \leq 2\left(x + \frac{1}{2}\right)$$

Question 36

The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in \mathbb{R}$, is always positive, is :
[Jan. 12, 2019 (II)]

Options:

A. 3

B. 8

C. 7

D. 6

Answer: C

Solution:

Let the given quadratic expression

$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, is positive for all $x \in \mathbb{R}$

then

$$1 + 2m > 0 \dots (i)$$

$$D < 0$$

$$\Rightarrow 4(1 + 3m)^2 - 4(1 + 2m)4(1 + m) < 0$$

$$\Rightarrow 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$$

$$\Rightarrow m^2 - 6m - 3 < 0$$

$$\Rightarrow m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$$

From (i)

$$\therefore m > -\frac{1}{2}$$

$$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$$

Then, integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$

Hence, number of integral values of $m = 7$